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# The Effects of Icing on the Longitudinal Dynamics of an Icing Research Aircraft 

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# THE EFFECTS OF ICING ON THE LONGITUDINAL DYNAMICS OF AN ICING RESEARCH AIRCRAFT 

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#### Abstract

Icing affects the aerodynamic performance of aircraft leading sometimes resulting in catastrophic crashes. The effects of icing have been studied in the past but from more of an aerodynamicist's viewpoint than a flight controls viewpoint. This paper develops models for aircraft dynamics in the presence of icing that can ultimately lead to a model based method of detecting icing. Flight test data from the NASA Lewis Twin Otter Icing Research Aircraft is analyzed in both case of a nominal un-iced configuration and a horizontal stabilizer experiencing icing due to a failed de-icing boot. Robust global nonlinear models were developed for each flap setting. The global nonlinear and localized linear models were analyzed for the differences between nominal and iced conditions with excellent agreement between model predictions and flight test data.


## Introduction

Icing affects the aerodynamic performance of aircraft by contaminating the aerodynamic surfaces. Without anti-icing equipment icing, if sufficiently severe, can relatively quickly lead to a situation in which controllable flight is impossible. Even aircraft with anti-icing equipment are potentially susceptible to icing under certain conditions. Some turboprop aircraft that have de-icing boots on the leading edge of aerodynamic surfaces may experience wing and tailplane icing behind the de-icing boots. Moreover, de-icing boots can fail due to a variety of conditions including holes produced by rocks etc. In particular

[^0]the failure of a horizontal stabilizer boot can occur under such circumstances that the flight crew is unaware of the failure.

The goal of the present paper is to develop robust global nonlinear models for aircraft dynamics under tailplane icing scenarios. A longer term goal, that is presented in a related paper [1] is to use these models to detect icing via the associated changes in aircraft dynamics and though changes in control effectiveness. In this paper robust nonlinear models are developed from flight test data collected while flying a suitably instrumented aircraft that as had modifications made to aerodynamic surfaces simulating actual icing. The models themselves are sufficiently robust to include a wide range of flight conditions and in various configurations. In particular we present models that represent approach and landing configurations with simulated icing(due e.g to failed boot) on the horizontal stabilizer. In addition these models are valid for the full range of flap deflections(in e.g. landing configuration) for which stable controllable flight is possible. This is particularly significant for horizontal stabilizer icing since the tail stall margin is greatly reduced due to downwash induced by the flap deflection. Specifically the test aircraft used to obtain the models experienced almost complete loss of pitch control with flaps at $40^{\circ}$. Clearly it would benefit flight safety to be able to warn the pilot of such a condition.

The application of the models developed in this paper[1] is to continuously estimate elevator effectiveness via a state estimator that is driven by existing aircraft sensors as inputs. In the case of a failed horizontal stabilizer boot the loss of elevator effectiveness has been shown to be predictable as ice accretes. The algorithm used to predict the loss of effectiveness are highly computational efficient such that warnings to the pilot can be initiated well be-
fore pitch control is lost.
The data set used to obtain the models presented was generously provided by NASA Lewis Research Center. The software packages, the Stepwise Regression, SWR[2], software package and Orthogonal Function Fit, OFFIT[3], software package, used to build the models were provided by NASA Langley Research Center. Funding for the research was provided by NASA Dryden Research Center.

This paper demonstrates the excellent fit of the model with the actual flight data. The models are quite robust to operating conditions including flight at 4 different flap settings and with and without ice. In addition, this paper shows that there is a significant trim change between the uniced and iced horizontal stabilizer. Such a trim change can provide one part of a method for detecting horizontal stabilzer icing.

## Flight Test

The NASA Lewis twin otter Icing Research Aircraft was used to conduct flight tests with different cases of tailplane icing. Various shapes were attached to the horizontal stabilizer of the twin otter to simulate different levels of tailplane icing. These simulated icing attachments were developed from actual ice accumulation in the NASA icing research wind tunnel.

Aircraft weight and balance were based on previous weight and balance information, [4], and computations to take into account the changes since the previous flight tests. In flight measurements of the fuel usage were used to compute the center of gravity and moments of inertia for post flight data processing.

The maneuvers were performed in a decoupled manner which allowed the longitudinal subset of the dynamics to be separated out. The control input was a classic elevator doublet. The power control was maintained at a nearly constant combined thrust coefficient of about $C_{T}=0.1$, yeilding a thrust of approximately 1000 lbf . Each maneuver last approximately 12 to 14 seconds. An example of the input is shown in Figure 1.

There were two distinct flight test configurations analyzed. The different cases are shown in Table 2. The un-iced configuration represents the nominal aircraft. The failed boot configuration represents the level of icing 22 minutes after a de-icing boot fails on an aircraft experiencing icing. The 22 minutes is derived from a FAA requirement.

## Results



Figure 1: Typical Elevator Input for Flight Tests

| Table 1 |  |  |
| :---: | :---: | :---: |
| Flight Test Maneuvers |  |  |
| Configuration | Maneuver | $\delta_{F}$ (Flap Angle) |
| Failed Boot | p 46 | $0,10,20,30$ |
| Nominal | p 52 | $0,10,20,30$ |

The following body axis equations, [5], were used to model the longitudinal dynamics of the twin otter icing research aircraft.

$$
\begin{align*}
& \dot{X}=f(X, V)  \tag{1}\\
& {\left[\begin{array}{c}
\dot{u} \\
\dot{w} \\
\dot{\theta} \\
\dot{q}
\end{array}\right]=\left[\begin{array}{c}
-w q+\frac{F_{z}}{m}-g_{x} \\
u q+\frac{F_{z}^{m}}{m}+g_{z} \\
q \\
\bar{q} S c C_{m} / I_{y}
\end{array}\right] }  \tag{2}\\
& \bar{q}=\frac{1}{2} p\left(u^{2}+w^{2}\right) \\
& F_{x}=\bar{q} S C_{x}+T_{x},
\end{align*} \quad \alpha=\tan ^{-1}\left(\frac{w}{u}\right), F_{z}=\bar{q} S C_{z}+T_{z} .
$$

where $C_{x}, C_{z}$, and $C_{m}$ are polynomials functions of $\alpha, q$, and $\delta_{e}$ (note the $q$ is dimensional and not the typical nondimensional). A more detailed explantion of the variables is in the Appendix. The functions for the SWR cases are of the form,

$$
\begin{align*}
C_{x} & =x_{1} \alpha+x_{2} \alpha^{2}+x_{3} \delta_{e}+x_{4}  \tag{3}\\
C_{z} & =z_{1} \alpha+z_{2} q+z_{3} \alpha^{2}+z_{4} \delta_{e}+z_{5}  \tag{4}\\
C_{m} & =m_{1} \alpha+m_{2} q+m_{3} \alpha^{2}+m_{4} \delta_{e}+m_{5} \tag{5}
\end{align*}
$$

The results of the SWR fit and the OFFIT fit are listed in the tables in the Appendix. The OFFIT
polynominals are also described in detail in the Appendix. An example of the fit for $C_{z}$ is shown in Figures 2 and 3.


Figure 2: SWR Fit of $C_{z}$


Figure 3: OFFIT Fit of $C_{z}$
Comparisons of open loop simulations of the SWR fitted polynominal functions versus the actual flight data for angle of attack are shown in Figure 4 and Figure 5. The pitch rate responses are shown in Figure 6 and Figure 7. The robustness of the polynominal fits discussed above is reflected in the excellent fit of the dynamic $\alpha$ and $q$ for both the nominal and failed boot cases. These two variables play an important role in the model based ice detection scheme presented in [1]. In particular the pitch rate fit for both nominal and failed boot case is important when estimating elevator effectiveness.


Figure 4: Nominal Angle of Attack $\alpha$


Figure 5: Failed Boot Angle of Attack $\alpha$

Figure 8 shows the elevator trim change bewteen the uniced and iced horizontal stabilzer at flaps $0^{\circ}, 10^{\circ}, 20^{\circ}$, and $30^{\circ}$. The flight conditions for these flap settings were the same(as much as is practically significant) for iced and uniced conditions. This change has the potential to provide one element of an ice detection method. For example in an aircraft flying on autopilot in icing conditions the elcvator/trim tab setting could be monitored. The indicated change could, for example, set a software flag that in combination with the state estimator output explained in the related paper, [1], could be used to detect a loss of elevator effectiveness.

## Acknowledgements

The authors would like to thank Thomas Ratvasky


Figure 6: Nominal Pitch Rate $q$


Figure 7: Failed Boot Pitch Rate $q$
of NASA Lewis Research Center and Judith Van Zante of NYMA, Inc., for their helpful discussions and comments. The flight test data was generously supplied by NASA Lewis Research Center. Also Dr. Gene Morelli of NASA Langley provided assistance with the SWR, stepwise regression, and the OFFIT, orthogonal function, flight test data fitting code from NASA Langley as well as some helpful comments.

## References

[1] Robert H. Miller and William B. Ribbens. Detection of the Loss of Elevator Effectiveness due to Icing. Number 99-0637 in 37th Aerospace Sciences. AIAA, Jan. 1999.


Figure 8: Elevator Trim Change
[2] Vladislav Klein, James G. Batterson, and Patrick C. Murphy. Determination of Airplane Model Structure from Flight Data using Modified Stepwise Regression. Technical Report 1916, NASA, 1981.
[3] Eugene A. Morelli. Global Nonlinear Aerodynamic Modeling Using Multivariate Orthogonal Functions. Journal of Aircraft, 32(2):270-277, March-April 1995.
[4] T.P. Ratvasky and R.J. Ranaudo. Icing Effects on the Aircraft Stability and Control Determined from Flight Data. Technical Report 105977, NASA, 1993.
[5] John H. Blakelock. Automatic Control of Aircraft and Missles. John Wiley, 2nd edition edition, 1991.

## Appendix OFFIT Fit

The OFFIT attempts to build a global aerodynamic model from flight data using multivariate orthogonal functions, [3]. The following tables contain the results of the offit fit. The aerodynamic coefficients are function based on the following variables,

$$
\begin{array}{r}
C_{x}\left(\alpha, \delta_{e}\right) \\
C_{z}\left(\alpha, q, \delta_{e}\right) \\
C_{m}\left(\alpha, q, \delta_{e}\right) \tag{10}
\end{array}
$$

The structure of each polynminal is of the form

$$
\begin{equation*}
C_{n}(\bullet)=\sum_{i=1}^{I} n_{i} \prod_{j=1}^{J} \nu_{n j}^{i j} \quad n=x, z, m \tag{11}
\end{equation*}
$$

where $n_{i}$ is the ith cocfficient and $\nu_{n j}=j$ th independent variable in $C_{n}$. For example Table 1 lists the coefficients for the functions in equations 8,9 , and 10 for zero flpas. The function $C_{x}$ is a function of two variables(ie $J=2, \nu_{x 1}=\alpha, \nu_{x 2}=\delta_{c}$ ). There are 8 terms in this polynominal with coefficients $x_{i}(i=1, \ldots, 8)$. The index column lists the exponent of each variable in sequence. An example of the a fit would be $C_{x}$ for the $0^{\circ}$ flap setting case.

$$
\begin{gathered}
C_{x}\left(\alpha, \delta_{e}\right)=-0.071+0.303 \alpha+0.157 \delta_{e}+3.281 \alpha^{2}(12) \\
-1.788 \alpha \delta_{e}-0.902 \delta_{e}^{2}+10.054 \delta_{e} \alpha^{2}+7.082 \delta_{e}^{2} \alpha
\end{gathered}
$$

| SWR FIT $C_{x}$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| $\delta_{F}=0$ |  |  |  |  |
|  | $x_{1}$ | $x_{2}$ | $x_{3}$ | $x_{4}$ |
| p 52 | 0.3900 | 2.9099 | 0.0961 | -0.0758 |
| $\sigma_{x}$ | 0.0728 | 0.2987 | 0.0162 | 0.0041 |
| p 46 | 0.714 | 1.7440 | 0.0682 | -0.0896 |
| $\sigma_{x}$ | 0.0894 | 0.3419 | 0.0155 | 0.0052 |
| $\delta_{F}=10$ |  |  |  |  |
| p 52 | 0.3364 | 4.4990 | 0.1174 | -0.1149 |
| $\sigma_{x}$ | 0.0259 | 0.2501 | 0.0166 | 0.0012 |
| p 46 | 0.3869 | 5.0679 | 0.1072 | -0.1178 |
| $\sigma_{x}$ | 0.0228 | 0.1933 | 0.0114 | 0.0010 |
| $\delta_{F}=20$ |  |  |  |  |
| p 52 | 0.3302 | 5.3376 | 0.2002 | -0.2055 |
| $\sigma_{x}$ | 0.0579 | 0.4078 | 0.0340 | 0.0031 |
| p 46 | 0.5175 | 5.4043 | 0.1710 | -0.1982 |
| $\sigma_{x}$ | 0.0352 | 0.2557 | 0.0231 | 0.0023 |
| $\delta_{F}=30$ |  |  |  |  |
| p 52 | 0.3357 | 5.7471 | 0.2438 | -0.3096 |
| $\sigma_{x}$ | 0.0962 | 0.7143 | 0.0753 | 0.0054 |
| p 46 | 0.5010 | 6.1872 | 0.2184 | -0.2967 |
| $\sigma_{x}$ | 0.0738 | 0.5936 | 0.0505 | 0.0050 |


| SWR FIT C |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\delta_{F}=0$ |  |  |  |  |  |
|  | $z_{1}$ | $z_{2}$ | $z_{3}$ | $z_{4}$ | $z_{5}$ |
| p 52 | -7.0186 | -0.1023 | 4.1109 | -0.2340 | -0.3112 |
| $\sigma_{z}$ | 0.2219 | 0.0356 | 0.8616 | 0.0500 | 0.0121 |
| p 46 | -7.5095 | -0.2375 | 6.5731 | -0.3541 | -0.3175 |
| $\sigma_{z}$ | 0.3491 | 0.0493 | 1.3377 | 0.0599 | 0.0208 |
| $\delta_{F}=10$ |  |  |  |  |  |
| p52 | -7.1429 | 0.2738 | 0.6482 | 0.0338 | -0.9135 |
| $\sigma_{z}$ | 0.0630 | 0.0365 | 0.7213 | 0.0486 | 0.0027 |
| p 46 | -7.4225 | 0.3008 | 1.9348 | 0.0467 | -0.9366 |
| $\sigma_{z}$ | 0.0782 | 0.0398 | 0.7689 | 0.0502 | 0.0034 |
| $\delta_{F}=20$ |  |  |  |  |  |
| p 52 | -7.7552 | 0.4784 | 4.5098 | 0.0611 | -1.4455 |
| $\sigma_{z}$ | 0.0758 | 0.0482 | 0.6768 | 0.0485 | 0.0037 |
| p 46 | -7.8163 | 0.6433 | 5.0107 | 0.3061 | -1.4858 |
| $\sigma_{z}$ | 0.0656 | 0.0483 | 0.5686 | 0.0507 | 0.0049 |
| $\delta_{F}=30$ |  |  |  |  |  |
| p 52 | -8.1446 | 0.7462 | 5.4151 | 0.1159 | -1.8940 |
| $\sigma_{z}$ | 0.0890 | 0.0778 | 1.1494 | 0.0697 | 0.0066 |
| p 46 | -8.2405 | 0.9358 | 5.3024 | 0.2610 | -1.8698 |
| $\sigma_{z}$ | 0.0505 | 0.0477 | 0.6017 | 0.0403 | 0.0034 |


| SWR FIT $C_{m}$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\delta_{F}=0$ |  |  |  |  |  |
|  | $z_{1}$ | $z_{2}$ | $z_{3}$ | $z_{4}$ | $z_{5}$ |
| p52 | -0.8789 | -0.6266 | -3.8520 | -1.8987 | -0.0108 |
| $\sigma_{m}$ | 0.0966 | 0.0180 | 0.3909 | 0.0252 | 0.0055 |
| p46 | -0.7899 | -0.6899 | -3.9300 | -1.8629 | -0.0255 |
| $\sigma_{m}$ | 0.1632 | 0.0206 | 0.6332 | 0.0273 | 0.0102 |
| $\delta_{F}=10$ |  |  |  |  |  |
| p52 | -1.2885 | -0.5672 | -3.0061 | -1.7991 | 0.0836 |
| $\sigma_{m}$ | 0.0347 | 0.0203 | 0.4118 | 0.0276 | 0.0015 |
| p46 | -1.2143 | -0.5047 | -3.2362 | -1.6219 | 0.0761 |
| $\sigma_{m}$ | 0.0441 | 0.0223 | 0.4543 | 0.0286 | 0.0018 |
| $\delta_{F}=20$ |  |  |  |  |  |
| p52 | -1.5091 | -0.6352 | -1.9476 | -1.7448 | 0.0654 |
| $\sigma_{m}$ | 0.0289 | 0.0203 | 0.2651 | 0.0203 | 0.0013 |
| p46 | -1.3633 | -0.4217 | -1.6774 | -1.3937 | 0.0415 |
| $\sigma_{m}$ | 0.0345 | 0.0255 | 0.3568 | 0.0267 | 0.0027 |
| $\delta_{F}=30$ |  |  |  |  |  |
| p52 | -1.8661 | -0.6122 | -2.1288 | -1.6955 | 0.0274 |
| $\sigma_{m}$ | 0.0352 | 0.0315 | 0.3836 | 0.0292 | 0.0019 |
| p46 | -1.3497 | -0.4467 | -3.2938 | -1.2787 | 0.0002 |
| $\sigma_{m}$ | 0.0467 | 0.0388 | 0.5609 | 0.0340 | 0.0035 |


| OFFIT FIT |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\delta_{F}=0$ |  |  |  |  |  |  |  |  |
| Nominal(p52) |  |  |  |  |  |  | Failed Boot(p46) |  |
| $x_{i}$ | $\sigma_{x_{i}}$ | index | $x_{i}$ | $\sigma_{x_{i}}$ | index |  |  |  |
| -0.071 | 0.005 | 0 | -0.084 | 0.006 | 0 |  |  |  |
| 0.303 | 0.091 | 1 | 0.582 | 0.103 | 1 |  |  |  |
| 0.157 | 0.087 | 10 | 0.066 | 0.095 | 10 |  |  |  |
| 3.281 | 0.424 | 2 | 2.442 | 0.442 | 2 |  |  |  |
| -1.788 | 0.866 | 11 | -1.090 | 0.940 | 11 |  |  |  |
| -0.902 | 0.609 | 20 | -0.849 | 0.488 | 20 |  |  |  |
| 10.054 | 4.148 | 12 | 8.953 | 3.677 | 12 |  |  |  |
| 7.082 | 4.118 | 21 | 6.882 | 3.462 | 21 |  |  |  |
| $z_{i}$ | $\sigma_{z_{i}}$ | index | $z_{i}$ | $\sigma_{z_{i}}$ | index |  |  |  |
| -0.373 | 0.012 | 0 | -0.359 | 0.016 | $\overline{0}$ |  |  |  |
| -5.766 | 0.226 | 1 | -6.346 | 0.269 | 1 |  |  |  |
| -0.446 | 0.127 | 10 | -0.292 | 0.127 | 10 |  |  |  |
| -1.262 | 0.149 | 100 | -0.924 | 0.163 | 100 |  |  |  |
| -1.831 | 1.031 | 2 | 0.141 | 1.116 | 2 |  |  |  |
| 11.484 | 2.121 | 11 | 9.815 | 2.034 | 11 |  |  |  |
| 18.459 | 2.310 | 101 | 13.124 | 2.376 | 101 |  |  |  |
| 0.751 | 0.277 | 20 | 0.615 | 0.226 | 20 |  |  |  |
| 0.215 | 0.389 | 110 | 0.529 | 0.335 | 110 |  |  |  |
| -0.313 | 0.362 | 200 | -0.236 | 0.260 | 200 |  |  |  |
| -62.875 | 8.664 | 12 | -63.167 | 8.404 | 12 |  |  |  |
| -78.176 | 9.271 | 102 | -63.473 | 9.189 | 102 |  |  |  |
| $m_{i}$ | $\sigma_{m_{i}}$ | index | $m_{i}$ | $\sigma_{m_{i}}$ | index |  |  |  |
| -0.023 | 0.008 | 0 | 0.008 | 0.009 | 0 |  |  |  |
| -0.636 | 0.135 | 1 | -1.097 | 0.130 | 1 |  |  |  |
| -0.814 | 0.127 | 110 | -0.447 | 0.091 | 10 |  |  |  |
| -2.175 | 0.170 | 100 | -1.376 | 0.130 | 100 |  |  |  |
| -4.985 | 0.597 | 2 | -3.494 | 0.532 | 2 |  |  |  |
| 4.141 | 1.261 | 11 | 0.600 | 0.941 | 11 |  |  |  |
| 4.130 | 1.797 | 101 | -2.875 | 1.253 | 101 |  |  |  |
| -0.233 | 0.577 | 20 | 0.168 | 0.355 | 20 |  |  |  |
| -1.747 | 1.573 | 110 | 1.414 | 0.770 | 110 |  |  |  |
| -1.329 | 0.902 | 200 | 1.465 | 0.558 | 200 |  |  |  |
| -13.475 | 4.325 | 12 | -8.098 | 3.243 | 12 |  |  |  |
| -7.729 | 5.908 | 102 | 3.765 | 4.574 | 102 |  |  |  |
| 11.221 | 3.585 | 21 | 5.307 | 2.318 | 21 |  |  |  |
| 6.688 | 9.516 | 111 | -7.805 | 4.742 | 111 |  |  |  |
| 2.800 | 2.792 | 120 | -2.015 | 1.239 | 120 |  |  |  |
|  | 3.332 | 210 | -9.599 | 1.493 | 210 |  |  |  |


| OFFIT FIT |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\delta_{F}=10$ |  |  |  |  |  |
| Nominal(p52) |  |  | Failed -0.Boot(p46) |  |  |
| $\boldsymbol{x}_{\boldsymbol{i}}$ | $\sigma_{x_{i}}$ | index | $x_{i}$ | $\sigma_{x_{2}}$ | index |
| -0.114 | 0.001 | 0 | -0.117 | 0.001 | 0 |
| 0.291 | 0.026 | 1 | 0.356 | 0.023 | 1 |
| 0.127 | 0.023 | 10 | 0.120 | 0.015 | 10 |
| 4.715 | 0.266 | 2 | 5.204 | 0.200 | 2 |
| 0.307 | 0.414 | 11 | 0.305 | 0.301 | 11 |
| -0.283 | 0.237 | 20 | -0.315 | 0.149 | 20 |
| -1.057 | 3.092 | 12 | -2.977 | 2.246 | 12 |
| 7.050 | 3.083 | 21 | 4.789 | 1.934 | 21 |
| $z_{i}$ | $\sigma_{z_{i}}$ | index | $z_{i}$ | $\sigma_{z_{i}}$ | index |
| -0.919 | 0.003 | 0 | -0.943 | 0.003 | 0 |
| -7.176 | 0.079 | 1 | -7.418 | 0.063 | 1 |
| 0.365 | 0.040 | 10 | 0.370 | 0.040 | 10 |
| 0.157 | 0.065 | 100 | 0.198 | 0.054 | 100 |
| 1.823 | 0.674 | 2 | 3.280 | 0.672 | 2 |
| 0.668 | 0.690 | 11 | -24.977 | 5.796 | 12 |
| 1.694 | 1.516 | 101 | -41.215 | 7.105 | 102 |
| -0.734 | 0.790 | 110 |  |  |  |
| -30.728 | 8.574 | 12 |  |  |  |
| -49.189 | 9.831 | 102 |  |  |  |
| $m_{i}$ | $\sigma_{m_{i}}$ | index | $m_{i}$ | $\sigma_{m_{i}}$ | index |
| 0.081 | 0.001 | 0 | 0.072 | 0.002 | 0 |
| -1.286 | 0.039 | 1 | -1.186 | 0.042 | 1 |
| -0.507 | 0.024 | 10 | -0.440 | 0.024 | 10 |
| -1.729 | 0.036 | 100 | -1.535 | 0.031 | 100 |
| -2.885 | 0.406 | 2 | -3.156 | 0.435 | 2 |
| 0.452 | 0.334 | 11 | 0.299 | 0.371 | 11 |
| -1.969 | 0.827 | 101 | -2.776 | 0.791 | 101 |
| 0.192 | 0.263 | 20 | 0.335 | 0.243 | 20 |
| 1.274 | 0.442 | 110 | 1.398 | 0.366 | 110 |
| -0.563 | 0.401 | 200 | -0.485 | 0.306 | 200 |
| -20.704 | 3.465 | 12 | -18.827 | 3.735 | 12 |
| -11.492 | 8.487 | 102 | -4.492 | 8.295 | 102 |
| 8.409 | 3.991 | 21 | 3.178 | 3.386 | 21 |
| -7.556 | 10.168 | 111 | -14.888 | 9.437 | 111 |
| 12.488 | 7.062 | 201 | 12.666 | 6.067 | 201 |
| 7.493 | 3.667 | 120 | 2.332 | 3.336 | 120 |
| -8.318 | 3.680 | 210 | -13.337 | 2.583 | 210 |


| OFFIT FIT |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\delta_{F}=20$ |  |  |  |  |  |
| Nominal(p52) |  |  | Failed Boot(p46) |  |  |
| $x_{i}$ | $\sigma_{x_{1}}$ | index | $x_{i}$ | $\sigma_{x_{i}}$ | index |
| -0.203 | 0.003 | 0 | -0.198 | 0.002 | 0 |
| 0.253 | 0.060 | 1 | 0.462 | 0.035 | 1 |
| 0.217 | 0.037 | 10 | 0.189 | 0.024 | 10 |
| 5.749 | 0.431 | 2 | 5.793 | 0.258 | 2 |
| 1.066 | 0.643 | 11 | 0.831 | 0.330 | 11 |
| -0.269 | 0.367 | 20 | 0.050 | 0.166 | 20 |
| -4.887 | 4.313 | 12 | -6.882 | 1.964 | 12 |
| 5.781 | 3.798 | 21 |  |  |  |
| $z_{i}$ | $\sigma_{z_{i}}$ | index | $z_{i}$ | $\sigma_{z_{\text {i }}}$ | index |
| -1.451 | 0.003 | 0 | -1.496 | 0.003 | 0 |
| -7.780 | 0.050 | 1 | -7.934 | 0.057 | 1 |
| 0.598 | 0.048 | 10 | 0.843 | 0.046 | 10 |
| 0.225 | 0.049 | 100 | 0.325 | 0.051 | 100 |
| 4.591 | 0.423 | 2 | 4.466 | 0.478 | 2 |
| 0.223 | 0.609 | 110 | -0.699 | 0.465 | 11 |
| -28.396 | 4.779 | 12 | 0.038 | 0.871 | 101 |
| -29.466 | 5.950 | 102 | 0.398 | 0.477 | 110 |
|  |  |  | 1.360 | 0.426 | 200 |
|  |  |  | -42.356 | 4.597 | 12 |
|  |  |  | -46.515 | 6.530 | 102 |
|  |  |  | -3.375 | 5.534 | 21 |
|  |  |  | -52.218 | 9.612 | 111 |
|  |  |  | -28.646 | 4.687 | 201 |
|  |  |  | -16.088 | 4.076 | 120 |
|  |  |  | -32.768 | 3.759 | 210 |
| $m_{i}$ | $\sigma_{m}$ | index | $m_{i}$ | $\sigma_{m_{i}}$ | index |
| 0.062 | 0.002 | 0 | 0.034 | 0.002 | 0 |
| -1.519 | 0.033 | 1 | -1.325 | 0.027 | 1 |
| -0.608 | 0.025 | 10 | -0.384 | 0.026 | 10 |
| -1.682 | 0.029 | 100 | -1.310 | 0.025 | 100 |
| -2.111 | 0.258 | 2 | -2.827 | 0.246 | 2 |
| 1.096 | 0.248 | 11 | -0.929 | 0.183 | 11 |
| -0.273 | 0.551 | 101 | -2.418 | 0.450 | 101 |
| 0.771 | 0.293 | 20 | 0.520 | 0.211 | 20 |
| 1.506 | 0.302 | 110 | 0.611 | 0.183 | 110 |
| -0.148 | 0.316 | 200 | -0.196 | 0.235 | 200 |
| -12.850 | 2.306 | 12 | -9.229 | 2.009 | 12 |
| -7.698 | 4.380 | 102 | -9.293 | 2.938 | 102 |
| -1.315 | 3.125 | 21 | -2.221 | 2.339 | 21 |
| -18.908 | 6.337 | 111 | -19.571 | 4.257 | 111 |
| 2.997 | 3.754 | 201 | -3.938 | 2.170 | 201 |
| -3.616 | 2.666 | 120 | -5.311 | 1.696 | 120 |
| -13.280 | 2.091 | 210 | -13.998 | 1.700 | 210 |


| OFFIT FIT |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\delta_{F}=30$ |  |  |  |  |  |
| Nominal(p52) |  |  | Failed Boot(p46) |  |  |
| $x_{i}$ | $\sigma_{x_{i}}$ | index | $x_{i}$ | $\sigma_{x_{i}}$ | index |
| -0.313 | 0.005 | 0 | -0.296 | 0.005 | 0 |
| 0.231 | 0.092 | 1 | 0.444 | 0.072 | 1 |
| 0.325 | 0.071 | 10 | 0.302 | 0.055 | 10 |
| 6.740 | 0.853 $=$ | 2 | 6.247 | 0.638 | 2 |
| 0.651 | 0.987 | 11 | 0.434 | 0.674 | 11 |
| 0.167 | 0.547 | 20 | -0.176 | 0.364 | 20 |
| -15.720 | 6.606 | 12 | -14.891 | 6.575 | 12 |
| 4.196 | 5.550 | 21 | 4.334 | 5.439 | 21 |
| $z_{i}$ | $\sigma_{z_{i}}$ | index | $z_{i}$ | $\sigma_{z_{i}}$ | index |
| -1.903 | 0.007 | 0 | -1.876 | 0.003 | 0 |
| -7.970 | 0.088 = | 1 | -8.293 | 0.053 | 1 |
| 0.764 | 0.073 | 10 | 1.151 | 0.066 | 10 |
| 0.163 | 0.056 | 100 | 0.351 | 0.054 | 100 |
| 6.466 | 0.946 | 2 | 5.004 | 0.601 | 2 |
| -6.984 | 0.811 | 11 | -2.910 | 0.694 | 11 |
| -6.421 | 1.312 | 101 | -2.361 | 0.772 | 101 |
| -0.829 | 0.759 | 20 | -0.942 | 0.739 | 110 |
| 0.897 | 0.580 | 110 | -51.461 | 7.991 | 12 |
| 0.343 | 0.604 | 200 | -30.673 | 12.980 | 102 |
| -22.968 | 6.700 | 12 | 4.119 | 10.278 | 21 |
| -54.806 | 10.479 | 102 | -30.820 | 20.595 | 111 |
| 4.096 | 9.706 | 21 | 3.111 | 8.237 | 201 |
| 48.554 | 12.299 | 111 | -20.589 | 7.850 | 120 |
|  |  |  | -42.077 | 6.595 | 210 |
| $m_{i}$ | $\sigma_{m_{i}}$ | index | $m_{i}$ | $\sigma_{m_{i}}$ | index |
| 0.021 | 0.003 | 0 | -0.003 | 0.003 | 0 |
| -1.858 | 0.038 | 1 | -1.307 | 0.035 | 1 |
| -0.597 | 0.035 | 10 | -0.468 | 0.034 | 10 |
| -1.687 | 0.026 | 100 | -1.238 | 0.032 | 100 |
| -1.979 | 0.456 | 2 | -4.575 | 0.386 | 2 |
| -0.308 | 0.375 | 11 | -1.877 | 0.402 | 11 |
| -1.727 | 0.763 | 101 | -4.338 | 0.542 | 101 |
| 1.033 | 0.411 | 20 | 0.488 | 0.275 | 20 |
| 1.893 | 0.273 | 110 | 0.417 | 0.314 | 110 |
| 0.184 | 0.306 | 200 | -0.730 | 0.232 | 200 |
| -10.323 | 3.263 | 12 | -5.049 | 4.853 | 12 |
| -10.383 | 4.934 | 102 | -9.287 | 7.719 | 102 |
| 1.092 | 3.877 | 21 | -20.504 | 5.872 | 21 |
| -8.503 | 6.578 | 111 | -11.693 | 10.779 | 111 |
| 4.788 | 3.677 | 201 | -0.278 | 4.702 | 201 |
| -5.922 | 2.951 | 120 | -21.994 | 4.508 | 120 |
| -15.882 | 2.492 | 210 | -16.635 | 3.474 | 210 |


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