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Technical Notes

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Stagnation-Point Heat Transfer Near the Continuum Limit

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Nomenclature

b	=	velocity coefficient
f	=	nondimensional stream function
h	=	convective heat transfer coefficient
K	=	nonequilibrium parameter
Pr	=	Prandtl number
T	=	temperature
U	=	external x velocity
u	=	x velocity
v	=	y velocity
Δ	=	step size
δ	=	boundary-layer thickness
γ	=	specific heat ratio
η	=	nondimensional position
λ	=	mean free path
σ	=	accommodation coefficient
υ	=	kinematic viscosity

Subscripts

g	=	gas
m	=	momentum
o	=	freestream
slip	=	slip
t	=	thermal
w	=	wall
δ	=	boundary-layer thickness

Superscript

*	=	nondimensional

I. Introduction

HEAT and momentum transfer at the stagnation point is a problem of theoretical and practical interest. Solution of the Navier–Stokes equations at the stagnation point is one of oldest known solutions to the Navier–Stokes equations [1,2] and is closely related to boundary-layer flow [3]. Once the fluid flow is computed, the heat transfer can be computed in both 2-dimensional [4] and axisymmetric [5] geometries. The importance of stagnation-point heat transfer in problems such as atmospheric reentry [6] and other rarefied hypersonic flows [7] make estimating the heat transfer a problem of practical engineering interest.

Initial attempts to solve stagnation-point flow and boundary-layer flow with a slip boundary condition using perturbation methods [8] suggested that the slip condition would not affect shear stress or heat transfer. A more complete thermal analysis partially contradicted this result, suggesting that heat transfer in a laminar boundary layer decreased in the presence of a slip boundary condition [9]. The apparent lack of a change in shear stress due to the slip condition led to the conclusion that the terms added by the slip boundary condition were smaller than the discarded second-order terms in the boundary-layer equations [10]. This led to the conclusion that slip could be ignored in both laminar boundary-layer and stagnation-point flows.

These conclusions were challenged by numerical results, including solution of the linearized Boltzmann equation for stagnation-point flow [11], solution of stagnation-point flow with slip [12], and solution of the Blasius boundary-layer equations with slip flow that incorporated the loss of self-similarity [13]. All of these analyses showed decreased shear stress and boundary-layer thickness. When heat transfer was incorporated in the boundary-layer analysis, the heat transfer decreased from the equilibrium values.

The present work extends previous analysis of the fluid flow and heat transfer in the presence of a slip boundary condition [12,14] to cover rarefied flow, in which the temperature jump and slip boundary conditions are coupled. This analysis provides an estimate for change in heat transfer due to rarefied-flow effects for a range of Knudsen and Prandtl numbers for both monatomic and diatomic gases.

II. Nondimensional Boundary Conditions and Solution

The geometry of the stagnation-point flow region is shown in Fig. 1. The external flow velocity $U(x)$ is given by the inviscid flow solution:

$$U(x) = bx \tag{1}$$

Analysis of this flow is simplified by using nondimensional velocities u^* and v^*, a nondimensional coordinate η, and a nondimensional stream function $f(\eta)$:

$$\eta = y\sqrt{b/\upsilon} \tag{2}$$

$$u^* = \frac{u}{U(x)} = f'(\eta) \tag{3}$$

$$v^* = \frac{v}{\sqrt{b/\upsilon}} = -f(\eta) \tag{4}$$

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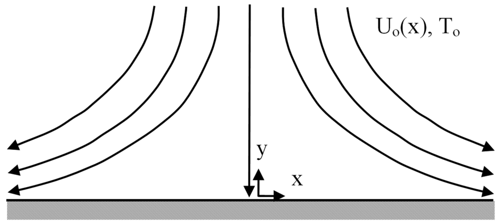

Fig. 1 Stagnation-point flow geometry.

The nondimensional temperature T^* is given by

$$T^* = \frac{T - T_w}{T_o - T_w} \tag{5}$$

These nondimensionalizations allow the Navier–Stokes and energy equations to be reduced to ordinary differential equations for both 2-dimensional and axisymmetric geometries [1–3].

For slightly rarefied flows, the velocity [15] and temperature [16] boundary conditions in this geometry are

$$u_{\text{slip}} = u_g - u_w = \lambda \frac{2 - \sigma_m}{\sigma_m} \frac{\partial u}{\partial y}\bigg|_w + \frac{3}{4} \frac{\upsilon}{T_{\text{gas}}} \frac{\partial T}{\partial x}\bigg|_w \tag{6}$$

$$T_{\text{gas}} - T_w = \frac{\lambda}{Pr} \frac{2 - \sigma_t}{\sigma_t} \frac{2\gamma}{\gamma + 1} \frac{\partial T}{\partial y}\bigg|_w \tag{7}$$

For an isothermal wall, Eq. (6) can be nondimensionalized to obtain

$$\frac{\partial f}{\partial \eta}\bigg|_{\eta=0} = \frac{(2 - \sigma_m)}{\sigma_m} \frac{\lambda}{x} \sqrt{\frac{b}{\upsilon}} \frac{\partial^2 f}{\partial \eta^2}\bigg|_{\eta=0} = K \frac{\partial^2 f}{\partial \eta^2}\bigg|_{\eta=0} \tag{8}$$

The nonequilibrium parameter K is defined as

$$K = \frac{(2 - \sigma_m)}{\sigma_m} \lambda \sqrt{\frac{b}{\upsilon}} \tag{9}$$

The boundary-layer thickness for a laminar boundary layer is given by

$$\delta = \eta_{99} \sqrt{\frac{b}{\upsilon}} \tag{10}$$

The no-slip solution for stagnation-point flow yields values of η_{99} of 2.4 for the 2-dimensional stagnation point and 2.0 for the axisymmetric geometries [1–3]. Previous research [12,13] shows that η_{99} decreases slightly for slip flows, but will not affect order-of-magnitude scaling such as Knudsen numbers.

Substituting Eq. (10) into Eq. (9) shows that K is proportional to the boundary-layer Knudsen number:

$$K = \frac{(2 - \sigma_m)}{\sigma_m} \eta_{99} K n_\delta \tag{11}$$

If the thermal accommodation coefficient is assumed to be approximately equal to the momentum accommodation coefficient, Eq. (7) simplifies to

$$T^*(\eta = 0) = \frac{1}{Pr} \frac{2\gamma}{\gamma + 1} K \frac{\partial T^*}{\partial \eta}\bigg|_{\eta=0} \tag{12}$$

This set of boundary conditions differs from those used in previous analysis of the laminar boundary layer with slip [13], in that K is not a function of position along the surface. This means that the self-similarity of the flow is preserved, and the x-momentum equation can be reduced to an ordinary differential equation [3]. Because the temperature jump and the velocity slip are coupled, they capture the rarefied-gas effects more accurately than in previous analysis, which assumed that the temperature jump was independent of the velocity slip [14].

The convective heat transfer coefficient h will be proportional to the thermal conductivity of the gas k multiplied by the wall temperature gradient. In nondimensional form, this becomes

$$h = k \sqrt{\frac{b}{\upsilon}} \frac{\partial T^*}{\partial \eta}\bigg|_{\eta=0} \tag{13}$$

Because the momentum and heat transfer equations are not coupled, the velocity field was solved for individual values of K from 0 to 1.0 using a shooting method to match the boundary equations. The ordinary differential equation for momentum [1–3] was integrated numerically using a Euler integration with step sizes $\Delta\eta$ of 1.0×10^{-6} and 5.0×10^{-6} to ensure grid independence. After solution of the flowfield, the energy equation was solved using a shooting method and for individual values of the specific heat ratio and Prandtl number. The heat transfer is computed for a specific heat ratio of $7/5$, representing a diatomic gas, and $5/3$, representing a monatomic gas. Three representative Prandtl numbers are used: 0.7, 1.0, and 1.4.

Figure 2 shows the nondimensional temperature gradient as a function of K for a two-dimensional stagnation point. The results show that the heat transfer decreases in a detectable manner with increasing values of K. For values in the slip-flow range, corresponding to K less than 0.1 [17], a decrease in heat transfer of up to 10% can be detected. In the transition range, where K is greater than 0.1 but less than 1.0, the breakdown of the continuum assumption leads to an error in the conduction and viscosity model used in these computations. However, these results suggest that the heat transfer continues to decrease in the transition range. These results show that the heat transfer is a strong function of the Prandtl number and is only weakly dependant on the specific heat ratio.

Figure 3 shows the temperature jump as a function of K for a two-dimensional stagnation point. These results suggest that the temperature jump will be measurable in the slip-flow regime and will increase as the Prandtl number and specific heat ratio decrease.

Figures 4 and 5 repeat this analysis for an axisymmetric stagnation point. These results confirm the trends of the 2-dimensional analysis. They suggest that rarefied-flow effects will be slightly larger in axisymmetric geometries.

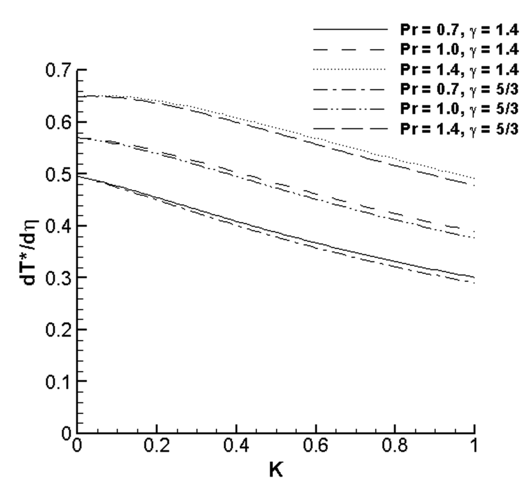

Fig. 2 Nondimensional wall temperature gradient versus K for a 2-D stagnation point.

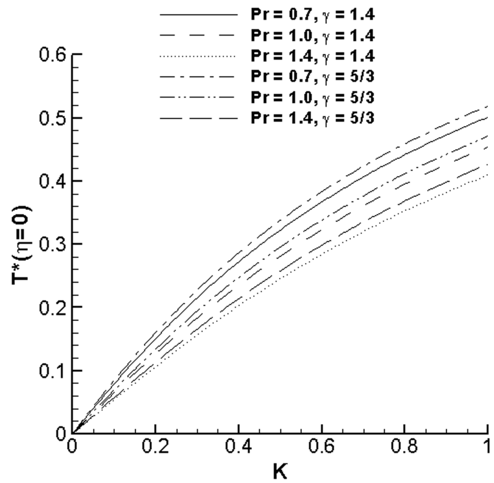

Fig. 3 Nondimensional wall temperature versus *K* for a 2-D stagnation point.

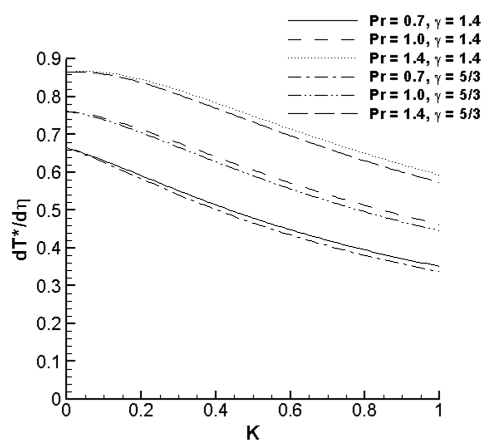

Fig. 4 Nondimensional wall temperature gradient versus *K* for an axisymmetric stagnation point.

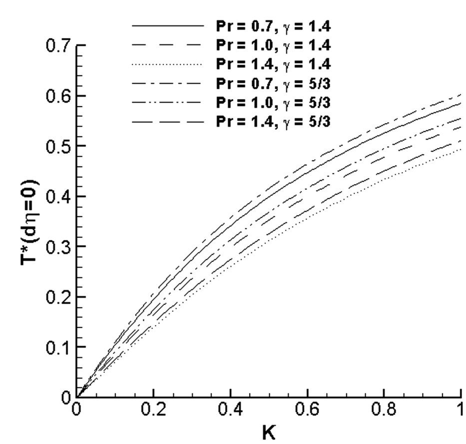

Fig. 5 Nondimensional wall temperature versus *K* for an axisymmetric stagnation point.

III. Conclusions

Use of modified velocity and temperature-jump equations allowed the effect of rarefaction on the heat transfer at a stagnation point to be computed. Although these results do not incorporate temperature recovery, viscous dissipation, or other effects that may be encountered in rarefied stagnation-point flows, they do provide an estimate for the impact of the slip and temperature boundary conditions on the heat transfer. In the slip-flow regime, these results show a change in the heat transfer and wall temperature jump on the order of several percent. They also show that the effect of rarefaction on the heat transfer will continue to increase in the transition regime. These results suggest that incorporating rarefied-flow effects into stagnation-point heat transfer models will result in a measurable increase in the accuracy of the estimated heat transfer.

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