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ABSTRACT

The transient behavior of two types of non-linear shock mounting has been determined theoretically and compared with the performance of an ideally linear mounting. Large departures of mount stiffness from linearity have been considered, and the influence of mount damping upon the transient motion of the mounted item discussed in detail. The performance of the mounting systems has been judged by their response to step-like foundation displacements possessing a wide range of rise times.

It is shown that a mount which stiffens considerably less rapidly upon compression than a linear mounting, and for which mount damping is small, reduces both the acceleration and displacement of the mounted item below the values which the mounted item is likely to experience when supported by a linear mounting or, as is normally the case in practice, a mount which stiffens more rapidly upon compression than a linear mounting. It is shown that the reduction in acceleration can be comparable with 10 db for quite a wide range of step rise-times if mount damping is small. It is frequently the magnitude of the acceleration experienced by the mounted item to which the likelihood of damage to the contents of the mounted item may be related.

1. INTRODUCTION

The terminology of this report relates to an item of equipment supported by a spring and damping element, as shown diagrammatically in Fig. 1. The transient motion of the mounted item, which is supposed to behave purely as a mass M , has been determined when the foundation of the mounting system experiences a rounded step-like displacement,^{1,2} $x_0(t)$, defined by the relations

$$\begin{aligned}x_0(t) &= 0 \quad \text{when } t < 0 \\x_0(t) &= x \left[1 - e^{-\gamma\omega_0 t} (1 + \gamma\omega_0 t) \right] \quad \text{when } t > 0, \quad (1.1)\end{aligned}$$

where t is time, γ is a parameter determining the rise time τ of the step, and x is a constant. The parameter ω_0 is the natural (angular) frequency of the mounting system when the peak value of the sinusoidally varying relative displacement across the mount at this frequency is small. The transient foundation displacement $x_0(t)$ is assumed to produce a time dependent displacement of the mounted item equal to $x_1(t)$.

The finite rise time τ of the step displacement is described by the parameter γ in terms of the natural frequency of the mounting system. If τ is defined^{1,2} as the time required for $x_0(t)$ to reach 82% of its final value, then:

$$\tau = \pi/\gamma\omega_0. \quad (1.2)$$

Figure 2 shows how the relative steepness of the leading edge of the step depends upon the value of γ .

The stiffness of the mount, $E(x_0, x_1)$, has not been considered a constant, as transient analyses of mounting systems normally assume, but a function of the displacements x_0 and x_1 . The results and conclusions obtained in this report, therefore, need not be limited in application to small foundation disturbances for which the mounts may be simply assumed to exhibit linear elasticity. It is convenient to write the stiffness of the mount in the following form:

$$E(x_0, x_1) = E_0 f(x_0, x_1), \quad (1.3)$$

where $f(x_0, x_1)$ is also a function of the displacements x_0 and x_1 , and E_0 is a constant equal to the stiffness of the mount when the relative displacement experienced by the mount is small. For an ideally linear mounting, the function $f(x_0, x_1)$ will equal unity.

The coefficient of viscosity η defining the behavior of the damping element is assumed to be constant, that is to say, the element is assumed to be

have in a linear manner. The coefficient η may be written more conveniently in terms of a damping ratio $\bar{\delta}$. This parameter is defined as the ratio of the coefficient η to the value of the coefficient which is required to damp the system critically. It may be shown that:

$$\bar{\delta} = \omega_0 \eta / 2E_0 \quad . \quad (1.4)$$

Following the definitions of Eqs. (1.3) and (1.4), the equation of motion for the mounted item may be written in the form:

$$\frac{\ddot{x}_1}{x} - 2\bar{\delta}\omega_0 \left(\frac{\dot{x}_0 - \dot{x}_1}{x} \right) - \omega_0^2 \left(\frac{x_0 - x_1}{x} \right) f(x_0, x_1) = 0 \quad , \quad (1.5)$$

remembering that x is the constant maximum displacement associated with the step-like motion of the foundation, and that the natural mounting frequency ω_0 is by definition equal to $(E_0/M)^{1/2}$. Dots above symbols indicate differentiations with respect to time.

While the great majority of the literature devoted to the isolation of mechanical shock is concerned with linear mount elasticity, the theoretical work of R. D. Mindlin³ is a notable exception, dealing with several characteristic types of non-linear mount elasticity. This analysis, however, was made specifically to provide a more rational guide to the design of package cushioning than the commonly employed method of drop testing.^{3,4} Moreover, the analysis neglected the presence of mount damping, except when discussing linear mount elasticity. Inclusion of mount damping in any transient analysis is felt to be essential, and detailed consideration is given by this report to its influence upon the transient motion of the mounted item.

2. LINEAR MOUNT ELASTICITY

The reaction of linear shock mountings to a rounded step displacement possessing a wide range of rise times has been discussed previously^{1,2} but, for convenience, the results obtained are reviewed briefly so that comparison may be made with the results obtained for two forms of non-linear mount elasticity.

Acceleration-, displacement-, and relative displacement-time relationships for a mounted item (Fig. 1), the foundation of which is disturbed by a series of rounded step displacements (Fig. 2), are shown in Figs. 3 and 4. The curves of these figures have been computed for values of the mount damping ratio $\bar{\delta} = 0.05$ and 0.5 respectively. It is apparent from these figures that:

- (1) The maximum acceleration of the mounted item is large, but of short duration, when mount damping is large and the rise time of the step displacement is small. In fact, the peak value of

the acceleration occurs before the foundation displacement has increased greatly, its magnitude being sensibly proportional¹ to γ , $\bar{\delta}$, and ω_0^2 .

- (2) The maximum displacement of the mounted item is smallest when either:
 - (a) the damping ratio is large, or
 - (b) the parameter γ is not large; that is, the mount is sufficiently stiff to ensure that half the period of natural vibration of the mounting system is comparable with, or less than the rise time of the foundation displacement.

- (3) The maximum relative displacement between the foundation and the mounted item is of the same order of magnitude as the transient displacement of the foundation when γ is greater than about 10. Large mount damping does not significantly reduce the maximum value of the relative displacement when γ is large.

3. NON-LINEAR (TANGENT) MOUNT ELASTICITY

Literature concerned with the isolation of mechanical shock tacitly assumes that shock mountings exhibit ideally linear behavior or, equivalently, that the mounts experience a compression which is sufficiently small to ensure that only the initial linear portion of their force-deflection curve is invoked, that is to say, the restoring force, $F(x_0, x_1)$, exerted by the spring pictured in Fig. 1 may simply be defined by the following equation:

$$\frac{F(x_0, x_1)}{E_0 x} = \frac{(x_0 - x_1)}{x} \quad (3.1)$$

In practice, shock mountings rarely exhibit completely linear behavior, and normally stiffen more rapidly upon compression than Eq. (3.1) predicts. In this report, mathematical functions have been employed to describe theoretically the non-linear behavior of the shock mounting depicted in Fig. 1. [The non-linearity of the resilient element is represented by one of two mathematical functions. The damping element is assumed to exhibit linear behavior (Section 2)]. Solution by numerical integration of the non-linear differential equations embodying these mathematical functions is feasible utilizing a digital computer (Section 7). Moreover, the solutions obtained are not limited to small departures from linearity, as would be the case had solution by perturbation methods been attempted.

In this section, it has been assumed that the manner in which shock mountings stiffen upon compression may be represented mathematically by the following relation between the mount stiffness, $E(x_0, x_1)$, and the relative displacement

experienced by the mount, namely,

$$E(x_0, x_1) = E_0 \frac{\tan \left[a \left(\frac{x_0 - x_1}{x} \right) \right]}{a \left(\frac{x_0 - x_1}{x} \right)} . \quad (3.2)$$

By suitable choice of the parameter "a," the degree of non-linearity exhibited by the mount may be varied considerably. A similar relation was employed by Mindlin³ to represent non-linear mount elasticity. It follows from Eq. (3.2) that the spring force exerted by the resilient element may be written:

$$\frac{F(x_0, x_1)}{E_0 x} = \frac{1}{a} \tan \left[a \left(\frac{x_0 - x_1}{x} \right) \right] . \quad (3.3)$$

This equation is plotted in Fig. 5 for three values of the parameter a referred to throughout the report. If ξ represents the greatest compression that the mount can sustain, namely, that value of $(x_0 - x_1)$ for which the restoring force exerted by the resilient element becomes extremely large, then the parameter a may be written:

$$a = \frac{\pi}{2} \left(\frac{x}{\xi} \right) , \quad (3.4)$$

where x is again the maximum displacement associated with the step-like motion of the foundation. Equation (3.4) indicates that as the parameter a approaches the value $\pi/2$, the mount will experience a compression approaching the maximum value of which it is capable.

The maximum acceleration experienced by the mounted item is likewise increased as the parameter a becomes larger. This is evident in Fig. 6, where the acceleration of the mounted item excited by a rounded step foundation displacement is shown as a function of time. The rise time of the step is defined by a value of $\gamma = 50$, and mount damping by the damping ratio $\bar{\delta} = 0.05$. The acceleration experienced by the mounted item when supported by a linear mounting ($a = 0$) appears in Fig. 6 as a broken line.

As previously discussed in Section 2, a peak value of acceleration occurs when the foundation of a viscously damped mounting is displaced rapidly. This peak value of acceleration is observed in Fig. 6 at a small value of time, its magnitude not altering as the mount stiffness becomes increasingly non-linear (the parameter a increasing in value). Again, at a small but subsequent time, the acceleration of the mounted item is increased in comparison to the value which it takes when $a = 0$. As the value of a increases, the acceleration exhibits a maximum value, which becomes comparable with and then exceeds the magnitude of the acceleration peak appearing initially.

The second maximum value of acceleration occurs at essentially the same time as the compression experienced by the mount reaches its greatest value. The mount stiffness also assumes its greatest value at this time [Eq. (3.2)], its non-linearity being directly responsible for the presence of the second region of maximum acceleration. In fact, the peak value of acceleration increases as the departure of the mount stiffness from linearity becomes greater. The subsequent peak values of deceleration and acceleration can also be of appreciable magnitude, as the curve labeled $a = 1.50$ of Fig. 6 indicates. The peak values again occur at the same instant as the stiffness of the mount in extension or compression, respectively, is greatest.

It is interesting to observe how the acceleration-time relations for a mounted item disturbed by a foundation displacement alter as the mount damping ratio $\bar{\delta}$ is increased. The curves of Figs. 7a, 8a, 8b, and 7b show how the acceleration varies when $\bar{\delta}$ takes the successive values 0, 0.05, 0.1, and 0.5, respectively, and the rise time of the rounded step is defined by $\gamma = 50$. The corresponding displacement-time relationships are shown in Fig. 9. The full curves refer to the performance of a mount for which $a = 1.5$, and the broken curves relate to a linear mounting for which $a = 0$. While the second initial peak value of acceleration is predominant when $\bar{\delta}$ is small, it becomes continuously less important as $\bar{\delta}$ increases until, when $\bar{\delta} = 0.5$, the acceleration of the mounted item is little different in magnitude or form from that observed when the mount is completely linear. This behavior may have been qualitatively anticipated since, as indicated previously, the occurrence of the first acceleration peak may be related to the resistance to motion exerted by the damping element, and this will naturally increase as $\bar{\delta}$ becomes larger.

The acceleration-time curves of Figs. 7 and 8, and the displacement-time curves of Fig. 9, illustrate how, when mount damping is not large, the rapid increase in mount stiffness upon compression and elongation causes the period of vibration of the mounting system to decrease. In fact, when mount damping is negligible, the period of vibration is reduced to nearly half the value observed when the mount is linear. As mount damping is increased, however, the resistance to motion of the damping element becomes proportionately greater, and the relative displacement experienced by the mount, and consequently the extent of the non-linearity which it exhibits, is decreased. The period of vibration therefore returns progressively to the value which it assumes when the mount possessed linear elasticity.

Acceleration-, displacement-, and relative displacement-time relationships for a mounted item, the foundation of which is disturbed by the series of step-like displacements shown in Fig. 2, are presented in Figs. 10 and 11. The curves of these figures relate to non-linear mountings for which $a = 1.5$, and values of the damping ratio $\bar{\delta} = 0.05$ and 0.1, respectively. The figures illustrate how, as the rise time of the step becomes long in comparison to the half-period of natural vibration of the mounting system (the parameter γ decreasing in value), the relative displacement experienced by the mounts, and the degree of non-linearity which they exhibit, becomes smaller. At the same time, the maximum values of acceleration experienced by the mounted items decrease rapid-

ly, and the period of vibration which the mounting systems possess lengthens appreciably.

4. NON-LINEAR (INVERSE TANGENT) MOUNT ELASTICITY

The second form of non-linear mount elasticity discussed in this report is such that the restoring force exerted by the spring pictured in Fig. 1 is always smaller than the force that an ideally linear spring would exert when experiencing the same relative displacement. The difference between the restoring forces is negligible when the relative displacement across the mount is small. As the relative displacement becomes progressively larger, however, the restoring force exerted by the non-linear spring becomes progressively smaller than the force which a linear spring would exert.

This form of non-linearity has been discussed because it was anticipated that a mounting possessing such characteristics would afford the contents of a mounted item appreciable protection from damage due to shock. The investigation described in this section was prompted by a theoretical treatment of package cushioning,³ which demonstrated that a cushioning pad (undamped) possessing similar non-linear characteristics was most favorably suited to offer protection from damage encountered in a drop test.^{3,4} Materials such as foamed polyurethane plastics and hair-filled rubber pads can possess non-linear elasticity of this form, providing that the relative displacement which they experience does not exceed a certain critical value, after which time they stiffen very rapidly as the relative displacement increases.

An endeavor has been made to demonstrate in a broad sense the value of this form of mount non-linearity. In so doing, it is hoped to encourage the development of a heavy-duty shock mount possessing the same or very similar non-linear characteristics, since the materials mentioned above are suitable only for applications such as package cushioning.

The behavior of the resilient element shown in Fig. 1 has been represented mathematically by a non-linear stiffness, $E(x_0, x_1)$, which varies in proportion to the inverse tangent of the relative displacement experienced by the mount, namely

$$E(x_0, x_1) = E_0 \frac{\tan^{-1} \left[b \left(\frac{x_0 - x_1}{x} \right) \right]}{b \left(\frac{x_0 - x_1}{x} \right)} . \quad (4.1)$$

The spring force, $F(x_0, x_1)$, exerted by the mount may therefore be written:

$$\frac{F(x_0, x_1)}{E_0 x} = \frac{1}{b} \tan^{-1} \left[b \left(\frac{x_0 - x_1}{x} \right) \right] \quad (4.2)$$

and this equation is plotted in Fig. 12 for three values of the parameter "b," which have been employed throughout the report. By suitable choice of b, the degree of non-linearity exhibited by the mount may be varied. It must be pointed out that the definition of the non-linear mount stiffness [Eq. (4.1)] implicitly assumes that the relative displacement across the mount does not reach the critical value for which the mount commences to stiffen rapidly as the relative displacement increases. It must also be emphasized, that providing the relative displacement experienced by the mount is small, the mount stiffness will be essentially identical to that of a linear mount or to that of a non-linear mount such as discussed in the preceding section. Consequently, the "softening" of the mount occurring for large values of the relative displacement should have no reflection upon the ability of such a mount of suitable design to function "normally" in supporting a given static load, or in providing adequate stability for the mounted item.

The maximum acceleration experienced by the mounted item decreases as the departure from linearity of the mount stiffness becomes larger (the parameter b increasing). For example, the acceleration of a mounted item supported by a mount for which the damping ratio $\bar{\delta} = 0.05$ is shown in Fig. 13 as a function of the parameter b. The acceleration of the mounted item is produced by a rounded step foundation displacement, the rise time of which is defined by a value of the parameter $\gamma = 5$. The broken line refers to the acceleration experienced by the mounted item when it is supported by a linear mounting.

It is evident from Fig. 13 that when $b = 4$, the maximum acceleration experienced by the mounted item initially, and the minimum and maximum values of acceleration experienced at subsequent values of time, are considerably less than would be experienced if the mount exhibited linear elasticity. Moreover, the maximum values of acceleration exhibited in the latter case ($b = 0$) will probably be less than the mounted item would experience in practice, since the majority of shock mountings will stiffen more rapidly upon compression than a linear mounting.

Because the value of γ considered in this example is not large, the acceleration peak which is associated with the resistance to motion of the damping element is not always evident at small values of time. In fact, only when $b = 4$ is a very small acceleration peak superimposed upon an initial broad region of maximum acceleration. The small acceleration peak becomes evident because, as the value of b increases, the restoring force exerted by the resilient element decreases with respect to the resistance to motion of the damping element.

The influence of mount damping upon the acceleration-time relationships for the mounted item may be observed in Figs. 14 and 15. Displacement-time relationships are presented in Fig. 16. These figures show how the acceleration and displacement of the mounted item vary as the damping ratio $\bar{\delta}$ takes the successive

values 0, 0.05, 0.1, and 0.5. The full curves relate to the performance of a mount for which $b = 4$, while the broken curves relate to the performance of a linear mounting. The rise time of the rounded step is again defined by a value of $\gamma = 5$.

It is seen that although the maximum values of acceleration experienced by the mounted item are effectively reduced by the non-linear mounting when mount damping is not large, they are not greatly different in magnitude or form from the values observed for a linear mounting if the mount damping ratio $\bar{\delta} = 0.5$. This may be ascribed to the increasing resistance to motion exerted by the damping element as $\bar{\delta}$ becomes larger—so that the influence of the resilient element upon the motion of the mounted item is progressively reduced—and also to the associated reduction of the relative displacement experienced by the mount, and hence the decrease in non-linearity which it exhibits.

The acceleration-time relations of Figs. 14 and 15, and the displacement-time relations of Fig. 16 indicate that when the mount damping is small, the reduction in the mount stiffness (or the "softening" of the mount) upon compression or elongation is responsible also for the increase in the period of vibration of the mounting system. In fact, when $\bar{\delta} = 0$, the period of vibration is seen to be approximately fifty per cent greater than the value it assumes when the mount possesses linear elasticity. As the value of $\bar{\delta}$ increases, with the attendant decrease in mount non-linearity, the period of vibration is seen to grow shorter and return to the value which a linear mounting would possess under similar circumstances.

Acceleration-, displacement-, and relative displacement-time relationships for a mounted item supported by a damped non-linear mount defined by values of the parameters $\bar{\delta} = 0.05$ and $b = 4.0$ are shown in Fig. 17. The transient motion of the mounted item is produced by a series of step-like foundation displacements which are shown in Fig. 2. When the rise time of the foundation displacement is defined by $\gamma = 50$, the acceleration experienced by the mounted item is equal to $2.05 (\omega_0^2 x)$. It is seen from this figure that, other than at small values of time, the maximum and minimum values of acceleration, displacement, and relative displacement change remarkably little as the rise time of the rounded step varies. Moreover, the phase of the oscillatory motions of the mounted item changes very little when the rise time of the step displacement varies, far less than observed for a mount that stiffens more rapidly upon compression than a linear mounting. (Compare the curves of Fig. 17 with those of Fig. 10.)

5. COMPARISON OF MOUNT PERFORMANCE

5.1. Introduction

Damage to equipment and its supporting shock mounts which may be caused by sudden foundation motions can be related,¹ in general, to the absolute values of the resulting maximum displacement and acceleration experienced by the mounted item, and to the magnitude of the maximum relative displacement between the mounted item and its foundation. In fact, the risk of damage to those elements within the mounted item possessing natural periods of vibration shorter than the rise time of the foundation displacement can be directly related¹ to the maximum acceleration of the mounted item. Again, the risk of damage to those elements possessing natural periods of vibration greater than the rise time of the foundation displacement can be directly related¹ to the maximum displacement of the mounted item. A large relative displacement between the mounted item and the foundation could result in mount failure, or collision of the mounted item with its surroundings. Consequently, if the contents of the mounted item are to receive the greatest possible protection from damage, the maximum acceleration, displacement, and relative displacement of the mounted item should be simultaneously small.

Quantities which have been employed here^{1,2} to describe the ratios of the maximum acceleration and displacement experienced by the mounted item to the corresponding maximum values of the acceleration and displacement experienced by the foundation are the shock acceleration ratio (SAR) and the shock displacement ratio (SDR) respectively. The ratio of the maximum relative displacement of the mounted item to the maximum foundation displacement is described by the relative displacement ratio (RDR).

The shock acceleration ratio, shock displacement ratio, and relative displacement ratio of the mounting systems discussed previously, and also of many other mounting systems which have been examined, are presented graphically in this section as a function of γ . It may be recalled that this parameter is a measure of the steepness of the leading edge of the foundation displacement (Fig. 2).

Because the quantities SAR, SDR, and RDR are of principal value in evaluating and comparing mount performance, the graphs to be presented provide a convenient method of summarizing the computations made during this study of the problem of isolating equipment from large transient foundation displacements.

5.2. The Shock Displacement Ratio and Relative Displacement Ratio

The shock displacement ratio of a mounted item supported by an ideally linear mounting is shown in Fig. 18. The ratio of the peak excursion of the mounted

item to the height of the disturbing step is shown in this figure as a function of γ for various values of the mount damping ratio $\bar{\delta}$. The SDR will always lie between 1 and 2, approaching these limiting values¹ when γ is small (stiff mounts) and γ is large (soft mounts), respectively.

The shock displacement ratios of the complete series of non-linear mounts discussed previously are shown in Figs. 19 and 20, for values of the mount damping ratio $\bar{\delta} = 0$ and 0.05, respectively. The considerable difference in the nature of the curves of the two figures demonstrates the danger of ignoring the presence of mount damping when determining the motion of the mounted item, even if the magnitude of the damping is small. The shock displacement ratios of the damped non-linear mounts for which $\bar{\delta} = 0.1$ and 0.5 are shown in Figs. 21 and 22, respectively. The broken curves refer to the SDR of linear mountings, which have been presented separately in Fig. 18.

It is evident from Figs. 20, 21, and 22 that other than for small values of γ , the SDR of a mount possessing a given damping ratio is increased when the mount stiffness increases more rapidly upon compression than observed for a linear mount, and is reduced if the mount "softens" upon compression. In fact, the SDR is seen in each figure to be beneficially least when $b = 4$, except when γ is small. The performance of the heavily damped non-linear mount defined by values of $\bar{\delta} = 0.5$ and $b = 4$ is seen to be particularly good.

The relative displacement introduced between the mounted item and its surroundings can be as large as the final height of the disturbing step-like foundation displacement. The relative displacement ratio of a linear mounting is shown in Fig. 23 as a function of γ for various values of the mount damping ratio $\bar{\delta}$. It is seen that the RDR is reduced by heavy mount damping, although it always becomes comparable with unity once γ is greater than about ten.

The relative displacement ratios of a series of non-linear mounts are presented in Figs. 24 and 25 for values of $\bar{\delta} = 0$ and 0.1, and $\bar{\delta} = 0.05$ and 0.5, respectively. Only the RDR of the non-linear mount defined by $b = 4$ has been labeled. Reading downward, the curves of the figures refer respectively to values of $b = 4.0, 1.5, 1.0$; $a = b = 0$; and $a = 1.0, 1.25, 1.5$. The broken curves refer to the performance of an ideally linear mounting.

It is evident from Figs. 24 and 25 that the RDR of mountings possessing even small mount damping, is little influenced by the degree of non-linearity exhibited by the mount stiffness. The relative displacement ratios of the non-linear mountings are practically identical when mount damping is large.

5.3. The Shock Acceleration Ratio

Although for step-like foundation displacements resilient mounting has been found to increase the absolute and relative displacement of the mounted item, it beneficially reduces the acceleration which the mounted item experiences. This

is evident in Fig. 26, where the SAR of a linear mounting is shown as a function of γ for different values of the mount damping ratio $\bar{\delta}$. The shock acceleration ratios of the non-linear mounts discussed in this section have been derived by dividing the maximum acceleration which the mounted item experiences by $\gamma^2 \omega_0^2 x$, the maximum acceleration associated with the transient motion of its foundation. When the rise time of the foundation displacement is short (large γ), mount damping finite, and the stiffness of the mount linear, the maximum value of the acceleration experienced by the mounted item is sensibly equal¹ to $2e^{-1}\gamma\bar{\delta}(\omega_0^2 x)$. Consequently, the SAR of such a linear mount decreases in direct proportion to the value of γ or, when plotted as in Fig. 26, decreases at 6 db/octave. Mount damping is seen to reduce the SAR of the mount when γ is very small, but to increase the SAR detrimentally when γ is large.

The shock acceleration ratios of the non-linear mountings discussed in Section 5.2 are shown in Figs. 27, 28, 29, and 30, which relate to values of the mount damping ratio $\bar{\delta} = 0, 0.05, 0.1, \text{ and } 0.5$, respectively. The form of the curves of these figures illustrates again the danger of drawing conclusions from analyses which consider only undamped shock mountings.

When mount damping may be neglected (Fig. 27), the SAR is seen to alter appreciably as the departure from linearity of the mount stiffness becomes larger. When the mount stiffens more rapidly upon compression than a linear mounting, the SAR is increased by an amount which, when the parameter $a = 1.5$, approaches 20 db for large values of γ . On the other hand, if the mount stiffens less rapidly upon compression than a linear mount, the SAR is reduced by an amount which is approximately the same for all values of γ and, for example, is equal to 10 db when the parameter $b = 4$.

From an initial consideration of the performance of a mount possessing negligible damping, it is interesting to observe how the form of the SAR changes as the damping ratio $\bar{\delta}$ of the mount is increased. Figures 28, 29, and 30 show that the difference between the shock acceleration ratios of mounts possessing different degrees of non-linearity is reduced until, when $\bar{\delta} = 0.5$, there is essentially no difference between the SAR of the non-linear mounts and the SAR of an ideally linear mounting. Figures 28 and 29 illustrate the intermediate situation, where the effect of the predominant resistance to motion of the damping element, such as observed in Fig. 30 when $\bar{\delta} = 0.5$, is apparent only at high values of γ . It should be noticed that the SAR possessed by mounts which stiffen very rapidly upon compression (such as a mount for which $a = 1.5$) is less influenced by mount damping than the SAR of mounts which "soften" upon compression (such as a mount for which $b = 4$). However, when γ and $\bar{\delta}$ are not large—for example, when γ is less than about 20 and $\bar{\delta} = 0.05$ —then the non-linear mount defined by a value of $b = 4$ possesses an appreciably smaller SAR than a non-linear mount defined by a value of $a = 1$, or greater. It is the latter type of non-linear behavior that shock mountings may be expected to exhibit in practice.

6. SUMMARY AND CONCLUSIONS

In this report, the performances of two types of non-linear shock mounting have been examined, and compared with each other and with the performance of an ideally linear mounting. Large departures of the mount stiffness from linearity have been considered, and detailed discussion made of the influence of mount damping upon the transient motion of the mounted item. The performance of the non-linear mountings has been judged by their response to step-like foundation displacements possessing a wide range of rise times. The results described in the report were obtained from the numerical integration of two non-linear differential equations. Integration was performed by a digital computer, and the computer program employed is described in Section 7.

The first type of mount non-linearity discussed in the report is commonly encountered in practice, describing the behavior of a mount which stiffens more rapidly upon compression than an ideally linear mounting. The second type of mount non-linearity which has been discussed is not, in general, observed in mounts capable of supporting items of equipment, being found only in materials suitable for applications such as package cushioning. The stiffness of such materials increases more slowly upon compression than the stiffness of a linear mounting. It has been attempted to illustrate in the report the merits of this type of mount non-linearity, with the aim of encouraging the development of a relatively heavy-duty mount, which will exhibit similar non-linear behavior. The first and second types of mount non-linearity have been simulated mathematically by assuming that the mount stiffness is proportional to the tangent, or to the inverse tangent of the relative displacement experienced by the mount, respectively.

It is pointed out that if the contents of a resiliently mounted item are to receive the greatest possible protection from damage due to shock, the shock acceleration ratio (SAR), the shock displacement ratio (SDR), and the relative displacement ratio (RDR) of the mounted item should be simultaneously small. These quantities have been plotted for a number of non-linear mounts, which possess different mount damping ratios, as a function of a parameter γ . This parameter describes the steepness of the leading edge of the step-like foundation displacement in terms of the period of natural vibration of the mounting system. The plots of SAR, SDR, and RDR versus γ provide a convenient method of comparing and summarizing the results of the computations made for this report.

From reference to the SAR, SDR, and RDR curves presented in the report, the following conclusions may be drawn:

1. The relative displacement of the mounted item is of similar magnitude to the applied step displacement for all mountings considered when γ is large. The influence of mount non-linearity upon the relative displacement is very small. Mount damping exerts

only slightly greater influence upon the magnitude of the relative displacement when γ is greater than about 10.

2. High mount damping detrimentally affects the performance of both non-linear and linear mountings by increasing the magnitude of the acceleration experienced by the mounted item. For abrupt foundation displacements, a predominant "spike" of acceleration is introduced while the foundation displacement is still small. Under these circumstances, the shock acceleration ratios of the non-linear mountings discussed in this report are found to be virtually identical. The "spike" of acceleration is responsible for damage to elements within the mounted item possessing natural periods of vibration shorter than the step rise time.
3. The SDR of a mount which stiffens considerably more slowly upon compression ($b = 4$) than a linear mounting, possesses a very low value if the mount is heavily damped ($\bar{\delta} = 0.5$). As mentioned previously, however, the SAR of this mounting will be as large as the SAR of an equally heavily damped linear mounting. Since damage to the contents of the mounted item will frequently be related to the maximum acceleration experienced by the mounted item (that is to say, the fragile elements within the mounted item are most likely to possess natural periods of vibration that are shorter than the step rise time), the use of such heavy mount damping to reduce the SDR of the mount greatly cannot, in general, be recommended.
4. When the same non-linear mount is less heavily damped ($\bar{\delta} = 0.05$), however, the SAR and SDR of the mount are smaller than the SAR and SDR of an equally damped linear mounting (except for the SAR at very large values of γ , and the SDR at very small values of γ). The beneficial reduction in the SAR and SDR is not immediately as striking as had been hoped for. The reduction obtained in practice may nevertheless be appreciable, since the mount performance should more realistically be compared with the performance of a mount which stiffens rapidly upon compression (the parameter a being greater than zero), this typifying the behavior exhibited by shock mountings in practice. The reduction of the SAR achieved with this mounting (defined by values of $b = 4$ and $\bar{\delta} = 0.05$) may therefore be of the order 10 db over quite a wide range of γ (Fig. 28), and it is commonly the magnitude of the SAR to which the likelihood of damage to the contents of a mounted item may be related.

7. APPENDIX I

by
D. M. Sinnett

Runge-Kutta-Milne Method for Solution of Second-Order Differential Equations of the Type $z'' = f(z, z', \theta)$

Among the best known methods for numerical integration are those of Runge Kutta and Milne.

Although each has many variations the underlying principle is the same. Each is an approximation, for a specific number of terms, to the Taylor's series expansion of the function to be integrated.

The program described here produces correct integration for four terms in the Taylor's series.

Each method has limitations, but they can be used effectively together.

The Runge Kutta is self starting, and is an effective way to reduce the increment, but it requires the evaluation of the function four times in a step from θ_n to θ_{n+1} . Also, there is no convenient check on the accuracy.

The Milne method, using the predictor and corrector equations requires the evaluation of the derivative only twice. By repeated applications of the corrector equation accuracy to a prescribed tolerance can be obtained; however, it is more sound mathematically to decrease the increment and use only the predictor equation. The computation time is decreased, and the corrector may be used as an occasional check. The Milne method is not self starting, and decreasing the increment size is cumbersome. However, the ability to increase or decrease the increment to take advantage of smoothness in the equation greatly reduces the running time.

For these reasons, the following method was used.

Four initial values were computed using the Runge Kutta method. They were then checked by the Milne predictor and corrector (see Fig. 31). The requirements were:

$$10^{-4} > |p - c| > 10^{-7}$$

If the requirements were not met, the increment was adjusted and the initial values were recomputed.

When the correct increment was found, the computation was carried on using the Milne predictor.

The corrector equation was applied to every fourth point computed, and if the error was not within the prescribed tolerance, the increment was adjusted.

If the increment was too large, it was halved and the Runge Kutta method was used to compute four additional points using the last previous correct values as initial values. The increment was doubled by applying the Milne predictor to alternate preceding values.

The actual equations follow.

Runge Kutta:

$$z'' = f(z, z', \theta) \quad (7.1)$$

$$z''_{11} = f(z_{11}, z'_{11}, \theta_{11}) \quad (7.2)$$

$$z''_{12} = f[z_{11} + (\Delta\theta/2) z'_{11}, z'_{11} + (\Delta\theta/2) z''_{11}, \theta_{11} + \Delta\theta/2] \quad (7.3)$$

$$z''_{13} = f[z_{11} + (\Delta\theta/2) z'_{12}, z'_{11} + (\Delta\theta/2) z''_{12}, \theta_{11} + \Delta\theta/2] \quad (7.4)$$

$$z''_{14} = f(z_{11} + \Delta\theta z'_{13}, z'_{11} + \Delta\theta z''_{13}, \theta_{11} + \Delta\theta) \quad (7.5)$$

$$z'_{21} = z'_{11} + (\Delta\theta/6) (z''_{11} + 2z''_{12} + 2z''_{13} + z''_{14}) \quad (7.6)$$

$$z_{21} = z_{11} + (\Delta\theta/6) (z'_{11} + 2z'_{12} + 2z'_{13} + z'_{14}) \quad (7.7)$$

Milne:

$$z^{(p)} = \text{predicted value of } z \quad (7.8)$$

$$z^{(c)} = \text{corrected value of } z \quad (7.9)$$

$$\epsilon = \text{error} \quad (7.10)$$

$$z'_{51}{}^{(p)} = z'_{41} + h(c_0 z''_{41} + c_1 z''_{31} + c_2 z''_{21} + c_3 z''_{11}) + \epsilon_p \quad (7.11)$$

$$z'_{51}{}^{(c)} = z'_{51} + h(c'_0 z''_{51} + c'_1 z''_{41} + c'_2 z''_{31} + c'_3 z''_{21}) + \epsilon_c \quad (7.12)$$

z_{51} is obtained by application of the same equations using values of z' .

Each method is correct integration for the first four terms of the Taylor's series, so ϵ is of the order

$$\frac{z^{(5)} h^5}{4!} .$$

In addition, ϵ_p and ϵ_c are of opposite signs, so the quantity $|\epsilon_p - \epsilon_c|$ is an overestimate of the truncation error.

8. APPENDIX II

The Response of Equipment to a Step-Like Change in Foundation Velocity

The transient motion of a mounted item supported by a linear mounting has been computed when the foundation of the mounting system experiences a step-like change in velocity $V_0(t)$, which is defined by the relations:

$$\begin{aligned} V_0(t) &= 0 \text{ when } t < 0 \\ V_0(t) &= V \left[1 - e^{-\gamma\omega_0 t} (1 + \gamma\omega_0 t) \right] \text{ when } t > 0, \end{aligned} \quad (8.1)$$

where t is time, ω_0 is the natural (angular) frequency of the mounting system, and V is a constant. The parameter γ determines the rise time of the step as is evident in Fig. 32, where Eq. (8.1) is plotted for a number of γ values. The displacement $x_0(t)$ of the foundation defined by these values of γ , and the displacement $x_1(t)$ of the mounted item for a value of $\gamma = 10$ and different values of the mount damping ratio $\bar{\delta}$ are plotted in dimensionless form in Figs. 33 and 34, respectively.

The acceleration, velocity and relative displacement of the mounted item resulting from a series of step-like changes in foundation velocity are shown in dimensionless form as functions of time in each of Figs. 35, 36, and 37, which relate to values of $\bar{\delta} = 0.05, 0.1, \text{ and } 0.5$, respectively.

Since it is the maximum values of acceleration, velocity and relative displacement experienced by the mounted item which are primarily of value in comparing the performance of the different mountings, the essential results of the computations so far described and many others may be depicted graphically as in Figs. 38, 39, and 40.

The shock velocity ratio (SVR), namely, the ratio of the maximum velocity experienced by the mounted item to the constant maximum velocity associated with the velocity step, is shown in Fig. 38. The product of the maximum relative displacement between the mounted item and the foundation and the quantity ω_0/V is shown in Fig. 39. The SVR is seen always to lie between 1 and 2, while the maximum relative displacement times (ω_0/V) is comparable with unity, unless mount damping is very large and the rise time of the velocity step long.

Although resilient mounting increases the value of the maximum velocity experienced by the mounted item, and introduces a relative displacement between the mounted item and the foundation, the maximum acceleration experienced by the mounted item is reduced for all but those velocity steps with long rise times. This is shown by Fig. 40, where the shock acceleration ratio (SAR) is plotted as a function of γ , the SAR again being defined as the ratio of the

maximum acceleration experienced by the mounted item to the maximum acceleration associated with the motion of its foundation.

The SAR is seen to decrease at essentially 6 db/octave for all values of γ greater than 5. The influence of mount damping upon the SAR is remarkably small for these values of γ , the difference in the shock acceleration ratios not exceeding 1.5 db.

9. REFERENCES

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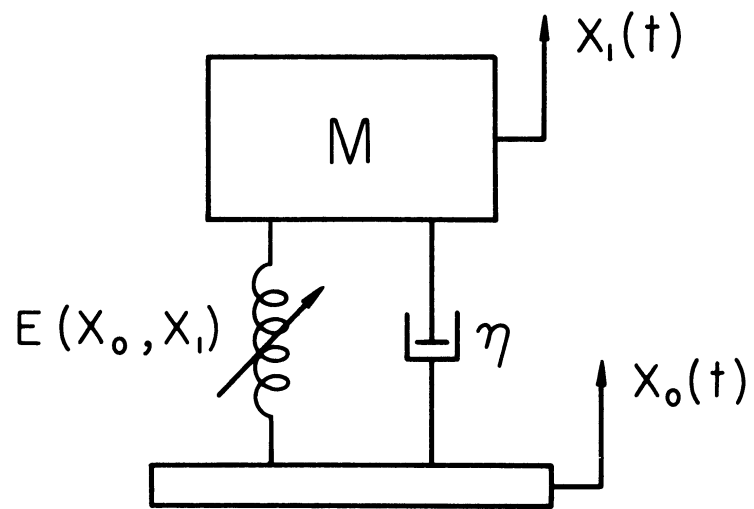


Fig. 1. An item of equipment supported by a damped non-linear spring upon a transiently displaced foundation.

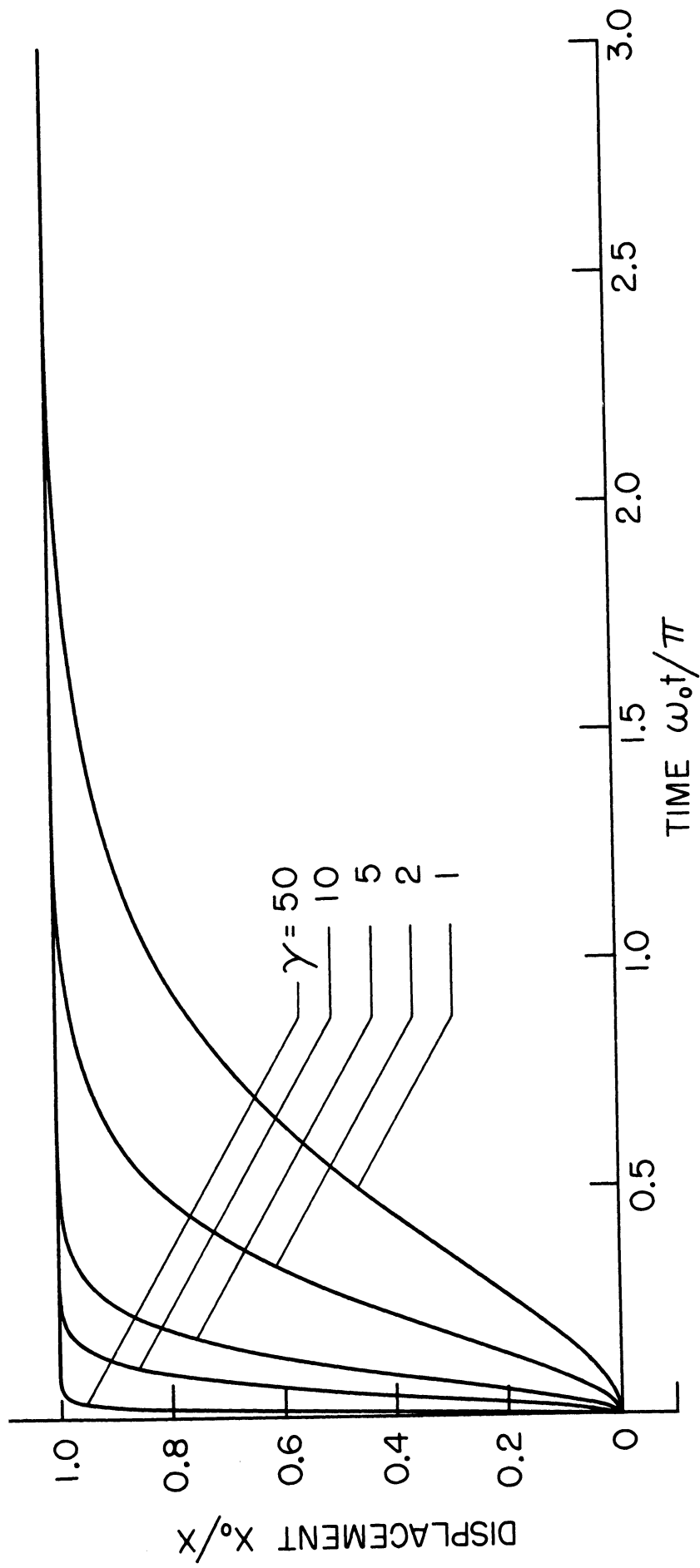


Fig. 2. The rounded step displacement.

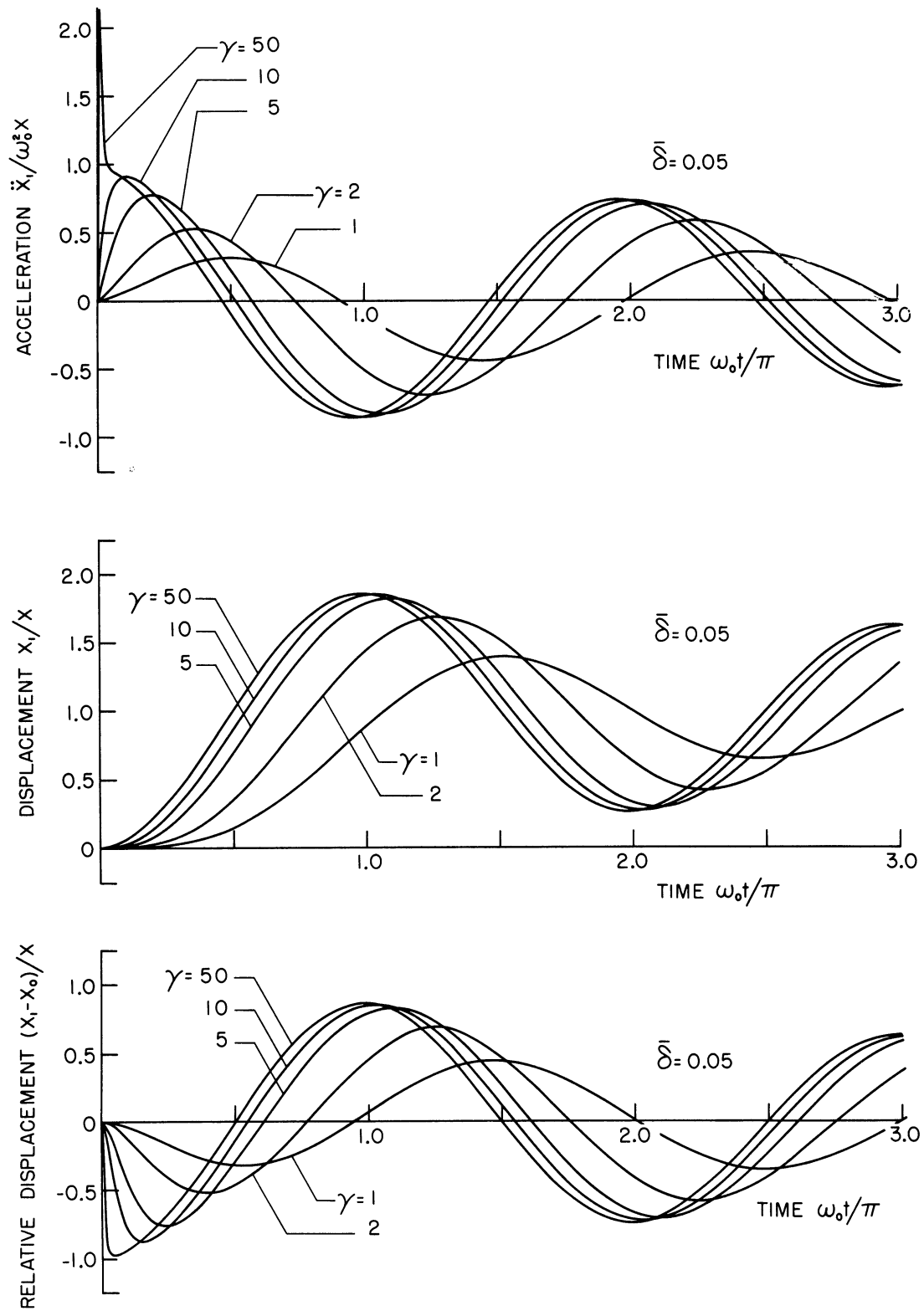


Fig. 3. Acceleration-, displacement-, and relative displacement-time relationships for a linear mounting. Mount damping ratio $\bar{\delta} = 0.05$.

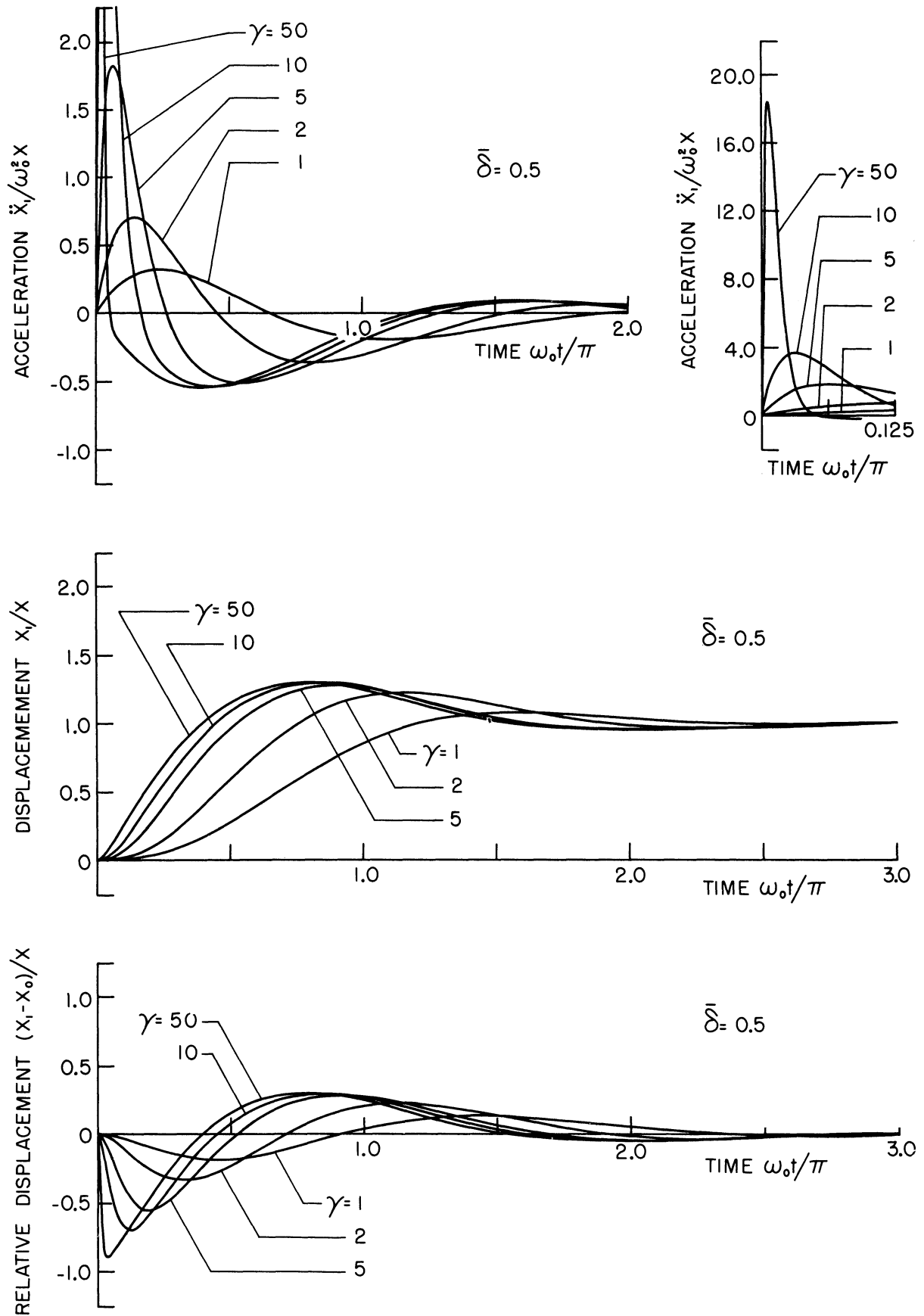


Fig. 4. Acceleration-, displacement-, and relative displacement-time relationships for a linear mounting. Mount damping ratio $\bar{\delta} = 0.5$.

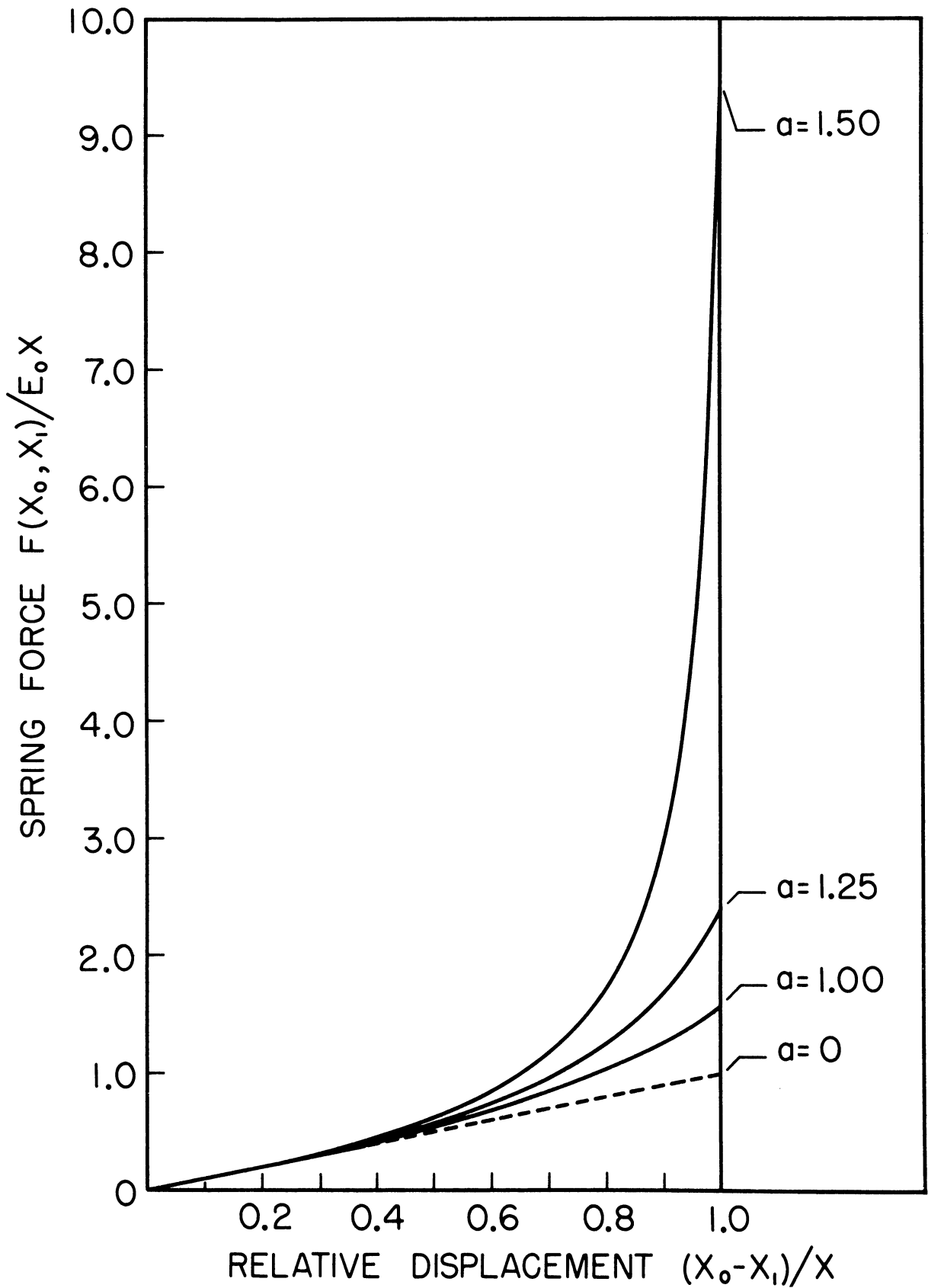


Fig. 5. The restoring force exerted by a mount, which stiffens more rapidly upon compression than a linear mounting, plotted as a function of the relative displacement experienced by the mounting.

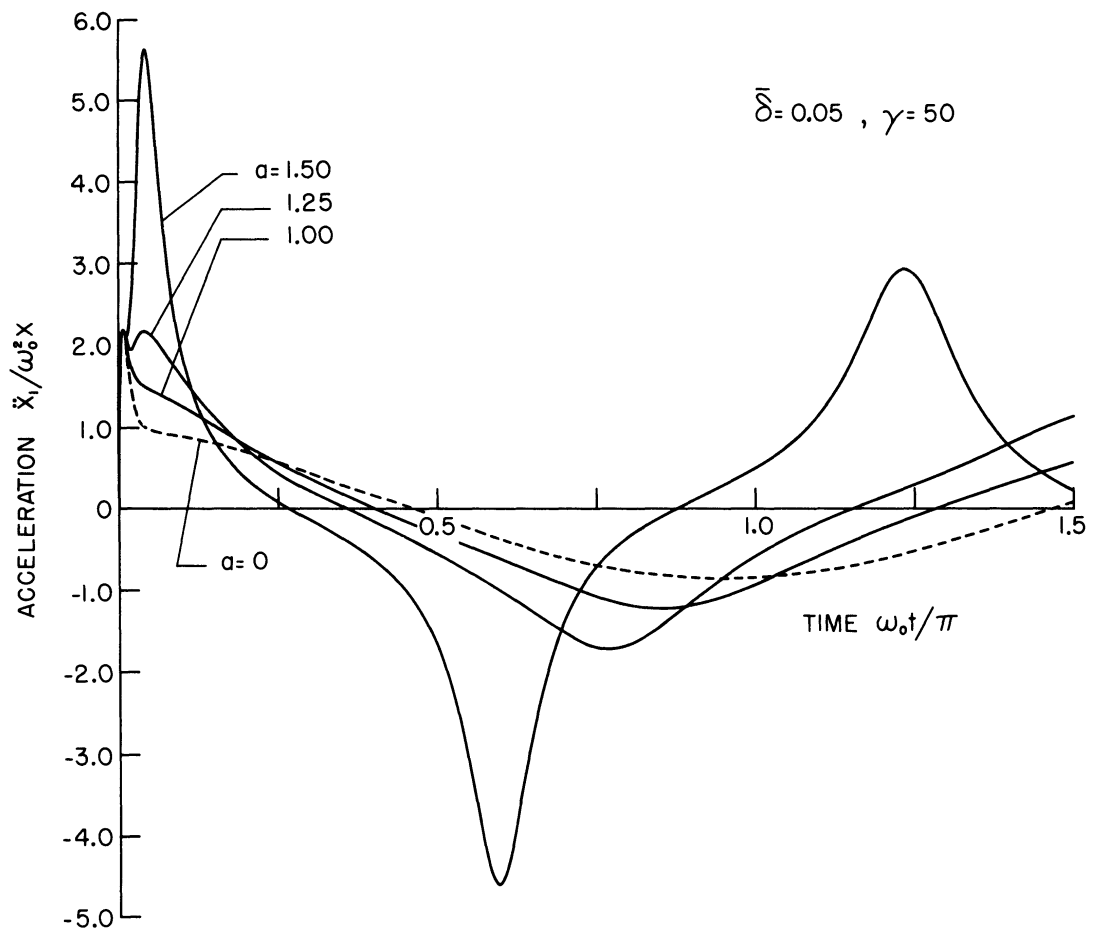
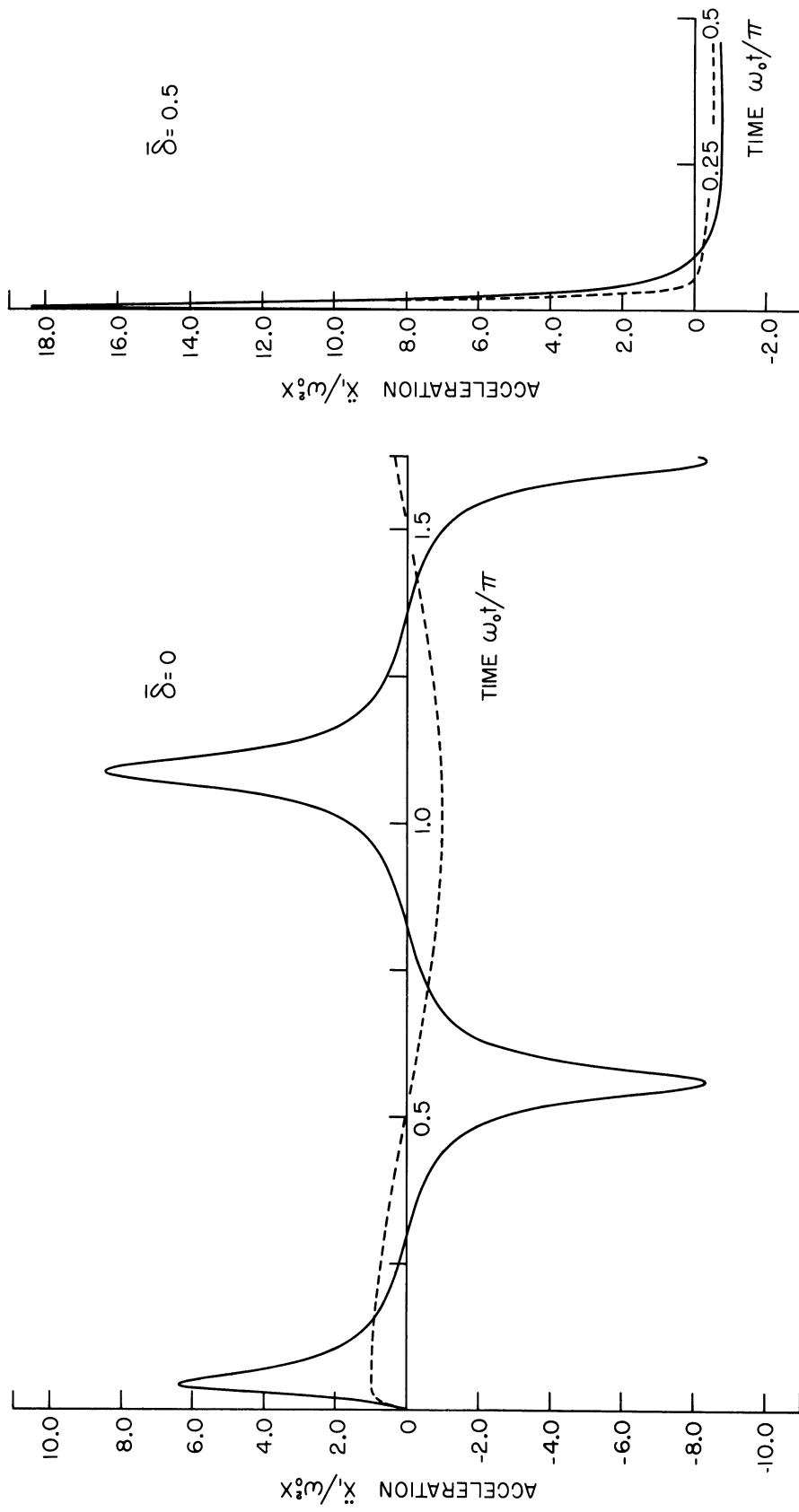
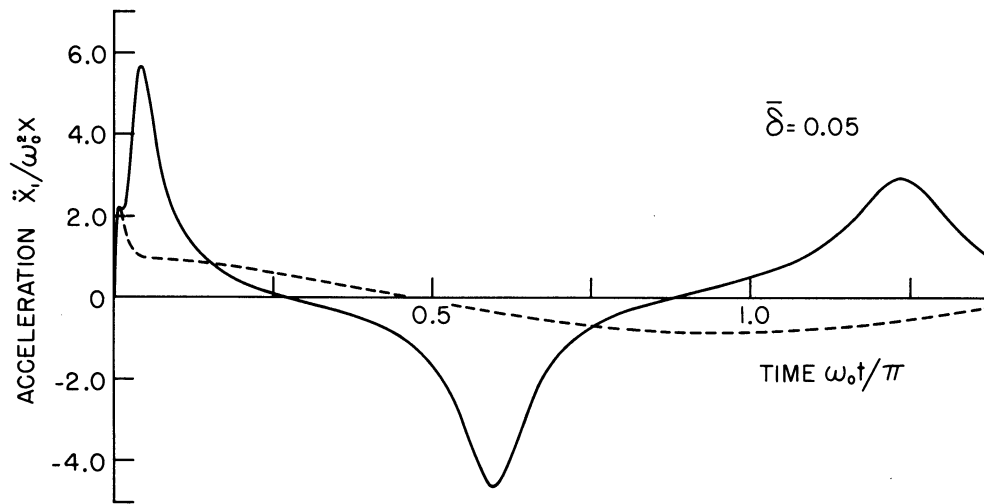


Fig. 6. Acceleration-time relationships for a series of damped non-linear mountings defined by values of $\bar{\delta} = 0.05$ and $a = 1.00, 1.25, \text{ and } 1.50$. The broken curve refers to the performance of a linear mounting. Rise time of the rounded step foundation displacement defined by $\gamma = 50$.

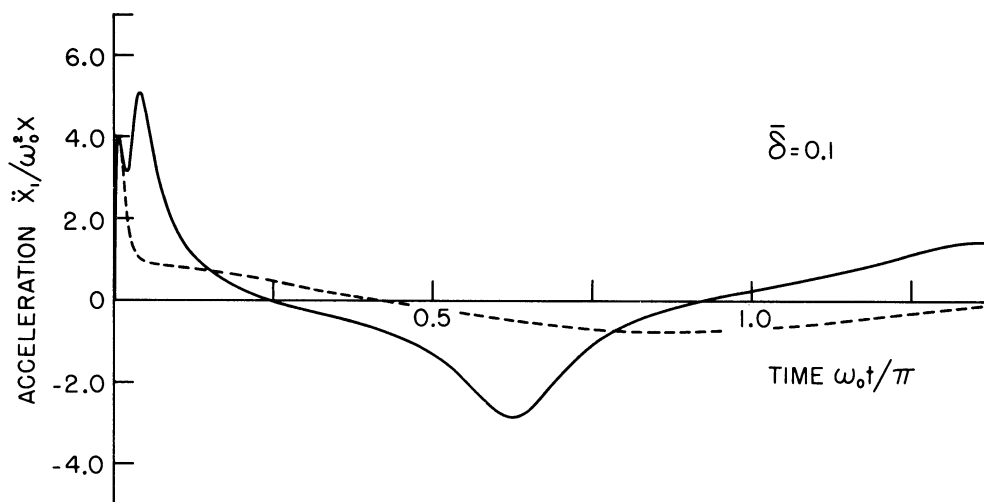


(a) (b)

Fig. 7. Acceleration-time relationships for damped non-linear mountings defined by a value of the parameter $a = 1.50$, and values of the damping ratio (a) $\bar{\delta} = 0$ and (b) $\bar{\delta} = 0.5$. The broken curves refer to the performance of a linear mounting. Rise time of the rounded step foundation displacement defined by $\gamma = 50$.



(a)



(b)

Fig. 8. Acceleration-time relationships for damped non-linear mountings defined by a value of the parameter $a = 1.5$, and values of the damping ratio (a) $\bar{\delta} = 0.05$ and (b) $\bar{\delta} = 0.1$. The broken curves refer to the performance of a linear mounting. Rise time of the rounded step foundation displacement defined by $\gamma = 50$.

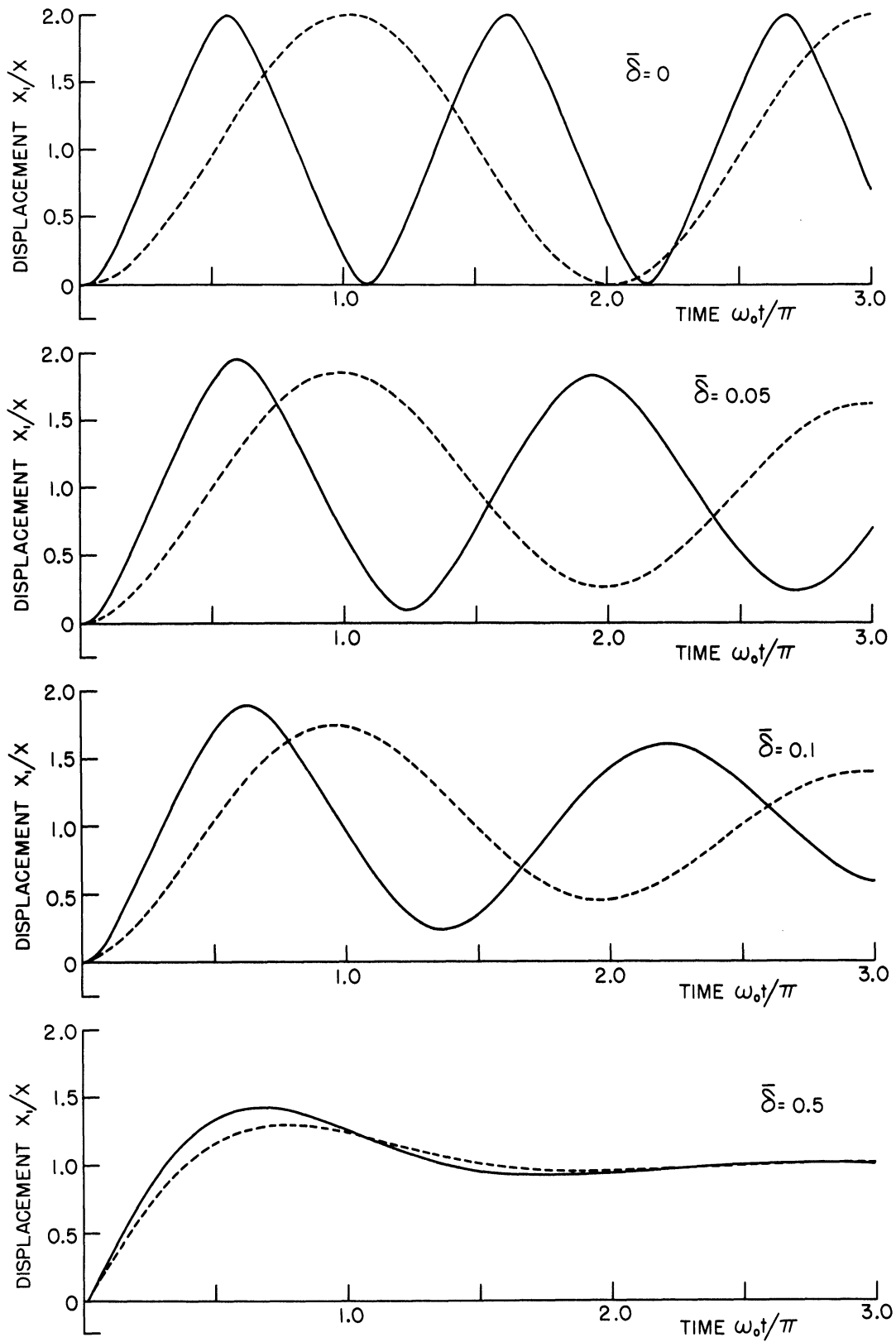


Fig. 9. Displacement-time relationships for non-linear mountings defined by a value of the parameter $a = 1.50$, and values of the damping ratio $\bar{\delta} = 0, 0.05, 0.1, \text{ and } 0.5$. The broken curves refer to the performance of a linear mounting. Rise time of the rounded step foundation displacement defined by $\gamma = 50$.

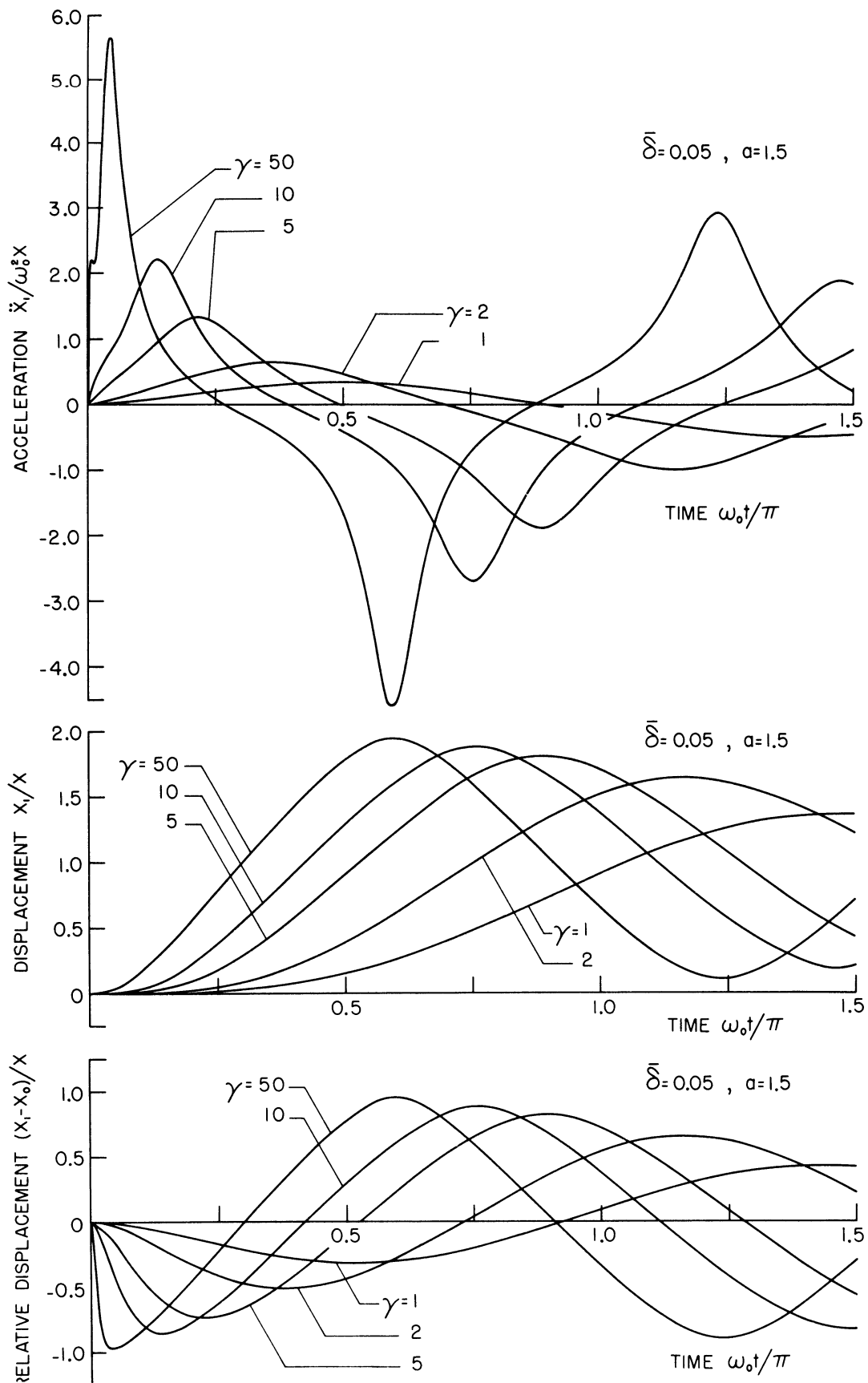


Fig. 10. Acceleration-, displacement-, and relative displacement-time relationships for a non-linear mounting defined by a value of the parameter $\alpha = 1.50$. Mount damping ratio $\bar{\delta} = 0.05$.

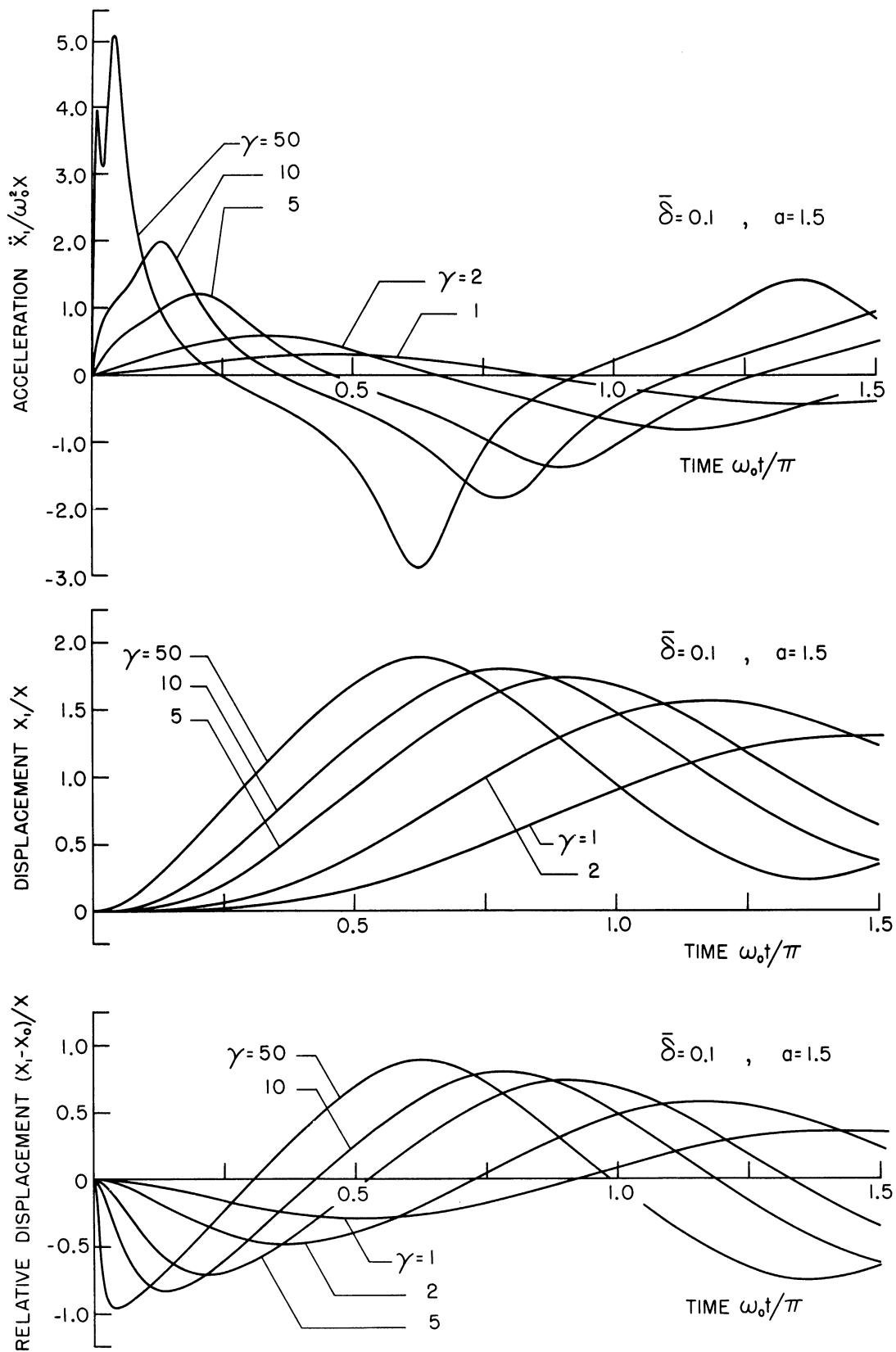


Fig. 11. Acceleration-, displacement-, and relative displacement-time relationships for a non-linear mounting defined by a value of the parameter $a = 1.50$. Mount damping ratio $\bar{\delta} = 0.1$.

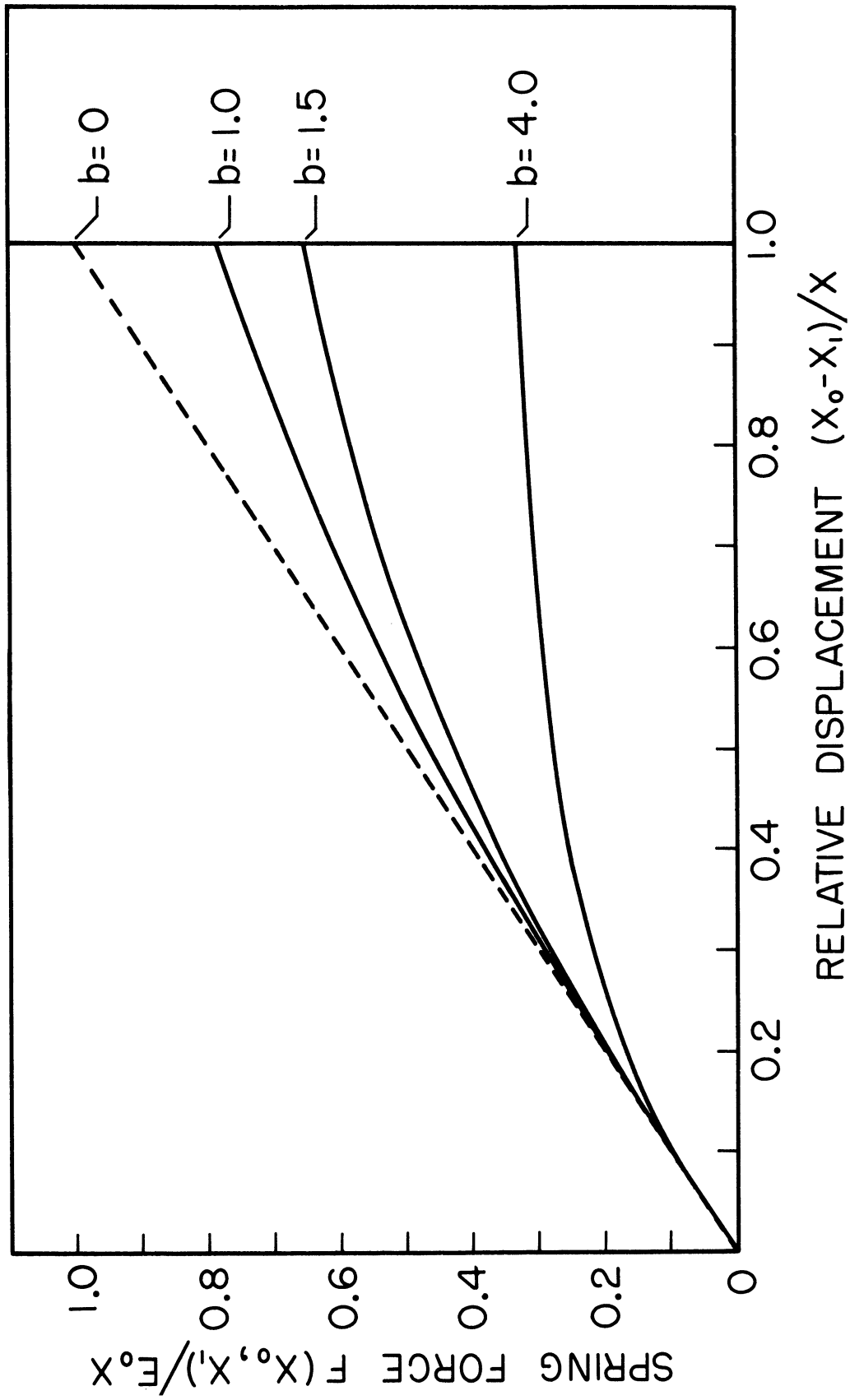


Fig. 12. The restoring force exerted by a mount, which stiffens less rapidly upon compression than a linear mounting, plotted as a function of the relative displacement experienced by the mounting.

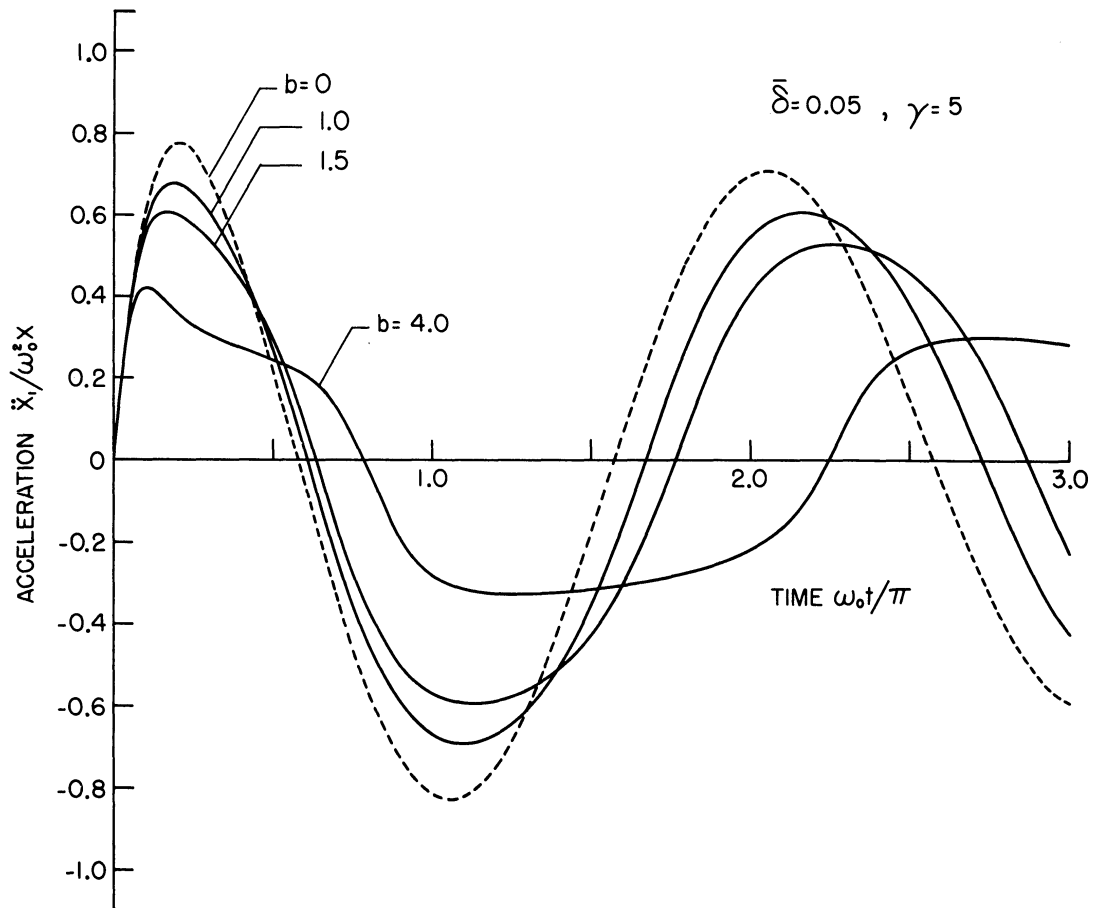
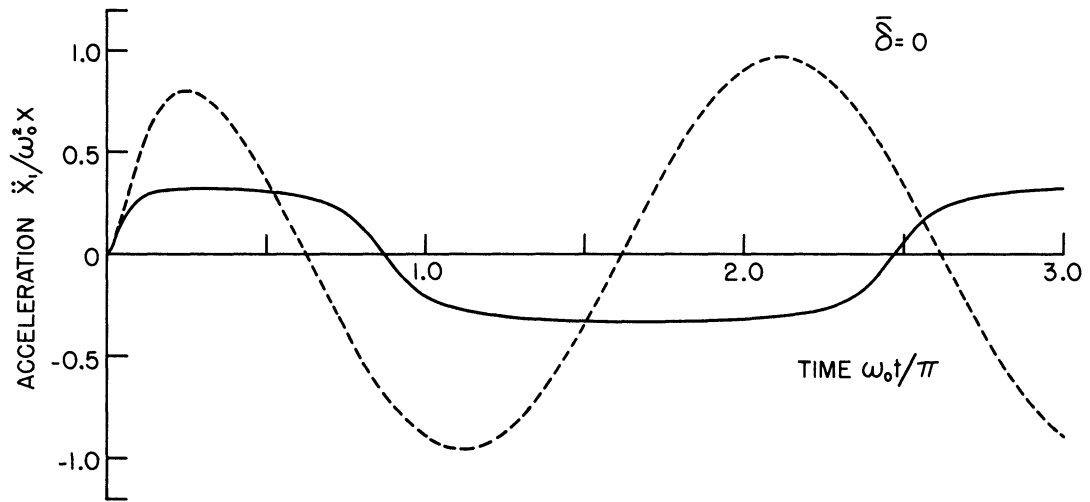
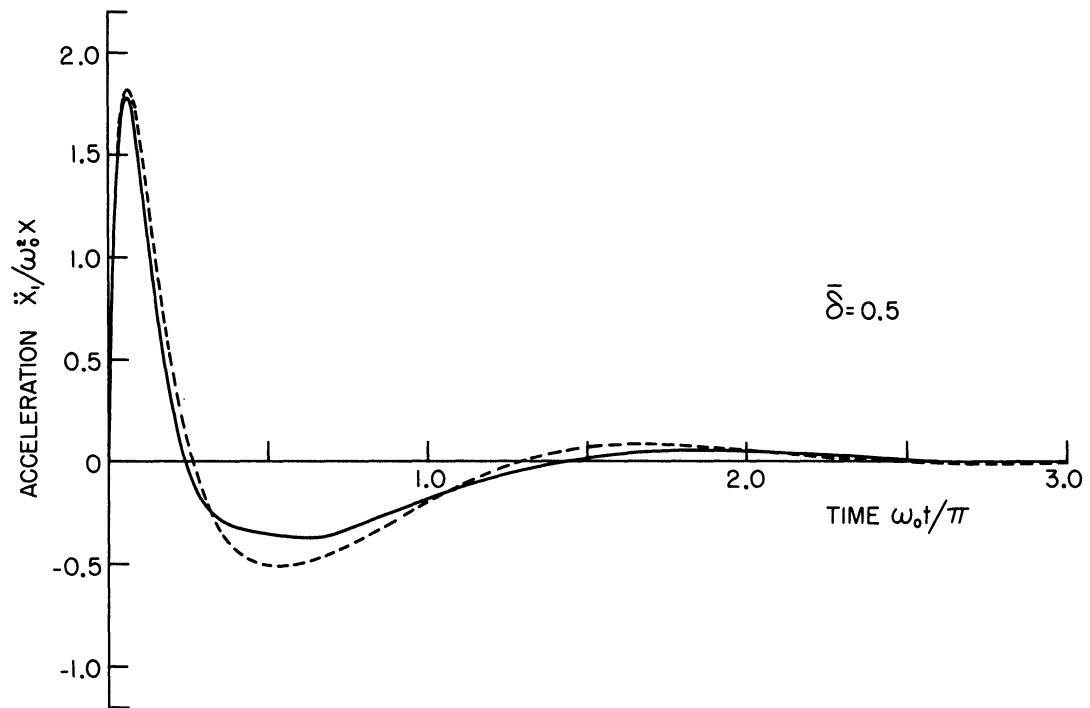


Fig. 13. Acceleration-time relationships for a series of damped non-linear mountings defined by values of $\bar{\delta} = 0.05$ and $b = 1.0, 1.5,$ and 4.0 . The broken curve refers to the performance of a linear mounting. Rise time of the rounded step foundation displacement defined by $\gamma = 5$.

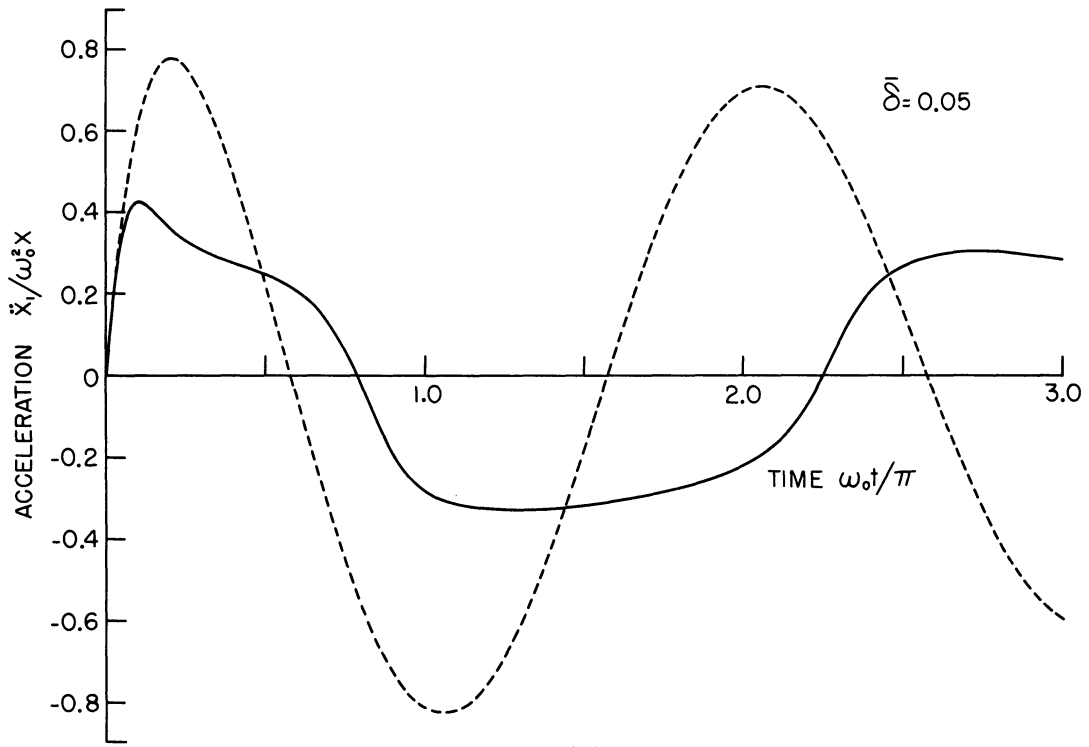


(a)

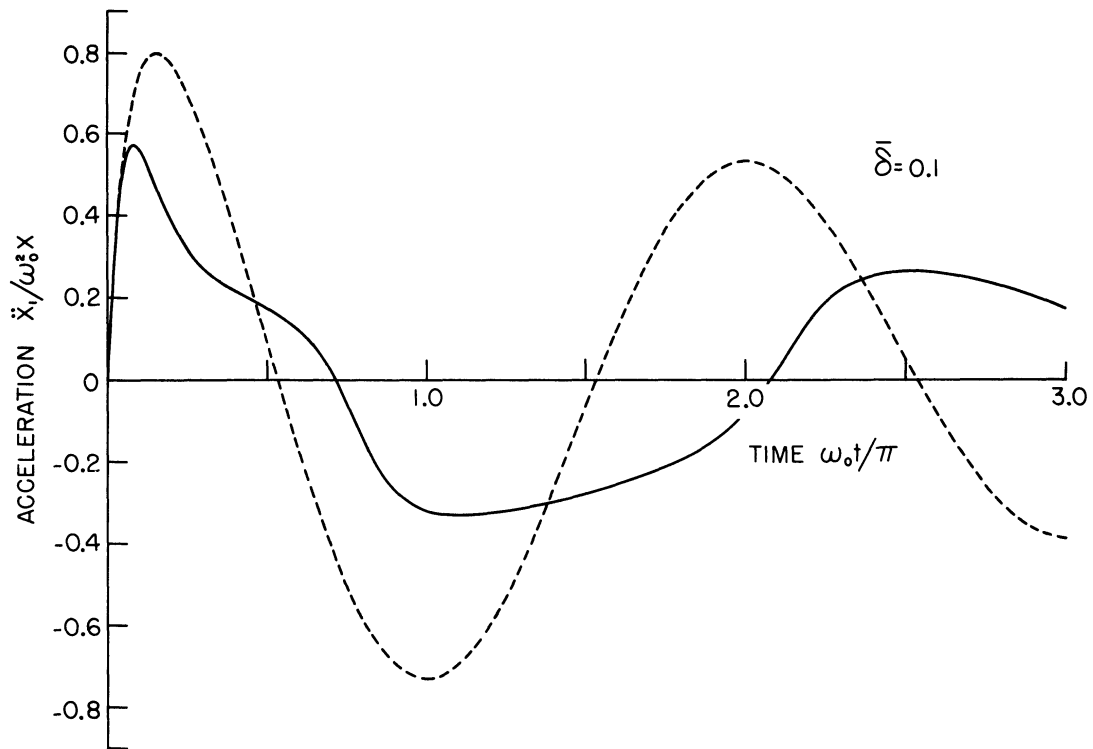


(b)

Fig. 14. Acceleration-time relationships for damped non-linear mountings defined by a value of the parameter $b = 4.0$, and values of the damping ratio (a) $\bar{\delta} = 0$ and (b) $\bar{\delta} = 0.5$. The broken curves refer to the performance of a linear mounting. Rise time of the rounded step foundation displacement defined by $\gamma = 5$.



(a)



(b)

Fig. 15. Acceleration-time relationships for damped non-linear mountings defined by a value of the parameter $b = 4.0$, and values of the damping ratio (a) $\bar{\delta} = 0.05$ and (b) $\bar{\delta} = 0.1$. The broken curves refer to the performance of a linear mounting. Rise time of the rounded step foundation displacement defined by $\gamma = 5$.

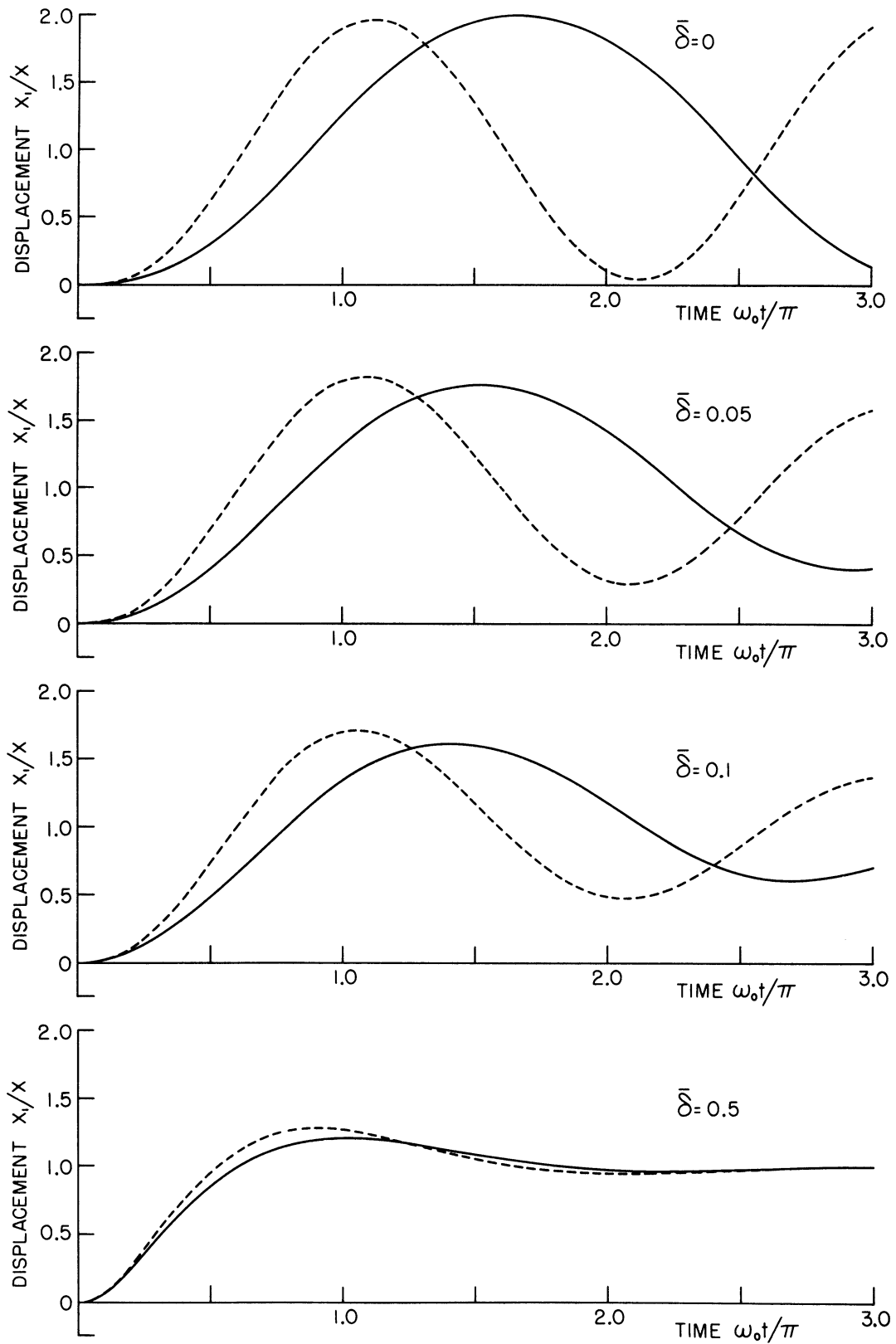


Fig. 16. Displacement-time relationships for non-linear mountings defined by a value of the parameter $b = 4.0$, and values of the damping ratio $\bar{\delta} = 0, 0.05, 0.1, \text{ and } 0.5$. The broken curves refer to the performance of a linear mounting. Rise time of the rounded step foundation displacement defined by $\gamma = 5$.

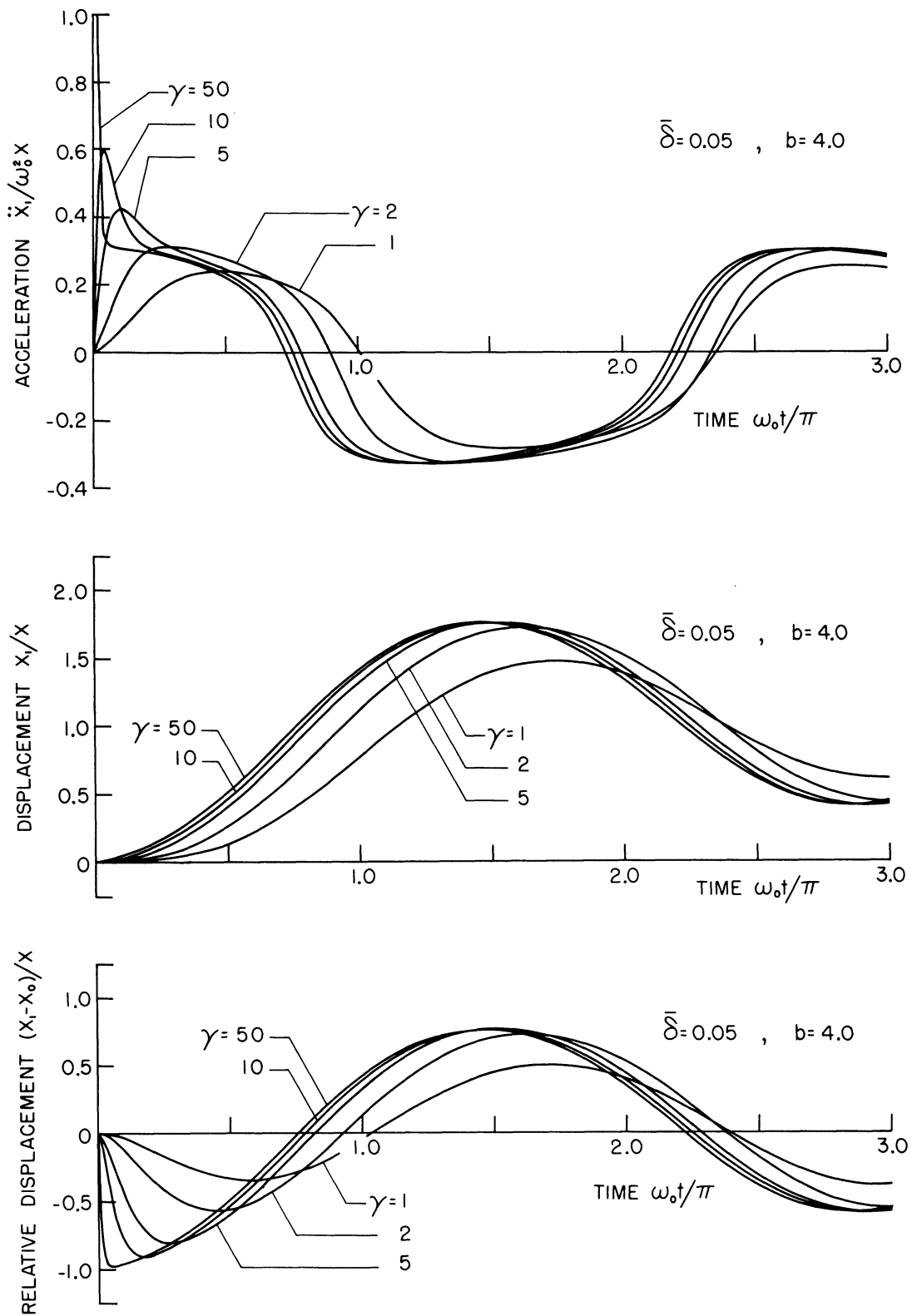


Fig. 17. Acceleration-, displacement-, and relative displacement-time relationships for a non-linear mounting defined by a value of the parameter $b = 4.0$. Mount damping ratio $\bar{\delta} = 0.05$.

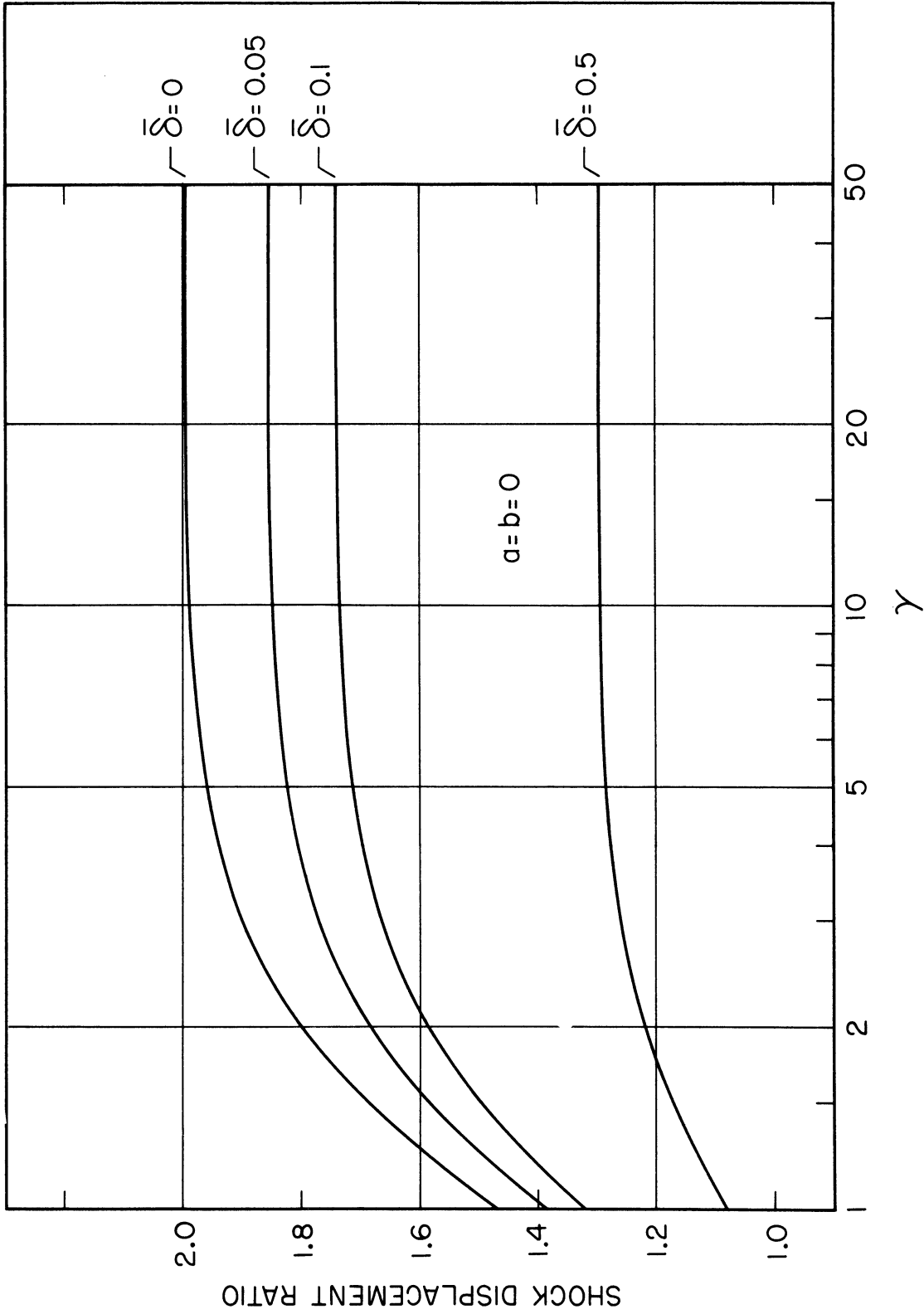


Fig. 18. The shock displacement ratio for a number of linear mountings plotted as a function of the parameter γ .

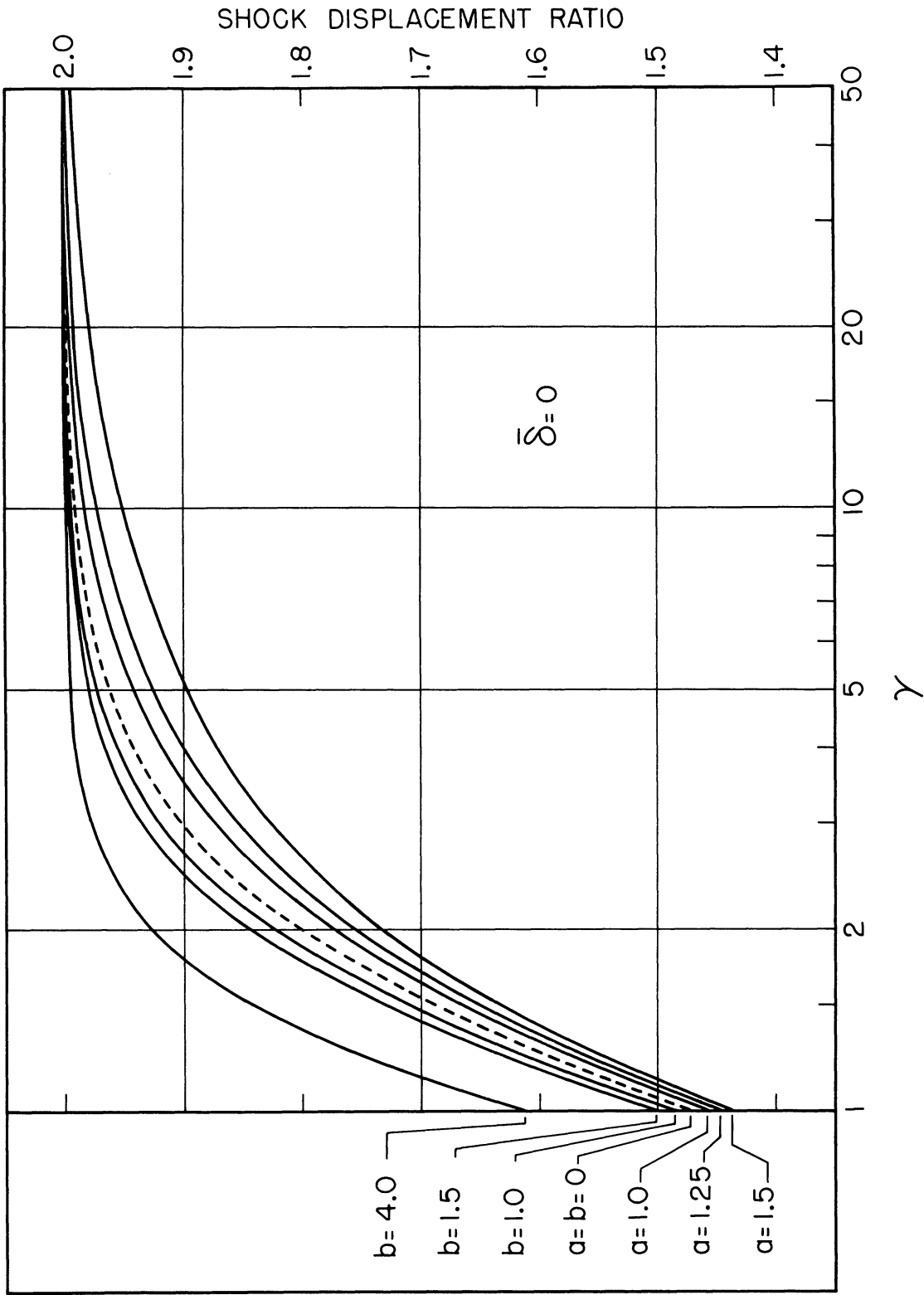


Fig. 19. The shock displacement ratio for a series of undamped non-linear mountings plotted as a function of the parameter γ .

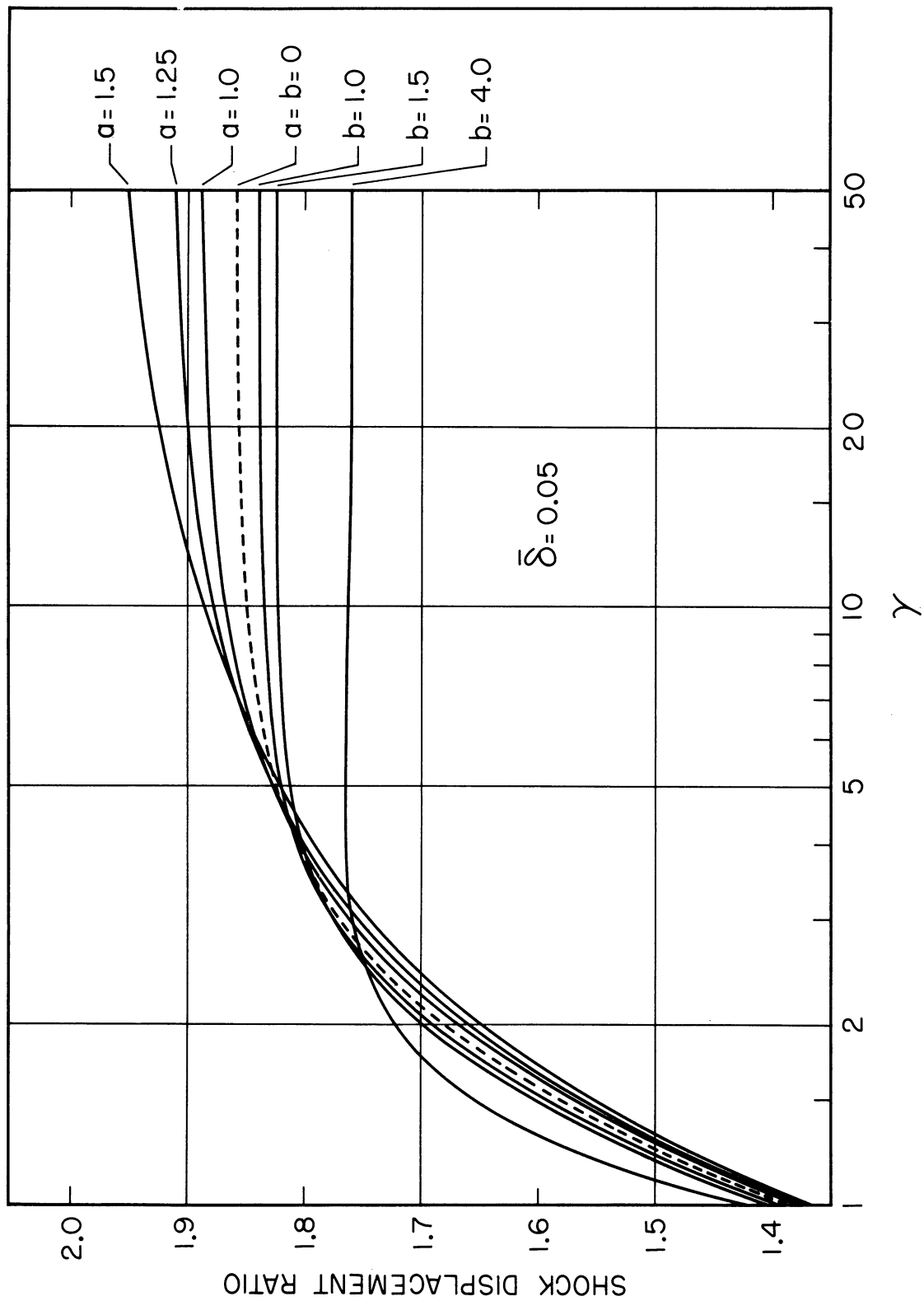


Fig. 20. The shock displacement ratio for a series of non-linear mountings plotted as a function of the parameter γ . Mount damping ratios $\bar{\delta} = 0.05$.

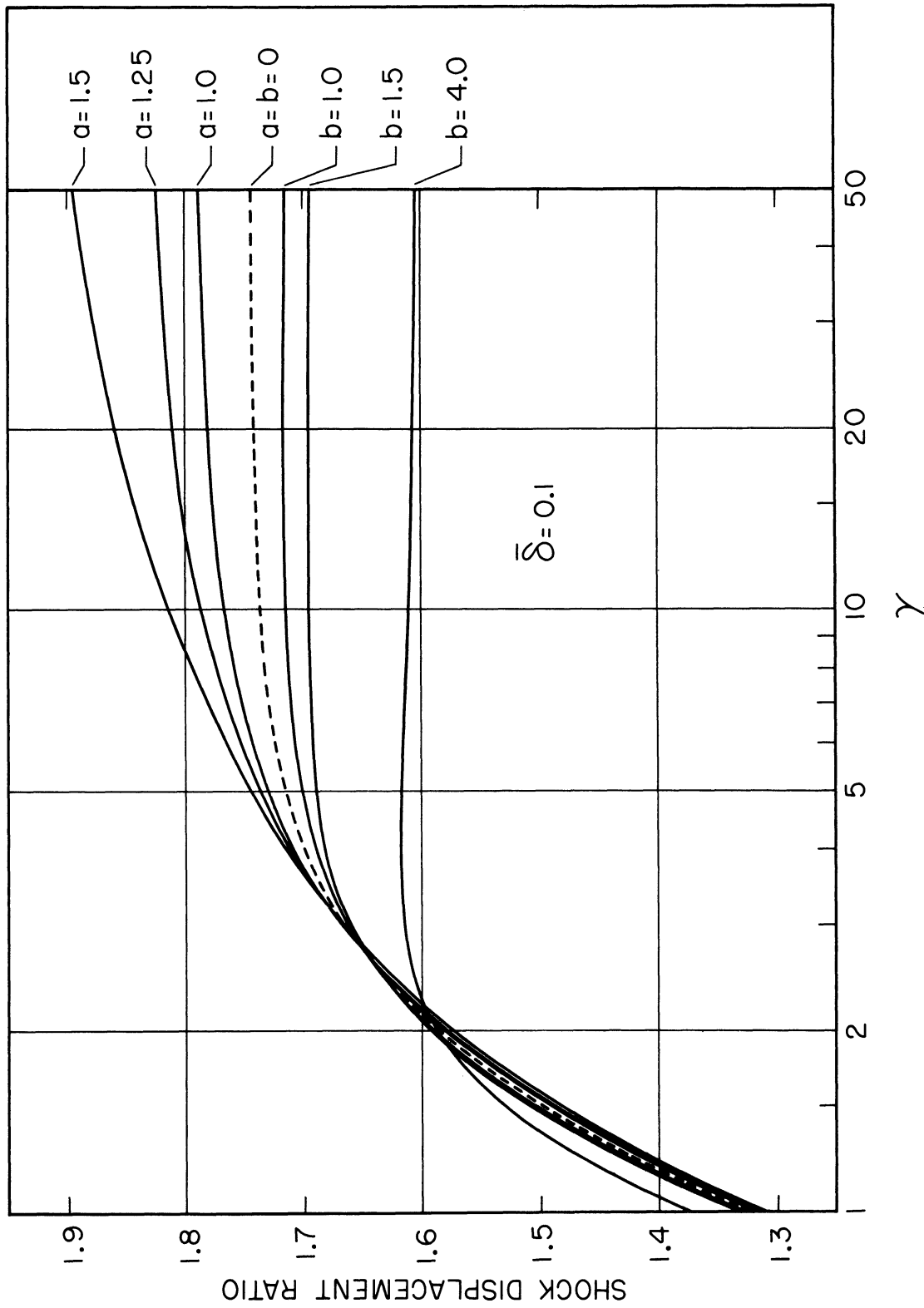


Fig. 21. The shock displacement ratio for a series of non-linear mountings plotted as a function of the parameter γ . Mount damping ratios $\bar{\delta} = 0.1$.

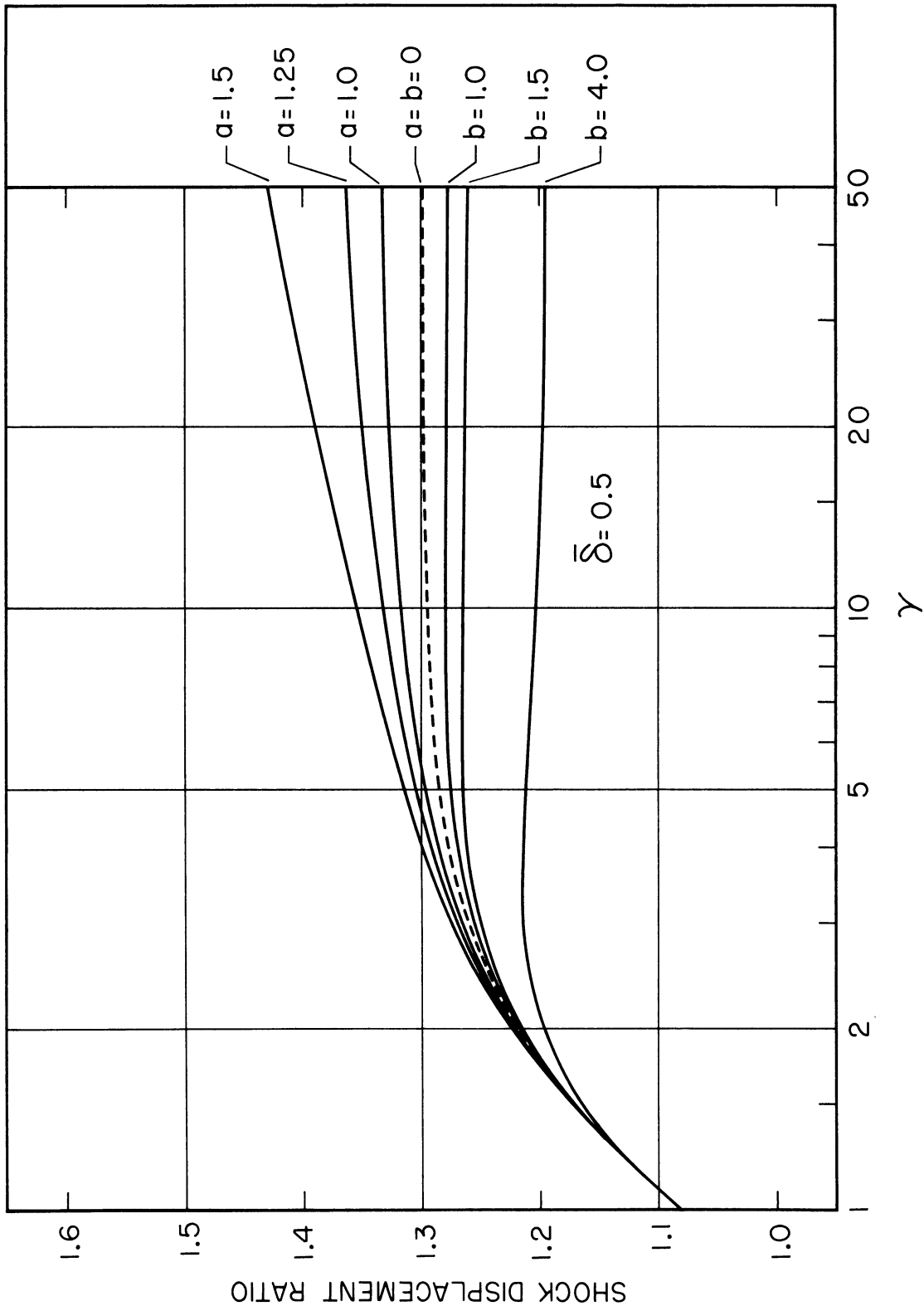


Fig. 22. The shock displacement ratio for a series of non-linear mountings plotted as a function of the parameter γ . Mount damping ratios $\bar{\delta} = 0.5$.

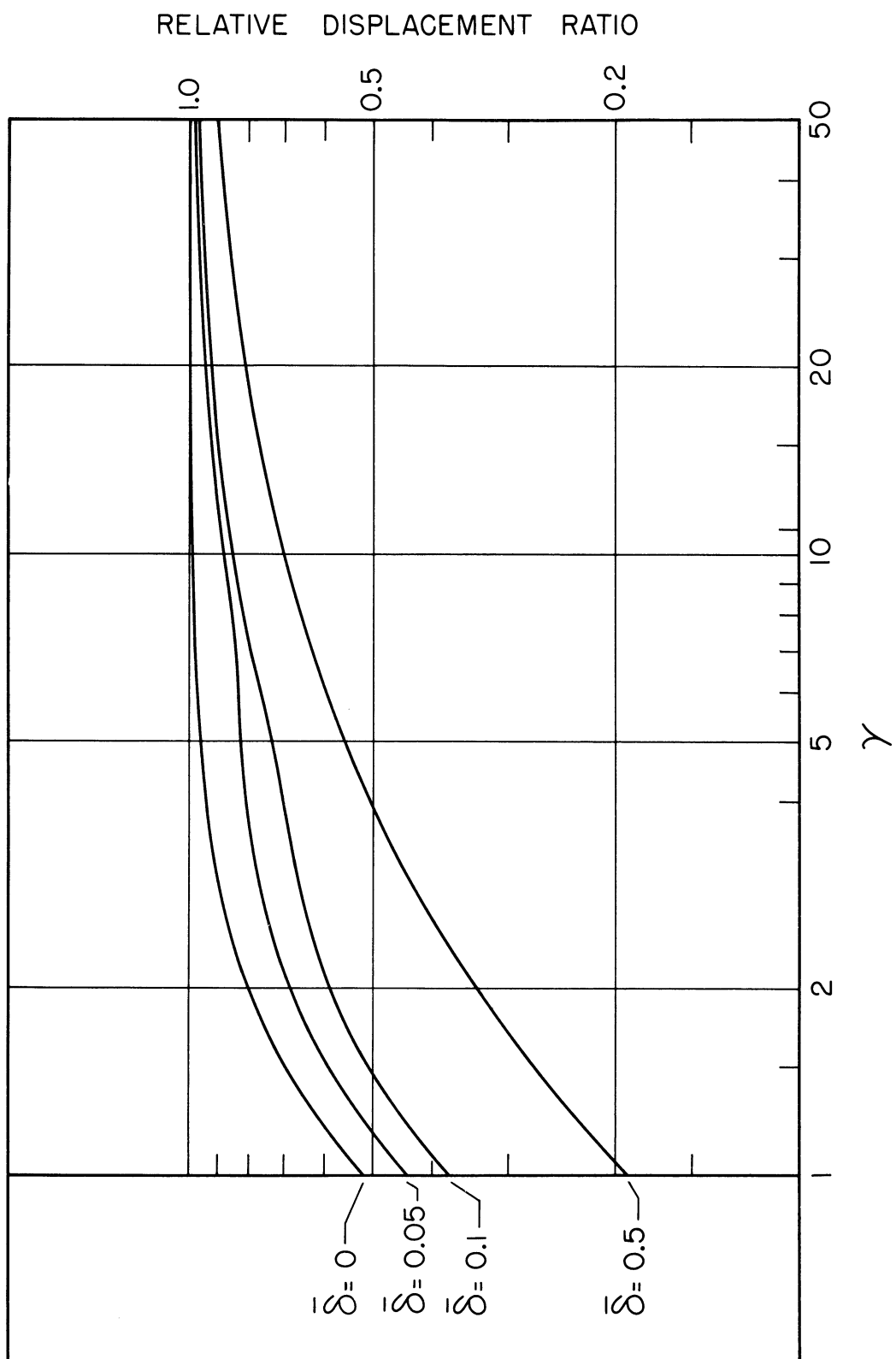


Fig. 23. The relative displacement ratio for a number of linear mountings plotted as a function of the parameter γ .

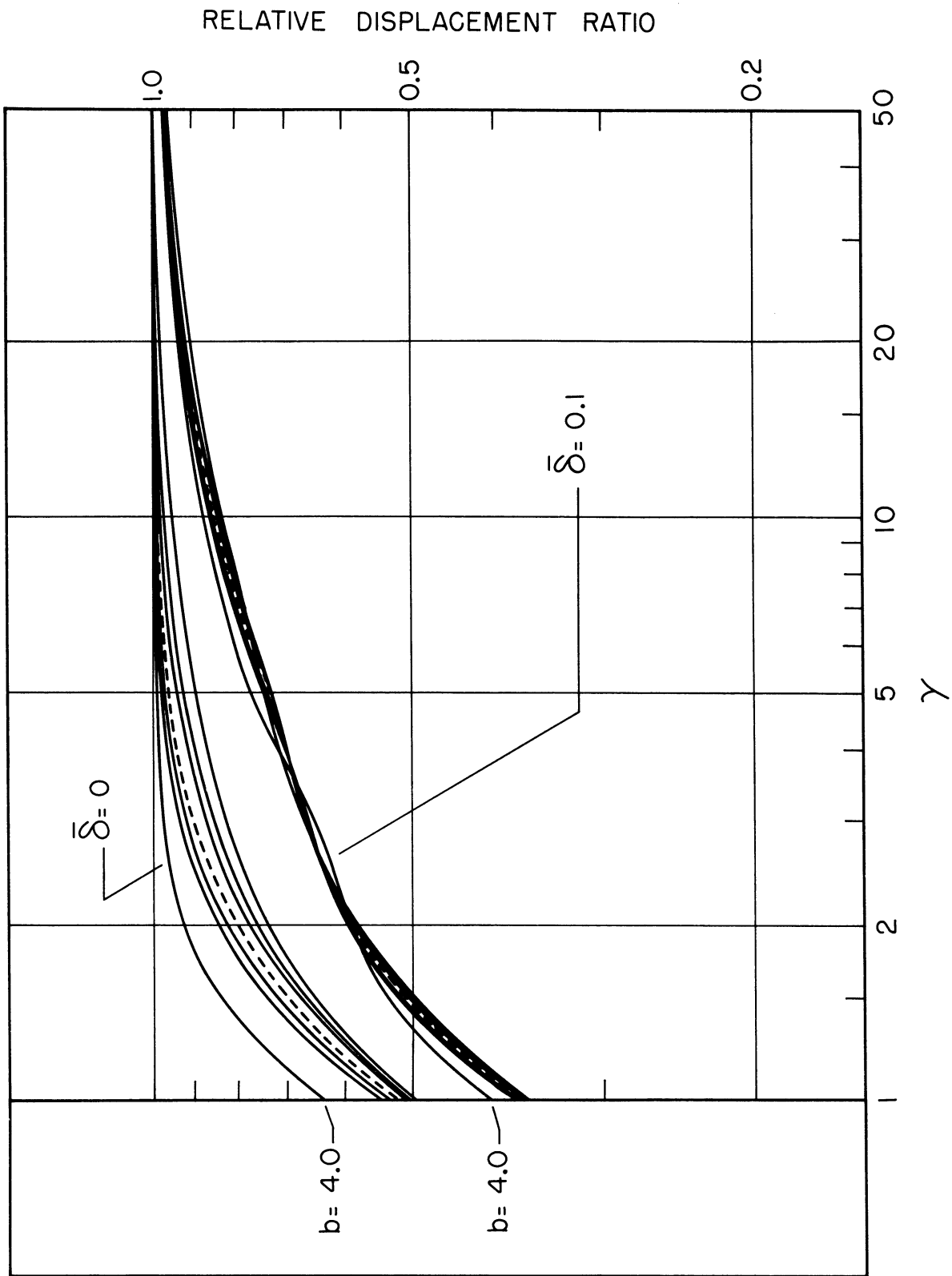


Fig. 24. The relative displacement ratio for a series of non-linear mountings plotted as a function of the parameter γ . Mount damping ratios $\bar{\delta} = 0$ and $\bar{\delta} = 0.1$.

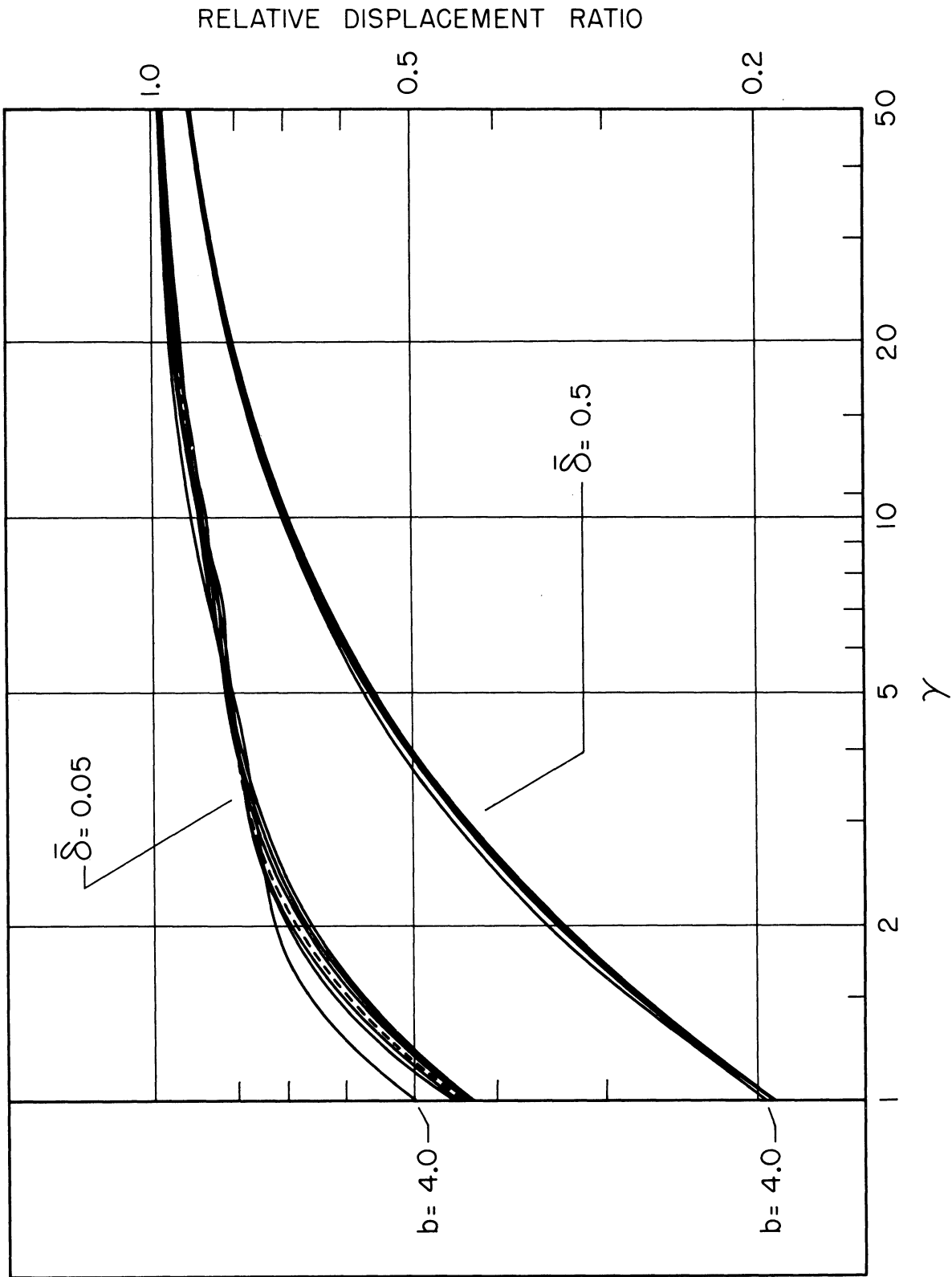


Fig. 25. The relative displacement ratio for a series of non-linear mountings plotted as a function of the parameter γ . Mount damping ratios $\bar{\delta} = 0.05$ and 0.5.

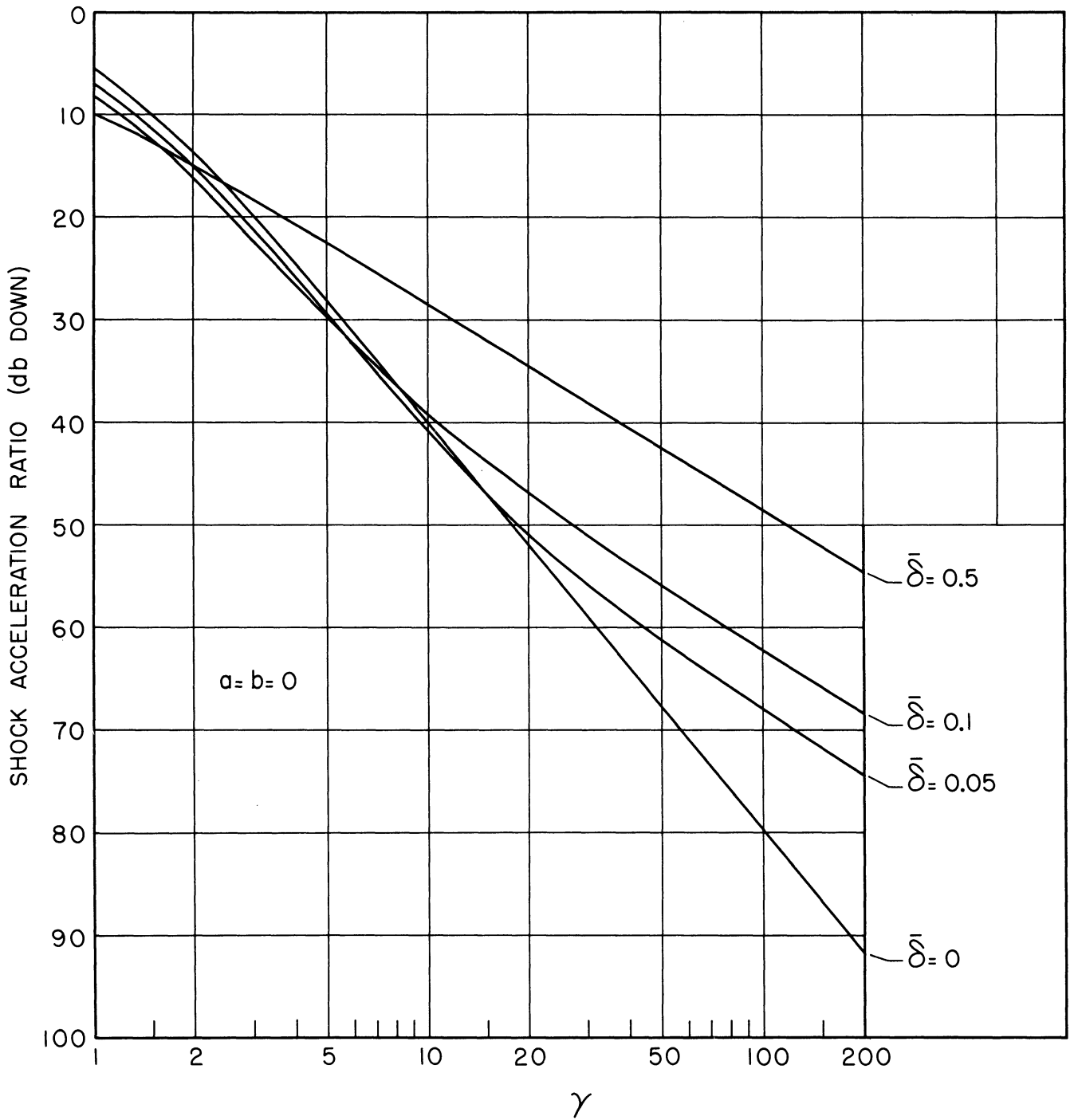


Fig. 26. The shock acceleration ratio for a number of linear mountings plotted as a function of the parameter γ .

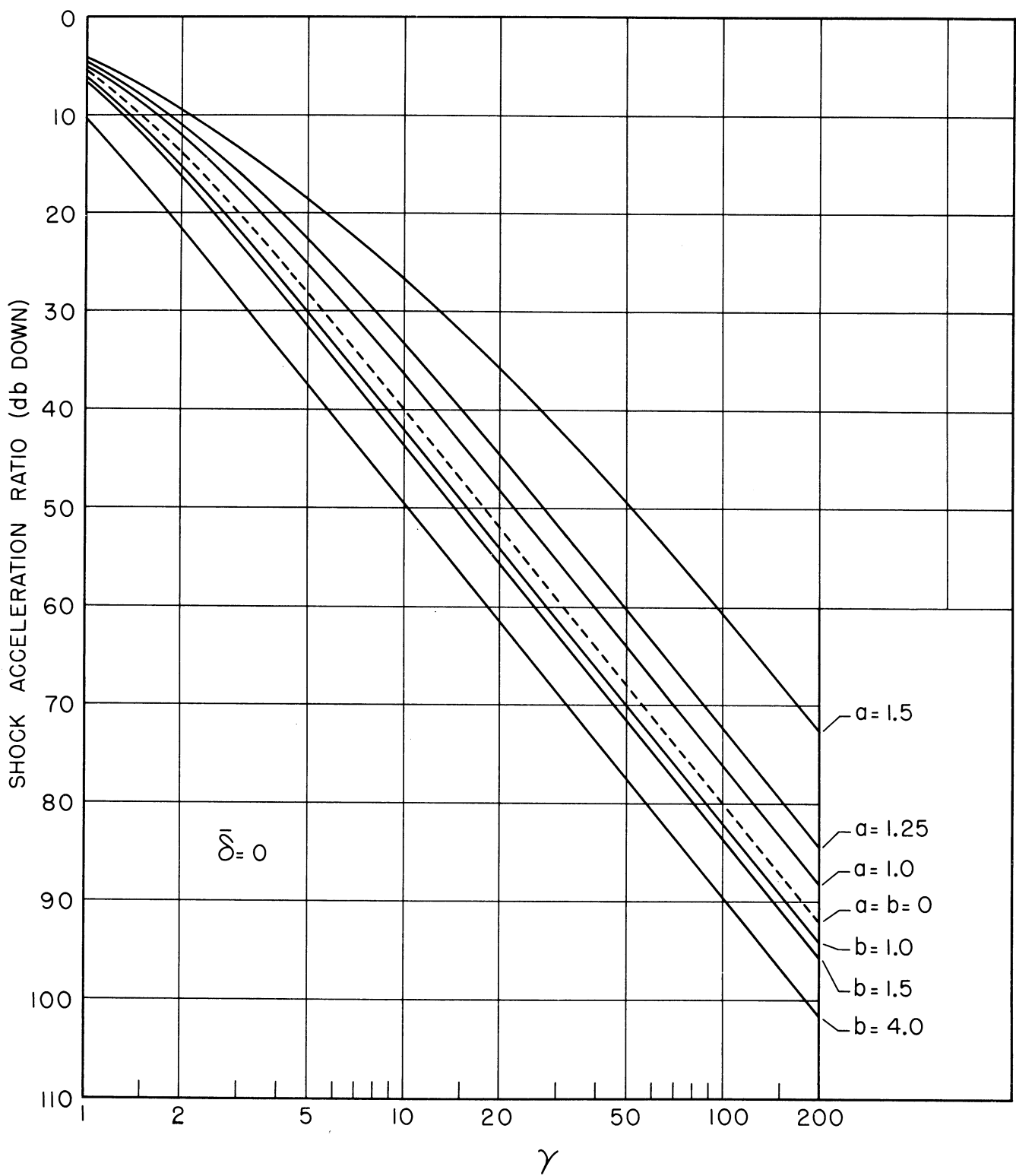


Fig. 27. The shock acceleration ratio for a series of undamped non-linear mountings plotted as a function of the parameter γ .

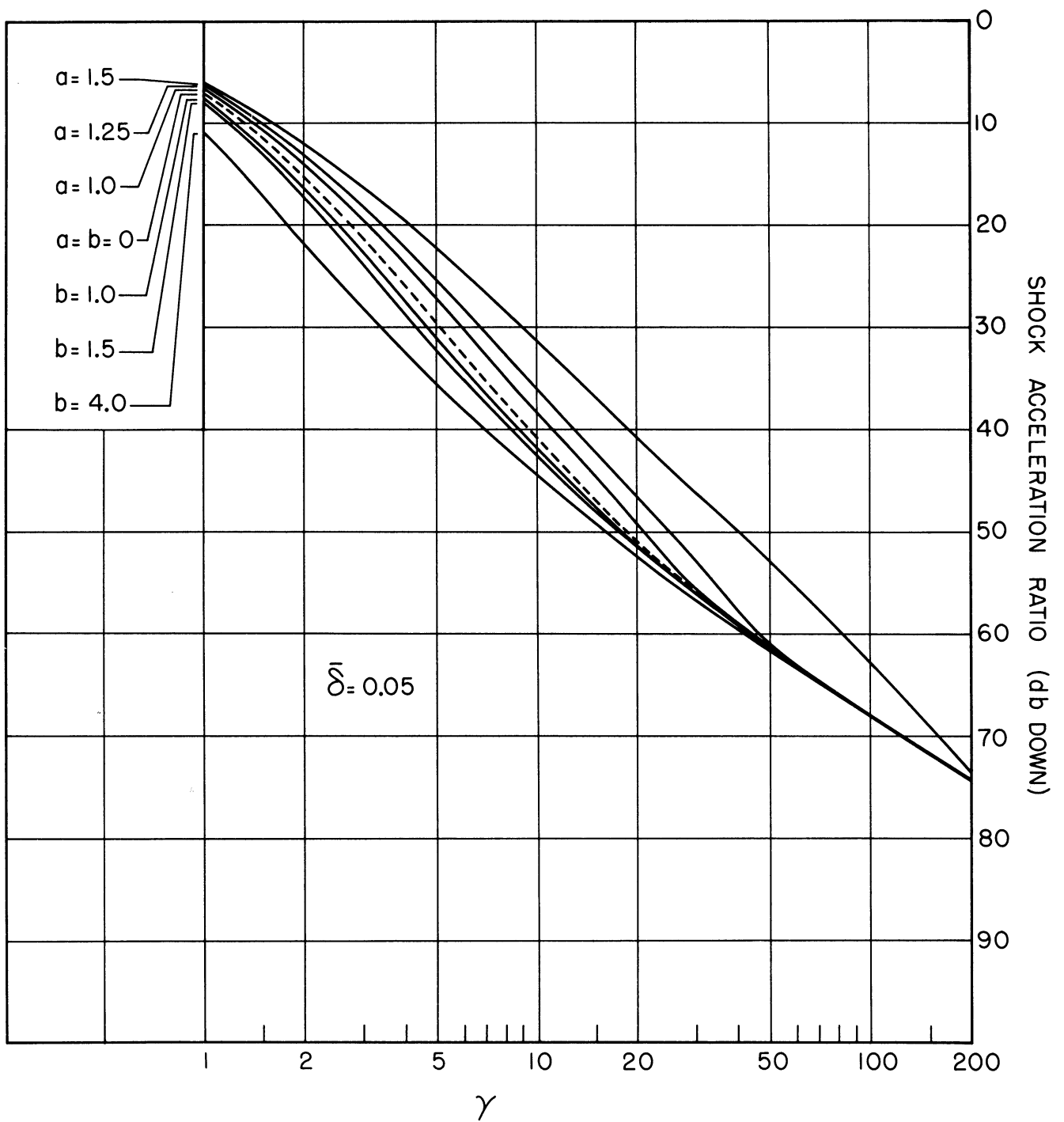


Fig. 28. The shock acceleration ratio for a series of non-linear mountings plotted as a function of the parameter γ . Mount damping ratios $\bar{\delta} = 0.05$.

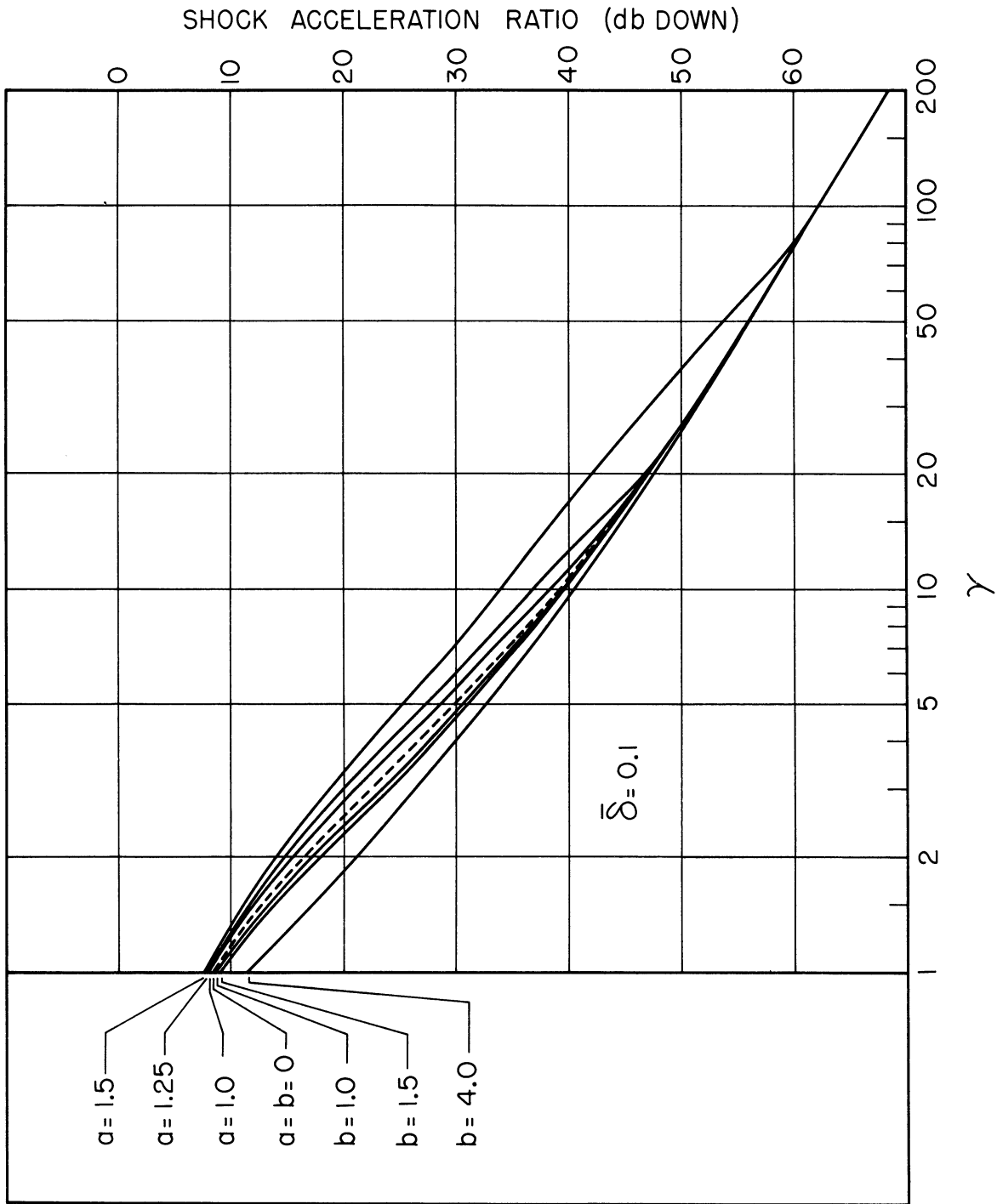


Fig. 29. The shock acceleration ratio for a series of non-linear mountings plotted as a function of the parameter γ . Mount damping ratios $\bar{\delta} = 0.1$.

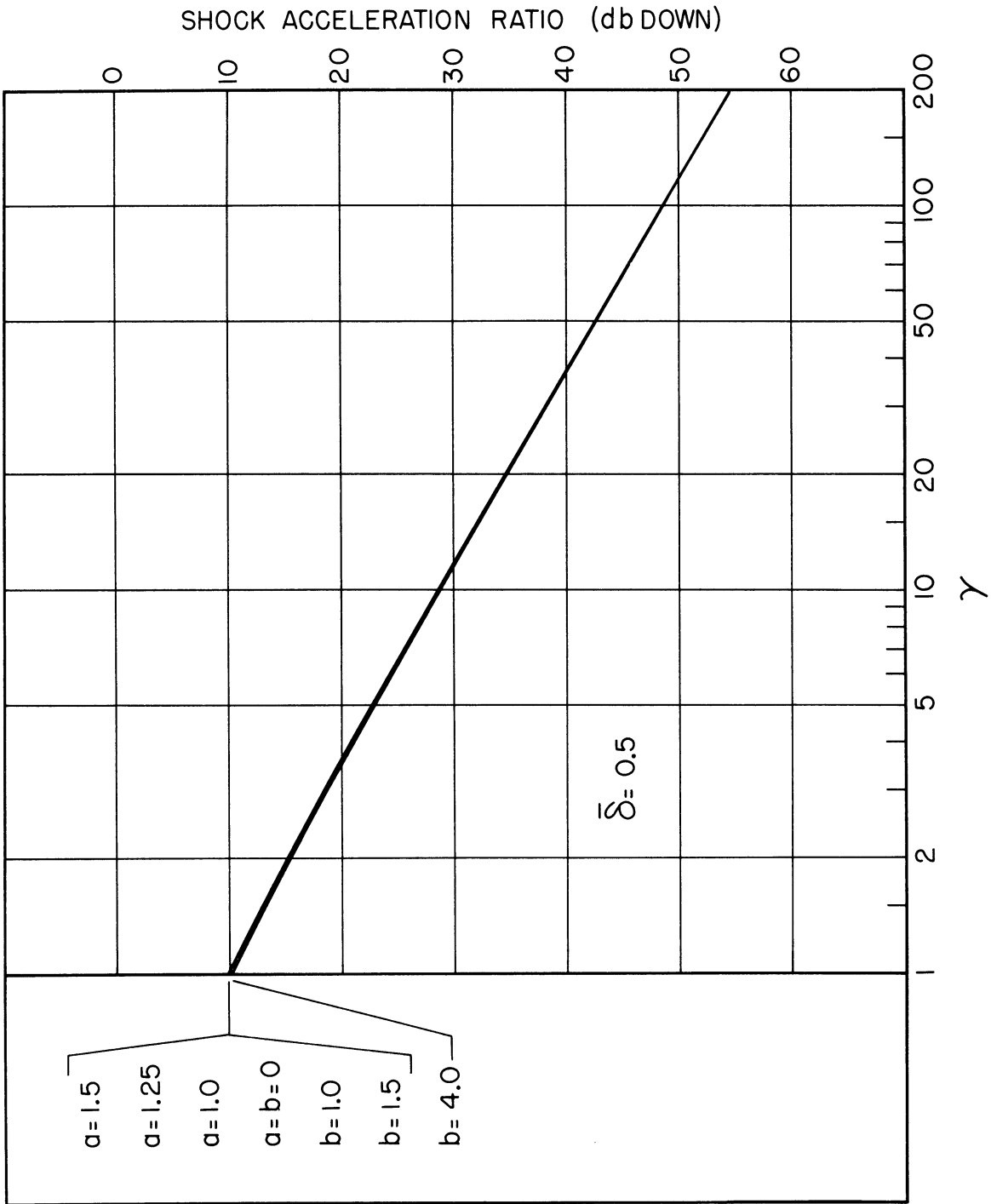


Fig. 30. The shock acceleration ratio for a series of non-linear mountings plotted as a function of the parameter γ . Mount damping ratios $\bar{\delta} = 0.5$.

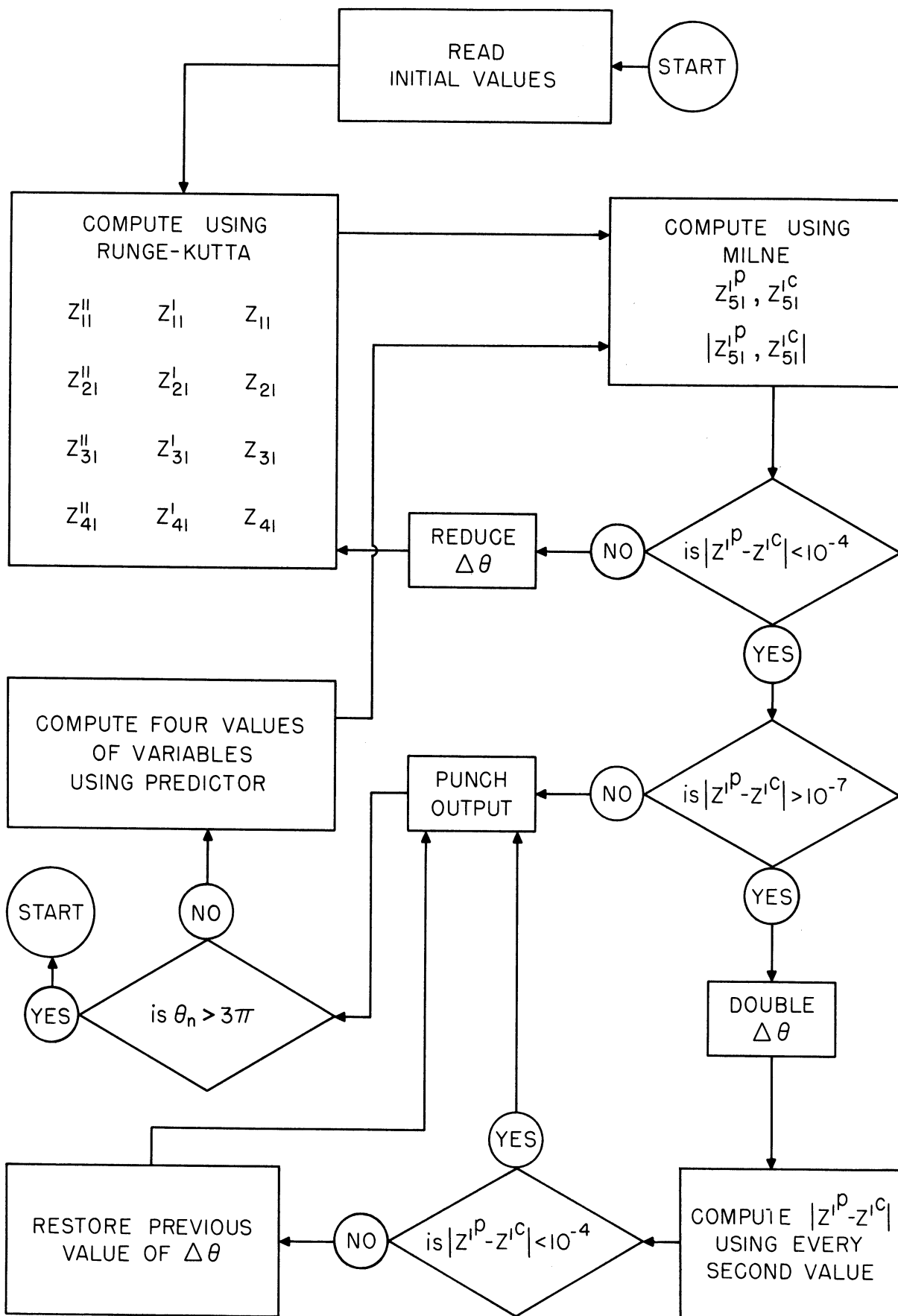


Fig. 31. Decision chart for the digital computer program.

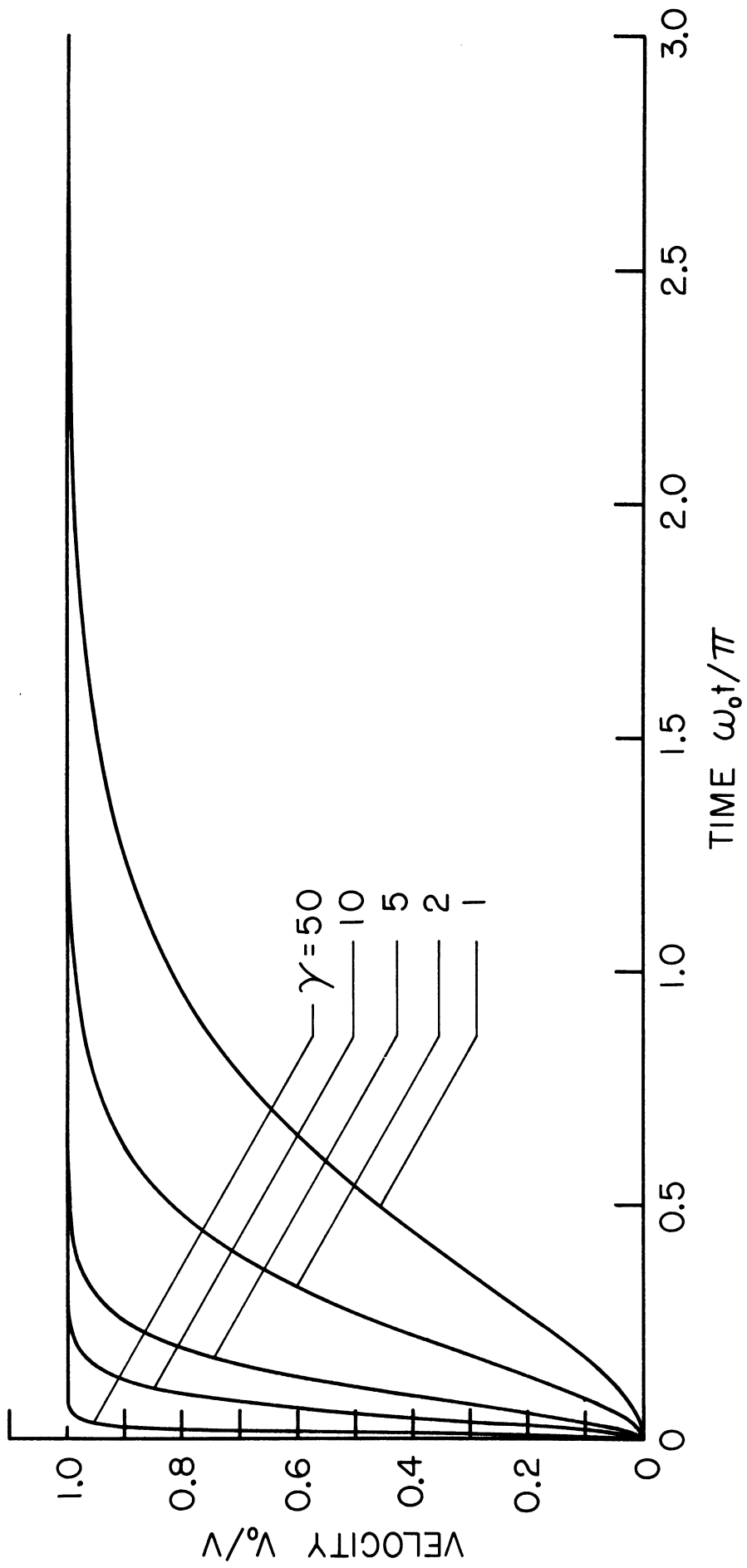


Fig. 32. The rounded step foundation velocity.

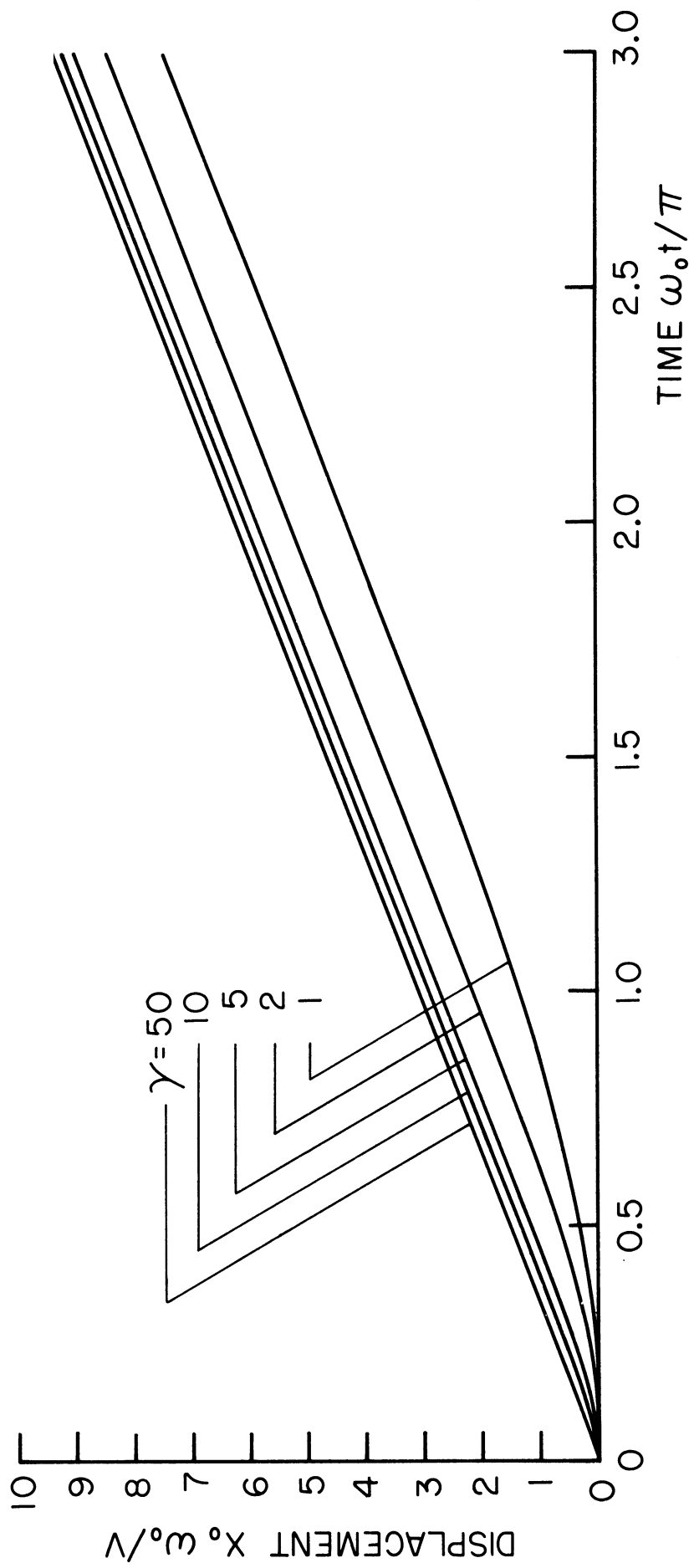


Fig. 33. The ramp-like foundation displacement.

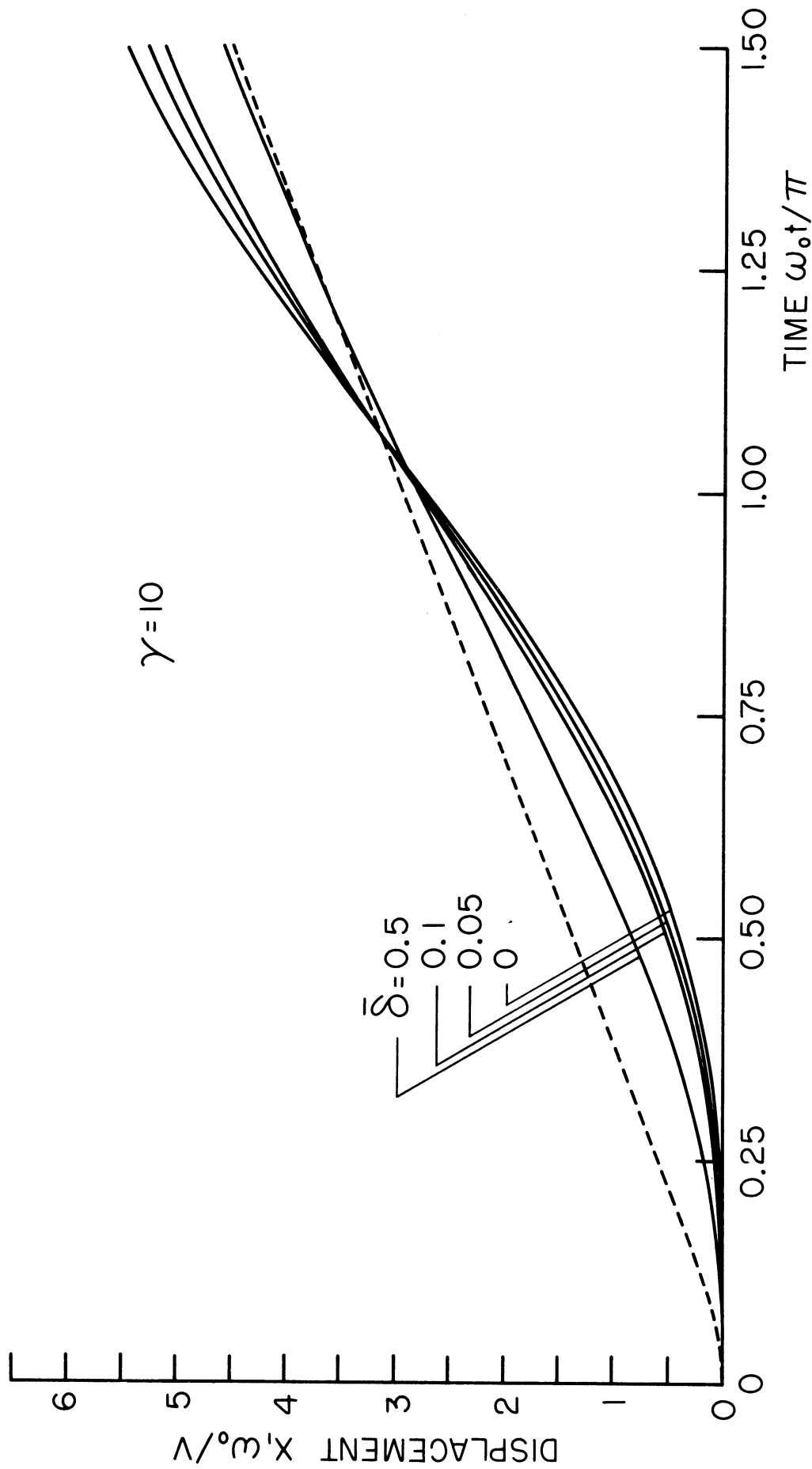


Fig. 34. Displacement-time relationships for linear mountings defined by values of $\bar{\delta} = 0, 0.05, 0.1, \text{ and } 0.5$. Rise time of the rounded step foundation velocity defined by $\gamma = 10$.

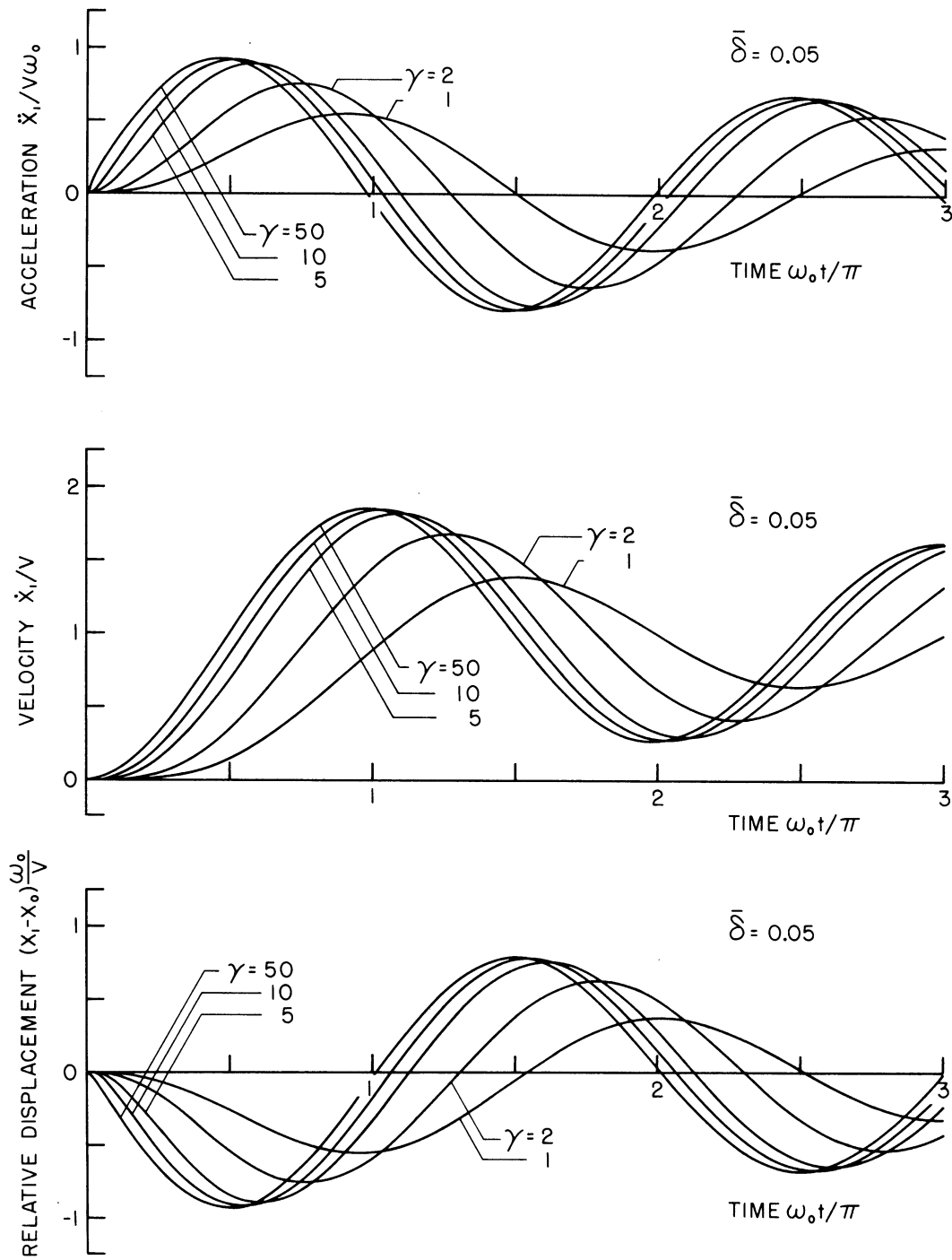


Fig. 35. Acceleration-, velocity-, and relative displacement-time relationships for a linear mounting. Mount damping ratio $\bar{\delta} = 0.05$.

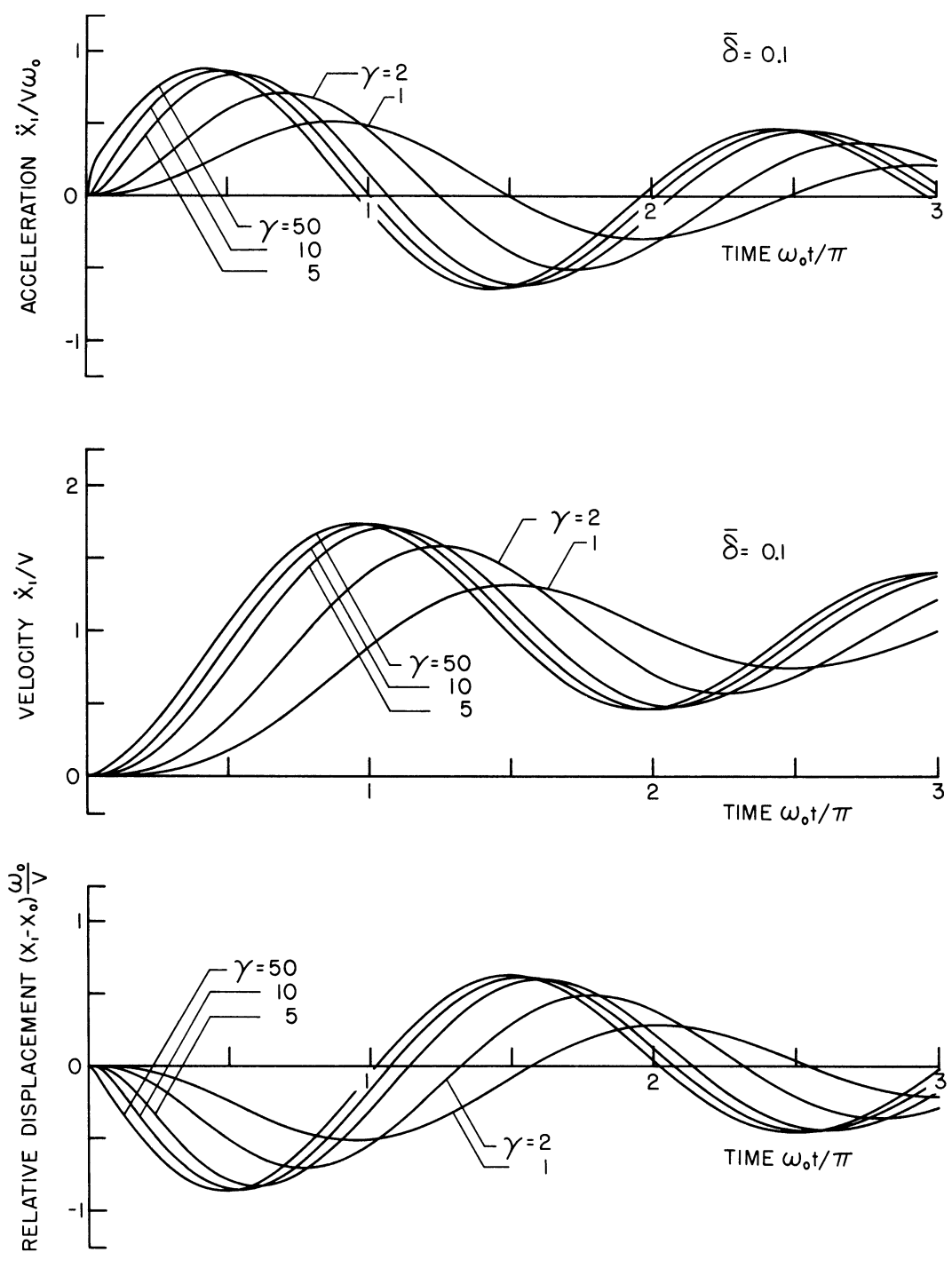


Fig. 36. Acceleration-, velocity-, and relative displacement-time relationships for a linear mounting. Mount damping ratio $\bar{\delta} = 0.1$.

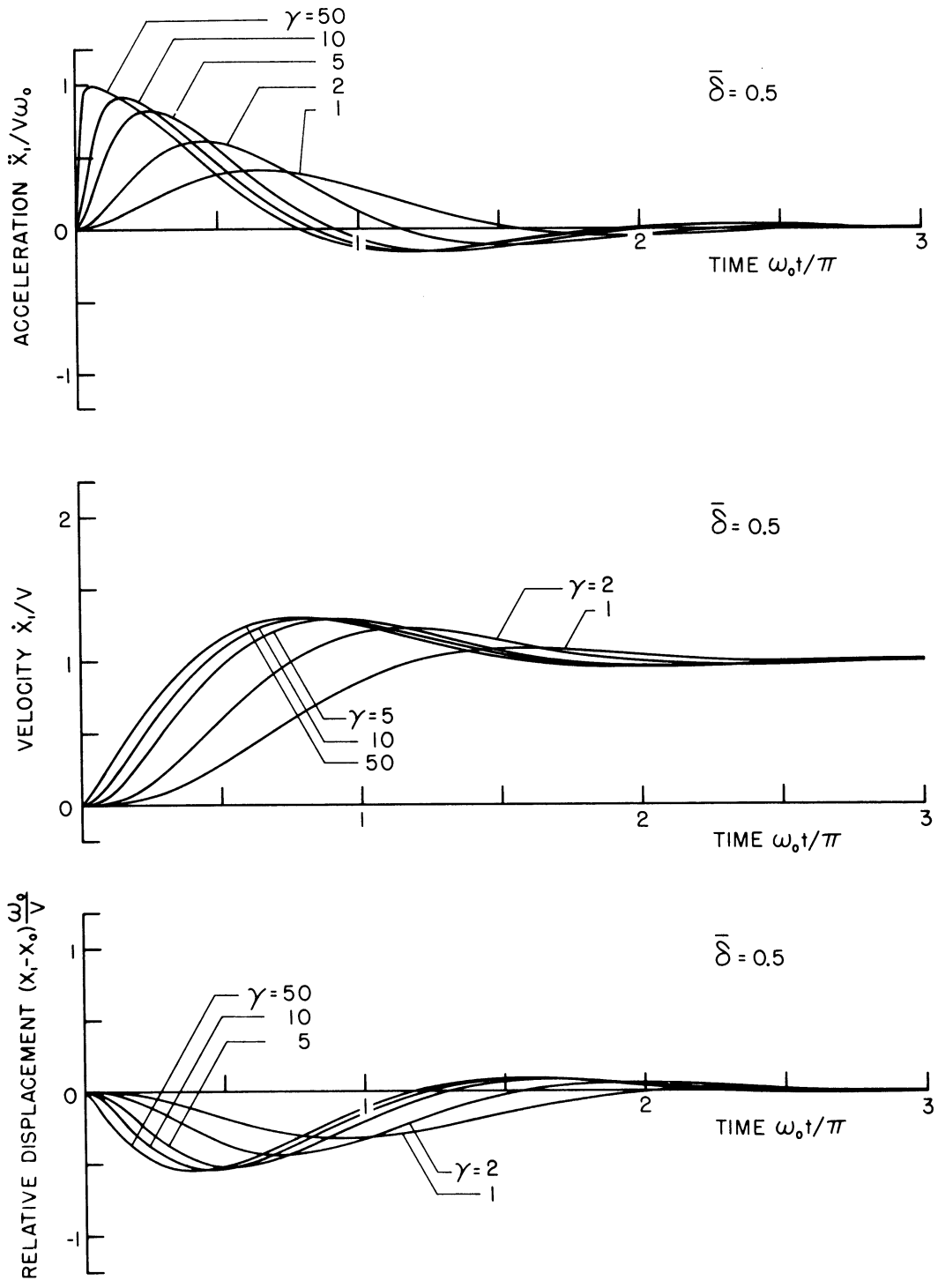


Fig. 37. Acceleration-, velocity-, and relative displacement-time relationships for a linear mounting. Mount damping ratio $\bar{\delta} = 0.5$.

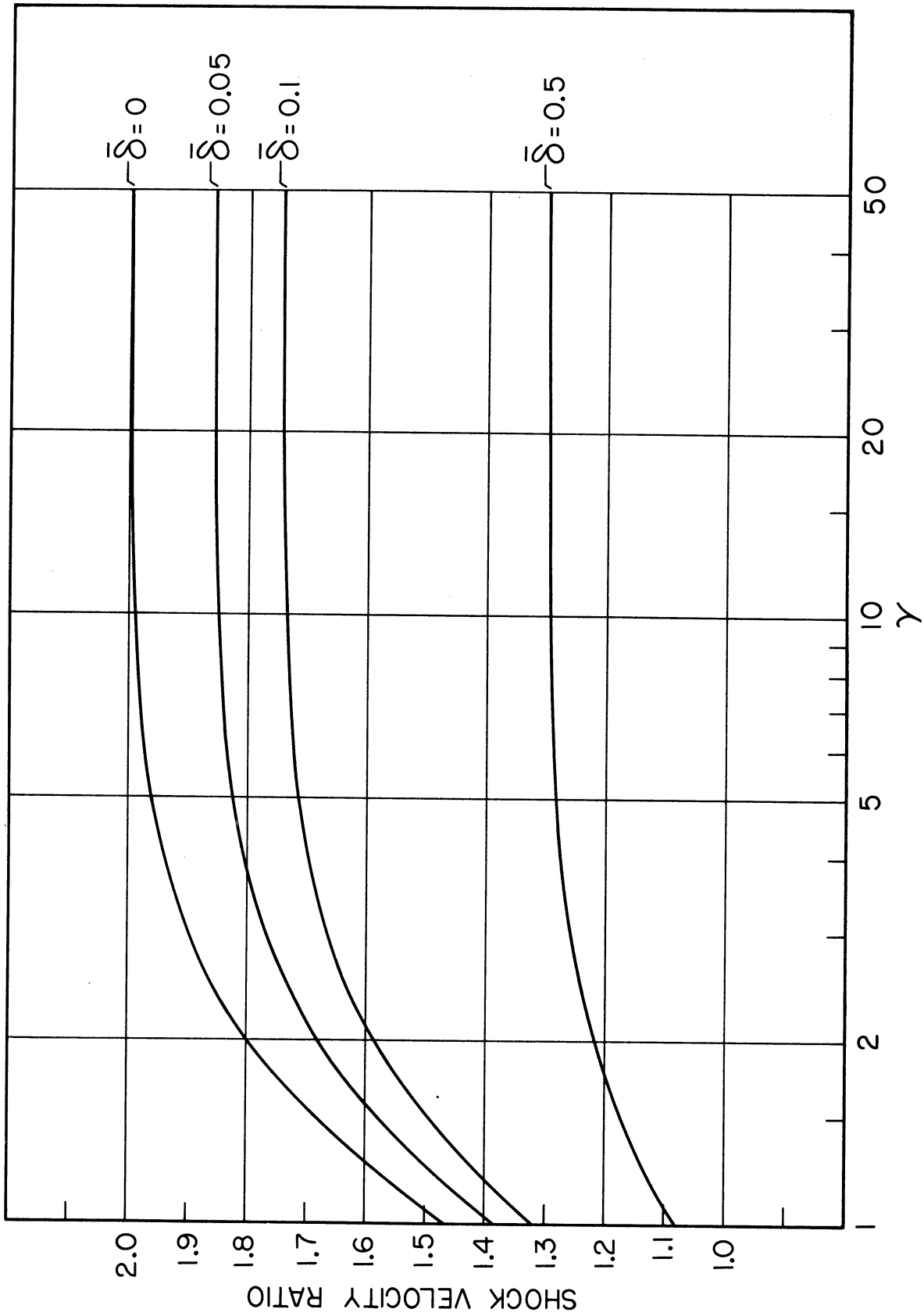


Fig. 38. The shock velocity ratio for a number of linear mountings plotted as a function of the parameter γ .

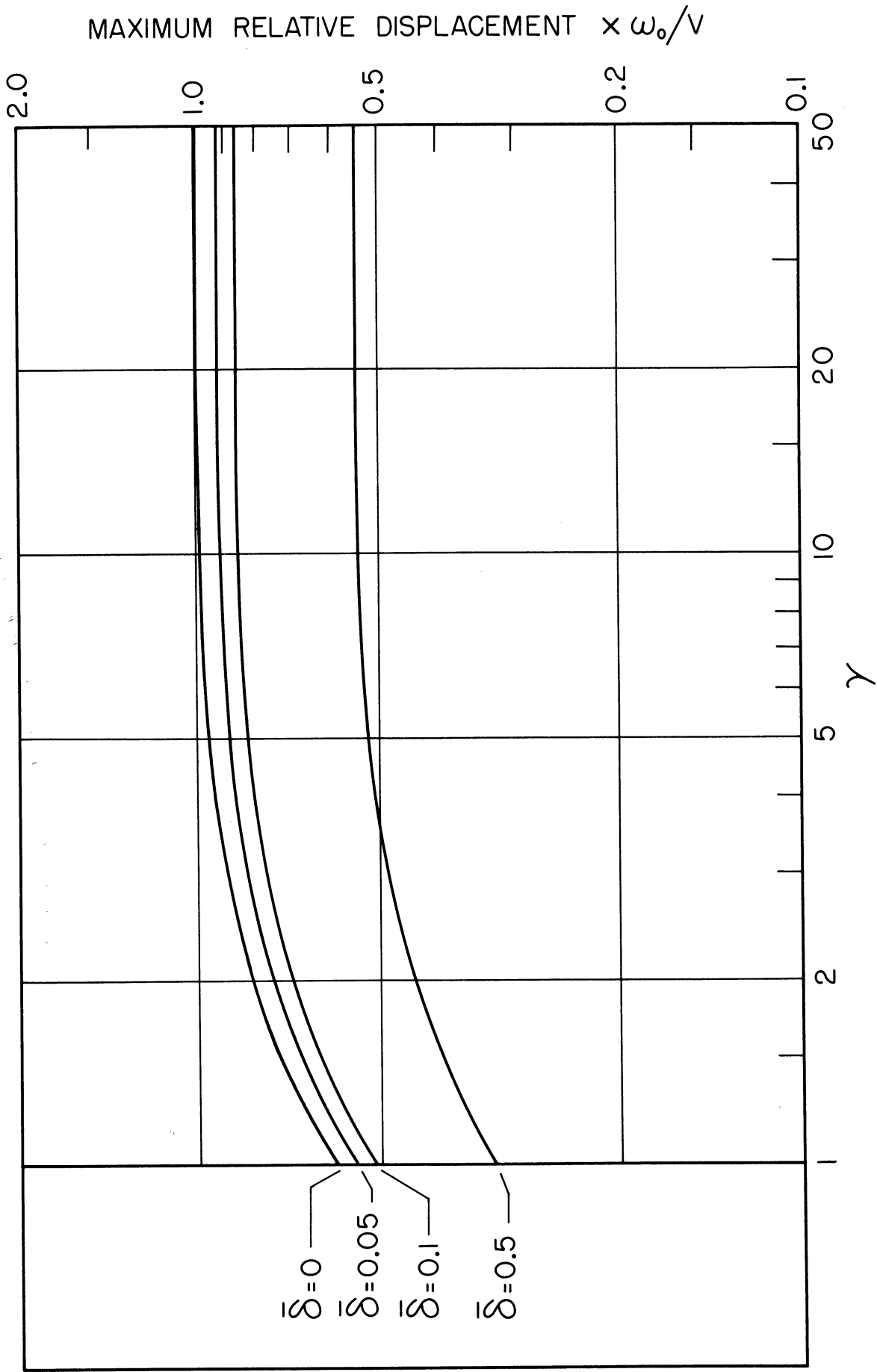


Fig. 39. The maximum relative displacement experienced by a number of linear mountings plotted as a function of the parameter γ .

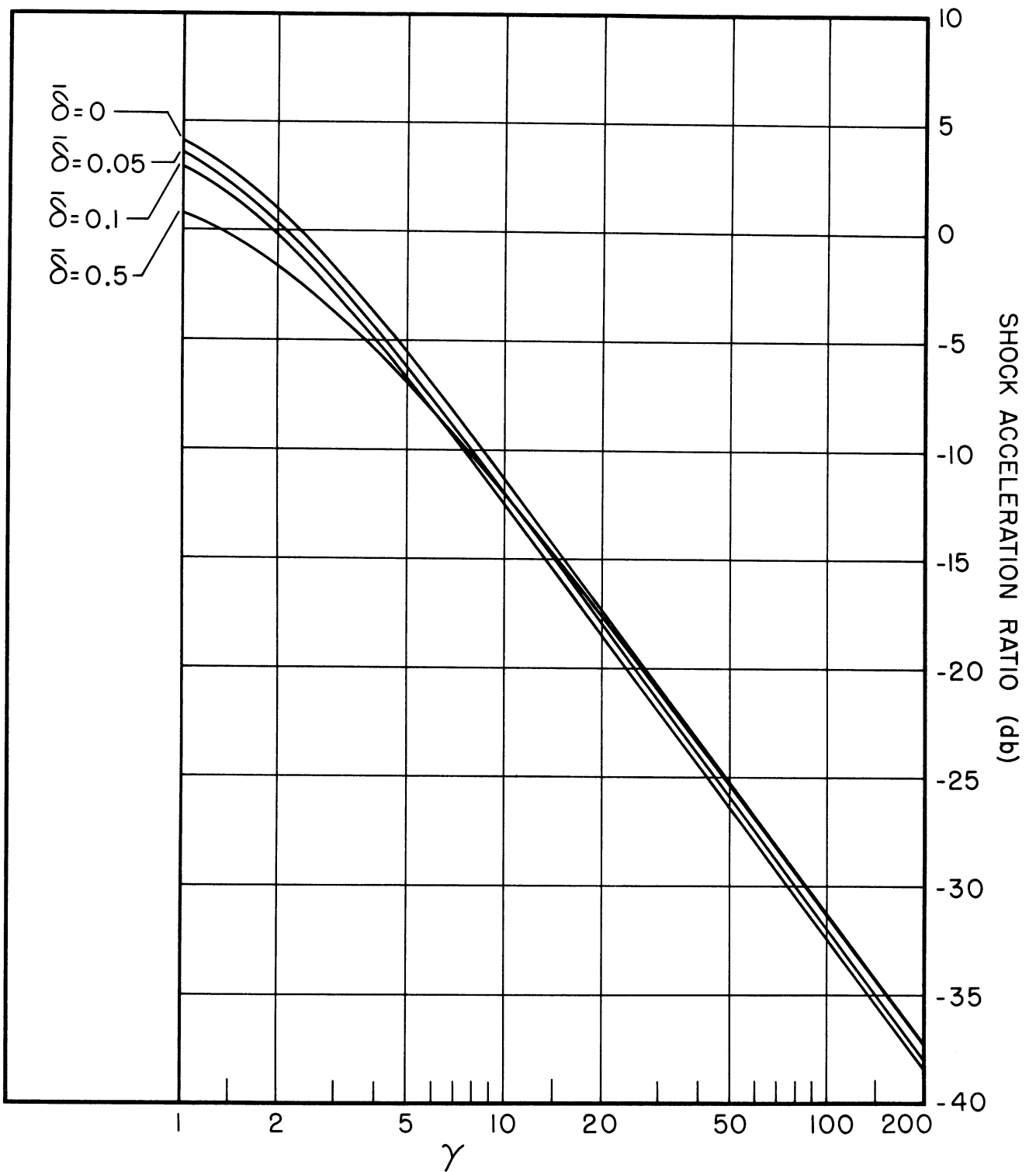


Fig. 40. The shock acceleration ratio for a number of linear mountings plotted as a function of the parameter γ .

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- Damping
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