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REDUCTION of the RESPONSE to VIBRATION of STRUCTURES POSSESSING FINITE MECHANICAL IMPEDANCE

Part II

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ABSTRACT

The performance of simple and compound mounting systems supported by a foundation of finite mechanical impedance, and the performance of the simple mounting supported by a mass-loaded foundation, have been theoretically determined and compared. A simply supported damped beam has been employed to simulate the behavior of the foundation. The dynamic mechanical properties of natural rubber and a high-damping rubber have been employed to describe the behavior of anti-vibration mount materials.

When the ratio of the mass of the mounted item to the mass of the foundation is large, the isolation afforded by the simple mounting is much less than predicted by its transmissibility curve, which relates to an ideally rigid foundation. The isolation provided by the simple mounting is increased significantly at high frequencies when the foundation of the mounting system is mass-loaded, being largest for a natural rubber mounting. In the example considered, large, but not greater, isolation is provided at high frequencies by the compound mounting utilizing a secondary mass equal to this loading mass, and mountings composed of natural and high-damping rubber in parallel.

1. INTRODUCTION

This report describes the second phase of a theoretical investigation[1] concerned with the isolation of machinery vibration from structures possessing finite mechanical impedance.

Particular attention has been devoted to representing realistically the dynamic mechanical properties of rubber-like materials employed as vibration isolators. The mechanical properties of natural rubber and a synthetic rubber, Thiokol R. D., have again been considered to typify the mechanical properties of low- and high-damping rubbers, respectively. The dynamic shear modulus and damping factor possessed by natural rubber (Fig. 1) and Thiokol R. D. (Fig. 2) have been deduced from the experimental results of other workers.[2] The parallel mounting[1] discussed here is comprised of elements of these materials in parallel, arranged such that both materials experience the same strain. The transmissibility[1] of simple mountings of natural rubber and Thiokol R. D. and the transmissibility[1] of three parallel mountings composed of the same materials are shown in Fig. 3 at a temperature of 20°C. It is assumed that the mounting systems possess natural frequencies of 5 cps. The parameter "a" represents the ratio of the cross-sectional area of the Thiokol R. D. element to that of the natural rubber element.

General equations have been derived from which the response ratios[1] of different mounting systems may be computed when the variation with frequency of the mechanical impedance Z of their foundations is known. The response ratios of the mounting systems considered here have been evaluated for a nonrigid foundation, the impedance of which is simulated by the driving point impedance of a supported-supported beam excited by a sinusoidal force at its mid-point. It has been assumed that the beam possesses damping of the solid type, which is described[1] by a constant damping factor δ_f. A series of response ratios for a natural rubber mounting have been determined (Fig. 4) for values of foundation damping defined by $\delta_f = 0.01$, 0.1, and 1. The mass of the mounted item M was assumed to be ten times greater than the mass of the foundation M_f. The magnitude of δ_f is seen to have little influence upon the value of the response ratio at frequencies between the resonant modes of foundation vibration. The isolation afforded by the mount at these frequencies appears to be determined primarily by the value of the mass ratio M/M_f, even when the foundation is heavily damped.

It has previously been shown[1] that the isolation afforded by a resilient mounting will become smaller as the mass ratio M/M_f becomes larger. The response ratio of a natural rubber mounting is plotted in Fig. 5 for values of $M/M_f = 2$, 10, and 50, with the assumption that foundation damping is negligible. As the ratio M/M_f increases, the isolation afforded by the mounting is seen to depart progressively further from the isolation obtained when the foundation is completely rigid (Fig. 3).

The mounted item is supposed to behave purely as a lumped mass. The anti-vibration mountings are assumed to employ only linear rubber-like materials. "Wave effects" which may occur in the mountings have been disregarded.

2. MASS-LOADING OF A FOUNDATION POSSESSING FINITE MECHANICAL IMPEDANCE

2.1. Introduction

An expression for the response ratio of a simple or parallel mounting system supported by a mass-loaded foundation of finite mechanical impedance is presented in the Appendix (Section 5.2). Substitution has been made in this expression for the driving point impedance of a damped supported-supported beam, and the resulting response ratios of several mounting systems have been computed. Before these results are discussed, however, an attempt will be made to explain in general terms the mechanism by which a mass m, placed between the bottom of a resilient mounting and a supporting nonrigid foundation, increases the isolation afforded by the mounting system.

Reference is made to any multi-resonant sub-structure of mechanical impedance Z. It is assumed that the structure responds to a sinusoidally varying force F_o at the driving point with a velocity V_o, which is shown diagrammatically as a function of frequency in Fig. 6a. When an item of mass M is supported by the structure, such that a sinusoidally varying force F_1 acting upon M, or generated within M, excites the structure in the same position and fashion as before, then the structure will respond with the velocity V_1 shown as a function of frequency in Fig. 6b. The response of the second mode of foundation vibration has been drawn with a broken line. The resonant frequencies of vibration possessed by the structure are removed to lower frequencies by an amount which will depend upon the magnitude of M, but which will not exceed the original (Fig. 6a) frequency separation of the particular mode of vibration and the next lower mode.

When the vibrating item is resiliently mounted, as in Fig. 6c, the resonant frequencies of the foundation return again to higher frequencies. In fact, the higher modes of vibration occur at frequencies which are essentially identical to those of the unloaded structure (Fig. 6a), because in this frequency region the inertia of the mounted item is large, and to a first approximation the motion of the structure is restrained only by a resilient element, the other end of which is attached to the "stationary" mass M. An additional region of high foundation velocity is introduced at low frequencies due to the resonant motion of the resiliently mounted item M. This resonance has been assumed to occur at a lower frequency than the fundamental mode of foundation vibration.

The response ratio of the mounting system is shown in Fig. 6d, this quantity having been defined[3] as the magnitude of the velocity ratio V_2/V_1. Essentially, the maximum and minimum values of the response ratio occur at the frequencies for which the velocities V_2 and V_1, respectively, possess maximum values. The response ratios which have been presented in Figs. 4 and 5 for simple systems employing natural rubber mounts are seen to be of this form. It follows that the greater the mass M, the greater the shift of the resonant modes of the foundation to higher frequencies occurring when M is mounted resiliently, and therefore the smaller the over-all reduction in foundation velocity afforded by the mounting system compared with the level observed when M is rigidly supported.

When a mass is employed to load the foundation (Fig. 6e), the resonant frequencies of the structure do not remove as far towards higher frequencies as before. In fact, when m is an appreciable fraction of the resiliently supported mass M—the case depicted in Fig. 6e—the resonant frequencies of the foundation only slightly exceed the values observed when M is rigidly supported (Fig. 6b). It follows that the response ratio of the mounting system, namely, the magnitude of the velocity ratio V_3/V_1 (Fig. 6f), is reduced by the introduction of m. In fact, when m becomes comparable in magnitude to M, the isolation afforded by the mounting system approaches the value predicted by the simple transmissibility curve (which refers to an ideally rigid foundation) except at frequencies in close proximity to the resonant modes of foundation vibration.

2.2. The Simple Mounting System

Following the qualitative description of the manner in which a mass situated below a resilient mounting favorably decreases the response ratio of the mounting system, a specific foundation, the mechanical impedance of which is represented by the driving point impedance of a damped beam (Section 1), is discussed. This foundation has previously been considered[1] to possess a mass M_f and solid-type damping described by a constant damping factor δ_f.

The response ratios of natural rubber and the high-damping rubber Thiokol R. D. have been computed from Eq. (5.14), assuming that the mass ratio M/M_f = 50 and that the damping inherently possessed by the foundation may be described by a damping factor $\delta_f = 0.01$. The response ratios are presented in Figs. 7 and 8, respectively, for values of the mass ratio m/M = 0.1, 0.2, and 1.0. The broken curves refer to the response ratios of the mounting systems when the foundation is not loaded by an additional mass m.

Figures 7 and 8 show that an extremely large gain in isolation is afforded by the mounting systems when the mass ratio m/M approaches unity. When m/M = 1, the foundation velocity is reduced (negative logarithmic values of the response ratio) by introduction of the natural rubber mount at all frequencies above 9 cps, and by the introduction of the Thiokol R. D. mount at all frequencies above 5 cps. Moreover, the isolation afforded by these mounts at frequencies above the fundamental resonant frequency of the foundation is essentially equal to the isolation they afford in a simple mounting system supported by an ideally rigid foundation (Fig. 3).

While the value of the response ratio R at the fundamental mode of foundation vibration is hardly influenced by the introduction of m, the magnitude of R at the second, and the higher modes of vibration is reduced significantly. It is interesting to note that the magnitude of the response ratio at the second mode of foundation vibration relative to the value taken by R at neighboring frequencies is only slightly influenced by the magnitude of the mount damping. As demonstrated previously,[1] the maximum compression experienced by the mounts and, consequently, the influence of mount damping at the higher modes of foundation vibration will be small.

It may be questioned if the response ratio of a mounting system would be improved equally well if the additional mass m were employed to increase the integral mass of the foundation. This is not the case, as is evident from Fig. 9, which presents the response ratios for a simple system utilizing natural rubber mounts. The two broken curves of this figure relate to foundations which are one-tenth and one-fiftieth as massive as the mounted item, namely, foundations for which $M/M_f = 10$ and 50. The full line relates also to the least massive foundation for which $M/M_f = 50$, but, in addition, this foundation is loaded by a mass m chosen such that the ratio $M/(m+M_f)$ is equal to 10. A value of the mass ratio $m/M = 0.08$ has therefore been considered. In consequence, it is possible to compare the response ratios for the mounting system when it is supported by foundations of "equal" mass, namely, foundations for which $M/M_f = 10$ (broken line) and $M/(m+M_f) = 10$ (full line). It is evident from Fig. 9 that while the response ratio at the fundamental mode of foundation vibration is of the same order of magnitude in each case, the response ratio at the second mode, and presumably at the higher modes of foundation vibration, is significantly less for the mass-loaded foundation. Again, the over-all level of the response ratio at frequencies greater than 35 cps is generally lower for the mass-loaded foundation, especially in the neighborhood of the resonant modes of foundation vibration.

The response ratios of natural rubber and Thiokol R. D. mountings have been computed also for a second foundation, and are presented in Figs. 10 and 11, respectively. The curves of these figures relate to a mass-loaded foundation to which a damping treatment has been applied. It has been assumed that the effect of the damping treatment may be described[1] by a damping factor $\delta_f = 0.1$. The response ratio curves have been computed for values of the mass ratios $M/M_f = 10$ and $m/M = 0.1$, 0.2, and 1.0. The response ratios of these mounting systems are seen to be very similar in form to those illustrated by Figs. 7 and 8, which have previously been discussed. The significant increase in isolation resulting from the introduction of the mass m is again apparent. In fact, when the ratio $m/M = 1$, the isolation afforded by the mounts becomes equal, as before, to the isolation which they afford in a simple mounting system supported by an ideally rigid foundation (Fig. 3).

The greater foundation damping favorably suppresses the response ratio of the natural rubber mounting at the fundamental mode of foundation vibration, particularly for the smaller values of the ratio m/M (compare with the curves of Figs. 4 and 7).

It is suggested by the results discussed in this section that the conclusion drawn from an elementary consideration of the transmissibility curves of simple mounting systems, namely, that high mount damping is undesirable, may in fact be completely valid when the mounting system is supported by a mass-loaded foundation of finite mechanical impedance. (Compare the response ratios of natural rubber mountings shown in Figs. 7 and 10 with the response ratios of the highly damped mounts shown in Figs. 8 and 11, respectively.)

2.3. The Parallel Mounting System

The response ratios for several mounting systems employing a parallel mount comprised of natural and Thiokol R. D. rubbers have been computed. The performance of the parallel mount, for which a = 0.2 (Section 1), has been evaluated solely at a temperature of 20°C. The results obtained relate to a mounting system which is supported by one of two mass-loaded foundations.

The response ratio of the parallel mounting supported by a mass-loaded foundation for which $m/M = 0.2$, $M/M_f = 50$, and $\delta_f = 0.01$ is shown in Fig. 12. Although the resonant motion of the mounted item and the fundamental mode of foundation vibration are favorably suppressed by the parallel mounting, the natural rubber mount is seen to afford the greatest isolation at higher frequencies. In fact, the relative values of the response ratio at any one frequency are then very similar to the relative values of the transmissibility which the mounts possess (Fig. 3) at the same frequency. It should be noticed, however, that the response ratios of the parallel and natural rubber mountings do not diverge appreciably until the absolute value of the isolation afforded by both mountings is quite large.

The response ratios of a parallel mounting supported by a mass-loaded foundation for which $M/M_f = 10$, $\delta_f = 0.1$, and $m/M = 0.1$ and 1.0 are shown in Figs. 13 and 14, respectively. Because of the high foundation damping, the amplitudes of the fundamental and second modes of foundation vibration are seen to be virtually unaffected by the magnitude of the mount damping, substantiating an earlier conclusion[1] that there is little merit in employing any mount material other than natural rubber when the foundation damping is large. The relative values of the response ratio at any frequency above the fundamental mode of foundation vibration are again practically identical to the relative values of mount transmissibility at the same frequency.

3. THE COMPOUND MOUNTING SYSTEM

3.1. Introduction

The transmissibility of a two-stage or compound mounting system[3,4] has been determined in general terms [Eq. (5.15)], assuming that different rubber-like materials are employed in each stage of the mounting system. The transmissibility of the compound mounting may therefore be computed when the dependence upon frequency of the dynamic elastic moduli and the damping factors of the materials is known, although only a reduced form of the equation referring to mounts of the same material has been employed here.

The transmissibility of mountings of natural rubber and Thiokol R. D., and of a parallel mounting composed of the same materials [for which a = 0.2 (Section 1)] has been evaluated for a temperature of 20°C. The response ratios of the compound systems utilizing mounts of these materials have also been determined, since mount performance cannot be judged satisfactorily[1] by reference to transmissibility alone. Expressions for the response ratio have been derived for any foundation of finite mechanical impedance Z [see Eq. (5.21)], and for the particular nonrigid foundation previously discussed [see Eq. (5.24)].

3.2. The Transmissibility of the Compound Mounting System

The transmissibility of compound mountings of natural rubber, Thiokol R. D., and the parallel mounting is shown in Figs. 15, 16, and 17, respectively. In each figure, the performance of a compound mounting employing three different secondary masses $\overline{M}$ defined by β = 0.1, 0.2, and 1.0 has been compared with the transmissibility of a simple mounting utilizing the same mount material. The parameter β is equal to $\overline{M}/M$, where M is the mass of the mounted item. The natural mounting frequencies of the simple systems are again assumed to occur at a frequency of 5 cps.

The transmissibility of the compound mountings utilizing hevea mounts (Fig. 15) compares closely with the form predicted by simple theory.[3,5] The high-frequency isolation afforded by the compound and simple systems does increase at essentially 24 db per octave and 12 db per octave, respectively, and the primary and secondary resonant frequencies ω_1 and ω_2 do comply with the relation:[5]

$$\frac{\omega_2^2}{\omega_1^2} = \frac{(1+\beta) + (1+\beta)^{1/2}}{(1+\beta) - (1+\beta)^{1/2}} , \qquad (3.1)$$

so that

β	0.1	0.2	1.0
ω_2/ω_1	6.48	4.69	2.41 .

The superior high-frequency isolation afforded by the compound mounting systems employing large intermediate masses may easily be recognized from Fig. 15. The reduction or loss in isolation at the secondary resonance is, however, an undesirable feature of the compound mounting system.[3,5]

The greater damping possessed by the Thiokol R. D. mounts effectively suppresses both the primary and secondary resonances of the compound systems shown in Fig. 16. The frequency ratio ω_2/ω_1 is seen to be greater than Eq. (3.1) predicts, since ω_2 is displaced towards higher frequencies by the increase in stiffness of the mount with increasing[1,5] frequency. The rapid increase in the dynamic modulus of high-damping mount materials with frequency is also responsible[1,5] for the relatively poor isolation afforded by these materials at high frequencies.

The transmissibility of the compound system utilizing parallel mountings for which the parameter a = 0.2 is shown in Fig. 17. The secondary resonance is favorably suppressed by the mount damping yet, because the mount stiffness increases more slowly[1] with frequency than a mount comprised solely of Thiokol R. D., the high-frequency isolation afforded by the mounting increases relatively rapidly with frequency, and the displacement of ω_2 to higher frequencies is small.

The performance of the three mountings discussed here may be compared more satisfactorily by reference to Figs. 18 and 19, where the transmissibility of the compound systems employing mass ratios β = 0.1 and 1.0, respectively, are redrawn. These figures confirm that the parallel mounting suppresses the secondary resonance of the compound systems while affording appreciably greater isolation at high frequencies than the Thiokol R. D. mount. In fact, the high-frequency isolation afforded by the parallel mounting resembles that afforded by the hevea mounts, the correspondence being least close when the isolation afforded by the mountings is very large.

3.3. The Response Ratio of the Compound Mounting System

The response ratios of compound mountings of natural rubber, Thiokol R. D., and a parallel mounting composed of these rubbers (for which a = 0.2) have been computed assuming that the same mount materials are employed in each stage of the compound systems. The response ratios have been determined from Eq. (5.24), and relate to a single foundation which is one-fiftieth as massive as the mounted item (M/M_f = 50) and to values of the mass ratio β = 0.1, 0.2, and 1.0. The damping inherently possessed by the foundation has again been described by a damping factor δ_f = 0.01.

The response ratios computed for natural rubber mountings are shown in Fig. 20, the broken curve referring to a simple mounting system. It is evident that the isolation afforded at high frequencies by the compound system increases very rapidly with frequency, and at any one frequency becomes larger when the mass ratio β is increased. In particular, the compound mounting greatly reduces the value of the response ratio at the fundamental and second modes of vibration of the foundation. As mentioned previously, however, an undesirable loss in isolation occurs at, and in the neighborhood of, the secondary resonant frequency of the compound mounting system. In fact, the peak values of the response ratio introduced at the frequencies 9.2 cps, 18.6 cps, and 25.5 cps, when β = 1.0, 0.2, and 0.1, respectively, are comparable to the value of the response ratio observed at the fundamental mode of foundation vibration.

Response ratios computed for compound systems utilizing high-damping rubber mounts and parallel mountings are shown in Figs. 21 and 22, respectively. The Thiokol R. D. mounts are seen to suppress greatly the primary and secondary resonances of the compound system, and the fundamental mode of vibration of the foundation. The isolation afforded by the compound mounting again increases rapidly with frequency in relation to the isolation afforded by the simple system but, for a given value of β, the absolute magnitude of the isolation is generally much less than provided by the compound system utilizing hevea mounts (Fig. 20). In fact, even though the second mode of foundation vibration is greatly suppressed by the Thiokol R. D. mountings, greater isolation is provided at this resonant frequency by the compound system employing hevea mounts. In contrast, the parallel mounting is seen (Fig. 22) to suppress both the mounting and the foundation resonances, and to afford an isolation at intervening frequencies which resembles the isolation provided by the hevea mounts more closely than the isolation afforded by the Thiokol R. D. mounts.

The relative performance of the hevea, Thiokol R. D., and parallel mountings can more easily be examined in Figs. 23 and 24, where the response ratios of the compound mountings defined by β = 0.1 and 1.0, respectively, are redrawn. These figures show clearly that while the compound system employing parallel mounts does not afford as large an isolation as the compound system employing hevea mounts—other than in the neighborhood of the resonant frequencies of the mounting system and the foundation—the difference is relatively small except at high frequencies, where the isolation afforded by both mountings is very large.

4. SUMMARY AND CONCLUSIONS

This report describes the second phase of a theoretical investigation that has suggested and examined methods by which machinery vibration can be isolated from structures possessing finite mechanical impedance.

Equations have been derived from which the performance of various mounting systems may be determined when the mechanical impedance of the nonrigid structure which supports them is known. In order to compare the performance of the mounting systems, substitution has been made in these equations for the driving point impedance of a damped supported-supported beam excited by a sinusoidally varying force at its mid-point. Substitution has also been made for the dynamic mechanical properties of anti-vibration mount materials, namely, for the dynamic elastic moduli and damping factors of natural rubber and the high-damping synthetic rubber Thiokol R. D. The mechanical properties of these rubbers have been deduced as functions of frequency from the experimental results of other workers.

It is shown that as the mass of a mounted item M increases with respect to the mass of its nonrigid foundation M_f, the isolation afforded by the mounting system will become smaller and depart further from the isolation that would be observed if the foundation were completely rigid.

The analysis suggests that a damping treatment applied to a nonrigid foundation will only increase the isolation afforded by the mounting system at the resonant frequencies of the foundation. The damping treatment will have little influence upon the isolation afforded at other frequencies, the isolation then being determined primarily by the magnitude of the mass ratio M/M_f. Since a damping treatment applied to the foundation would probably have only secondary influence upon the over-all level of the isolation provided by the mounting system, and since it would probably be impractical to damp the foundation heavily, most of the results refer to an "untreated" foundation, the inherent solid-type damping of which is defined by a damping factor $\delta_f = 0.01$.

When the ratio of the mass of the mounted item to the mass of the foundation is large, the over-all level of the isolation afforded by a simply mounted item is found to be appreciably less than predicted by the transmissibility curve of the mounting system (the curve relating to an ideally rigid foundation), not regarding the loss in isolation occurring as expected at the resonant frequencies of the foundation. This report discusses two ways in which the over-all level of the isolation afforded by a mounting system, for which the ratio M/M_f is large, can be made to approach the isolation predicted by its transmissibility curve. Both methods introduce a mass which is employed either to load the foundation supporting the mounting system, or to form a compound mounting[3,4] as an intermediate or secondary mass. It is desirable in both cases that the mass introduced be as large a fraction of the mass of the mounted item as possible.

It is shown that the isolation afforded by the simple mounting system supported by a mass-loaded foundation becomes larger as the loading mass m is increased. In fact, when the loading mass is equal to the mass of the mounted item, the mounting system affords the isolation which is predicted by its transmissibility curve at frequencies above the fundamental resonant frequency of the mass-loaded foundation. The conventional conclusion drawn from simple consideration of the transmissibility curves of rubber-like materials is therefore valid in these circumstances, that is to say, low-damping rubbers such as natural rubber will be the most suitable anti-vibration mount materials.

It is also shown that the isolation afforded by the compound mounting system becomes larger as the secondary mass is increased. In fact, when the secondary mass is equal to the mass of the mounted item, the isolation afforded at frequencies above the secondary resonant frequency of the mounting system is very large. In the example considered, it approaches that provided by the simple mounting supported by the mass-loaded foundation (for which m = M) discussed above. The isolation afforded by the compound system is detrimentally reduced at the resonant frequencies of the nonrigid foundation, but the loss in isolation at the fundamental mode of vibration is not large when parallel mountings are employed, although at most other frequencies natural rubber mountings would provide somewhat greater isolation.

The relative performance of the simple mounting, the foundation of which is mass-loaded, and the compound mounting may be compared more readily with the help of Figs. 25 and 26, which refer to a loading mass or a secondary mass which is one-fifth and equal to the mass of the mounted item, respectively. Figure 26 illustrates the extremely large isolation afforded by the simple mounting system when supported by a heavily mass-loaded foundation (for which m = M). This level of isolation is greater than that afforded by either the compound or simple mounting systems at nearly all frequencies above the fundamental frequency of the mass-loaded foundation. The performance of the compound system is superior at some high frequencies, but the isolation is theoretically so large at these frequencies that comparison is trivial, since it is most probable that the isolation will be impaired by mechanical "shorts" linking the vibrating machine to the foundation other than through its resilient mountings.

It should be realized that when the mass ratio M/M_f decreases below the value considered in this example, the over-all isolation afforded by the compound mounting system may increase more rapidly than that of the simple mounting with a mass-loaded foundation. For sufficiently smaller values of M/M_f, therefore, the compound mounting may provide the greatest over-all isolation, particularly at high frequencies.

5. APPENDIX

5.1. Introduction

It has been shown[1] that the response ratio of a parallel mounting supported by a foundation of finite mechanical impedance Z is given by the following equation:

$$R^2 = \frac{(1 + \Delta_\omega^2)}{\left|\left\{1 - \frac{\omega^2}{\omega_0^2}\left(\frac{G_{10} + aG_{20}}{G_{1\omega} + aG_{2\omega}}\right)\left(\frac{1}{1 + j\omega M/Z}\right) + j\Delta_\omega\right\}\right|^2} , \qquad (5.1)$$

where the dynamic moduli and damping factors of the low- and the high-damping rubbers comprising the mounting are $G_{1\omega}$ and $\delta_{1\omega}$, and $G_{2\omega}$ and $\delta_{2\omega}$, respectively. The parameter "a" represents the ratio of the cross-sectional area of the high-damping material to that of the low-damping rubber. The cross-sectional areas are assumed to be uniform. The parameters G_{1o} and G_{2o} refer, respectively, to the values of $G_{1\omega}$ and $G_{2\omega}$ at the natural mounting frequency ω_o. This angular frequency is given by the relation:

$$\omega_o^2 = \frac{k_1 \left(G_{1o} + aG_{2o}\right)}{M} , \tag{5.2}$$

where M is the mass of the mounted item and k_1 is a constant equal to the ratio of the cross-sectional area of the low-damping rubber to the mount thickness. The damping factor of the parallel mounting is given by the following relation:

$$\Delta_\omega = \left[\frac{G_{1\omega}\,\delta_{1\omega} + aG_{2\omega}\,\delta_{2\omega}}{G_{1\omega} + aG_{2\omega}}\right] . \tag{5.3}$$

The equation for the response ratio of a simple mounting supported by a foundation of mechanical impedance Z may simply be obtained by equating the parameter a to zero.

The impedance of a damped supported-supported beam excited by a sinusoidal force at its mid-point has been employed[1] to simulate the mechanical properties of a nonrigid foundation. The beam was assumed to possess a complex elastic modulus with constant real and imaginary parts and, consequently, a constant damping factor δ_f.[1] It has been shown[1] that:

$$\left[1 + \frac{j\omega M}{Z}\right] = 1 - \left(\frac{M}{M_f}\right)\left(\frac{n_d b}{2}\right)\left[\frac{\sin(n_d b)\cosh(n_d b) - \cos(n_d b)\sinh(n_d b)}{\cos(n_d b)\cosh(n_d b)}\right] , \tag{5.4}$$

where M_f is the mass of the beam, b the half-length of the beam, and n_d a parameter defined by the relation:

$$n_d^4 = \frac{\omega^2 \rho}{k^2 E\,(1 + j\,\delta_f)} . \tag{5.5}$$

The quantities ρ, k, and E are constants representing the density, the radius of gyration of the cross section, and the real part of the complex Young's modulus of the beam, respectively.

It is convenient to express the product $(n_d b)$ as the complex number $(p + jq)$, where p and q are given by the relations:

$$p = (nb)\left[\frac{1}{2\Delta_f^{1/2}} + \frac{(1+\Delta_f)^{1/2}}{2\sqrt{2}\ \Delta_f}\right]^{1/2} \qquad (5.6)$$

$$q = -(nb)\left[\frac{1}{2\Delta_f^{1/2}} - \frac{(1+\Delta_f)^{1/2}}{2\sqrt{2}\ \Delta_f}\right]^{1/2} , \qquad (5.7)$$

where $n = (\omega^2\rho/k^2E)^{1/4}$ and $\Delta_f = (1 + \delta_f^2)^{1/2}$. Equation (5.4) may then be written in the form:

$$\left[1 + \frac{j\omega M}{Z}\right] = [1 - (A + jB)] , \qquad (5.8)$$

where

$$(1\text{-}A) = \Bigg[1 - \frac{p}{2PQ}\left(\frac{M}{M_f}\right)(P \sin p \cos p - Q \sinh p \cosh p)$$

$$- \frac{q}{2PQ}\left(\frac{M}{M_f}\right)(Q \sin q \cos q - P \sinh q \cosh q)\Bigg] , \qquad (5.9)$$

$$B = \Bigg[\frac{q}{2PQ}\left(\frac{M}{M_f}\right)(P \sin p \cos p - Q \sinh p \cosh p)$$

$$- \frac{p}{2PQ}\left(\frac{M}{M_f}\right)(Q \sin q \cos q - P \sinh q \cosh q)\Bigg] , \qquad (5.10)$$

and

$$P = (\cosh^2 p - \sin^2 q) \qquad (5.11)$$

$$Q = (\cosh^2 q - \sin^2 p) . \qquad (5.12)$$

5.2. Mass-Loading of a Foundation Possessing Finite Mechanical Impedance

A qualitative interpretation of the mechanism by which a mass m placed between the bottom of a resilient mounting and a nonrigid foundation (Fig. 6e) increases the isolation afforded by the mounting system has been given in Section 2.

It can be shown that the response ratio of a parallel mounting supported by a mass-loaded foundation of finite mechanical impedance Z is given by the following equation:

$$R^2 = \frac{(1 + \Delta_\omega^2)}{\left|\left\{\left[1 - \frac{\omega^2}{\omega_0^2}\left(\frac{G_{10}+aG_{20}}{G_{1\omega}+aG_{2\omega}}\right)\left(\frac{1}{1+j\omega M/Z}\right)\right] + \left(\frac{m}{M}\right)\left(\frac{j\omega M/Z}{1+j\omega M/Z}\right)\left[1 - \frac{\omega^2}{\omega_0^2}\left(\frac{G_{10}+aG_{20}}{G_{1\omega}+aG_{2\omega}}\right)\right] + j\Delta_\omega\left[1 + \left(\frac{m}{M}\right)\left(\frac{j\omega M/Z}{1+j\omega M/Z}\right)\right]\right\}\right|^2} \quad . \tag{5.13}$$

From this equation the response ratio of the parallel mounting system (or that of any simple mounting if the parameter a is equated to zero) may be evaluated when the dependence of the mechanical impedance Z upon frequency is known. Substitution in this expression for the mechanical impedance possessed by the non-rigid foundation discussed previously [Eq. (5.8)] leads to the equation:

$$R^2 = \frac{(1 + \Delta_\omega^2)\,[(1-A)^2 + B^2]}{\left\{\frac{\omega^4}{\omega_0^4}(\lambda^2+\xi^2)\left(\frac{G_{10}+aG_{20}}{G_{1\omega}+aG_{2\omega}}\right)^2 - 2\frac{\omega^2}{\omega_0^2}\left[\lambda(\lambda-A) + \xi(\xi+B) + B\delta_\omega\right]\left(\frac{G_{10}+aG_{20}}{G_{1\omega}+aG_{2\omega}}\right) + (1 + \Delta_\omega^2)\,[(\lambda-A)^2 + (\xi+B)^2]\right\}} \quad , \tag{5.14}$$

where the parameters A and B are defined by Eqs. (5.9) and (5.10), respectively, and $\lambda = [1 - (m/M)A]$ and $\xi = (m/M)B$.

5.3. Transmissibility of the Compound Mounting System

The transmissibility of the compound mounting has been determined most generally with the assumption that a different rubber-like material is employed in each stage of the mounting system. The materials for the primary and secondary mountings are described by dynamic moduli and damping factors equal to $G_{3\omega}$ and $\delta_{3\omega}$, and $G_{4\omega}$ and $\delta_{4\omega}$, respectively. (The secondary mounting supports the intermediate mass $\overline{M}$.)

The optimum value for the ratio of the secondary to the primary mount stiffnesses assumed in the derivation of the expression for transmissibility ensures that the separation of the two resonant frequencies of the mounting system is a minimum, providing that the mount damping factors are not large. The optimum value of this stiffness ratio equals $(1+\beta)$, where $\beta = \overline{M}/M$, M again being the mass of the mounted item. The general transmissibility equation is:

$$T^2 = \frac{\left(\frac{G_{3\omega}}{G_{3o}}\right)^4 \left(\frac{G_{4\omega}}{G_{3\omega}}\right)^2 [(1+\delta_{3\omega}^2)\ (1+\delta_{4\omega}^2)]}{[E^2 + F^2]}, \tag{5.15}$$

where

$$E = \left[\left(\frac{\omega^4}{\omega_o^4}\right)\left(\frac{\beta}{2+\beta}\right)\left(\frac{1+\beta}{2+\beta}\right)\left(\frac{G_{4o}}{G_{3o}}\right)\left(\frac{G_{3o}}{G_{3\omega}}\right)^2 - \left(\frac{\omega}{\omega_o}\right)^2\left(\frac{1+\beta}{2+\beta}\right)\left(\frac{G_{3o}}{G_{3\omega}}\right)\left(\frac{G_{4o}}{G_{3o}} + \frac{G_{4\omega}}{G_{3\omega}}\right) + \left(\frac{G_{4\omega}}{G_{3\omega}}\right)(1-\delta_{3\omega}\delta_{4\omega})\right] \tag{5.16}$$

$$F = \left[\left(\frac{\omega^2}{\omega_o^2}\right)\left(\frac{1+\beta}{2+\beta}\right)\left(\frac{G_{3o}}{G_{3\omega}}\right)\left(\frac{G_{4o}}{G_{3o}}\delta_{3\omega} + \frac{G_{4\omega}}{G_{3\omega}}\delta_{4\omega}\right) - \left(\frac{G_{4\omega}}{G_{3\omega}}\right)(\delta_{3\omega} + \delta_{4\omega})\right]. \tag{5.17}$$

The quantities G_{3o} and G_{4o} refer, respectively, to the values of $G_{3\omega}$ and $G_{4\omega}$ at the natural mounting frequency ω_o, which is given by the relation:

$$\omega_o^2 = \frac{k_3 G_{3o}}{M}\left(\frac{1+\beta}{2+\beta}\right), \tag{5.18}$$

where k_3 is a constant equal to the ratio of the cross-sectional area of the primary mount to the mount thickness.

When both primary and secondary mounts utilize the same rubber-like material —so that $G_{3\omega}$ and $G_{4\omega}$ become equal to $G_{1\omega}$, say, and $\delta_{3\omega}$ and $\delta_{4\omega}$ become equal to $\delta_{1\omega}$—then Eq. (5.15) may be written:

$$T^2 = \frac{(1 + \delta_{1\omega}^2)^2}{\left[\left(\frac{\omega}{\omega_o}\right)^4\left(\frac{\beta}{2+\beta}\right)\left(\frac{1+\beta}{2+\beta}\right)\left(\frac{G_{1o}}{G_{1\omega}}\right)^2 - 2\left(\frac{\omega}{\omega_o}\right)^2\left(\frac{1+\beta}{2+\beta}\right)\left(\frac{G_{1o}}{G_{1\omega}}\right) + (1-\delta_{1\omega}^2)\right]^2 + \left[\left(\frac{\omega}{\omega_o}\right)^2\left(\frac{1+\beta}{2+\beta}\right)\left(\frac{G_{1o}}{G_{1\omega}}\right) - 1\right]^2 (4\delta_{1\omega})^2}, \tag{5.19}$$

where

$$\omega_o^2 = \frac{k_1\ G_{1o}}{M}\left(\frac{1+\beta}{2+\beta}\right). \tag{5.20}$$

The transmissibility of identical parallel mountings in the compound system may be obtained from this equation simply by substituting $[(G_{1o}+aG_{2o})/(G_{1\omega}+aG_{2\omega})]$ for $(G_{1o}/G_{1\omega})$ and Δ_ω for $\delta_{1\omega}$, where these parameters have the same significance as in Section 5.1.

5.4. Response Ratio of the Compound Mounting System

The response ratio of the compound mounting system supported by a foundation of finite mechanical impedance Z has been determined with the assumption that the same rubber-like material is utilized in both the primary and secondary mountings. The optimum value of the mount stiffness ratio (Section 5.3) has been employed in the derivation of the following equation:

$$R^2 = \frac{(1 + \delta_{1\omega}^2)^2 \; |(1 + j\omega M_1/Z)|^2}{|(H - jI)|^2} \; , \tag{5.21}$$

where

$$H = \left\{ \left[\left(\frac{\omega}{\omega_o}\right)^4 \left(\frac{G_{1o}}{G_{1\omega}}\right)^2 \left(\frac{\beta}{2+\beta}\right) \left(\frac{1+\beta}{2+\beta}\right) - 2\left(\frac{\omega}{\omega_o}\right)^2 \left(\frac{G_{1o}}{G_{1\omega}}\right) \left(\frac{1+\beta}{2+\beta}\right) + (1 - \delta_{1\omega}^2) \right] - \frac{j\omega M_1}{Z} \left[\left(\frac{\omega}{\omega_o}\right)^2 \left(\frac{G_{1o}}{G_{1\omega}}\right) \left(\frac{\beta}{2+\beta}\right) - (1 - \delta_{1\omega}^2) \right] (1+\beta) \right\} \tag{5.22}$$

$$I = \left\{ \left[\left(\frac{\omega}{\omega_o}\right)^2 \left(\frac{G_{1o}}{G_{1\omega}}\right) \left(\frac{1+\beta}{2+\beta}\right) - 1 \right] (2\delta_{1\omega}) + \frac{j\omega M_1}{Z} \left[\left(\frac{\omega}{\omega_o}\right)^2 \left(\frac{G_{1o}}{G_{1\omega}}\right) \left(\frac{\beta}{2+\beta}\right) - 1 \right] (1+\beta)\delta_{1\omega} \right\} \tag{5.23}$$

Equation (5.21) relates to a single rubber-like material, but again the response ratio for the system employing the parallel mounting may be obtained when appropriate substitution is made for the ratio $(G_{1o}/G_{1\omega})$ and $\delta_{1\omega}$. When the mechanical impedance of the nonrigid foundation considered previously [Eq. (5.8)] is substituted for Z in this equation, the response ratio may be written:

$$R^2 = \frac{(1 + \delta_{1\omega}^2)^2 \; [(1-A)^2 + B^2]}{[J^2 + K^2]} \tag{5.24}$$

where

$$J = \left\{ A(1+\beta) \left[\left(\frac{\omega}{\omega_o}\right)^2 \left(\frac{G_{1o}}{G_{1\omega}}\right) \left(\frac{\beta}{2+\beta}\right) - (1-\delta_{1\omega}^2) \right] - B(1+\beta)\delta_{1\omega} \left[\left(\frac{\omega}{\omega_o}\right)^2 \left(\frac{G_{1o}}{G_{1\omega}}\right) \left(\frac{\beta}{2+\beta}\right) - 2 \right] + \left[\left(\frac{\omega}{\omega_o}\right)^4 \left(\frac{G_{1o}}{G_{1\omega}}\right)^2 \left(\frac{\beta}{2+\beta}\right) \left(\frac{1+\beta}{2+\beta}\right) - 2\left(\frac{\omega}{\omega_o}\right)^2 \left(\frac{G_{1o}}{G_{1\omega}}\right) \left(\frac{1+\beta}{2+\beta}\right) + (1-\delta_{1\omega}^2) \right] \right\} \tag{5.25}$$

$$K = \Bigg\{ A(1+\beta)\delta_{1\omega} \left[\left(\frac{\omega}{\omega_o}\right)^2 \left(\frac{G_{1o}}{G_{1\omega}}\right) \left(\frac{\beta}{2+\beta}\right) - 2 \right] + B(1+\beta) \left[\left(\frac{\omega}{\omega_o}\right)^2 \left(\frac{G_{1o}}{G_{1\omega}}\right) \left(\frac{\beta}{2+\beta}\right) - (1-\delta_{1\omega}^2) \right]$$

$$- 2\delta_{1\omega} \left[\left(\frac{\omega}{\omega_o}\right)^2 \left(\frac{G_{1o}}{G_{1\omega}}\right) \left(\frac{1+\beta}{2+\beta}\right) - 1 \right] \Bigg\} , \qquad (5.26)$$

and the parameters A and B are defined by Eqs. (5.9) and (5.10).

REFERENCES

1. J. C. Snowdon, Reduction of the Response to Vibration of Structures Possessing Finite Mechanical Impedance, Part I, The University of Michigan Willow Run Laboratories Technical Report 2892-4-T, November, 1959.

2. W. P. Fletcher and A. N. Gent, Brit. J. Appl. Phys., 8, 194 (1957).

3. J. C. Snowdon, Akust. Beih., No. 1, 118 (1956).

4. J. C. Snowdon and G. G. Parfitt, J. Acoust. Soc. Am., 31, 967 (1959).

5. J. C. Snowdon, Brit. J. Appl. Phys., 9, 461 (1958).

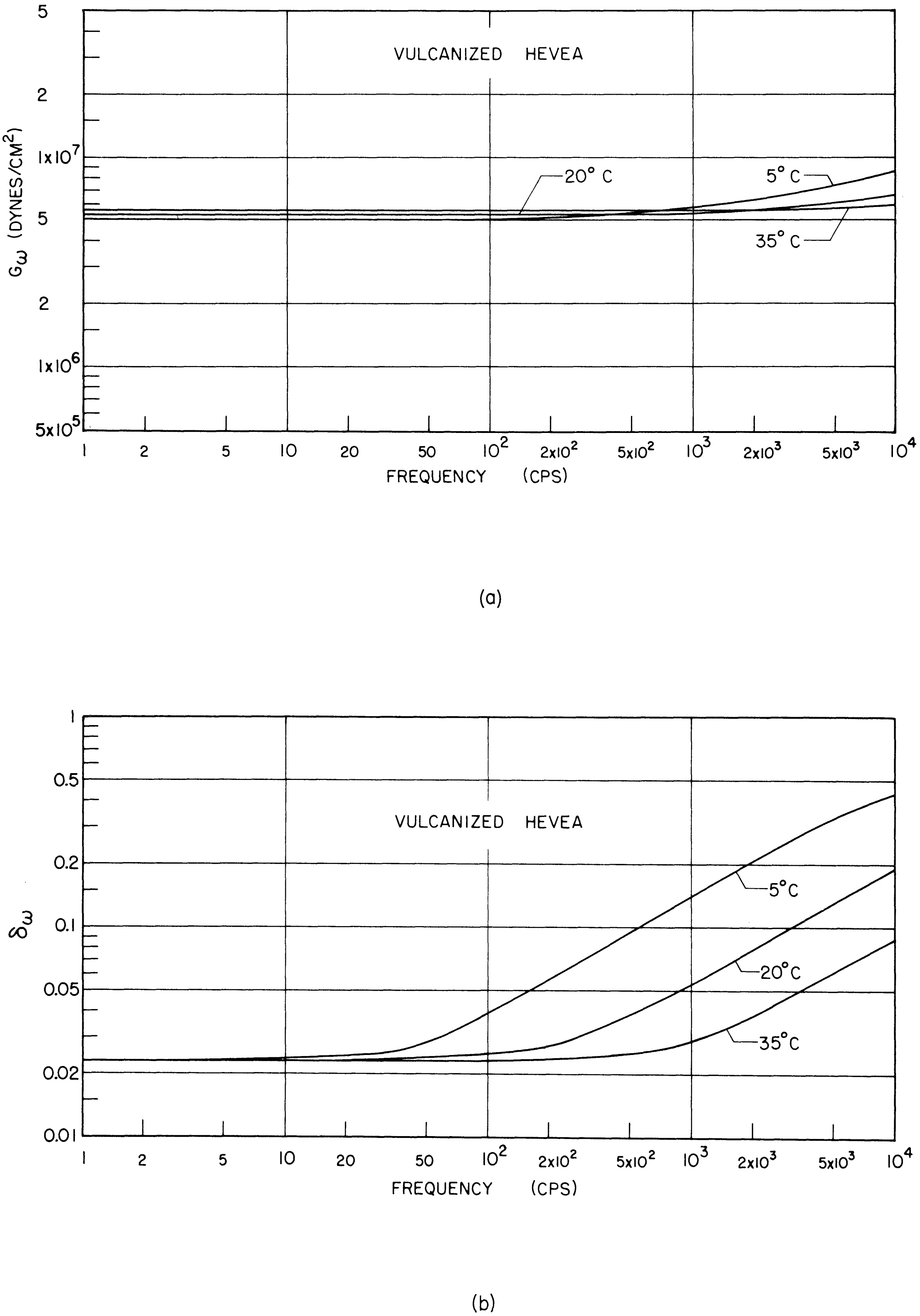
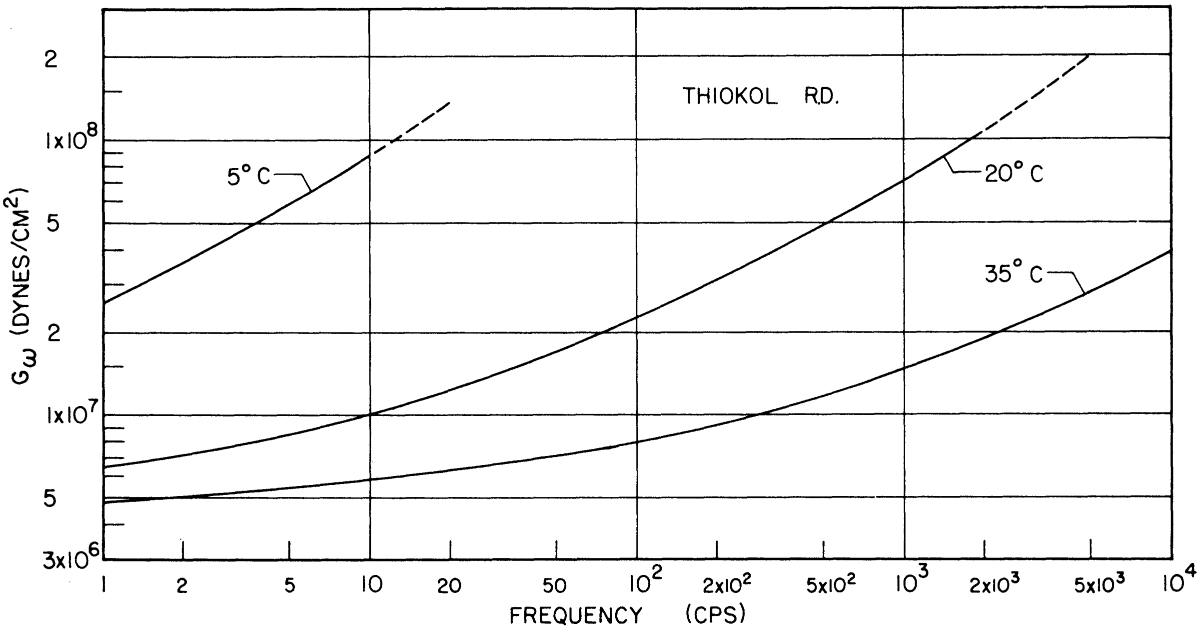
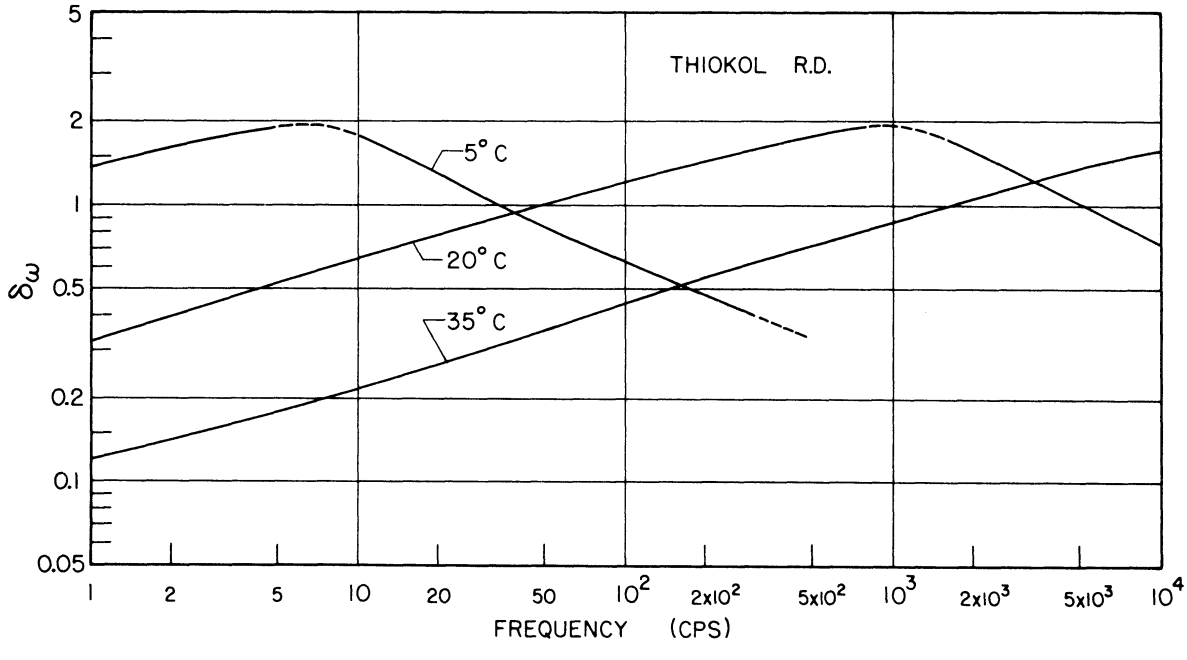


Fig 1. The frequency dependence of (a) the dynamic shear modulus and (b) the damping factor possessed by a natural rubber vulcanizate.



(a)



(b)

Fig. 2. The frequency dependence of (a) the dynamic shear modulus and (b) the damping factor possessed by Thiokol R. D.

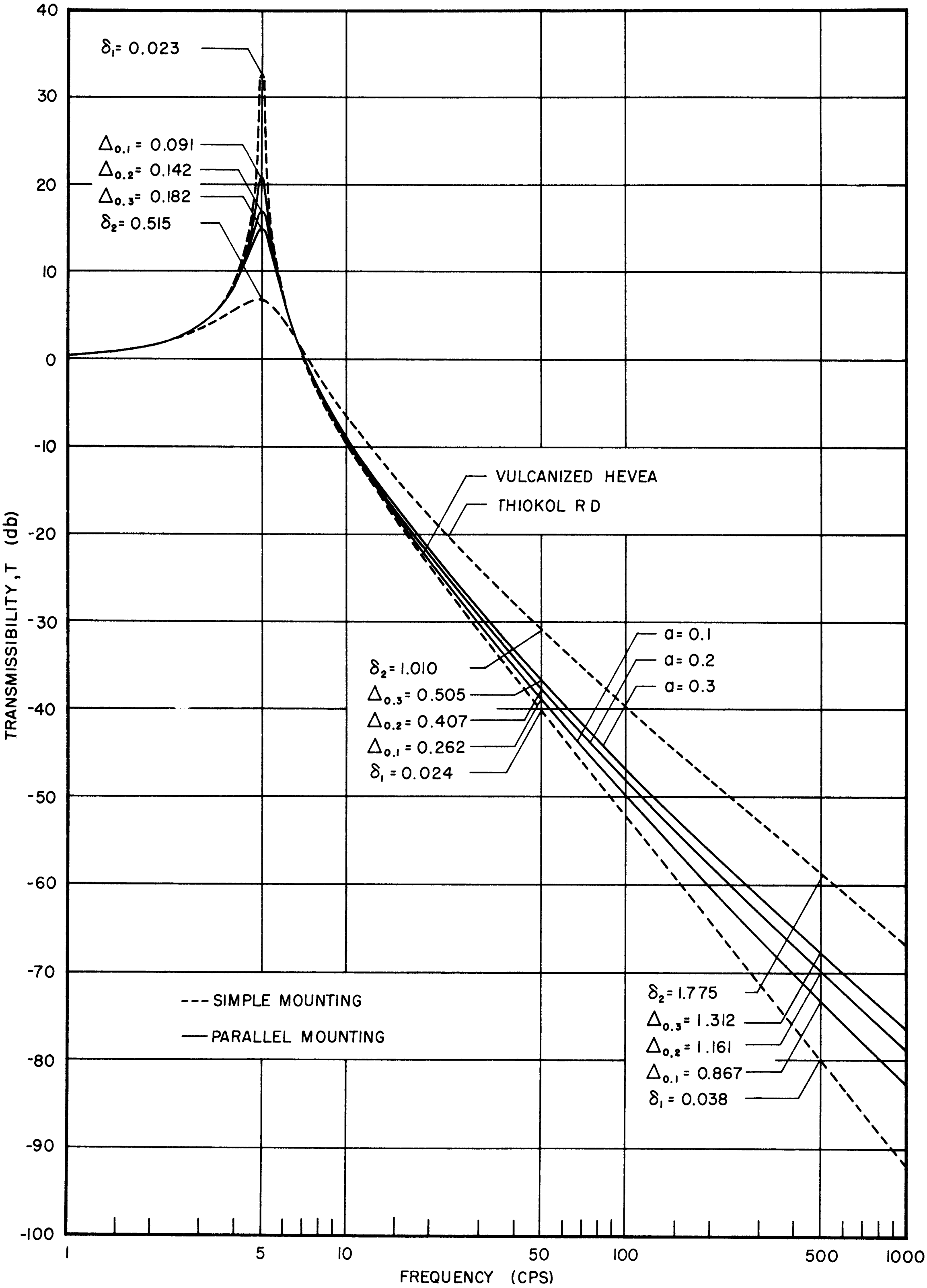


Fig. 3. The transmissibility of simple and parallel mountings of vulcanized hevea and Thiokol R. D. at a temperature of 20°C.

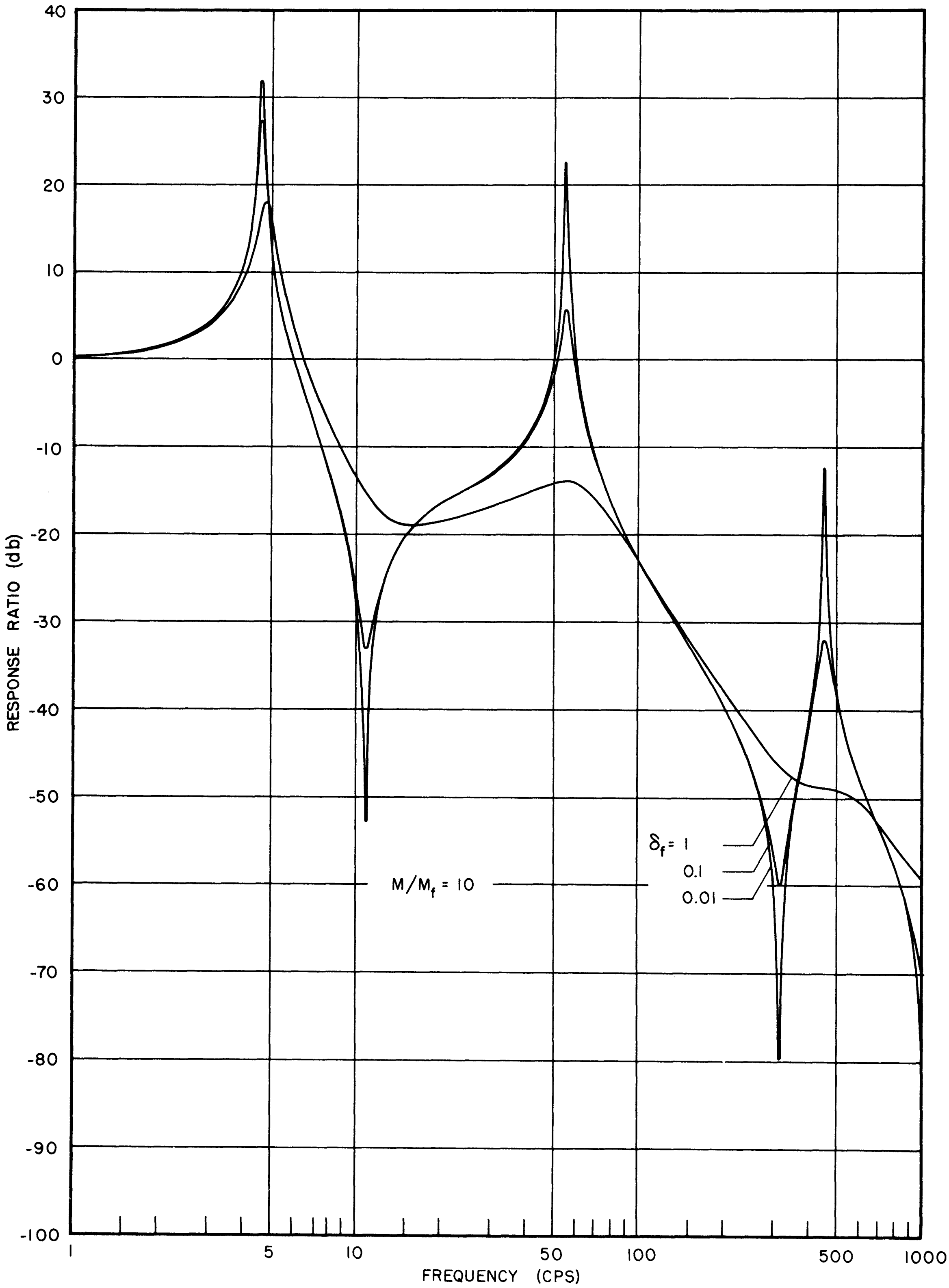


Fig. 4. The response ratio of a simple mounting of vulcanized hevea. Mounted item ten times more massive than the foundation. Foundation damping defined by δ_f = 0.01, 0.1, and 1.0

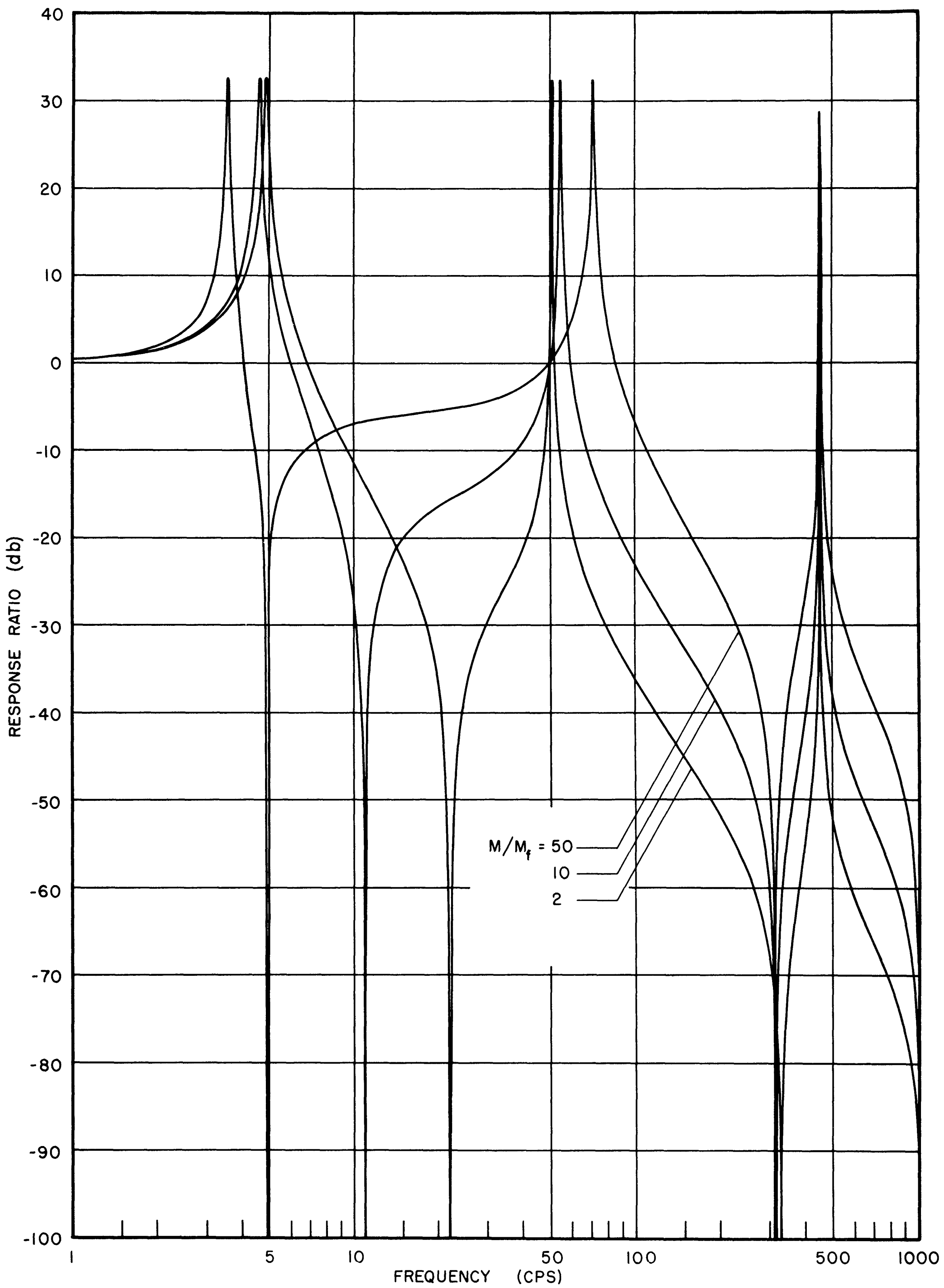


Fig. 5. The response ratio of a simple mounting of vulcanized hevea. Mounted item twice, ten, and fifty times more massive than the undamped foundation.

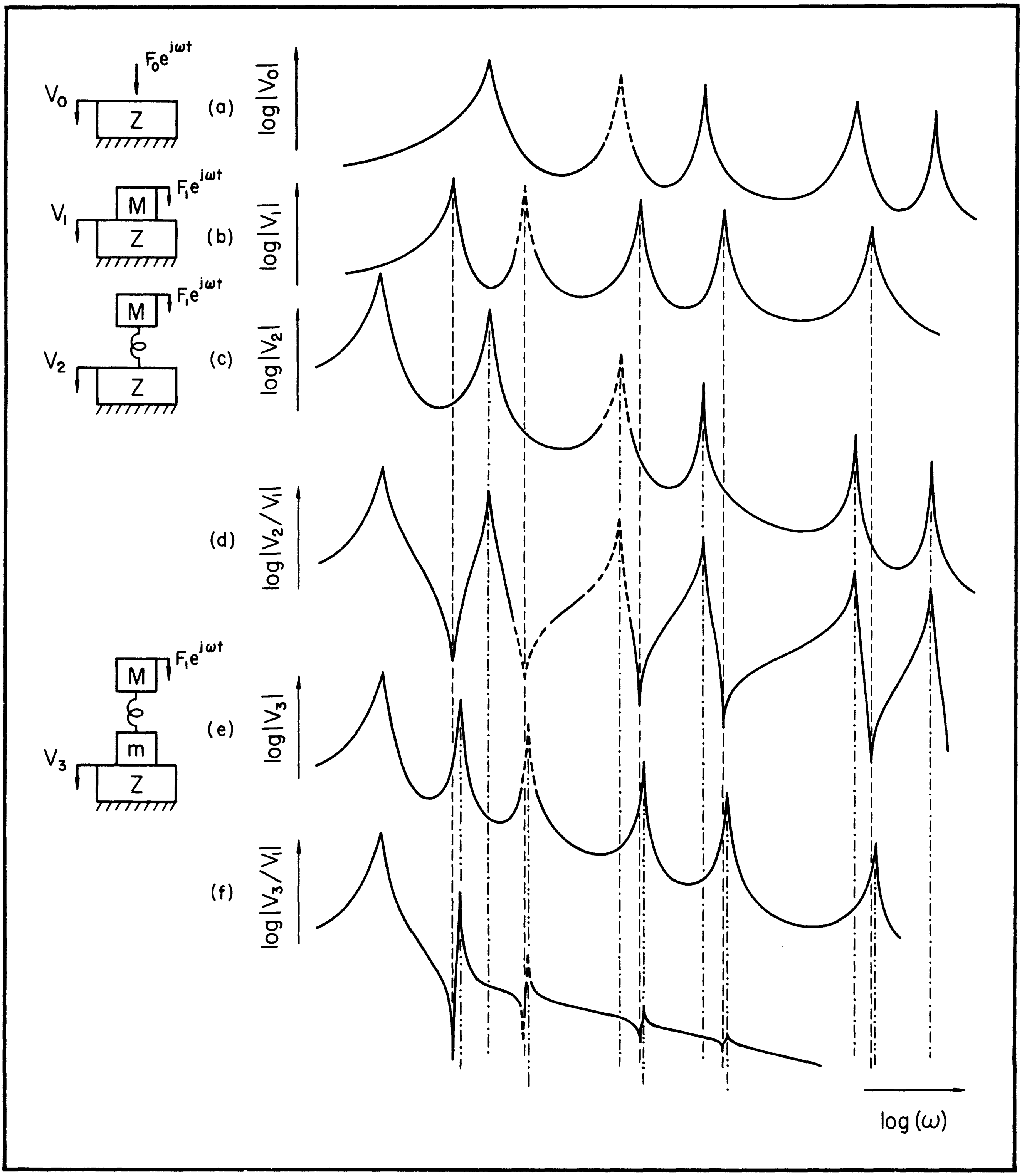


Fig. 6. The frequency dependence of the velocity with which a nonrigid foundation responds to mechanical vibration.

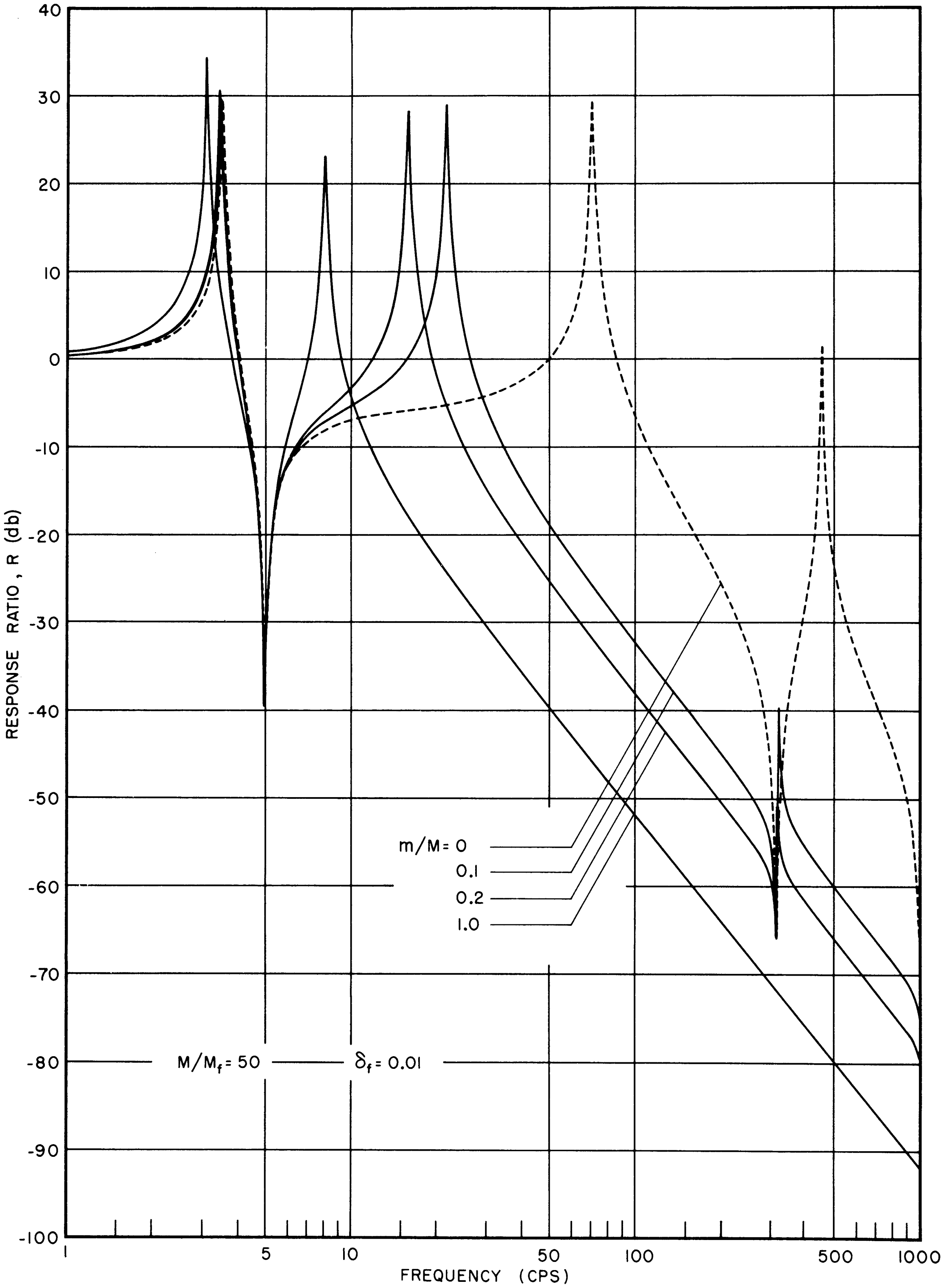


Fig. 7. The response ratio of a simple mounting of vulcanized hevea supported by a mass-loaded foundation. Mounted item fifty times more massive than the foundation. Loading mass one-tenth, one-fifth, and equal to the mass of the mounted item. Foundation damping defined by $\delta_f = 0.01$.

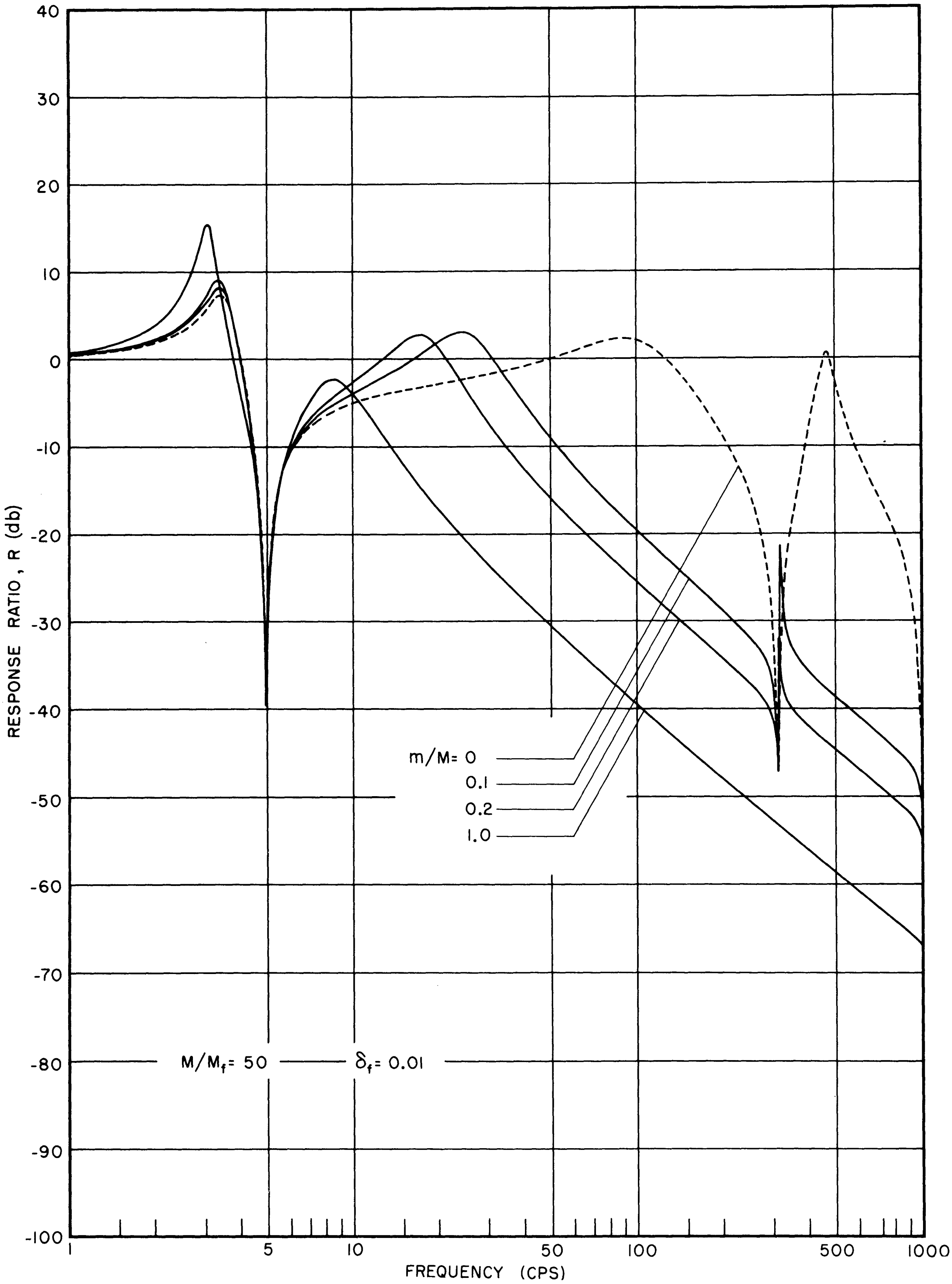


Fig. 8. The response ratio of a simple mounting of Thiokol R. D. supported by a mass-loaded foundation. Mounted item fifty times more massive than the foundation. Loading mass one-tenth, one-fifth, and equal to the mass of the mounted item. Foundation damping defined by δ_f = 0.01.

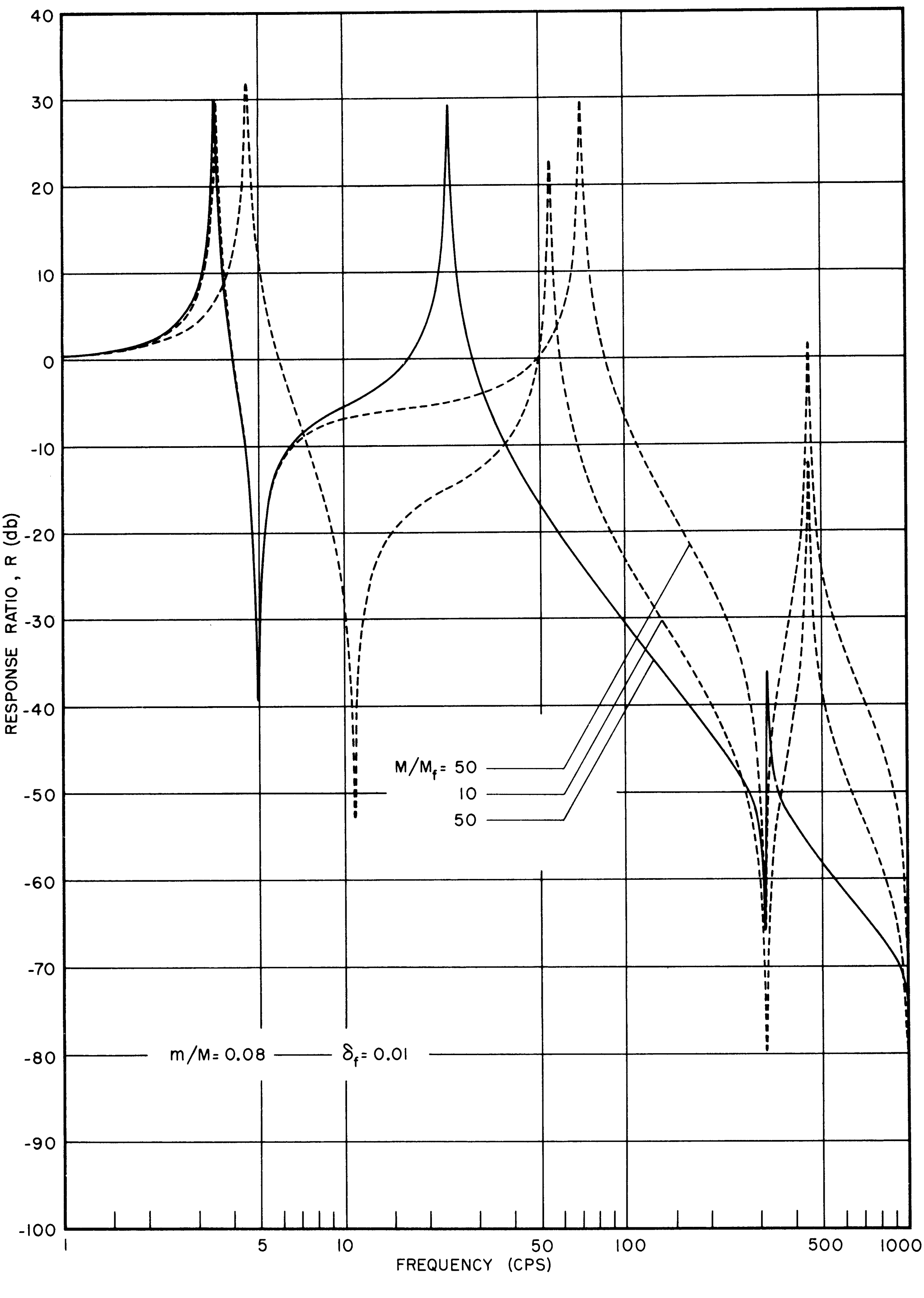


Fig. 9. The response ratio of a simple mounting of vulcanized hevea supported by a mass-loaded foundation. Mounted item ten and fifty times more massive than the foundation. Loading mass equal to 0.08 of the mass of the mounted item. Foundation damping defined by δ_f = 0.01.

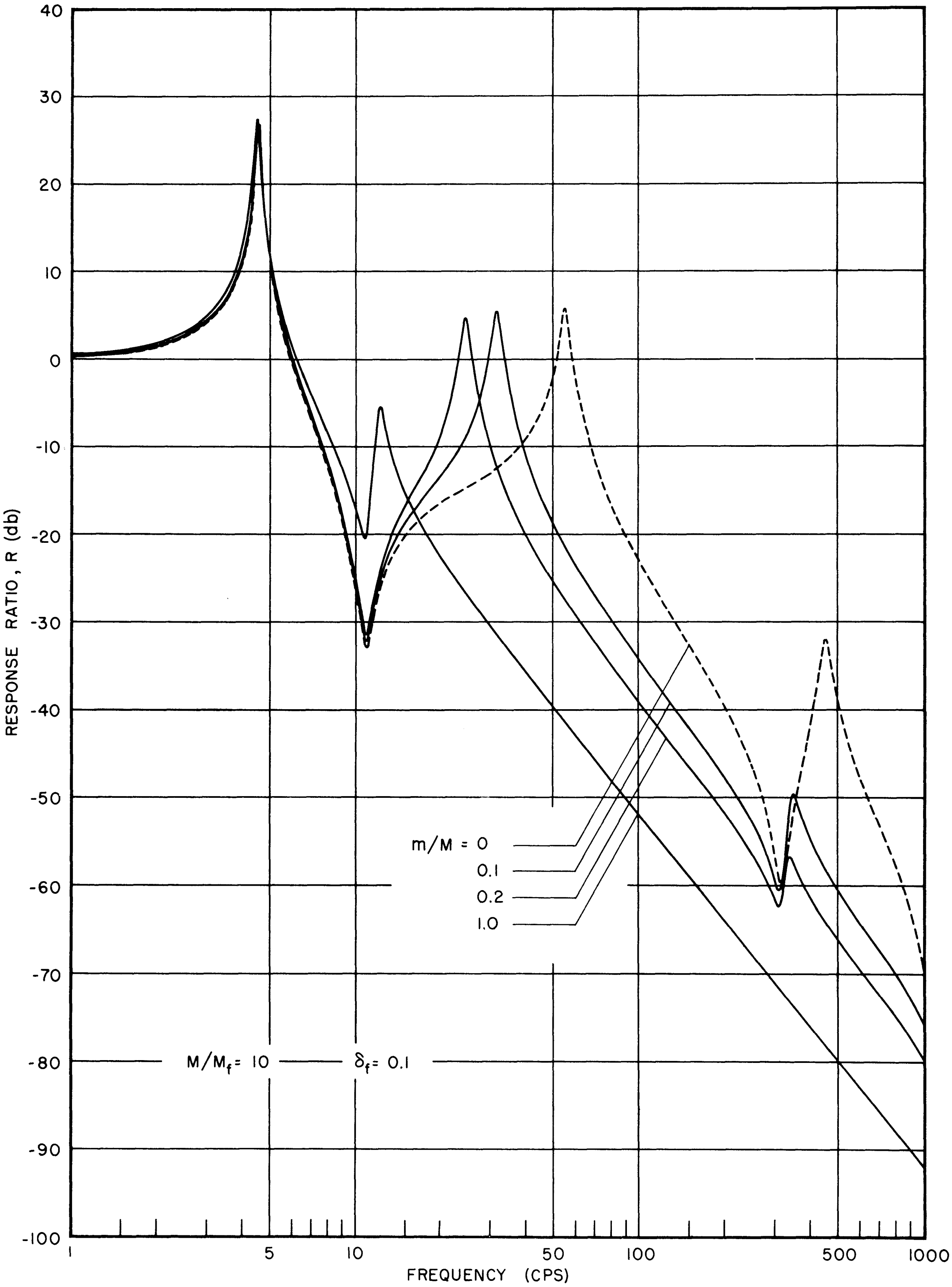


Fig. 10. The response ratio of a simple mounting of vulcanized hevea supported by a mass-loaded foundation. Mounted item ten times more massive than the foundation. Loading mass one-tenth, one-fifth, and equal to the mass of the mounted item. Foundation damping defined by $\delta_f = 0.1$.

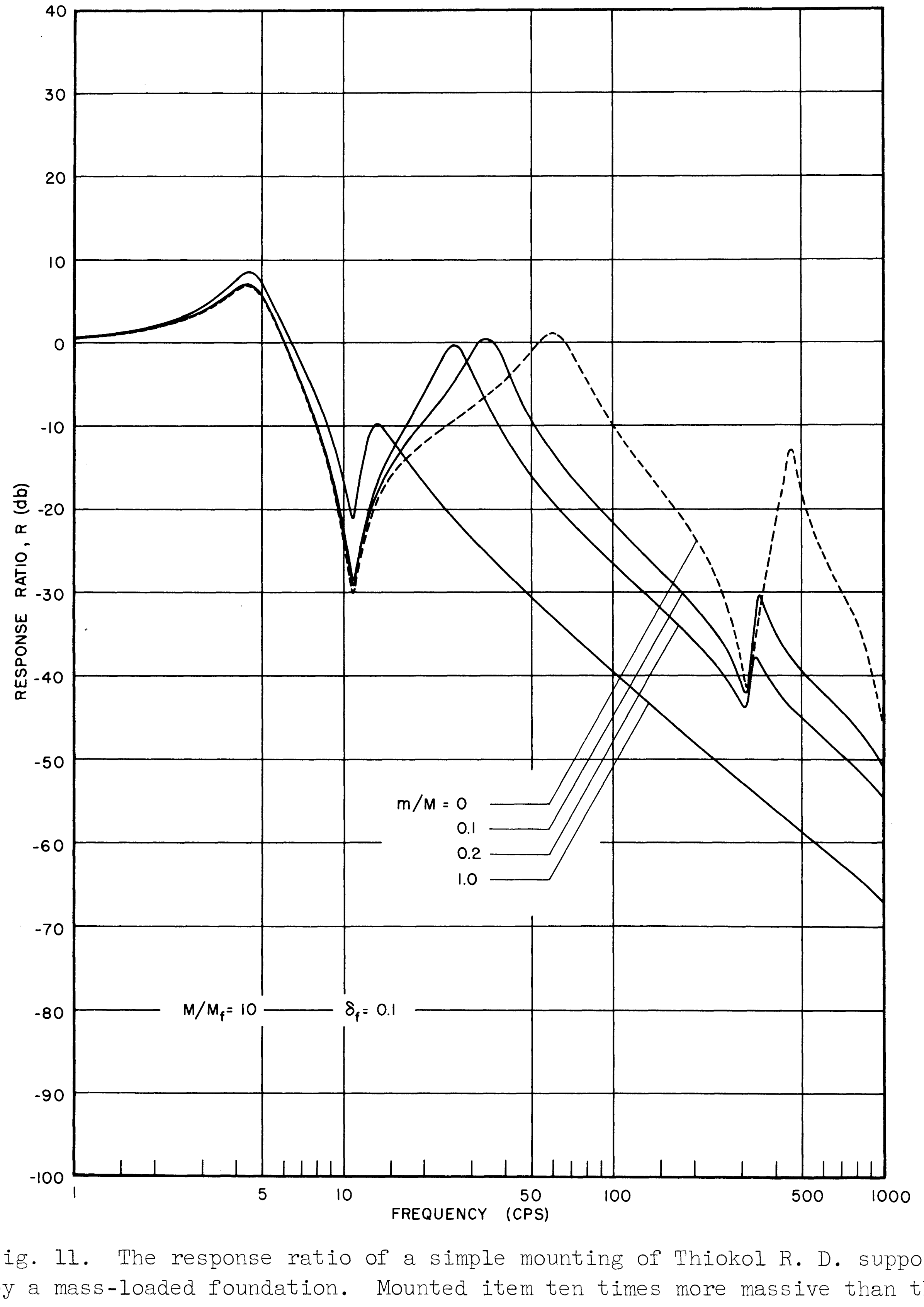


Fig. 11. The response ratio of a simple mounting of Thiokol R. D. supported by a mass-loaded foundation. Mounted item ten times more massive than the foundation. Loading mass one-tenth, one-fifth, and equal to the mass of the mounted item. Foundation damping defined by $\delta_f = 0.1$.

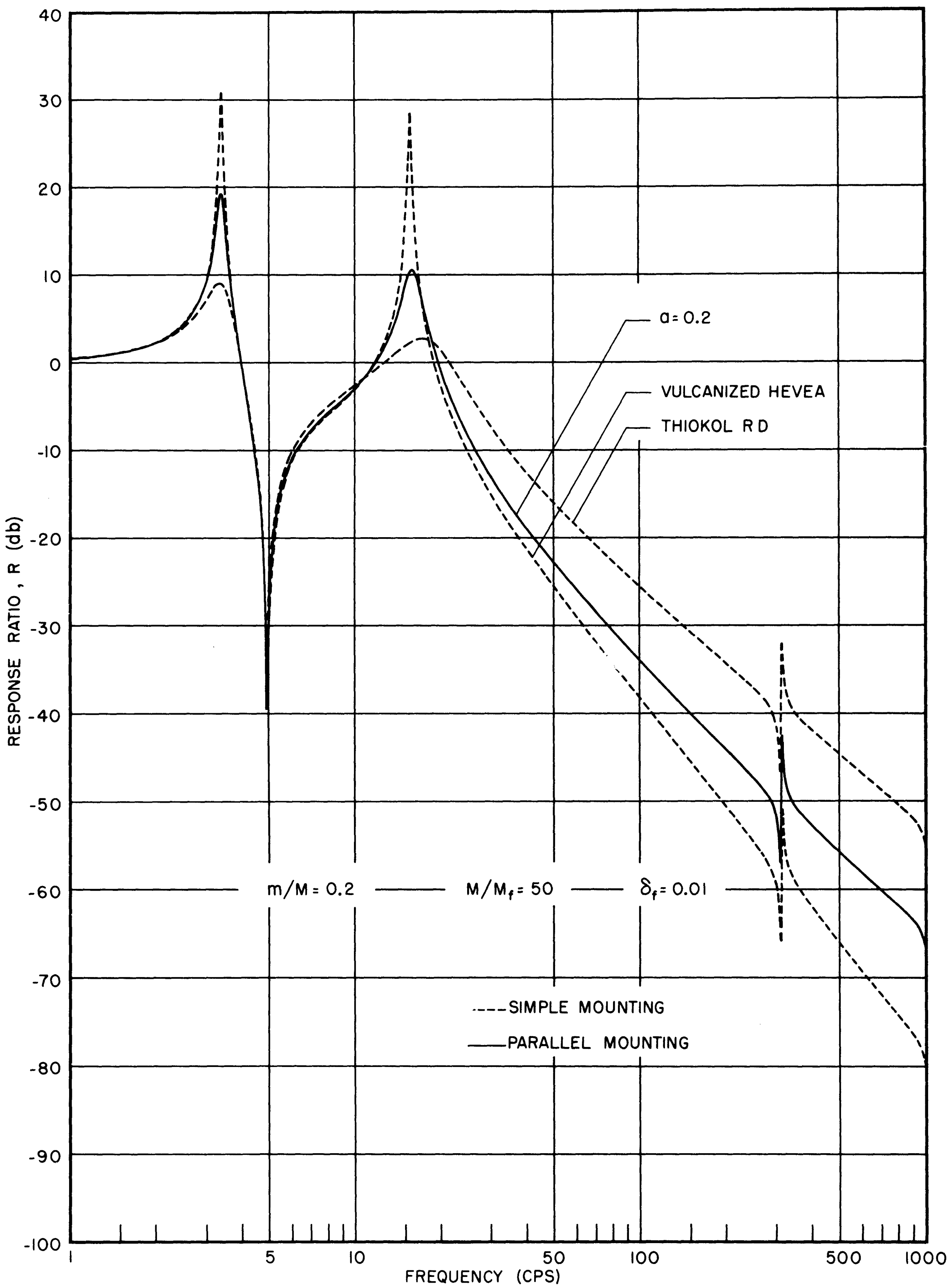


Fig. 12. The response ratio of simple and parallel mountings of vulcanized hevea and Thiokol R. D. supported by a mass-loaded foundation. Mounted item fifty times more massive than the foundation. Loading mass one-fifth of the mass of the mounted item. Foundation damping defined by $\delta_f = 0.01$.

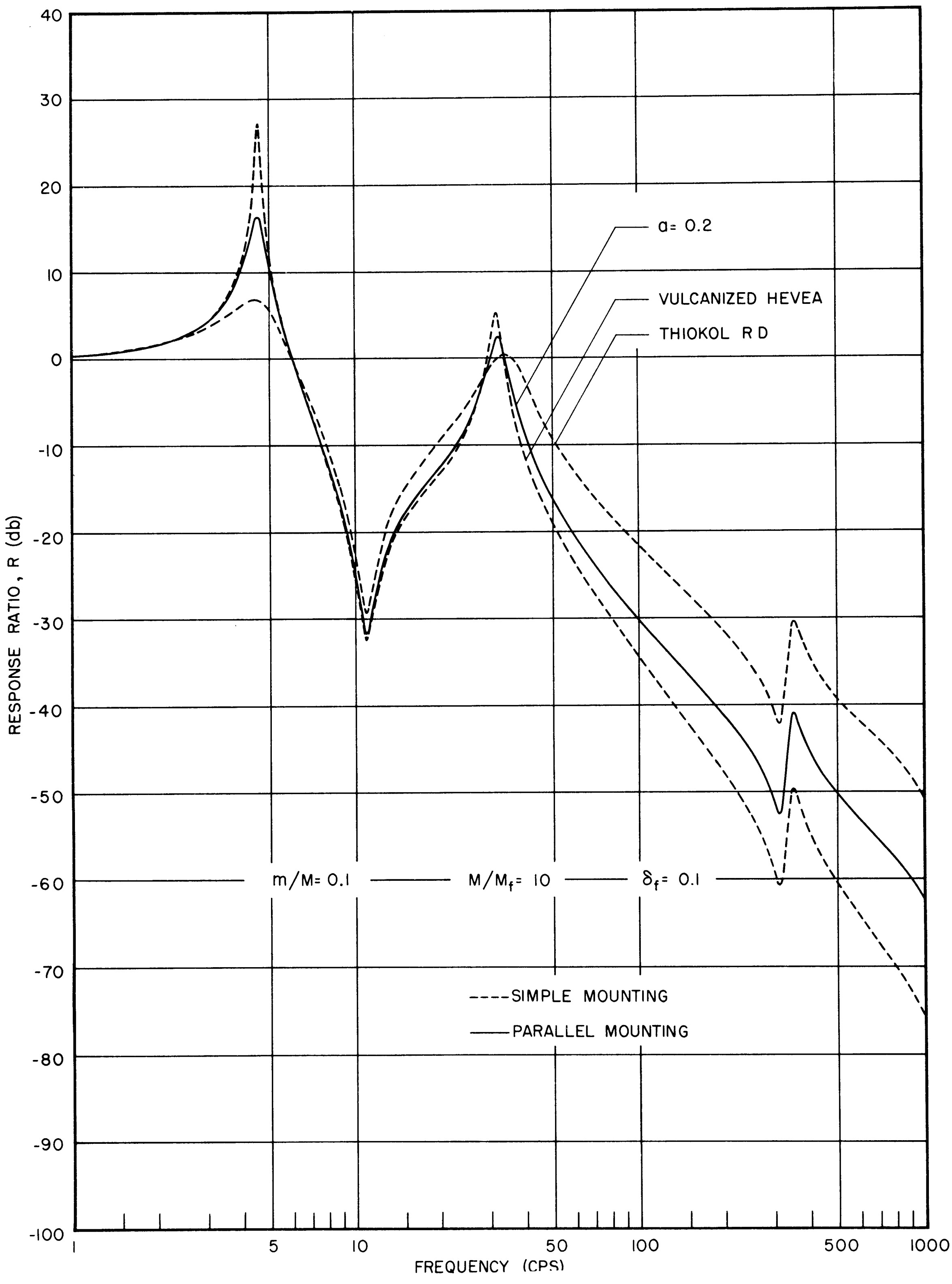


Fig. 13. The response ratio of simple and parallel mountings of vulcanized hevea and Thiokol R. D. supported by a mass-loaded foundation. Mounted item ten times more massive than the foundation. Loading mass one-tenth of the mass of the mounted item. Foundation damping defined by $\delta_f = 0.1$.

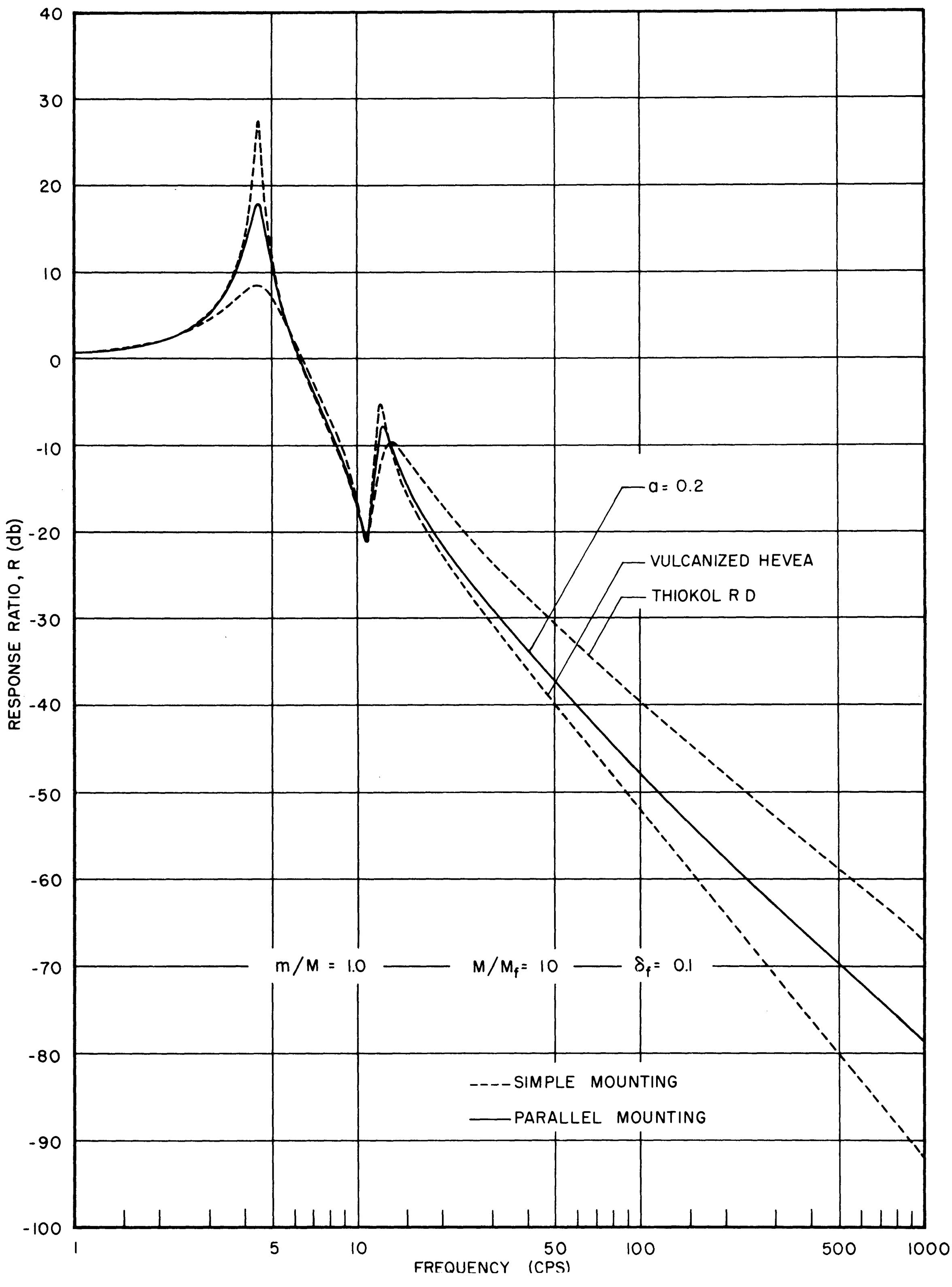


Fig. 14. The response ratio of simple and parallel mountings of vulcanized hevea and Thiokol R. D. supported by a mass-loaded foundation. Mounted item ten times more massive than the foundation. Loading mass equal to the mass of the mounted item. Foundation damping defined by $\delta_f = 0.1$.

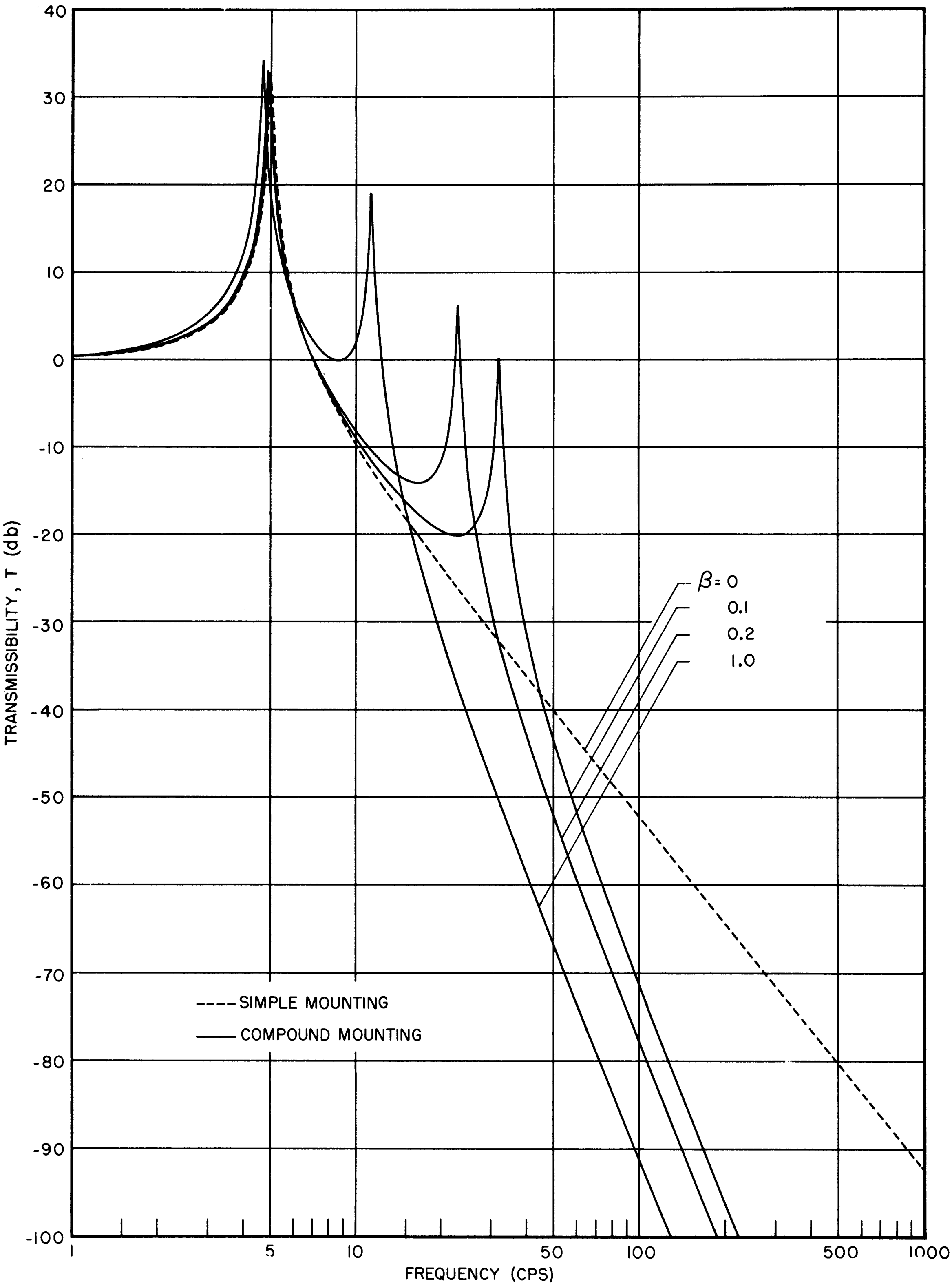


Fig. 15. The transmissibility of a compound system employing vulcanized hevea mounts. Secondary mass one-tenth, one-fifth, and equal to the mass of the mounted item.

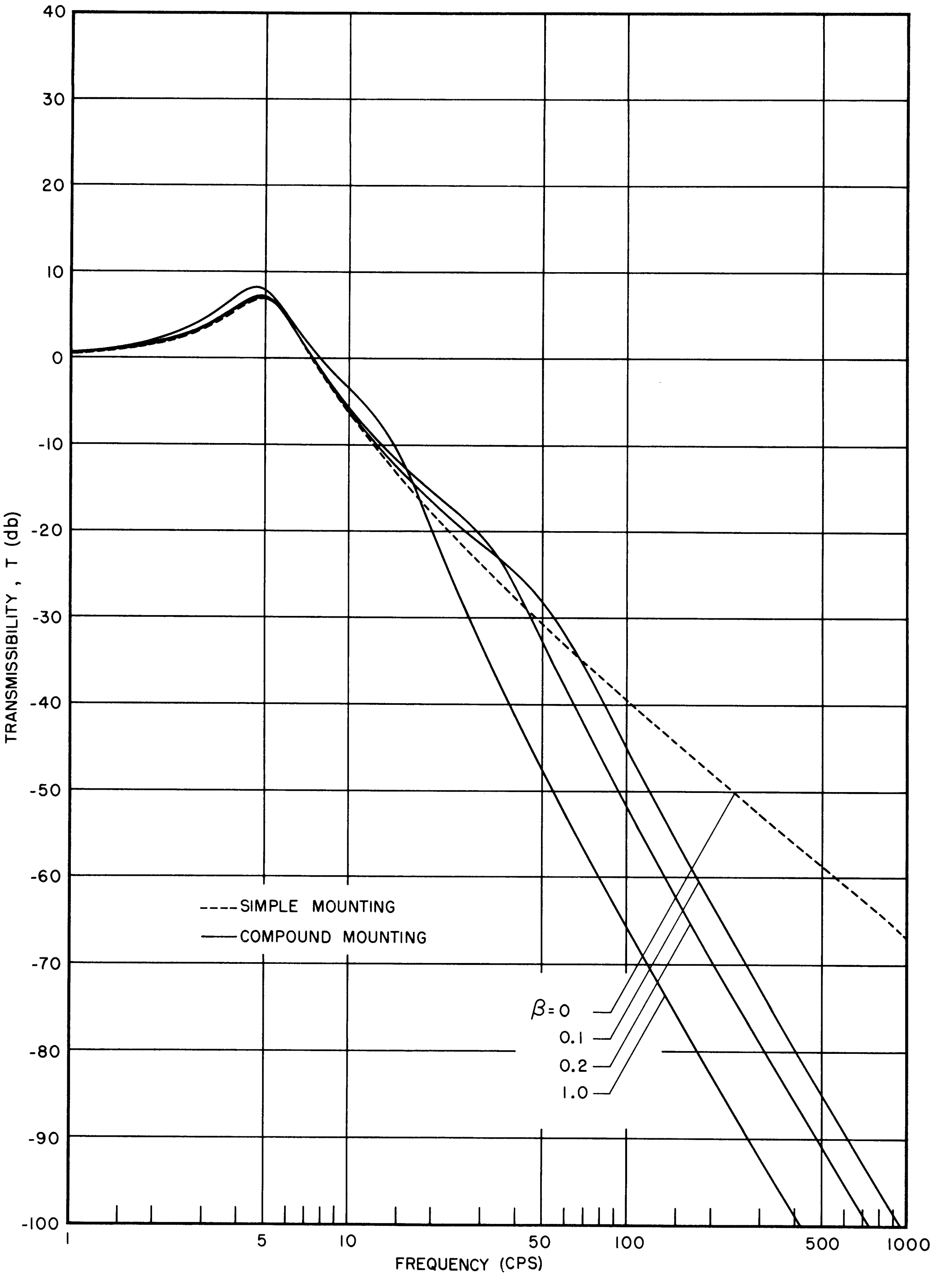


Fig. 16. The transmissibility of a compound system employing Thiokol R. D. mounts. Secondary mass one-tenth, one-fifth, and equal to the mass of the mounted item.

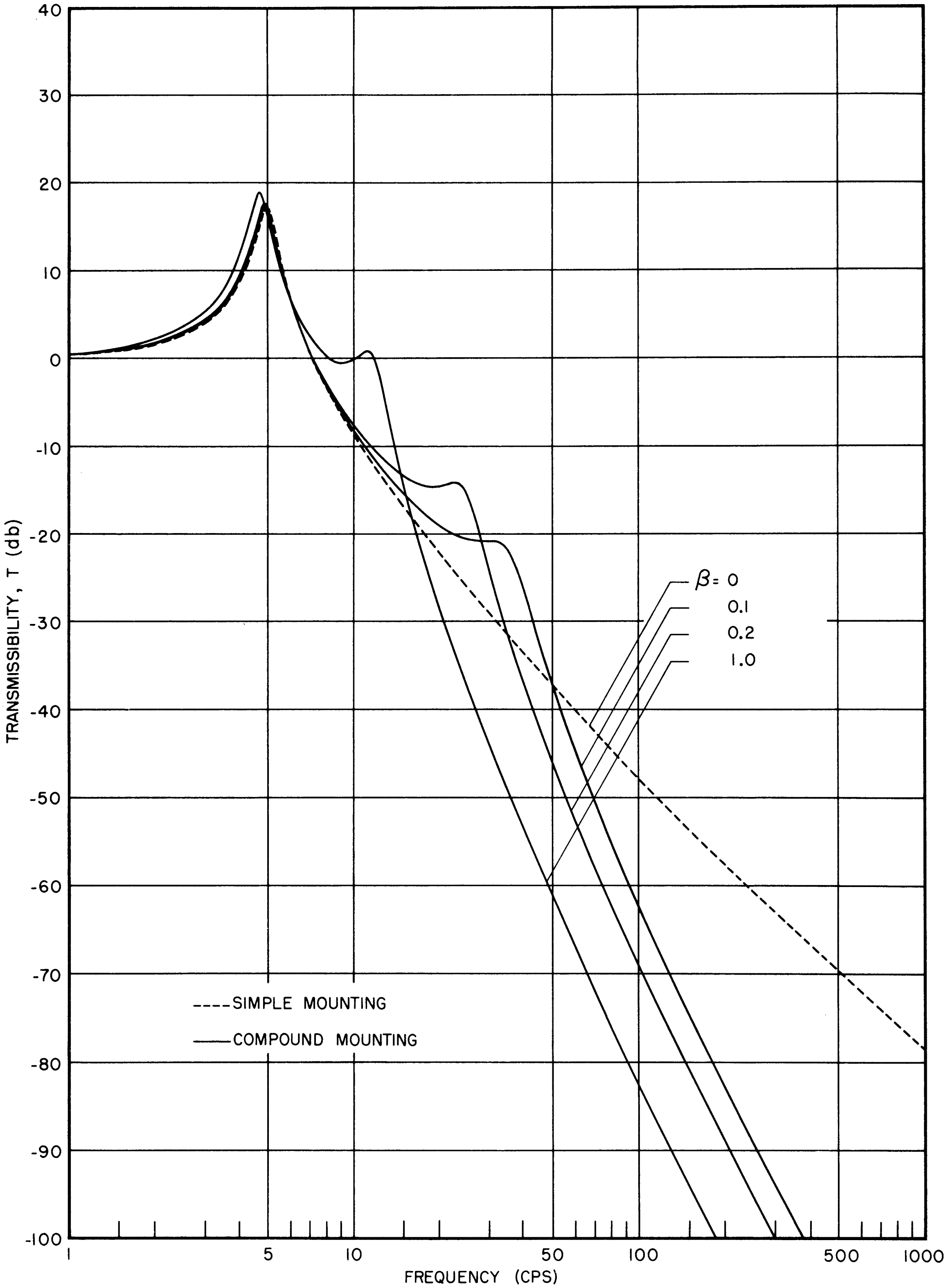


Fig. 17. The transmissibility of a compound system employing parallel mounts comprised of vulcanized hevea and Thiokol R. D. Secondary mass one-tenth, one-fifth, and equal to the mass of the mounted item.

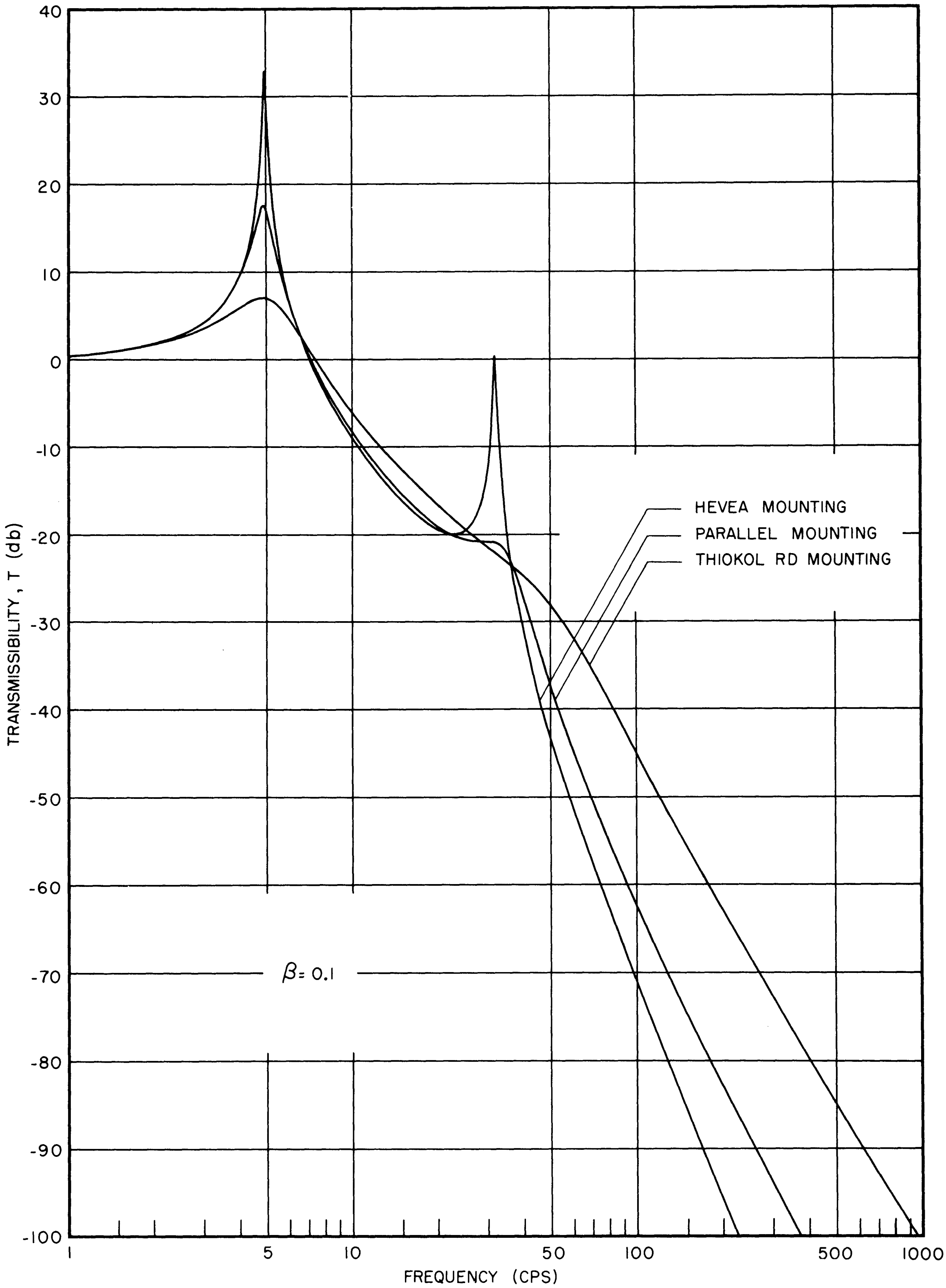


Fig. 18. The transmissibility of a compound system employing vulcanized hevea, Thiokol R. D., and parallel mounts. Secondary mass one-tenth of the mass of the mounted item.

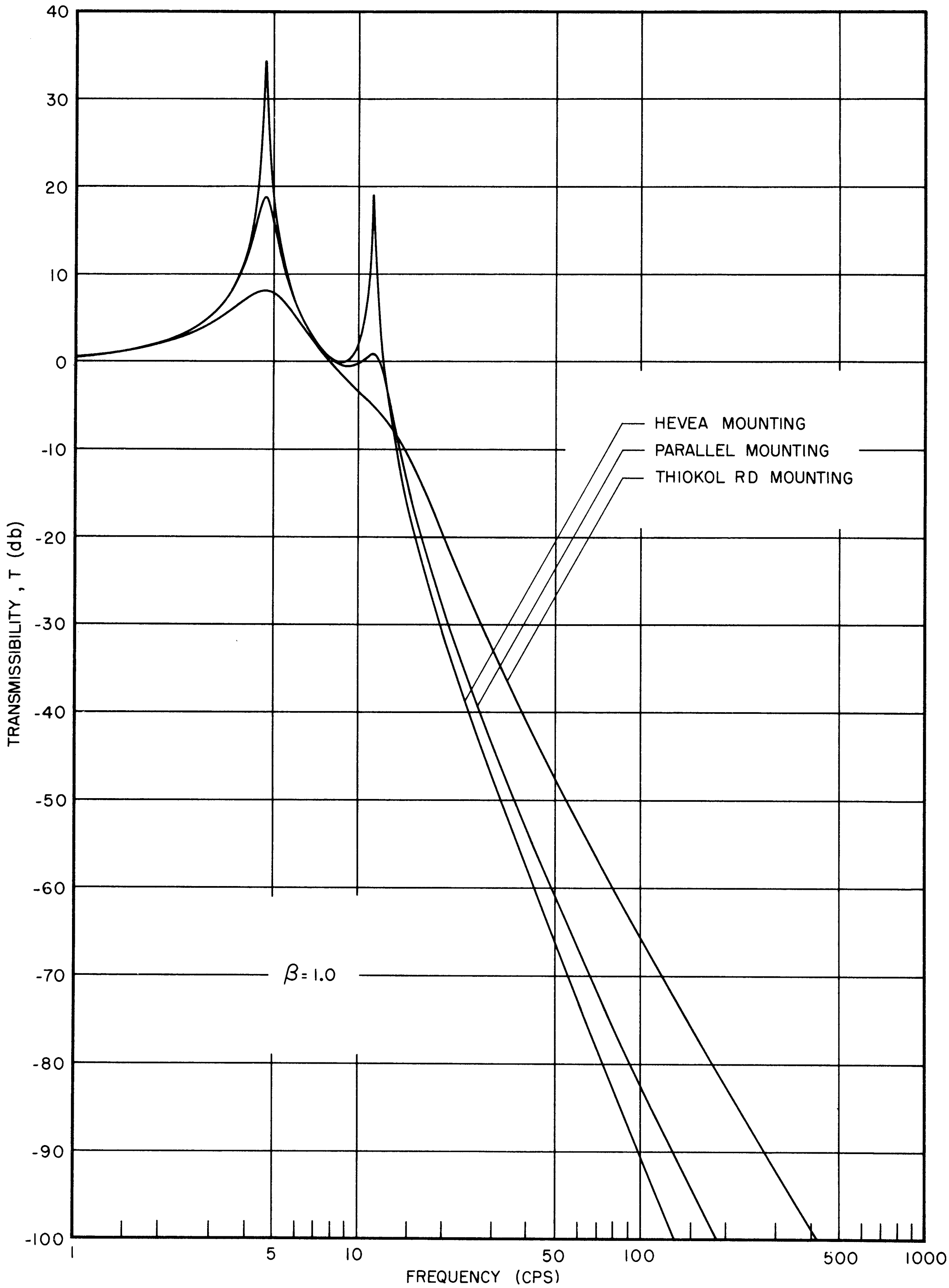


Fig. 19. The transmissibility of a compound system employing vulcanized hevea, Thiokol R. D., and parallel mounts. Secondary mass equal to the mass of the mounted item.

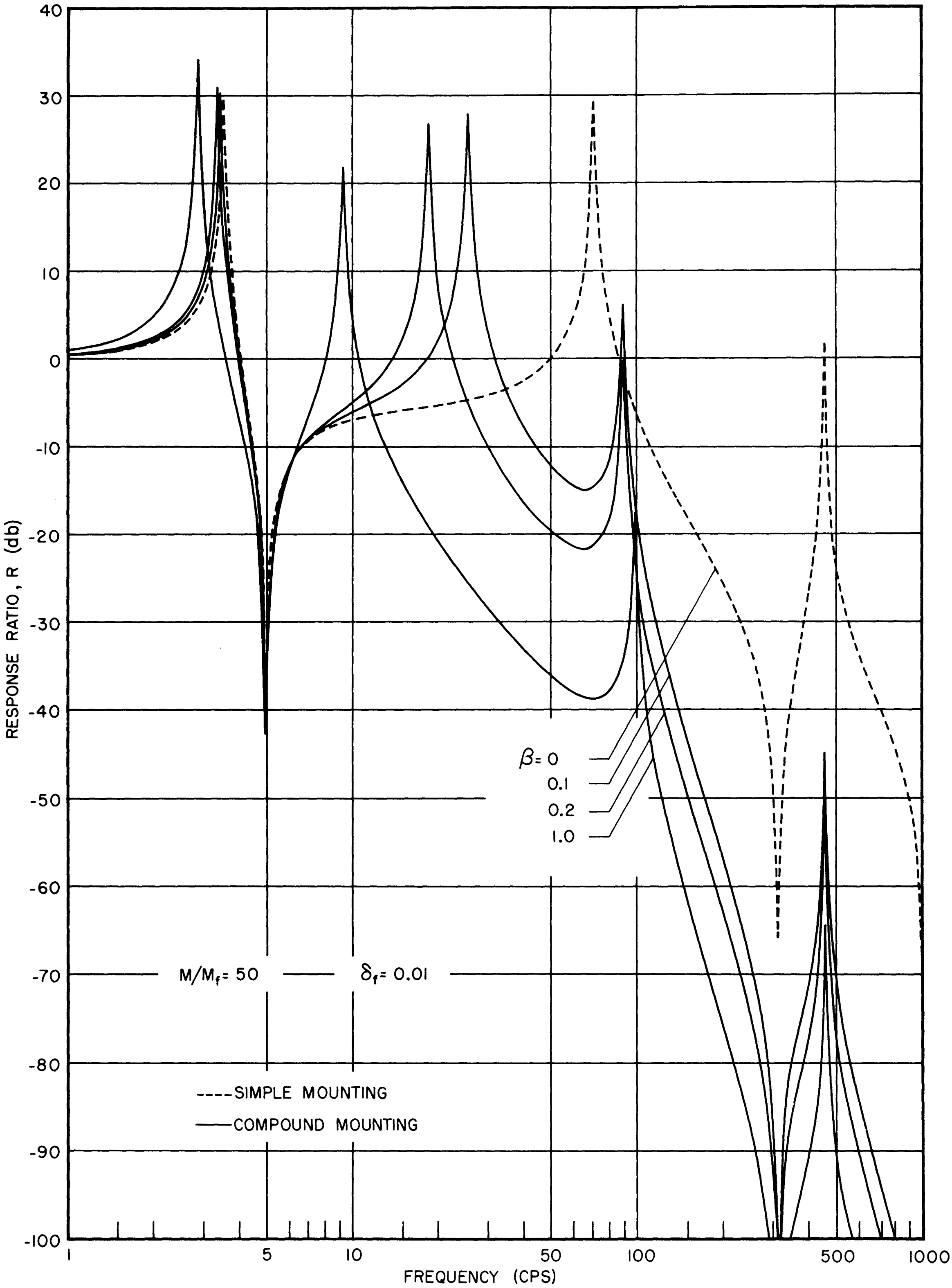


Fig. 20. The response ratio of a compound system employing vulcanized hevea mounts. Mounted item fifty times more massive than the foundation. Secondary mass one-tenth, one-fifth, and equal to the mass of the mounted item. Foundation damping defined by $\delta_f = 0.01$.

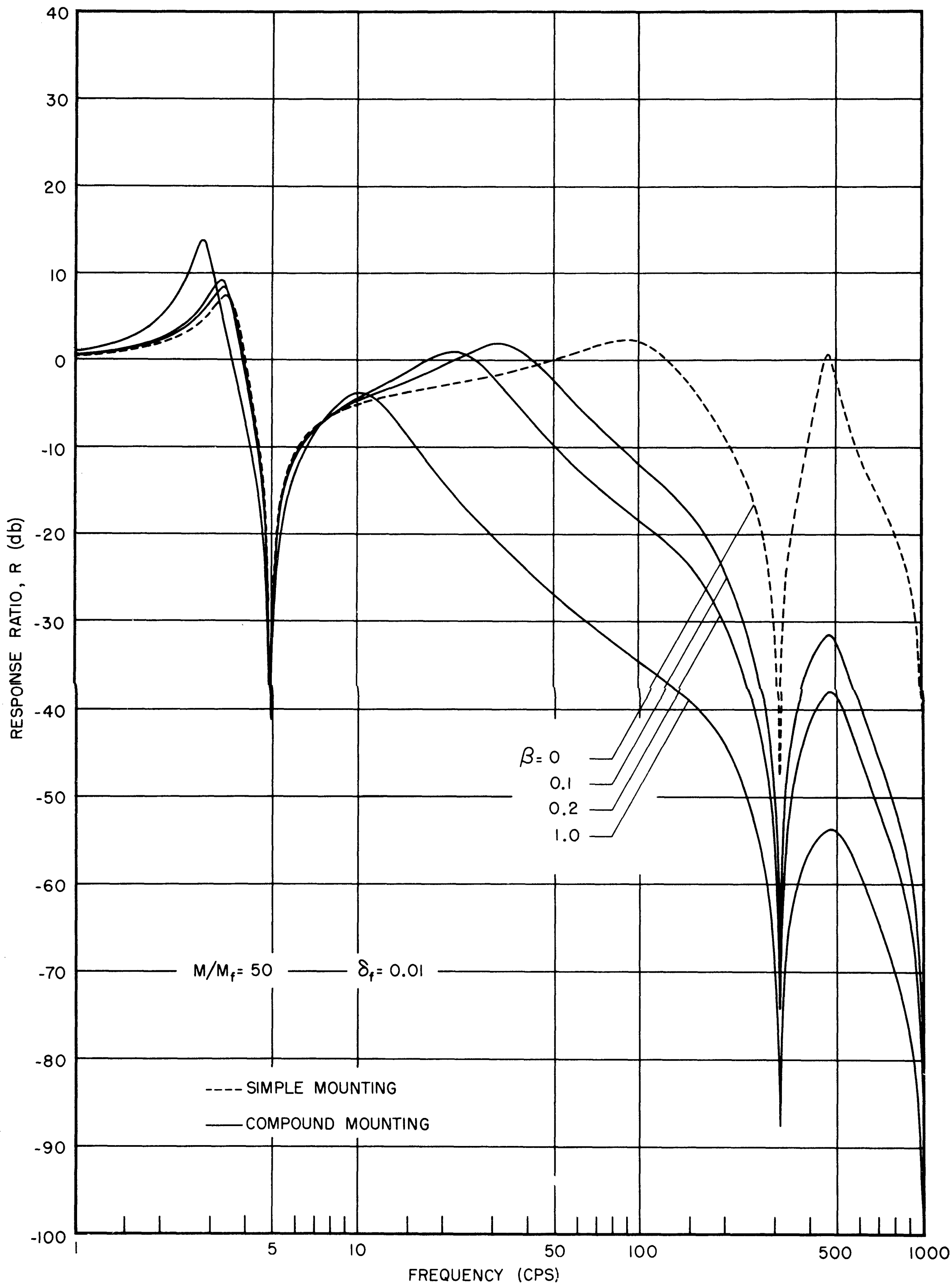


Fig. 21. The response ratio of a compound system employing Thiokol R. D. mounts. Mounted item fifty times more massive than the foundation. Secondary mass one-tenth, one-fifth, and equal to the mass of the mounted item. Foundation damping defined by $\delta_f = 0.01$.

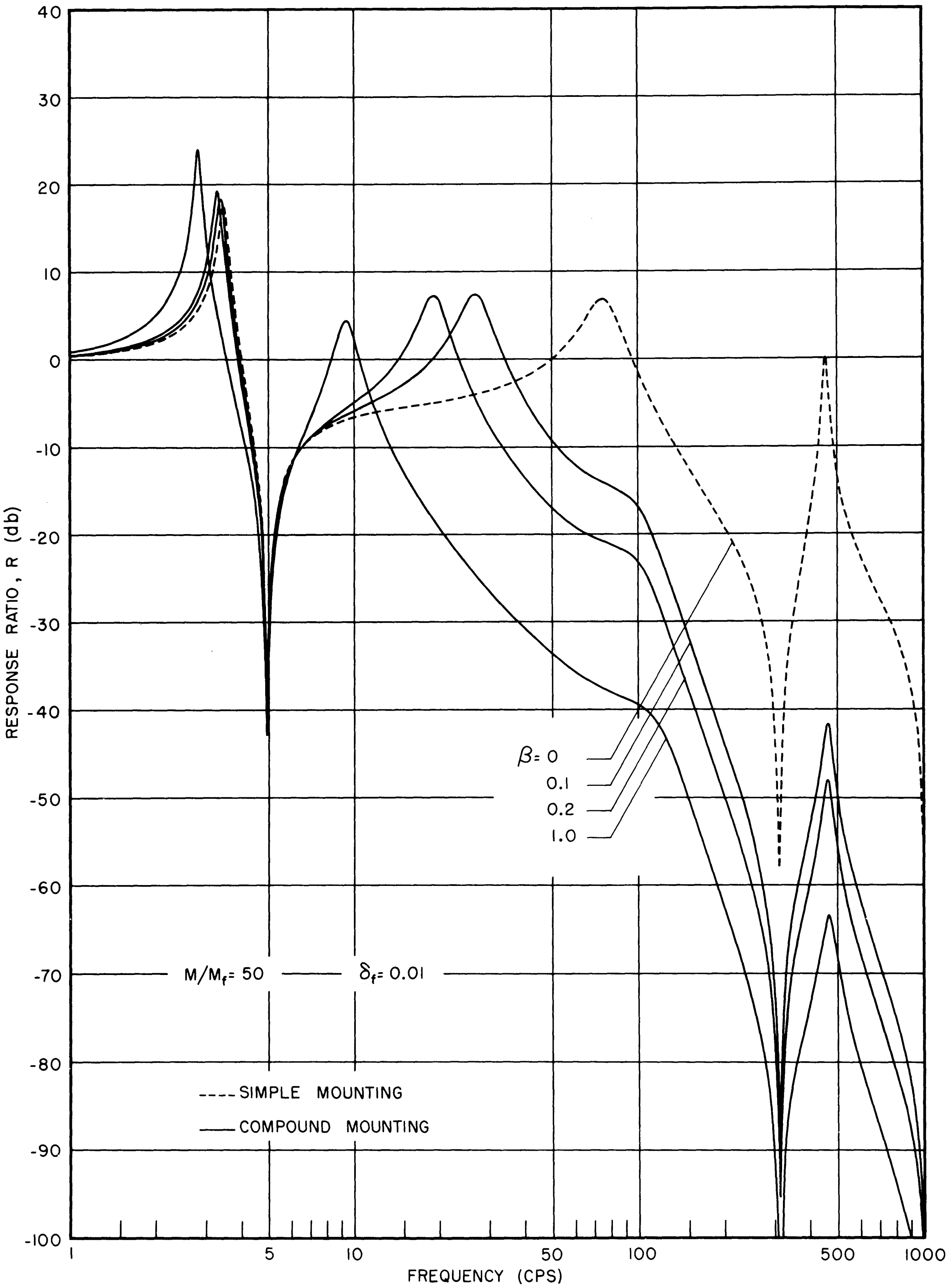


Fig. 22. The response ratio of a compound system employing parallel mounts comprised of vulcanized hevea and Thiokol R. D. Mounted item fifty times more massive than the foundation. Secondary mass one-tenth, one-fifth, and equal to the mass of the mounted item. Foundation damping defined by $\delta_f = 0.01$.

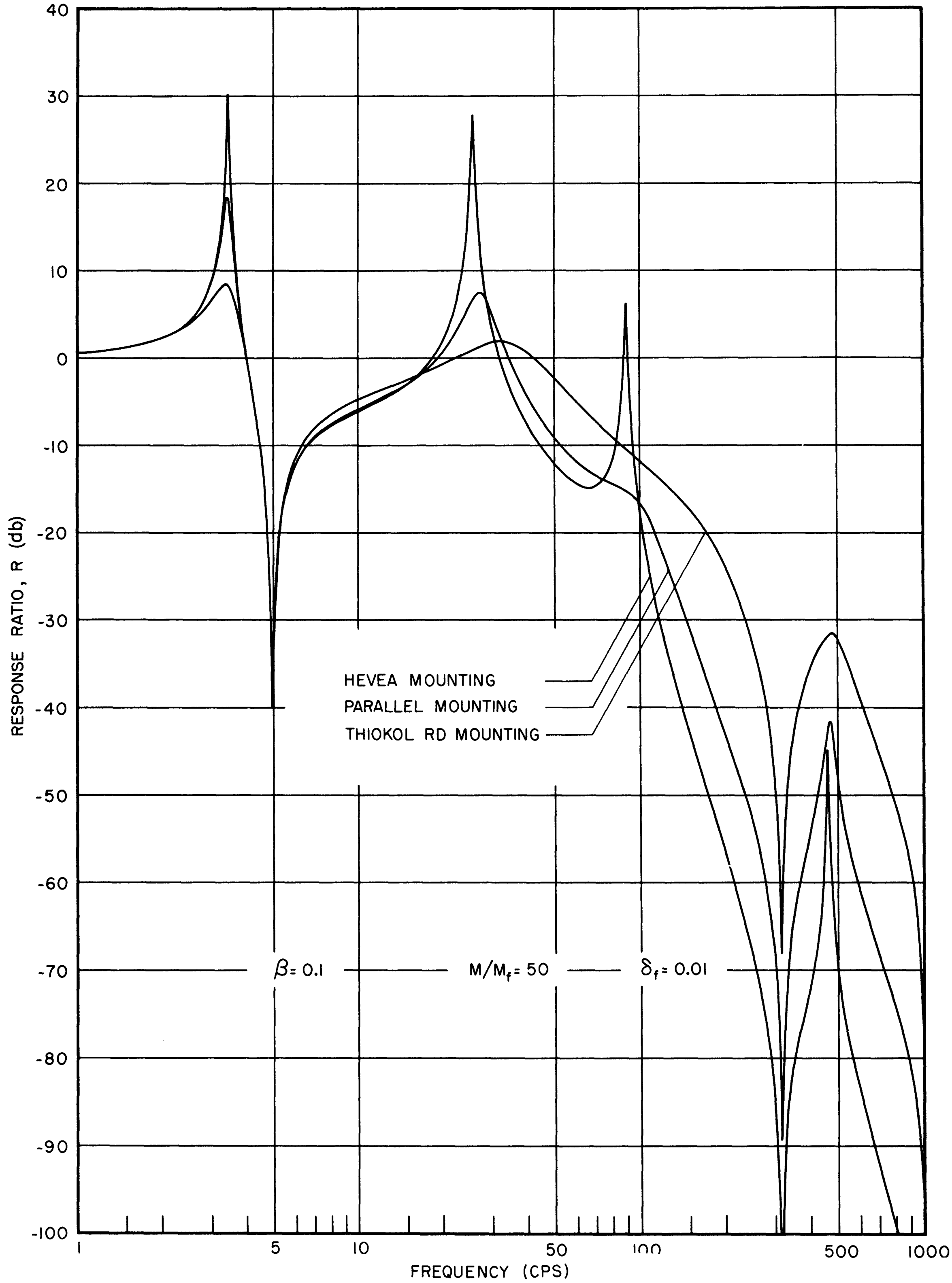


Fig. 23. The response ratio of a compound system employing vulcanized hevea, Thiokol R. D., and parallel mounts. Mounted item fifty times more massive than the foundation. Secondary mass one-tenth of the mass of the mounted item. Foundation damping defined by $\delta_f = 0.01$.

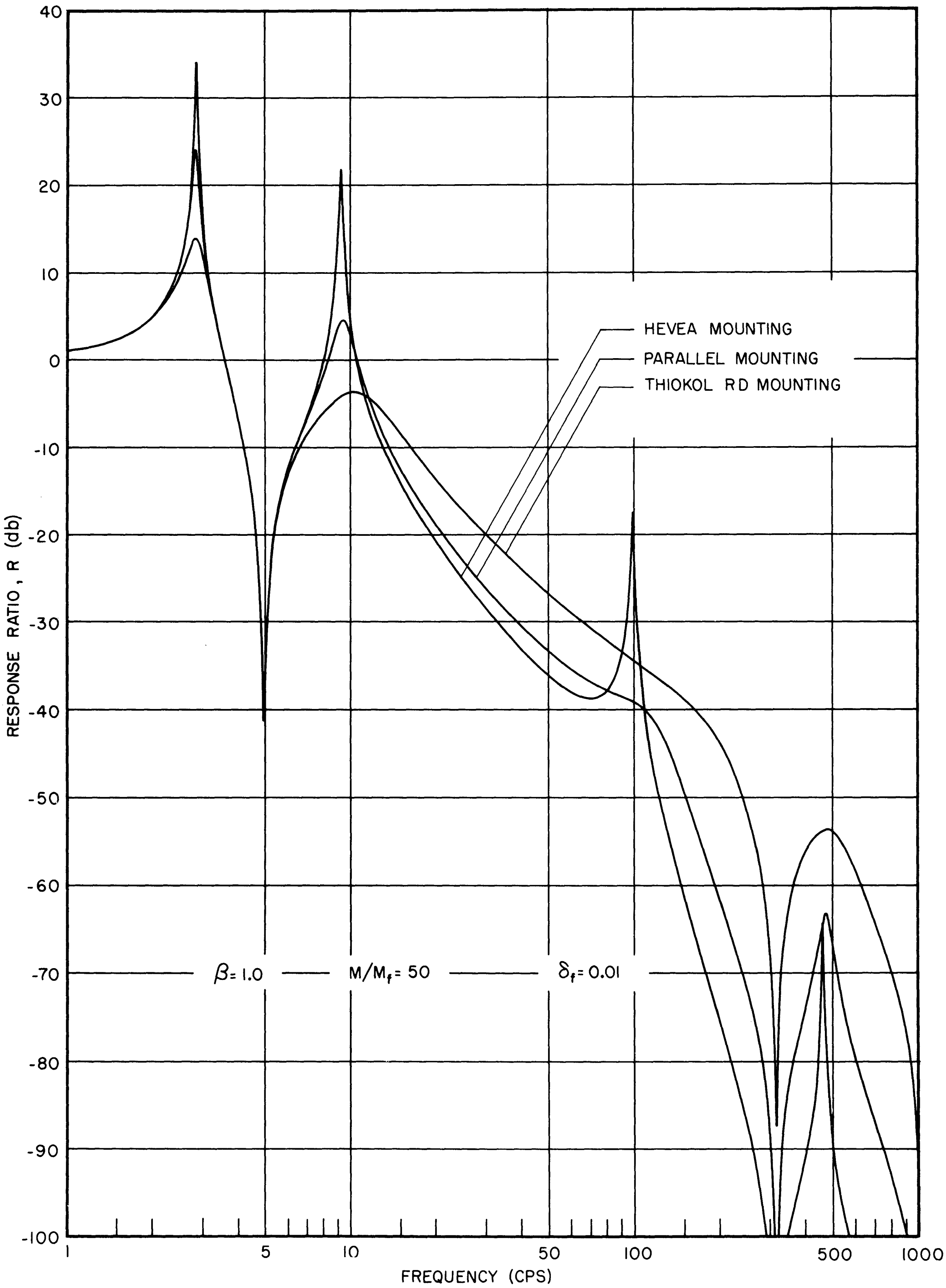


Fig. 24. The response ratio of a compound system employing vulcanized hevea, Thiokol R. D., and parallel mounts. Mounted item fifty times more massive than the foundation. Secondary mass equal to the mass of the mounted item. Foundation damping defined by $\delta_f = 0.01$.

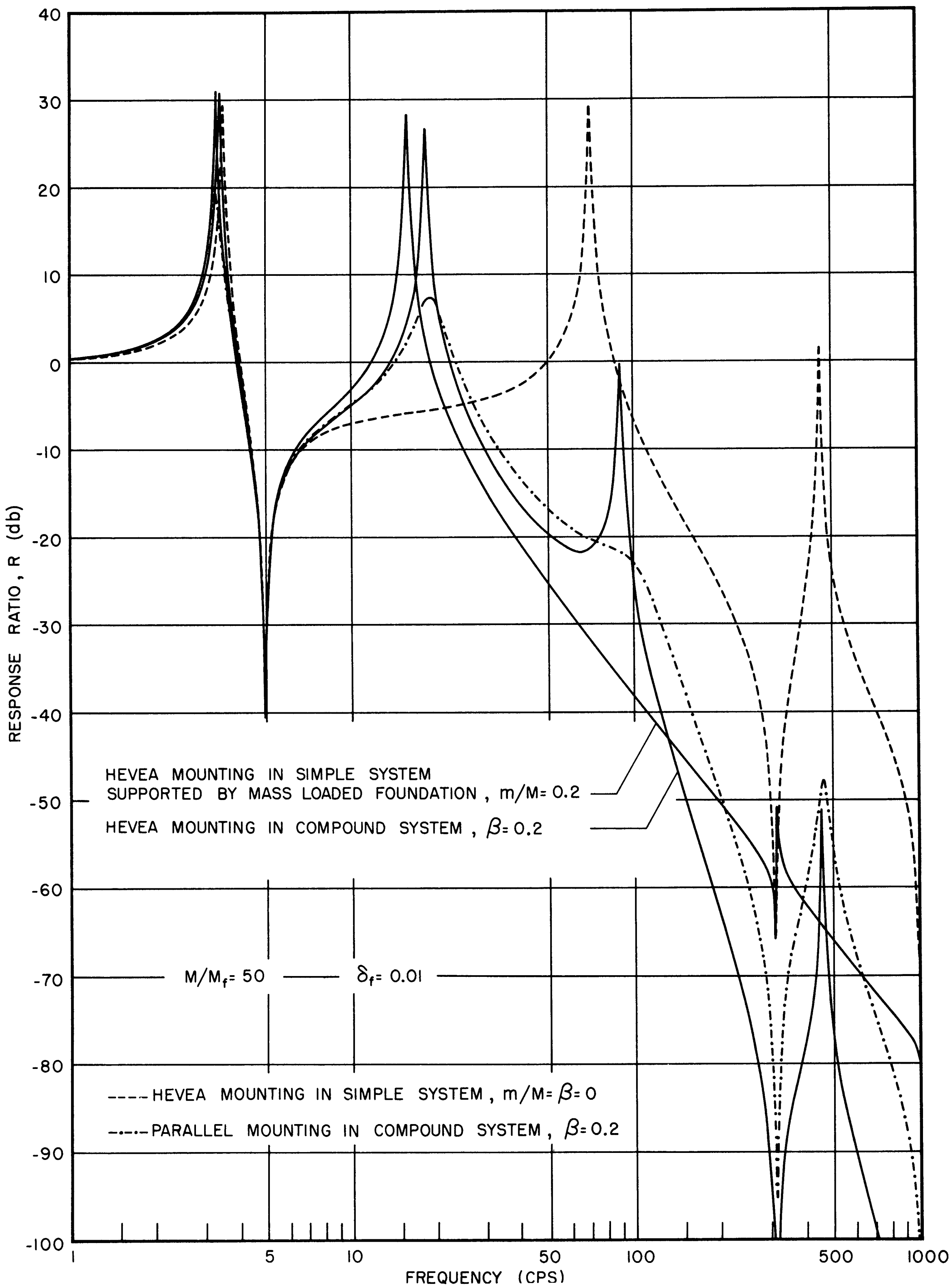


Fig. 25. The response ratio of a compound system employing vulcanized hevea mounts, and the response ratio of a simple mounting of vulcanized hevea supported by a mass-loaded foundation. Mounted item fifty times more massive than the foundation. Secondary mass one-fifth of the mass of the mounted item ($\beta = 0.2$). Loading mass one-fifth of the mass of the mounted item ($m/M = 0.2$). Foundation damping defined by $\delta_f = 0.01$.

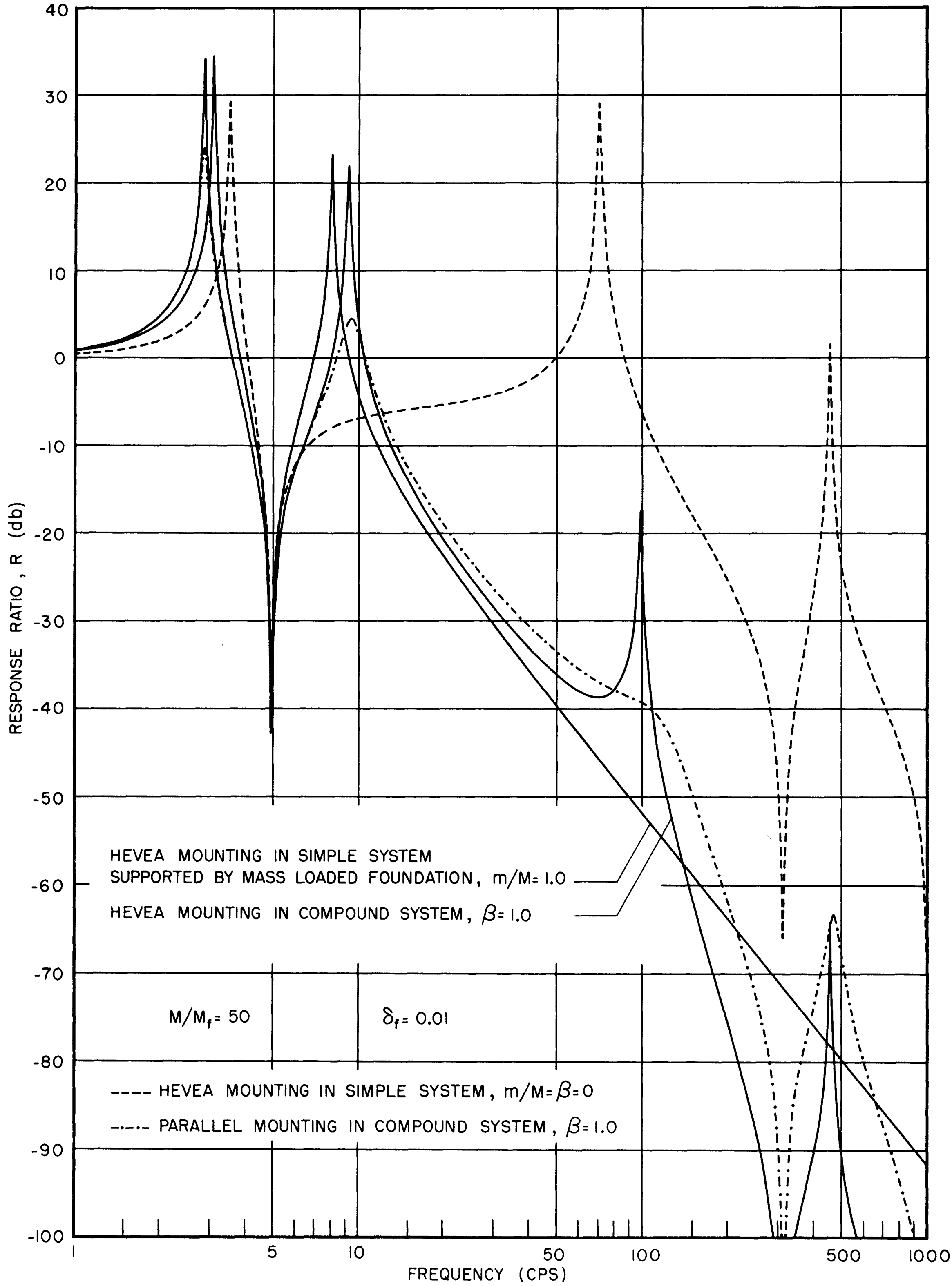


Fig. 26. The response ratio of a compound system employing vulcanized hevea mounts, and the response ratio of a simple mounting of vulcanized hevea supported by a mass-loaded foundation. Mounted item fifty times more massive than the foundation. Secondary mass equal to the mass of the mounted item (β = 1.0). Loading mass equal to the mass of the mounted item (m/M = 1.0). Foundation damping defined by δ_f = 0.01.

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44 p. incl. illus., 5 refs.
(Rept. no. 2892-6-T)
(Contract NObs 77072) Unclassified report

The performance of simple and compound mounting systems supported by a foundation of finite mechanical impedance, and the performance of the simple mounting supported by a mass-loaded foundation, have been theoretically determined and compared. A simply supported damped beam has been employed to simulate the behavior of the foundation. The dynamic mechanical properties of natural rubber and a high-damping rubber have been employed to describe the behavior of anti-vibration mount materials.

When the ratio of the mass of the mounted item to the mass of the foundation is large, the isolation (over)

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2. Rubber—Mechanical properties
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REDUCTION OF THE RESPONSE TO VIBRATION OF STRUCTURES POSSESSING FINITE MECHANICAL IMPEDANCE, PART II, by J. C. Snowdon. Technical rept. January 1960.
44 p. incl. illus., 5 refs.
(Rept. no. 2892-6-T)
(Contract NObs 77072) Unclassified report

The performance of simple and compound mounting systems supported by a foundation of finite mechanical impedance, and the performance of the simple mounting supported by a mass-loaded foundation, have been theoretically determined and compared. A simply supported damped beam has been employed to simulate the behavior of the foundation. The dynamic mechanical properties of natural rubber and a high-damping rubber have been employed to describe the behavior of anti-vibration mount materials.

When the ratio of the mass of the mounted item to the mass of the foundation is large, the isolation (over)

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2. Rubber—Mechanical properties
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