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THE STEADY-STATE AND TRANSIENT BEHAVIOR OF THE DYNAMIC ABSORBER

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This report has been composed from notes and analyses made over a period of several years. It is not easy to attribute portions of this study accurately to specific periods of time, but it is true to say that the majority of the conclusions and analyses within Sections 2, 4.2, 4.3, and 9 were made at The University of Michigan Research Institute since April, 1958. Likewise, the contents of Sections 7 and 8.2, and the initial portion of Section 4.2 were completed at the Imperial College of Science and Technology, London, prior to October, 1956. The analyses and computations described in Section 8.3 were completed during the intervening period, and are thought to represent one major portion of the total effort and time devoted to this study.

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ABSTRACT

The resonant vibration of machinery and resiliently mounted equipment can be reduced with a dynamic absorber of suitable design. The necessary analytical and graphical information for an optimum design is presented here. It is assumed that the absorber mass is attached resiliently to the vibrating item with a rubber-like material, and not with a spring as considered in the classical theory of Ormondroyd and Den Hartog. The dynamic absorber utilizing a material with a stiffness proportional to frequency and a constant damping factor can reduce the resonant vibration of machinery and equipment items considerably. Its performance is superior to that of the classical dynamic absorber which possesses damping of the viscous type.

The transient motion of a resiliently mounted item can also be reduced with a dynamic absorber of optimum design. An original treatment of this problem is given here for absorber damping of the viscous type. The transient motion of the mounted item is assumed to result from step-like foundation displacements which possess a wide range of rise times. This motion is characterized by the rapid fashion in which it decays with time. In general, if the contents of a mounted item are to receive the greatest possible protection from damage due to shock, the maximum acceleration and displacement of the mounted item must simultaneously be small. The dynamic absorber with large absorber masses comes close to satisfying this requirement, in contrast to the simple mounting system.

A. THE STEADY-STATE BEHAVIOR OF THE DYNAMIC ABSORBER

1. INTRODUCTION

The analysis and conclusions of this section are phrased in terms of translational rather than rotational motion, but they are equally applicable to both. In fact, some of the more important applications of the dynamic absorber are concerned with the reduction of rotational vibration. The steady-state theory of the dynamic absorber with viscous-type damping has long been known.[1,2,3] Viscous damping, however, only characterizes the damping of a dashpot or an air-damped mounting, for example. No attempt has been made previously to consider natural or synthetic rubbers as the damped resilient "spring" attaching the absorber to the vibrating item. Such an analysis is presented below.

The assumptions made here about the behavior of the dynamic stiffness and damping possessed by rubber-like materials have been employed before,[4] and shown to provide results comparing quite favorably with experimental data over nearly two decades in frequency. Since the dynamic absorber is effective over a somewhat smaller frequency range, these assumptions should be even more sound in the following applications. The short discussion of the viscously damped absorber is included merely to introduce the solution to the conventional problem, and to provide a standard against which the performance of the dynamic absorber with rubber-like damping may be compared.

In common with other discussions of the dynamic absorber, damping in the resilient element which supports the vibrating item is neglected early in the analysis. This is necessary if criteria for the design of the dynamic absorber are to be obtained simply and in a relatively concise form.

2. THE GENERAL TRANSMISSIBILITY EQUATION OF THE DYNAMIC ABSORBER

Figure 1a shows an item of equipment or machinery of mass M_1 which is supported resiliently by an isolator of complex stiffness $\overline{E}_1$[5,4] upon a sinusoidally vibrating foundation. If the complex amplitudes of M_1 and the foundation are x_1 and x_0, respectively, then the absolute value of the ratio x_1/x_0 is defined as the transmissibility, T, of the system. If the resiliently supported mass M_1 were vibrating upon a foundation possessing infinite impedance, then the absolute value of that fraction of the force exciting M_1 which is transmitted to the foundation would also equal transmissibility. The dual significance of the expressions obtained for T should always be borne in mind. Shown also in Fig. 1a are the absorber mass M_2 and resilient absorber element of complex stiffness $\overline{E}_2$. The displacement amplitude of M_2 is x_2. It can be shown that:

$$\frac{x_1}{x_0} = \frac{\bar{E}_1(\bar{E}_2/M_2 - \omega^2)}{[(\bar{E}_2/M_2 - \omega^2)(\bar{E}_1 - M_1\omega^2) - \bar{E}_2\omega^2]} , \qquad (2.1)$$

where ω is the angular frequency of the sinusoidal foundation vibration, hereafter known simply as frequency.

Let $\mu = M_1/(M_1+M_2)$, $y = \omega/\omega_0$, and $n = \bar{\omega}/\omega_0$, where ω_0 and $\bar{\omega}$ are defined as the natural frequencies of the loaded primary mounting and the absorber, respectively. Assuming that the stiffness of $\bar{E}_1$ is entirely real and equal to E_1, then $\omega_0^2 = E_1/(M_1+M_2) = \mu E_1/M_1$. Since the complex stiffness $\bar{E}_2$ at any frequency ω may be written:

$$\bar{E}_2 = E_\omega(1+j\delta_\omega) ,$$

then $\bar{\omega}^2 = E_{\bar{\omega}}/M_2$. The damping factor,[4] δ_ω, of the isolator is defined as the ratio of the imaginary to the real part of its complex stiffness.

It follows from Eq. (2.1) that:

$$T^2 = \left|\frac{x_1}{x_0}\right|^2 = \frac{[n^2-y^2(E_{\bar{\omega}}/E_\omega)]^2 + (n^2\delta_\omega)^2}{[\mu y^4(E_{\bar{\omega}}/E_\omega)-y^2(n^2+E_{\bar{\omega}}/E_\omega)+n^2]^2 + (n^2\delta_\omega)^2(1-y^2)^2} . \qquad (2.2)$$

From this general equation the performance of any dynamic absorber can be obtained once the correct variation with frequency of the stiffness and damping factor of the absorber "spring" are inserted.

It is apparent from Eq. (2.2) that when the frequency ratio, y, is large, transmissibility is inversely proportional to y^2 and the mass ratio μ. When absorber damping, δ_ω, becomes infinitely large, the transmissibility becomes that of the undamped simple system,[6] namely:

$$T = \pm\frac{1}{(1-y^2)} . \qquad (2.3)$$

When δ_ω is zero, the general transmissibility equation simplifies to:

$$T = \frac{\pm(n^2-y^2\alpha)}{[\mu y^4\alpha - y^2(n^2+\alpha) + n^2]} , \qquad (2.4)$$

where $\alpha = E_{\bar{\omega}}/E_\omega$.

The transmissibility becomes independent of the damping factor δ_ω at two frequencies, namely, y_a and y_b, which are given by the relation:

$$\frac{1}{(n^2-y^2\alpha)^2} = \frac{(1-y^2)^2}{[\mu y^4\alpha - y^2(n^2+\alpha) + n^2]^2} \tag{2.5}$$

At these frequencies, for all values of δ_ω, the transmissibility takes the same values, namely, T_a and T_b (see Fig. 2). The transmissibility of the dynamic absorber for any value of δ_ω lies between the two curves shown in this figure, and in general possesses either one maximum within the frequency range y_a to y_b, or two maxima outside this range.

The dynamic absorber is said to be most favorably tuned and damped when the two maximum values of transmissibility are equal. The classical procedure[1,2,3,7] is to assume that the maximum values of T actually occur at the frequencies y_a and y_b. The parameter n is then chosen so that T_a and T_b become equal, and δ_ω so that the maxima of the transmissibility curve take the values T_a and T_b. It follows from Eq. (2.3) that T_a and T_b become equal when:

$$y_a^2 + y_b^2 = 2 \quad . \tag{2.6}$$

From Eqs. (2.5) and (2.6) it is therefore possible to derive an optimum value of n equal to n_o, say.

Determination of the optimum value of δ_ω is algebraically tedious. Fundamentally, it involves writing the general equation in the form:

$$T^2 = \frac{A + B\,\delta_\omega^2}{C + D\,\delta_\omega^2} \;,$$

where, for optimum tuning conditions, A, B, C, and D are functions of n_o and y. Solving for δ_ω gives:

$$\delta_\omega^2 = \frac{CT^2 - A}{B - DT^2} \tag{2.7}$$

Substituting for A, B, C, and D, and $T_{a,b}$ in Eq. (2.7) to obtain optimum damping $(\delta_\omega)_o$, say, provides only an indeterminate expression o/o for $(\delta_\omega)_o$, since the frequencies y_a and y_b are independent of δ_ω. If however, L'Hopital's Rule[8] is applied to Eq. (2.7) or a slightly different frequency substituted in the equation such as $(y+c)$ or sometimes (y^2+c) where c is a very small quantity, then a finite ratio of two "second order" terms can be obtained for δ_ω.

3. ABSORBER DAMPING OF THE VISCOUS TYPE

Viscous or "dashpot" damping is commonly discussed in the literature concerned with vibration isolation. The analysis presented in this section is essentially similar to that of earlier workers.[1,2,3,7] The resilient absorber element is considered to be a spring of constant stiffness E_2 (equal to $\bar{\omega}^2 M_2$) placed in parallel with a dashpot possessing constant coefficient of viscosity η.

It is more convenient in the case of viscous damping to define a damping ratio $\bar{\delta}$ which is characteristic of the mounting system as a whole. This quantity is defined as the ratio of the coefficient of viscosity possessed by the dashpot to that required to damp the system critically. It can be shown[4] that:

$$\delta_\omega = 2y\,\bar{\delta} \ .$$

Substituting for δ_ω and equating α, which is the ratio $E_{\bar{\omega}}/E_\omega$ to unity in Eq. (2.2) gives:

$$T^2 = \frac{(n^2-y^2)^2 + (2n^2y\bar{\delta})^2}{[\mu y^4 - y^2(n^2+1) + n^2]^2 + (2n^2y\bar{\delta})(1-y^2)^2} \ . \qquad (3.1)$$

Equating α to unity in Eq. (2.5) and solving for $y_{a,b}$ gives:

$$y^2_{a,b} = \frac{(1+n^2) \mp \sqrt{[(1+n^2)^2 - 2n^2(1+\mu)]}}{(1+\mu)} \ . \qquad (3.2)$$

By substituting for y_a and y_b in Eq. (2.6), it is possible to obtain the optimum value of the tuning parameter n, namely:

$$n_o = \sqrt{\mu} \ . \qquad (3.3)$$

For this value of n:

$$y^2_{a,b} = 1 \mp \sqrt{\frac{1-\mu}{1+\mu}} \ , \qquad (3.4)$$

and

$$T_{a,b} = \sqrt{\frac{1+\mu}{1-\mu}} \qquad (3.5)$$

Figure 3 shows the resonant transmissibility* of the dynamic absorber computed from Eq. (3.1). The curves are drawn for the most favorably tuned absorber one-fifth as massive as the vibrating item. Different values of the damping ratio $\bar{\delta}$ are considered, but unless the optimum value is employed, the maximum values of transmissibility remain unequal. It can be seen that the optimum value of $\bar{\delta}$ lies between 0.2 and 0.4.

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*Transmissibility is presented here on a decibel scale; thus a force ratio equal to T appears on a decibel scale as $20 \log_{10}(T)$ decibels.

Optimum absorber damping is derived by the method outlined in Section 2. It can be shown that:

$$\bar{\delta}_o^2 = \left(\frac{1-\mu}{1+\mu}\right)\left[\frac{(1+\mu)}{2\mu} - \frac{1}{4y_{a,b}^2}\right] . \tag{3.6}$$

The two optimum values of $\bar{\delta}$ given by this equation are such as to make the maxima of the transmissibility curve equal to T_a or T_b. The mean of these values is usually quoted as "optimum" damping, and this quantity is plotted in Fig. 4 for a wide range of absorber masses. It can be computed from the expression:

$$\bar{\delta}_m = \frac{\bar{\delta}_{oa} + \bar{\delta}_{ob}}{2} = \frac{1}{2}\sqrt{\frac{1-\mu}{8\mu}}\left(\sqrt{2+y_a^2} + \sqrt{2+y_b^2}\right) , \tag{3.7}$$

although this may be written:

$$\bar{\delta}_m = \sqrt{\frac{3(1-\mu)}{8\mu}} \tag{3.8}$$

with less than one-half percent error providing that $\mu < 0.5$.

Once calculated, the values of optimum tuning and damping can be used to determine the transmissibility of the dynamic absorber for any given mass ratio, providing what is hereafter known as optimum transmissibility. Figure 5 shows three such curves calculated for values of μ = 10/11, 5/6, and 1/2. Mass ratios such as the first two might be used to reduce the resonant vibration from a machine reaching its foundation. A mass ratio of a half might be employed when isolating a delicate item of equipment from the vibration of its support. The broken curves are included for comparison. They refer to the simple mounting system with relatively low and high damping mounts. It can be seen that, as the frequency of vibration increases, the isolation afforded by the dynamic absorber improves more rapidly than that of the simple system.

The relative displacement between the principal and absorber masses at and near resonance is plotted in Fig. 6 as a ratio to the vibrational foundation amplitude. The curves are calculated for the most favorably tuned and damped absorbers.

4. ABSORBER DAMPING OF THE SOLID TYPE

4.1. Introduction.—A number of materials, including natural rubber, are found experimentally to have a damping factor which is approximately constant over quite a large frequency range. These materials are said to possess solid-type damping. It follows that the real and imaginary parts of the complex stiffness possessed by such materials must either show a similar frequency dependence or be approximately constant.

A damping factor $\delta_\omega = 0.1$ typifies the damping possessed by natural rubber. The damping factors of some synthetic rubbers are of this magnitude, but δ_ω is commonly the order unity for others. When δ_ω is relatively small, experiments show that the real part of the complex stiffness, E_ω, is almost constant, although when δ_ω is large, it increases very rapidly with frequency. In general, neither the relatively small nor the large values of δ_ω change very much with frequency.

Quite good theoretical agreement with experimental data over nearly two decades in frequency has been obtained previously[6] by assuming, for a low damping rubber, that δ_ω and E_ω are constants, and for a high damping rubber, that δ_ω is constant and E_ω is proportional to frequency. These assumptions are made here also, being employed under the headings of damping of the solid type I and solid type II, respectively.

No limitation is placed theoretically upon the values which δ_ω can take. In practice, it is quite unlikely that δ_ω will be large if damping is of the solid type I where E_ω is constant. It is equally unlikely that δ_ω will be small if damping is of the solid type II where E_ω is increasing rapidly with frequency.

4.2. Absorber Damping of the Solid Type I.—The absorber "spring" is considered here to be a rubber-like material possessing constant stiffness, E_2, and damping factor δ. The complex stiffness and the natural frequency of the absorber can therefore be written:

$$\bar{E}_2 = E_2\,(1+j\delta)$$

$$\bar{\omega}^2 = E_2/M_2 \quad .$$

Because the stiffness ratio $E_{\bar{\omega}}/E_\omega$ is now equal to unity, the general transmissibility equation simplifies to:

$$T^2 = \frac{(n^2-y^2)^2 + (n^2\delta)^2}{[\mu y^4 - y^2(n^2+1) + n^2]^2 + (n^2\delta)^2(1-y^2)^2} \quad , \qquad (4.1)$$

and the solution of Eq. (2.5) for $y_{a,b}$ yields the same result as before [Eq.(3.2)]. Since the relationship between y_a and y_b also remains unaltered [Eq. (2.6)], the optimum value of the tuning parameter is identical to that found for the viscously damped absorber, namely,

$$n_o = \sqrt{\mu} \quad . \qquad (4.2)$$

It follows that:

$$y^2_{a,b} = 1 \mp \sqrt{\frac{1-\mu}{1+\mu}} \quad , \tag{4.3}$$

and

$$T_{a,b} = \sqrt{\frac{1+\mu}{1-\mu}} \quad . \tag{4.4}$$

Optimum absorber damping is determined in the conventional manner discussed in Section 2. It can be shown that the two optimum values of δ are given by the equation:

$$\delta_o^2 = \left(\frac{1-\mu}{1+\mu}\right)\left[\frac{2\ (1+\mu)}{\mu}\ y_{a,b} - 1\right] \quad , \tag{4.5}$$

and their mean by the equation:

$$\delta_m = \frac{\delta_{oa} + \delta_{ob}}{2} = \frac{1}{2}\sqrt{\frac{1-\mu}{2\mu}}\left[y_a\ \sqrt{2+y_a^2} + y_b\ \sqrt{2+y_b^2}\right] \quad . \tag{4.6}$$

Although the two optimum values of viscous-type absorber damping are similar—their mean provides good results in practice (Fig. 4)—the two values of optimum damping determined here are significantly different unless the absorber mass is very small compared with the main mass. Consequently, it is found that when their arithmetic mean, δ_m, is employed to calculate optimum transmissibility the two maximum values of T are far from equal.

Reference to Fig. 7 enables this to be more clearly understood. In the figure the optimum and mean values of $\bar{\delta}$ are shown at y=1. Relatively little difference exists between them. Because of the linear frequency dependence shown by viscous damping, the optimum absorber damping at y_a and y_b is equal to $y_a\bar{\delta}_{oa}$ and $y_b\bar{\delta}_{ob}$, respectively. These values do not depart far from the values of the arithmetic mean $y_a\bar{\delta}_m$ or $y_b\bar{\delta}_m$. On the other hand, the optimum values of the frequency independent damping δ_{oa} and δ_{ob} are not similar since $\delta_{oa}/2$ and $\delta_{ob}/2$ are of the order $y_a\bar{\delta}_{oa}$ and $y_b\bar{\delta}_{ob}$, respectively. Consequently, although their mean value, δ_m, is practically identical with $\bar{\delta}_m$, it differs significantly from both $\delta_{oa}/2$ and $\delta_{ob}/2$.

A modified average of optimum absorber damping is therefore required. It was felt intuitively that this should involve the quantities $(1-y_a)$, (y_b-1), and the optimum tuning parameter. It was not difficult to discover a multiplier, ξ, such that an "optimum" damping $\xi\delta_m$ would give two sensibly equal values of maximum transmissibility. A very satisfactory relation was found to be:

$$\xi = \left(\frac{10.5 - n_o^2}{10}\right)\sqrt{\frac{1-y_a}{y_b-1}} \quad . \tag{4.7}$$

"Optimum" damping (namely, $\xi\delta_m$) is plotted in Fig. 4. The damping factor of a material required for an absorber "spring" can therefore be obtained simply when the ratio of absorber to principal masses is known. In practice, however, it will probably be the damping possessed by a suitable material which will guide the choice of absorber mass.

Figure 8 shows three optimum transmissibility curves computed for mass ratios $\mu = 10/11, 5/6, 1/2$. The broken curves refer to the simple system with mount damping of the solid type I. The high-frequency isolation of the dynamic absorber and simple system increase at an identical rate. The accompanying variation of the relative displacement at and near resonance between the principal and absorber masses is shown in Fig. 6.

Because the unadjusted mean value of optimum absorber damping is unsatisfactory, the expression obtained [Eq. (4.4)] for maximum transmissibility is not accurate. It underestimates the actual value calculated from the transmissibility equation [Eq. (4.1)]. Compared with absorber damping of the viscous type, the difference is not more than 0.5 db when $\mu \geqq 5/6$, but is equal to 1.25 db when $\mu = 1/2$. The relative displacement between the absorber and principal masses also differs by slightly larger amounts.

4.3. Absorber Damping of the Solid Type II.—The resilient material employed as the absorber "spring" is here considered to possess a stiffness which is proportional to frequency, and a damping factor which is constant. In this case, the complex stiffness of the material and the parameter α can be written:

$$\overline{E}_2 = E_\omega (1+j\delta)$$

$$\alpha = \frac{E_{\bar{\omega}}}{\overline{E}_\omega} = \frac{\bar{\omega}}{\omega} \quad ,$$

while the natural frequency of the dynamic absorber and the product αy become:

$$\bar{\omega}^2 = E_{\bar{\omega}}/M_2$$

$$\alpha y = \bar{\omega}/\omega_o = n \quad .$$

Substitution of these relations in the general transmissibility equation gives:

$$T^2 = \frac{(n-y)^2 + (n\delta)^2}{(\mu y^3 - ny^2 - y + n)^2 + (n\delta)^2 (1-y^2)^2} \quad . \tag{4.8}$$

Since the stiffness E_ω is proportional to ω, the natural frequency of the absorber is proportional to $\omega^{1/2}$, and therefore not constant as before but removed to progressively higher or lower frequencies when ω varies. It is not difficult to see that this absorber is "effective" over an increased range of frequencies because of the dependence of its stiffness upon frequency. Subsequent analysis shows that its performance is markedly superior to that of an absorber with a "spring" of constant stiffness.

The optimum value of the tuning parameter is found from Eqs. (2.5) and (2.6). The frequencies y_a and y_b form the two positive roots of the cubic equation obtained when n is substituted for αy in Eq. (2.5), namely,

$$(1+\mu)y^3 - 2ny^2 - 2y + 2n = 0 \quad . \tag{4.9}$$

If the values of T_a and T_b are to be equal (Section 2) then the roots, y_a and y_b, of this equation have to satisfy the Eq. (2.6), namely:

$$y_a^2 + y_b^2 = 2 \quad .$$

The optimum value of n for which this is true is given by the relation:

$$n_o = \mu \sqrt{\frac{2(1+\mu)}{(1+3\mu)}} \quad . \tag{4.10}$$

It can be shown that for this value of n:

$$y_{a,b}^2 = 1 \mp \sqrt{\left(\frac{1-\mu}{1+\mu}\right)\left(\frac{1+3\mu}{1+\mu}\right)} \quad , \tag{4.11}$$

and

$$T_{a,b} = \sqrt{\left(\frac{1+\mu}{1-\mu}\right)\left(\frac{1+\mu}{1+3\mu}\right)} \quad . \tag{4.12}$$

Therefore, compared to an absorber with constant "spring" stiffness, the separation of the frequencies y_a and y_b is increased by a factor approaching $\sqrt{2}$, and the maximum transmissibility reduced by nearly 3 db if the absorber mass is small.

Optimum damping can again be determined in the manner outlined in Section 2. The two optimum damping factors obtained differ significantly from their mean, δ_m, although not as seriously as in Section 4.2. It is therefore necessary to employ another multiplier such that its product with δ_m really provides optimum damping. A suitable quantity is found simply to be $\sqrt{\xi}$, where, as before:

$$\xi = \left(\frac{10.5 - n_\delta^2}{10}\right)\sqrt{\left(\frac{1-y_a}{y_b-1}\right)} \quad . \tag{4.13}$$

The use of the multiplier here means that the expression (4.12) underestimates the maximum value of transmissibility. It is found that the discrepancy is less than 0.5 db when $\mu = 1/2$, and decreases to a negligible amount when μ increases towards unity.

It can be shown that the two optimum values of δ are given by the equation:

$$\delta_o^2 = \frac{1}{2}\left(\frac{1-\mu}{1+\mu}\right)\left[\frac{(1+3\mu)}{2\mu} \; y_{a,b}^2 + 1\right] \quad , \tag{4.14}$$

so that their mean is simply:

$$\delta_m = \frac{\delta_{oa} + \delta_{ob}}{2} = \sqrt{\left(\frac{1-\mu}{1+\mu}\right)}\left[\sqrt{1 + \frac{(1+3\mu)}{4\mu} \; y_a^2} + \sqrt{1 + \frac{(1+3\mu)}{4\mu} \; y_b^2}\,\right] . \tag{4.15}$$

Values of optimum damping (namely, $\sqrt{\xi}\,\delta_m$) are plotted in Fig. 4 as a function of absorber mass.

Figure 9 shows the optimum transmissibility of the dynamic absorber, and Fig. 6, the accompanying variation of the relative displacement between the absorber and principal masses for the mass ratios $\mu = 10/11$, 5/6, and 1/2. The broken curves of Fig. 9 refer to the simple system with mount damping of the solid type II. The isolation offered by the dynamic absorber above resonance is superior to that of the simple system, the transmissibility curves diverging at approximately 6 db/octave.

The favorably greater frequency range over which this absorber is effective is evident in both Figs. 6 and 9. The increased effectiveness of the absorber is due to the dependence of its stiffness upon frequency, as discussed earlier in this section.

The maximum values of T obtained for absorbers with variable "spring" stiffness are smaller than the resonant values previously obtained by at least 2.5 db if $\mu \geqq 5/6$, and by almost 3 db if $\mu \geqq 10/11$. When μ equals 0.5, maximum transmissibility is 3 db positive; this extremely low value is almost 2 db less than values obtained for absorbers with constant stiffness. The performance of this particular absorber is excellent because its maximum transmissibility equals that of a heavily damped ($\delta = 0.5$ or $\delta = 1$) simple system, and yet at high frequencies its isolation increases rapidly at 12 db/octave. It might be thought from Fig. 8 that high damping of the solid type I ($\delta = 1$) in the simple system would provide better overall isolation. As mentioned in Section 4.1, however, it is unlikely in practice that the stiffness of the mount material would be constant if δ were so large, although this was assumed when the curve was computed. It is more realistic, therefore, to compare the absorber performance only with the high damping curves of the simple system found in Figs. 5 and 9.

5. SUMMARY AND CONCLUSIONS

(1) The dynamic vibration absorber consists of a mass attached resiliently to a vibrating item, which resonates upon separate resilient supports or because of its own inherent elasticity. The absorber is normally "tuned" to within the resonant frequency range of the vibrating item. Since the absorber mass is in effect greatly magnified at resonance, it adds considerably to the inertia of the vibrating item over this frequency range, thus reducing its resonant transmissibility.

(2) The resilient element attaching the absorber to the vibrating item is assumed to possess a complex stiffness of the form:

$$\overline{E}_\omega = E_\omega (1+j\delta_\omega) \quad .$$

In general, both the dynamic stiffness, E_ω, and the damping factor, δ_ω, possessed by the resilient element are functions of the angular frequency ω. Three separate assumptions have been made about the dependence of E_ω and δ_ω upon frequency under the following headings:

Absorber Damping of the Viscous Type: δ_ω proportional to ω, E_ω constant.
Absorber Damping of the Solid Type I: δ_ω constant, E_ω constant.
Absorber Damping of the Solid Type II: δ_ω constant, E_ω proportional to ω.

The resilient absorber element invariably discussed by other workers is comprised of a spring in parallel with a dashpot. It possesses the properties listed above with damping of the viscous type, and is considered merely to provide a standard against which the performance of other absorbers can be judged.

The headings "Absorber Damping of the Solid Type I" and "Absorber Damping of the Solid Type II" cover analysis introduced here describing the performance of a dynamic absorber with a rubber-like material for the absorber "spring." The assumptions listed opposite these headings have been shown elsewhere to describe quite well the behavior of certain rubbers. It is more important to realize, however, that if certain material characteristics are found to be theoretically desirable, which is true here, then high-polymer technology admits a good chance of "tailor-making" materials with these characteristics.

(3) The transmissibility of the dynamic absorber is characterized, in general, by two maximum values which fall below and above the resonance which the absorber is originally "tuned" to suppress. When the two maximum values of transmissibility are equal, the dynamic absorber is said to be most favorably tuned and damped.

The tuning parameter, n, which is simply the ratio of the natural frequency of the absorber to that of the vibrating item, takes the optimum values, n_o, as follows:

Absorber Damping of the Viscous Type and Solid Type I	Absorber Damping of the Solid Type II
$n_o = \sqrt{\mu}$	$n_o = \mu \sqrt{2\left(\frac{1+\mu}{1+3\mu}\right)}$,

where $\mu = M_1/(M_1+M_2)$, M_1 and M_2 being the masses of the vibrating item and the dynamic absorber, respectively.

The values of optimum absorber damping can be calculated from somewhat cumbersome equations, but it is far more convenient to refer to Fig. 4 where optimum damping is plotted for a wide range of the ratio M_2/M_1. For absorber damping of the solid type, however, the choice of this ratio will be guided in practice by the damping factors of suitable rubber-like materials.

The discussions of absorber damping of the solid type assume that the rubber-like materials exhibit linear behavior regardless of the amplitude of vibration they experience. This assumption may not always be correct, but so little has been published on the nonlinear properties of these materials other than heavily filled natural rubber that it is not possible to judge how the above results will be affected.

(4) Transmissibility curves for the dynamic absorber are shown in Figs. 5, 8, and 9. They are computed for absorbers with optimum tuning, and damping of the viscous type, solid type I, and solid type II, respectively. Each figure shows three such optimum curves calculated for values of the ratio M_2/M_1 equal to 0.1, 0.2, and 1.0 The accompanying variation in the relative displacement between M_1 and M_2 is shown in Fig. 6, and is seen not to be excessive. The curves of this figure indicate, however, that very large relative motions can be expected for values of M_2/M_1 not far below 0.1.

The greatest resonant transmissibility is seen to be associated with absorber damping of the solid type I (Fig. 8). This transmissibility is always inferior to that of the conventional absorber with viscous-type damping. The difference is appreciable for large values of M_2/M_1, although it decreases to a very small amount for values of M_2/M_1 less than 0.1, say. An additional drawback of this absorber is the very large damping factor which is necessary to achieve optimum conditions for any worthwhile ratio of absorber to primary mass. This is unfortunate since it is unlikely that a material could be found having at the same time a dynamic stiffness which is independent of frequency.

The maximum transmissibility of an absorber with damping of the solid type II is seen (Fig. 9) to be significantly less than for an absorber with viscous-type damping. This difference approaches a maximum value of 3 db as the ratio

M_2/M_1 becomes very small. Additionally important, there seems to be a very good chance of obtaining absorber materials behaving in the required optimum manner. The use of equal absorber and primary masses is seen to offer exceptionally good isolation throughout the resonant frequency region and at higher frequencies, where the isolation increases at some 12 db/octave. This overall isolation is nearly always superior to that of the simple system with either high mount damping of the viscous type ($\bar{\delta} = 0.5$), or of the solid type II ($\delta = 1$). It is sometimes inferior over small ranges of frequency, but only when transmissibility is less than the maximum value of 3 db positive.

B. THE TRANSIENT BEHAVIOR OF THE DYNAMIC ABSORBER

6. INTRODUCTION

The transient theory of the dynamic absorber has received surprisingly little attention in past years. The only previous discussion has been given by Haringx[7] which deals, inexhaustively, with the free decay of vibrations induced by an impulse at some arbitrary time origin.

The analysis introduced here is concerned only with absorber damping of the viscous type in common with the treatment given by Haringx. The dynamic absorber is therefore of the form shown in Fig. 1b, where a spring placed in parallel to a dashpot forms the absorber element. Both the spring stiffness and the coefficient of viscosity of the dashpot are considered constant. A value of the damping ratio $\bar{\delta}$ (Section 3) equal to 0.1 is thought to be typical of the damping possessed by commercial shock mountings.

The dynamic absorber is considered to be disturbed at its foundation by a "step-like" displacement known here as the rounded step. This displacement[9] possesses a finite rise time and is defined by the relations:

$$x_o = 0 \quad \text{when time, } t, < 0$$

$$x_o = x\,[1 - e^{-\gamma\omega_o t}\,(1 + \gamma\omega_o t)] \quad \text{when } t > 0 \quad , \tag{6.1}$$

where ω_o is the natural frequency of the mounting system when the absorber mass is attached rigidly to the mounted item (Section 2), x is a constant, and γ is a parameter determining the rise time, τ, of the step. For convenience τ is defined as the time required for the displacement x_o to reach eighty-two percent of its final value,[9] and so:

$$\tau = \pi/\gamma\omega_o \quad . \tag{6.2}$$

The parameter γ is indicative of the relative steepness of the step as can be seen in Fig. 10, where Eq. (6.1) is plotted for a range of γ values which are of the same order and shorter than the half period of natural vibration of the dynamic absorber.

It can be shown that if the time-varying velocity and acceleration of the mounted item and its foundation are to be described realistically, then the disburbing displacement must be represented by an expression possessing a first time derivative which is equal to zero at the time origin. The rounded step displacement fulfils this condition.[9]

In general, when the transient behavior of the dynamic absorber is discussed, the absorber is assumed to be most favorably tuned and damped according to the relations derived in Section 3. The same three mass ratios $\mu = 10/11$, $5/6$, and $1/2$ are considered here and the analysis is simplified by the assumption that the mountings possess linear load-deflection characteristics.

7. THE UNDAMPED STEADY-STATE AND TRANSIENT BEHAVIOR OF THE DYNAMIC ABSORBER

The two maximum values of transmissibility possessed by the dynamic absorber for optimum absorber tuning and zero absorber damping can be simply shown [Eq. (3.1)] to occur at the frequencies:

$$\sqrt{(A \mp B)}\, \omega_o \quad , \tag{6.3}$$

where $A = (1 + \mu)/2\mu$ and $B = \sqrt{[(1+\mu)^2 - 4\mu^2]}/2\mu$.

When the foundation of the undamped dynamic absorber is disturbed by an ideal step displacement, namely, the displacement defined by an infinitely large value of γ in Eq. (6.1), so that

$$\begin{aligned} x_o &= 0 \quad \text{when } t < 0 \\ x_o &= x \quad \text{when } t > 0 \quad , \end{aligned} \tag{6.4}$$

then the transient displacement of the mounted item is given by:

$$\frac{x_1}{x} = \left\{ 1 - \frac{[\mu - (A-B)]}{2\mu B\,(A-B)} \cos \sqrt{A-B}\, \omega_o t - \frac{[\mu - (A+B)]}{2\mu B\,(A+B)} \cos \sqrt{A+B}\, \omega_o t \right\} , \tag{6.5}$$

where A and B are identical to the quantities defined above. Since the oscillatory terms of this equation are identical to those of Eq. (6.3), it appears that the step displacement of the foundation excites the frequencies at which the dynamic absorber resonates when disturbed by continuous vibration.

The expression for the displacement x_1 is composed of one constant and two undamped oscillatory terms. The coefficients of these terms are of similar magnitude and their ratio takes, for example, the values:

μ	1/2	5/6	10/11	20/21
Ratio	2.62	1.56	1.47	1.25

Figure 11 presents displacement-time curves for the undamped absorber which have been computed from Eq. (6.5). Also shown are the corresponding time-dependent variations of the relative displacement between the absorber and primary masses.

When μ possesses a value near unity, the frequencies $\sqrt{(A \mp B)}\,\omega_0$ are not widely separated. The oscillatory terms become in anti-phase after some time and since these terms are of comparable magnitude, the net displacement of the mounted item is then small. At a still later time the oscillatory terms will be in phase, but if damping is introduced into the absorber "spring," the displacement of the mounted item will be reduced correspondingly. These terms are also in phase originally, so that the initial form of the displacement is only slightly influenced by the mass of the absorber unless it is comparable to that of the mounted item.

As μ approaches the value 1/2, the separation of the frequencies $\sqrt{(A \mp B)}\,\omega_0$ increases. The first maximum value of displacement is reduced because the oscillatory terms are then already in anti-phase. They rapidly become in phase, however, but then suitably high absorber damping would ensure that x_1 did not exceed its original maximum value.

Because of the absorber action described above, the transient motion of the mounted item can be expected to decay extremely rapidly. It will be shown that this expectation is realized. As an immediate illustration, however, the performance of a most favorably tuned and damped absorber, one-fifth as massive as the mounted item, is compared with that of two other equally massive absorbers. These are also tuned most favorably but possess the extreme values of zero and critical damping. The acceleration and displacement of the primary mass due to a rounded step foundation displacement are shown as a function of time in Fig. 12 for these values of absorber damping. The rise time of the rounded step disturbance is defined by a value of γ equal to 50 (Fig. 10). Although the first maximum value of acceleration or displacement is not significantly affected by the value of absorber damping, the subsequent motion of the absorber possessing optimum damping decays very rapidly with time.

8. THE TRANSIENT RESPONSE OF THE DYNAMIC ABSORBER TO THE ROUNDED STEP DISPLACEMENT

8.1. Introduction.—The displacement and acceleration-time relations for the dynamic absorber have been developed with the help of the Laplace transformation. The notation of Carslaw and Jaeger has been followed.[10]

To determine the displacement x_1 of the mounted item (Fig. 1b), the subsidiary equation[10] for the ratio x_1/x_0 must first be obtained. The product of the Laplace transformation of x_0 and the subsidiary equation is then equal to the Laplace transformation of x_1, namely, $\tilde{x}_1$. This transformation may be inverted to give x_1 using a form of Heaviside's Expansion Theorem.[10] It is not possible, however, to invert the transformation in general terms, even when the simplifying assumptions of optimum absorber tuning and damping are made. Before inversion can be performed, it is necessary to specify particular values for the parameters μ and $\bar{\delta}$.

8.2. Acceleration and Displacement-Time Relationships for the Simple System.—To provide a basis for comparison of the acceleration and displacement-time relationships obtained for the dynamic absorber, similar relationships obtained previously for the simple mounting system[9] are presented here. Figures 13 and 14 refer to the motion of a simply mounted item produced by a rounded step foundation displacement. Mount damping ratios are equal to 0.1 and 0.5, respectively. The relative displacement between the mounted item and the foundation can be obtained simply, once the displacement of the mounted item has been determined.

It is apparent from Figs. 13 and 14 that:

(1) For step displacements with short rise times, high mount damping is responsible for large initial accelerations of the mounted item.

(2) Small values of the maximum displacement of the mounted item can be obtained only if either:
   (a) the damping ratio is large, or
   (b) the parameter γ is not large, that is, the mount is sufficiently stiff to ensure that the half natural period of the mounting system is comparable with, or less than, the rise time of the foundation displacement.

(3) The maximum relative displacement between the foundation and the mounted item is of the same order as the transient displacement of the foundation, unless γ is small (γ equal to two or less, say). High mount damping does not significantly affect the maximum value of the relative displacement when γ is large.

8.3. Acceleration and Displacement-Time Relationships for the Dynamic Absorber.—Acceleration and displacement-time relationships which describe the motion of the dynamic absorber resulting from a rounded step foundation displacement have been determined by the method outlined in Section 8.1. The most favorable values of absorber tuning and damping are employed for the mass ratios μ = 10/11, 5/6, and 1/2. The optimum damping ratios are equal to 0.19, 0.27, and 0.61, respectively. Acceleration, displacement, and relative displacement-time curves have been computed for these mass and damping ratios and are shown in Figs. 15, 16, and 17.

The broken curves depict the response of the simple system to an ideal step foundation displacement. The maximum displacement of the simply mounted item is made to equal that of the dynamic absorber disturbed by a rounded step. The rise time of this step is defined by the value of γ equal to 50. These curves illustrate again how rapidly the motion of the most favorably tuned and damped absorber decays.

Reference to Figs. 15, 16, and 17 indicates that:

(1) When γ is small, the maximum acceleration of the mounted item is similar in magnitude to that of the simple systems discussed above with relatively low and high mount damping. When γ is large, however, the maximum acceleration

of the mounted item is considerably less than that of the simple systems. In fact, when γ is very large the maximum acceleration approaches the limiting value of $(\omega_0^2 x)/\mu$, while in the simple system[9] it increases sensibly as $2e^{-1} \gamma \bar{\delta}(\omega_0^2 x)$. It is interesting to note that the former value of acceleration is virtually independent of the step rise time when this is sufficiently short.

Particularly when the absorber mass is large, the oscillatory acceleration of the mounted item decays very quickly with time once the first maximum and first minimum values have been reached.

(2) For a given value of γ, the maximum displacement of the mounted item decreases as the absorber mass increases. The oscillatory displacement of the mounted item decays rapidly after the initial maximum has occurred.

When γ is small, very low values of maximum displacement are observed, and if μ equals 0.5, the performance of the heavily damped simple system is rivaled.

When γ is large, the maximum displacement is also quite large and for small absorber masses compares unfavorably with the simple system possessing as low a mount damping ratio as 0.1. When the absorber mass becomes larger, the maximum displacement is reduced, for example, to a limiting value just greater than 1.5 x when μ equals 0.5.

Although the maximum values of displacement are in general disappointingly large when the absorber mass is small, it is felt that the subsequent rapid decay of oscillatory displacement is most beneficial. It is difficult to express this in mathematical terms, but there is little doubt that when the foundation of the mounted item is disturbed repeatedly, the risk of failure by fatigue of fragile elements within the mounted item is significantly reduced.

(3) In common with the simple system, the maximum relative displacement between the mounted item and its foundation is of the same order as the transient foundation displacement unless γ is less than 2, say. The relative displacement subsequently decays with time in the rapid fashion of the displacement discussed above.

9. THE USE OF THE DYNAMIC ABSORBER TO REDUCE DAMAGE DUE TO SHOCK

9.1. Introduction.—A study of the transient behavior of the dynamic absorber was originally undertaken because it appeared to fulfill, at least partially, the requirements for a good shock-isolating system. Reference to Fig. 5 indicates that the transmissibility curves for the dynamic absorber are characterized by a rapidly increasing isolation of 12 db/octave at frequencies well above resonance. This isolation increases at the same rate as that of the simple system with negligible mount damping. On the other hand, the resonant transmissibility of the dynamic absorber is in many ways characteristic of the resonant transmissibility of the simple system with relatively high mount damping, particularly when the absorber mass is large. The dynamic absorber is therefore likely to be a good shock mounting because it is largely true to say that:

(1) To protect the high natural frequency elements (those greater than ω_0) within a resiliently mounted item from damage due to shock (brittle items such as the electrode assembly of a tube), the maximum acceleration of the mounted item must be as small as possible. Low maximum acceleration, however, is associated with rapidly increasing high-frequency isolation in the steady-state case.

(2) To protect the low natural frequency elements (those less than ω_0) from damage, the absolute displacement of the mounted item should be as small as possible. Minimum displacement, however, is associated with maximum suppression of the mounting resonance in the steady-state case.

There is no doubt that ideally a mounting system should protect both the high and the low natural frequency elements simultaneously, but normally only one of these conditions can be satisfied at the expense of the other. It can be seen, however, that the dynamic absorber probably goes further towards protecting in both ways the contents of a resiliently mounted item from damage than either the simple or the compound systems.[9] A critical comparison of the performance of the dynamic absorber with the simple system will now be made.

9.2. The Shock Displacement Ratio.—The shock displacement ratio, hereafter known simply as S.D.R., is indicative of the maximum displacement experienced by a resiliently mounted item when its foundation is transiently disturbed. The S.D.R. is defined as the ratio:

$$\frac{\text{maximum displacement experienced by the item when resiliently mounted}}{\text{maximum displacement experienced by the item when rigidly mounted}} .$$

For a given mass ratio and absorber or mount damping ratio, the S.D.R. for the rounded step displacement increases to a constant value as γ increases. Reference to Fig. 18 indicates that the S.D.R. is largest when the rise time of the disturbing step is shortest, or when the mount is stiffest. Conversely, the greater the rise time of the step or the softer the mounting, the smaller the S.D.R. becomes. The S.D.R. of the systems considered here is always less than two.

The S.D.R. of the dynamic absorber with the smaller absorber masses ($\mu <$ 10/11, say) is inferior to practically all other systems discussed here, the only exception being the simple system with negligible mount damping. On the other hand, the S.D.R. of the dynamic absorber with equal absorber and primary masses is inferior only to that of the simple systems with high mount-damping ratios. It lies significantly below the curves relating to the simple systems with relatively low mount damping.

9.3. The Shock Acceleration Ratio.—A quantity very often indicating the value of resiliently protecting an item from its transiently disturbed foundation is the shock acceleration ratio. This quantity, hereafter known simply as S.A.R., is defined as the ratio:

$$\frac{\text{maximum acceleration experienced by the item when resiliently mounted}}{\text{maximum acceleration experienced by the item when rigidly mounted}} .$$

The dynamic absorbers and the simple systems considered previously (Section 9.2) possess an S.A.R. for the rounded step displacement which is shown as a function of the parameter γ in Fig. 19. It has been found[9] that when γ is large, the S.A.R. of the simple system is inversely proportional to γ. Because the maximum acceleration of the dynamic absorber approaches a limiting value as γ increases (Section 8.3), the S.A.R. it possesses for sufficiently large values of γ is inversely proportional to γ^2. The S.A.R. of the dynamic absorber therefore decreases more rapidly than that of the simple system, the difference in the rates approaching 6 db/octave as γ increases.

The nature of the S.A.R. for the simple system and the dynamic absorber is strongly reminiscent of the high-frequency regions of the transmissibility curves possessed by these systems in the steady-state case (Section 3), where transmissibility decreases in proportion to frequency and the square of frequency respectively.

Reference to Fig. 19 indicates that:

(1) When γ is small, the S.A.R. possessed by the simple system with low mount damping is slightly superior to the S.A.R. of the undamped system. It becomes increasingly inferior as γ increases above 10, for example, and eventually decreases in proportion to γ only.

(2) The S.A.R. of the simple system with high mount damping ($\delta = 0.5$) is inferior to any system considered here, once γ is greater than 2.5. The difference is very large for high values of γ.

(3) The S.A.R. of the dynamic absorber decreases in proportion to γ^2 when γ is large. For a given value of γ, the S.A.R. is then least when the absorber mass is smallest.

(4) The difference between the S.A.R. of the dynamic absorbers with mass ratios $\mu < 5/6$, say, is very small. For example, the difference in isolation afforded by the dynamic absorbers with mass ratios $\mu = 10/11$ and $5/6$ is never greater than 0.76 db.

(5) The S.A.R. of the dynamic absorber with equal absorber and primary masses is never more than 6 db above the S.A.R. for any smaller absorber mass. Except for low values of γ, it is also somewhat

greater than the S.A.R. of the simple system with relatively small mount damping until γ is approximately equal to 30. However, it always lies significantly below the S.A.R. of the heavily damped simple system.

10. SUMMARY AND CONCLUSIONS

(1) The transient motion of the dynamic absorber resulting from a step-like displacement of its foundation has been determined as a function of the step rise time. This quantity is indicative of the "severity" of the shock. Purely as a basis for comparison, the response of the simple system to similar disturbances is also discussed here.

The relative performance of the dynamic absorber and the simple system can be examined most easily by reference to Figs. 18 and 19, where the shock displacement ratio (S.D.R.) and the shock acceleration ratio (S.A.R.) of several mounting systems are plotted as a function of the parameter γ. This parameter is indicative of the relative steepness of the disturbing rounded step displacement. The S.D.R. and the S.A.R. curves depict the maximum values of displacement and acceleration experienced by a resiliently mounted item in terms of the corresponding maximum values experienced when the item is attached rigidly to its foundation.

(2) In many cases the damage suffered by the contents of a mounted item can be related either to the maximum acceleration or the maximum displacement of the mounted item, or equivalently to the value of the S.D.R. or the S.A.R. For maximum protection from damage, however, the values of both the S.D.R. and the S.A.R. must simultaneously be small.

It is seen that the S.D.R. of the dynamic absorber and the simple system can be reduced only if an increase in their S.A.R. is acceptable, or conversely, their S.A.R. can be reduced only if an increase in the S.D.R. is acceptable. It is thought, however, that a satisfactory compromise can be obtained with the dynamic absorber, particularly with the larger absorber masses.

An important characteristic of the dynamic absorber is the very rapid fashion in which the transient motion of the mounted item decays with time, once an initial maximum has occurred. If the systems are disturbed repeatedly, therefore, the dynamic absorber will offer the more fragile contents of the mounted item greater protection from failure due to fatigue. In addition, the motion of the mounted item is less likely to be excessive when the disturbances are regularly spaced at some integral multiple of its half period of natural vibration.

(3) It is evident from Figs. 18 and 19 that, although the heavily damped simple system exhibits the lowest S.D.R., it generally possesses the largest S.A.R. The S.A.R. decreases at only 6 db/octave when γ is greater than 5, and even more slowly when γ is smaller. The simple system with negligible mount damping possesses the greatest S.D.R., but the lowest S.A.R. once γ is greater than 16.

The S.A.R. then decreases at 12 db/octave. It is mostly inferior to that of other systems for smaller values of γ.

The performance of the dynamic absorber with equal absorber and primary masses is seen to be a compromise between these two extremes, particularly for isolation from the "more severe" shocks defined by γ greater than 10, for example. Although the S.D.R. of this dynamic absorber is greater than that of the simple system with a mount damping ratio $\bar{\delta} = 0.5$, the difference is quite small when γ is of the order unity and, more important, it lies significantly below that of all other systems discussed here. The S.A.R. of this dynamic absorber decreases rapidly as γ increases, being never more than 6 db inferior to the S.A.R. of the idealized simple system with zero mount damping. It diverges rapidly from the S.A.R. of the heavily damped simple system, being some 8 db superior when γ equals 10, and increasingly so thereafter at a rate approaching 6 db/octave. It can be argued that other systems offer more isolation through all or part of this range of γ values, but it should be appreciated that when γ is in the neighborhood of 10, the S.A.R. is already approximately 40 db down.

For "less severe" shocks where γ is perhaps less than 10, the S.A.R. of the dynamic absorber with equal absorber and primary masses does not appear greatly different from that of the other systems considered here. The heavily damped simple system is an exception, being notably inferior when γ is greater than 2. Some of the lowest values of the S.A.R. in and above this range of γ values can be obtained with a dynamic absorber having a mass ratio $\mu = 5/6$, but this is compensated by the higher overall S.D.R. possessed by the system.

Initially it might be thought that the performance of a simple system with mount damping $\delta = 0.22$ would rival that of the dynamic absorber with equal absorber and primary masses, since, when γ is large, the S.D.R. of the dynamic absorber and this simple system will coincide. It is felt, however, that the conclusions above remain unaltered for the two following reasons:

(a) The S.A.R. of the dynamic absorber would be superior to that of the simple system when γ is approximately 10 and larger, becoming increasingly so at a rate of 6 db/octave as γ increases. For example, when γ is 40, the S.A.R. of the dynamic absorber would be nearly 12 db superior to that of the simple system. The S.A.R. of the systems would be practically identical when γ is less than 10, say.

(b) The S.D.R. of the simple system would be inferior to that of the dynamic absorber through a similarly low range of γ values.

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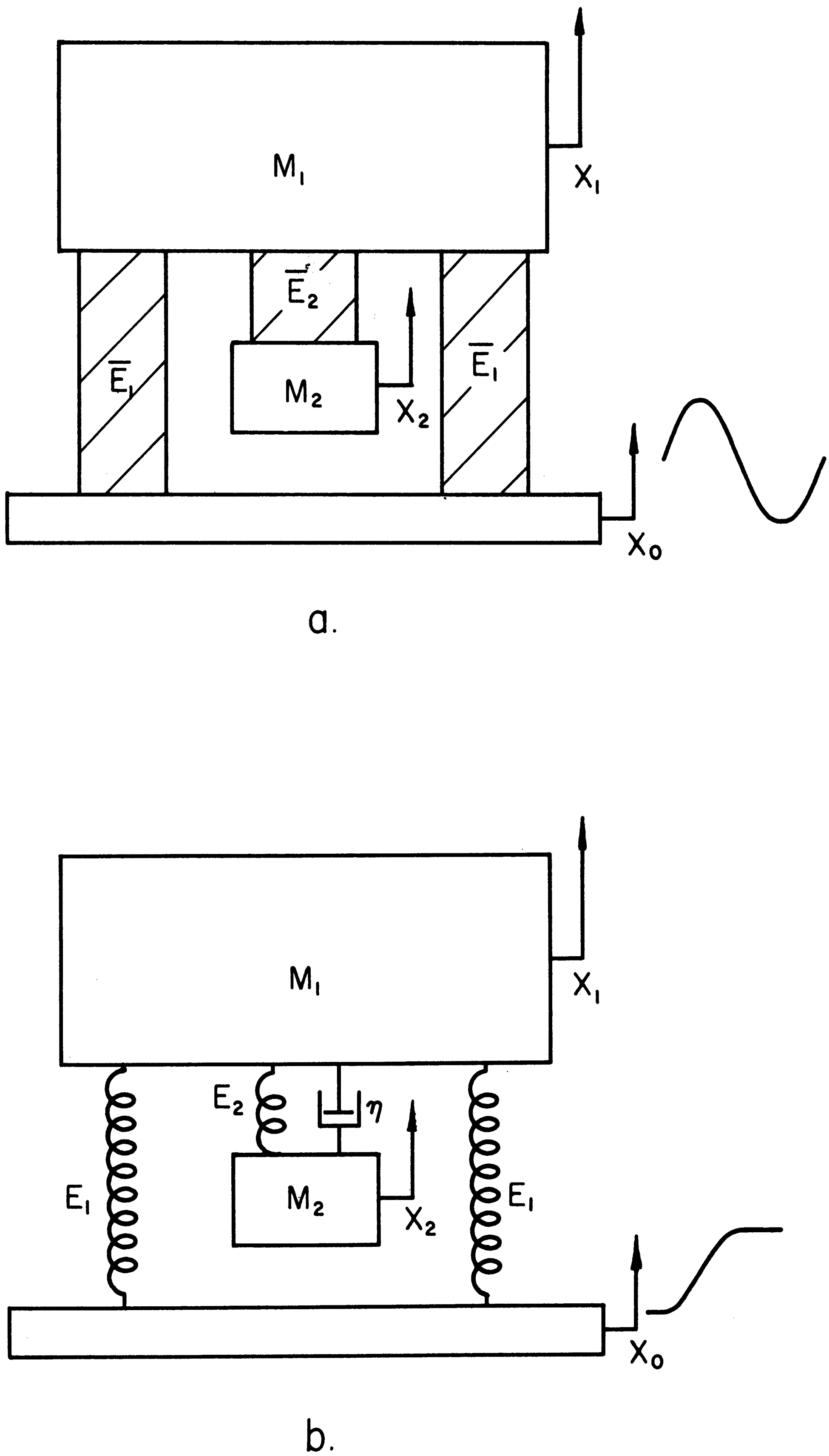


Fig. 1. The dynamic absorber.

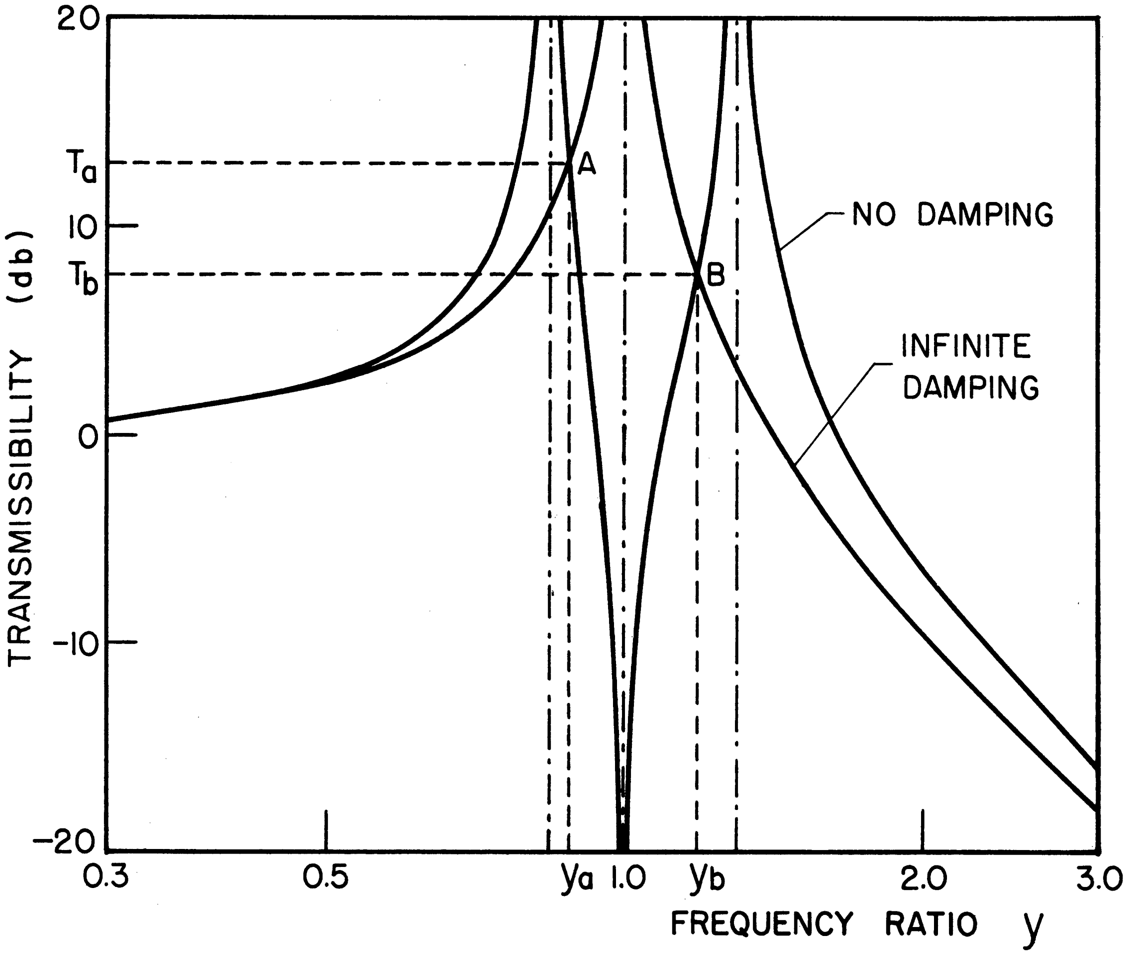


Fig. 2. The transmissibility of the dynamic absorber with zero and infinite absorber damping.

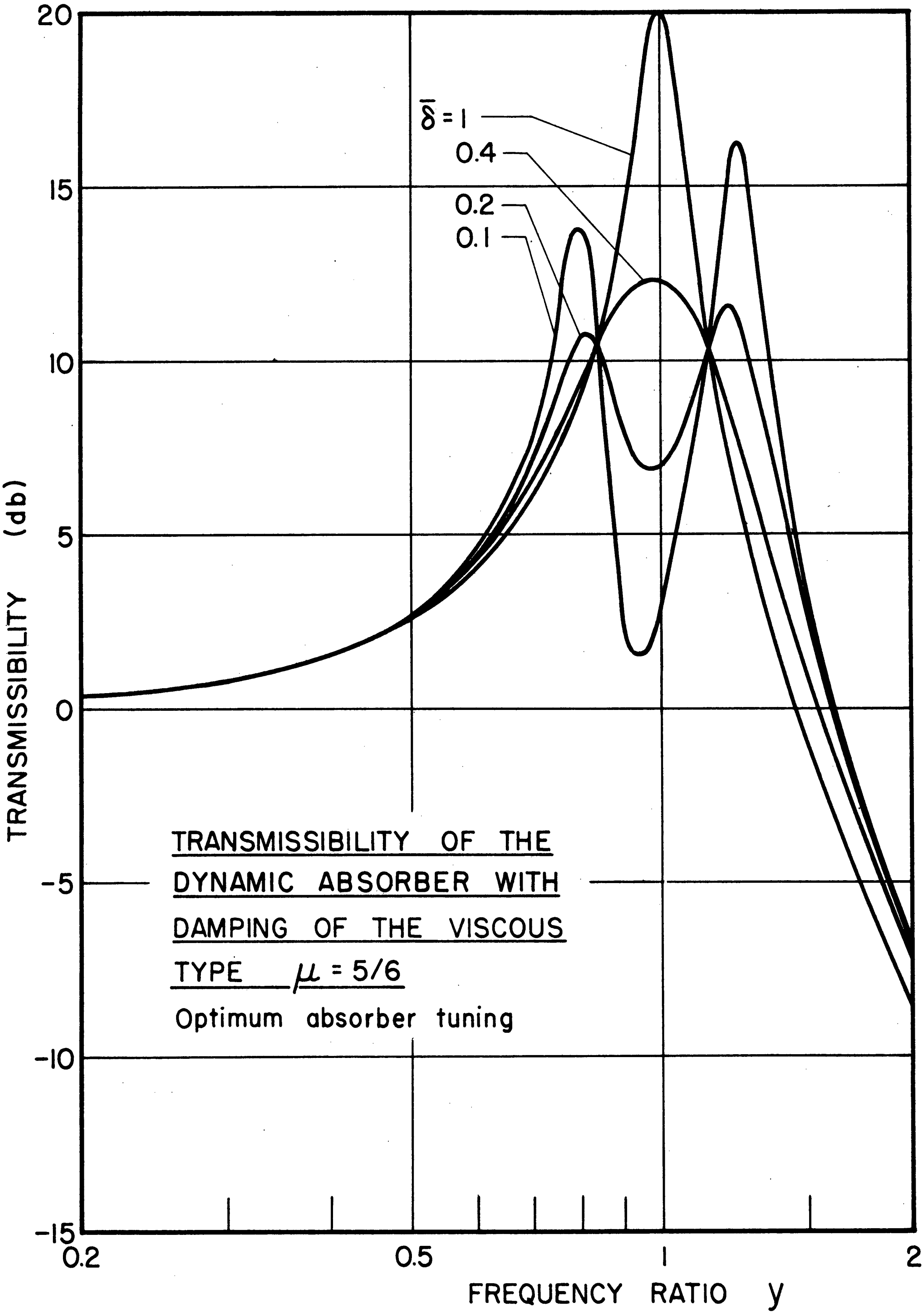


Fig. 3. The transmissibility of the dynamic absorber with absorber damping of the viscous type. Optimum absorber tuning for the mass ratio $\mu = 5/6$.

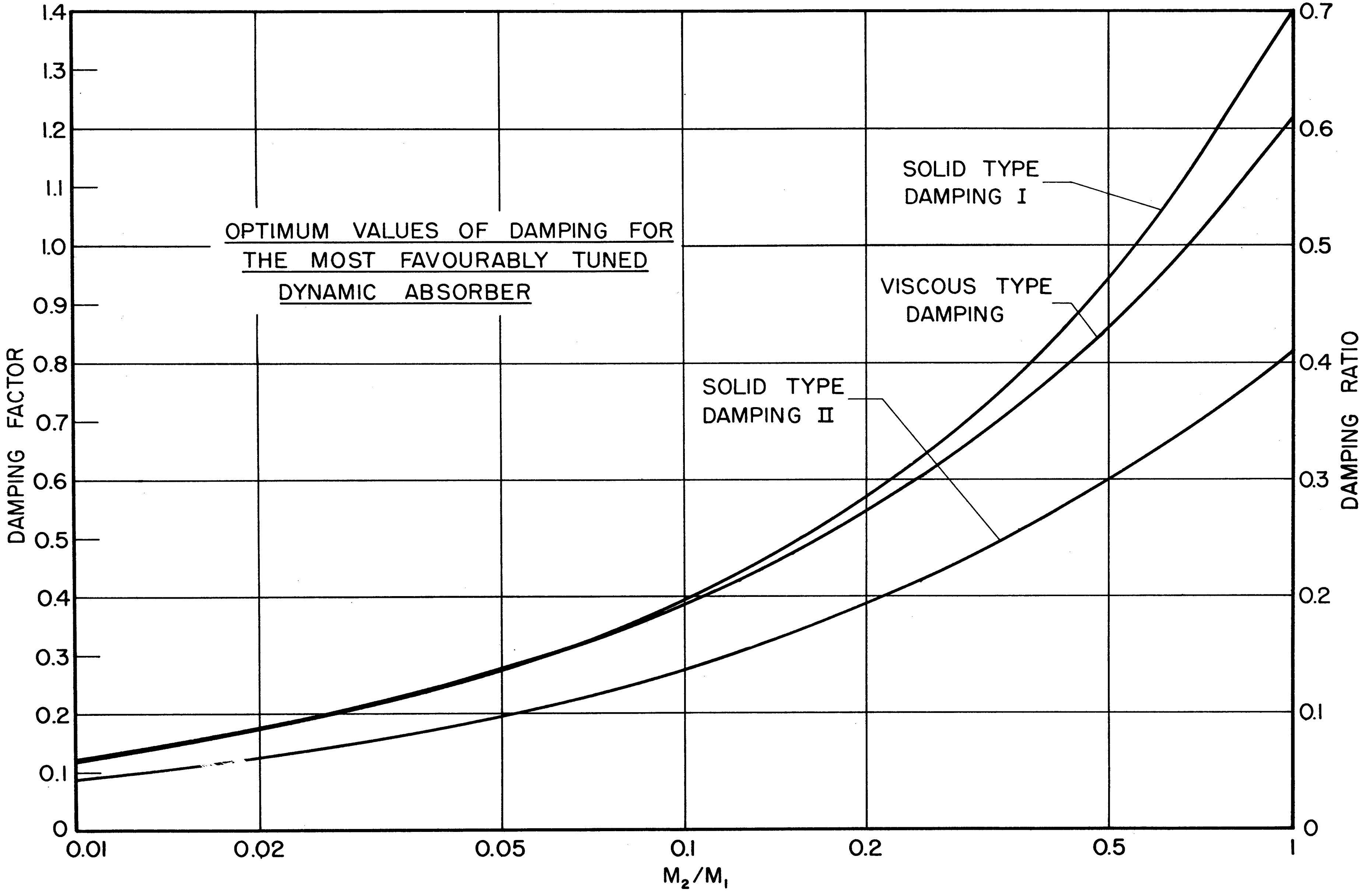


Fig. 4. Values of optimum damping for the most favorably tuned dynamic absorber.

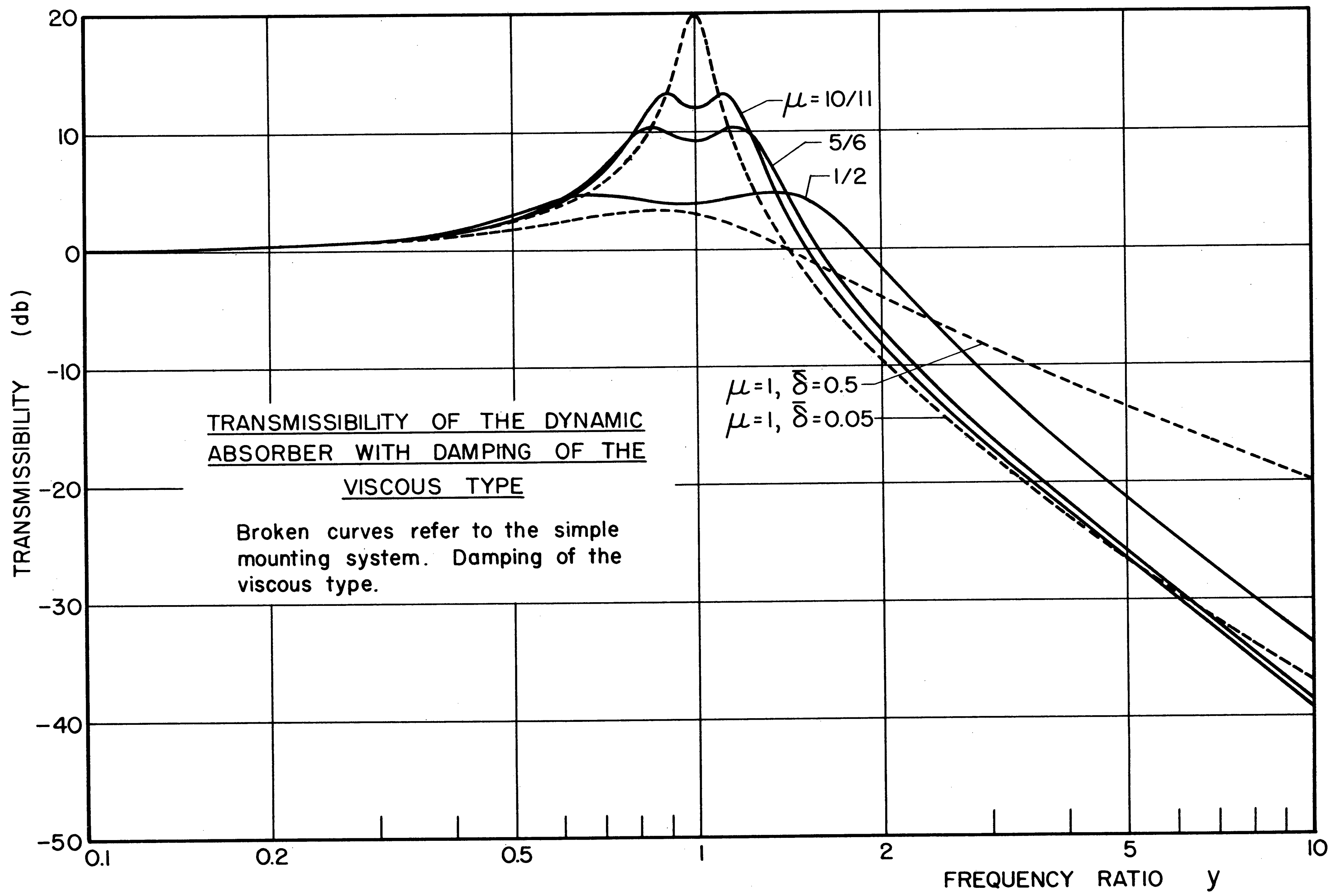


Fig. 5. The transmissibility of the dynamic absorber with damping of the viscous type. Mass ratio μ = 10/11, 5/6, 1/2. Optimum absorber tuning and damping.

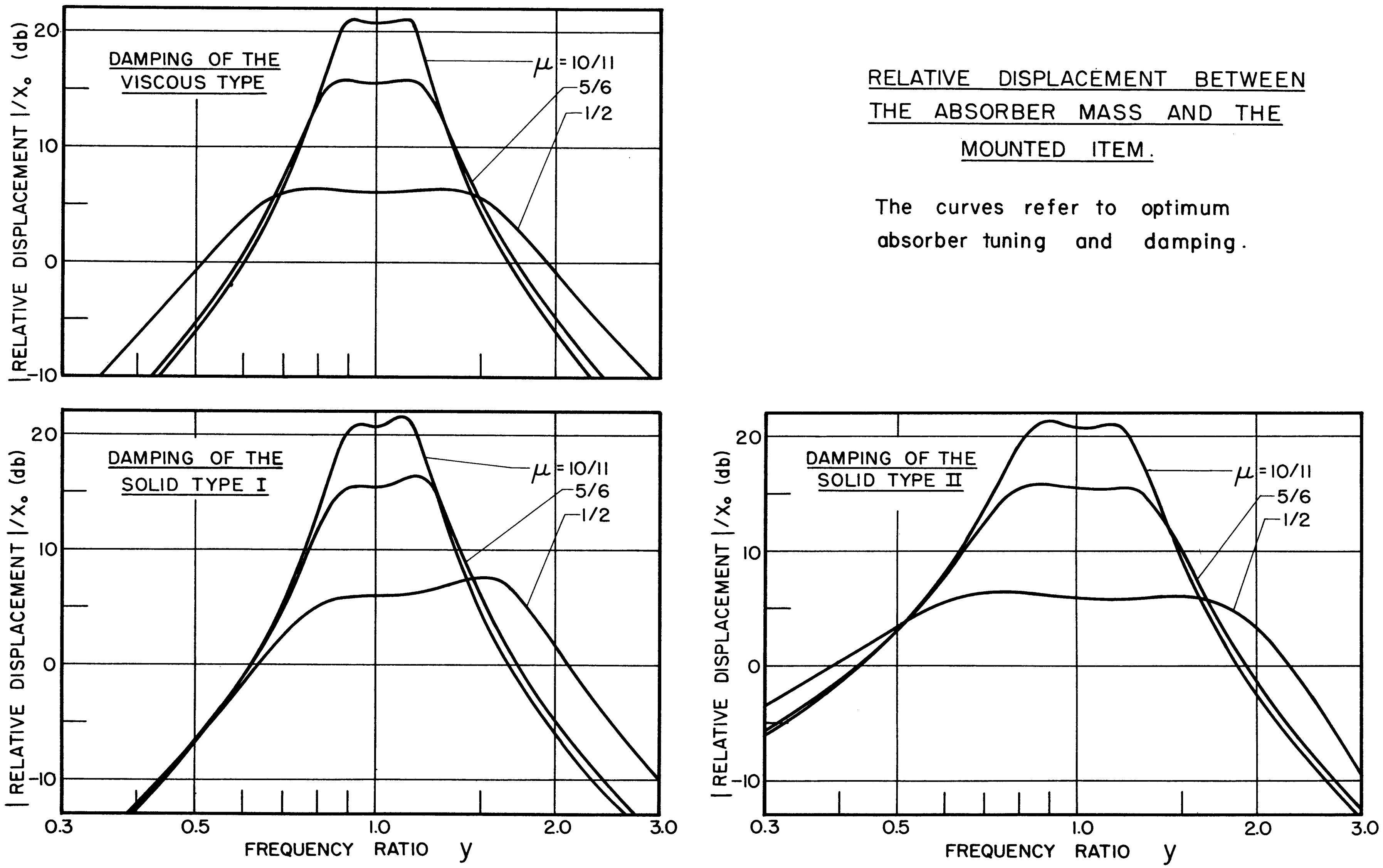


Fig. 6. The relative displacement between the absorber mass and the mounted item. Mass ratio μ = 10/11, 5/6, 1/2. Optimum absorber tuning and damping.

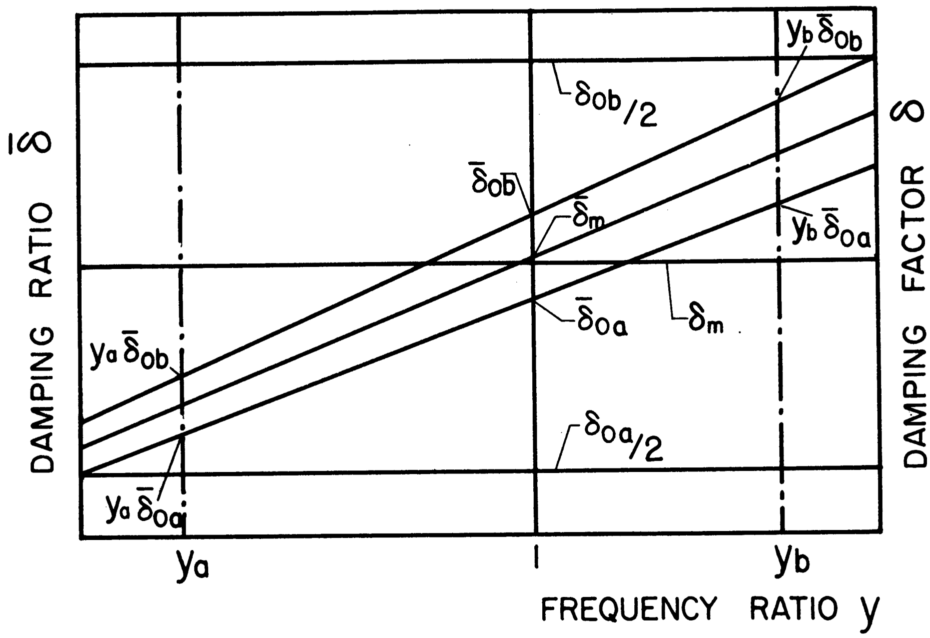


Fig. 7. The frequency dependence of optimum viscous-type absorber damping compared to the optimum values of solid-type damping.

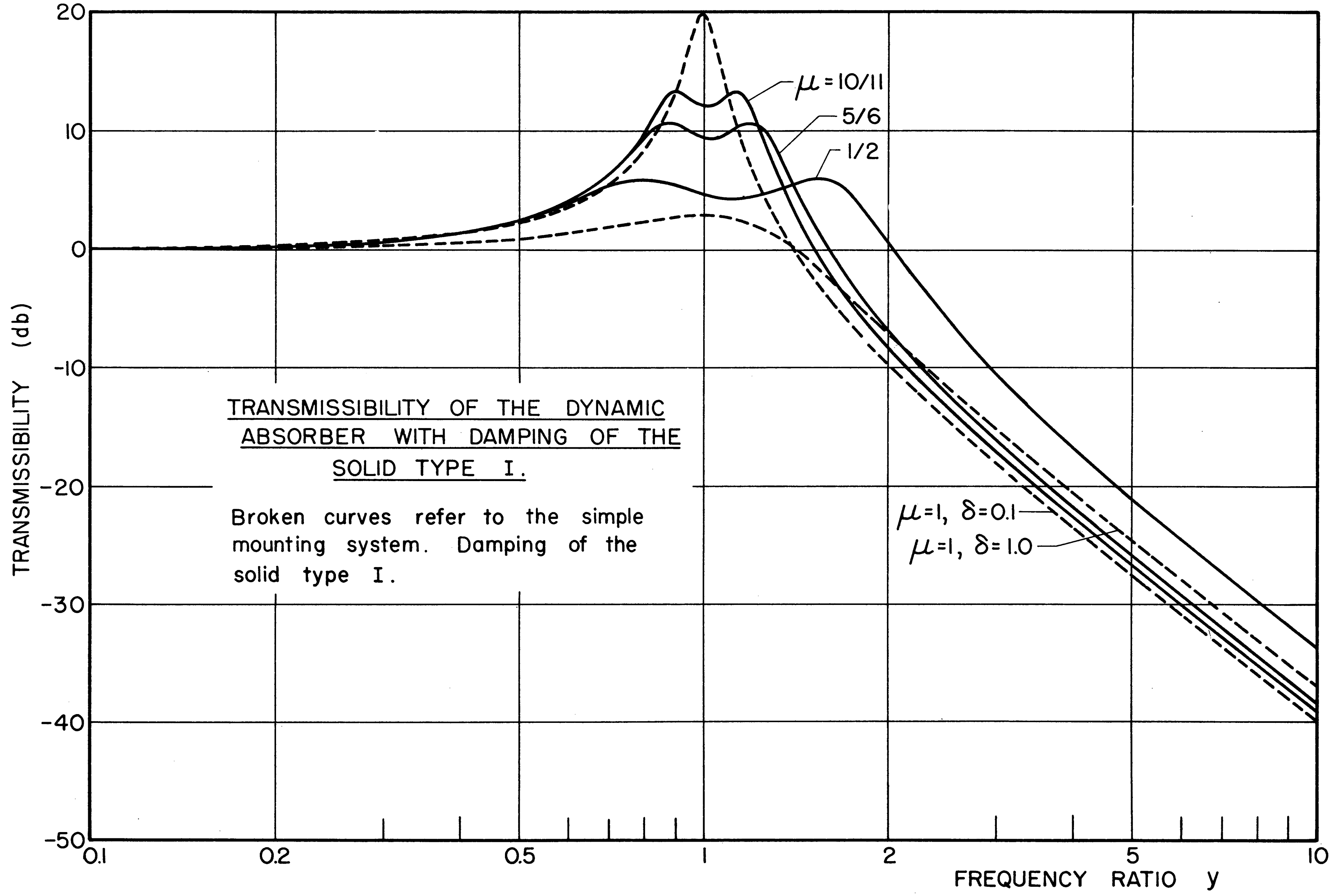


Fig. 8. The transmissibility of the dynamic absorber with damping of the solid type I. Mass ratio $\mu = 10/11$, $5/6$, $1/2$. Optimum absorber tuning and damping.

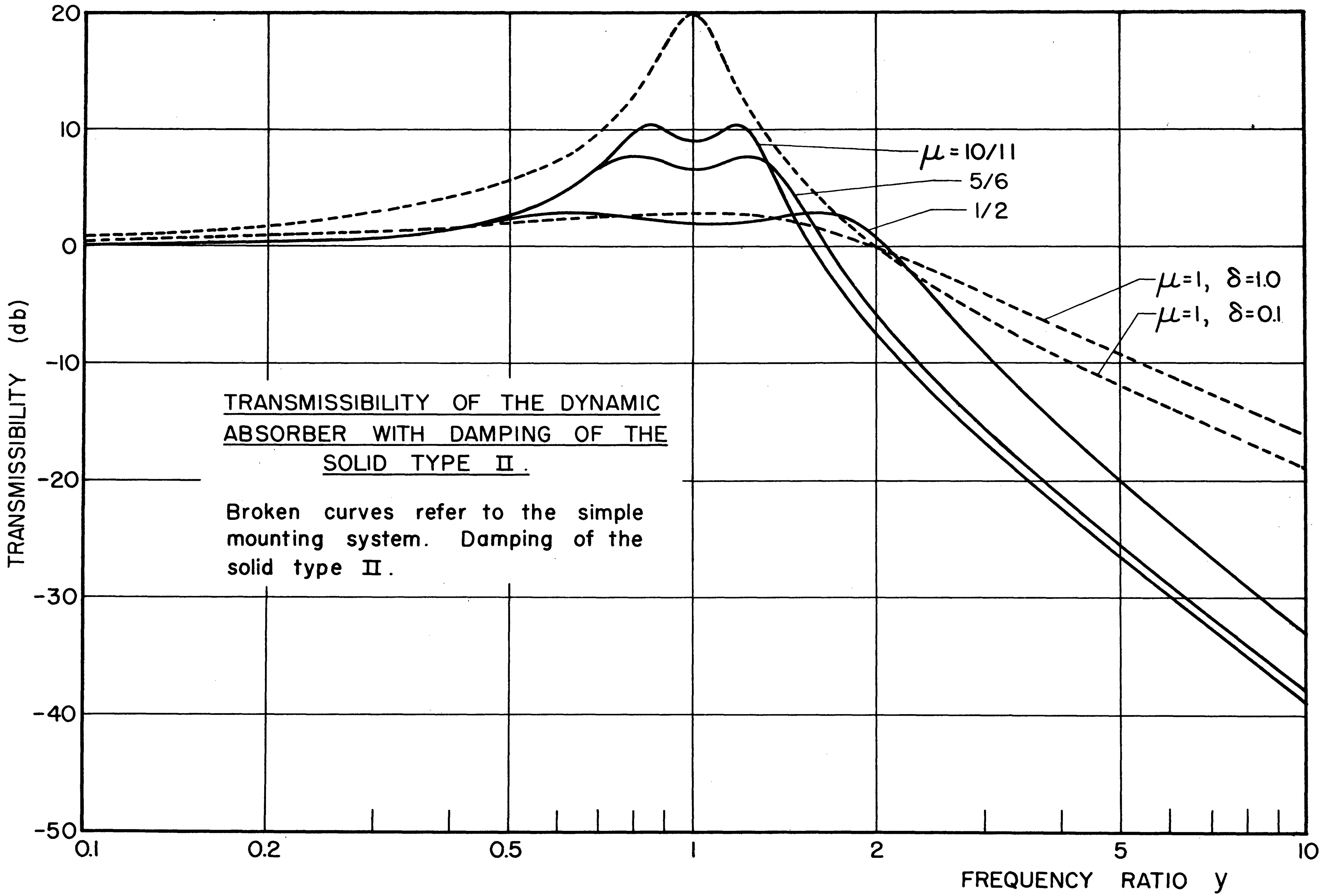


Fig. 9. The transmissibility of the dynamic absorber with damping of the solid type II. Mass ratio $\mu = 10/11, 5/6, 1/2$. Optimum absorber tuning and damping.

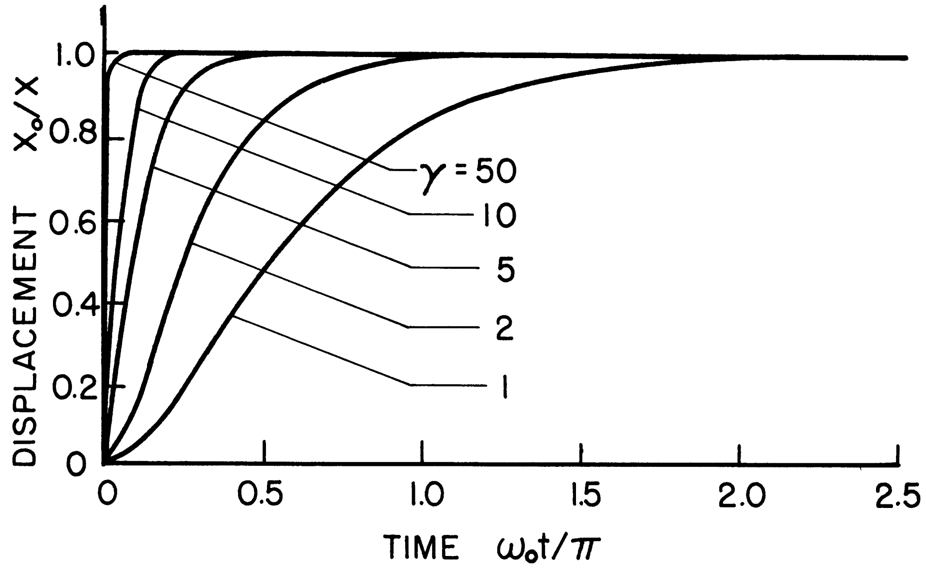
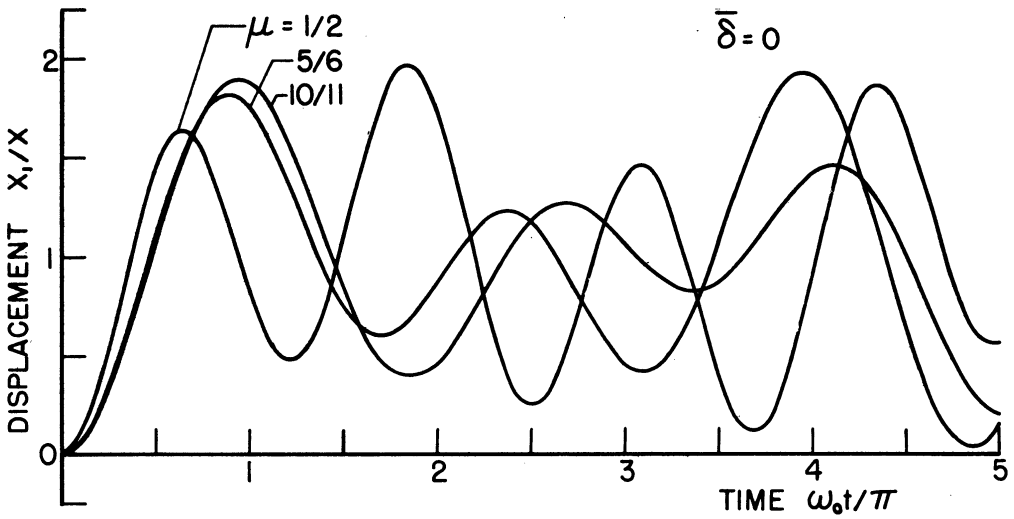


Fig. 10. The rounded step displacement.



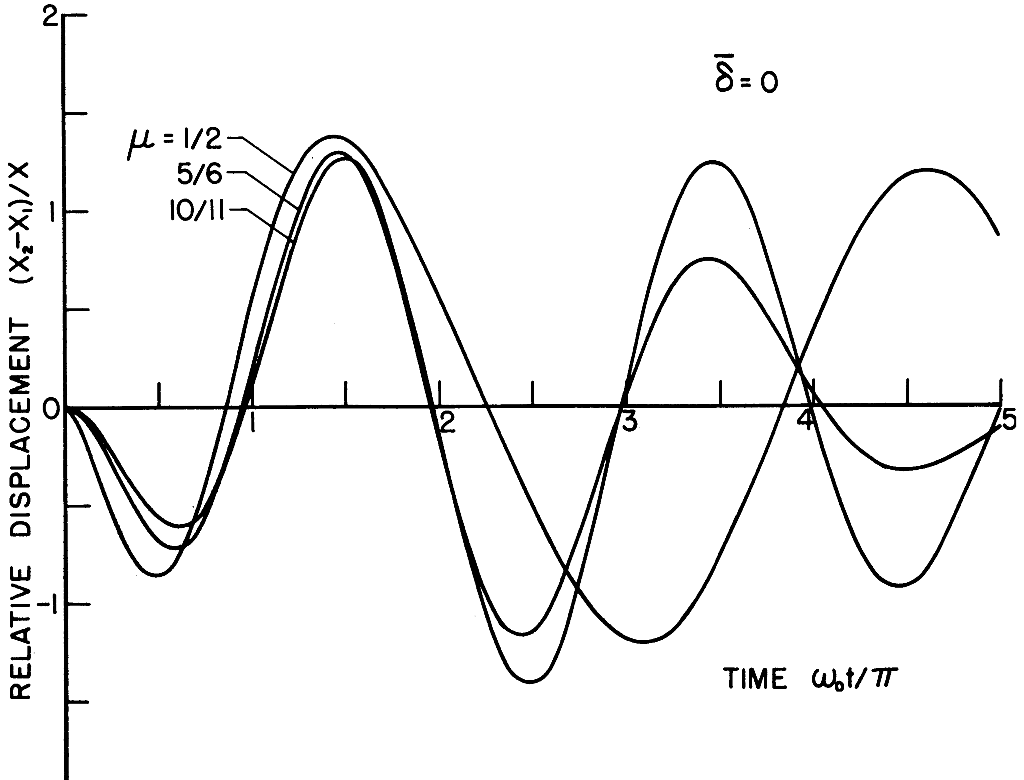
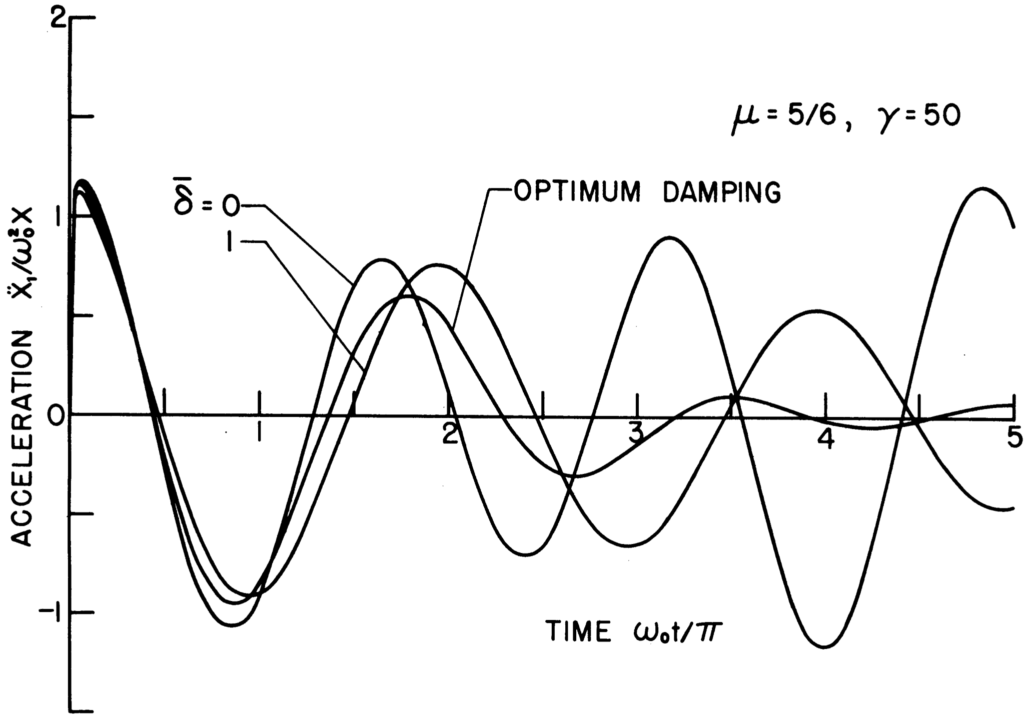


Fig. 11. The displacement and relative displacement-time relationships for the undamped dynamic absorber. Mass ratio μ = 10/11, 5/6, 1/2. Optimum absorber tuning.



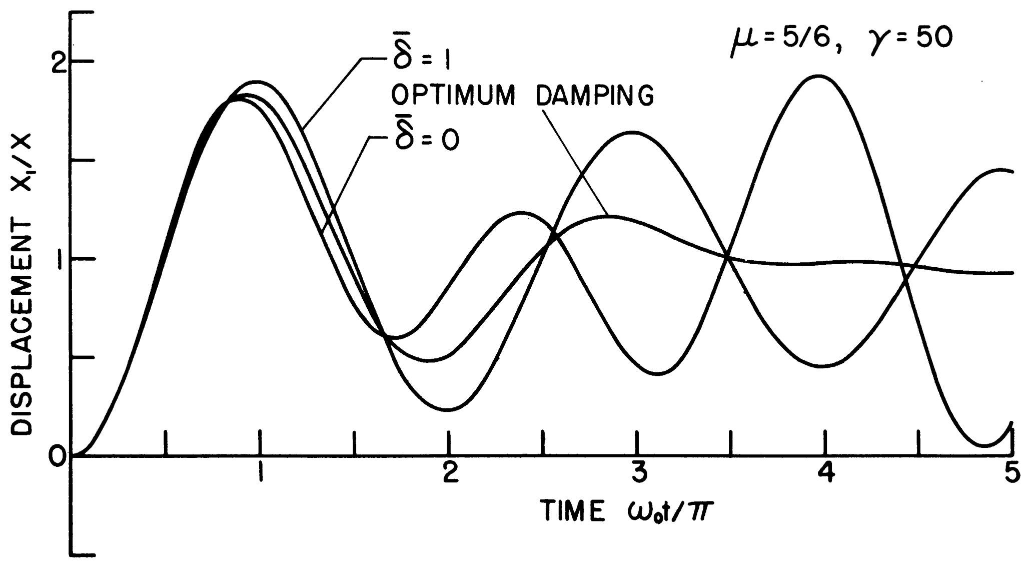


Fig. 12. Acceleration and displacement-time relationships for the dynamic absorber. Rise time of the rounded step foundation displacement defined by γ = 50. Optimum absorber tuning for the mass ratio μ = 5/6. Absorber damping ratio equal to its optimum value and the extreme values of zero and unity.

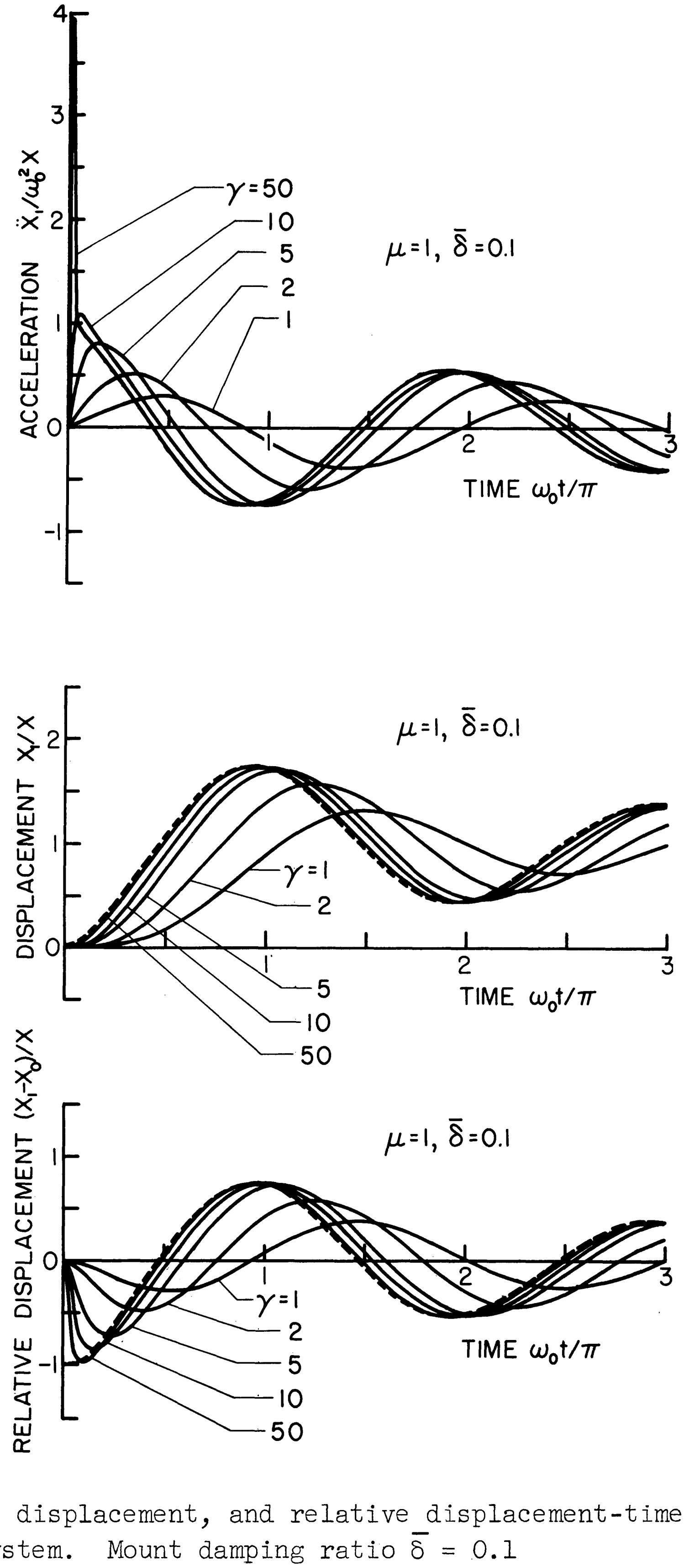


Fig 13. Acceleration, displacement, and relative displacement-time relationships for the simple system. Mount damping ratio $\bar{\delta}$ = 0.1

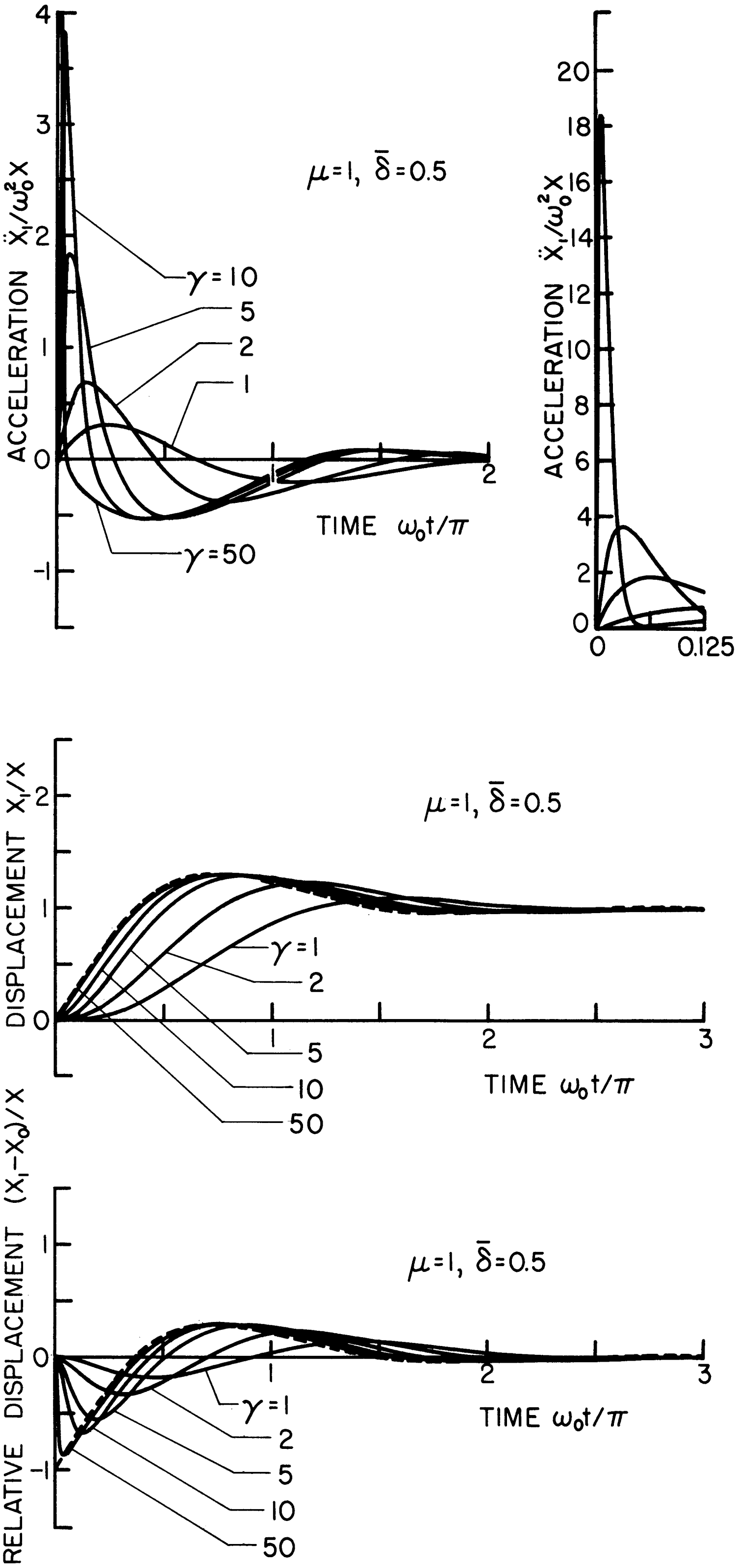


Fig. 14. Acceleration, displacement, and relative displacement-time relationships for the simple system. Mount damping ratio $\bar{\delta}$ = 0.5.

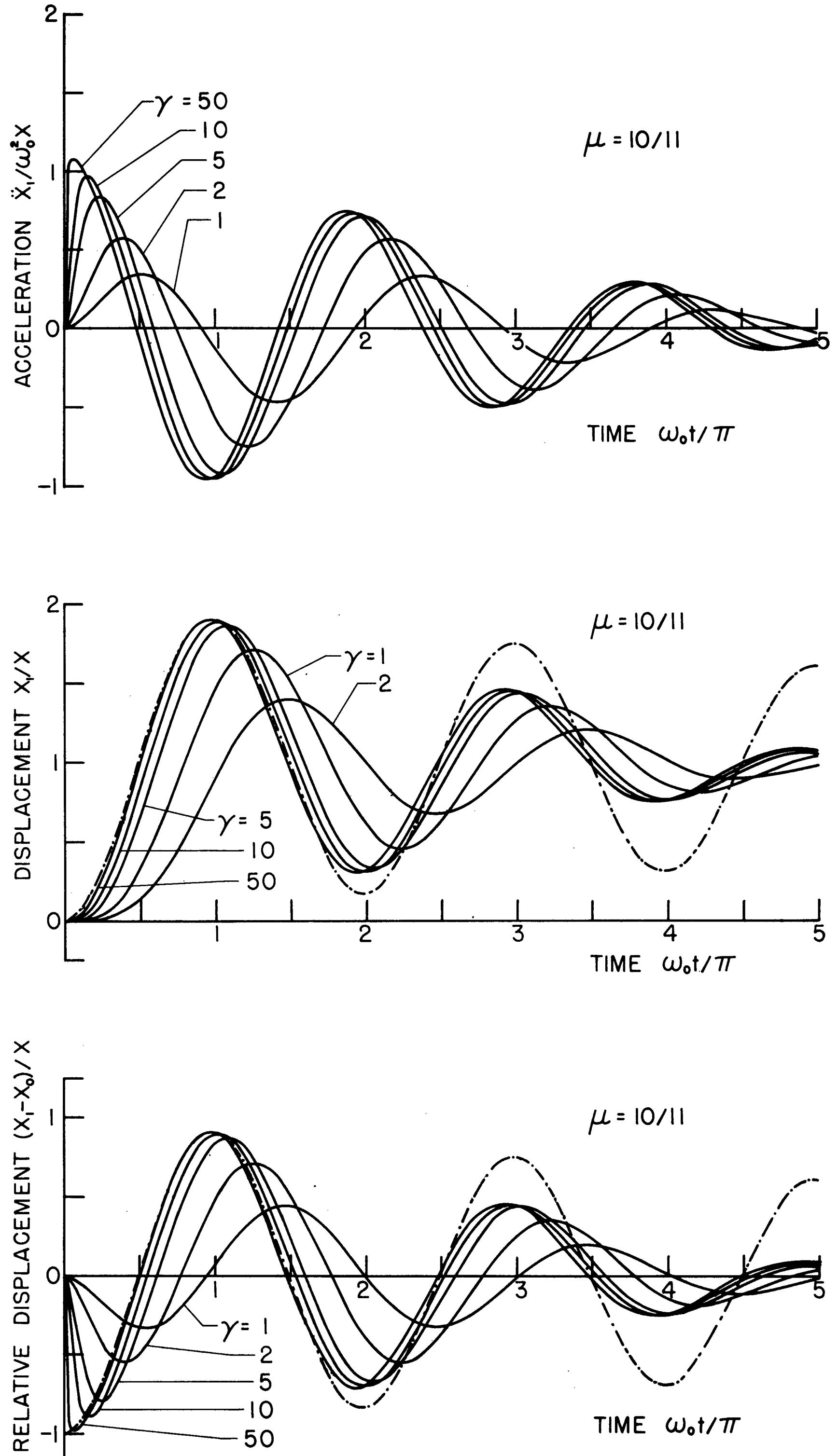


Fig. 15. Acceleration, displacement, and relative displacement-time relationships for the dynamic absorber. Mass ratio $\mu = 10/11$. Optimum absorber tuning and damping.

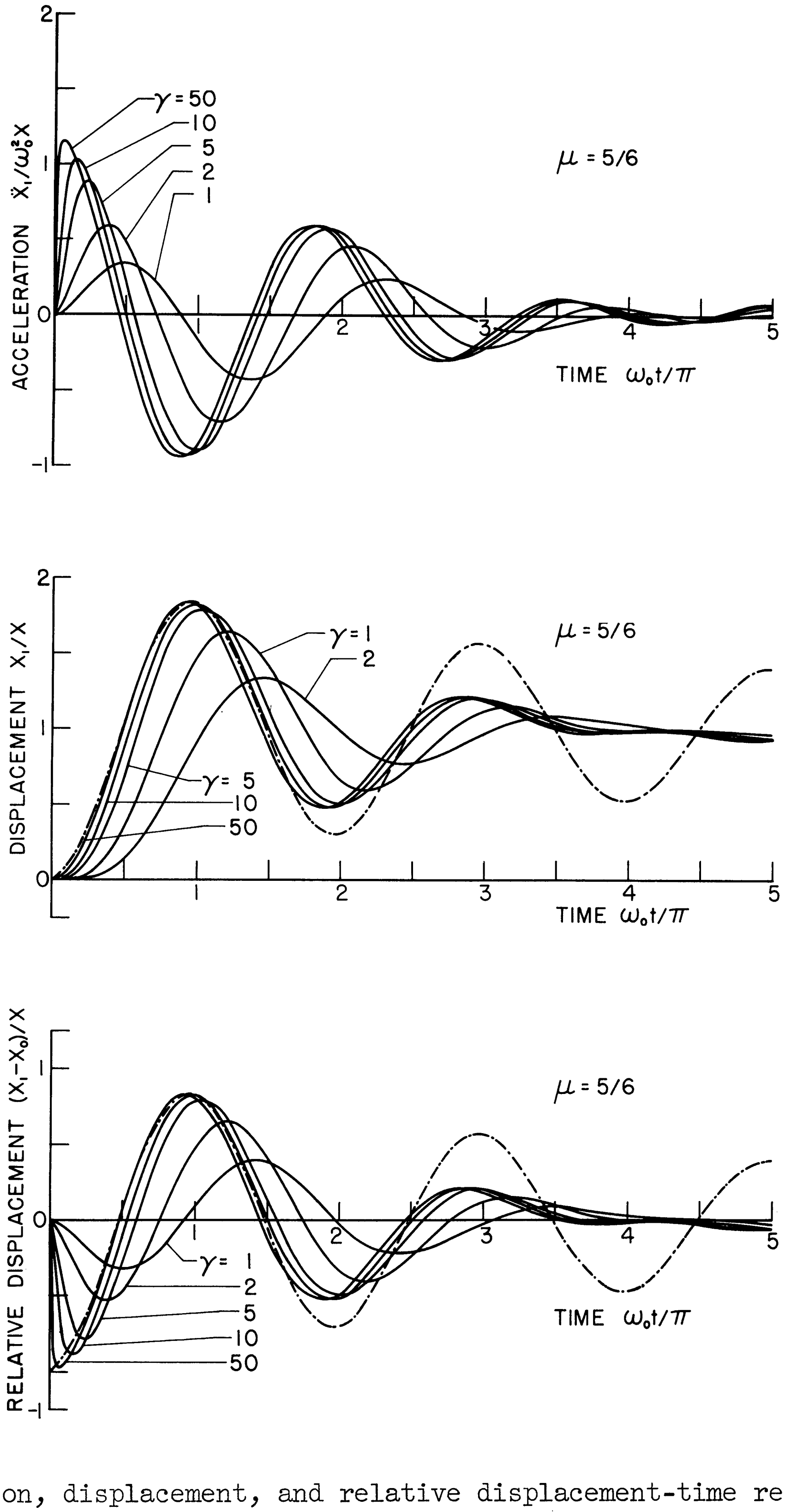


Fig. 16. Acceleration, displacement, and relative displacement-time relationships for the dynamic absorber. Mass ratio $\mu = 5/6$. Optimum absorber tuning and damping.

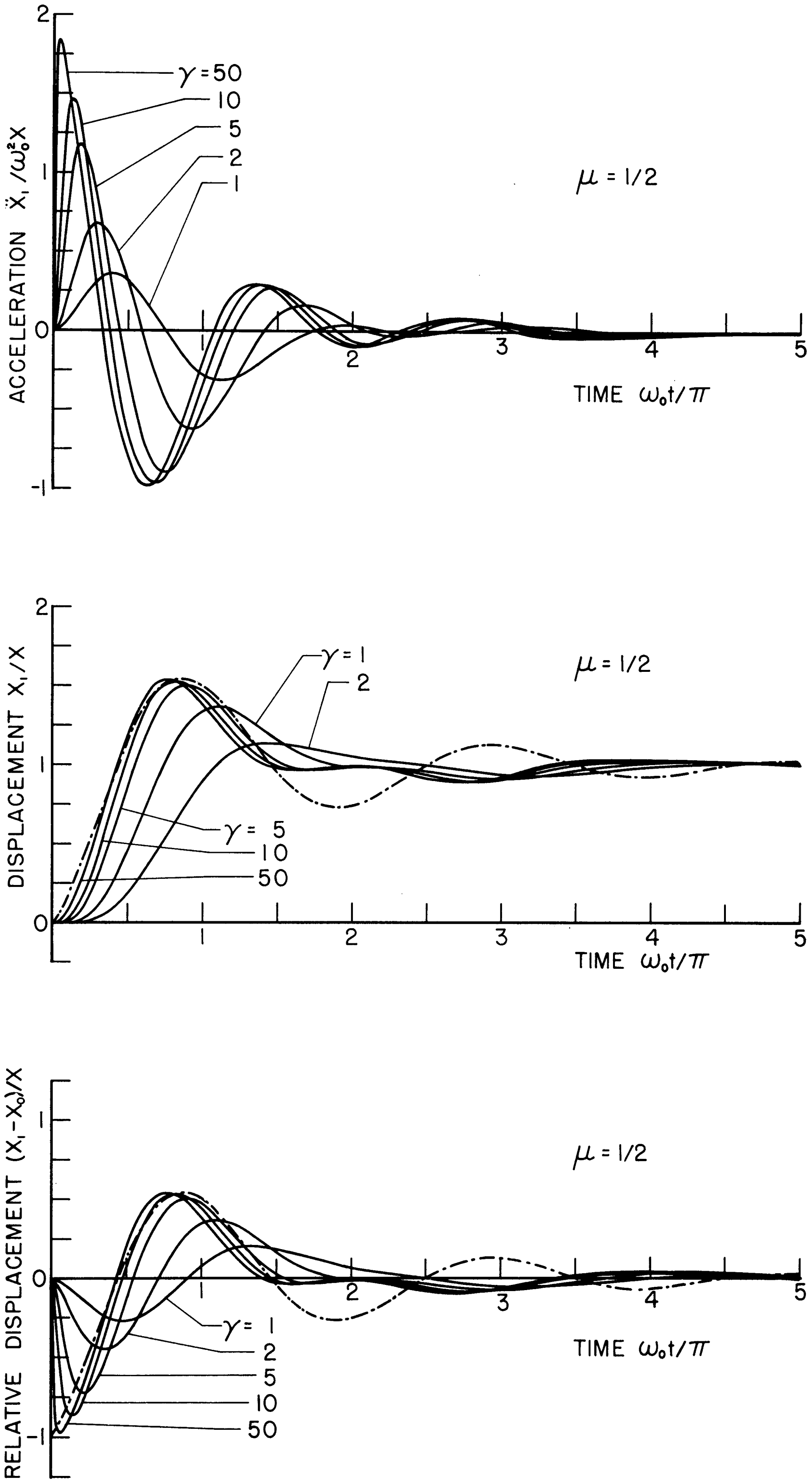


Fig. 17. Acceleration, displacement, and relative displacement-time relationships for the dynamic absorber. Mass ratio $\mu = 1/2$. Optimum absorber tuning and damping.

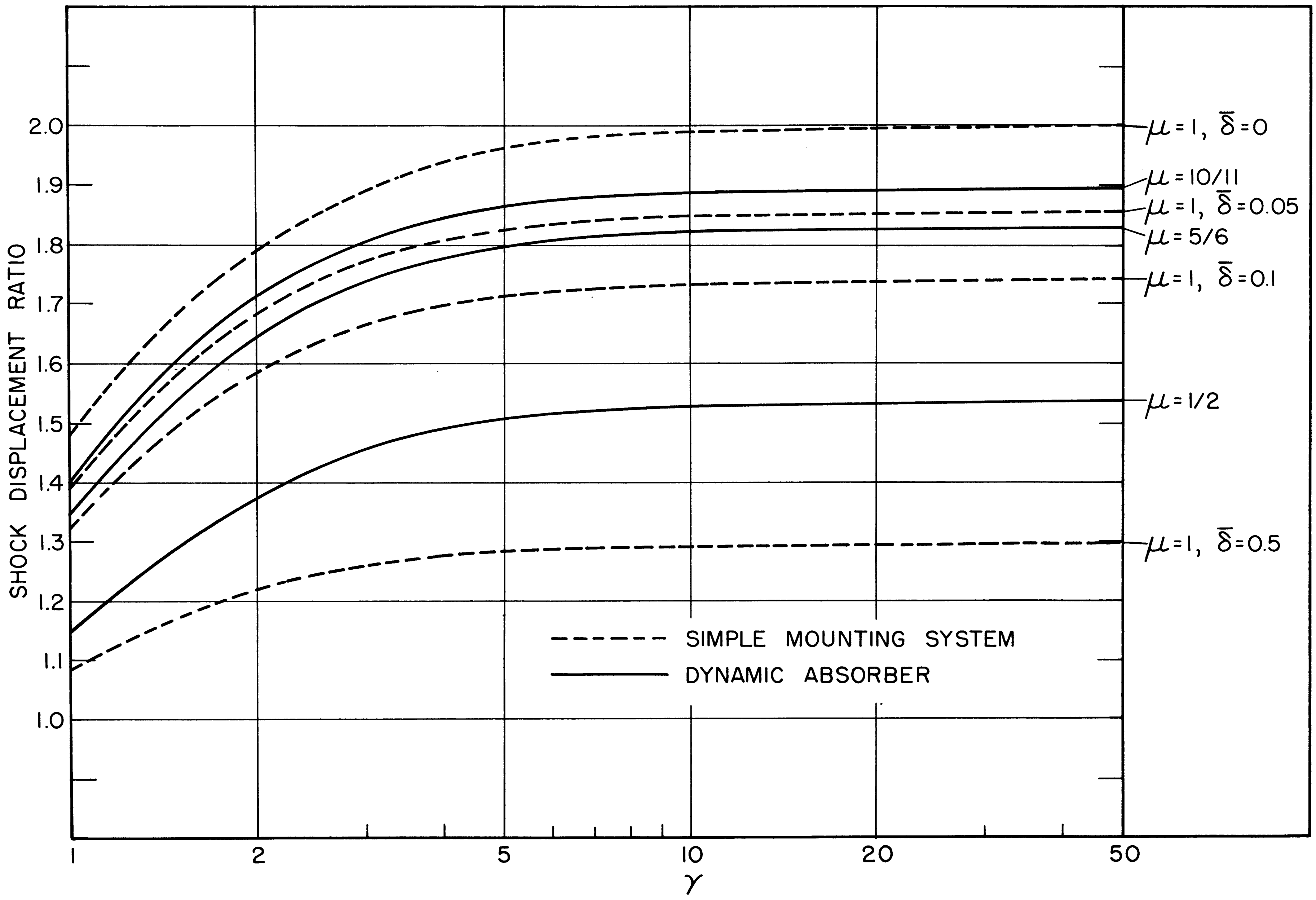


Fig. 18. The shock displacement ratio for a number of dynamic absorbers and simple systems plotted as a function of the parameter γ. Optimum absorber tuning and damping.

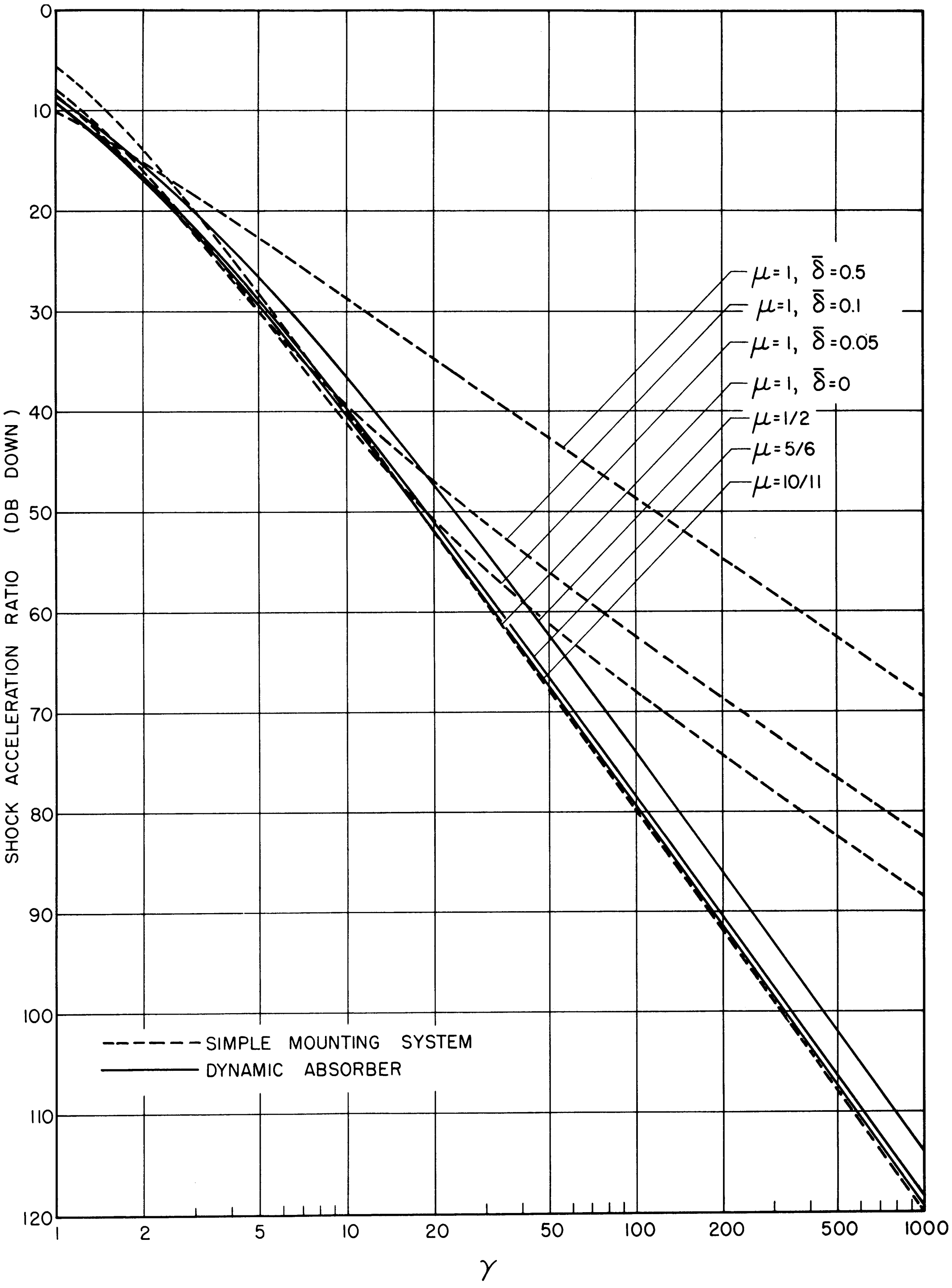


Fig. 19. The shock acceleration ratio for a number of dynamic absorbers and simple systems plotted as a function of the parameter γ. Optimum absorber tuning and damping.

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