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ANN ARBOR, MICHIGAN

EXTENSION OF LONGITUDINAL-BEAM PARAMETRIC-AMPLIFIER THEORY

TECHNICAL REPORT NO. 31

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Project 2750

CONTRACT NO. AF30(602)-1845
DEPARTMENT OF THE AIR FORCE
PROJECT NO. 4506, TASK NO. 45152
PLACED BY: THE ROME AIR DEVELOPMENT CENTER
GRIFFISS AIR FORCE BASE, NEW YORK

September, 1959

ABSTRACT

The differential equations are presented describing the longitudinal-beam parametric-amplifier in which the upper and lower sidebands around the pump frequency are considered. The solution is shown and discussed for the case of a very thin beam and for one of finite thickness. The results give better correlation with experiments than previous analyses.

LIST OF ILLUSTRATIONS

EXTENSION OF LONGITUDINAL-BEAM PARAMETRIC-AMPLIFIER THEORY*

The longitudinal-beam parametric amplifier has been previously investigated[1,2] using a model in which coupling between only the lower sideband and the signal was considered. Experiments by Ashkin[3] et al which showed large discrepancies between theoretical and measured values of gain raised the question as to whether or not the original model accurately described the system. Another model which had been used in a study of high-frequency beam modulation of traveling-wave tubes is now proposed. In the latter model, coupling is allowed between the signal and the first upper and lower sidebands around the fundamental of the pump frequency. The coupling to the sidebands around the second harmonic of the pump may be an important factor, however this is neglected in the present study.

The assumptions made in this "multimode" theory are very similar to those used by Louisell and Quate[1], that is the pump frequency is twice the signal frequency, only one space-charge wave (either fast <u>or</u> slow) is excited at each frequency, and the pump amplitude is much greater than the sideband and signal amplitudes. All amplitudes are assumed much less than the beam average quantities. Under these conditions the pump signal will propagate unperturbed with a wave number $\beta_e(1 - a'\omega_q/\omega)$ where

$$a' = \frac{\omega_{q2}}{2\omega_q} = \frac{\text{reduced plasma frequency at pump frequency}}{2\text{x reduced plasma frequency at signal frequency}} \qquad (1)$$

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* Published as letter to the editor in the <u>Proc</u>. <u>IRE</u>.

and $a' > 0$ for the fast wave and $a' < 0$ for the slow wave. Introduce the following definitions

$$b' = \frac{\omega_{q3}}{3\omega_q} = \frac{\text{reduced plasma frequency at upper sideband}}{\text{3x reduced plasma frequency at signal frequency}} \qquad (2)$$

$$m = |m|e^{j\varphi} = \frac{i}{I_o} = \begin{array}{l}\text{complex modulation index}\\ \text{of pump excitation}\end{array} , \qquad (3)$$

where i is component of r-f convection current at the pump frequency and I_o is the average beam current. Also v_1 is defined as the velocity at the signal frequency and, v_{11} the velocity at the upper sideband and finally, $\beta_q = \omega_q/u_o$ the reduced plasma frequency radian wave number. Assume that the velocities can be described by

$$v_1 = u_1(z)\exp - j\beta_e\left(1 - \frac{a'\omega_q}{\omega}\right) z ,$$

$$v_{11} = u_{11}(z)\exp - 3j\beta_e\left(1 - \frac{a'\omega_q}{\omega}\right) z . \qquad (4)$$

The propagation at the sideband frequencies can then be described in terms of the velocity amplitudes by two simultaneous linear differential equations,

$$\frac{d^2u_1}{dz^2} + 2j\beta_q a' \frac{du_1}{dz} - \beta_q^2\left[a'^2\left(1 + \frac{|m|^2}{4}\right) - 1 + \frac{a'^2}{b'^2}\frac{|m|^2}{4}\right] u_1$$

$$- \beta_q^2 \frac{m}{2}(1+2a'^2)u_1^* + j\beta_q a' \frac{m^*}{2}\left[1 + \frac{1}{3b'^2}\right]\frac{du_{11}}{dz}$$

$$- \beta_q^2 \frac{m^*}{2}\left[1 + a'^2\left(1 + \frac{1}{b'^2}\right)\right] u_{11} - a'^2 \beta_q^2 \frac{m^2}{4} u_{11}^* = 0 \qquad (5)$$

and

$$\frac{d^2u_{11}}{dz^2} + 6j\beta_q a' \frac{du_{11}}{dz} - 9\beta_q^2 \left[a'^2 (1+b'^2) \frac{|m|^2}{4} - b'^2\right] u_{11}$$

$$+ j\beta_q a' \frac{3m}{2} \left[1 + 3b^2\right] \frac{du_1}{dz} - 9\beta_q^2 \frac{m}{2} \left[a'^2 (1+b'^2) + b'^2\right] u_1$$

$$- 9a'^2b'^2\beta_q^2 \frac{m^2}{4} u_1^* = 0 \quad . \qquad (6)$$

The asterisk indicates the complex conjugate of a quantity and the assumption $a'\omega_q/\omega \ll 1$ has been made. Equations 5 and 6 can be integrated by standard linear methods using the operator $d/dz = \beta_q\mu$. There will be eight waves propagating in this system. An approximate solution for the case of a thin beam ($a'=b'=1$) and for small pump amplitudes, $|m| < 1$, is given as

$$\mu_{1,2} \approx \pm j\, 6.0$$

$$\mu_{3,4} \approx \pm j\, 2.0$$

$$\mu_{5,6} \approx 0.375|m| \pm j\, 1.25|m|$$

$$\mu_{7,8} \approx -\, 0.375|m| \pm j\, 1.25|m| \qquad (7)$$

More exact values of the propagation constants are shown in Figs. 1 and 2 respectively for a thin beam and a beam of finite thickness. The wave amplitudes can be calculated by using the boundary conditions that the signal is initially excited in the beam as a pure single space-charge wave and that the initial upper sideband velocity is zero. It can then be shown that only the growing or declining waves are appreciably excited. Gain can be obtained by adjusting the pump phase to $\varphi = \pi/2$. The velocity wave amplitudes for a thin beam are then described as

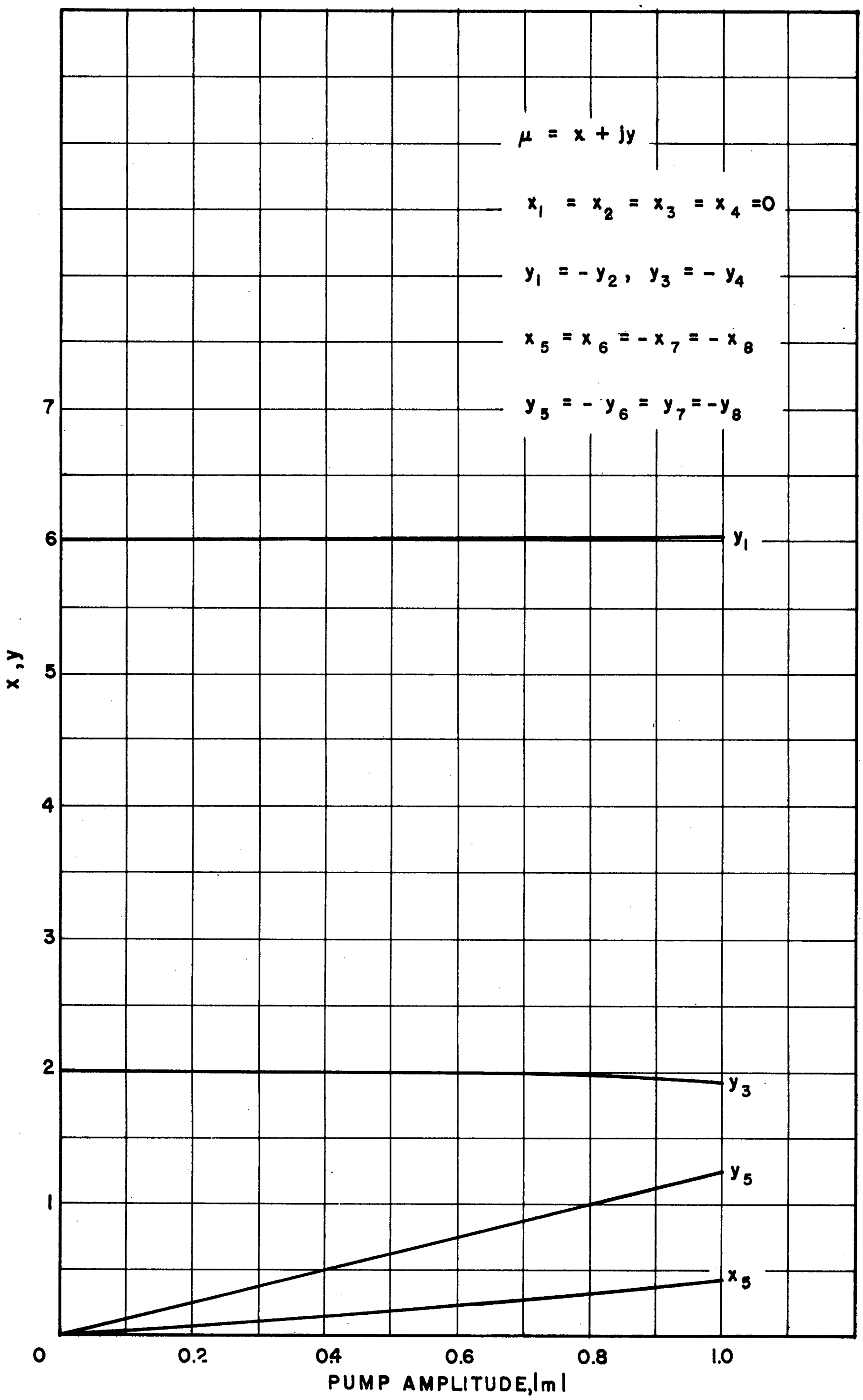

FIG. 1 LONGITUDINAL-BEAM PARAMETRIC-AMPLIFIER PROPAGATION CONSTANTS AS FUNCTIONS OF PUMP AMPLITUDE $a' = b' = 1$

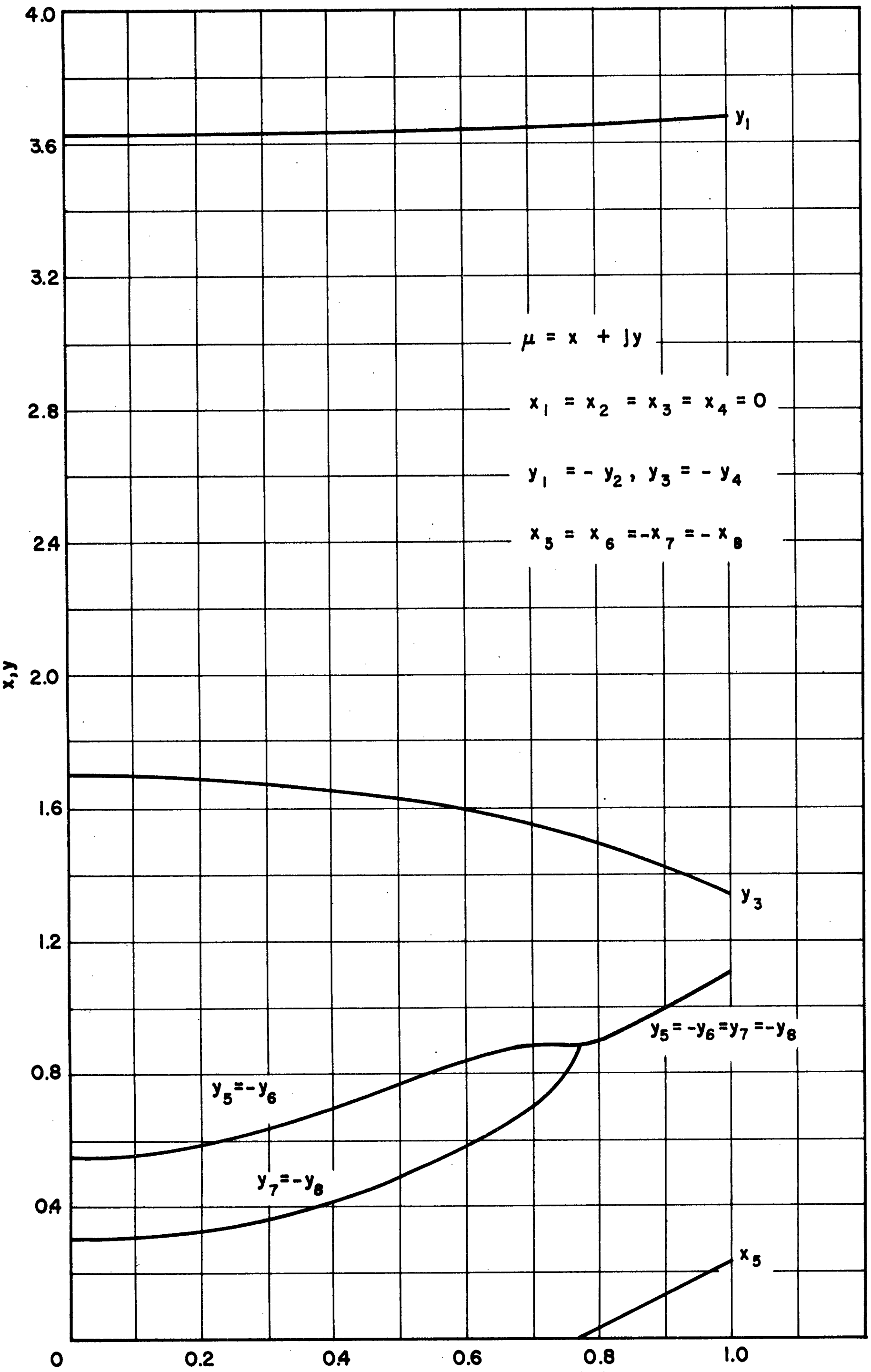

FIG. 2 LONGITUDINAL-BEAM-PARAMETRIC-AMPLIFIER PROPAGATION CONSTANTS AS FUNCTIONS OF PUMP AMPLITUDE $a' = 0.697$, $b' = 0.513$

$$u_1 \approx 1.05 \cos(1.25|m|\beta_q z - 17.84°) \exp 0.375|m|\beta_q z \exp - j\beta_e z \left(1 - \frac{a'\omega_q}{\omega}\right) \tag{8}$$

$$u_{11} \approx 1.822 \sin(1.25|m|\beta_q z) \exp 0.375|m|\beta_q z \exp - 3j\beta_e z \left(1 - \frac{a'\omega_q}{\omega}\right) . \tag{9}$$

It can be concluded that the upper sideband is heavily excited in the beam. The growth constant is one half of that obtained by Louisell and Quate[1] and therefore the theoretical gain will be much lower than theirs and hence closer to the experimental value. The next refinement in the analysis would be the inclusion of the sidebands around the second harmonic of the pump. It is expected that this will provide still better agreement between theory and experiment since the growth rate will be further reduced.

The complete derivation of the above equations along with a thorough discussion of the longitudinal-beam parametric-amplifier will be contained in Technical Report 33.

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3. Ashkin, A., Bridges, T. J., Louisell, W. H., Quate, C. F., "Parametric Electron Beam Amplifiers", paper presented at IRE WESCON Meeting; August, 1958.

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