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THEORETICAL POWER OUTPUT AND BANDWIDTH OF TRAVELING-WAVE AMPLIFIERS

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ABSTRACT

Expressions are developed to calculate the theoretical power output of traveling-wave amplifiers using any type of r-f structure. Calculations are made for helix-type tubes and it is shown how to calculate the power output of tubes using more dispersive structures in terms of calculations made for helix tubes.

The principle factors accounting for higher power output of dispersive structures are presented and discussed. The gain and bandwidth of forward-wave helix amplifiers are derived from the small-signal theory as functions of frequency and it is shown that the gain in db times the frequency bandwidth is a constant as a function of helix length for high $\gamma_o a'$ and the gain times the bandwidth squared is a constant for low $\gamma_o a'$.

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THEORETICAL POWER OUTPUT AND BANDWIDTH OF TRAVELING-WAVE AMPLIFIERS

Introduction

In a previous paper[1] the authors have developed a general design procedure for high-efficiency traveling-wave amplifiers. The design equations and procedure were developed primarily for helix-type devices, but correction factors were also derived which permitted the design of tubes with other r-f structures. The procedure was to select one parameter such as electron stream perveance and then determine the other parameters consistent with highest possible saturation efficiency. The consequences of design compromises are readily apparent.

The general curves and theoretical relations developed in that paper are used here to draw other composite curves which summarize the information and afford an opportunity to calculate the theoretical power output achievable from a traveling-wave amplifier as a function of any parameter. The power output calculations are predicted for any type of r-f propagating structure by relating its characteristics to a tube with a sheath-helix structure. They do not, however, account for thermal limitations of the r-f structure. Equations are derived from which the power output can be determined using a theoretical efficiency figure. The results indicate the principal dependence of efficiency on operating parameters and show how efficiency may be optimized.

The small-signal theory of traveling-wave amplifiers is used to calculate the bandwidth of helix-type tubes. This is facilitated by expanding the growing-wave propagation constant in a power series in C, QC, d and b. The effects, on gain and bandwidth, of circuit dispersion and loss as well as beam space charge are shown. The developed expressions for gain and bandwidth are then used to obtain a useful gain-bandwidth product for traveling-wave tubes.

Derivation of Power Output Relations

With the aid of appropriate reduction factors to relate the actual impedance of the helix to the impedance of a sheath helix, expressions have been obtained[1] for the space-charge parameter and the electron stream perveance in terms of other operating parameters. These expressions for helix-type amplifiers are as follows.

$$QC = \frac{60}{\left[\frac{B}{R_h}\left(\frac{FK_s'}{C}\right)^{1/2}\frac{(1+Cb)^{1/2}}{1-Cy} + 15.5\ C\right]^2} \qquad (1)$$

and

$$P_\mu = \frac{132\ (CB)^2\ (QC)}{\left[1 - 2C(QC)^{1/2}\right]^2 R_h^2(1-Cy_1)^2}, \qquad (2)$$

where R_h = the radian plasma-frequency reduction factor for a helix-type tube,

C = gain parameter,

QC = small-signal space-charge parameter,

B = $\gamma b'$ = $\gamma a'(b'/a')$, the space-charge range parameter ($B \propto$ stream diameter),

P_μ = electron-stream microperveance,

K_s' = $K_s(\beta_o/\gamma)[(1 + (\beta_o/\gamma)^2]^{3/2}$, proportional to the sheath-helix impedance, F = sheath-helix impedance-reduction factor, ratio of actual helix impedance to sheath-helix impedance,

b = $(u_o - v_o)/Cv_o$, the electron velocity parameter, and

y_1 = small-signal wave-velocity parameter.

The electron-stream perveance can also be written as

$$P_\mu = \frac{7905\ C^3}{FK_s'\ (1+Cb)} = \frac{P_o}{\eta_s\ V_o^{5/2}} \cdot 10^6\ , \qquad (3)$$

where P_o = r-f power output at saturation,

η_s = saturation efficiency, and

V_o = electron accelerating potential.

Equations 1 and 3 are combined to obtain the following expression for the saturated power output of a helix-type tube in terms of the beam voltage, saturation efficiency and other operating parameters of the tube.

$$P_o = \frac{2.5 \times 10^5\ (CB)^2\ \eta_s\ V_{okv}^{5/2}}{R^2(1-Cy)^2 \left[\left(\frac{60}{QC}\right)^{1/2} - 15.5\ C\right]^2}\ , \qquad (4)$$

where V_{okv} = stream accelerating voltage in kilovolts.

Large-signal calculations[2,3,4] of the saturation efficiency as a function of the operating parameters are used in conjunction with Eq. 4 to compute the theoretical power output as a function of the stream diameter parameter B, stream microperveance P_μ, accelerating voltage V_o, gain parameter C and the space-charge parameter QC.

Calculations of the nonlinear performance of traveling-wave amplifiers have been made using a one-dimensional model of the device. The efficiency results are applicable to all forms of these amplifiers, independent of frequency range or type of r-f structure used. The principal assumptions made in the theory are that the electron stream is a confined flow and that the electric field due to the circuit and the space charge does not vary across the stream.

This latter assumption is considered reasonable for values of the stream diameter parameter B up to 0.5 or slightly higher, as indicated by various experimental studies. For larger B values it is necessary to use correction factors on the efficiency. Rowe[4] has investigated theoretically the first-order effects of radial electric field variations across a solid beam. The theoretical efficiency correction factors agree well with experimental observations[5,6]. The efficiency correction factors for solid and hollow beams are discussed in the next section.

A summary of efficiency calculations made with this one-dimensional theory is presented in Fig. 1. Considerable comparison with experiment has been made, with satisfactory results when consistent experimental data can be obtained.

Efficiency results such as those presented in Fig. 1 can be used in conjunction with Eq. 4 to calculate curves of theoretical power output for an amplifier as a function of such operating parameters as the stream diameter, the stream perveance, the accelerating voltage, the gain parameter and the saturation efficiency. In order to determine maximum power output the operating voltage is assumed to be adjusted for maximum saturation power output rather than for maximum gain. Typical theoretical power output curves for a helix-type traveling-wave amplifier are shown in Figs. 2 and 3. Additional theoretical power output curves are presented in Appendix A.

The power output curves are restricted to a helix-type device because of the fact that the sheath-helix impedance and the appropriate impedance reduction factors were used in calculating the values of C and

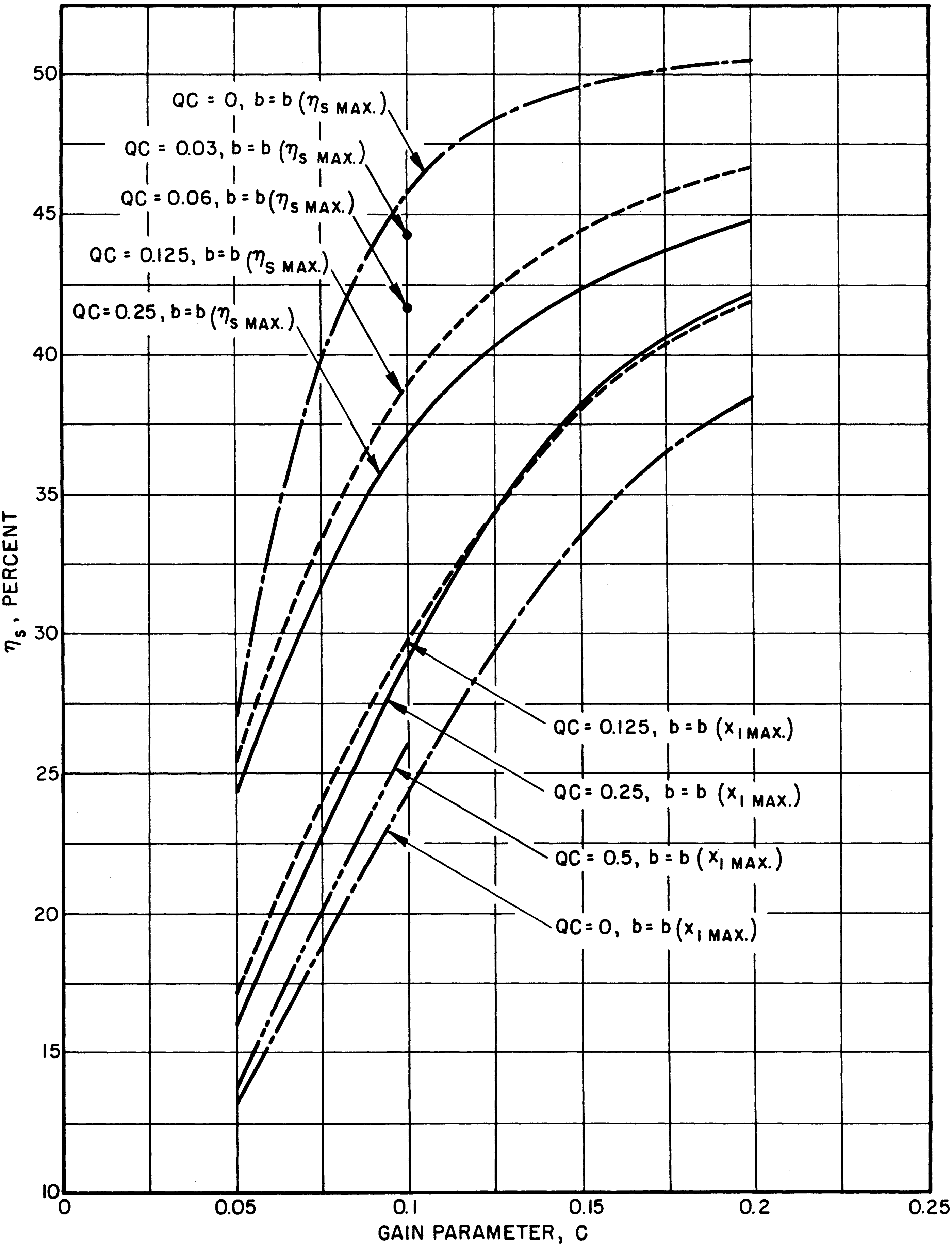

FIG. 1 SATURATION EFFICIENCY VS. GAIN PARAMETER. (B = 1.0, d = 0)

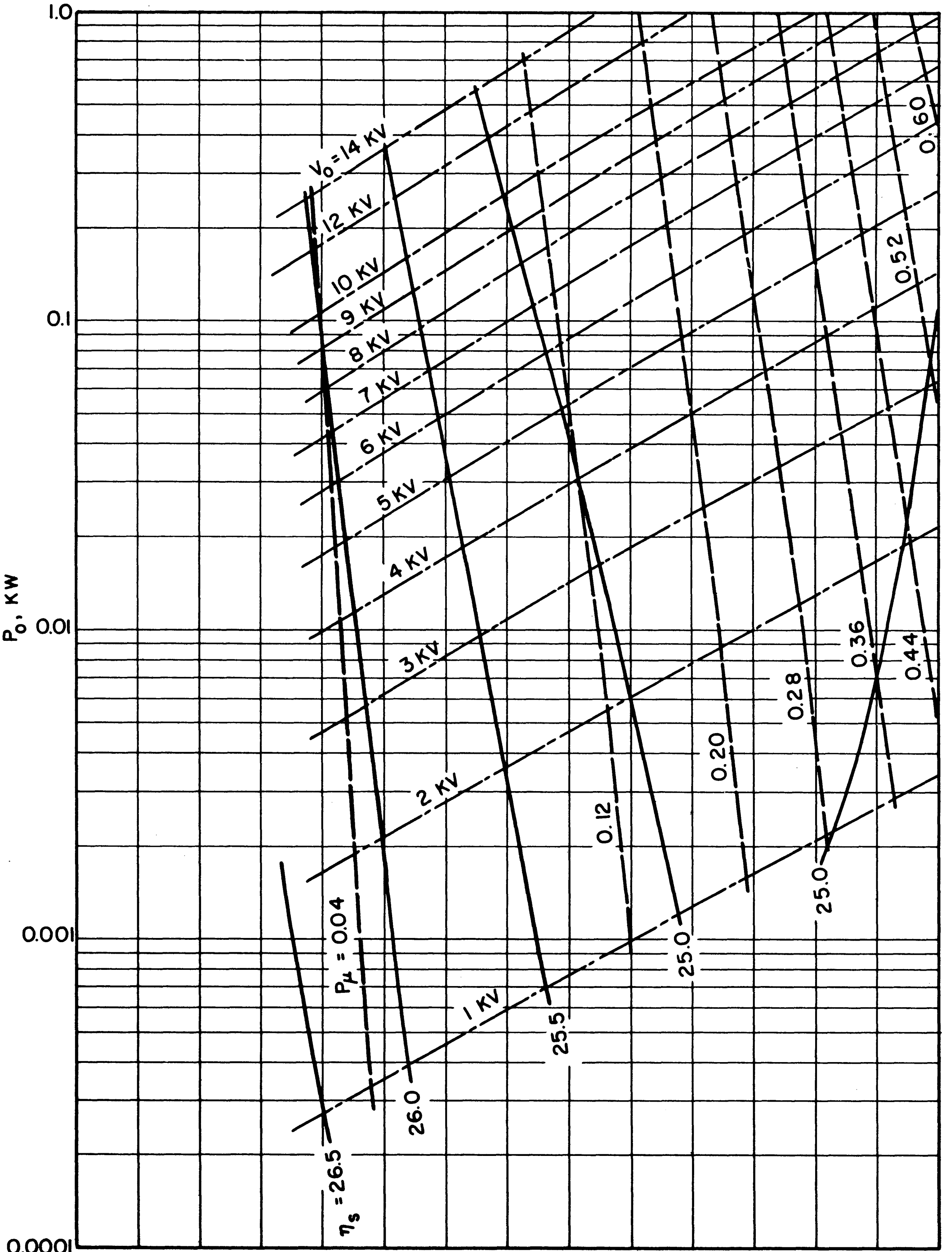

FIG. 2 THEORETICAL POWER OUTPUT FOR A HELIX-TYPE TRAVELING-WAVE AMPLIFIER. (C = 0.05, a'/b' = 1.4, DLF = 80 %, d = 0)

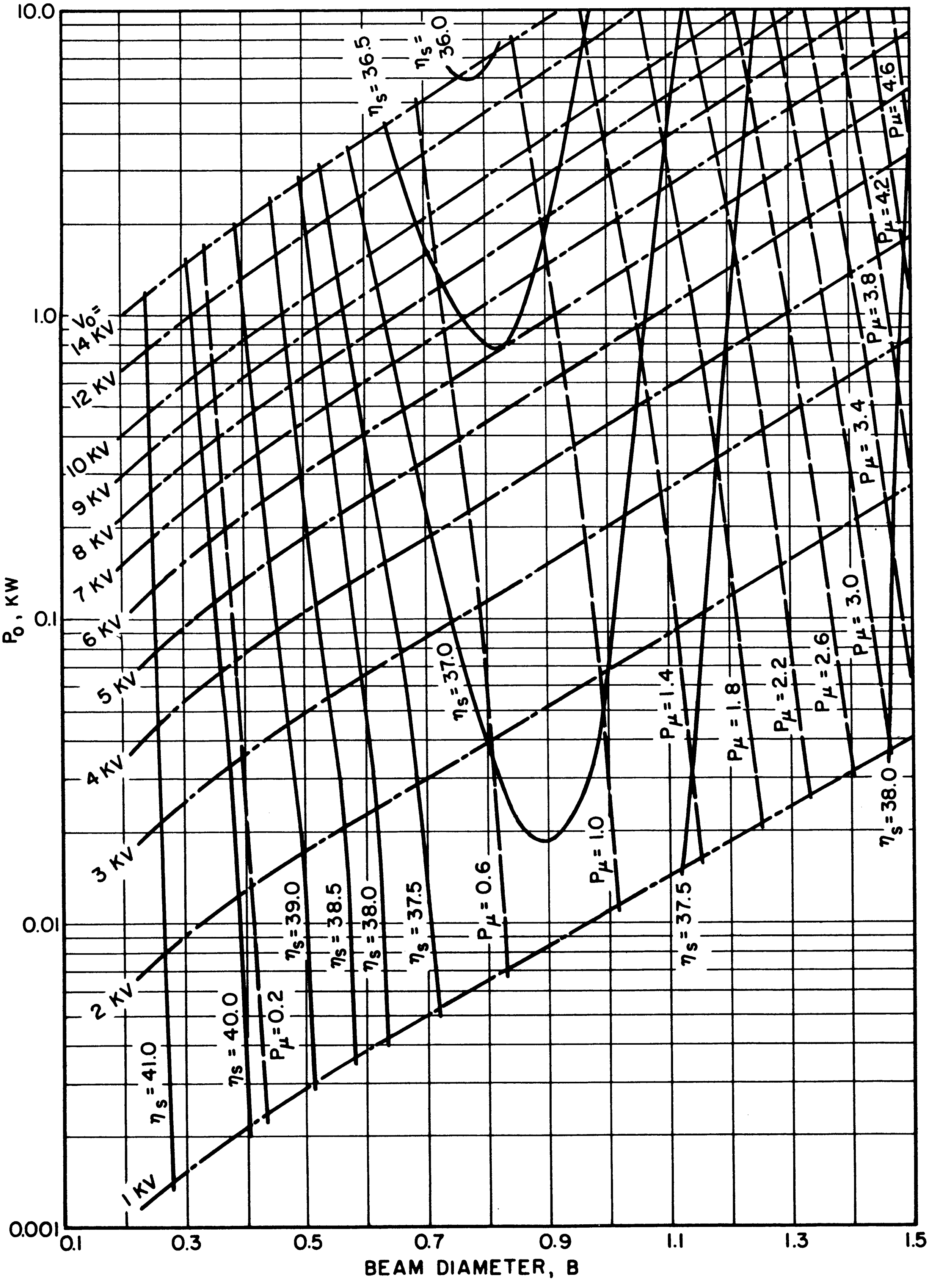

FIG. 3 THEORETICAL POWER OUTPUT FOR A HELIX-TYPE TRAVELING-WAVE AMPLIFIER. (C = 0.10, a'/b' = 1.4, DLF = 80 %, d = 0)

QC. In a later section correction factors will be derived which permit the use of these curves in determining the power output of traveling-wave amplifiers utilizing periodically loaded waveguide r-f circuits.

The helix voltage curves were carried to only 14 kv, as it is believed the higher voltages would result in a very low impedance and hence a low value of C at most frequencies due to the openness of the helix pitch. The limitation on stream perveance was taken as 3×10^{-6} owing to the impracticability of constructing solid streams with higher values of perveance. In addition, it is noted that the saturation efficiency is relatively low at high perveance, independent of the accelerating voltage. In all curves of the type shown in Figs. 2 and 3 the maximum saturation efficiency occurs at relatively low power output values. The perveance is low at high saturation efficiencies, which indicates that the high efficiency is due to high circuit impedance giving a relatively large value of C rather than to high stream perveance or current.

Since each set of power curves is for a constant a'/b', the abscissa is directly proportional to $\gamma a'$ and the fact that the high efficiencies occur at low values of $\gamma a'$ indicates a dispersive helix giving a relatively narrow-band operation. It will be shown in a later section of this paper how the bandwidth depends upon each of the operating parameters and in particular how the choice of B for a fixed a'/b' is dependent upon bandwidth considerations.

The effect of radial electric field variations of the circuit field on the saturation efficiency has been investigated theoretically by solving the nonlinear equations assuming confined flow and no radial motion of the electrons. It was found that this radial circuit field dependence had a marked effect on the efficiency for large values of B. The results indicate that the saturation efficiency for devices with large-diameter streams can be expressed as follows.

$$\eta_s(B) \approx \eta_s\Big|_{f=1} f^{1/2}(B) \qquad \text{for } b = b(x_{1max}) \tag{5a}$$

and

$$\eta_s(B) \approx \eta_s\Big|_{f=1} f^{1/4}(B) \qquad \text{for } b = b(\eta_{smax}) \tag{5b}$$

where $\eta_s\Big|_{f=1}$ = the saturation efficiency for no radial circuit field variation,

$$f(B) = \frac{I_0(\gamma b')}{I_0(\gamma a')} \text{ for thin hollow streams, and}$$

$$f(B) = \frac{[I_0^2(\gamma b') - I_1^2(\gamma b')]^{1/2}}{I_0(\gamma a')} \text{ for solid streams.}$$

A graphical presentation of this dependence is given in Fig. 8 of Ref. 4. This correction factor may then be used in Eq. 4 when calculating the power output.

Theoretical Power Output of Modified Helix and Periodically Loaded Waveguide Circuits

The previously developed power output curves may be used to determine the theoretical power output from a traveling-wave amplifier using any other type of r-f structure. All nonhelix structures will be described by writing their impedance characteristics as a factor times the sheath-helix impedance, as in Eq. 6.

$$K_d = GK_s , \tag{6}$$

where K_d, G and K_s are all functions of frequency.

In many cases G is dependent upon frequency and hence point-by-point power output calculations have to be made across the frequency band. If the structure is of the modified helix form and there is a dielectric material present that loads the helix, then the G factor (impedance gain factor) takes into account the effect of the dielectric. The ratio of the space-charge parameter for an arbitrary dispersive structure to that for a helix tube has been derived as[1]

$$\frac{\left[\frac{QC}{1+Cb}\right]_d}{\left[\frac{QC}{1+Cb}\right]_h} = \xi \left[1 + \frac{31}{\nu} C \left(\frac{R_d}{G^{1/2}}\right)\left(\frac{1}{\xi^{1/2}} - 1\right)\right], \tag{7}$$

where $\xi \triangleq \left(\frac{R_d}{R_h}\right)^2 \frac{F}{G}$,

$\nu \triangleq \left(\frac{BK_s'}{C}\right)^{1/2}$, and

R_d = the radian plasma-frequency reduction factor for the resonant or dispersive structure.

The power output for a dispersive r-f structure relative to the power output of a helix tube may be written from Eq. 4 as

$$\frac{P_{od}}{P_{oh}} \doteq \left(\frac{C_d}{C_h}\right)^2 \left(\frac{\eta_{sd}}{\eta_{sh}}\right) \left(\frac{R_h}{R_d}\right)^2 \frac{(1+C_h b_h)^2}{(1+C_d b_d)^2} \left(\frac{V_{od}}{V_{oh}}\right)^{5/2} \frac{\left[\left(\frac{60}{(QC)_h}\right)^{1/2} - 15.5\, C_h\right]^2}{\left[\left(\frac{60}{(QC)_d}\right)^{1/2} - 15.5\, C_d\right]^2}, \tag{8}$$

where C_d/C_h = the ratio of the gain parameter for a dispersive structure to that for a helix,

η_{sd}/η_{sh} = ratio of the saturation efficiency for a dispersive structure to that for a helix,

R_h/R_d = ratio of the radian plasma-frequency reduction factors for a dispersive structure and a helix, and

V_{od}/V_{oh} = ratio of voltages for dispersive and helix structures.

It has been assumed in the development of the above equation that the stream diameter parameter, B, is kept the same for the dispersive structure as for the helix structure. Since all the parameters for a helix tube can be determined from the theoretical power output curves, any hypothetical helix design can be used as the reference tube in Eq. 8 and the power output of the dispersive structure may be computed in reference to this tube.

In general the impedance of the dispersive structure will be higher than that of the helix tube but its frequency passband will be narrower. It is extremely convenient to write Eq. 8 assuming that the value of the gain parameter C is the same for the two structures. This, of course, means that the design and power output curves for a helix structure can be used. Also, for dispersive structures $R_d > R_h$, since the effective equivalent drift-tube radius for a periodically loaded waveguide circuit will be larger than that for a helix.

Equation 8 can then be written as

$$\frac{P_{od}}{P_{oh}} = \left(\frac{\eta_{sd}}{\eta_{sh}}\right)\left(\frac{R_h}{R_d}\right)^2\left(\frac{V_{od}}{V_{oh}}\right)^{5/2} \cdot \frac{\left[\left(\frac{60}{(QC)_h}\right)^{1/2} - 15.5\ C\right]^2}{\left[\left(\frac{60}{(QC)_d}\right)^{1/2} - 15.5\ C\right]^2} \cdot \qquad (9)$$

Equation 9 may be simplified to the following form:

$$\frac{P_{od}}{P_{oh}} = \frac{\eta_{sd}}{\eta_{sh}}\left(\frac{V_{od}}{V_{oh}}\right)^{5/2} \xi\kappa \left[\frac{1 - 2C(QC)_h{}^{1/2}}{1 - 2C(QC)_d{}^{1/2}}\right]^2 , \qquad (10)$$

where $\kappa = (R_h/R_d)^2$.

The appropriate values of ξ and κ for any structure may be calculated or obtained from the curves of Ref. 1. Since the second term in the last factor in the numerator of Eq. 10 is usually small compared to unity, a further simplification of Eq. 10 may be carried out:

$$\frac{P_{od}}{P_{oh}} = \frac{\eta_{sd}}{\eta_{sh}} \left(\frac{V_{od}}{V_{oh}}\right)^{5/2} \frac{F}{G} \left[1 + 4C(QC)_h^{\ 1/2} (\xi^{1/2} - 1)\right] . \qquad (11)$$

In general the last factor of Eq. 11 is near unity, since it is proportional to C, and hence the equation may be further simplified. Since the efficiency ratio is usually near one or somewhat greater, and F/G is less than one, it can be seen from Eq. 11 that the prime factor in obtaining greater power output is increased voltage. Here the voltage ratio has to be greater than approximately 2 to obtain more power from the dispersive structure than from the helix structure. The basic power-ratio equation (8) can be reduced to various simpler forms by assuming equality between some parameter of the dispersive structure and the corresponding parameter of the helix structure.

Interaction Bandwidth

General

In an earlier section of this paper, it was mentioned that the choice of B (or $\gamma a'$, since $B = \gamma a' \cdot b'/a'$) was in part dictated by bandwidth requirements. A discussion of bandwidth is therefore in order. The interaction bandwidth is determined only by the internal gain limitations and is not influenced by transducer cutoff or loading effects. When considering the overall bandwidth of a tube one must consider all external as well as internal phenomena; however, the following discussion will be limited to the interaction bandwidth.

The interaction bandwidth is determined by the range of frequencies over which the electrons keep in step with the circuit wave. This requires a consideration of circuit dispersion. The sheath helix will be the only circuit considered.

When designing a helix-type traveling-wave tube, one usually refers to the approximate variation of gain with frequency given by Pierce[7] and decides upon a center frequency value of $\gamma_o a'$ for wideband operation. It is one of the purposes of this paper to consider the gain as a function of frequency in a more detailed manner than in the above reference. Space-charge forces, loss, finite values of C and nonsynchronism will be considered.

While a nonlinear analysis of bandwidth characteristics would be desirable for this discussion, the complications introduced are quite formidable; hence the analysis presented here is a linearized one. The results are further limited to $QC < 1/4$, which is consistent with high saturation efficiency.

Gain Equations for the Sheath-Helix Tube

The small-signal gain as a function of frequency will be studied by choosing the center frequency values of the parameters, calculating their variations with frequency, determining the propagation constants as frequency functions and, finally, using the standard growing-wave gain equation

$$G = \exp A \exp \beta_e C x_1 L . \tag{12}$$

The gain as found by this method actually may cover a somewhat narrower range of frequencies than the actual gain, since it is possible to have a gain due to a beating of the three forward waves after growth has ceased. This is the phenomenon which has been named the Crestatron[8] effect and it will be neglected here since it occurs near the band edge and the gain contributed is probably less than the minimum used in defining the band. The band will usually be referred to as the range of frequencies in the vicinity of the center frequency over which the gain does not vary more than 3 db or 10 db[9].

Sensiper[10] has derived explicit forms of the propagation constants as functions of the traveling-wave tube parameters. This particular formulation is valid only for QC $<$ 1/4. The leading terms of Sensiper's expansion are

$$x_1 \approx \sqrt{3}\left[\frac{1}{2}\left(1+\frac{C}{2}\right)-\frac{d}{3\sqrt{3}}\left(1+\frac{C}{2}-\frac{d}{2\sqrt{3}}+\frac{b}{3}+\frac{4}{3}\,QC\right)-\frac{2}{3}\,QC\left(1-\frac{2}{3}\,b\right)\right.$$
$$\left.+\frac{b}{6}\left(C-\frac{b}{3}\right)\right] . \tag{13}$$

Substitution of Eq. 13 into Eq. 12 gives the gain as a product of four terms, hereafter called the gain factors. Therefore if we define

$$F_1(\omega) \triangleq \frac{\omega C}{2}\left(1+\frac{C}{2}\right), \tag{14}$$

$$F_2(\omega) \triangleq -\frac{\omega C d}{3\sqrt{3}}\left(1+\frac{C}{2}-\frac{d}{2\sqrt{3}}+\frac{b}{3}+\frac{4}{3}\,QC\right), \tag{15}$$

$$F_3(\omega) \triangleq -\frac{2\omega C}{3}\,QC\left(1-\frac{2}{3}\,b\right), \tag{16}$$

and

$$F_4(\omega) \triangleq \frac{\omega b C}{6}\left(C-\frac{b}{3}\right), \tag{17}$$

and take logarithms, the gain can be expressed as

$$\frac{u_o}{\sqrt{3}\,L}\,\ln\left(\frac{G}{\exp A}\right) = F_1+F_2+F_3+F_4 . \tag{18}$$

The gain factor F_1 is determined by the small-signal gain parameter C. This factor is the fundamental one in determining gain; the remaining factors introduce the effects due to other parameters. As will be seen later on, the factors F_2, F_3 and F_4 actually define the operating range but their effect is small in the region of active gain. F_2 vanishes for no loss, F_3 vanishes for no space charge, and F_4 vanishes for synchronism and no dispersion.

It will now be assumed that the initial loss A is constant with frequency. The results published by Brewer and Birdsall[11] indicate that A does vary with variation of the parameters. However, for the range of parameters chosen in this study, the variation of A will be small for large C. For small C, the loss of gain due to nonsynchronism will be so great as to outweigh all other effects. Equation 18 is equivalent to:

$$G_{db}(\omega) = 15.1 \left(\frac{L}{u_o}\right) [F_1 + F_2 + F_3 + F_4] + 8.68\, A \quad . \tag{19}$$

The gain factors are evaluated by substituting Eqs. 5, 11, 14 and 16 of Appendix B into Eqs. 14 through 17 above. The following normalizing parameters are introduced:

$$\xi = \frac{\omega}{\omega_o} = \text{normalized frequency} \ ,$$

$$\eta = \frac{\gamma a'}{\gamma_o a'} = \text{normalized dispersion} \ ,$$

C_o, ω_o, γ_o are the indicated quantities at the center or design frequency.

$$\chi = C_o^3 \exp 2 \left(1 - \frac{b'}{a'}\right) \gamma_o a'$$

$$\tau = \frac{181}{30} \left(\frac{b'}{a'}\right)^2 (\gamma_o a')^2 \ ,$$

and

$$g = f' \gamma_o a' \left(\frac{b'}{a'}\right) \ , \tag{20}$$

where f' is the initial slope of the plasma-frequency reduction factor curve ($g = 0.6\, \gamma_o a'$ for $b'/a' = 0.7$), and b'/a' is the beam-to-helix radii ratio.

The gain factors are then given in two regions, $\gamma a' \gtrsim 1$ and $\gamma a' \lesssim 1$. For the region in the vicinity of $\gamma a' \approx 1$, one must interpolate; however, the values in each region agree quite well near $\gamma a' = 1$. For $\gamma a' < 1$

$$F_1(\omega) = \frac{\omega_o}{2} \xi (\chi\tau)^{1/3} \left[\exp(-0.59\, \gamma_o a'\eta)\right] \left[1 + \frac{1}{2}(\chi\tau)^{1/3} \exp -0.59\, \gamma_o a'\eta\right] \tag{21}$$

$$F_2(\omega) = -\frac{\omega_o C_o d_o \sqrt{\xi}}{3\sqrt{3}} \Bigg[1 + \frac{1}{2}(\chi\tau)^{1/3} \exp(-0.59\, \gamma_o a'\eta)$$

$$+ \left\{-\frac{1}{2\sqrt{3}}\left(\frac{1}{\chi\tau\xi^{3/2}}\right)^{1/3} C_o d_o + \frac{1}{3}\left(\frac{1+C_o b_o}{\xi}\eta - 1\right)\left(\frac{1}{\chi\tau}\right)^{1/3}\right.$$

$$\left. + \frac{4}{3} C_o^2 Q C_o \left[\frac{1 - \exp -g\eta}{1 - \exp -g}\right]^2 \frac{\exp 0.59\, \gamma_o a'\eta}{(\xi^3\chi\tau)^{2/3}}\right\} \exp 0.59\, \gamma_o a'\eta\Bigg] \tag{22}$$

$$F_3(\omega) = -\frac{2\omega_o}{3} C_o^2 Q C_o \left[\frac{1 - \exp -g\eta}{1 - \exp -g}\right]^2 \frac{\exp 0.59\, \gamma_o a'\eta}{(\xi^3\chi\tau)^{1/3}} \Bigg\{1$$

$$- \frac{2}{3}\left(\frac{1}{\chi\tau}\right)^{1/3}\left[\frac{1+C_o b_o}{\xi}\eta - 1\right] \exp 0.59\, \gamma_o a'\eta\Bigg\} \tag{23}$$

$$F_4(\omega) = \frac{\omega_o \xi}{6}\left[\frac{1+C_o b_o}{\xi}\eta - 1\right]\Bigg[(\chi\tau)^{1/3} \exp -0.59\, \gamma_o a'\eta$$

$$- \frac{1}{3}\left(\frac{1}{\chi\tau}\right)^{1/3}\left(\frac{1+C_o b_o}{\xi}\eta - 1\right) \exp 0.59\, \gamma_o a'\eta\Bigg] . \tag{24}$$

For $\gamma a' \gtrapprox 1$

$$F_1(\omega) = \frac{\omega_o}{2}\left[\frac{\chi\xi^2}{\eta}\right]^{1/3}\left[1 + \frac{1}{2}\left(\frac{\chi}{\xi\eta}\right)^{1/3} \exp - \frac{2}{3}\left(1 - \frac{b}{a}\right)\gamma_o a'\eta\right]$$

$$\exp - \frac{2}{3}\left(1 - \frac{b}{a}\right)\gamma_o a'\eta \qquad (25)$$

$$F_2(\omega) = - \frac{\omega_o C_o d_o \sqrt{\xi}}{3\sqrt{3}} \quad 1 + \frac{1}{2}\left(\frac{\chi}{\xi\eta}\right)^{1/3} \exp - \frac{2}{3}\left(1 - \frac{b'}{a'}\right)\gamma_o a'\eta$$

$$+ \left\{- \frac{d_o C_o}{2\sqrt{3}}\left(\frac{\eta}{\chi\xi^{1/2}}\right)^{1/3} + \frac{1}{3}\left(\frac{1+C_o b_o}{\xi}\eta - 1\right)\left(\frac{\xi\eta}{\chi}\right)^{1/3}\right.$$

$$\left. + \frac{4}{3} C_o^2 Q C_o \left(\frac{1 - \exp -g\eta}{1 - \exp -g}\right)^2 \left(\frac{\eta}{\chi\xi^2}\right)^{2/3} \exp \frac{2}{3}\left(1 - \frac{b}{a}\right)\gamma_o a'\eta\right\}$$

$$\left.\exp \frac{2}{3}\left(1 - \frac{b}{a}\right)\gamma_o a'\eta\right] \qquad (26)$$

$$F_3(\omega) = - \frac{2\omega_o}{3} C_o^2 Q C_o \left(\frac{1 - \exp -g\eta}{1 - \exp -g}\right)^2 \left(\frac{\eta}{\chi\xi^2}\right)^{1/3} \exp \frac{2}{3}\left(1 - \frac{b'}{a'}\right)\gamma_o a'\eta \left\{1\right.$$

$$\left. - \frac{2}{3}\left(\frac{\xi\eta}{\chi}\right)^{1/3}\left[\frac{1+C_o b_o}{\xi}\eta - 1\right] \exp \frac{2}{3}\left(1 - \frac{b'}{a'}\right)\gamma_o a'\eta\right\} \qquad (27)$$

$$F_4(\omega) = \frac{\omega_o \xi}{6}\left(\frac{1+C_o b_o}{\xi}\eta - 1\right)\left[\left(\frac{\chi}{\xi\eta}\right)^{1/3} \exp - \frac{2}{3}\left(1 - \frac{b'}{a'}\right)\gamma_o a'\eta\right.$$

$$\left. - \frac{1}{3}\left(\frac{\xi\eta}{\chi}\right)^{1/3}\left(\frac{1+C_o b_o}{\xi}\eta - 1\right) \exp \frac{2}{3}\left(1 - \frac{b'}{a'}\right)\gamma_o a'\eta\right] \qquad (28)$$

The relation between η and ξ can be obtained from the characteristic equation of the structure.

Gain Factors

F_1 is shown in Fig. 4 as a function of the normalized frequency, with C_o and $\gamma_o a'$ assuming several values at the center frequency ($\xi = 1$). These curves represent the gain for a hypothetical tube with no space charge, no loss and with the unperturbed circuit wave and the beam in synchronism at all frequencies. Figure 4 indicates that in this case an optimum selection of $\gamma_o a'$ for wideband operation would be approximately 1.2.

This value is lower than that given in Fig. 3.6 of Pierce's book[7] (for $b'/a' = 0.7$). Pierce's curve is a plot of the proportionality

$$CN \propto \gamma a' \, F(\gamma a') \quad , \tag{29}$$

where N is the number of circuit wavelengths. The quantity CN, however, is not exactly proportional to the gain. The gain is instead proportional to the number of stream wavelengths N_s. For a gain calculation at a particular frequency the error involved in using the circuit wavelengths in place of the stream wavelengths would be small. However, when the gain variation with frequency is being studied the error can increase significantly, since the number of stream wavelengths varies indirectly with frequency whereas, due to dispersion, the number of circuit wavelengths does not. Therefore, a better approximation to the gain in this very idealized tube would be

$$\text{Gain} \propto CN_s \;=\; \gamma a' \, F(\gamma a') \, \frac{v(\gamma_o a')}{u_o} \, \frac{v(\gamma a')}{v(\gamma_o a')} \quad . \tag{30}$$

The ratio $v(\gamma a')/v(\gamma_o a')$ compares the velocity at a given $\gamma a'$ to that at the center frequency $\gamma_o a'$. Pierce presents a curve in Fig. 3.2 showing this dependence. A multiplication of the curve in Fig. 3.6 with that in

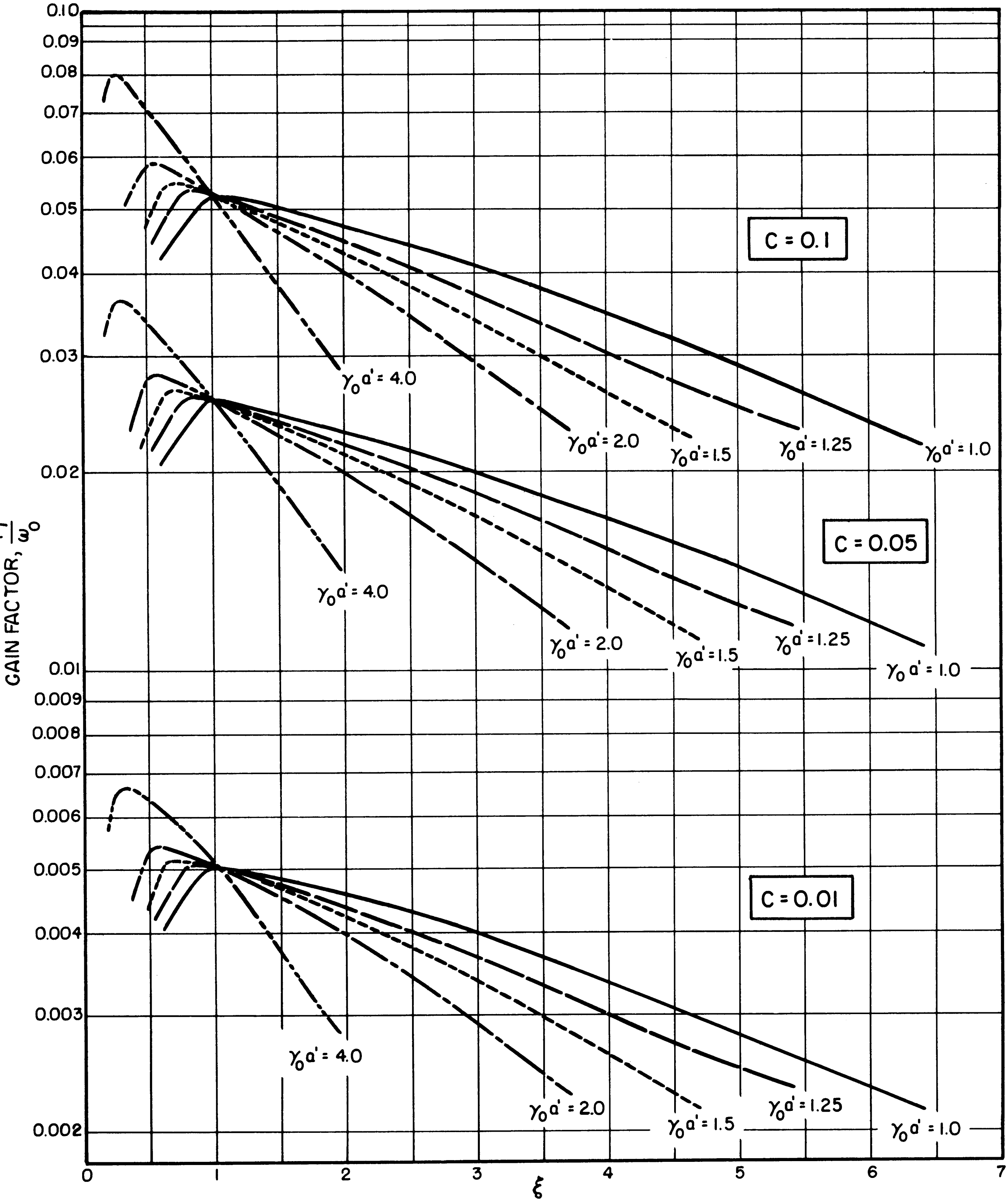

FIG. 4 GAIN FACTOR, $\frac{F_1}{\omega_0}$ VS. NORMALIZED FREQUENCY. (b'/a' = 0.7)

Fig. 3.2 results in a shift of the maximum toward the value shown in Fig. 4 of this paper.

Representative plots of F_2 as a function of frequency are shown in Figs. 5 and 6. For all cases considered, F_2 becomes more negative as the frequency increases. The inclusion of space charge tends to accentuate the effect of F_2 in decreasing the gain. The value of b used in the calculations is the one producing maximum small-signal gain at the center frequency. It is possible to get an increase in gain for $C_0 = 0.01$ at very low $\gamma_0 a'$. This occurs because of the large negative b value that results at the lower frequencies with small C. Actually, this increase is never realized, since at this negative b value F_4 is a very large negative number and there is no gain.

F_3 is shown in Figs. 7 and 8. The calculations of F_3 as well as all other factors involving QC were carried out only until QC reached 1/4. This restriction is imposed by the approximation for x_1 given in Eq. 13.

F_4 is shown in Figs. 9 and 10. For low C, F_4 restricts the gain to a very small range of frequencies. C represents the coupling between the beam and the circuit. For very loose coupling the interaction will be negligibly small unless the beam travels approximately at circuit velocity. For tighter coupling, the beam can perturb the circuit wave sufficiently to produce interaction even though the velocities may be quite far from synchronism.

Gain as a Function of Frequency

The expression for the gain is given in Eq. 19. A summation of the gain factors describes the gain as a function of the center frequency parameters. The following sums are considered:

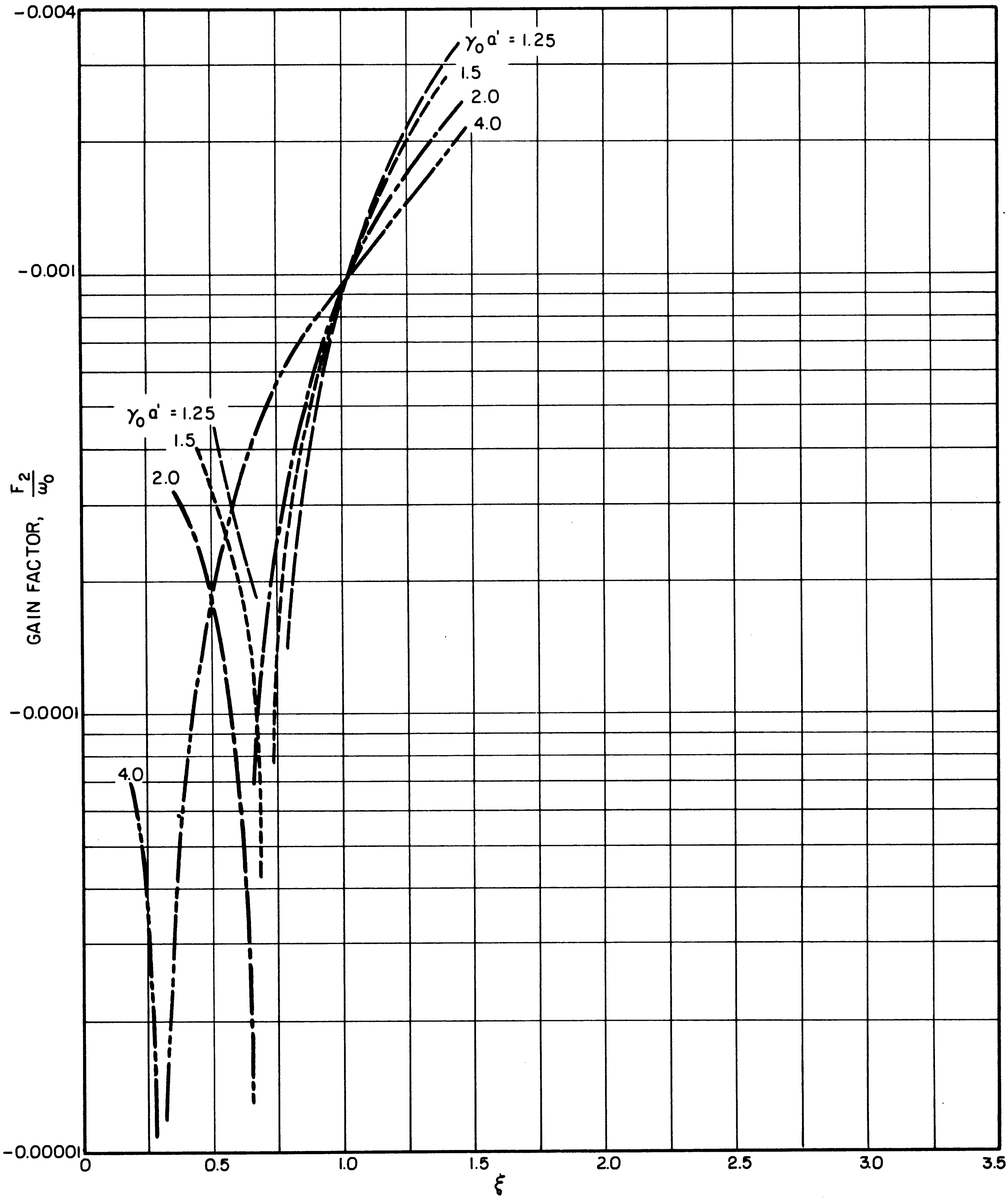

FIG. 5 GAIN FACTOR, $\frac{F_2}{\omega_0}$ VS. NORMALIZED FREQUENCY. (C = 0.01, QC = 0.125, d = 0.125, b'/a' = 0.7)

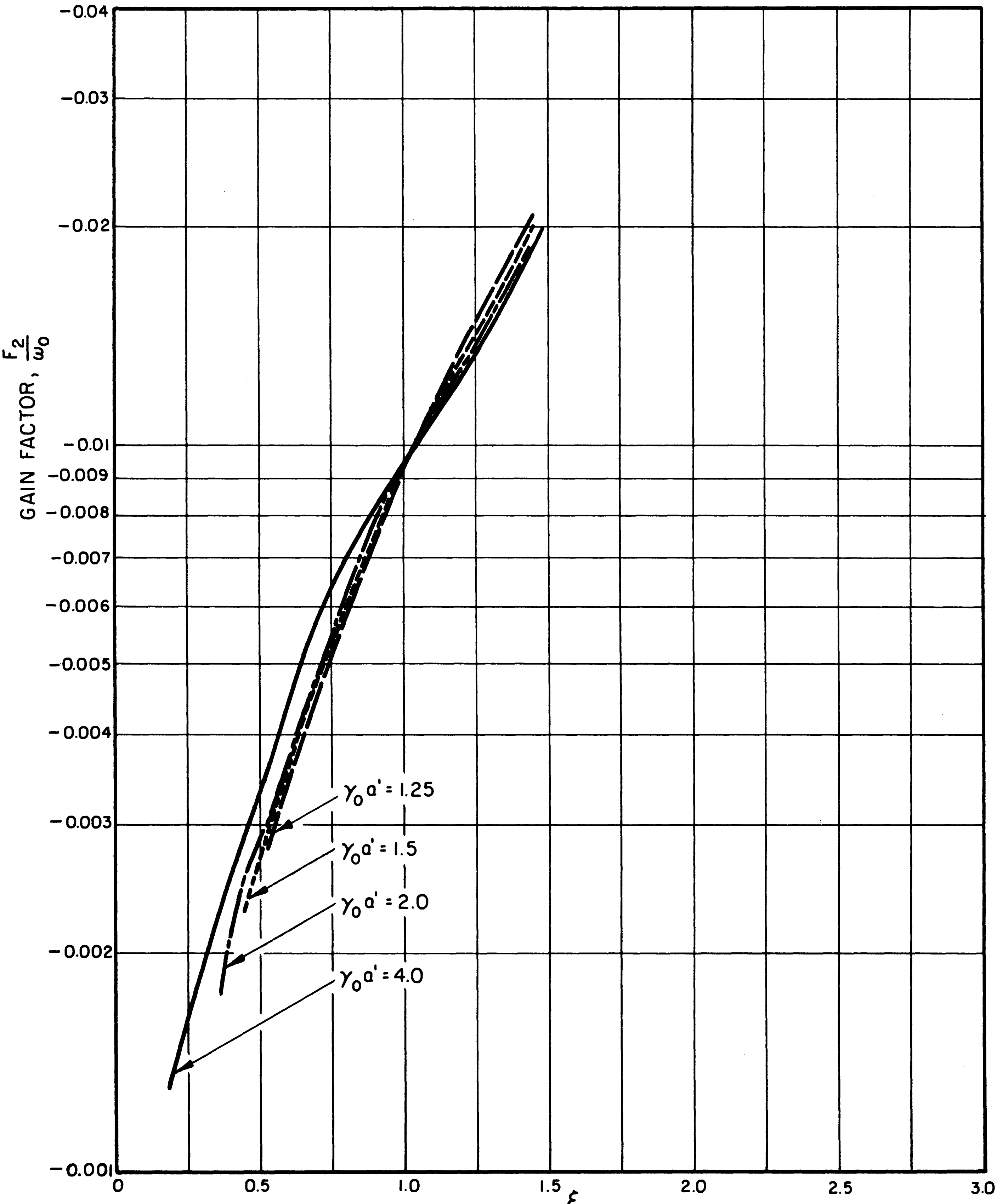

FIG. 6 GAIN FACTOR, $\frac{F_2}{\omega_0}$ VS. NORMALIZED FREQUENCY. (C = 0.1, QC = 0.125, d = 0.125, b'/a' = 0.7)

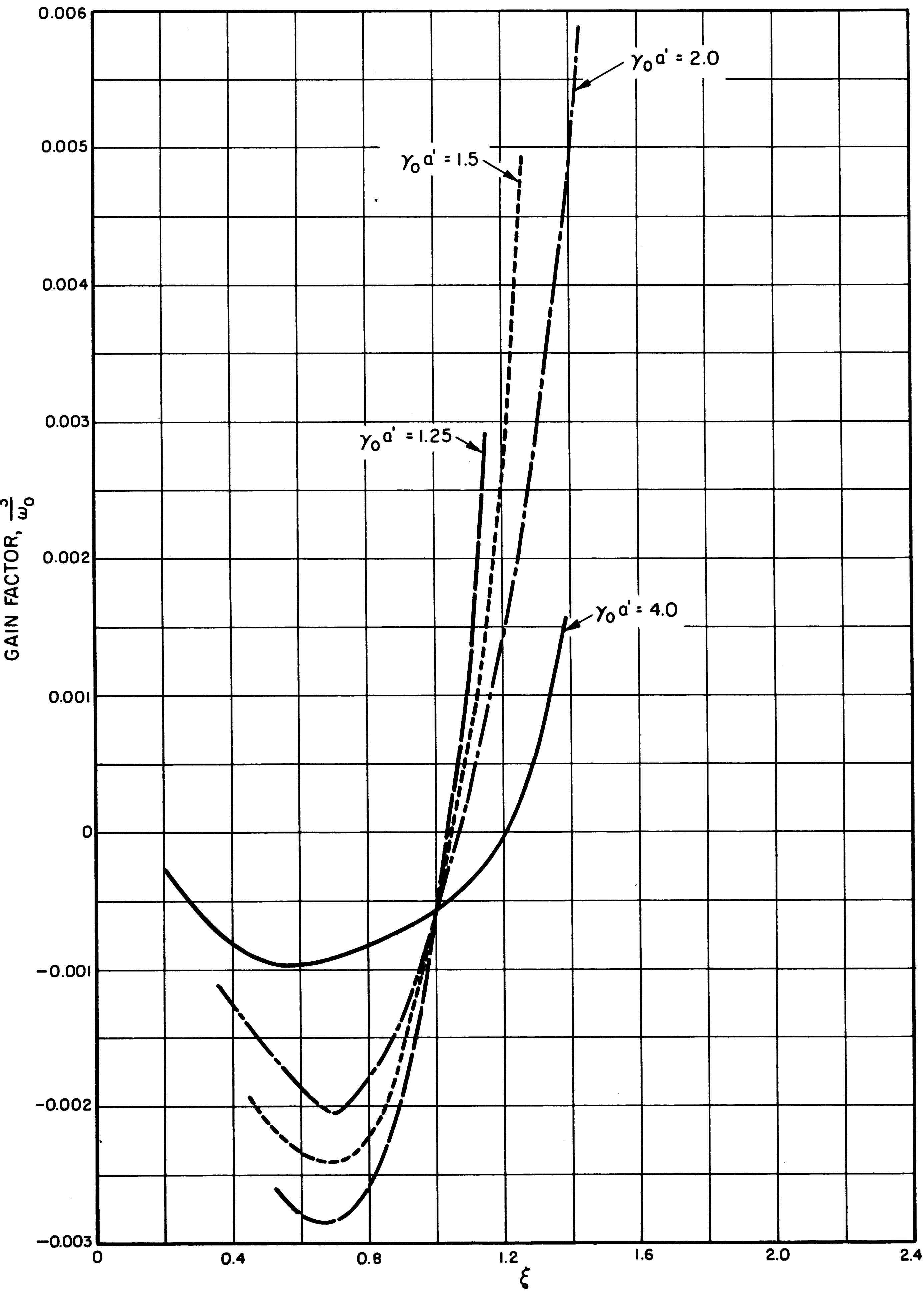

FIG. 7 GAIN FACTOR, $\frac{F_3}{\omega_0}$ VS. NORMALIZED FREQUENCY. (C = 0.01, QC = 0.125, b'/a' = 0.7. INDEPENDENT OF d)

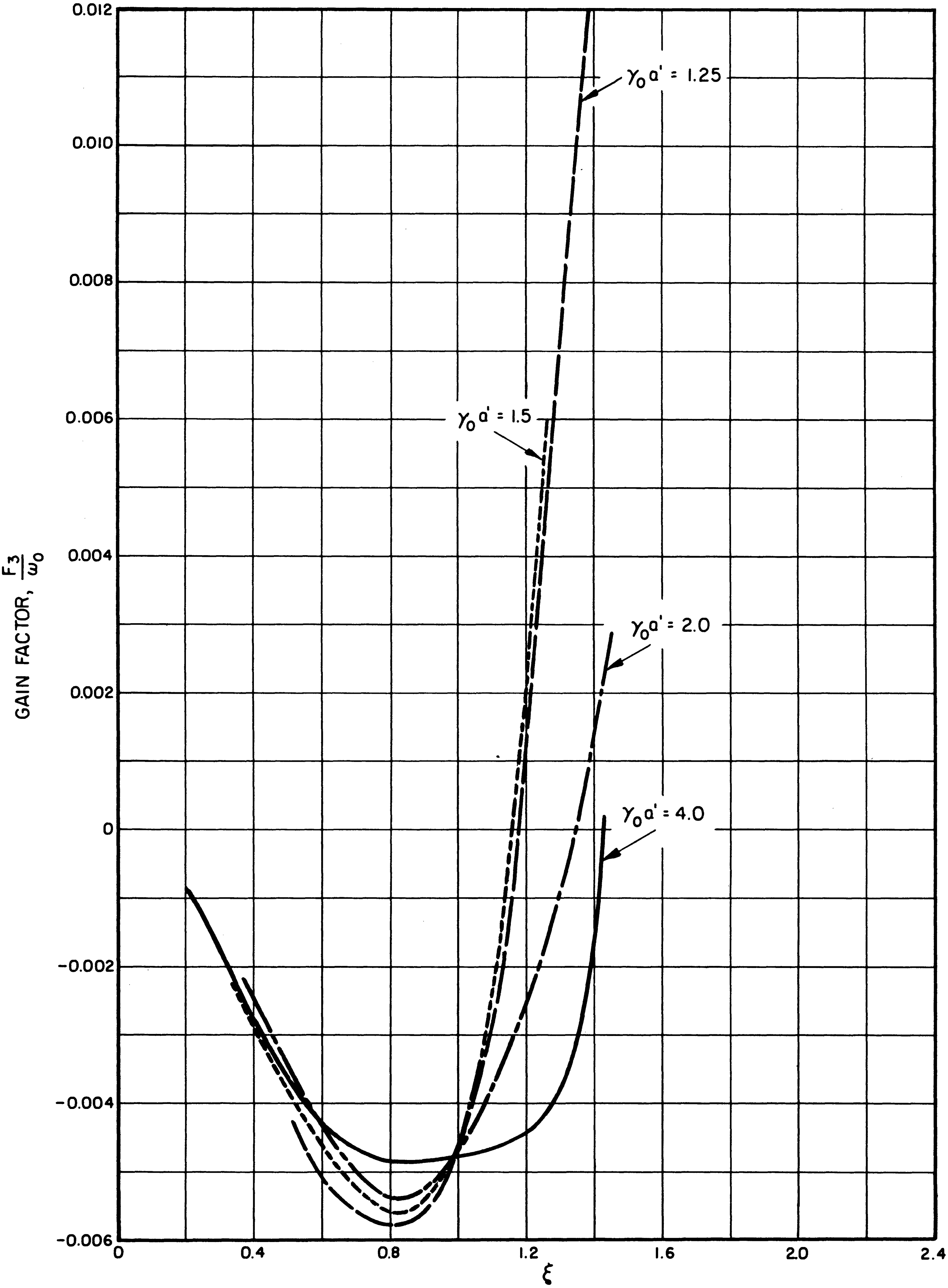

FIG. 8 GAIN FACTOR, $\frac{F_3}{\omega_0}$ VS. NORMALIZED FREQUENCY. (C = 0.1, QC = 0.125, b'/a' = 0.7. INDEPENDENT OF d)

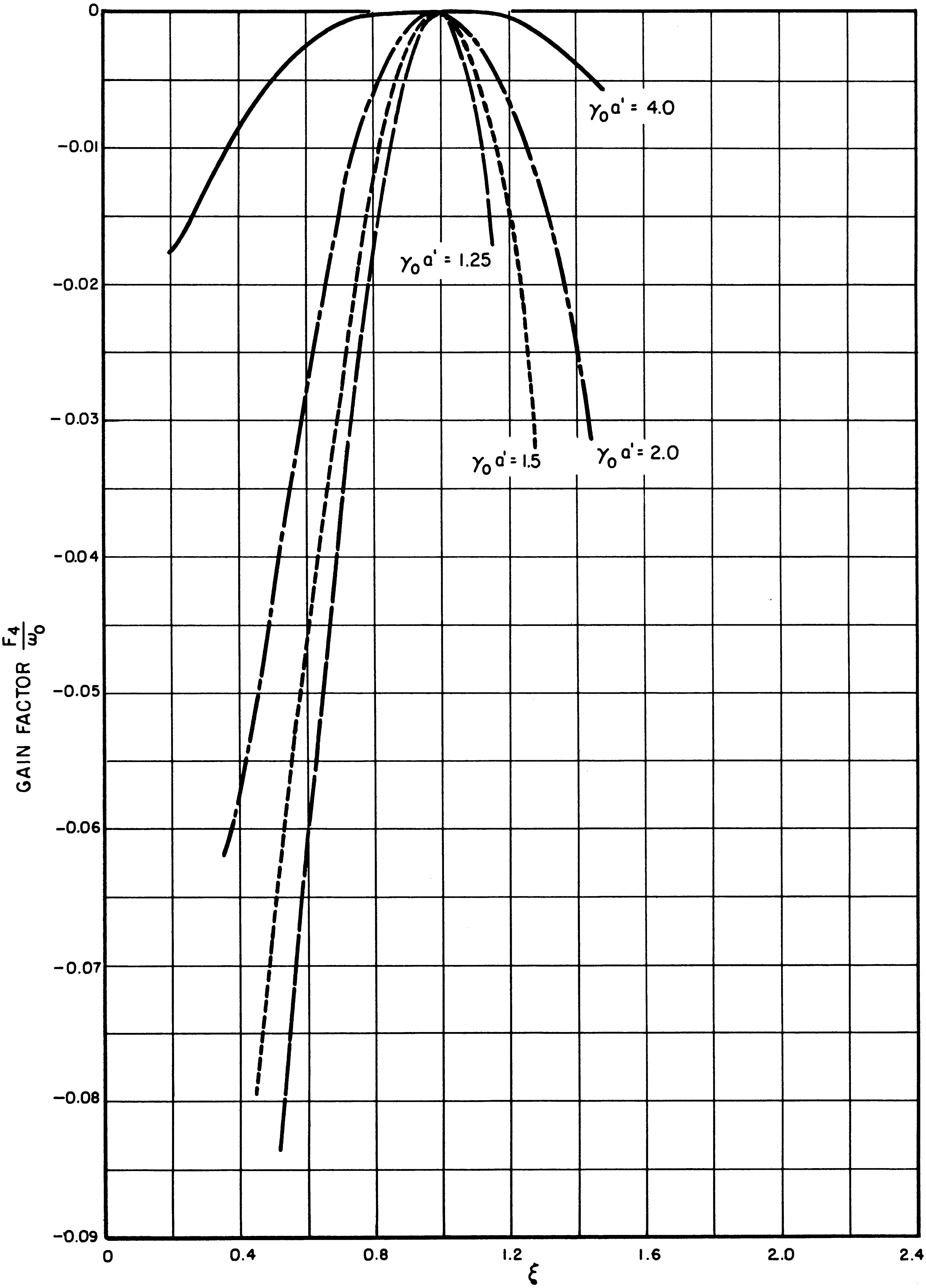

FIG. 9 GAIN FACTOR, $\frac{F_4}{\omega_0}$ VS. NORMALIZED FREQUENCY. (C = 0.01, QC = 0.125, b'/a' = 0.7. INDEPENDENT OF d)

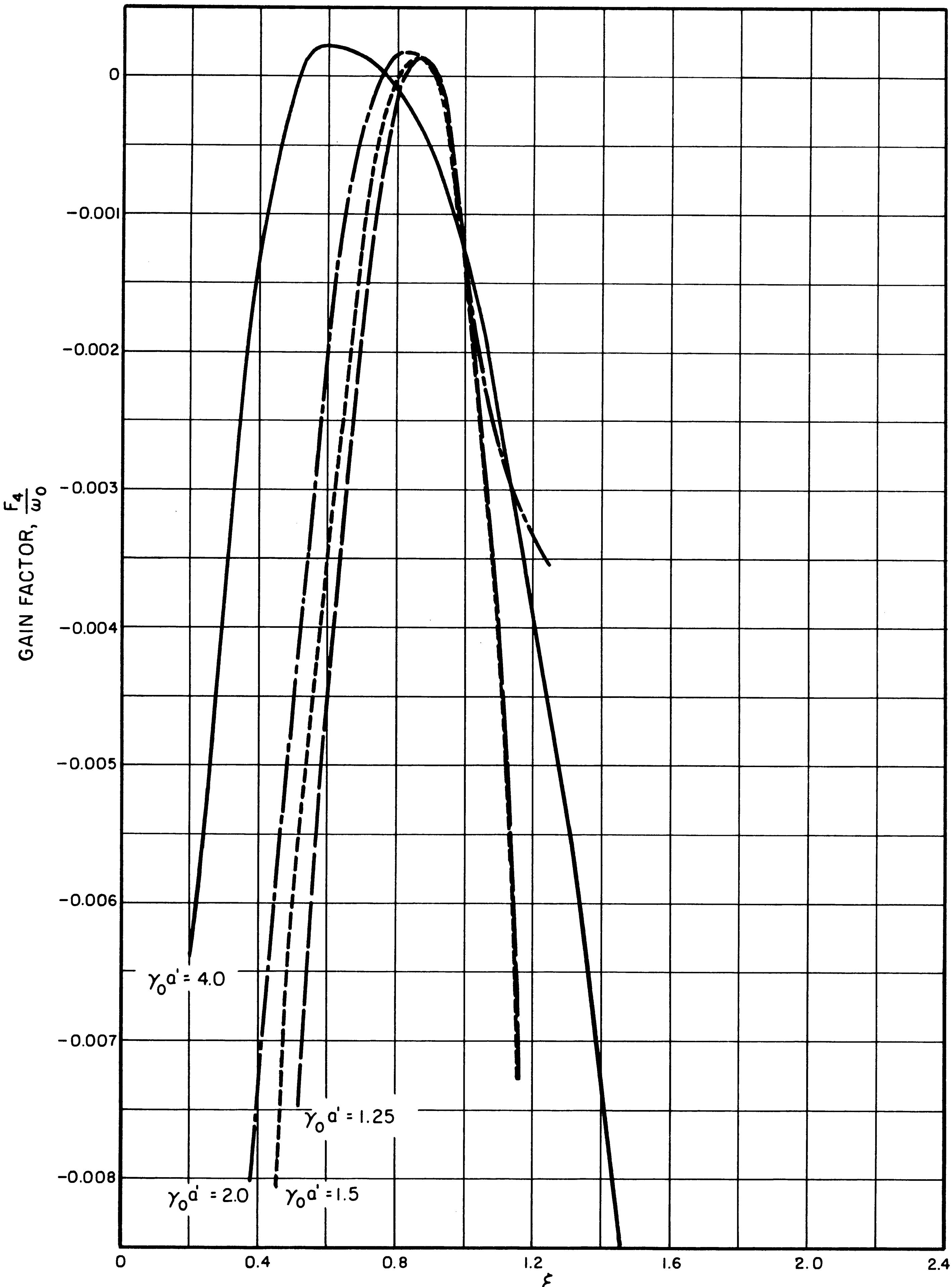

FIG. 10 GAIN FACTOR, $\frac{F_4}{\omega_0}$ VS. NORMALIZED FREQUENCY. (C = 0.1, QC = 0.125, b'/a' = 0.7. INDEPENDENT OF d)

I) $F_1 + F_4(QC=0)$; $C \neq 0$, $QC = d = 0$

II) $F_1 + F_3 + F_4(QC\neq 0)$; $C \neq 0$, $QC \neq 0$, $d = 0$

III) $F_1 + F_2(QC=0) + F_4(QC=0)$; $C \neq 0$, $d \neq 0$, $QC = 0$

IV) $F_1 + F_2(QC\neq 0) + F_3 + F_4(QC\neq 0)$; $QC \neq 0$, $C \neq 0$, $d \neq 0$.

The variation of gain with frequency for two values of C_o and several values of $\gamma_o a'$ is shown in Figs. 11 and 12. The ordinate of these figures can be converted to db through multiplication by

$$\frac{G_{db}(\xi=1) - 8.68\,A}{\sum_{i=1}^{4} \frac{F_i(\xi=1)}{\omega_o}}$$

The summation of $F_i(\xi)$ is valid only so long as it is positive. This is necessary because the approximation for x_1 given in Eq. 12 is valid only for positive x_1.

For small C values (C = 0.01), curves of Type I are smooth functions in the vicinity of $\xi = 1$. Inclusion of space charge (Type II) preserves the regularity but narrows the band. However, introducing loss as in cases III and IV brings about the irregularities observed in Fig. 8 for $\gamma_o a' = 4$. The general shape of the gain curves in the vicinity of $\xi = 1$, for small C, varies from parabolic (for small $\gamma_o a'$) to linear (for large $\gamma_o a'$). The gain variation for large C(C = 0.10) is more irregular near $\xi = 1$ (especially so for $\gamma_o a' = 1.25$); however, the same "average" shape is obtained as for the small-C curves. It is immediately evident that the

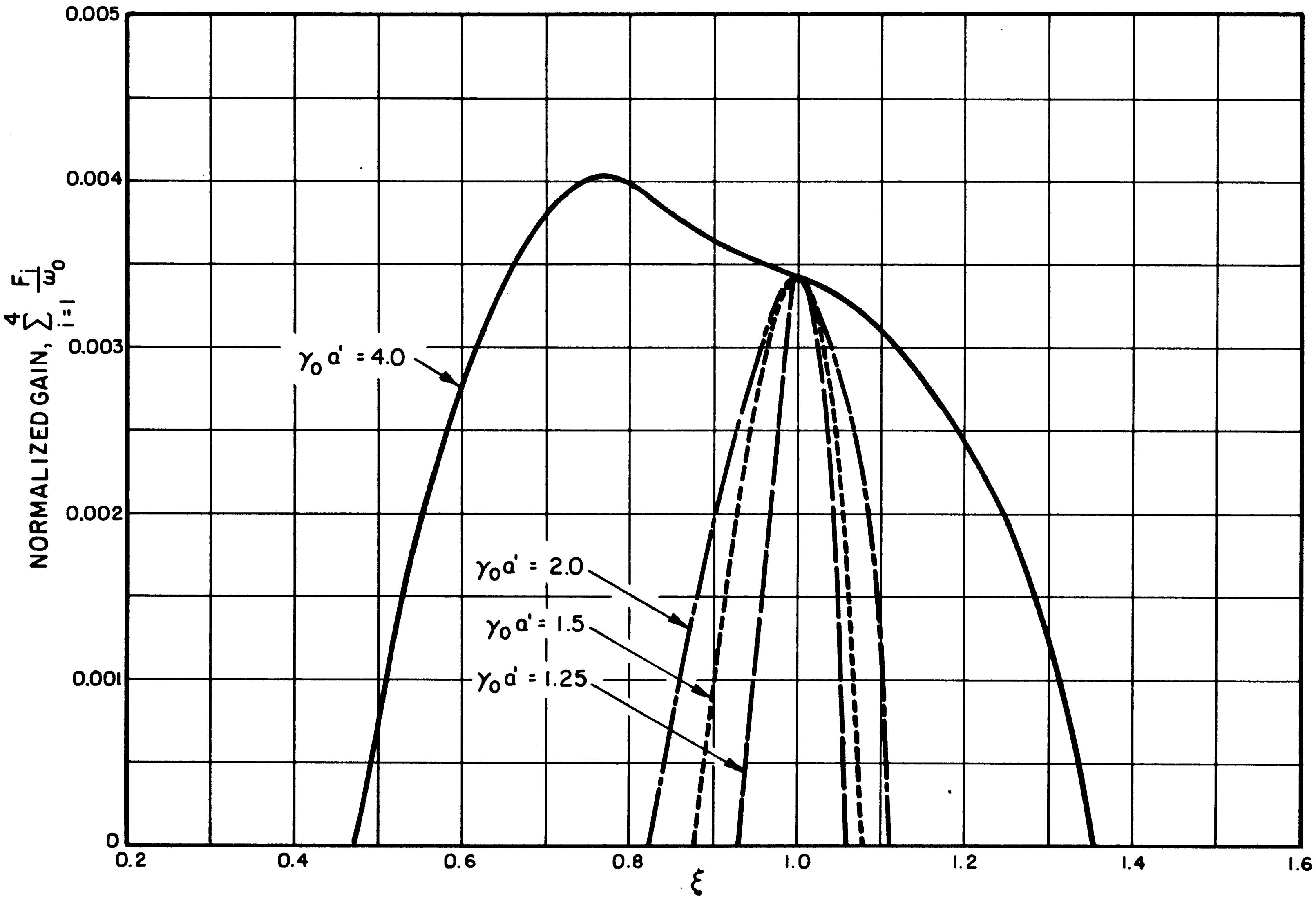

FIG. 11 NORMALIZED GAIN VS. NORMALIZED FREQUENCY. (C = 0.01, QC = 0.125, d = 0.125, b'/a' = 0.7)

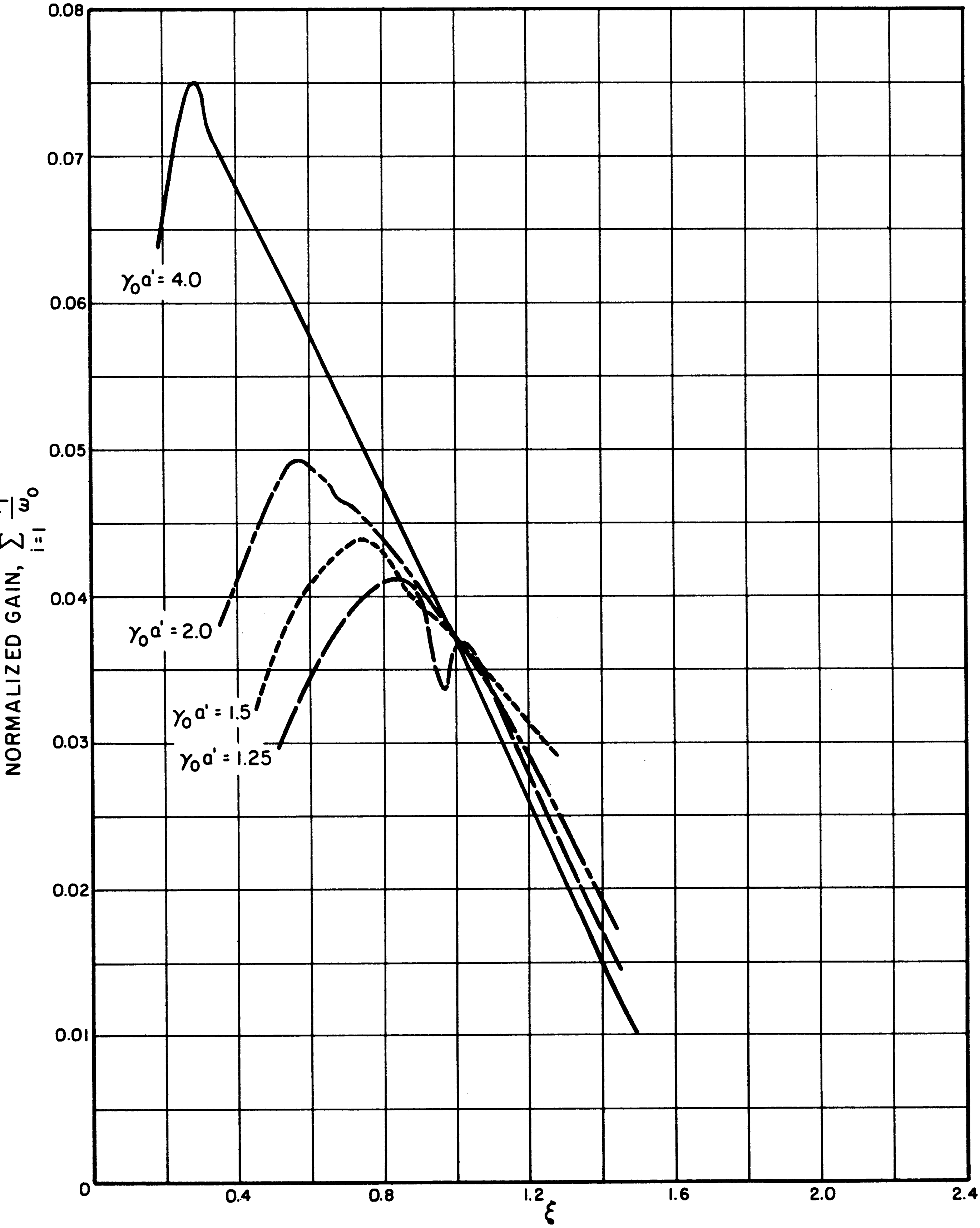

FIG. 12 NORMALIZED GAIN VS. NORMALIZED FREQUENCY. (C = 0.1, QC = 0.125, d = 0.125, b'/a' = 0.7)

bandwidth is much greater for the C = 0.10 tube.

Interaction Bandwidth

If a center frequency gain is specified, the ordinates of the curves shown in Figs. 11 and 12 can be converted to db, as mentioned above, and the theoretical bandwidth can be measured directly from the curves. The percent bandwidth as a function of $\gamma_0 a'$, for different center frequency parameters, is shown in Figs. 13 and 14 for a tube with 30 db gain at the center frequency.

Figures 13 and 14 can now be used to draw the following conclusions. In order to obtain a large bandwidth for a small-C tube, high values of $\gamma_0 a'$ are required. In fact, a $\gamma_0 a'$ in the vicinity of 3 or 4 would be desirable to get 3-db bandwidths of about 10 percent, for a C = 0.01 tube, which would correspond to 50 percent with a 10-db bandwidth. However, using a high $\gamma_0 a'$ value inherently raises the ka' value and in doing so makes possible backward-wave oscillation. If $\gamma_0 a'$ is lowered to values between 2 and 3, it is still possible to obtain 3-db bandwidths between 5 and 10 percent, but now the 10-db bandwidth is lowered to about 20 percent.

For large-C tubes with space charge and loss, a $\gamma_0 a'$ of 1.5 or less results in 3-db bandwidths of greater than 20 percent and 10-db bandwidths greater than 60 percent.

It can be seen from Figs. 13 and 14 that loss tends to decrease bandwidth, as does space charge, except for the rather anomolous behavior in Fig. 11 of QC = 0.125, d = 0.

When fabrication is considered, the result requiring a larger $\gamma_0 a'$ for low-C tubes is favorable, since the lower C values are usually associated with high frequencies.

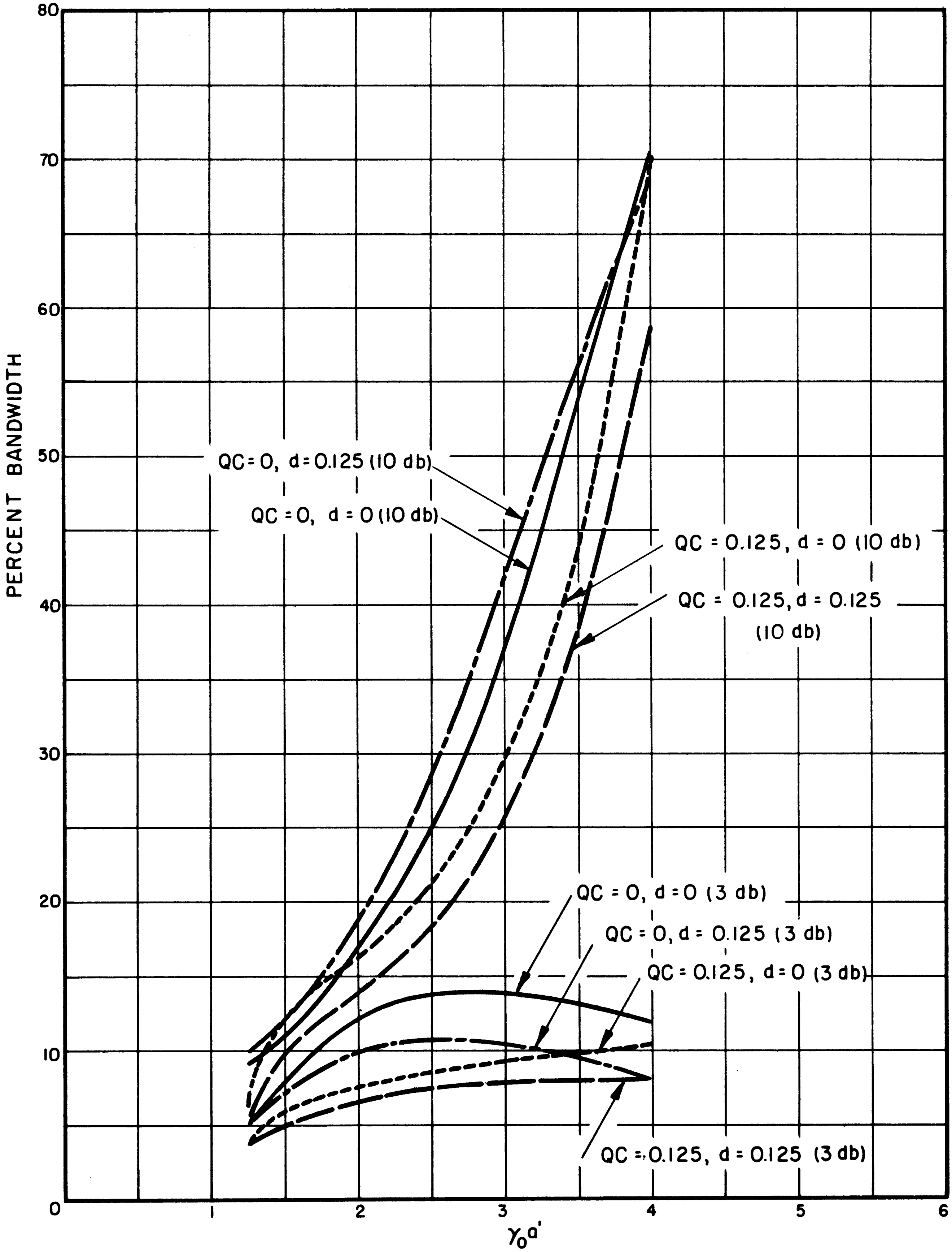

FIG. 13 PERCENTAGE BANDWIDTH VS. $\gamma_0 a'$. ($C = 0.01$, $b'/a' = 0.7$)
$G + |A| = 40$ db AT $\xi = 1$.

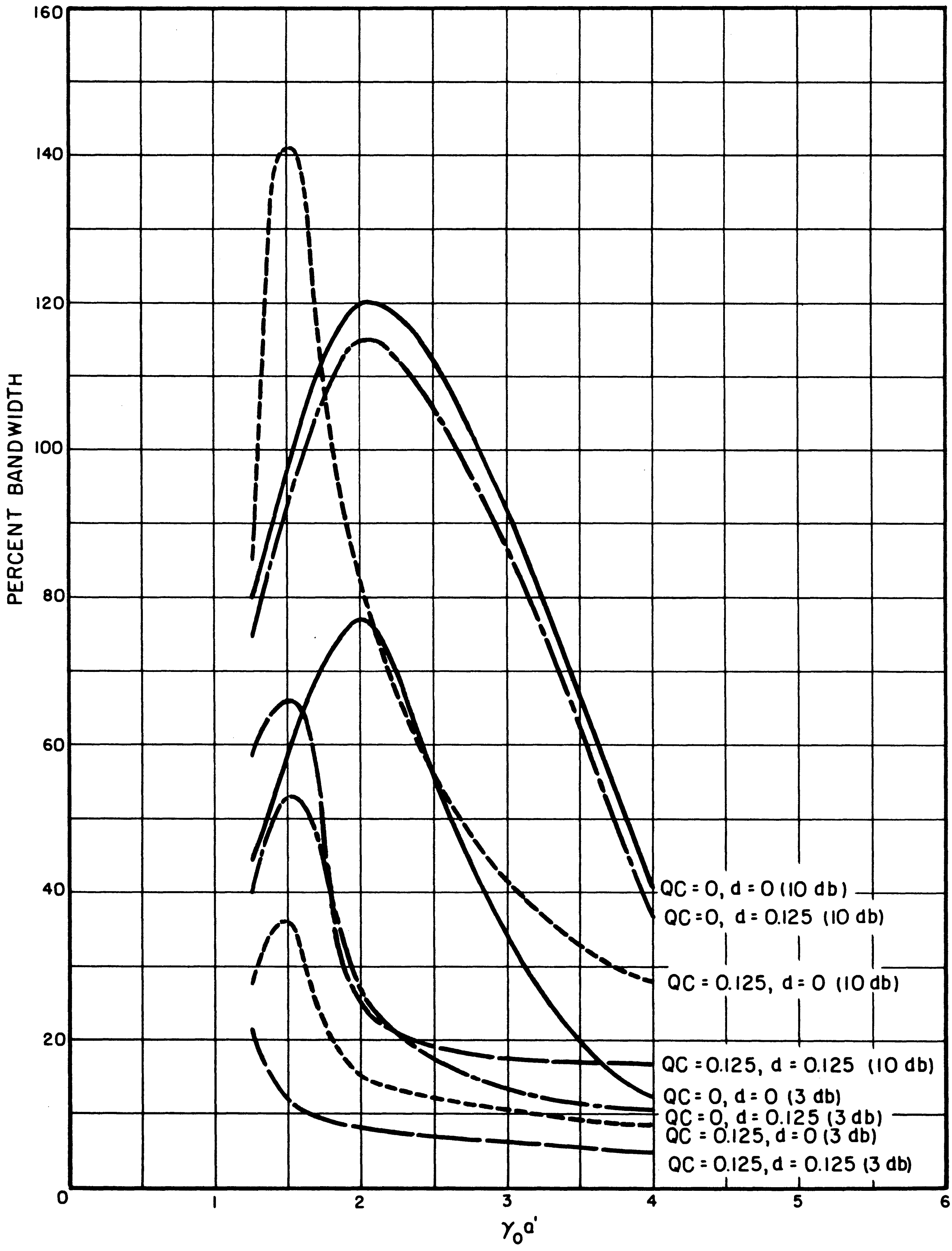

FIG. 14 PERCENTAGE BANDWIDTH VS. $\gamma_0 a'$. ($C = 0.1$, $b'/a' = 0.7$)
$G + |A| = 40$ db AT $\xi = 1$.

External circuitry such as attenuators, couplers and loads may be used to flatten the gain characteristics by dissipating power in regions of high gain but passing the low-gain frequencies.

Gain-Bandwidth Product

The concept of a constant gain-bandwidth product has been proven to be of great importance in the applications of low-frequency tubes. It is worthwhile to give some consideration to a possible similar product for traveling-wave tubes.

The gain of a traveling-wave tube was expressed in Eq. 12. If we once again neglect the frequency variation of A, the gain may be written as

$$\ell n \left(\frac{G}{\exp A} \right) = f(\omega)\, L \quad , \tag{31}$$

where $f(\omega) = \beta_e C x_1$.

The factor $f(\omega)$ is determined by the applied signal frequency, beam voltage, current and cross section, transverse dimensions, periodicity and loss of the circuit. The quantity $\ell n(G/\exp A)$ can be considered as a "pseudo" gain (in db). $\ell n(G/\exp A)$ varies linearly with the helix length at a fixed frequency.

The band edge frequencies will be defined as those frequencies at which the gain differs from the center frequency gain by a specified number of db (3 or 10 as used in this report). This may be expressed mathematically as:

$$|f(\xi) - f(1)| = \frac{M}{L} \quad , \tag{32}$$

where $M \triangleq \dfrac{\text{db difference}}{8.68}$ and

$L \triangleq$ structure length in meters.

The solutions of Eq. 32 nearest $\xi = 1$ are the desired ones. Expanding $f(\xi)$ in a Taylor series about $\xi = 1$ yields

$$\left| \sum_{n=1}^{\infty} a_n (\xi - 1)^n \right| = \frac{M}{L} \quad . \tag{33}$$

The form of $f(\xi)$ and hence that of a_n is determined partly by the selection of $\gamma_o a'$. From Figs. 8 and 9 it is seen that if $\gamma_o a'$ is large the gain varies linearly with frequency in the vicinity of $\xi = 1$; hence for this case one need only use the $n = 1$ terms of the above series. Solving Eq. 33 for the band edge frequencies and taking their difference gives the bandwidth as:

$$\Delta\xi = \frac{2M}{a_1 L} \quad . \tag{34}$$

The "gain"-bandwidth product is therefore

$$G_{db} BW = \frac{2\, f(\omega)\, M}{a_1} \quad . \tag{35}$$

Hence, for a helix designed with high $\gamma_o a'$, varying the gain by adjusting the helix length gives a constant "gain"-bandwidth product, since none of the variables in the right side of Eq. 35 are functions of length. We must remember that this gain-bandwidth product corresponds to a product of a frequency band and a gain in db.

Unfortunately, a similar product does not yield the same results for lower $\gamma_o a'$ values. Consider a hypothetical case where $f(\omega)$ may be approximated by a parabola centered at $\xi = 1$. Under these circumstances only the $n = 2$ term appears in Eq. 33. The bandwidth in this case is

$$\Delta\xi = \sqrt{\frac{M}{a_2 L}} \tag{36}$$

and the gain-bandwidth product

$$G_{db}BW = 2\frac{M}{a_2} f(\omega) \sqrt{L} \quad . \qquad (37)$$

This product varies as the square root of the gain as the helix length is adjusted. A constant may be obtained by using the product of the gain and the square of the bandwidth:

$$G_{db}(BW)^2 = \frac{4M}{a_2} f(\omega) \quad . \qquad (38)$$

As a rough approximation, one might say that the product of the gain in db and the bandwidth is a constant for large $\gamma_o a'$, while the product of the gain in db and the squared bandwidth is constant for small $\gamma_o a'$.

Conclusions

Analytical expressions have been derived and theoretical power output curves have been presented for helix-type traveling-wave tubes as a function of the principal operating parameters. It has been shown that highest efficiency occurs at relatively low perveance and that the saturation efficiency falls off markedly at large values of B. Also expressions are developed for determining the power output of tubes with high-impedance dispersive structures.

An investigation of traveling-wave tube bandwidth has indicated that the gain-bandwidth product is a constant with respect to length for high $\gamma_o a'$ and the gain times the bandwidth squared is a constant for low $\gamma_o a'$. The gain and bandwidth functions are plotted for representative values of the operating parameters.

Acknowledgments

The authors wish to express their appreciation to Messrs. R. Y.Y. Lee and N. Masnari for their work in calculating and preparing the curves.

APPENDIX A. THEORETICAL POWER OUTPUT FOR A HELIX-TYPE TRAVELING-WAVE AMPLIFIER

C	a'/b'	DLF
0.05	1.4	80
	1.6	80
	1.4	100
	1.6	100
0.10	1.4	80
	1.6	80
	2.0	80
	1.4	100
	1.6	100
	2.0	100

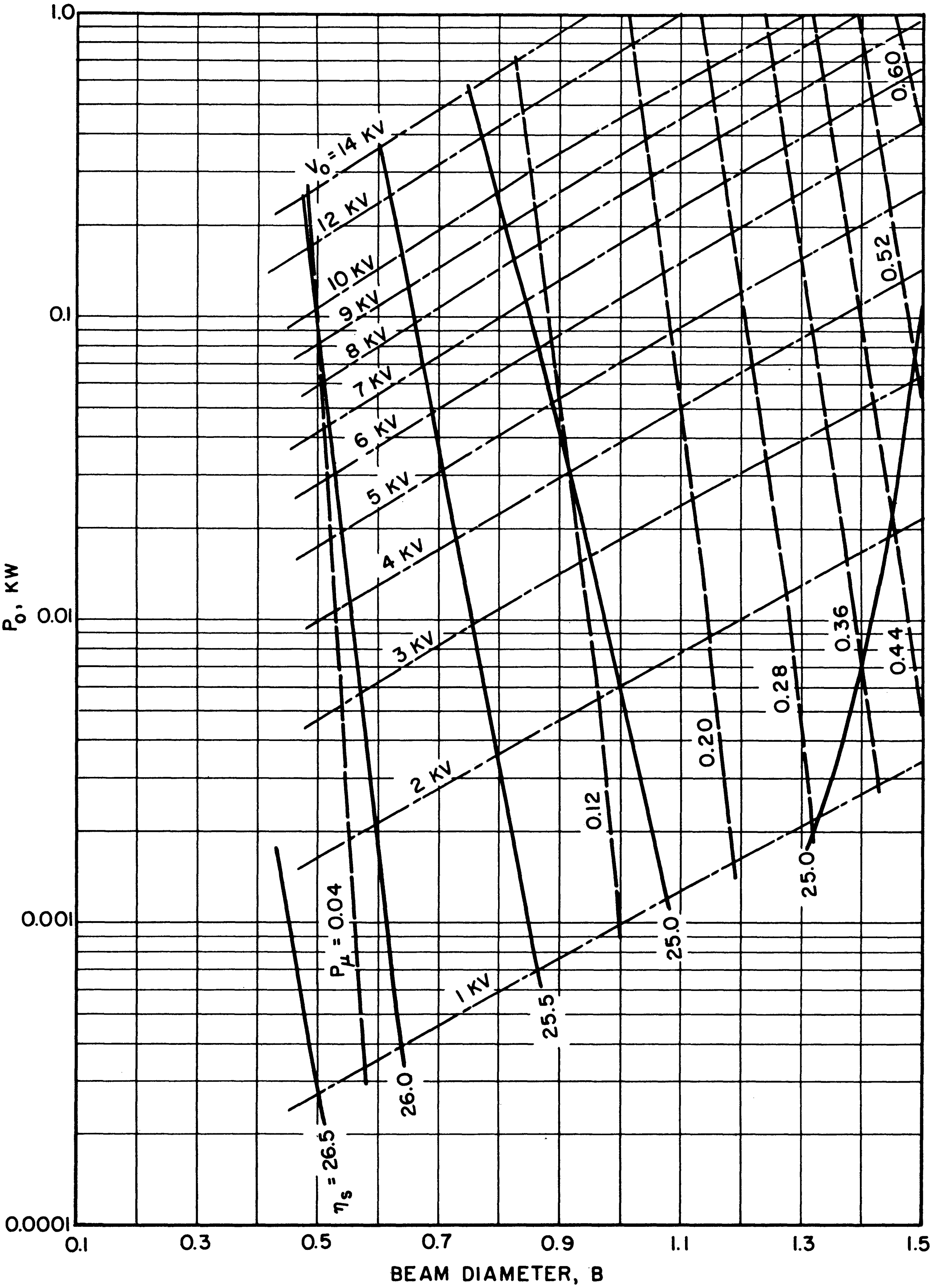

FIG. A.1 THEORETICAL POWER OUTPUT FOR A HELIX-TYPE TRAVELING-WAVE AMPLIFIER. (C = 0.05, a'/b' = 1.4, DLF = 80 %, d = 0)

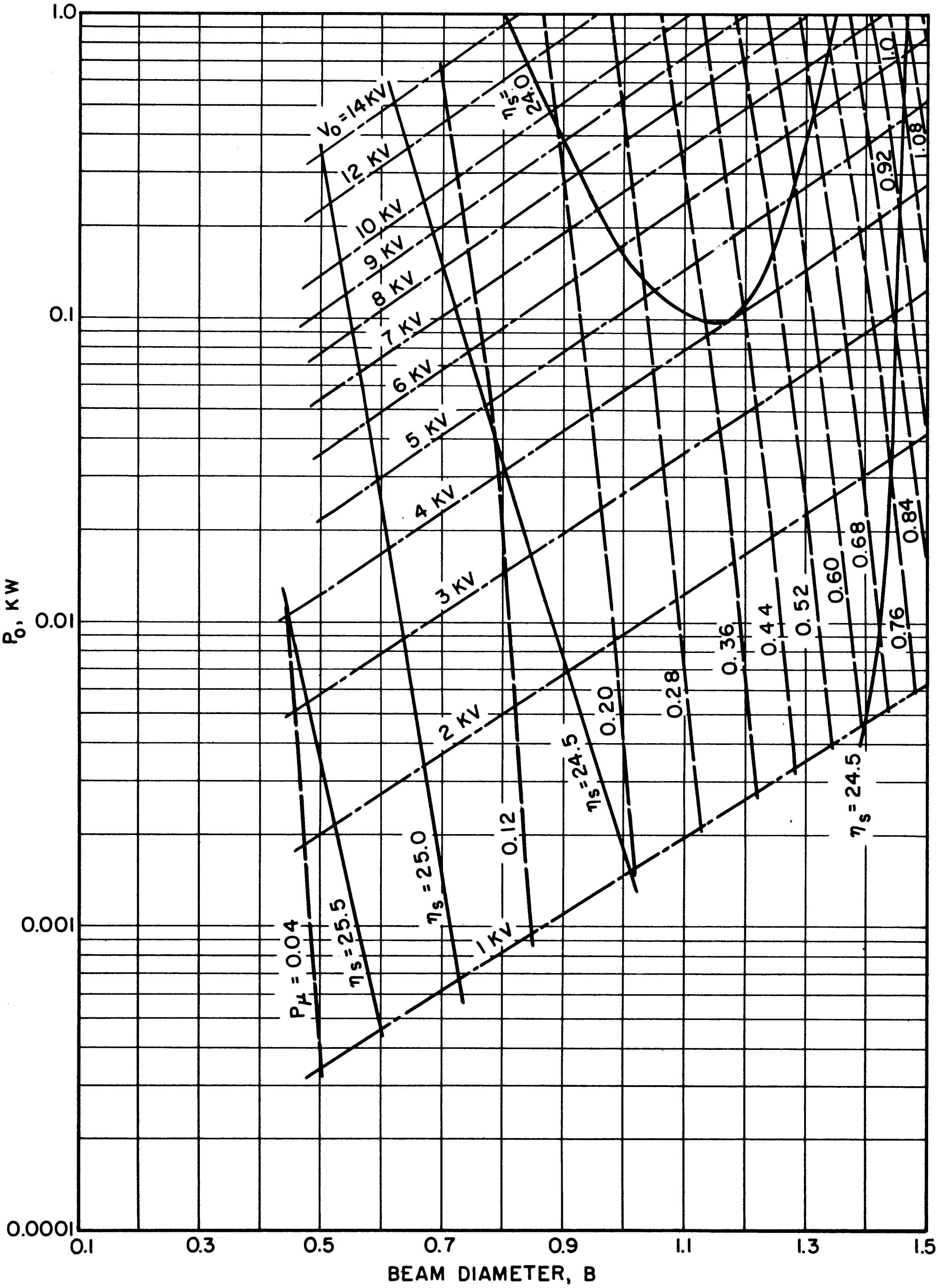

FIG. A.2 THEORETICAL POWER OUTPUT FOR A HELIX-TYPE TRAVELING-WAVE AMPLIFIER. (C = 0.05, a'/b' = 1.6, DLF = 80 %, d = 0)

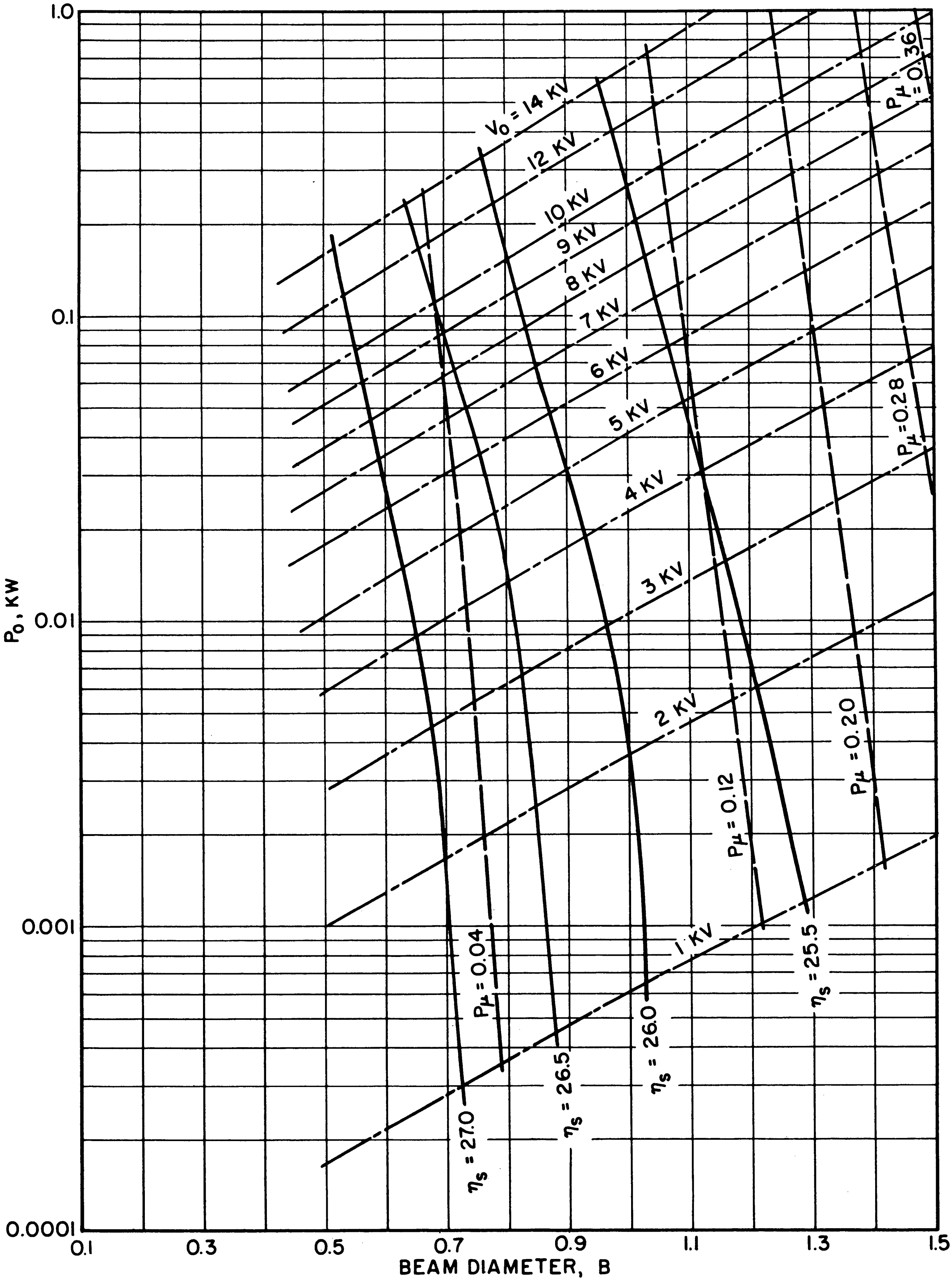

FIG. A.3 THEORETICAL POWER OUTPUT FOR A HELIX-TYPE TRAVELING-WAVE AMPLIFIER. (C = 0.05, a'/b' = 1.4, DLF = 100 %, d = 0)

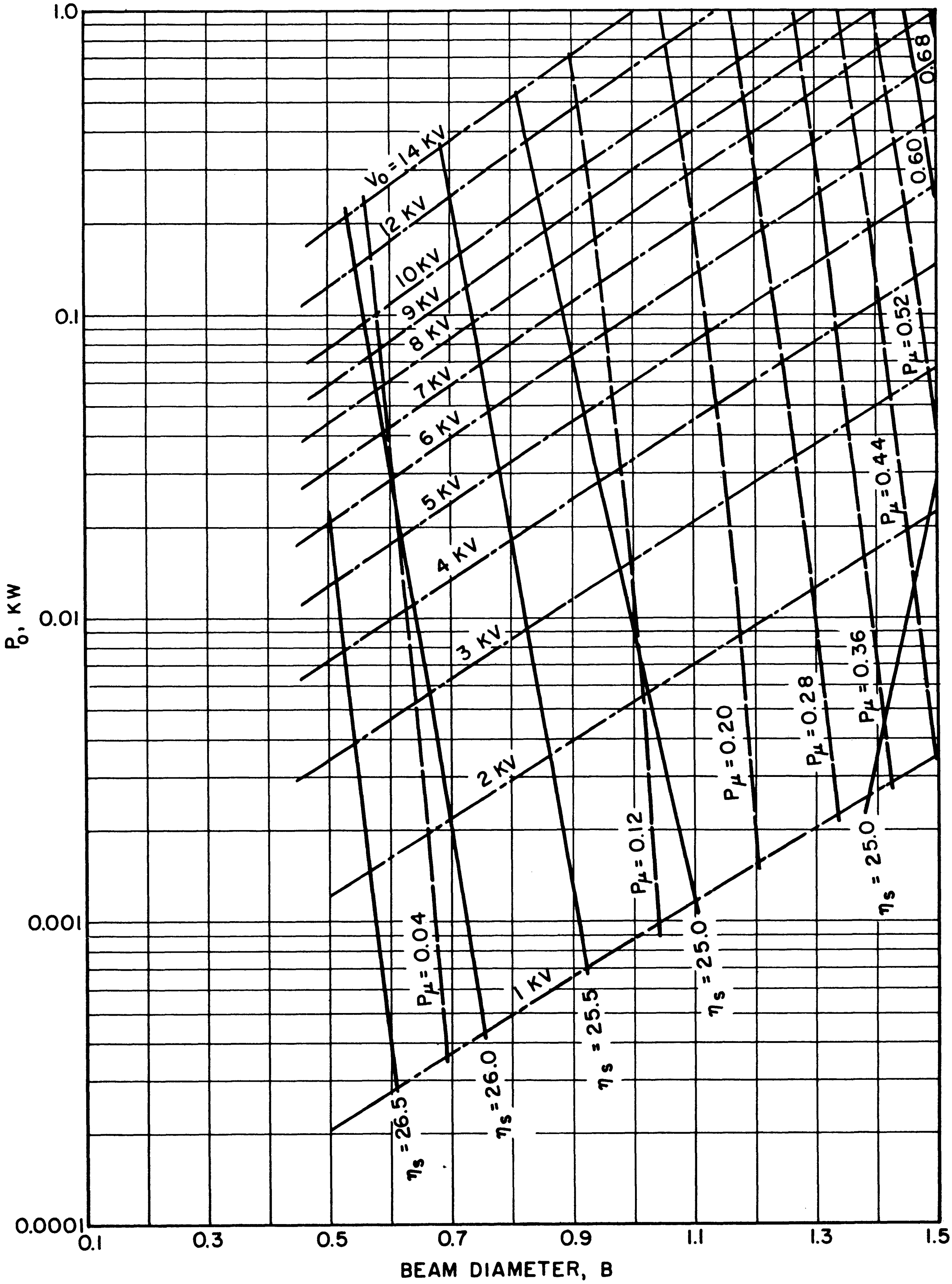

FIG. A.4 THEORETICAL POWER OUTPUT FOR A HELIX-TYPE TRAVELING-WAVE AMPLIFIER. (C = 0.05, a'/b' = 1.6, DLF = 100 %, d = 0)

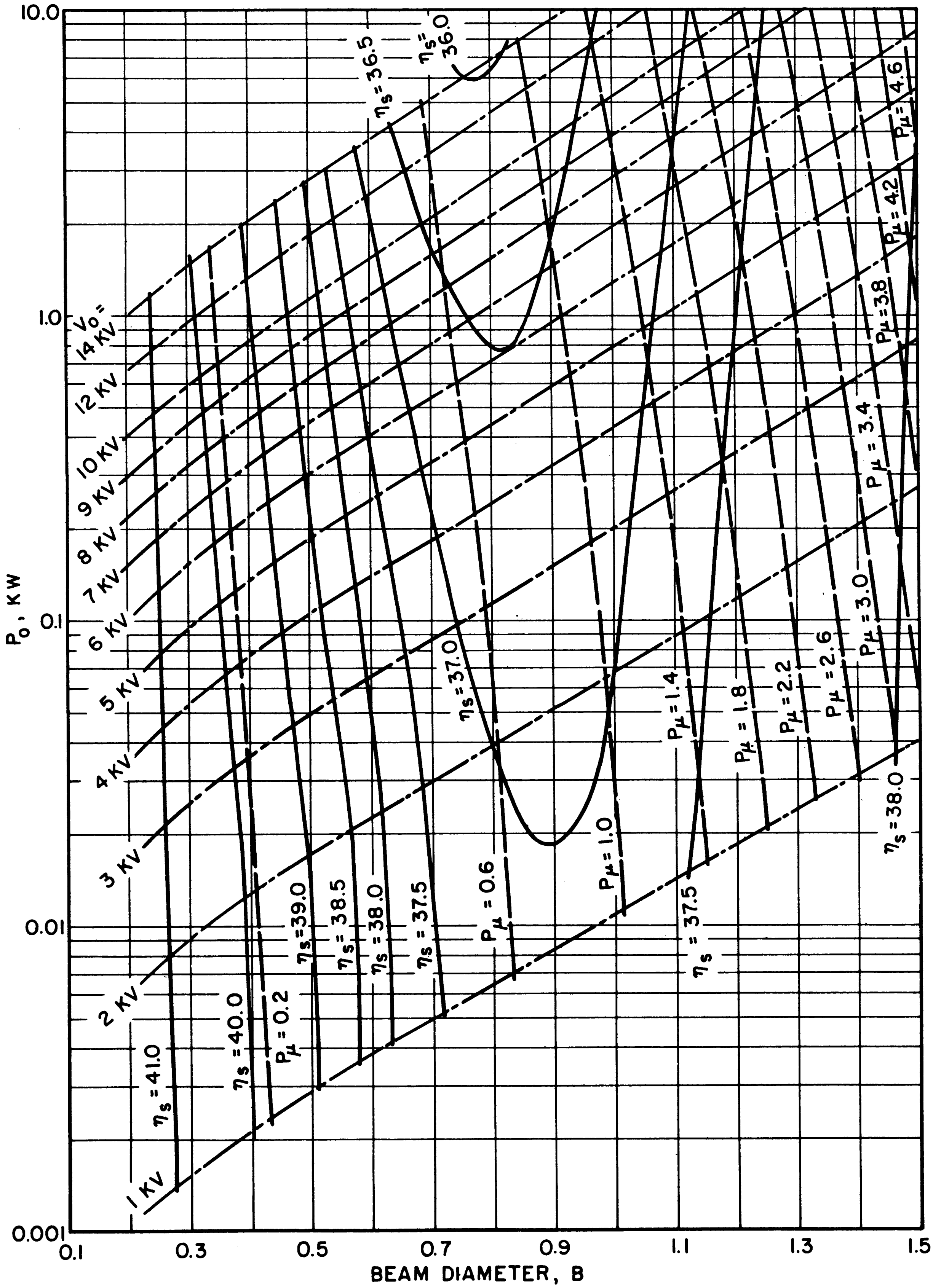

FIG. A.5 THEORETICAL POWER OUTPUT FOR A HELIX-TYPE TRAVELING-WAVE AMPLIFIER. (C = 0.10, a'/b' = 1.4, DLF = 80 %, d = 0)

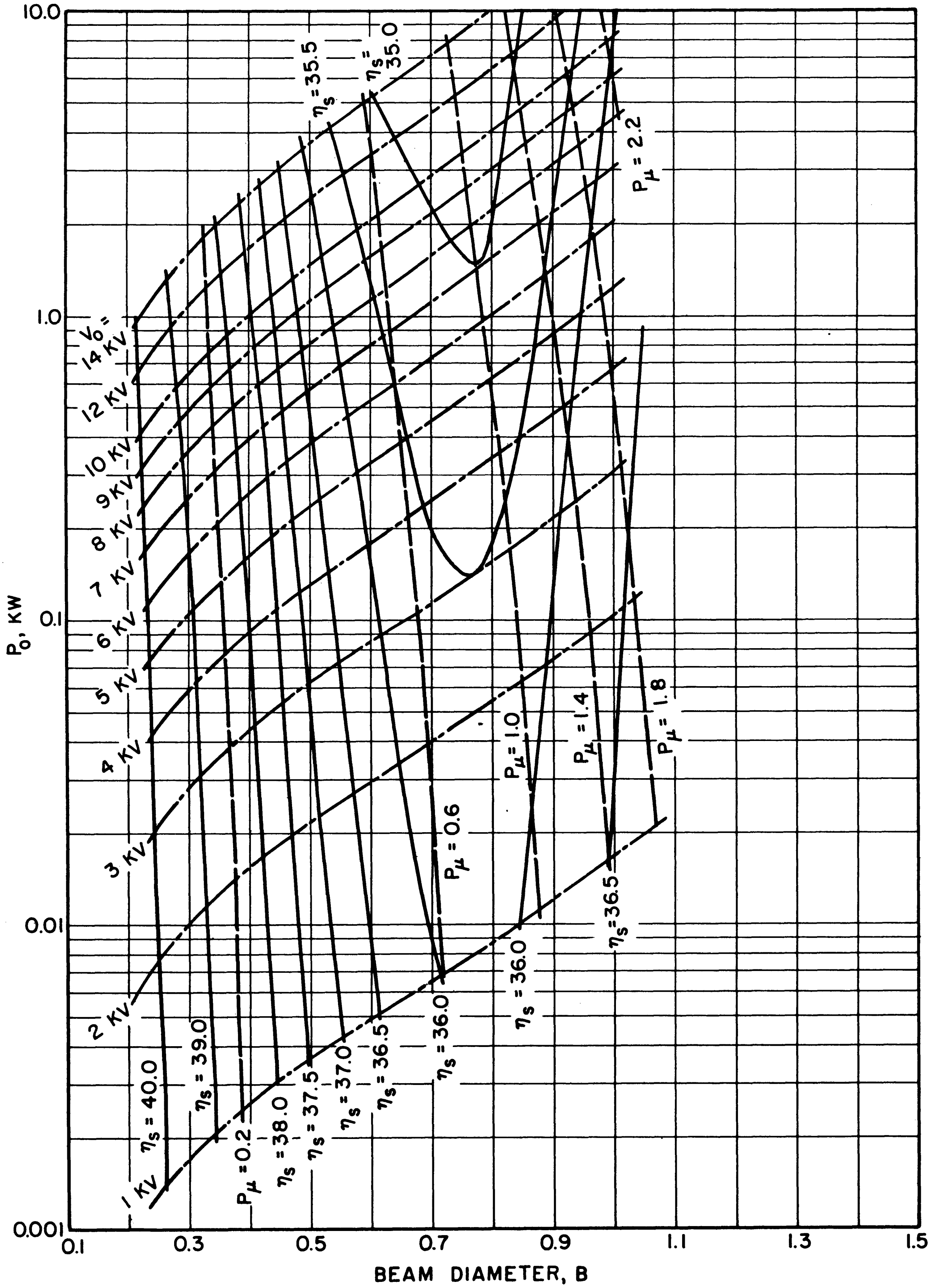

FIG. A.6 THEORETICAL POWER OUTPUT FOR A HELIX-TYPE TRAVELING-WAVE AMPLIFIER. (C = 0.10, a'/b' = 1.6, DLF = 80 %, d = 0)

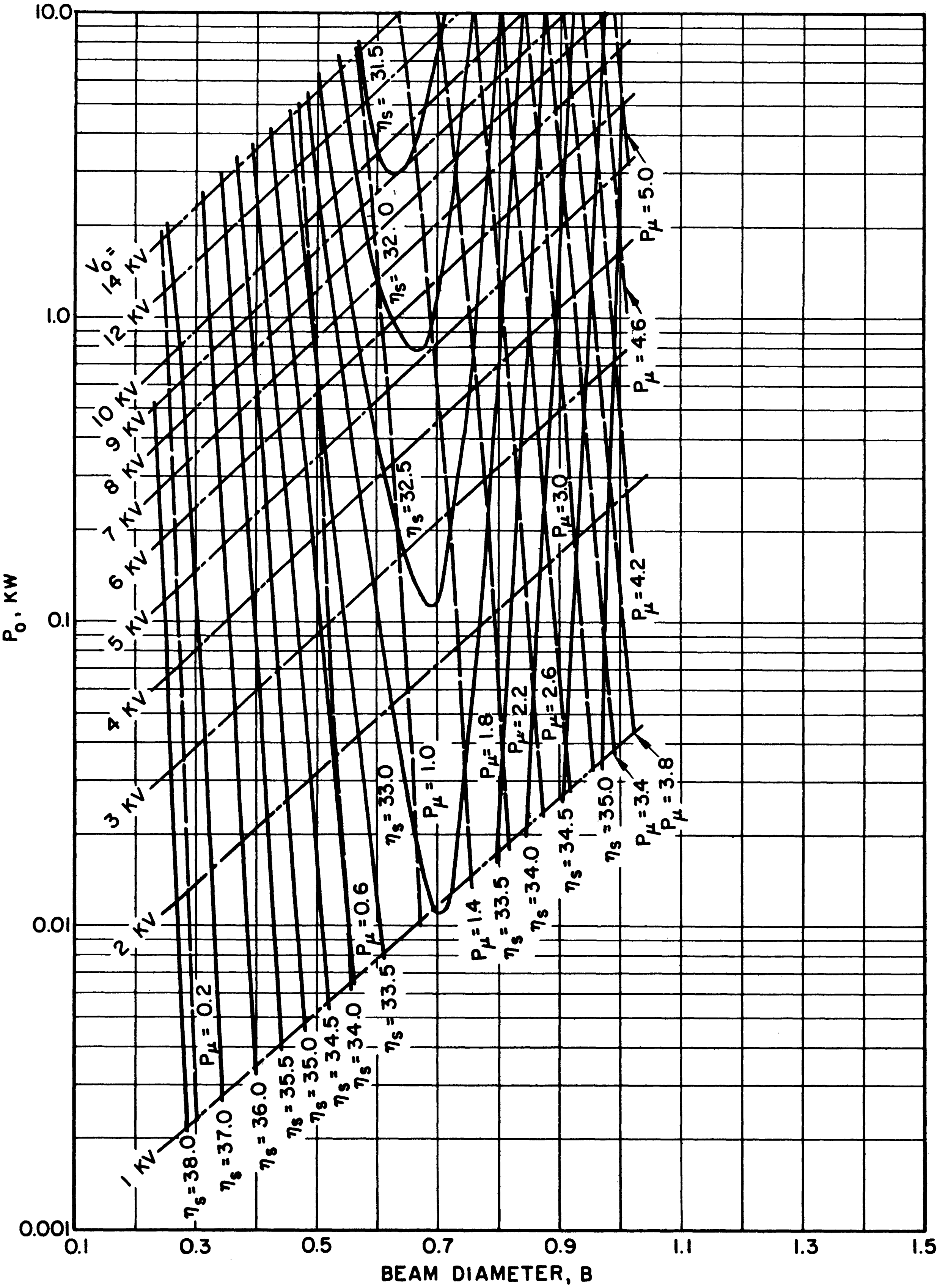

FIG. A.7 THEORETICAL POWER OUTPUT FOR A HELIX-TYPE TRAVELING-WAVE AMPLIFIER. (C = 0.10, a'/b' = 2.0, DLF = 80%, d = 0)

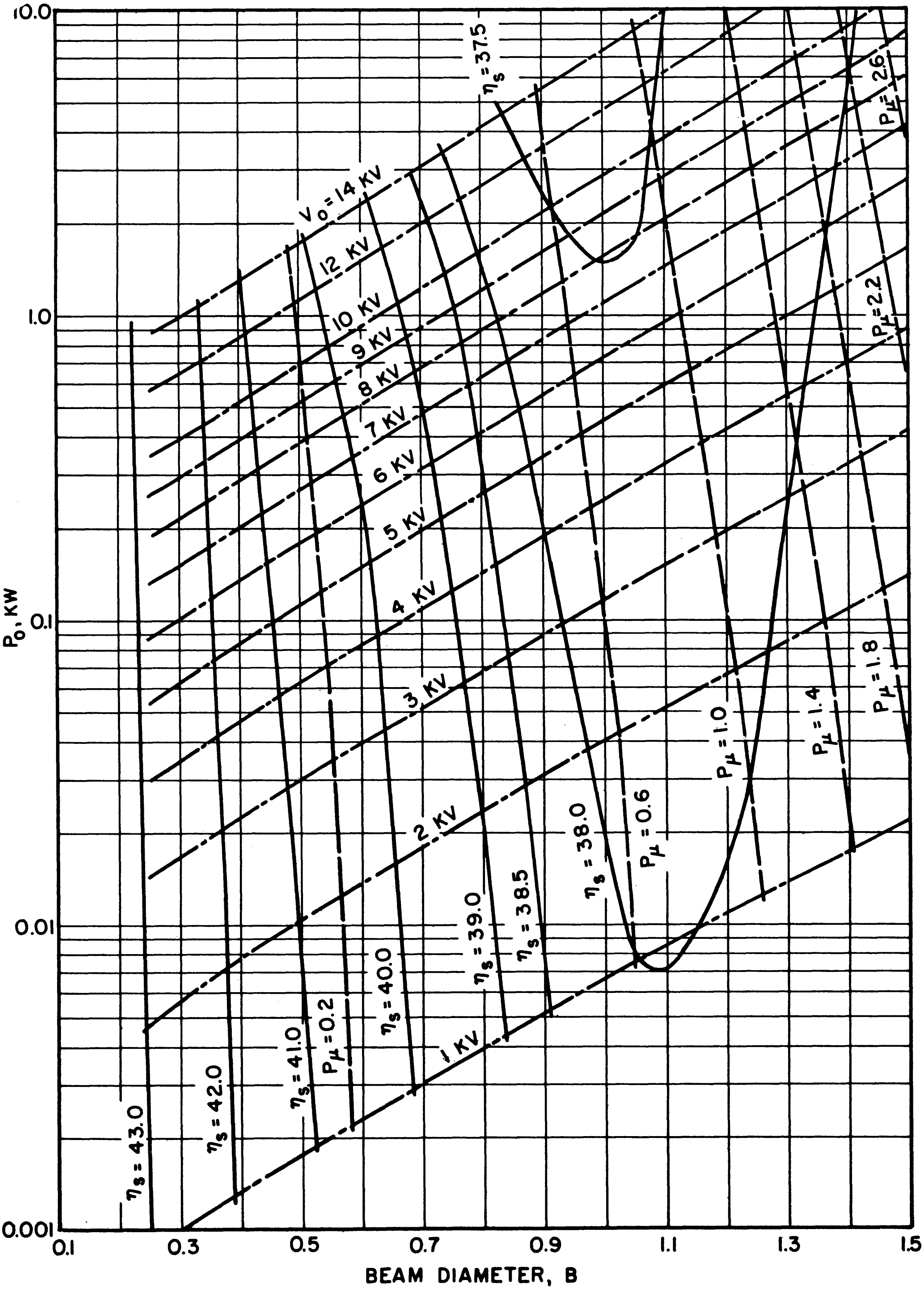

FIG. A.8 THEORETICAL POWER OUTPUT FOR A HELIX-TYPE TRAVELING-WAVE AMPLIFIER. (C = 0.10, a'/b' = 1.4, DLF = 100 %, d = 0)

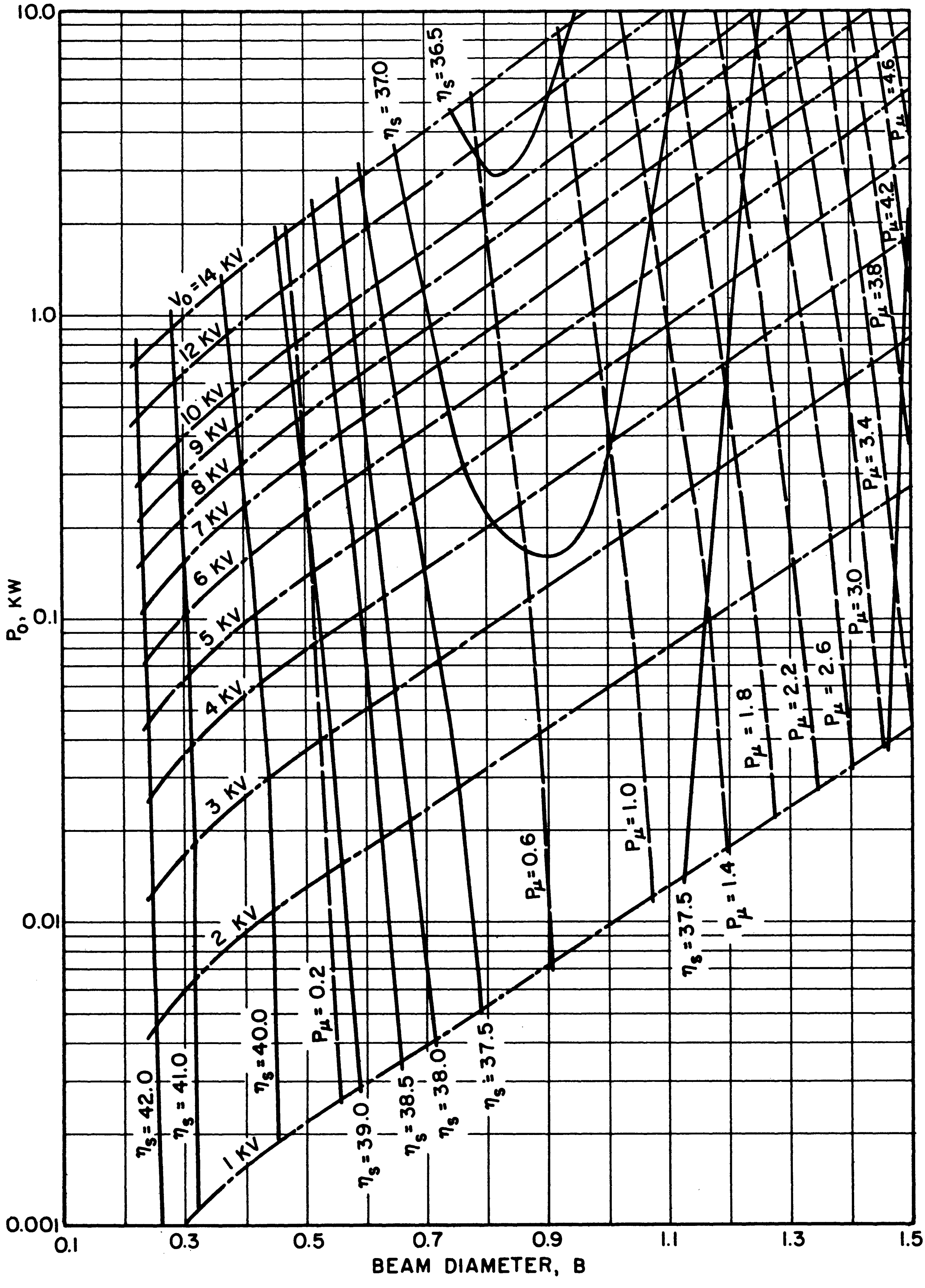

FIG. A.9 THEORETICAL POWER OUTPUT FOR A HELIX-TYPE TRAVELING-WAVE AMPLIFIER. (C = 0.1, a'/b' = 1.6, DLF = 100 %, d = 0)

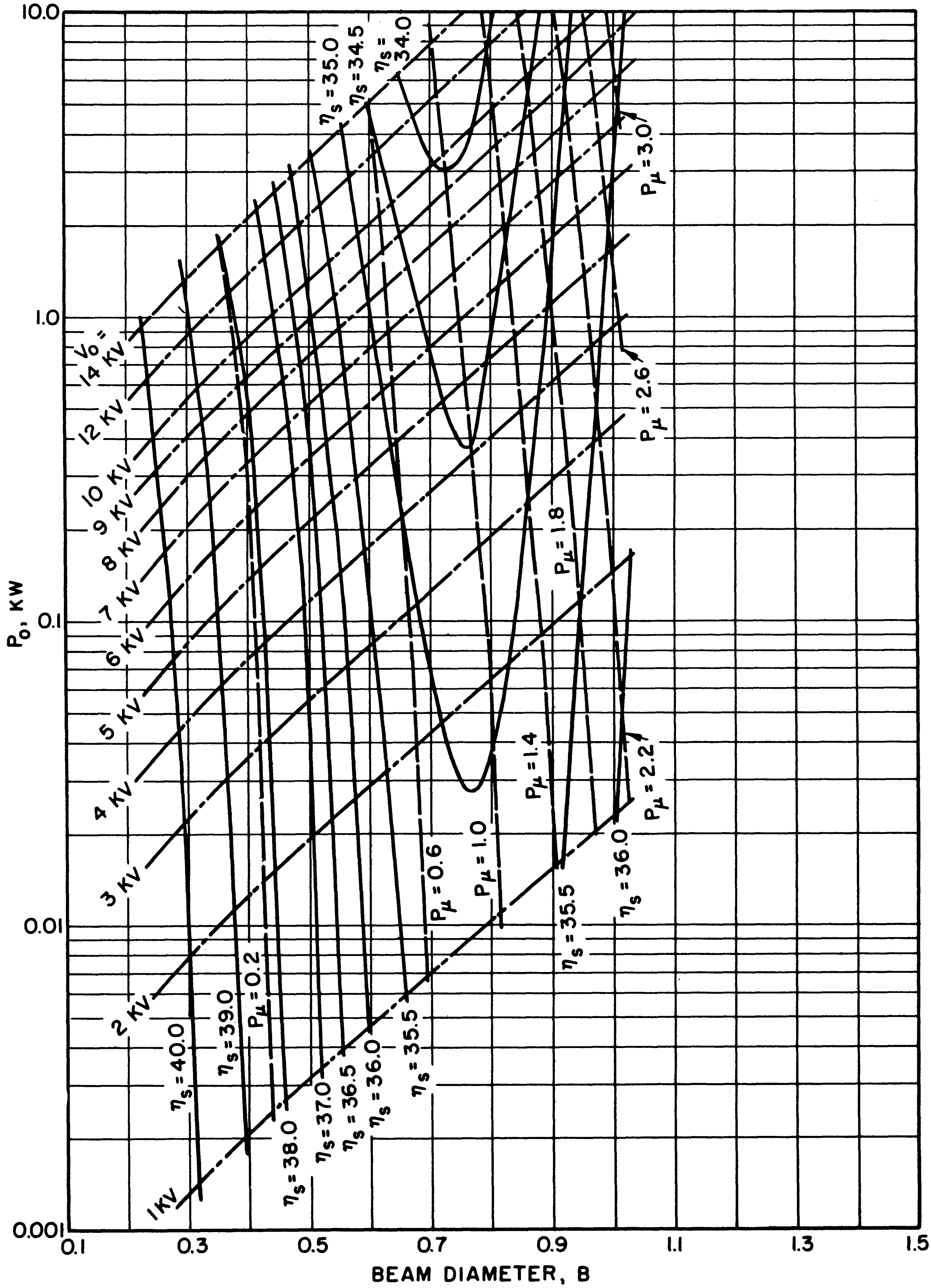

FIG. A.10 THEORETICAL POWER OUTPUT FOR A HELIX-TYPE TRAVELING-WAVE AMPLIFIER. (C = 0.1, a'/b' = 2.0, DLF = 100 %, d = 0)

APPENDIX B. TRAVELING-WAVE TUBE PARAMETERS AS A FUNCTION OF FREQUENCY

The impedance of a sheath helix of radius a' surrounding a solid beam of radius b' is given as:

$$K = \frac{120}{(\gamma a')^2} \left[I_0^2(\gamma b') - I_1^2(\gamma b')\right] \frac{\beta}{k} \left(\frac{\gamma}{\beta}\right)^4 \left[\left(1 + \frac{K_1(\gamma a')\, I_0(\gamma a')}{I_1(\gamma a')\, K_0(\gamma a')}\right) \cdot \right.$$

$$\left(I_1^2(\gamma a') - I_0(\gamma a')\, I_2(\gamma a')\right) + \left(\frac{I_0(\gamma a')}{K_0(\gamma a')}\right)^2 \cdot$$

$$\left.\left(1 + \frac{K_0(\gamma a')\, I_1(\gamma a')}{I_0(\gamma a')\, K_1(\gamma a')}\right) \left(K_0(\gamma a')\, K_2(\gamma a) - K_1^2(\gamma a')\right) \right]^{-1} \quad . \qquad (B.1)$$

For large $\gamma a'$ the asymptotic expansions of the Bessel functions may be used. Therefore

$$K = \frac{30}{(\gamma a')^2} \frac{\beta}{k} \left(\frac{a'}{b'}\right)^2 \exp\left[-2\gamma a'\left(1 - \frac{b'}{a'}\right)\right], \qquad \text{for large } \gamma a'. \qquad (B.2)$$

For regions of small $\gamma a'$, the curve of the impedance of a sheath helix as given by Fletcher[12] can be approximated by the following equations.

$$K = 181 \frac{\beta}{k} \exp\left(-1.77\gamma a'\right), \qquad \text{for small } \gamma a' \quad . \qquad (B.3)$$

Now a value of the gain parameter C_0 at the design frequency ω_0 is defined. The definition will be based on the large-$\gamma a'$ expression

$$C_0^3 = \frac{I_0}{4V_0} \left\{ \frac{30}{(\gamma_0 a')^2} \frac{\gamma_0 a'}{k_0 a'} \left(\frac{a'}{b'}\right)^2 \exp\left[-2\gamma_0 a'\left(1 - \frac{b'}{a'}\right)\right] \right\} \quad . \qquad (B.4)$$

Introducing the normalizing parameters as defined in Eq. 19 of the text gives the gain parameter C as:

$$C^3 = \frac{\chi}{\xi\eta} \exp\left[-2\left(1 - \frac{b'}{a'}\right)\gamma_o a'\eta\right], \qquad \text{for } \gamma a' > 1$$

$$C^3 = \chi\tau \exp\left(-1.77\,\gamma_o a'\eta\right), \qquad \text{for } \gamma a' < 1 \quad . \tag{B.5}$$

The space-charge parameter QC is given approximately as

$$QC \approx \frac{R^2}{4C^2}\left(\frac{\omega_p}{\omega}\right)^2 \quad . \tag{B.6}$$

The plasma-frequency reduction factor R^2 may be approximated as[13]

$$R^2 \approx (1 - e^{-Bf})^2 \quad , \tag{B.7}$$

where f is the initial slope. Now we define

$$g \triangleq \gamma_o a' f\left(\frac{b'}{a'}\right) \quad . \tag{B.8}$$

The plasma-frequency reduction factor at any frequency is then

$$R^2 = [1 - \exp(-g\eta)]^2 \quad . \tag{B.9}$$

The center frequency space-charge parameter is

$$QC_o = \frac{(1 - \exp -g)^2}{4C_o^{\,2}}\left(\frac{\omega_p}{\omega_o}\right)^2 \quad . \tag{B.10}$$

Therefore the space-charge parameter at any frequency is

$$QC = C_o^2 QC_o \left(\frac{\eta}{\chi\xi^2}\right)^{2/3}\left(\frac{1 - \exp -g\eta}{1 - \exp -g}\right)^2 \exp\left[\frac{4}{3}\left(1 - \frac{b'}{a'}\right)\gamma_o a'\eta\right] \quad \text{for } \gamma a' \gtrsim 1$$

$$QC = \frac{C_o^2\, QC_o}{(\xi^3\chi\tau)^{2/3}}\left[\frac{1 - \exp -g\eta}{1 - \exp -g}\right]^2 \exp\left[\frac{2}{3}\,(1.77\,\gamma_o a')\eta\right] \quad \text{for } \gamma a' \lesssim 1 \; . \tag{B.11}$$

The velocity injection parameter b is defined as

$$b = \frac{1}{C}\left[\frac{u_o}{v_p} - 1\right] = \frac{1}{C}\left[\frac{u_o}{v_{po}}\ \frac{v_{po}}{v_p} - 1\right] \quad . \tag{B.12}$$

The velocity ratios are

$$\frac{u_o}{v_{po}} = 1 + C_o b_o$$

and

$$\frac{v_{po}}{v_p} = \frac{\eta}{\xi} \quad . \tag{B.13}$$

Therefore,

$$b = \left[\frac{1 + C_o b_o}{\xi}\eta - 1\right]\left(\frac{\xi\eta}{\chi}\right)^{1/3} \exp\left[\frac{2}{3}\left(1 - \frac{b'}{a'}\right)\gamma_o a'\eta\right] \quad \text{for } \gamma a' \gtrapprox 1$$

$$b = \left[\frac{1 + C_o b_o}{\xi}\eta - 1\right]\left(\frac{1}{\chi\tau}\right)^{1/3} \exp\left(\frac{0.59}{3}\gamma_o a'\eta\right) \quad \text{for } \gamma a' \lessapprox 1 \quad . \tag{B.14}$$

If $T_L(\omega_o)$ is defined as the total cold-circuit loss at ω_o, and the loss is assumed to vary as the square root of frequency, the loss parameter is given as

$$d = \frac{0.1155\, T_{L_o} \sqrt{\xi}}{C\,\xi\,\frac{L\omega_o}{u_o}} \quad , \tag{B.15}$$

which becomes

$$d = \frac{d_o C_o}{\left(\frac{\chi\xi^{1/2}}{\eta}\right)^{1/3}} \exp\left[\frac{2}{3}\left(1 - \frac{b'}{a'}\right)\gamma_o a'\eta\right] \quad \text{for } \gamma a' \gtrapprox 1$$

$$d = \frac{d_o C_o}{(\chi\tau\xi^{3/2})^{1/3}} \exp\left(0.59\,\gamma_o a'\eta\right) \quad \text{for } \gamma a' \lessapprox 1 \quad . \tag{B.16}$$

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