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B. C. Fan, M. Sichel and C. W. Kauffman, The University of Michigan, Ann Arbor, MI

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ANALYSIS OF OBLIQUE SHOCK-DETONATION WAVE INTERACTIONS IN THE SUPERSONIC FLOW OF A COMBUSTIBLE MEDIUM

B. C. Fan, M. Sichel and C. W. Kauffman
Department of Aerospace Engineering
The University of Michigan
Ann Arbor, MI 48109-2140

Abstract

Shock polar analysis is used to analyze the interaction which occurs when a detonation propagates past a bounding layer of inert or explosive gas. The oblique shock or detonation which is transmitted into the bounding medium is reflected from the lower bounding wall either regularly or as a Mach reflection. Such interactions occur in layered explosives and will also arise in supersonic combustors or in proposed detonative ramjet engines. Wave angles, velocities, and pressures are computed in the case of both regular and Mach reflection for a primary detonation propagating through a stoichiometric H_2-O_2 mixture bounded by an explosive mixture, also consisting of H_2-O_2 but with an equivalence ratio varying from 0 to 4.5. The computed results were in reasonable agreement with interaction parameters determined experimentally.

1. Introduction

Oblique shock-detonation wave interactions occur in many situations:

1) There has been recent interest in the detonative ramjet engine for use in supersonic and hypersonic flight.[11,12] Other supersonic combustor designs may also require the consideration of oblique detonations and their interaction with oblique shock waves.

2) In fuel air explosions detonations propagate through nonhomogeneous fuel-oxidizer clouds. Oblique detonations and shock-detonation interactions will arise in regions of where the fuel-oxidizer ratio changes rapidly and at the edge of the cloud.

3) In layered solid explosives detonations propagating through a primary explosive layer induce oblique shock waves or detonations in the secondary or bounding explosive.

The above processes have many features in common. However, it is difficult to conduct experimental laboratory studies of the detonative ramjet or to generate oblique detonations under steady flow conditions. The problem which provided the motivation of the present study is the propagation of detonations in layered solid explosives. It is very difficult to observe the interactions involved in the case of solids; however, because of the close analogy between gaseous and solid explosives detailed studies of layered detonations[1-5] have been made using various gaseous fuel oxidizer mixtures, and the results of these experiments also provided information relative to the other problems mentioned above. Constant-velocity oblique detonation waves along with many other complicated interactions have been observed in these experiments.

A detailed analysis of these observed oblique shock-detonation interactions is described in this paper. The experimental and analytical results obtained previously are first summarized, followed by a detailed description of the analytical approach. The analytically determined interaction parameters are then compared with experiment.

2. Experimental and Previous Analytical Results

As already indicated interactions between oblique shock and detonation waves can play an important role in supersonic combustion and are a dominating factor in the detonation and initiation of layered explosives. Experimental study of such interactions under steady flow conditions is difficult and very little oblique detonation data is currently available. Such interactions have, however, been studied experimentally[1-5] for gaseous explosive mixtures by taking advantage of the fact that, when an established detonation propagating along a layer of explosive comes in contact with an adjacent explosive layer, an oblique shock or detonation may be established in the secondary bounding mixture.

The experimental arrangement, which is described in detail elsewhere,[3] consists of two adjacent 1.6-cm square shock tubes each of which can be filled by a different explosive mixture. At the test section the two mixtures are separated by a very thin 50 nanometer cellulose film in order to avoid interference with the interaction. Pulsed laser Schlieren photography was used to obtain framing photographs of the interaction at 2 µsec intervals and provided very sharp detailed pictures of the interaction process.

Typical interaction patterns are shown schematically in Fig. 1 and some typical Schlieren framing photographs are shown in Fig. 4. As shown in Fig. 1, regular or Mach reflection of the transmitted wave, which could be an oblique shock or detonation, can occur at the wall of the lower shock tube. The normal detonation in the primary explosive or Explosive 1 is followed by an expansion wave.

Downstream of this wave the interface between the combustion products of the primary detonation and the bounding gas acts like a wedge or ramp at angle δ_3 to the horizontal, moving supersonically with the velocity of the primary detonation relative to the bounding explosive. Relative to coordinates fixed to the primary detonation the bounding mixture appears as a supersonic premixed fuel-oxidizer stream passing through an oblique detonation or shock wave – this is the feature which relates the layered detonation to a supersonic combustor.

The nature of the interaction which is observed depends strongly on the properties of both the primary and secondary or bounding explosive and an infinite number of combinations is possible. Most of the experiments[3] have been conducted using a stoichiometric mixture of H_2-O_2 as the primary explosive and using H_2-O_2 mixtures of various equivalence ratios as the secondary explosive. Several distinct interaction regimes were observed and a detailed description and analysis of these regimes is presented in the present paper and compared to experimental results. It is shown that a relatively simple analysis using the properties of oblique shocks and detonations can be used to calculate the interaction parameters with reasonable accuracy.

3. Analysis

Analysis of the interactions described above requires a consideration of oblique detonations. Oblique detonation polars have been computed by Rutkowski and Nicholls, and Gross[6,9] by assuming a constant value of the ratio of specific heats γ across the wave. The Mach reflection of oblique detonations has been observed experimentally and analyzed by Ong[13] using the one-gamma method and by assuming that the Mach Stem is a planar-discontinuous front perpendicular to the wall.

Because of the large changes in gas properties across detonations this analytical approach is relatively inaccurate. Consequently, a two-gamma method similar to that used by Woolard[10] was developed by Liou, et. al[1,5] to provide a relatively accurate means of computing oblique detonation and shock polars which then can be used for graphical determination of the interaction parameters. This technique has also been used by Liou[5] to study the Mach reflection of oblique detonations based on the same assumption as used by Ong.[13]

The shock polar analysis requires the tedious construction of oblique shock and detonation polars and graphical analysis for each set of interaction parameters. In the present paper, analytical solutions of the interaction problem based on the Gordon-McBride code[7] combined with the two-gamma method are developed to replace the polar analysis. On the basis of these relations a code was developed which rapidly computes interaction configurations for a wide range of primary and secondary explosives.

The basic configuration considered here is shown in Fig. 1. A normal C-J detonation wave propagates through Explosive 1 at glancing incidence to Explosive 2. Consequently, a shock or detonation wave is transmitted into Explosive 2 and is reflected from the lower wall.

In the two-gamma model used in the calculations, it is assumed that the gas is perfect upstream and downstream of the wave but that the upstream and downstream ratios of specific heats γ_u and γ_d are different. the main problem then is determination of the downstream value γ_d. Liou[5] has shown that best agreement with exact calculations using the Gordon-McBride code[7] is obtained if γ_d is taken as the ratio of specific heats γ_j for a Chapman-Jouguet detonation obtained from exact calculations.

Using the conservation equations across the wave it then follows that the dimensionless heat release $q = Q/a^2$ can be expressed in terms of γ_u and γ_j so that:

$$q = \frac{(\gamma_u - \gamma_j(a_u/u_j)^2)^2}{2(\gamma_j^2-1)\gamma_u^2(a_u/u_j)} - \frac{(\gamma_j - \gamma_u)(\gamma_j - 1)}{\gamma_u(\gamma_u-1)(\gamma_j^2+1)} \qquad (1)$$

where u_j = the C-J detonation velocity, and a = the velocity of sound. As already indicated γ_j, as well as the Chapman-Jouguet velocity u_j, can be obtained by using the Gordon-McBride code.

Given the value of q, expressions for other detonation parameters are readily determined as shown in Refs. (4) and (5). For oblique detonations it also is necessary to relate the flow deflection angle δ and the wave angle α (Fig. 1) to detonation parameters. Thus, for example, the deflection angle is given by

$$\delta = \tan^{-1} \frac{\{(p_d-p_u)A/\rho_u - [(p_d-p_u)/(\rho_u u_u)]^2\}^{1/2}}{u_u - [(p_d-p_u)/\rho_u u_u]} \qquad (2)$$

while the wave angle α is given by

$$\alpha = \sin^{-1}[p_d - p_u)^{1/2} / \rho_u u_u A] \qquad (3)$$

The function A is given by the expression

$$A = 1 - [(\gamma_d-1)/(\gamma_u-1)] \frac{(\gamma_u-1)p_d + (\gamma_u+1)p_u + 2\gamma_u(\gamma_u-1)p_u q}{(\gamma_d+1)p_d + (\gamma_d-1)p_u}$$

These equations are supplemented by the relation between deflection angle and pressure across the Prandtl-Meyer expansion wave immediately behind the primary detonation[8] where $\gamma = \gamma_d = \gamma_j$. The transmitted wave is determined by the requirement that both flow deflection and pressure must be equal at the interface between regions 2 and 3 so that:

$$\delta_2 = \delta_3 \qquad p_2 = p_3 \qquad (4)$$

When the transmitted shock angle α is less than a critical value α_c, the transmitted wave reflects regularly from the lower wall. The flow behind the reflected shock then will be parallel to the lower bounding wall so that the reflected shock is determined by the condition:

$$\delta_4 = \delta_3 \quad (5)$$

The above algebraic relations have been programmed for rapid computation of the transmitted and reflected waves when the the transmitted wave is either an oblique shock or a detonation.

When $\alpha > \alpha_c$ Mach reflection occurs at the lower wall requiring a different treatment. From experimental framing pictures[4] it has been observed that, at the initial stage of interaction, an explosive bubble of increasing size is formed and propagates toward the lower wall as either an oblique shock or a detonation wave. As shown schematically in Fig. 2. the point where the oblique shock or detonation first contacts the wall can then be regarded as the apex of a wedge of angle $(\pi/2-\alpha)$, where α is the angle of the transmitted wave. The Mach reflection at the lower wall is thus similar to that which occurs when a moving incident shock or detonation wave impinges on a sharp compressive corner as also shown in Fig. 2. A detailed view of the Mach reflection from a coordinate system fixed at the triple point is shown in Fig. 3. The critical value α_c can be determined using the detachment or mechanical equilibrium criterion.[14,15] The present problem differs from the conventional one in that the various angles which determine the Mach reflection are determined by the interaction which occurs at the interface between the two explosives.

It is assumed that the Mach Stem is a planar-discontinuous front, without regard to the effect of the finite reaction zone length. Then, assuming that the Mach Stem is perpendicular to the lower wall and that the coordinates are fixed on the triple point to equalize the particle velocities upstream of both the incident wave and the Mach Stem it follows that:

$$u_n = u_{ja} \sin\alpha = u_o \sin(\alpha - \chi) \quad (6)$$

so that the normal component of the velocity of the incident wave, u_n, remains fixed. Here u_{ja} is the C-J detonation velocity of Explosive 1 and χ is the triple point trajectory angle which is a measure of the Mach Stem growth rate.

In terms of the coordinates fixed to the triple point the following relations must be satisfied and determine the strength of the reflected wave and the Mach Stem:

$$p_4 = p_5 ; \qquad \delta_3 = \delta_4 \quad (7)$$

and from the assumption that the Mach Stem is perpendicular to the wall it follows that

$$\alpha_1 + \chi = \pi/2 \quad (8)$$

where α_1 is the angle between Mach Stem and triple-point trajectory.

From the above equations the parameters of the Mach reflection, that is p_3, p_4, p_5, ρ_3, ρ_4, ρ_5, and χ can be determined. Generally $\chi \neq 0$ as shown by the calculated results which means that the width of the Mach Stem increases with time. This type of reflection is sometimes called "unstable Mach reflection." If it is assumed that $\chi = 0$, the Mach Stem near the triple point will no longer be perpendicular to the lower wall; however, the width of the Mach Stem now remains constant. This type of reflection is called "stable Mach reflection."

3. Comparison with Experiment

A stoichiometric H_2-O_2 mixture was used as the primary explosive and H_2-O_2 mixtures with equivalence ratios ranging from 0.15 to 4.5 were used as the bounding secondary explosive in the experimental arrangement[1-5] already described above. Four distinct interaction regimes were observed with increasing values of the equivalence ratio of the bounding explosive, and these are described in detail below. A set of typical Schlieren framing photographs is shown in Fig. 4 with each frame representing a typical example of each regime.

When the equivalence ratio $\phi < 0.35$, the secondary mixture is so lean that detonation cannot be initiated directly. The transmitted wave is then a shock which reflects regularly from the lower wall as shown in Fig. 4(a).

When $0.35 < \phi < 1.0$, direct initiation of the bounding explosive is often observed so that the transmitted wave is a C-J detonation as shown in Fig. 4(b). The incidence angle α then exceeds critical angle α_c, and a Mach Stem, which is an overdriven detonation, is observed at the wall.

When $1.0 < \phi < 3.1$, the transmitted wave is a shock which should reflect regularly from the wall according to the detachment criterion. The pressure behind the reflected wave is much higher than the C-J pressure and a detonation is often initiated at the impact point. This detonation is perpendicular to the lower wall and propagates with the C-J detonation velocity of the bounding explosive which generally exceeds that of the primary mixture. The resulting configuration is similar to that of Mach reflection and is shown in Fig. 4(c).

When $\phi > 3.1$, the incidence angle of the transmitted wave exceeds α_c based on the detachment criterion, resulting in the formation of a Mach Stem. The pressure behind the reflection, which drops dramatically at $\phi = 3.1$, is nearly equal to the C-J pressure. In this case, initiation of a detonation at the impact point is no longer possible so that both the transmitted wave and the Mach Stem are shock waves as shown in Fig. 4(d).

The analysis described above has been used to calculate the propagation velocities and angles of the waves involved in the above interactions, and these are compared to experimentally measured values in Figs. 5 and 6.

When $\phi < 0.35$, the observed transmitted wave is an oblique shock, and the normal component of the velocity relative to the shock and the incident angle are represented by the oblique shock curves shown in Figs. 5 and 6. It can be seen that the measured velocities and angles are greater than those computed until $\phi > 3$, suggesting that some reaction was present behind the shock even when a full detonation was not observed in the bounding mixture. The propagation velocity of the impact point on the wall is, in this case equal to the C-J detonation velocity of the primary Explosive 1 which is displayed by curve 2.

When $0.35 < \phi < 1.0$, the observed transmitted wave was either an oblique shock or an oblique C-J detonation wave; however, for thinner values of the separating film an oblique detontation was observed for all experiments[4]. The normal component of the velocity relative to the oblique detonation is higher than that relative to the shock and is, represented by the detonation wave curve in Fig.5. The angle of incidence as shown in Fig. 6 is, then, much larger resulting in Mach reflection. The Mach stem is then an overdriven detonation, and the Mach Stem velocity corresponding to this case is shown in Fig. 5 as curves 3 and 4, which have been computed using the one and two gamma analysis, respectively. Agreement between measured and computed velocities and angles appears reasonable in this regime.

For $1.0 < \phi < 3.1$, the oblique shock curve in Fig 5 represents the normal component of the velocity of the transmitted wave which is a shock wave, and the C-J detonation curve represents the propagation velocity of the impact point on the wall since it now moves with the same velocity as the primary wave. The incident angles for both an unstable and stable Mach Stem are shown in Fig. 6, and the experimental data supports the existence of an unstable Mach Stem. Near $\phi = 3.1$, transition occurs from regular to Mach reflection, based on the detachment criterion[14].

For $\phi > 3.1$, the oblique shock curve represents the normal velocity component to the transmitted oblique shock, curve 1 represents the propagation velocity of the impact point on the wall for "unstable Mach reflection," i.e. $\chi \neq 0$, and curve 2 represents the impact point velocity for "stable Mach reflection," i.e. $\chi = 0$. The experimental points again suggest that the Mach reflection is unstable for $\phi > 3.1$. In the range $1.0 < \phi < 3.1$ the experimentally determined Mach Stem velocity exceeds the C-J detonation velocity of the bounding mixture suggesting that the detonative Mach Stem reflection also is unstable.

The variation of the triple-point trajectory angle, which is a measure of the rate of Mach Stem growth, with equivalence ratio has also been computed in the case of Mach Reflection and the results shown in Fig. 7 are in reasonable agreement with measured values. The trajectory angle increases with increasing Mach Stem velocity. When $0.35 < \phi < 1.0$ the Mach Stem is an overdriven detonation and the trajectory angle is shown as curves 1 and 2 computed using the one-gamma and two-gamma analysis respectively. When $1.0 < \phi < 3.1$ the Mach Stem is a C-J detonation represented by the Mach detonation curve in Fig.7. When $\phi > 3.1$ the Mach Stem is a shock rather than a detonation wave and the trajectory angle is then represented by the Mach shock curve. The experimental results were obtained by Liu.[16]

The variation of theoretically computed pressures with the equivalence ratio of the bounding H_2-O_2 mixture are shown in Fig. 8; reliable pressure data was unfortunately not available for purposes of comparison. The two lower curves show the variation of the pressure behind an oblique shock induced in the bounding gas and the Chapman-Jouguet pressure of the bounding mixture. The pressures behind the regular and Mach reflections when the induced wave is an oblique shock are represented by the regular and Mach reflection curves. The critical point beween regular and Mach reflection occurs at $\phi = 1.26$ according to the mechanical equilibrium criterion. In this case the curve is continuous although the derivative is discontinuous at the critical point. The pressure decreases monotonically beyond this point for both stable and unstable Mach reflection. If the detachment criterion is used, Mach reflection does not start until $\phi = 3.1$ and the pressure then drops discontinuously from 25.64 atm to 17.56 atm for unstable Mach reflection and to 16.45 atm for stable Mach reflection.

Curves ABE and CDE represent the pressures behind reflected oblique detonation waves computed using the one- or two-gamma methods. The critical point between regular and Mach reflection is almost the same for the one- and two -gamma methods at $\phi = 0.35$ and $\phi = 0.53$ respectively. As the equivalence ratio increases the pressure increases during regular reflection but then decreases for Mach reflection as the strength of the transmitted detonation increases.

4. Discussion

Analytical solutions have been developed for the interaction which occurs when a detonation propagates past a bounding explosive layer. Both the transmitted shock or detonation wave and regular or Mach reflection from the lower wall have been considered. The computed results are in reasonable agreement with experiment and the analysis provides a rapid means of dealing with such interaction problems. Based on comparison with experiment transition to Mach reflection appears to be governed by the detachment criterion. The data indicates that when Mach reflections occur they are of the unstable type

The results developed here, and validated by comparison with experiment, are not only applicable to layered detonations but can also provide a basis for the study detonative interactions in supersonic combustion.

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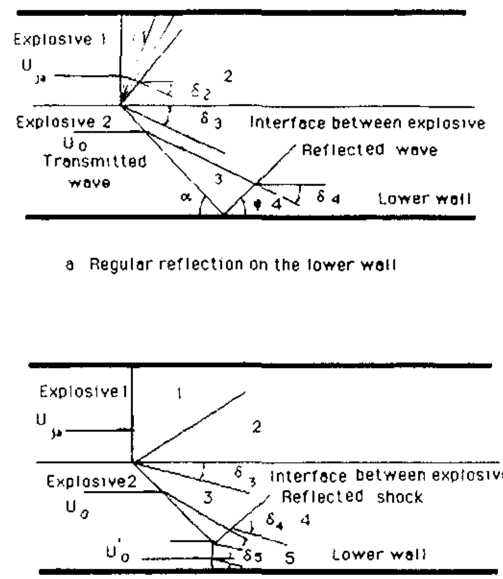
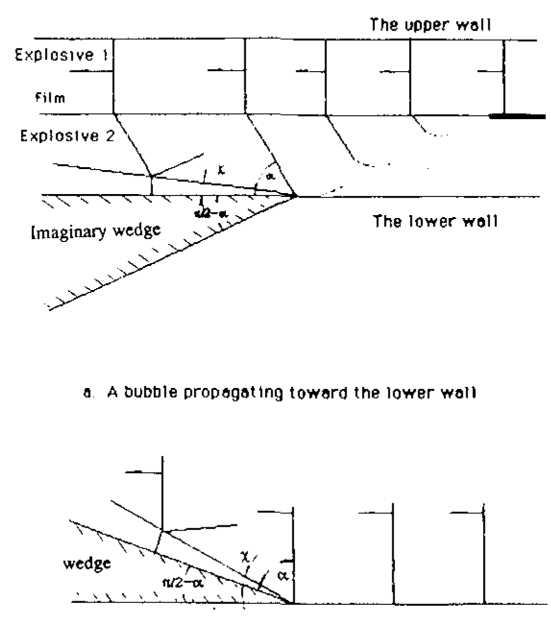


a Regular reflection on the lower wall

b Mach reflection on the lower wall

Fig 1. Glancing interaction between detonation and explosive boundary.



a. A bubble propagating toward the lower wall

b. A moving incident shock wave impinging on a sharp compressive corner

Fig 2. Pulsed laser and high speed camera system schematic diagram.

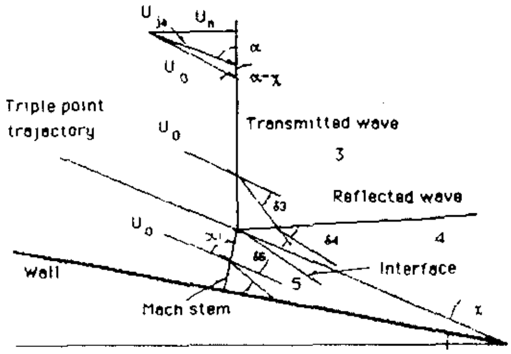


Fig 3. Mach reflection on the lower wall, coordinate fixed on the triple point

(c) Bounding explosive H_2-O_2, $\phi = 2.0$ an oblique shock and normal detonation

(a) Bounding explosive H_2-O_2, $\phi = 0.2$ an oblique shock with regular reflection.

(d) Bounding explosive H_2-O_2, $\phi = 4.5$ an oblique shock with Mach reflection.

(b) Bounding explosive H_2-O_2, $\phi = 0.45$ an oblique detonation with Mach reflection.

Fig 4. Typical Schlieren spark photographs (Behind detonation wave, the typical transverse wave structure can be seen.)

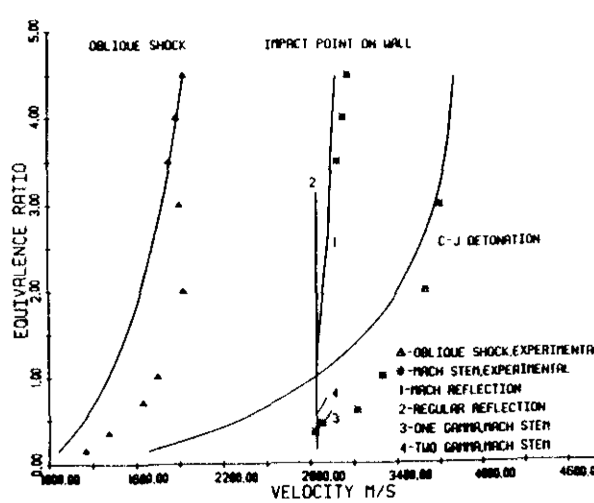


Fig 5. Variation of velocities with equivalence ratio.

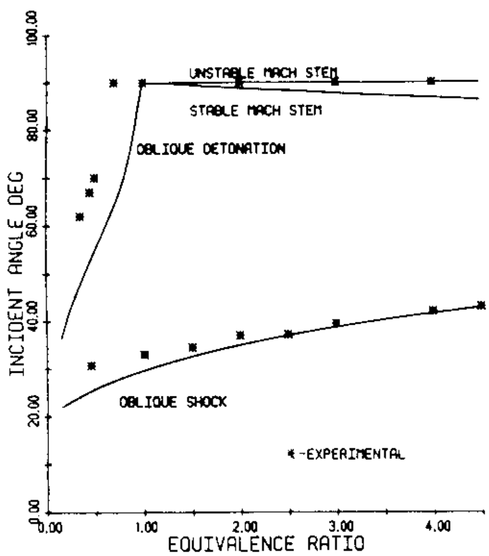


Fig 6. Variation of incident angle with equivalence ratio.

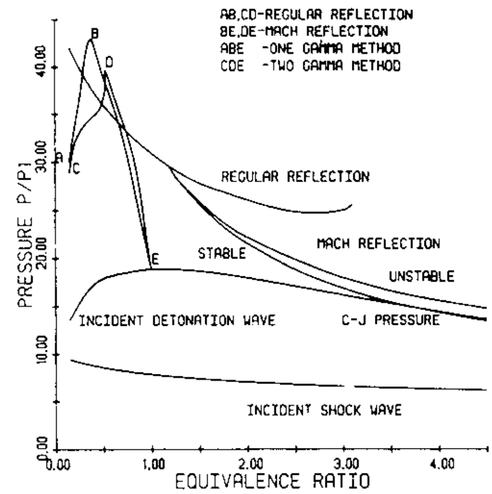


Fig 8. Variation of pressure with equivalence ratio.

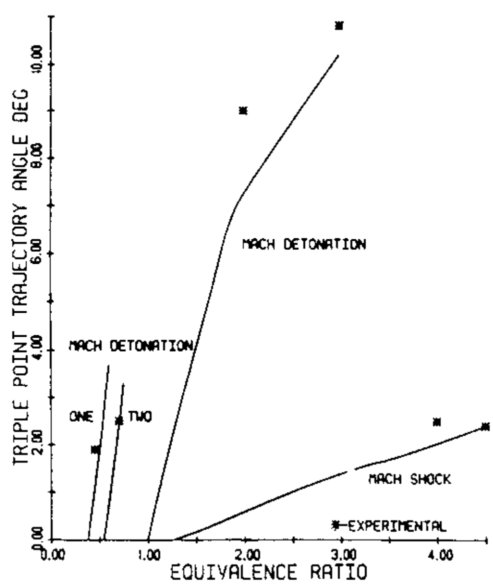


Fig 7. Variation of triple point trajectory angle with equivalence ratio.