THE UNIVERSITY OF MICHIGAN

COLLEGE OF ENGINEERING
Department of Mechanical Engineering

Student Project Report

SPRING-DAMPER SUSPENSION SYSTEM ANALYSIS FOR HORIZONTAL AXIS WASHING MACHINES

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ABSTRACT

A major concern of washing machine design is controlling walking and load transmission to the floor. A suspension system must do this without large deflection. Given preliminary design data about a horizontal axis washing machine, a procedure was developed to design a spring-damper suspension system. Parts of the procedure were given limited testing and accuracy within 35% was found. Suggestions for improving the design procedure and suspension system conclude the report.

INTRODUCTION

The horizontal axis washing machine, while having advantages over vertical axis machines, has some disadvantages. One of the problems encountered in these machines is controlling the dynamics of the spin cycle when the load in the basket is not evenly distributed. Machine movement (walking) and large load transmission to the floor (i.e., causing dishes to shake off kitchen shelves) are the two basic concerns of the spin cycle dynamics problem.

There are at least two methods for dealing with these conditions. The first is to balance the basket during the spin cycle to eliminate the imbalance. This has been investigated by Whirlpool, and the last production model contained a balancing mechanism. Unfortunately, the mechanism was very expensive and therefore was discontinued.

The second method is to suspend the chassis (i.e., springs) to absorb deflections without walking and transmitting excessive loads to the floor. This method is the subject of this report. A mathematical model of a horizontal axis washing machine is constructed and tested and a design procedure formulated.

THE MATHEMATICAL MODEL

A mathematical model of a spring suspended chassis was formulated. It describes the physical machine of Figure 1. This is a six-degree-of-freedom system with a rotating imbalance.

Ordinarily a ground reference frame would be chosen at A. From A, a vector $\vec{Q}$ describes the position of the machine, as represented by coordinate system B imbedded in the machine halfway along the drum's axis of rotation (or at any convenient point). The rotation of B relative to A is described by a transformation matrix (i.e., the Euler transformation). A vector $\vec{D}$ in frame B locates the imbalance in the basket. A vector describing the imbalance relative to ground, $\overrightarrow{QD}$, requires three components of vector $\vec{Q}$, a transformation matrix and three components of vector $\vec{D}$. I decided that the machine could be modeled adequately with a simple set of linear, ordinary differential equations whose solutions could be obtained analytically. Although numerical methods were feasible, it is easier to work with an explicit expression of the form x = FUNCTION (machine parameters), where x is the machine displacement and the parameters are damping coefficients, masses, spring constants, etc. This form of equation is more flexible.

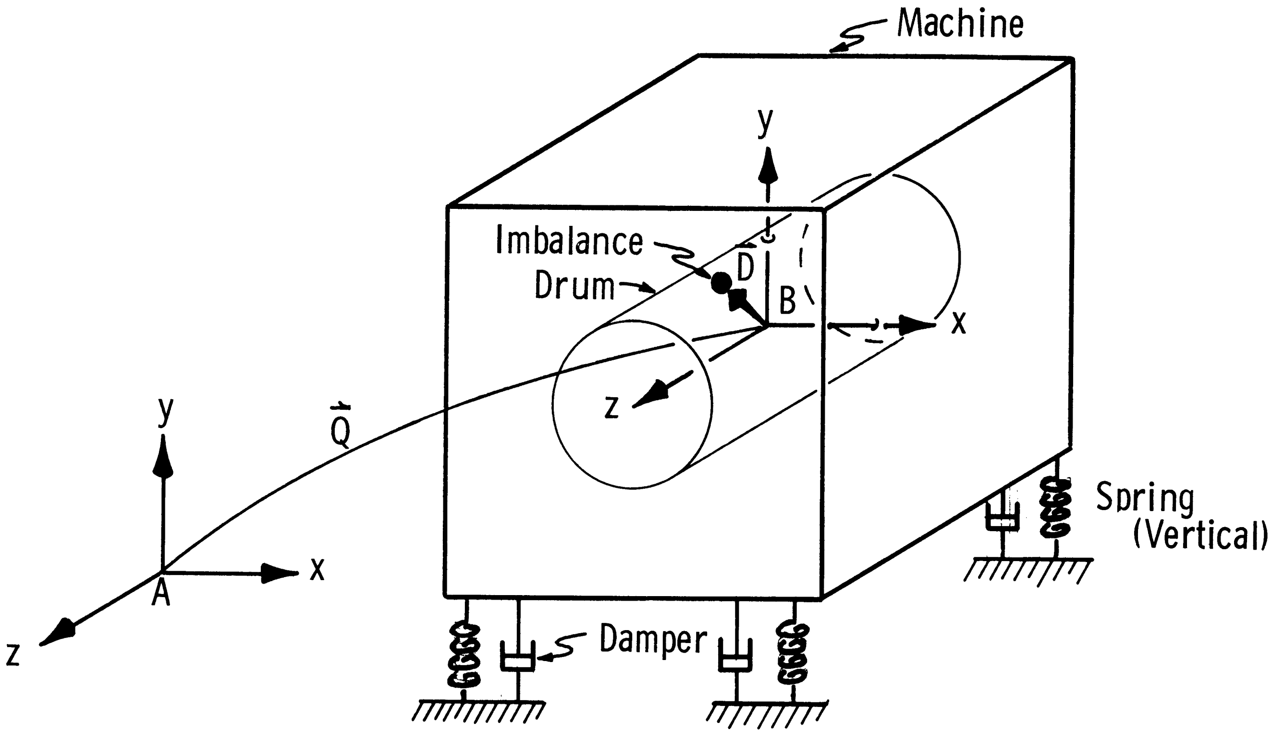


Figure 1

The mathematical model formulation was attempted using the standard reference frame, and was abandoned because the complexity of expressions of $\overrightarrow{QD}$ appeared to require a very tedious derivation of the governing equations. To simplify $\overrightarrow{QD}$, the coordinate system A for the ground frame was to consist of three axes fixed to ground located in the center of the drum axis such that at static equilibrium the z axis runs along the axis of rotation of the drum. Coordinate systems A and B are superimposed at static equilibrium and vector Q becomes the deflection of the machine in the dynamic state.

Since $|\vec{Q}|$ is small compared to $|\vec{D}|$, $|\vec{R}|$, or other vectors of the problem ($|\vec{Q}| \approx .5$ in., $|\vec{D}| \approx 15$ in., $|\vec{R}| \approx 8$ in.), the coordinate systems A and B are assumed superimposed DURING THE DYNAMIC STATE. This does not mean deflections are assumed to be zero; it is the deflections we wish to solve for. It means deflections are assumed not to influence the position vectors locating the imbalance masses. Note also, that the angles of machine rotation θ_x, θ_y, θ_z are also assumed small to justify the above assumption. These assumptions will reduce the former vector $\overrightarrow{QD}$ to only three components (see Figure 2).

This unusual approach is taken because the Lagrange equation will be used.

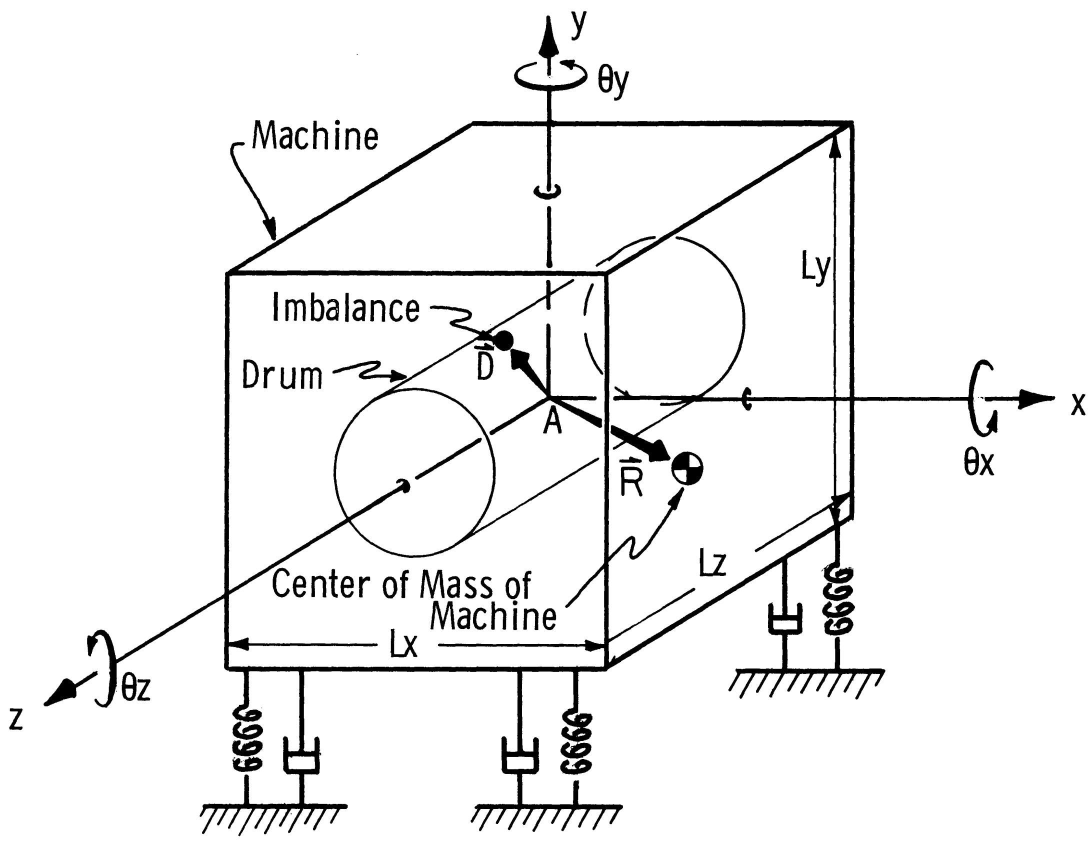


Figure 2

$$\frac{\partial}{\partial t}\left(\frac{\partial(T-V)}{\partial \dot{q}_i}\right) - \frac{\partial(T-V)}{\partial q_i} = Q_i \qquad (1)$$

where T = kinetic energy of system

V = potential energy of system

Q_i = nonconservative forces associated with system

This equation will require three vectors of the type $\overrightarrow{QD}$ to locate two imbalances in the basket and the center of mass of the machine in a six-degree-of-freedom system. These vectors must be used for the kinetic and potential energy of the system and therefore were chosen to be as simple as possible. The Lagrange approach with the first coordinate system would consume a much larger portion of the project time. The angular coordinates are the rotations observed around the individual axes if no other deflections were present in the system. Due to the deflections being small, this is a reasonable assumption which further simplifies the problem.

DERIVATION OF EQUATIONS OF MOTION

The kinetic energy of the system (see Figure 3) is:

$$T = \frac{1}{2} M_1 \dot{\rho}_1^2 + \frac{1}{2} M_2 \dot{\rho}_2^2 + \frac{1}{2} M \dot{\rho}_3^2 + \frac{1}{2} I_x \dot{\theta}_x^2 + \frac{1}{2} I_y \dot{\theta}_y^2 + \frac{1}{2} I_z \dot{\theta}_z^2$$

$$\vec{\rho}_1 = \vec{r}_1 + \vec{r}_3 = r_1 \hat{r}_1 + r_3 \hat{r}_3 \qquad (2)$$

where $\hat{r}_1, \hat{r}_3$ = unit vectors

Similarly $\rho_2 = \vec{r}_2 + \vec{r}_4$

r_4, r_3 are fixed magnitudes on the basket: $dr_3/dt = 0$

ϕ = angle of drum rotation relative to machine

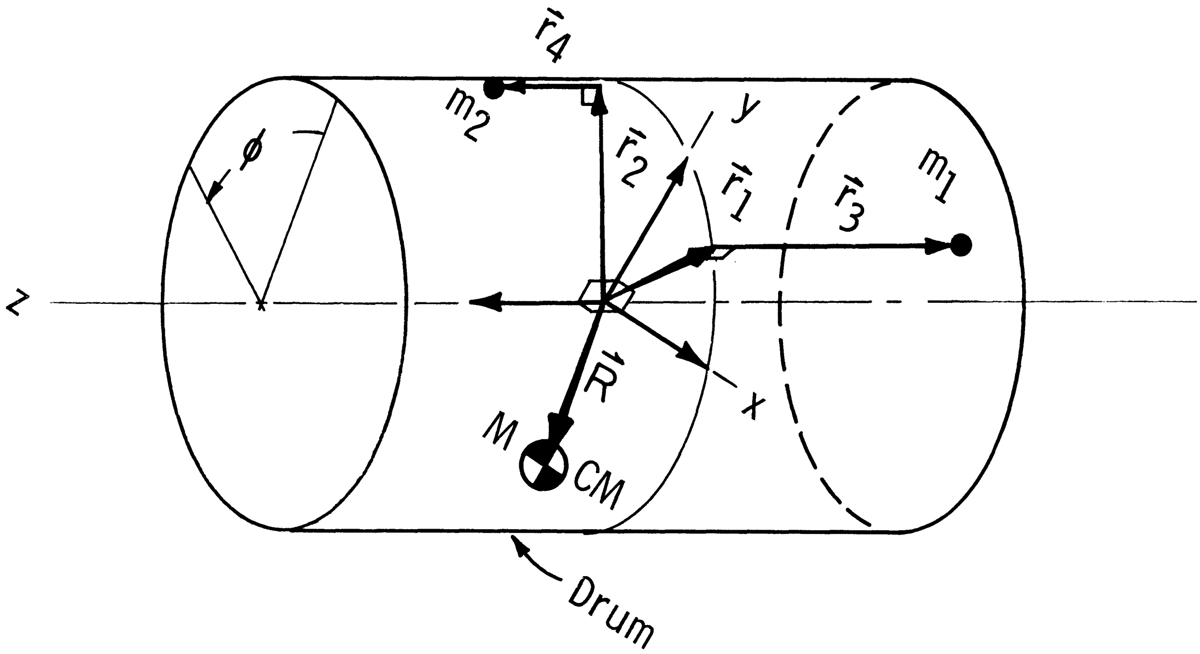


Figure 3

If the spring rates in the x and z directions are known in terms of the spring rate in the y direction, the potential energy of the x, y, and z coordinates is

$$V = \frac{1}{2} k_x x^2 + \frac{1}{2} k_y y^2 + \frac{1}{2} k_z z^2 \qquad (3)$$

The rotational potential energy is

$$V_{\theta_i} = \frac{1}{8} L_i^2 k_i \sin^2 \theta_i \approx \frac{1}{2} k_{\theta_i} \theta_i^2 \qquad (4)$$

Because deflections are small, it is assumed there is no coordinate coupling in the potential energy terms.

Once T and V are known, the tedious job of differentialing T-V is done.

If the damping coefficients in the x, y, and z directions are known, the nonconservative forces are:

$$Q_x = -C_x \dot{x}$$

$$Q_y = -C_y \dot{y}$$

$$Q_z = -C_z \dot{z}$$

$$Q_{\theta_x} = -\frac{L_z^2 C_y + L_y^2 C_z}{2} (\dot{\theta}_x) = -C_{\theta_x} \dot{\theta}_x$$

$$Q_{\theta_y} = -\frac{L_z^2 C_x + L_x^2 C_z}{2} (\dot{\theta}_y) = -C_{\theta_y} \dot{\theta}_y$$

$$Q_{\theta_z} = -\frac{L_y^2 C_x + L_x^2 C_y}{2} (\dot{\theta}_z) = -C_{\theta_z} \dot{\theta}_z \qquad (5)$$

The resulting differential equation of motion in the x-direction is:

$$m_1\ddot{r}_{x_1} + m_2\ddot{r}_{x_2} + M\ddot{X} - m_1\left[\left(\dot{\phi}_z^2 + \dot{\phi}_y^2\right)r_{x_1} - \dot{\phi}_x\dot{\phi}_z r_{z_1} - \dot{\phi}_x\dot{\phi}_y r_{y_1}\right]$$

$$- m_2\left[\left(\dot{\phi}_z^2 + \dot{\phi}_y^2\right)r_{x_2} - \dot{\phi}_x\dot{\phi}_z r_{z_2} - \dot{\phi}_x\dot{\phi}_y r_{y_2}\right]$$

$$- M\left[\left(\dot{\theta}_z^2 + \dot{\theta}_y^2\right)X - \dot{\theta}_x\dot{\theta}_z Z - \dot{\theta}_x\dot{\theta}_y Y\right] + k_x = -C_x\dot{x} \qquad (6)$$

where $\dot{\phi}_x$, $\dot{\phi}_y$, and $\dot{\phi}_z$ are the components of the vector $\dot{\vec{\phi}}$.

$$X = R_x + x, \qquad Y = R_y + y, \qquad Z = R_z + z \qquad (7)$$

where R_x, R_y, R_z = components of $\vec{R}$ (constants)

x, y, z = machine deflections

Now r_{x1}, r_{y1}, r_{z1}, r_{x2}, r_{y2}, r_{z2} are the components of single vectors from the origin of the ground frame to m_1 and m_2, respectively. This equation is still not simple enough for an analytical solution. The following assumptions are made:

$$M \gg m_1 + m_2$$

$$\dot{\phi}_z = \omega \gg \dot{\phi}_x, \dot{\phi}_y, \dot{\theta}_x, \dot{\theta}_y, \dot{\theta}_z$$

$$r_x \approx r \cos \omega t \qquad (8)$$

The resulting equation is:

$$M\ddot{x} + C_x\dot{x} + k_x x = m_1\omega^2 r \cos \omega t + m_2\omega^2 r \cos(\omega t + \Delta) \qquad (9)$$

This equation is solvable.

The expressions in the other coordinate systems are of the same form.

$$m_1\ddot{r}_{y_1} + m_2\ddot{r}_{y_2} + M\ddot{Y} - m_1\left[\left(\dot{\phi}_z^2 + \dot{\phi}_x^2\right)r_{y_1} - \dot{\phi}_y\dot{\phi}_z r_{z_1} - \dot{\phi}_y\dot{\phi}_x r_{x_1}\right]$$
$$- m_2\left[\left(\dot{\phi}_z^2 + \dot{\phi}_x^2\right)r_{y_2} - \dot{\phi}_y\dot{\phi}_z r_{z_2} - \dot{\phi}_y\dot{\phi}_x r_{x_2}\right]$$
$$- M\left[\left(\dot{\theta}_z^2 + \dot{\theta}_x^2\right)Y - \dot{\theta}_y\dot{\theta}_z Z - \dot{\theta}_y\dot{\theta}_z X\right] + k_y Y = Q_y \qquad (10)$$

$$m_1\ddot{r}_{z_1} + m_2\ddot{r}_{z_2} + M\ddot{Z} - m_1\left[\left(\dot{\phi}_x^2 + \dot{\phi}_y^2\right)r_{z_1} - \dot{\phi}_z\dot{\phi}_y r_{y_1} - \dot{\phi}_z\dot{\phi}_x r_{x_1}\right]$$
$$- m_2\left[\left(\dot{\phi}_x^2 + \dot{\phi}_y^2\right)r_{z_2} - \dot{\phi}_z\dot{\phi}_y r_{y_2} - \dot{\phi}_z\dot{\phi}_x r_{x_2}\right]$$
$$+ M\left[\left(\dot{\theta}_y^2 + \dot{\theta}_x^2\right)Z - \dot{\theta}_z\dot{\theta}_y Y - \dot{\theta}_z\dot{\theta}_x X\right] + k_z Z = Q_z \qquad (11)$$

$$\frac{d}{dt}\left(I_x\dot{\theta}_x + M\left[\left(Z^2 + Y^2\right)\dot{\theta}_x - ZX\dot{\theta}_z - YX\dot{\theta}_y\right]\right)$$
$$+ m_1\left(2\omega^2 r_{z_1} r_{y_1} + 2\omega^2 r_{z_3} r_{y_3}\right) + m_2\left(2\omega^2 r_{z_2} r_{y_2} - 2\omega^2 r_{z_4} r_{y_4}\right)$$
$$+ \frac{1}{4}\left(L_y^2 k_z + L_z^2 k_y\right)\theta_x = Q_{\theta_x} \qquad (12)$$

$$\frac{d}{dt}\left(I_y\dot{\theta}_y + M\left[\left(Z^2 + X^2\right)\dot{\theta}_y - YZ\dot{\theta}_2 - YX\dot{\theta}_x\right]\right)$$
$$+ m_1\left(2\omega^2 r_{z_1} r_{x_1} + 2\omega^2 r_{z_3} r_{x_3}\right) + m_2\left(2\omega^2 r_{z_2} r_{x_2} - 2\omega^2 r_{z_4} r_{x_4}\right)$$
$$+ \frac{1}{4}\left(L_x^2 k_z + L_z^2 k_x\right)\theta_y = Q_{\theta_y} \qquad (13)$$

$$\frac{d}{dt}\left(I_z\dot{\theta}_z + M\left[\left(Y^2 + X^2\right)\dot{\theta}_z - YZ\dot{\theta}_y - XZ\dot{\theta}_x\right]\right)$$
$$+ \frac{1}{4}\left(L_x^2 k_y + L_y^2 k_x\right)\theta_z = Q_{\theta_z} \qquad (14)$$

These equations are suitable to numerical or modal solutions. However, assumptions similar to the x-direction were used to reduce the equations further.

$$M\ddot{y} - Q_y + K_y Y = m_1\omega^2 r \sin \omega t + m_2\omega^2 r \sin(\omega t + \Delta) \tag{15}$$

$$M\ddot{z} - Q_z + K_z Z = 0 \tag{16}$$

$$I_x\ddot{\theta}_x - Q_{\theta_x} + \frac{1}{4}\left(L_y^2 k_z + L_z^2 k_y\right)\theta_x = -m_1\omega^2 r_3 r_1 \sin \omega t - m_2\omega^2 r_4 r_2 \sin(\omega t + \Delta) \tag{17}$$

$$I_y\ddot{\theta}_y - Q_{\theta_y} + \frac{1}{4}\left(L_x^2 k_z + k_x L_z^2\right)\theta_y = -m_1\omega^2 r_3 r_1 \sin \omega t - m_2\omega^2 r_4 r_2 \sin(\omega t + \Delta) \tag{18}$$

$$I_z\ddot{\theta}_z - Q_{\theta_z} + \frac{1}{4}\left(L_x^2 k_y + L_y^2 k_x\right)\theta_z = -MR_y^2\ddot{\theta}_z \tag{19}$$

Because the force generated by the rotating imbalances acts directly through the center of the coordinate system, it is impossible for these forces to gen- a torque in the θ_z direction. The forcing function in the θ_z direction is, therefore, implicit in $-MR_y^2\ddot{\theta}_z$. To show this, a free body diagram of the force on the machine (assumed only to include the effects of the rotating unbalance, i.e., spring force is small compared to this force) should be drawn. This force will act at a distance $R = |\vec{R}|$ from the center of mass. By writing Newton's motion equations, one obtains

$$F = M\ddot{x}, \qquad T = -FR = I\ddot{\theta} \tag{20}$$

These equations yield $\ddot{\theta} = -MR\ddot{x}/I$. The θ_z equation becomes

$$I_z\ddot{\theta}_z - Q_{\theta_z} + \frac{1}{4}\left(k_x L_y^2 + k_y L_x^2\right)\theta_z = \frac{M^2 R_y^2}{I}\ddot{x} \tag{21}$$

The second derivative of the solution of the x equation is proportional to the forcing function of the θ_z direction. I_z is taken about the center of mass.

Furthermore, an important assumption must be included to get the last set of differential equations. That assumption is:

$$R_x = R_z = 0 \tag{22}$$

i.e., the center of mass of the machine is on the vertical centerline of the machine (the y axis).

These uncoupled equations can allow an unlimited number of imbalances to be accounted for. This is accomplished by a geometric exercise that reduces the effect of two imbalances into a single effective imbalance. In the uncoupled x-y plane (see Figure 4):

$$\overbrace{m_1\omega^2 r}^{a} \sin \omega t + \overbrace{m_2\omega^2 r}^{b} \sin(\omega t + \Delta) = \overbrace{m\omega^2 r}^{c_L} \sin(\omega t + A) \tag{23}$$

$$A_L = \tan^{-1}\left(\frac{1}{\dfrac{a}{b \sin \Delta} + \dfrac{1}{\tan \Delta}}\right)$$

$$C_L = \frac{b \sin \Delta}{\sin[A_L]} \tag{24}$$

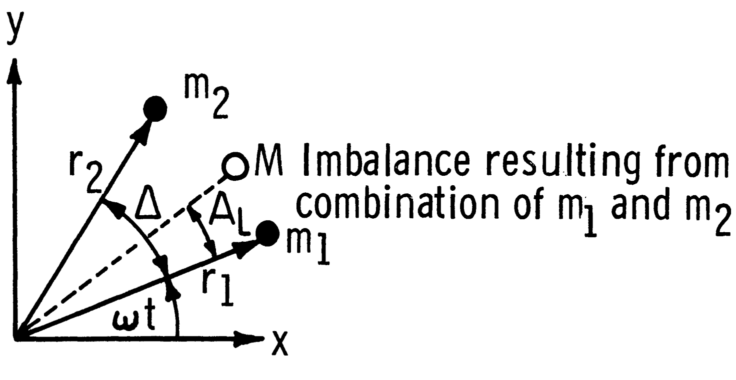


Figure 4

In the same manner, the torsional effect of two imbalances can be resolved into a single effect.

$$\underbrace{m_1\omega^2 r_1 r_3}_{a} \sin \omega t + \underbrace{m_2\omega^2 r_2 r_4}_{b} \sin(\omega t + \Delta) = \underbrace{m\omega^2 r_5 r_6}_{c_\theta} \sin(\omega t + A_\theta) \qquad (25)$$

The above formulas apply again for this A_θ and c_θ.

Plugging this result into the differential equations results in a set of six uncoupled equations with a single forcing function for each. The solutions of these equations are

$$x = X \sin(\omega t + \alpha_1) \qquad (26)$$

$$y = Y \sin(\omega t + \alpha_2) \qquad (27)$$

$$z = 0 \qquad (28)$$

$$\theta_x = \Theta_x \sin(\omega t + \alpha_3) \qquad (29)$$

$$\theta_y = \Theta_y \sin(\omega t + \alpha_4) \qquad (30)$$

$$\theta_z = \Theta_z \sin(\omega t + \alpha_5) \qquad (31)$$

where α_i = phase angles, see equations (39)-(43)

$$X = \frac{mr_a\omega^2}{\sqrt{\left(4c_x\omega\right)^2 + \left(4k_x - M\omega^2\right)^2}} \qquad (32)$$

$$Y = \frac{mr_a\omega^2}{\sqrt{\left(4c_y\omega\right)^2 + \left(4k_y - M\omega^2\right)^2}} \qquad (33)$$

where $mr_a\omega^2$ = the result of resolving the two imbalances

$r_a = r_1 = r_2$, so m is an equivalent imbalance

$$\Theta_x = \frac{-mr_a r_b\omega^2}{\sqrt{\left(2c_{\theta_x}\omega\right)^2 + \left(2k_{\theta_x} - I_x\omega^2\right)^2}} \qquad (34)$$

$$\Theta_y = \frac{-mr_a r_b\omega^2}{\sqrt{\left(2c_{\theta_y}\omega\right)^2 + \left(2k_{\theta_y} - I_y\omega^2\right)^2}} \qquad (35)$$

mr_b is the equivalent torsional effect

$$Z = 0 \tag{36}$$

$$\textcircled{H}_z = \frac{-mr_a R_y^3 \omega^2 M^2/I_z}{\sqrt{\left(2c_{\theta_z}\omega\right)^2 + \left(2k_{\theta_z} - I_z\omega^2\right)}\sqrt{\left(4c_x\omega\right)^2 + \left(4k_x - M\omega^2\right)^2}} \tag{37}$$

WALKING BEHAVIOR

The mathematical model should also contain a routine for predicting if a washing machine will walk. On a smooth floor, an experimental chassis with a spring suspension (see Figure 5) was observed to begin walking with an oscillating sliding motion (all four feet moved the same distance in the same direction simultaneously). The machine did not pivot about one foot. It was assumed the total loads on the individual feet of the machine produce the same effect as the average load applied to all four feet simultaneously (because the feet are connected by a rigid structure). The problem then becomes:

-u(vertical forces on the feet) $\geq$ (horizontal forces on the feet)

u $\equiv$ frictional coefficient

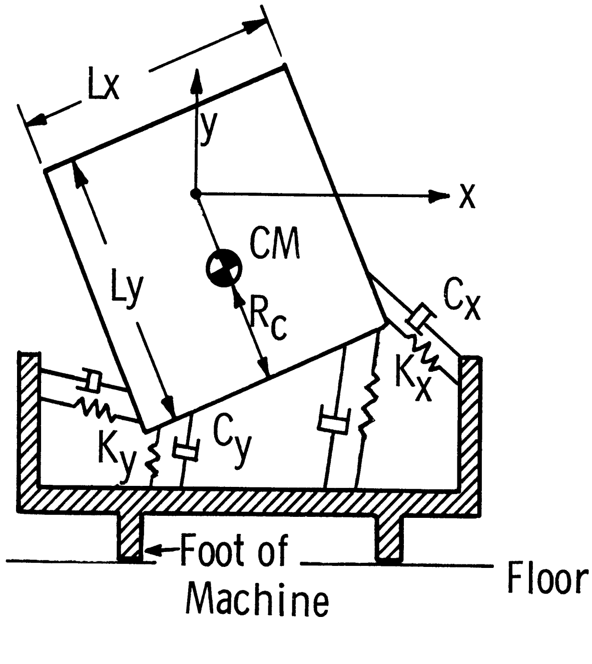


Figure 5

If this equation holds, the machine will not walk. Since the forces on the feet of the machine are functions only of the weight of the machine and deflections of the springs in the suspension, ($F = k_x$), the walking equation becomes:

$$-u\left(k_y y + c_y \dot{y} - \frac{W}{4}\right) \geq \left\{\left[k_x\left(x + \theta_y \frac{Lz}{2} + \theta_z R_c\right) + c_x\left(\dot{x} + \dot{\theta}_y \frac{Lz}{2} + \dot{\theta}_z R_c\right)\right]^2 + \left[k_z\left(\theta_x R_c + \theta_y \frac{Lx}{2}\right) + c_z\left(\dot{\theta}_x R_c + \dot{\theta}_y \frac{Lx}{2}\right)\right]^2\right\}^{1/2} \tag{38}$$

R_c is defined in Figure 5. This is a time dependent equation. The quantities in it are defined

$$x = X \cos(\omega t + A_L - \alpha) \qquad \alpha = \tan^{-1}\left(\frac{4c_x \omega}{4k_x - M\omega^2}\right) \tag{39}$$

$$\dot{x} = \frac{dx}{dt} \qquad A_L \equiv \text{defined previously}$$

$$y = Y \sin(\omega t + A_L - \beta) \qquad \beta = \tan^{-1}\left(\frac{4c_y \omega}{4k_y - M\omega^2}\right) \tag{40}$$

$$\dot{y} = \frac{dy}{dt}$$

$$\theta_x = Ⓗ_x \sin(\omega t + A_\theta - \gamma) \qquad \gamma = \tan^{-1} \frac{2c\theta_x \omega}{2k\theta_x - I_x \omega^2} \tag{41}$$

$$\dot{\theta}_x = \frac{d\theta_x}{dt}$$

$$\theta_y = Ⓗ_y \cos(\omega t + A_\theta - \delta) \qquad \delta = \tan^{-1} \frac{2c\theta_y \omega}{2k\theta_y - I_y \omega^2} \tag{42}$$

$$\dot{\theta}_y = \frac{d\theta_y}{dt}$$

$$\theta_z = Ⓗ_z \cos(\omega t + A_L - \varepsilon) \qquad \varepsilon = \tan^{-1}\left(\frac{2c\theta_z \omega}{2k\theta_z - I_z \omega^2}\right) \tag{43}$$

A_L and A_θ defined from equations (24) and (25).

SUPPLEMENTARY EQUATIONS

In addition to the derived equations, some additional equations were needed to relate the mathematical model to the physical machine.

In the book Mechanical Springs by A. M. Wahl, springs are treated as elastic columns in two cases of interest. The first case deals with the lateral spring rate (p. 70) (see Figure 6).

k_x = lateral spring rate

$$k_x = \frac{10^6 d^4}{c_L nD\left(.204\ h_s^2 + .265\ D^2\right)} \qquad (44)$$

k_y is the vertical spring rate; the spring was designed for ($F_y = k_y Y$).

$$\frac{k_y}{k_x} = 1.44\ c_L\left(.204\ \frac{h_s^2}{D^2} + .265\right) \qquad (45)$$

These equations are for round wire springs where $E = 30 \times 10^6$ psi and $G = 11.5 \times 10^6$ psi or $E/G \approx 2.6$.

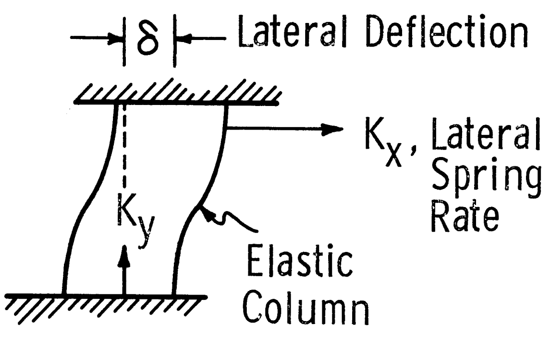


Figure 6

Observation of the experimental chassis on springs showed still another problem area. Certain springs are incapable of supporting the weight of the machine without buckling. This instability occurred if the spring was long or the spring rate, k, was small. The second case is where Wahl provides two equations to predict if a spring will buckle under a given load (pp. 279-82).

$$\frac{\delta cr}{\ell o} = .812\left[1 \pm \sqrt{1 - 6.87(D/\ell o)^2}\right]$$

$$Pcr = \frac{\delta cr}{\ell o}\left[\frac{G\, d^4 \ell o}{64\, r^3 n}\right] \qquad (46)$$

where G = modulus of rigidity (steel $\approx 11.5 \times 10^6$ psi)

ℓo = free length of spring

n = number of active coils

d = wire diameter

r = mean radius of coils, $D = 2r$

h_s = compressed length of spring

C_L = given by graph in Mechanical Springs, p. 74

Pcr = vertical load on springs that produces buckling

δcr = lateral deflection which produces bucling with load Pcr

Because coulomb damping is used more on washing machines than viscous damping, one needs an equivalent viscous damping coefficient developed from the coulomb damping factor. This exercise is carried out in Mechanical Vibrations by Tse, Morse, and Hinkle on p. 177. The result is:

$$Ceq = \frac{4Fk(1 - r^2)}{\omega\, \pi\, Fo \sqrt{1 - (4F/\pi\, Fo)^2}} \qquad (47)$$

where r = frequency ratio = $\omega/\omega n$; F is the frictional resisting force of the damper, and $Fo = me\omega^2$ for a rotating imbalance. Also e = eccentricity.

SUSPENSION DESIGN

With the mathematical model of the suspension complete, how does one use it? The pupose of this project originally was: Given the machine parameters, design a suspension that does not walk, does not transmit vibration to the floor, and deflects within allowable limits.

So a design procedure was formulated to try and do this. The first step was to try to show that for the given machine parameters there is a single function of the following form:

$$\text{function value} = \text{function (displacement, walking behavior)} \qquad (48)$$

This function value should have a maximum or minimum value when the best compromise is found between walking and deflection behavior. The damping coefficient, c, and spring constants, k, would be the variables.

Optimization routines generally take a complicated function and find the maximum or minimum points of the function. It was thought that by combining the walking equation and displacement equations directly, the result would be something of the form

$$\text{function value} = A + B(\text{deflection})^n + C(\text{walking behavior})^m$$

$$A,B,C = \text{constants} \quad m,n = \text{powers} \qquad (49)$$

and this could be optimized. The problem was how to find A,B,C,n, and m. This is when the problem of spring buckling was first noted. This would have to be included too. Formulating this single function seemed difficult, the task was given up and another method sought.

The next step was to consider optimizing the deflection alone and using walking behavior and stability equations as constraints. This would mean writing and optimization computer program from scratch, as no optimization computer programs I found could handle constraints.

This appears reasonable, until you realize that the deflection equations can be optimized by inspection. The larger the values of c and k (the operating point is above the natural frequency), the smaller the deflections. The result of optimizing deflection would be k and c values so large, that if they would be any larger, the machine would walk (see Figure 7). Washing machines are not to be designed on such "borderlines." You need a factor of safety.

Optimization was discarded as a design procedure. The only alternative was to admit defeat to the original objectives of the research. The following compromise can be made.

For any given machine parameters, determine the values of the spring constants and damping coefficients which will make the machine stable and will not let the machine walk. This amounts to describing the region labeled "solutions" in Figure 8.

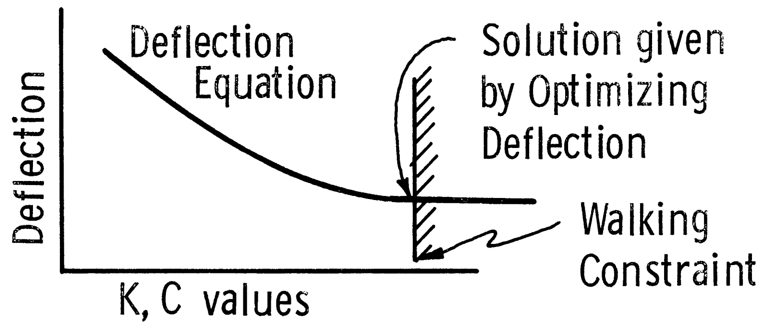


Figure 7

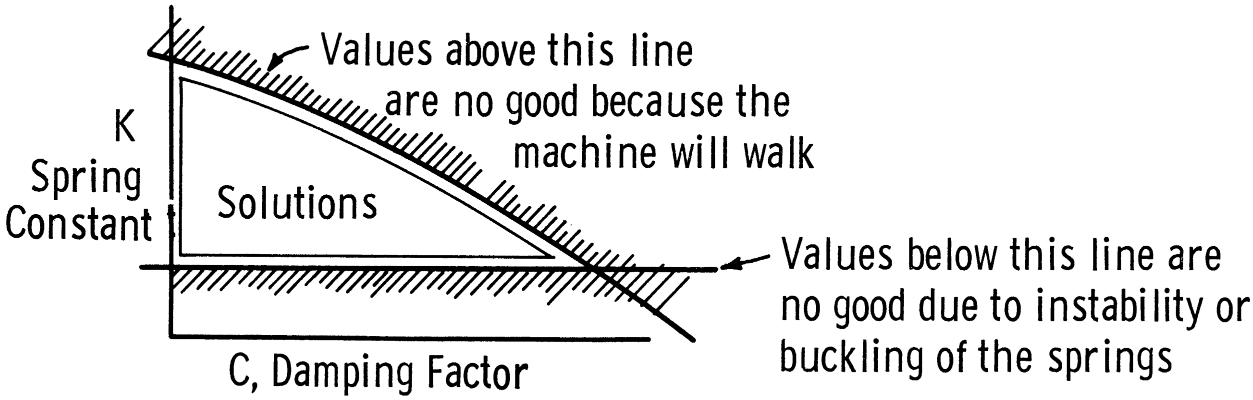


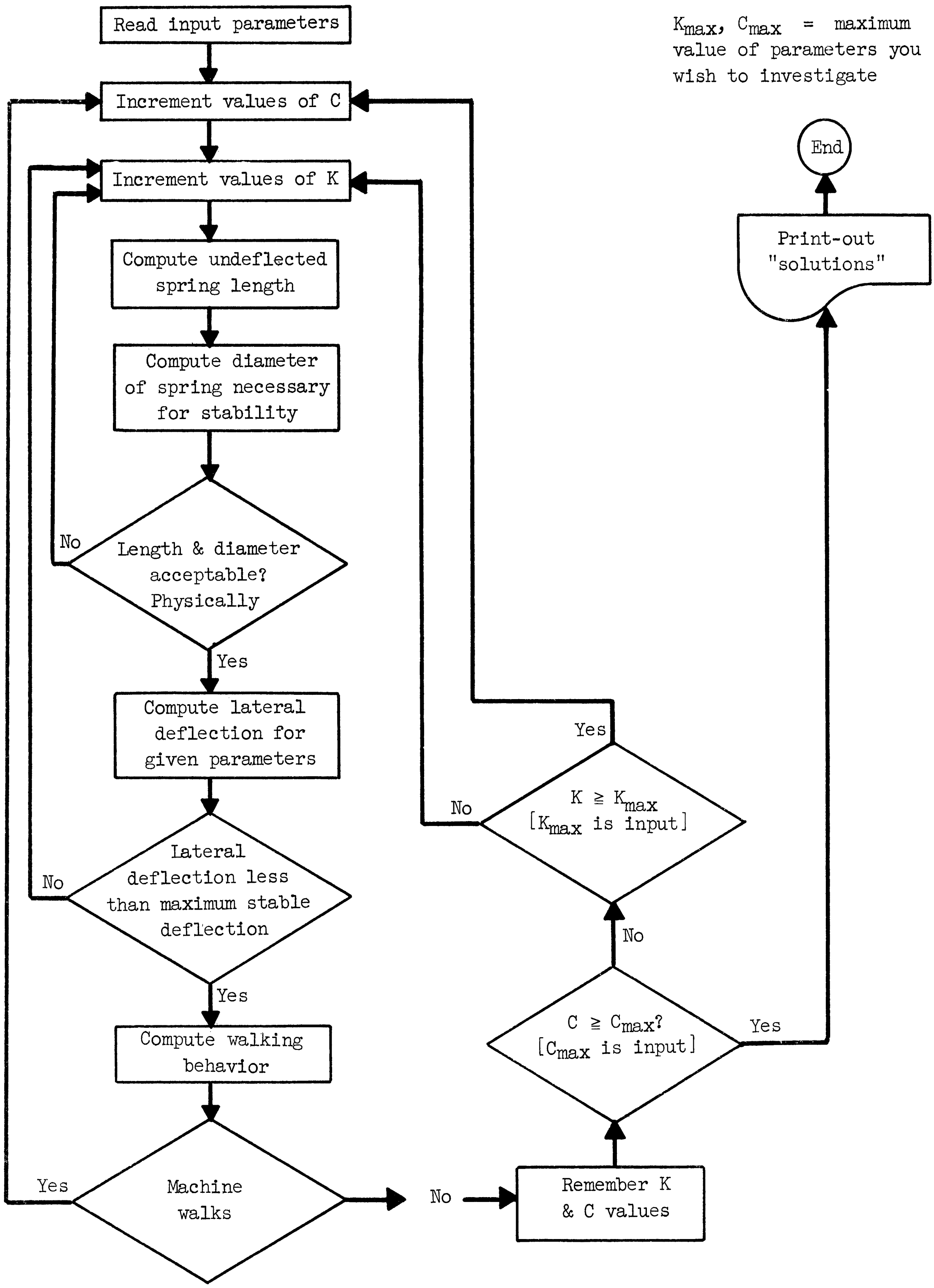
Figure 8

A close examination of the stability equations reveals that very soft springs, in general, will be unstable under the weight of the machine and, therefore, unsuitable. Examination of the walking equations (or force transmission curves of rotating imbalances—found in many dynamics textbooks) will show that when spring constant and damping coefficient combinations become large, the machine will walk. Using physical reasoning, if $k = \infty$ and $c = \infty$, i.e., no suspension at all, when a 4-lb rotating imbalance reaches 500 rpm, the centripetal load generated (approximately 300 lbf) could lift a 200-lb machine completely off the floor. This is the extreme case of walking.

If a designer of a washing machine were given the region labeled "solutions," he could pick the values he feels is the best comprmise between walking and instability and then see if the deflections with these k and c values are acceptable.

The procedure designed to produce the "solutions" region picks values of k and c in an orderly fashion, and tests the combination for walking behavior or instability with the machine parameters given. It can be changed into a computer program where the acceptable values are remembered and printed as output.

FLOW CHART


Read input parameters
Increment values of C
Increment values of K
Compute undeflected spring length
Compute diameter of spring necessary for stability
Length & diameter acceptable? Physically
No
Yes
Compute lateral deflection for given parameters
Lateral deflection less than maximum stable deflection
No
Yes
Compute walking behavior
Machine walks
Yes
No
Remember K & C values
C ≥ Cmax? [Cmax is input]
No
Yes
K ≥ Kmax [Kmax is input]
No
Yes
Print-out "solutions"
End
Kmax, Cmax = maximum value of parameters you wish to investigate

BASIC PROCEDURE

An explanation of the following routine follows it.

(1) Read $(M, g, \delta c, \delta k, \delta A, d, n, G, D, m, e, \omega, C_{max}, K_{max}, D_{max}, Lo_{max})$

(2) $C = -\delta C$

(3) $m = 0$

(4) $C = C + \delta C$

(5) $m = m + 1$

(6) $K = -\delta K$

(7) $N = 0$

(8) $K = K + \delta K$

(9) $N = N + 1$

(10) $\delta = Mg/K$

(11) $Lo_{MN} = (\delta + \delta A) + d \cdot n$

(12) $\delta cr/\ell o = Mg/K \; Lo_{MN}$

(13) $D_{MN} = Lo_{MN}((1 - ((\delta cr/\ell o)/.812 - 1)^2)/6.87)^{1/2}$

(14) If $(D_{max} - D_{MN})$ 8, 15, 15

(15) If $(Lo_{max} - L_{MN})$ 8, 16, 16

(16) If D_{MN} is complex, go to 8

(17) Choose value of C_L

(18) $h_s = Lo_{MN} - \delta$

(19) $K_x = 10^6 d^4 / \left(C_L \, n \, D \left(.204 \, h_s^2 + .265 \, D^2\right)\right)$

(20) $X = me\omega^2/((c\omega)^2 + (K - M\omega^2)^2)^{1/2}$

(21) $\delta cr = \delta cr/\ell o \cdot Lo_{MN}$

22 If $(\delta cr - X)$ 8, 8, 23

23 If $(-u(K_y y + C_y \dot{y} - Mg/4) - \{[K_x(x + \theta_y\ Lz/2 + \theta_z R_c)$
$+ C_x(\dot{x} + \dot{\theta}_y\ Lz/2 + \dot{\theta}_z R_c)]^2 + [K_z(\theta_x R_c + \theta_y\ Lx/2)$
$+ C_z(\dot{\theta}_x R_c + \dot{\theta}_y\ Lx/2)]^2\}^{1/2})$ 4, 24, 24

24 $C = C_{MN}$

25 $K = K_{MN}$

26 If $(C - C_{max})$ 27, 100, 100

27 If $(K_{max} - K)$ 4, 4, 8

100 Print Lo_{MN}, D_{MN}, C_{MN}, K_{MN} for all values which are solutions

101 End

where g = acceleration of gravity

δc = arbitrary increment of the damping coefficient

δk = arbitrary increment of the spring constant

δA = expected maximum deflection; δA may require a separate equation to evaluate it if real accuracy is to be obtained

e = radius of rotation of imbalance

C_{max} = maximum range of C you want to evaluate

K_{max} = maximum range of K you want to evaluate

D_{max} = maximum acceptable spring diameter

Lo_{max} = maximum acceptable free spring length

Lines (1) through (9) read the data, increment C and K, and increment subscript indices M and N.

Lines (10) and (11) define the free length of the spring.

Lines (12) and (13) define the stable diameter of the spring.

Lines (14) through (17) check to see if diameters and lengths of spring are acceptable.

Lines (17) through (20) compute expected lateral deflection.

Lines (21) and (22) check to see that lateral deflection does not exceed the critical buckling deflection of the spring.

Line (23) insures machine does not walk.

Lines (24) and (25) index C and K to be remembered.

Lines (26) and (27) check whether C_{max} or K_{max} have been exceeded.

EXPERIMENT

Due to the many assumptions made to arrive at the final form of the deflection equations, it seemed appropriate to seek correlation with the actual deflection of an experimental machine. A chassis with motor and drum was obtained and mounted on springs. Two sets of springs were obtained; one set having a large spring constant (86 lb/in. for each of four springs) and the other set having a small spring constant (8 lb/in. for each spring). Deflections were to be measured at angular drum velocities lower than and exceeding the natural frequency of the washing machine mass on the springs. The 8 lb/in. springs, however, were unstable (which prompted the search for a stability routine) and had poor fasteners at each end due to the large diameter of the spring. Tests were conducted only with the 86 lb/in. springs below the natural frequency.

The simple case of one imbalance in the axially centered plane of the drum was tested ($m_2 = 0$, $r_3 = 0$). Then the x and y displacements were measured. Displacement was measured with a load cell; an amplifier bridge circuit boosted the signal; and the displacement was drawn on a strip chart recorder.

Because the deflection readings are so small, it is difficult to obtain accurate data. Barring some basic misunderstanding of the experiment, it is estimated the recorded data should not be more than 10% in error. The following were the machine parameters:

Drum radius = 12.5 in.

m = 2.1 lbm

M = 92 lbm

k = 86 lb/in. per spring (4 springs),

(lateral spring rate, K_x, was calculated)

c = 0

The experimental data:

ω	1.7 cycles/sec	2.1 cycles/sec
y	.0346 in.	.062 in.
x	.0825 in.	.135 in.

Calculated values for these conditions:

ω	1.7 cycles/sec	2.1 cycles/sec
y	.0234 in.	.04 in.
x	.0445 in.	.079 in.

The calculated values are about 61% of the measured values on the average. I discovered that the weight distribution on each side of the drum axis measured at the springs was 44 lb on one side and 48 lb on the other. This means that the center of mass of the machine is not on the vertical centerline of the machine, a basic assumption of the derived equations. The machine was balanced and the experiment repeated. M = 100 lbm. The experimental data:

ω	1.57 cycles/sec	1.5 cycles/sec	2.12 cycles/sec
y	.0299 in.	---	.0543 in.
x	---	.0469 in.	.115 in.

The calculated values:

ω	1.57 cycles/sec	1.5 cycles/sec	2.12 cycles/sec
y	.0208 in.	---	.0406 in.
x	---	.0356 in.	.0816 in.

The calculated values are now 73% of the measured values on the average. The small center of mass offset cost about 12% of the accuracy.

Better agreement would be welcome, but it does suggest that the displacement equations of the type derived should be checked at drum rotation velocities above the natural frequencies. The equations derived may be suitable, however, for rough estimates of spring constants and dampers.

The lateral spring constant and stability equations were checked. The lateral spring rate was computed to be 287 lb/in. (for all four springs together). It was difficult to measure the lateral spring rate on the test machine and recorded values fluctuated between 200 and 280 lb/in.

For the weight of the machine, the stability equations predicted that springs of .188 in. diameter round wire ($G = 20 \times 10^6$ psi) should be 10.9 in. in free length. The 8 lb/in. springs were 12 in. long and unstable. These springs were cut down to 7-1/2 in. and were very stable. The lateral spring constant equations and stability equations are, therefore, considered to be fairly accurate.

Obtaining data for evaluating the walking equation's accuracy was difficult and time would not permit this experiment. The mathematical model appears good enough to get a "ball park" value of springs and dampers. It is suggested that additional testing be carried out, especially for drum rotation velocities above the natural frequency.

CONCLUSIONS

The problems associated with spring suspensions applied to horizontal axis washing machines are: deflection control; load transmission (walking); and stability under the machine weight. Finding a spring-damper combination which will satisfy all three of these conditions simultaneously may be possible, but difficult. An alternative is to satisfy the walking and stability conditions, and see if the resulting deflection of the machine can be accommodated. If the deflection is too large, some of the machine parameters will have to be changed.

Changing the parameters to reduce deflection is an intuitive operation, and the following examples should help illustrate how it is done ($\omega > \omega_n$).

1. From the walking equation (38), as R_c becomes smaller it is possible to use stiffer springs without the machine walking. This means the center of gravity should be as low as possible.

2. From the deflection equation (37), if R_y is made small, the angular θ_z deflections hould be reduced. In other words, the drum axis should be close to the center of mass.

3. From the deflection equations in general (32)-(37), increasing the sprung mass of the machine will reduce deflections once beyond the natural frequency.

Choosing spring rates and damping coefficients can be done using the routine given in this report. The values received should be rough approximations due to the difficulty of getting an exact mathematical model of the machine. A more accurate mathematical model may be obtained using the coupled differential equations given by equations (9)-(14) or one may start from scratch and derive his own equations. These equations should require more detailed development and solution by computer.

It is felt that further testing of the deflection equations should be done, especially for drum rotation velocities above the natural frequencies of the system. The importance of having the center of mass on the vertical centerline of the machine, equations (22), should be emphasized also.

Anticipating problems, the start-up phase, where drum rotation velocity goes from 0 to 50 rad/sec, will be difficult. Some informal testing was done with the experimental machine. It was found that excessive deflections resulted when trying to go through the natural frequency at a moderate pace. When the machine was rushed through the natural frequency, the d'Alembert reaction torque ($J\ddot{\phi}$) on the drum caused the machine to "dance" around the floor. A separate device, such as a dual spring rate or damping coefficient, may be needed to cope with this problem.

THE BELLEVILLE SPRING SUSPENSION

Belleville springs offer an unusual load-deflection relationship which might be exploited in this situation. The curve in Figure 9 is the one of interest in washing machine suspensions.

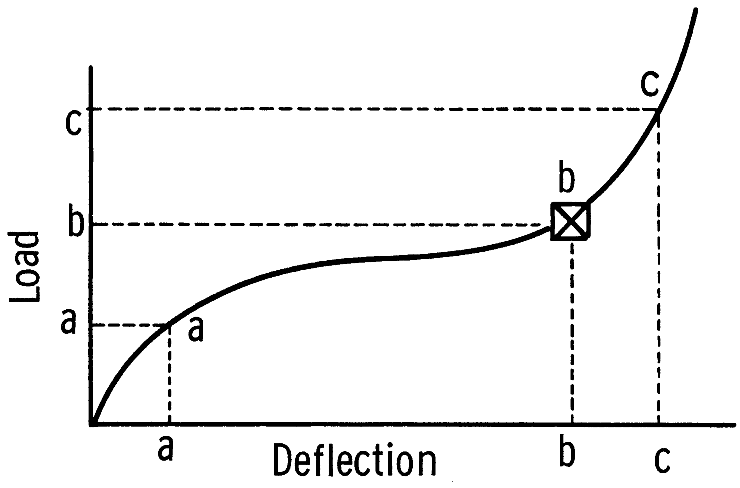


Figure 9

A brief study was done into Belleville spring design, and it is considered possible to design a spring with the following properties. The weight of the machine will load the spring to the static deflection located at b in Figure 9. With the rotating unbalance spinning at frequencies above the natural frequency of the system, the machine can deflect from b to a when it is lifting up on the suspension. This is the moment when the machine will walk with present suspensions. The Belleville spring, however, has a very "soft" spring rate from b to a; there is a lot of deflection allowed without much changing of the normal force on the feet of the machine. This will help keep the machine with Belleville springs from walking by maintaining a high frictional force on the floor while the machine is deflecting the suspension.

When the machine is traveling downward on the suspension, the Belleville spring deflects from b to c. The spring rate from b to c is much "stiffer" than a to b. This will help keep deflections small, but also will help keep the machine stable. From the stability equations already presented, it can be seen, in general, that larger spring rates are more stable than softer ones for a given diameter and free length of the spring.

The shape of the Belleville spring also enhances the stability. It is a slightly conical circular disk with a hole in the center (see Figure 10). The load is applied on the circumference of the hole on the inside. This shape cannot buckle as coil springs do. It is also a very compact spring. A rough guess for the size spring needed to support a washing machine might be 5 in. in diameter and 1/2 in. tall.

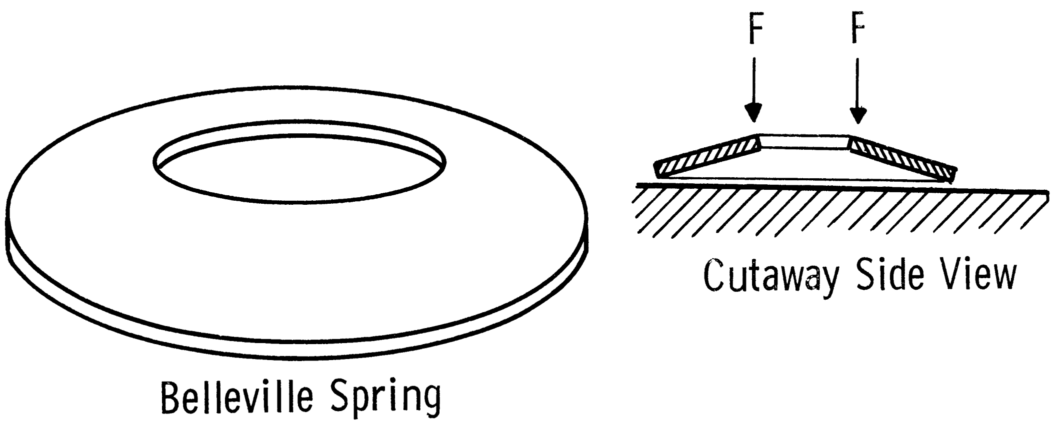


Figure 10

There is a small amount of damping action associated with deflection of a Belleville spring, especially when they are stacked on top of each other. It is also suspected that since the spring rate changes so drastically as the spring deflects, a Belleville spring will not have a true natural frequency. The instantaneous natural frequency will fluctuate over large ranges as the spring constant changes with deflection. This should aid in keeping deflections reasonable when running up through the natural frequency to the operating speed of the machine.

In summary, a properly designed Belleville spring offers many advantages over coil springs. Although no actual designing of the spring was done, the design parameters are considered flexible enough to allow design of the spring to produce these davantages.