A01-25292

42nd AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference and Exhibit
Seattle, WA 16-19 April 2001

AIAA-2001-1574

Compressive Splitting Failure in Fiber Reinforced Unidirectional Composites using Modified Shear Lag Theory

Chandra S. Yerramalli and Anthony M. Waas*
Composite Structures Laboratory,
Department of Aerospace Engineering,
University of Michigan, Ann Arbor, MI 48109-2140

Abstract

Compressive failure by splitting is studied herein via an analytical model. The use of a shear lag model to determine the stress state at the crack tip and the modeling of the region away from the crack tip by the 3D elasticity equations leads to a simple analytical expression is used to determine the compliance changes for both unsteady crack growth as well as steady state crack growth under compressive loading. Certain modifications to the assumptions used in the classical shear lag model have been made to increase the accuracy of the predictions for the rate of change of compliance with respect to crack length, $\frac{dc}{dl}$. The present approach leads to closed form expressions for the compressive strength of unidirectional fiber reinforced composites.

1 Introduction

Previous experimental work [1, 2] has shown splitting to be the dominant mode of failure in glass/vinyl ester unidirectional composites . Optical photomicrographs show a clean fracture between the fiber and matrix indicating that splitting failure is akin to interfacial fiber/matrix fracture. Lee and Waas [1] have developed expressions for the compressive strength of polymer matrix composites using linear elastic fracture mechanics and the assumption of steady state crack propagation. The stress analysis included only the areas in the cracked and uncracked regions and excluded a small region of size ϵ around the crack tip. In this region the stress state is a function of the crack tip field. When the crack is propagating under steady state conditions, this region translates with the crack tip. Thus, the rate of change of compliance with crack length is unaffected. The expression for the rate of change of compliance $\frac{dc}{dl}$ is also independent of the crack length or the initial fiber length. For short cracks, where the crack propagation is unsteady, Lee and Waas [1] used the finite element method to extract the dependency of $\frac{dc}{dl}$ on crack length. In the present work a modified shear lag model has been used to study the stress state at the crack tip. The local shear lag based stress field has been superposed on the far field stress state of the composite obtained from the 3D elasticity equations, to obtain expressions for compliance and compliance change as a function of crack length. The use of this method helps in developing an integrated expression for the compressive strength in terms of the fracture toughness of the material, which can be used for both short cracks as well as long cracks.

2 Stress Analysis

Consider a representative volume element (RVE) of the composite containing a single fiber of length 2L with a crack of length 2l embedded in it as shown in Figure 1. The single fiber is divided into four regions for the purpose of getting the expression for compliance and the rate of change of compliance. By symmetry, only one side of the fiber ($0 < z < L$), containing the crack region ($0 < z < l$) is modeled. Assume that a region ϵ extends from the crack tip in the positive Z axis direction as well as in the negative Z axis direction. The region extending beyond ϵ is

*Graduate Research Assistant and Professor of Aerospace Engineering, respectively, University of Michigan. Assoc. Fellow AIAA, Copyright ©2001 by Anthony M. Waas. Published by the AIAA, with permission.

modeled as in the case of the steady state splitting model as shown in Lee and Waas[1].

2.1 Shear Lag Analysis

The main assumptions in the traditional shear lag model of Cox [3], are that the matrix carries only shear and any axial straining in the matrix is only for the purpose of load introduction. The fiber can only undergo axial contraction or extension and the fiber axial stress is zero at the fiber ends. The classical shear lag method based on the above assumptions leads to a reasonably accurate calculation of the compliance of a cracked fiber/matrix system. But the rate of change of compliance with crack length, $\frac{dc}{dl}$ turns out to be inaccurate. This in turn leads to the inaccurate calculation of strain energy release rate. Nairn [4] has indicated the limitations of using classical shear lag methods for strain energy release rate calculations. Time dependent effects in the matrix are captured in a non-linear viscoelastic shear lag model by Thuruthimatttam, Waas and Wineman [5]. For the present work, we assume that the fiber ends do carry some load and that the shear lag method is valid over the small region extending from $l-\epsilon$ to $l+\epsilon$. These modifications are introduced into the shear lag model by ensuring that the fiber axial stress at z = $l+\epsilon$ matches with the fiber axial stress of the uncracked region calculated using the 3D equations of elasticity. Also, at z = $l-\epsilon$ the fiber axial stress is equated with the steady state splitting model stress in the crack region, which is similar to what is obtained via a simple rule of mixtures based stress prediction.

Taking a small segment dz of the composite containing the single fiber as shown in Figure 2, we can get the radial variation of shear stress in the matrix ,$\hat{\tau}$ by equating the shear forces on neighboring annuli with radii r_1 and r_2 of length dz. Then,

$$2\pi r_1 \hat{\tau}_1 dz = 2\pi r_2 \hat{\tau}_2 dz$$

From the above equation we get the relation between $\hat{\tau}_1$ and $\hat{\tau}_2$ as $\frac{\hat{\tau}_1}{\hat{\tau}_2}=\frac{r_2}{r_1}$. Thus at any radius r, we can relate the shear stress $\hat{\tau}(r)$ to the interface shear stress $\hat{\tau}_i$.

To obtain the relation between shear strain and shear stress, consider the displacement $\hat{u}_r(z)$ of the matrix with respect to the unstressed position. Then,

$$\frac{d\hat{u}}{dr} = \gamma = \frac{\hat{\tau}_r}{G_m} = \frac{\hat{\tau}_i}{G_m}(\frac{r_0}{r}) \text{ or}$$

$$d\hat{u} = \frac{\hat{\tau}_i}{G_m}(\frac{r_0}{r})dr$$

Integrating the above equation between r=r_0 and r=R, we obtain

$$\int_{\hat{u}_r=(r_0)}^{\hat{u}_r=(R)} d\hat{u} = \frac{\hat{\tau}_i r_0}{G_m} ln(\frac{R}{r_0}) = (\hat{u}_R - \hat{u}_{r_0})$$

where $\hat{u}_R$ is the matrix displacement at a distance R from the fiber and $\hat{u}_{r_0}$ is the matrix displacement at interface $r = r_0$. The value of R is based on the assumption that the matrix strain is uniform, remote from the fiber-matrix interface. Thus the appropriate value of R is dictated by the proximity of fibers which in turn depends on the fiber packing and fiber volume fraction V_f. Assuming a hexagonal packing of fibers and also taking note of the fact that the ratio R/r_0 appears as a logarthmic term and is relatively insensitive to the details of geometry, we can write the following expression relating fiber volume fraction V_f and the ratio R/r_0.

$$(\frac{R}{r_0})^2 \approx \frac{1}{V_f}$$

From Figure 2b, by equating the forces acting along the axial direction of the fiber we get the relation between the axial stress in the fiber and the interfacial shear stress.

$$2\pi r_0 \hat{\tau}_i dz = -\pi r_0^2 d\hat{\sigma}_f$$
$$\frac{d\hat{\sigma}_f}{dz} = \frac{-2\hat{\tau}_i(z)}{r_0}$$

Combining the above expression with the expression for interfacial shear stress $\hat{\tau}_i$ in terms of the displacements we get

$$\frac{d\hat{\sigma}_f}{dz} = \frac{-2E_m}{(1+\nu_m)} \frac{(\hat{u}_R - \hat{u}_{r_0})}{r_0^2 ln(1/V_f)} \quad (1)$$

differentiating Equation(1), we get

$$\begin{aligned} \frac{d^2\hat{\sigma}_f}{dz^2} &= \frac{-2E_m}{(1+\nu_m)} \\ &\quad \frac{(\hat{u}_R - \hat{u}_{r_0})}{r_0^2 ln(1/V_f)}(\frac{d\hat{u}_R}{dz} - \frac{d\hat{u}_{r_0}}{dz}) \\ &= \frac{\eta^2}{r_0^2}(\hat{\sigma}_f - \hat{\epsilon}_m E_f) \quad (2) \end{aligned}$$

where the following substitutions have been made

$$\frac{d\hat{u}}{dz}\Big|_{r=r_0} = \hat{\epsilon}_f = \frac{\hat{\sigma}_f}{E_f}$$

$$\frac{d\hat{u}}{dz}\Big|_{r=r_R} = \hat{\epsilon}_m$$
$$\eta^2 = \frac{2E_m}{E_f(1+\nu_m)ln(1/V_f)}$$

Equation(2) is a second order ordinary differential equation, whose solution is

$$\hat{\sigma}_f(z) = E_f\hat{\epsilon}_m + B\sinh(\frac{\eta z}{r_0}) + D\cosh(\frac{\eta z}{r_0}) \quad (3)$$

2.2 Splitting Model - Shear Lag Method

In this section we present the details of the application of the shear lag model to the splitting model analysis of Lee and Waas [1]. For this purpose, the RVE can be divided into two regions - the crack region and the uncracked region.

2.2.1 Crack Region

In this region it is reasonable to assume that the fiber axial stress is equal to that of the axial stress obtained from the splitting model with increasing distance from the crack tip. Also in the crack region according to the shear lag model there is no shear stress hence the fiber axial stress is constant. Thus,

$$\frac{d\hat{\sigma}_{f(1)}}{dz} = 0$$
$$\Rightarrow \hat{\sigma}_{f(1)} = A$$
$$\Rightarrow \hat{\epsilon}_{f(1)} = \frac{A}{E_f}$$

Integrating the expression for fiber axial strain we get $\hat{u}_{f(1)} = \frac{A}{E_f}z$ where the constant of integration has been taken to be zero since it represents a rigid body translation. The expressions for the stress and displacement field from the splitting model for the crack region are as given in Lee and Waas [1].

$$\hat{\sigma}_{f2D} = \frac{PE_f}{\pi r_0^2\delta}$$

from which we can obtain

$$\hat{u}_{f2D} = \frac{P}{\pi r_0^2\delta}z$$

2.2.2 Uncracked Region

In the uncracked region, we can take the expression of normal fiber stress as given by Equation(3). However in the present analysis this expression is valid in the region $l < z < l+\epsilon$. For $z > l+\epsilon$, the normal stress expression corresponding to the 3D equations of elasticity, is taken from the splitting model. Thus, at $z = l+\epsilon$, we equate the fiber normal stress $\hat{\sigma}_{f(2)}$ obtained from the shear lag analysis with that of the fiber normal stress obtained from the splitting model. Also, at the interface between the crack and uncracked region continuity of fiber normal stress and axial displacements is enforced. The expression for normal fiber strain can be obtained from Equation(3) and is as follows.

$$\hat{\epsilon}_{f(2)} = \hat{\epsilon}_m + \frac{B}{E_f}\sinh(\frac{\eta z}{r_0}) + \frac{D}{E_f}\cosh(\frac{\eta z}{r_o})$$

On integrating the above, we get

$$\hat{u}_{f(2)} = \hat{\epsilon}_m z + \frac{B}{E_f}\frac{r_0}{\eta}\cosh(\frac{\eta z}{r_0}) + \frac{D}{E_f}\frac{r_0}{\eta}\sinh(\frac{\eta z}{r_0}) + C2$$

Similarly an expression for the displacement field in the region beyond $z = l+\epsilon$ can be obtained from the 3D splitting model analysis and is as follows

$$u_{3d} = \frac{\beta P}{\pi r_0^2}z + C2$$

The expression for the stress σ_{3d} is as follows [1],

$$\sigma_{3d} = \frac{\beta P}{\pi r_0^2}(E_m - 4\alpha\nu_m(\nu_f - \nu_m))$$

The following boundary conditions can be written for the present boundary value problem.

Crack Region

at $z = l-\epsilon$

$$\hat{\sigma}_{f(1)} = \sigma_{f2D}$$
$$\Rightarrow A = \frac{PE_f}{\pi r_0^2\delta}$$

and hence

$$\hat{u}_{f(1)} = \frac{P}{\pi r_0^2\delta}(l-\epsilon)$$

Further, at $z = l$

$$\hat{u}_{f(1)} = \hat{u}_{f(2)}$$

From the condition for continuity of displacement across the crack interface the constant C2 is evaluated which is then used to get the displacement field

$\hat{u}_{f(2)}$ to be,

$$\begin{aligned} \hat{u}_{f(2)} = & \frac{P}{\pi r_0^2 \delta} + \\ & \frac{B}{E_f}\frac{r_0}{\eta}[\cosh(\frac{\eta z}{r_0}) - \cosh(\frac{\eta l}{r_0})] + \\ & \frac{D}{E_f}\frac{r_0}{\eta}[\sinh(\frac{\eta z}{r_0}) - \sinh(\frac{\eta l}{r_0})] \end{aligned} \quad (4)$$

At $z = l$, the continuity of normal stresses provides

$$\hat{\sigma}_{f(1)} = \hat{\sigma}_{f(2)}$$

$$\frac{PE_f}{\pi r_0^2 \delta} = E_f \epsilon_m + B\sinh(\frac{\eta l}{r_0}) + Dcosh(\frac{\eta l}{r_0}) \quad (5)$$

Uncracked Region

$$\text{at } z = l + \epsilon$$

$\hat{\sigma}_{f(2)} = \sigma_{f3D}$, which translates to

$$\begin{aligned} & E_f \epsilon_m + B\sinh(\frac{\eta(l+\epsilon)}{r_0}) \\ & +D\cosh(\frac{\eta(l+\epsilon)}{r_0}) = \\ & \frac{\beta P}{\pi r_0^2}[E_m - 4\alpha\nu_m(\nu_f - \nu_m)] \end{aligned} \quad (6)$$

To ensure compatibility, we equate the displacements $u_{f(2)}$ and u_{f3D}. This provides,

$$\hat{u}_{f(2)} = u_{f3D}$$

$$\begin{aligned} \epsilon_m(l+\epsilon) + & \frac{B}{E_f}\frac{r_0}{\eta}\cosh(\frac{\eta(l+\epsilon)}{r_0}) + \\ & \frac{D}{E_f}\frac{r_0}{\eta}\sinh(\frac{\eta(l+\epsilon)}{r_0}) = \\ & \frac{\beta P(l+\epsilon)}{\pi r_0^2} + C3 \end{aligned} \quad (7)$$

Equation(7) enables to obtain the constant C3 in terms of the remaining constants B and D. The constants B and D can be evaluated from the two Equations(5) and (6). Knowing B and D, C3 and C2 can be determined. Expressions for these constants and expression for β, α and δ are given in the appendix.

The total axial displacement can be written in terms of the integrals of the fiber axial strains as follows.

$$\begin{aligned} \Delta = & \underbrace{2\int_0^l \hat{\epsilon}_{f(1)} dz}_{CrackRegion(1)} \\ + & \overbrace{2\int_l^{(l+\epsilon)} \hat{\epsilon}_{f(2)} dz + 2\int_{(l+\epsilon)}^L \epsilon_{f3D} dz}^{UncrackedRegion(2)} \end{aligned}$$

On substituting the expressions for the axial strains in the above integrals results in,

$$\begin{aligned} \Delta = & 2\int_0^l \frac{P}{\pi r_0^2 \delta} dz + \\ & 2\int_l^{(l+\epsilon)} [\epsilon_m + \frac{B}{E_f}\sinh(\frac{\eta z}{r_0}) \\ + & \frac{D}{E_f}\cosh(\frac{\eta z}{r_0})]dz + \\ & 2\int_{l+\epsilon}^L [\frac{\beta P}{\pi r_0^2}]dz \end{aligned} \quad (8)$$

We can observe from the above equations for total displacement Δ that as $\epsilon \to 0$ (meaning the region ϵ around the crack tip vanishes), the second integral vanishes and also the third reduces to the same expression as given in Lee and Waas [1] giving us the displacement expression obtained with steady state crack propagation assumption.

3 Strain Energy Release Rate

The total potential energy Π of the RVE under consideration, when applied with a compressive load, P is $U - W$, where U is the strain energy stored in the RVE and W is the work done. The expression for strain energy release rate is

$$G = \frac{d\Pi}{dA}$$

where $A = 4\pi r_0 l$ is the crack surface area and r_0 is the fiber radius. The overall compliance of the RVE, c is defined as;

$$c = \frac{\Delta}{P}$$

where Δ is the axial compressive displacement of the composite and P is the external compressive load. The expression for Δ is substituted from Equation(8). For either case of load control or displacement control, the strain energy release rate can be written as

$$G = \frac{P^2}{8\pi r_0}\frac{dc}{dl}$$

The fracture toughness γ_f is half of the value of G at the time of initiation of crack propagation.

$$G = 2\gamma_f$$

The compressive stress σ_c can be related to the fracture toughness of the material γ_f by the following the expression

$$\sigma_c = \sqrt{\left(\frac{16\gamma_f V_f^2}{\pi r_f^3}\frac{dc}{dl}\right)} \tag{9}$$

4 Solution

The expression for Δ in the form of the integral given in Equation(8) is evaluated to get the displacement.

$$\begin{aligned}\Delta &= 2\Big\{\frac{Pl}{\pi r_o^2\delta} + \frac{P\epsilon}{\pi r_o^2\delta} \\ &+ \Big\{\frac{B}{E_f}[\cosh(\frac{(l+\epsilon)\eta}{r_o}) - \cosh(\frac{l\eta}{r_o})] \\ &+ \frac{D}{E_f}[\sinh(\frac{(l+\epsilon)\eta}{r_o}) - \sinh(\frac{l\eta}{r_o})]\Big\}\frac{r_o}{\eta} \\ &+ \frac{P\beta(L-l-\epsilon)}{\pi r_o^2}\Big\}\end{aligned} \tag{10}$$

From Equation(10), we get the relation for compliance c by dividing the above equation by the load term P.

$$c = \frac{\Delta}{P}$$

$$\begin{aligned}c &= 2\Big\{\frac{l}{\pi r_o^2\delta} + \frac{\epsilon}{\pi r_o^2\delta} \\ &+ \Big\{\frac{B}{E_f}[\cosh(\frac{(l+\epsilon)\eta}{r_o}) - \cosh(\frac{l\eta}{r_o})] \\ &+ \frac{D}{E_f}[\sinh(\frac{(l+\epsilon)\eta}{r_o}) - \sinh(\frac{l\eta}{r_o})]\Big\}\frac{r_o}{\eta} \\ &+ \frac{\beta(L-l-\epsilon)}{\pi r_o^2}\Big\}\end{aligned} \tag{11}$$

Equation(11), is an expression for compliance of the system in terms of the crack length, l, and crack tip influence zone ϵ. The rate of change of compliance, $\frac{dc}{dl}$ is obtained by differentiating Equation(11) with respect to the crack length, l.

$$\begin{aligned}\frac{dc}{dl} &= \frac{2}{\pi r_o^2}\Big[\frac{1}{\delta} - \beta\Big] \\ &+ \Big[\frac{r_o}{\eta E_f}\Big\{(\frac{dB}{dl} + D)(\cosh(\frac{(l+\epsilon)\eta}{r_o}) \\ &- \cosh(\frac{l\eta}{r_o})) \\ &+ (\frac{dD}{dl} + B)(\sinh(\frac{(l+\epsilon)\eta}{r_o}) \\ &- \sinh(\frac{l\eta}{r_o}))\Big\}\Big]\end{aligned} \tag{12}$$

It can be seen in the above equation that the first term corresponds to the steady state behavior and the second term, which contains the crack length and the crack tip influence zone ϵ, is due to the shear lag model. The constants B and D in the above expression are functions of crack length l and are given in the appendix. For determining the rate of change of compliance with crack length, the previously described expressions were coded in MAPLE (symbolic math package) and evaluated for a glass/epoxy composite system as given in Lee and Waas [1]. For the ease of calculation the total length of fiber L, crack length l and the crack tip influence zone ϵ were all expressed as a factor of the fiber radius r_o. Plots of compliance and rate of change of compliance were obtained as a function of the crack length factor $n1$. The following material properties of the glass fiber and epoxy matrix were used in the analysis.

Glass Fiber

$$\begin{aligned}r_o &= 0.012mm \\ E_f &= 72000MPa \\ \nu_f &= 0.22\end{aligned}$$

Vinyl-ester Resin

$$\begin{aligned}E_m &= 3585MPa \\ \nu_m &= 0.36\end{aligned}$$

5 Discussion and Conclusions

As the equation for the $\frac{dc}{dl}$ indicates, compliance change is no longer independent of the crack length l or the crack tip influence zone parameter ϵ. However, the value of ϵ is initially unknown since the crack tip zone advances with the crack and also the region of influence does not remain constant. Thus the first step in evaluating the compliance changes would be to determine the value of ϵ. It has to be kept in mind that the value of ϵ cannot exceed the crack length l and at the same time ϵ should be smaller than L -l. As shown in figure 3, the compliance of the fiber/matrix system is plotted as a function of the crack length paramater for different values of ϵ. For a particular ϵ we can see that the compliance vs crack length plots are straight lines. If we mark the lower tip of the straight lines with black dots as shown in figure 3, we can observe that the slope of a imaginary line joining these points is initially varying and then becomes a constant after some crack length paramater nl indicating the steady state crack growth region. Similarly, for the compliance change $\frac{dc}{dl}$, the curve of $\frac{dc}{dl}$ vs ϵ as a function of n becomes nearly flat beyond $\epsilon \geq 1.5r_o$ as seen in figure 4. The value of rate of change of compliance obtained with the present integrated approach using shear lag model is found to be 3.59715 X 10^-5 $(N/mm)^-1$ which is 0.001% less than the value obtained from the steady state analysis, 3.597186 X 10^-5 $(N/mm)^-1$ given in Lee and Waas [1]. This value of $\frac{dc}{dl}$ was obtained for a ϵ of 0.024mm and crack length, l of 0.096mm. The total length of fiber considered was about $40r_o$. As seen in figure 4, for this crack length and ϵ, the $\frac{dc}{dl}$ value has reached its asymptotic value. Thus the present analysis incorporating the shear lag model to account for the crack tip stress state provides a simple analytical approach to study both unsteady crack growth of short cracks and steady state crack behavior as crack length increases.

6 ACKNOWLEDGMENT

The authors are grateful to the Army Research Office for supporting this research. Dr. Mohammed Zikry is the ARO scientific monitor.

7 Appendix

$$\delta = E_f + E_m(\frac{1}{V_f} - 1)$$

$$\begin{aligned}
\alpha &= [\frac{2(1+\nu_f)(1-2\nu_f)}{E_f}(V_f^{-1} - 1) \\
&+ \frac{2(1+\nu_m)(1-2\nu_m + V_f^{-1})}{E_m}]^{-1} \\
\beta &= [E_f + (V_f^{-1} - 1) \\
&\quad \{E_m + 4\alpha(\nu_f - \nu_m)^2\}]^{-1}
\end{aligned}$$

$$\begin{aligned}
D &= \frac{\theta}{\left[\frac{\tau\psi}{\gamma} - \phi\right]} \\
B &= \frac{-\tau\theta}{\gamma\left[\frac{\tau\psi}{\gamma} - \phi\right]} \\
\gamma &= \sinh(\frac{\eta l}{r_o}) \\
\tau &= \cosh(\frac{\eta l}{r_o}) \\
\psi &= \sinh(\frac{\eta(l+\epsilon)}{r_o}) \\
\phi &= \cosh(\frac{\eta(l+\epsilon)}{r_o}) \\
\theta &= \frac{P}{\pi r_o^2}\Big[\frac{E_f}{\delta} - \beta \\
&\quad \Big[E_f + 4\alpha\nu_f(nu_f - nu_m)(\frac{1}{V_f} - 1)\Big]\Big]
\end{aligned}$$

References

[1] Lee, S. H. and Waas, A. M., "Compressive Response of Fiber Reinforced Unidirectional Composites", International Journal of Fracture, Vol.100, 1999, pp.275-306.

[2] Oguni, K and Ravichandran, G., "An energy-based model of longitudinal splitting in unidirectional fiber-reinforced composites",Journal of Applied Mechanics, Vol.67, 2000,pp. 437-443.

[3] Cox, H.L., "The elasticity and strength of paper on other fibrous materials",Br.J.Appl.Phys, Vol.3, pp.72.

[4] Nairn, J.A., "On the use of shear-lag methods for analysis of stress transfer in unidirectional composites",Mechanics of Materials, Vol.26,1997,pp.63-80.

[5] Thuruthimattam,B.J., Waas,A.M. and Wineman,A.S.,"Stress transfer modeling in viscoelastic polymer matrix composites," International Journal of Non-Linear Mechanics, Vol.36, 2001,pp.69-87.

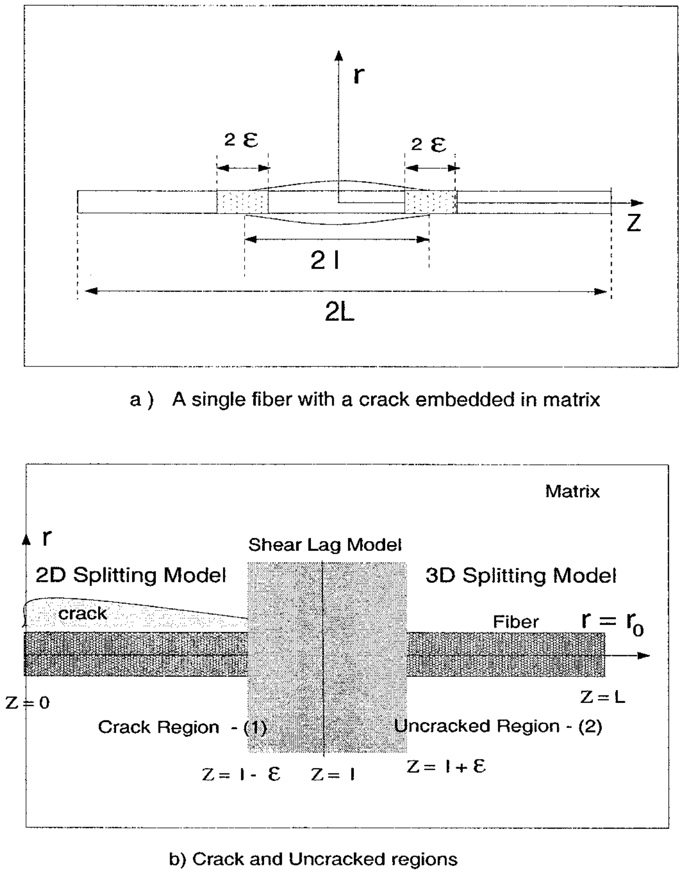

Figure 1: RVE showing the various regions of analysis

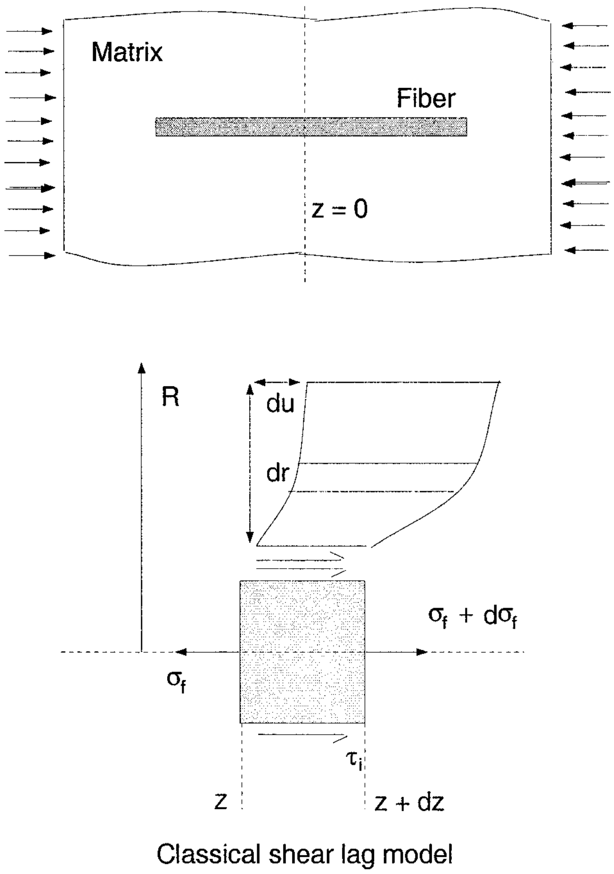

Figure 2: Free body diagram of a small segment of fiber and attached matrix

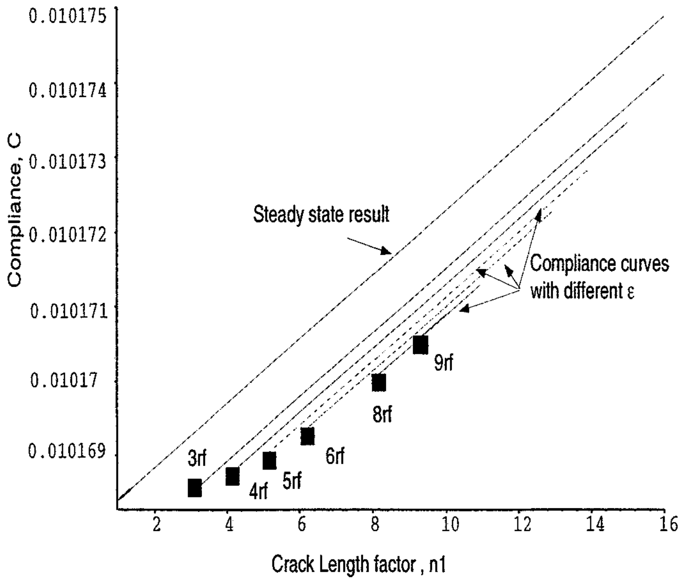

Figure 3: Curves of compliance as a function of crack length for different values of ϵ and L $= 20r_o$

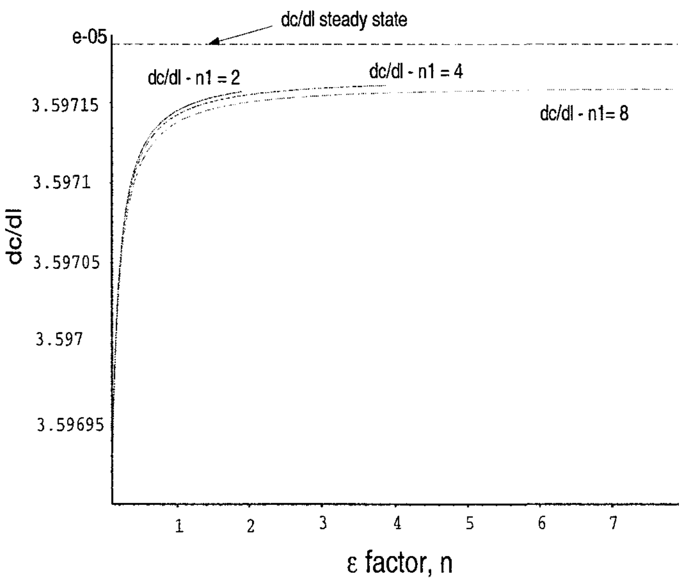

Figure 4: Curves of rate of change of compliance with crack length $\frac{dc}{dl}$ as a function of crack length with varying ϵ and L $= 20r_o$