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SIMULTANEOUS DESIGN OF STRUCTURAL TOPOLOGY AND MATERIAL PROPERTIES

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ABSTRACT

A brief review of different strategies for designing simultaneously the global structural topology and the local material properties is presented. Different treatments have been developed in the last decade to design the stiffest continuum structure using the concept of material distribution introduced in 1988 by Bendsøe and Kikuchi. The comparison of such treatments shows a trend towards simplification, as expected, and also towards unification of metrics that allows the expression of design variables, objective functions, and constraints in a single basis. Numerical examples are presented to show the capabilities of the 'natural basis' treatment introduced by Taylor in 1998.

INTRODUCTION

The concept of `optimum material allocation' was introduced in 1988 by Bendsøe and Kikuchi in order to compute optimum topologies of continuum structures. The goal was then to obtain the global shape and topology of a structure made of isotropic material. Bendsøe and Kikuchi used orthotropic materials to expand the space of possible solutions and to be able to attain a better optimum. Homogenization approaches were used to compute 'average' properties of such composite materials and perform finite element analyses. This approach was proved to work well, and has been widely studied. Later, Bendsøe et al (1994) proposed a refinement of the idea in which all components of the elasticity tensor were considered as design variables, with the intention of reaching the stiffest structure using the locally stiffest material. The following is a presentation of different ways to formulate the minimum compliance problem, considering both the global structural design (as intended by Bendsøe and Kikuchi in 1988) and the local material design (as intended by Bendsøe *et al* in 1994).

TREATMENTS FOR SIMULTANEOUS STRUCTURAL AND MATERIAL DESIGN

The Free Tensor Design Model

The first paper addressing the design of local material properties in the context of a global design domain subject to generic loads was written by Bendsøe, *et al* (1994). The optimization problem was formulated in a way that has the unrestricted modulus tensor E_{ijkl} appear in the role of design variable. This introduced a total of 6 design fields for 2D elasticity. There are two important points to notice in this formulation. First, the topology optimization problem was posed without the use of any homogenization approach. This fact, until today, has been surprisingly down played. Second, the isoperimetric constraint is measured in terms of the trace of the E tensor, rather than a measure of material of given form, as was originally posed by Bendsøe and Kikuchi. Therefore, there are three different units of measure, namely, one for the design variables (entries of the E tensor), one for the objective function (compliance, work), and one for the constraint (the argument for the latter stated in terms of a norm of the E tensor).

The Unit Cost Design Model

As an attempt to incorporate some means for technical (e.g., manufacturing) constraints in the previous formulation, Guedes and Taylor (1997) introduced a unit cost coefficient in the material resource constraint plus an algorithm that allowed them to enforce solid/void structure in order to predict optimal topology results. The novel idea of this formulation, which is the preamble for the next one, is the introduction of a unit cost distribution. An elaboration of this approach was used in Rodrigues, *et al* (1999) for the prediction of optimal topology design for composites, based on a mix of two materials.

The Natural Basis Design Model

This formulation addressed the weaknesses pointed out above. First, and most important, a single basis, called

here 'natural basis', is used to measure all three quantities, namely, the basic design variable (material properties), the objective function, and the constraints. Second, a set of unit weights for each entry in the E tensor, plus another weight for the global structural topology were introduced in a hierarchical form. The formulation for compliance minimization reads as follows (Taylor, 1998, 2000):

$$\max_{B_\gamma;\,B} \quad \{local \quad problem\}$$

subject to

$$\begin{cases} 0 < \underline{B}_\gamma \le B_\gamma \le \overline{B}_\gamma, (\gamma = 1, \cdots M) \\ 0 < \underline{B} \le B \le \overline{B} \\ \sum_{\gamma=1}^{M} b_{L\gamma} B_\gamma - B \le 0 \\ \int_\Omega bBdV - R \le 0 \end{cases}$$

where the local problem is

$$\left[\min_{c_\alpha;\,u_k} \left\{ \int_\Omega \sum_{\gamma=1}^{M} e_\gamma B_\gamma dV \right\} \right]$$

$$subject\ to \qquad (1)$$

$$\begin{cases} \underline{W} - \int_\Omega f_k u_k dV \le 0 \\ \frac{1}{2}(u_{i,j} + u_{j,i}) - \sum_{\alpha=1}^{L} c_\alpha \eta_{ij}^{\alpha} = 0 \end{cases}$$

where M is the total number of the components of local material properties, L corresponds to the number of independent components of strain. For 2D problems, $M = 6$, $L = 3$. B_γ is the γ^{th} component of the natural (energy) basis, which will be used to identify the local properties. When a set of reference strains η_{ij}^{α} are given, B_γ will have uniquely linear relation with the elasticity modulus tensor E_{ijkl}. B is the measure of the natural basis, which could be used to identify the global materials distribution. $b_{L\gamma}$ and b represent the unit cost of local properties and global materials distribution, respectively.

COMPUTATIONAL IMPLEMENTATION OF THE NATURAL BASIS MODEL

Since B_γ and B are relatively independent design variables, in the computational implementation, the original problem could be treated as two independent parts: the design of local properties and the design of global material distribution :

Part I - Design of local properties for fixed global material distribution : given $B(x)$, find $B_\gamma(x)$ that

$$\max_{B_\gamma} \left\{ \int_\Omega \sum_{\gamma=1}^{M} e_\gamma B_\gamma dV \right\}$$

$$subject\ to \begin{cases} 0 < \underline{B}_\gamma \le B_\gamma \le \overline{B}_\gamma, (\gamma = 1, \cdots, M) \\ \sum_{\gamma=1}^{M} b_{L\gamma} B_\gamma - B \le 0 \end{cases} \qquad (2)$$

$$(x \in \Omega)$$

The converse problem, i.e., the design of $B(x)$ for specified and fixed $B_\gamma(x)$, is expressed as :

Part II - Design of global material distribution for fixed local properties : given $B_\gamma(x)$, find $B(x)$

$$\max_{B} \left\{ \int_\Omega eBdV \right\}$$

$$subject\ to \begin{cases} 0 < \underline{B} \le B \le \overline{B} \\ \int_\Omega bBdV - R \le 0 \end{cases},$$

$$(where: \ e = \frac{\sum_{\gamma=1}^{M} e_\gamma \hat{B}_\gamma}{\sum_{\gamma=1}^{M} b_{L\gamma} \hat{B}_\gamma}) \qquad (3)$$

where $\hat{B}_\gamma$ represents the normalized local properties.

Description of the computational algorithm

The algorithm can be described in five main steps. The two optimization problems can be solved in sequence iteratively until convergence is attained.

Step 1. Define the initial value of normalized local properties $\hat{B}_\gamma^{(0)}$ and global material distribution $B^{(0)}$.

Step 2. Solve a sub-problem of optimization of global material distribution for fixed local properties.

Step 3. Solve a sub-problem of optimization of local properties in each point(or element, for discrete computation) for fixed global material distribution.

Step 4. Compute the new normalized local properties.

Step 5. Check Convergence : If converged, stop; If not converged, go back to step 2 and begin a new cycle.

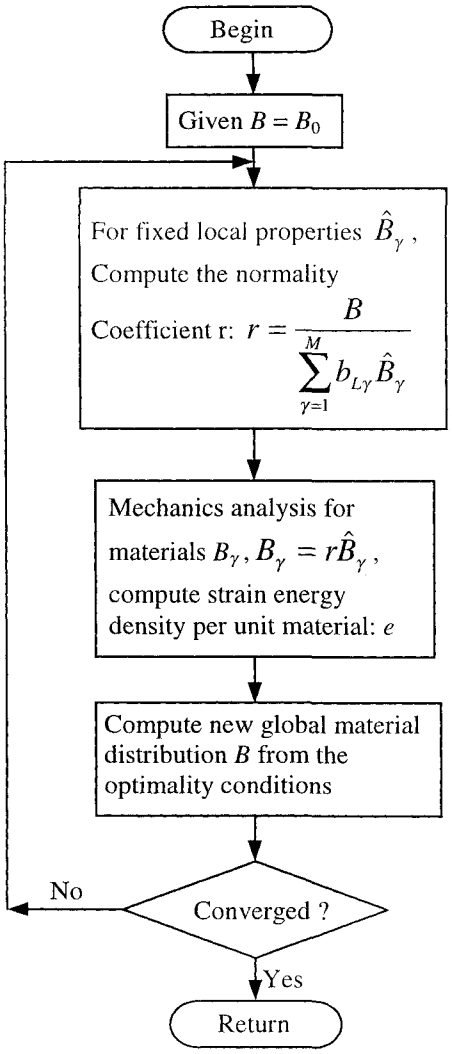

Figure 1: global design. Find optimum B for given B_γ.

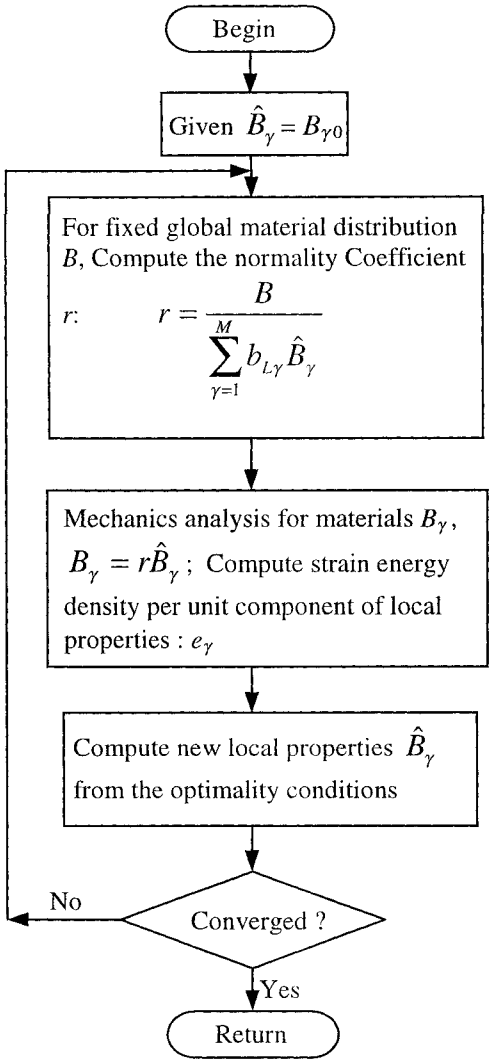

Figure 2: local design. Find optimum B_γ for given B.

Optimality Conditions

Optimality conditions are used in the update scheme of B (Figure 1). They are:

$$B_{(k+1)} = \frac{e_{(k)}}{b\,\tilde{e}_{(k)}} B_{(k)} \tag{4}$$

where, $\tilde{e}_k$ could be defined as the strain energy per unit resource :

$$\tilde{e}_{(k)} = \frac{\int_\Omega e_{(k)} B_{(k)} dV}{R} = \frac{U_{(k)}}{R} \tag{5}$$

Optimality conditions are used in the update scheme of B_γ (Figure 2) :

$$B_{\gamma(k+1)} = \frac{e_{\gamma(k)}}{b_{L\gamma}\tilde{e}_{\gamma(k)}} B_{\gamma(k)} \tag{6}$$

Several parameters were introduced into the computation to control the speed of convergence and the stability of iteration. Based on the similar technique of the paper (Rodrigues, Soto, Taylor, 1999), the "shades of gray" results could be developed further to a "black-white" (or void/solid) topology by properly adjusting the global cost distribution b.

NUMERICAL EXAMPLES

In all of the examples presented below, the 2D design areas are discretized by 3-nodes triangular constant strain elements. A technology of taking the average value of the strain of the adjacent elements is introduced in order to improve the precision of the strain energy of elements.

Multiple load design of a rectangular sheet with a prescribed hole

Figure 3 shows a rectangular sheet with a prescribed square hole. Three kinds of symmetric load cases are considered. Case 1: horizontal load. Case 2: vertical load. Case 3: horizontal and vertical loads simultaneously applied. Since the loads are symmetric, the structure will be able to keep equilibrium without additional boundary constraints.

Values in Figure 3 are: b = 2, $p_1 = p_2 = 1$, t = 0.01, where t is the thickness of the sheet. From the symmetry, only one quarter of the structure is considered in design, with proper constraints applied on the horizontally and vertically symmetry plane. The material used for the initial design is isotropic and distributed uniformly over the global design area. The initial value of local properties are: $E = 10^{11}$, $\nu = 0.3$, for the given reference strain (Taylor, 1998), the corresponding value of the basis of strain energy are : $B_1 = 5.495\times10^{10}$, $B_2 = 3.846\times10^{10}$, $B_3 = 5.495\times10^{10}$, $B_4 = 7.143\times10^{10}$, $B_5 = 4.670\times10^{10}$, $B_6 = 4.670\times10^{10}$. The upper bound of the measure of material properties $\overline{B} = 2.0\times10^{12}$. In topology design, this value will make the resource available equals 28% of the volume of the design domain. The lower bound of the measure of material properties, $\underline{B} = \overline{B}\times10^{-9}$. The unit cost of local material properties : $b_r = 1$, and uniformly distributed over design area. One quarter of the design area is divided by a 47×23 element mesh.

A Matlab code was written to implement the computation. Linear springs are introduced at the boundary to simulate different boundary conditions, $k = +\infty$ indicates boundary with fixed support; $k = 0$ indicates free boundary (k is stiff coefficient of spring).

For the multiple load design the objective is a linear combination of strain energies of all load cases :

$$\overline{U} = \sum_{i=1}^{3} \alpha_i \left(\int_\Omega \sum_{\gamma=1}^{M} e_\gamma^i B_\gamma dV\right) \qquad (7)$$

where e_γ^1, e_γ^2 and e_γ^3 are corresponding to load case 1, case 2, and case 3, respectively. α_1, α_2 and α_3 are weights of strain energy, which could be varied from 0 to 1.

Example 1

In this example, 31 iteration steps were executed. The total computational time is 15 minutes. α coefficients are:

$\alpha_1 = \alpha_2 = \frac{1}{2}$, $\alpha_3 = 0$ (Only show one quarter of the original structure from the symmetry)

Initial weighted strain energy : $\overline{U} = 2.456\times10^{-9}$, and final weighted strain energy : $\overline{U} = 0.520\times10^{-9}$

From figures 4 and 5, we could see that the solution converged very quickly in this example. After about 15 iteration steps, the strain energy history and the structural topology don't change substantially.

Figures 6 and 7 show the distribution of the natural basis and the corresponding material components. The former is invariant for a group of fixed reference strains, the latter varies with coordinates. For the single load design, the optimal material is orthotropic, Projected on the principle direction of material, the E_{1112} and E_{1222} components are zeros. On the other hand, for multiple loads design, the optimal material is usually not orthotropic, it means there are no orthogonal axes where the E_{1112} and E_{1222} components are zero.

Example 2

Figure 8 shows the three load cases to be studied independently.

From figure 9, in multiple load conditions, the final design is 79% stiffer than the initial design. In the single load case, the final design is 81% stiffer than the initial design for case 1; 85% stiffer than the initial design of case 2; and 85% stiffer than initial design of case 3.

Fig 10 compares the optimal topology in multiple loads design and three kinds of single load design. For the multiple loading case, the structure along horizontal and vertical directions is strong, which will provide enough stiffness for both load cases. For single load cases the

design is optimized to perform better only for the load in question.

From figures 11 and 12 it can be concluded that, for single load cases, the optimal local properties are basically orthotropic (E_{1112} and E_{1222} are very small),and at the same time, E_{1212} is also very small, that means, in the principle coordinates, the optimal distribution of local properties make the materials concentrate in the component that mainly sustain the tension and pressure, i.e. E_{1111}, E_{2222}, E_{1122} components.

However, for multiple load cases, the optimal local properties are usually not orthotropic everywhere. This is shown in figure 13.

CONCLUSIONS

We presented the evolution of the idea of designing the stiffest structure with the stiffest material. Three formulations were briefly presented. The 'natural basis' approach was implemented and an example for several load cases was solved to show the capabilities of this new algorithm to predict optimum topology and material design simultaneously.

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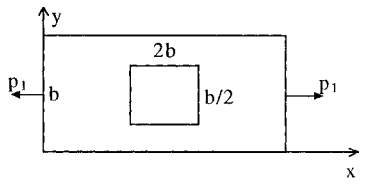
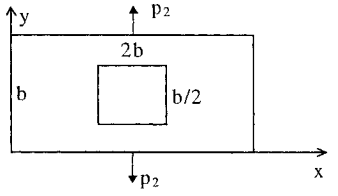
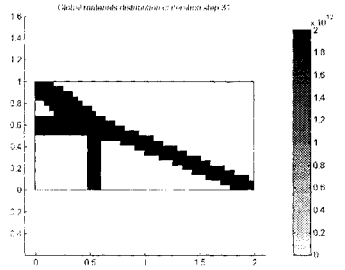


(a)

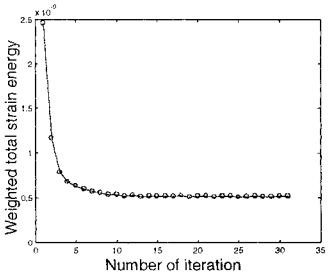


(b)

Figure 3 (a) Load case 1. (b) Load case 2.



(a)



(b)

Figure 4 (a) Optimal topology. (b)Weighted total strain energy vs. iterations.

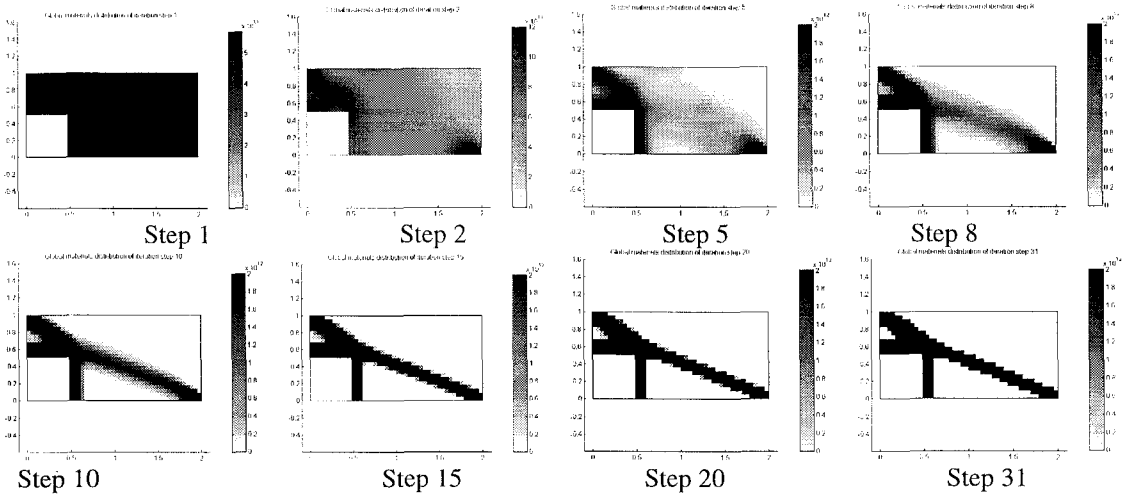


Figure 5 Topology results for some iteration steps

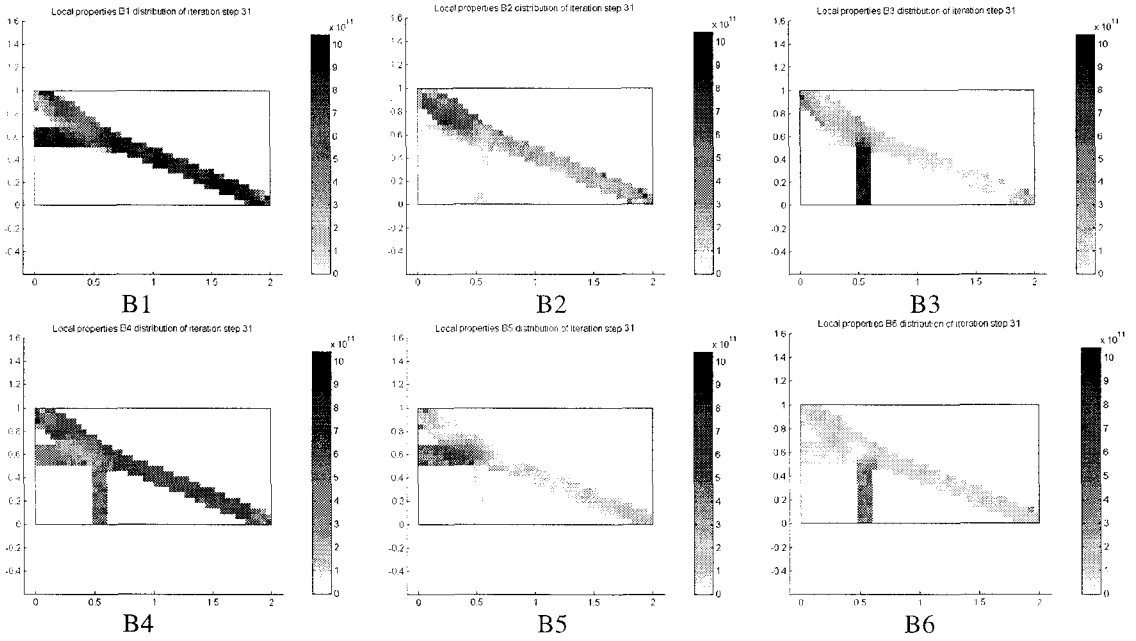
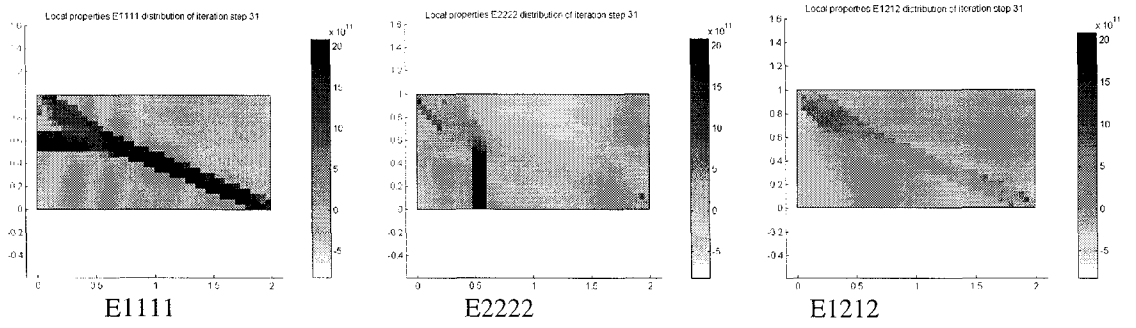
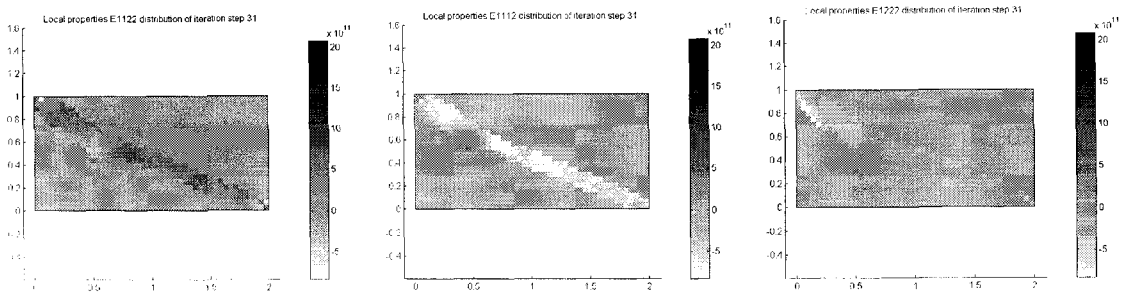


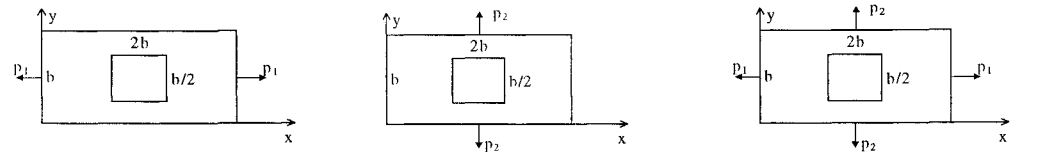
Figure 6 Optimal distribution of strain energy basis for topology design.





E1122 E1112 E1222

Figure 7 Optimal local properties in the original x-y coordinates



case 1 : horizontal load case 2 : vertical load case 3 : simultaneously horizontal and vertical loads

Figure 8 Three load cases considered.

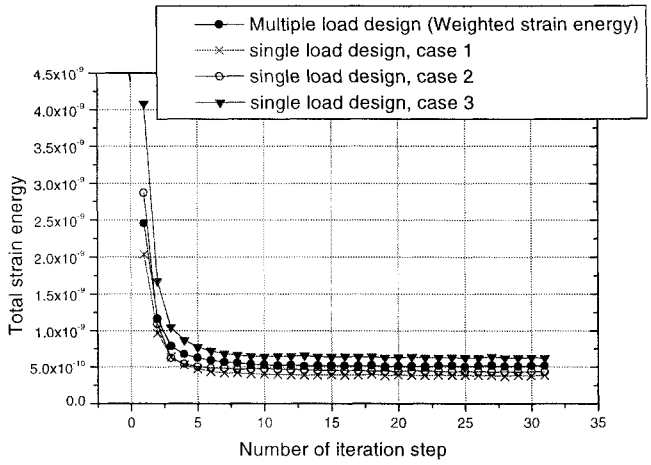
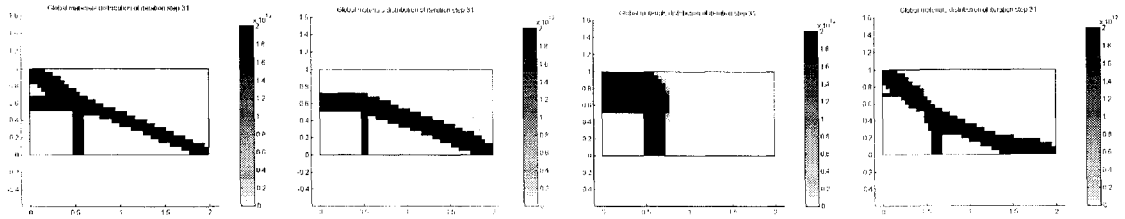


Figure 9 Strain energy iteration history



Multiple load design ($\overline{U} = 0.520\times10^{-9}$) Single load design(1) ($U = 0.387\times10^{-9}$) Single load design(2) ($U = 0.430\times10^{-9}$) Single load design(3) ($U = 0.624\times10^{-9}$)

Figure 10 Comparison of topology results for different load cases.

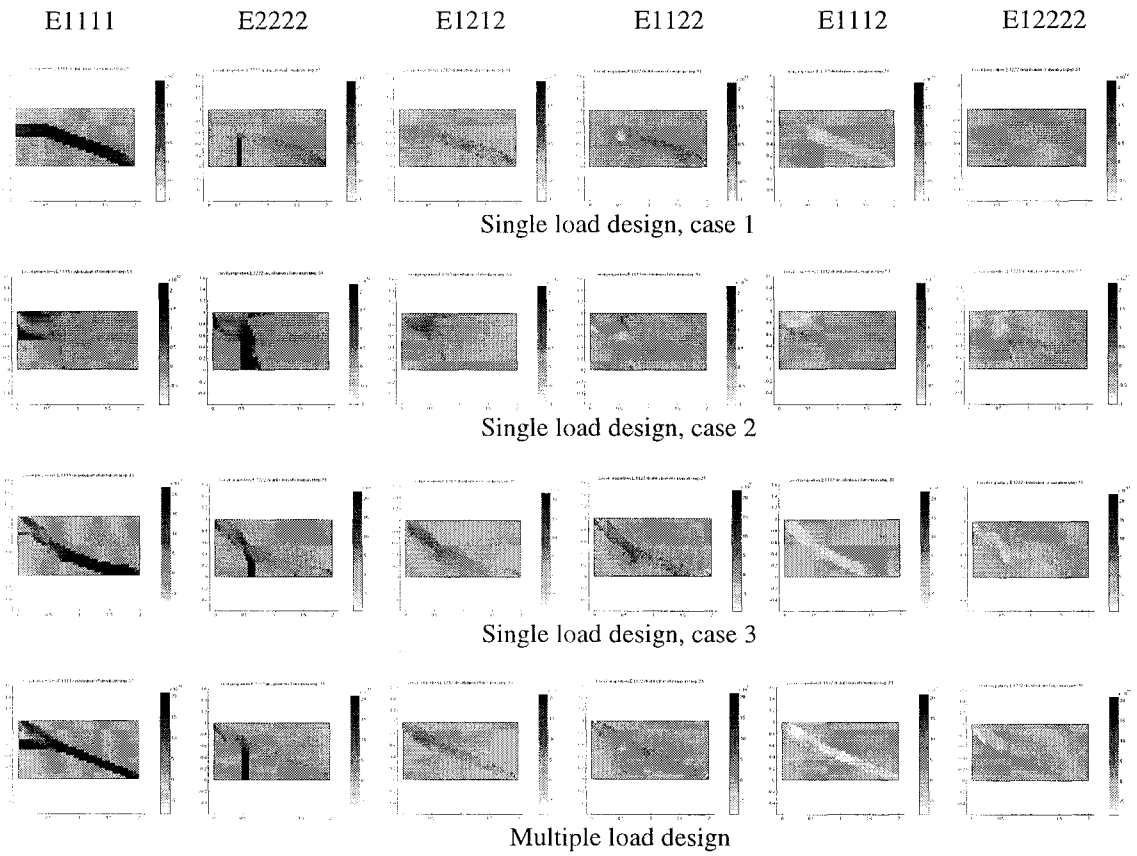


Figure 11 Compare of optimal local properties in different design. Optimal local properties in original x-y coordinates

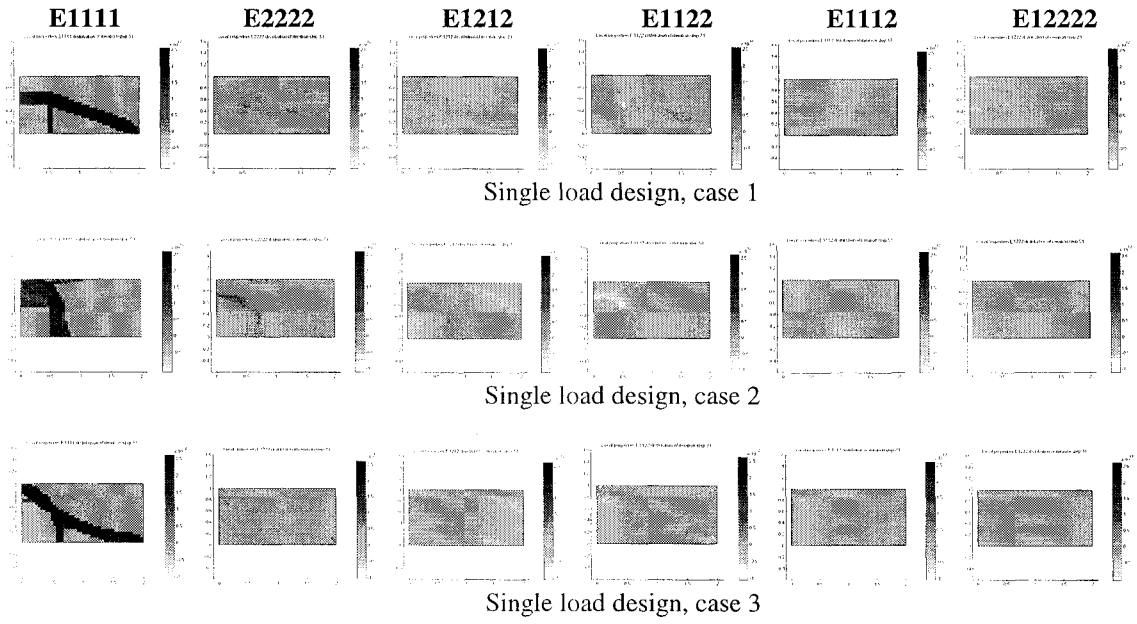


Figure 12 Optimal local properties projected onto directions identified with principle stresses.

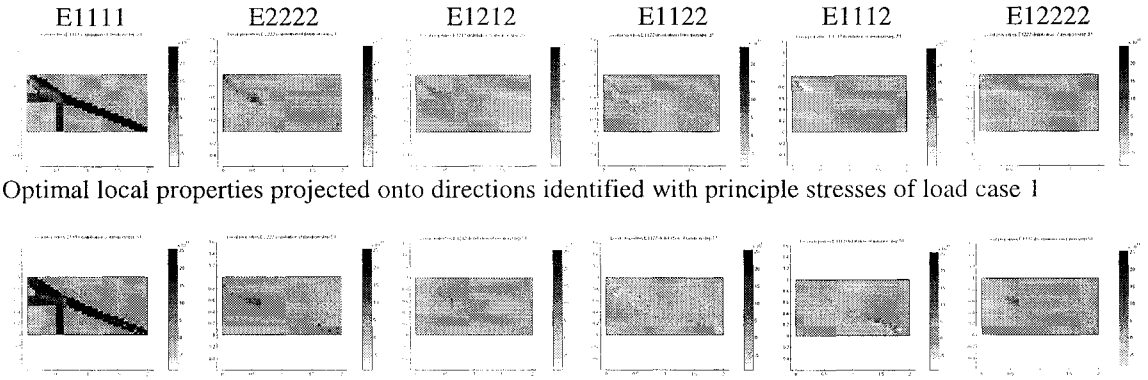


Figure 13 Multiple load design