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Final Report

ON THE USE OF IONIZATION-GAGE DEVICES AT VERY HIGH ALTITUDES

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ABSTRACT

The use of an ionization-gage—omegatron combination in rockets and satellites at very high altitudes is considered. Appropriate instrumentation employing these devices is believed to offer good possibilities for the measurement of atmospheric pressure, temperature, density, and composition.

OBJECTIVE

The purpose of this investigation was to conduct an analytical study of the feasibility of employing ionization-gage devices in the measurement of very-high-altitude atmospheric pressure, temperature, and density.

INTRODUCTION

The measurement of atmospheric pressure, temperature, and density in regions of the high atmosphere where the mean free path of typical particles is long as compared with the dimensions of the instrument has interested upper-air researchers for many years. Early attempts^{1,2,3,4} using V-2 rockets were largely inconclusive because of instrument contamination by rocket gas or because the altitudes attained were insufficient. Some significant measurements to 120 km were made but could not be considered as entirely fulfilling desired conditions of a long mean full-path environment. No further significant measurements were made until the Viking rocket, and still later the Aerobee Hi rocket, became available.^{5,6,7,8}

The methods employed in these measurements have made use of local density determining devices which permit investigation of the pressure or density at selected points on the surface of a rocket. This allows interpretation in terms of ambient conditions, provided there is knowledge of rocket velocity, altitude, air composition, and other such factors.

Advances in instrumentation, the availability of rockets with an increased capability for high altitudes, and the imminent availability of satellite vehicles have stirred a compelling new interest in these measurement areas.

This report thus considers the employment of ionization-type devices for performing measurements of ambient pressure, temperature, density, and in addition, composition, in the light of the newer developments.

I. MEASUREMENTS WITH A SINGLE IONIZATION GAGE

Each gas in a mixture of gases exerts a partial pressure upon the walls of the confining vessel in proportion to its concentration. Since pressure is defined in terms of the rate of momentum transfer, these partial pressures are also determined by the mass, velocity, and hence temperature of the various constituents.

The presence of particles of different masses implies a velocity spectrum and thus suggests that some form of mass spectrometry approach is applicable to the measurement of structural parameters at altitudes where the

mean free paths of various particles are large in comparison with the dimensions of an intruding object.

Consider a body moving in space at velocities comparable to the random molecular speeds of the constituent gases. Assume also that the body's velocity and orientation are known. If an opening to a body-borne chamber is exposed to the outside, an interchange of particles will take place in a manner that is dependent upon the particle velocity distribution function, the body velocity, its orientation, the characteristic temperature, the composition, and other effects.

If the body velocity V_c is reduced to zero, then the interchange will be altered only in degree. The relationship between the internal and external pressure will be governed by the thermal transpiration effect as investigated by Maxwell,⁸ Reynolds,⁹ and Knudsen.¹⁰ If the body velocity is not zero, then the pressure relationship is less easily stated, but readily so upon assumption of a velocity distribution function. Several experimentors^{7,11,12} have derived a relation which is seen to be the transpiration equation modified by a factor dependent upon the ratio of body to particle velocity. Thus,

$$\frac{P_i}{P_o} = \sqrt{\frac{T_i}{T_o}} [e^{-s^2} + s \sqrt{\pi} (1 - \text{erf } s)] , \quad (1.1)$$

where

$$f(s) = e^{-s^2} + s \sqrt{\pi} (1 - \text{erf } s),$$

P_i = internal pressure,

P_o = ambient pressure,

T_i = internal temperature,

T_o = external temperature, and

$$s = \frac{V_x}{V_g} = \frac{\text{component of body velocity normal to gage opening}}{\text{most probable particle velocity}}$$

P_i above was defined only as the pressure in the chamber, implying total pressure. It can be considered as the sum of the partial pressures of the component gases; thus

$$P_i = p(N_2) + p(N) + p(O_2) + p(O) + p(A) + p(H_2) + p(H) + p(He) + \dots , \quad (1.2)$$

considering some of the possibilities, including dissociation. Similarly, P_o

may be considered as the sum of the external partial pressures, and accordingly an equation like (1.1) can be written for each component.

Since

$$V_g = \sqrt{\frac{2kT_0}{m_0}} \quad (1.3)$$

where

k = gas constant per molecule

= 1.381×10^{-16} erg per degree K.

T = temperature in °K, and

m_0 = mass per particle, grams,

an $f(s)$ exists for each component. Thus, the internal partial pressure ratios will differ from the external ratios, the difference being a function of the body velocity component normal to the chamber opening.

Thermal equilibrium can be assumed externally¹³ as well as internally with small error. Internally, however, it is assumed that the particle temperatures will be set by the wall temperature, adopting a unity accommodation coefficient. A Maxwellian velocity distribution in the region being sampled follows the equilibrium assumption.

The above considerations suggest the possibility of composition measurement by observation of the change in pressure in a chamber as a function of V_x , the body velocity component normal to the chamber opening. As the relative velocity increases (or decreases), the total pressure will, in principle at least, increase (or decrease) gas by gas, so to speak, as required by the above considerations. The rate of change will be governed by the distribution function as well as by the normal acceleration component which presumably can be known. The maximum relative velocity must appreciably exceed the velocity of most of the particles to make this measurement possible. Expected relative values of most probable external particle velocity corresponding to the various components are readily determined upon assumption of external temperature and temperature equilibrium. Typical values are presented in Table I, which has been computed from Equation (1.3) for three temperatures. Since m can vary relatively from 1 (atomic hydrogen) to as much as perhaps 40 (argon), V_g can have a range of about 6.5 to 1 for a given temperature. For purposes of evaluating the feasibility of the experimental approach, the minimum expected particle velocity can be taken as 1300 fps (argon at approximately 400°K) and the maximum for any particle except the electron as 15,000 fps (hydrogen atom at 1200°K). Considering both rockets and satellites as possible research vehicles, satellite velocities are adequate for all particles, while rocket velocities of present upper-air experimentation are adequate only for the more massive particles. Thus, in general, this procedure is discussed

TABLE I
 MOST PROBABLE VELOCITY OF VARIOUS ATMOSPHERIC COMPONENTS AS A FUNCTION OF THE KINETIC TEMPERATURE

Component	300°K		600°K		1200°K	
	cm/sec	ft/sec	cm/sec	ft/sec	cm/sec	ft/sec
air	4.17×10^4	1.36×10^3	5.9×10^4	1.93×10^3	8.34×10^4	2.75×10^3
N ₂	4.2	1.38	5.95	1.95	8.4	2.77
N	5.95	1.95	8.4	2.77	1.19	3.9
O ₂	3.95	1.3	5.58	1.83	8.0	2.62
O	5.56	1.82	7.9	2.6	1.11×10^5	3.65
He	1.12×10^5	3.7	1.58×10^5	5.2	2.24	7.39
H	2.23	7.3	3.15	1.04×10^4	4.46	1.47×10^4
H ₂	1.58	5.2	2.23	7.3×10^3	3.01	9.9
A	3.52×10^4	1.16	5.0×10^4	1.65	7.04×10^4	2.3
e	5.18×10^7	1.7×10^6	7.3×10^7	2.4×10^6	1.03×10^8	3.4×10^6

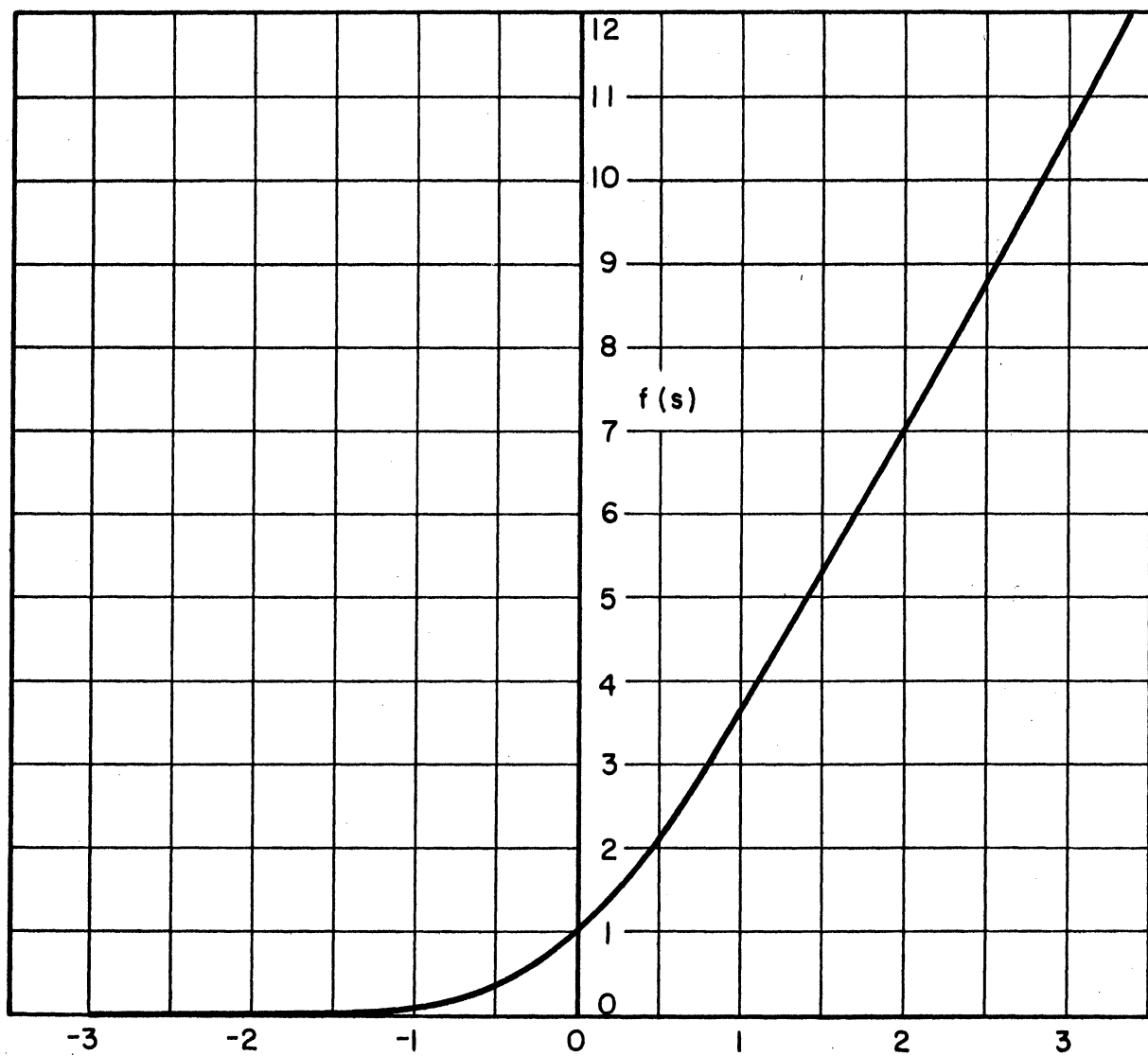
from the point of view of an orbiting vehicle.

Consider application to a satellite, where the velocity may be taken as 26,000 fps. Thus, if the chamber opening is opposite to the direction of motion, only a few of the least massive components, as determined by the distribution function, could enter. On the other hand, if the opening is oriented in the forward direction, then all particles can be overtaken, again excepting a few of the lighter components. It follows that particular intermediate positions enable partial collection of the heavier atoms.

In the expected situation of rotation that would cyclically orient the opening in the direction of motion and then opposite to the direction of motion, velocity "scanning" would occur. The measured total pressure would, in principle, vary in accordance with the concentration of the various constituents and their associated velocities, thus offering the possibility of a particle velocity measurement.

The s function (Fig. 1) effect upon the internal partial pressure of a gas as compared with the external pressure is to reduce the internal pressure to zero as s approaches -1 or to increase the internal pressure significantly as s assumes larger positive values. Since s is greater for the more massive particles, contributions to total pressure from N_2 , N , O_2 , and O can, under high-velocity conditions, tend to overwhelm the lighter and less plentiful components out of proportion to their ambient composition stature. For negative velocities, the effect is opposite, possibly enabling detection of the lighter components. This is illustrated in Fig. 2, which shows the ratios of internal to external partial pressures of nitrogen and hydrogen as functions of the angle of attack which is a measure of the normal velocity component. The curves are valid only for equal concentrations of the gases, a condition assumed for the sake of illustration. Some of the following data were used in computing the curves:

Altitude:	300 km
Pressure:	2.0×10^{-8} mb
Density:	5×10^{-12} kg/cu m
Temperature:	1000°K
Body velocity:	26,000 fps
Rotation:	1 rps with rotational axis perpendicular to plane of orbit
Internal temperature:	300°K



$$f(s) = e^{-s^2} + s \sqrt{\pi} (1 + \operatorname{erf} s)$$

$$s = \frac{V_x}{V_g} = \frac{\text{COMPONENT OF BODY VELOCITY NORMAL TO GAGE OPENING}}{\text{MOST PROBABLE PARTICLE VELOCITY}}$$

Fig. 1. Curve of f(s) versus s.

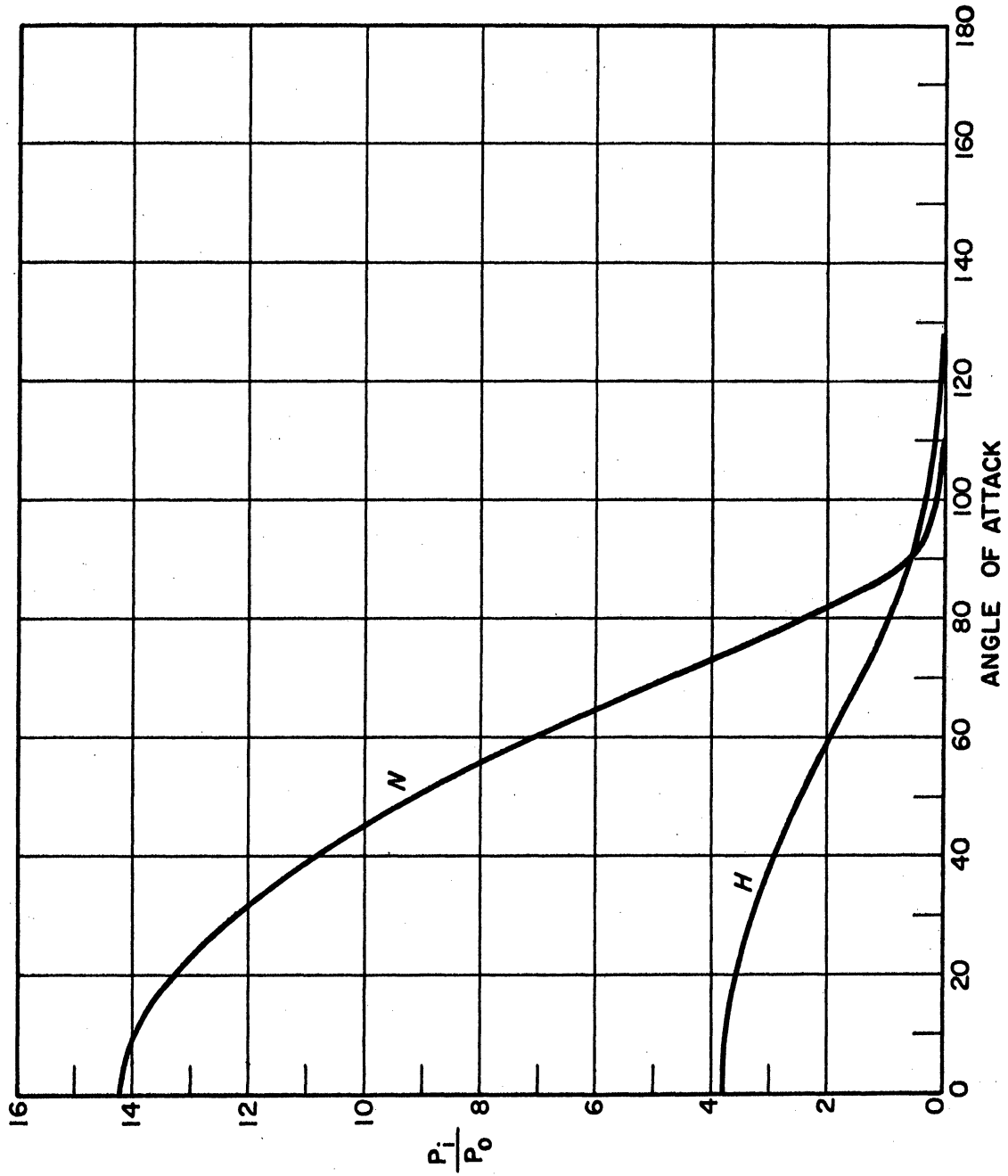


Fig. 2. Ratio of external to internal pressure for nitrogen and hydrogen with angle of attack.

Most probable velocity: of atomic nitrogen at 1000°K = 3.56×10^3 fps

of atomic hydrogen at 1000°K = 1.33×10^4 fps

Because the chamber wall temperature is likely to be substantially less than the external temperatures, the internal pressures can be less by as much as a factor of 2. However, the reduction will take place for all components and thus has a bearing on sensor sensitivity considerations rather than on relative partial pressures.

Although velocity scanning occurs as a result of rotational motion and in principle enables differentiation between the various component gases, as a practical matter there is a considerable overlap of the distribution functions primarily because the region is populated with gases of comparable mass. As a consequence, it is probably not possible to determine relative concentration by this method. It may be feasible, however, to determine the most probable velocity and hence the characteristic temperature. If the assumption of external thermal equilibrium is valid, the characteristic temperatures of the various gases should be the same. Thus determination of the most probable velocity for any component indicates the external temperature. Knowing the temperature, it is then possible to return to Eq. (1.1) and compute the external pressure.

Summarizing, a pressure (or density) measuring device such as an ionization gage in a body which is moving at an appropriate and known altitude and velocity, with a prescribed rotational rate, makes certain measurements possible. Preferably, the maximum velocity should exceed, by an adequate margin, say 2:1, the most probable velocity of the lightest component to be measured, thus indicating the need for a vehicle of satellite capabilities, if the lighter gases are to be considered.

With such an arrangement, it appears that it may be feasible to measure the temperature directly which would enable deduction of the ambient pressure and density.

II. IONIZATION GAGE PLUS OMEGATRON

The addition of another instrument which permits an additional and independent measurement of the chamber gas offers considerably greater promise of fruitful experimentation. Assume that a small mass spectrometer-type device is placed within the ionization chamber, so that the same gases measured by the ion gage are measured by the spectrometer. A suitable instrument is the omegatron, a device first developed and reported by Sommer, Thomas, and Hipple,¹⁴ who were concerned primarily with measurements of charge to mass ratio. A version of the device applicable in the sense required here was later developed and reported by Alpert and Buritz,¹⁵ who suggested its usefulness as a

pressure-measurement tool.*

The addition of an omegatron, which appears quite feasible, would enable determination of (a) the composition, (b) verification of the shape of the distribution function, and (c) the relative concentration of the constituent gases, provided the associated sensor had adequate sensitivity to detect the less plentiful gases. Thus, the "total" pressure determined by the ion gage could be resolved in terms of the various partial pressures.

Considering Eq. (1.1) in partial pressure form,

$$P_{ai} = P_{ao} \sqrt{\frac{T_i}{T_o}} f(s) \quad (2.1)$$

where

$$f(s) = e^{-s^2} + s \sqrt{\pi} (1 + \operatorname{erf} s) \quad , \quad (2.2)$$

$$s = \frac{V_x}{V_a} \quad , \quad (2.3)$$

$$V_a = \sqrt{\frac{2kT_o}{m_a}} \quad , \quad (2.4)$$

P_{ai} = internal partial pressure of gas "a,"

P_{ao} = external partial pressure of gas "a,"

and

$$s = V_x \sqrt{\frac{m_a}{2kT_o}} \quad .$$

Substituting in Eq. (2.1),

$$P_{ai} = P_{ao} \sqrt{\frac{T_i}{T_o}} \left[e^{-\left(V_x \sqrt{\frac{m_a}{2kT_o}} \right)^2} + V_x \sqrt{\frac{m_a \pi}{2kT_o}} \left(1 + \operatorname{erf} V_x \sqrt{\frac{m_a}{2kT_o}} \right) \right] \quad . \quad (2.5)$$

This is the general form for any value of V_x and hence any value of s . In simplification, as

$$s \rightarrow \infty, \quad \operatorname{erf} s \rightarrow 1, \quad \text{and}$$

*Possible usefulness at high altitudes was suggested by H. S. Sicinski.

$$f(s) \cong 3.55 V_x \sqrt{\frac{m_a}{2kT_0}} .$$

Thus Eq. (2.5) becomes

$$P_{ai} = P_{ao} \sqrt{\frac{T_i}{T_0}} \left(3.55 V_x \sqrt{\frac{m_a}{2kT_0}} \right) , \quad (2.6)$$

which, by combining, becomes

$$P_{ai} = 3.55 V_x \sqrt{\frac{m_a T_i}{2k}} \times \frac{P_{ao}}{T_0} . \quad (2.7)$$

It is considered here that the only unknowns are P_{ao} and T_0 , which appear as a ratio. Consequently, since the pressure can also be defined as

$$P_{ao} = n_a k T_0 , \quad (2.8)$$

where

n_a = number of particles of gas "a,"

one can substitute

$$n_a k = \frac{P_{ao}}{T_0} . \quad (2.9)$$

Equation (2.7) becomes an expression for the number density in space of the gas "a" as a function of the internal partial pressure, the normal component of the body velocity, and the internal chamber temperature. Thus:

$$n_a = \frac{1}{\sqrt{0.58 k m_a T_i}} \times \frac{P_{ai}}{V_x} . \quad (2.10)$$

It follows that the ambient number density of any constituent can be determined from the internal partial pressure. In addition, knowledge of the external temperature, so obtained from a characteristic velocity measurement with an ion gage or similarly with an omegatron, will permit computation of the external partial pressures.

Thus through employment of an ion gage and an omegatron, which admittedly involves redundancy, several quantities can be evaluated, including

- a) pressure, temperature, and density of ambient gas;
- b) type and concentration of the constituent gases.

III. VELOCITY DISTRIBUTION FUNCTION

The preceding comments have been based upon the assumption of a Maxwellian distribution of velocities among the various gas particles. Although the literature substantiates this assumption,¹³ it is of course desirable to obtain experimental verification, which the presence of the omegatron as indicated above makes possible.

Consider again the omegatron in the body whose orientation pattern is such as to bring the chamber opening cyclically into and away from the "stream." Assuming diffusive equilibrium between the chamber and the outside, the internal number density of a particular gas, which is the quantity measured by the omegatron, would vary as a function of the normal velocity component and the distribution function. Since the body velocity is known, determination of the distribution function follows.

A plot of the change in internal number density (or partial pressure) as a function of angle of attack, or normal velocity component, should appear like Fig. 2, provided, of course, that the distribution is Maxwellian. Other distributions would alter the shape and would thus be definable.

IV. ATMOSPHERIC VELOCITY

All previous considerations have assumed that the relative velocity between the body or chamber and the gas was due to (a) that characteristic of the gas and (b) that of the instrument-carrying body itself. Accordingly, the assumption is implied that the atmospheric velocity is negligible. This is not the case, for if one considers that the atmosphere moves with the earth, the velocity in the equatorial plane at an altitude of 300 km is 1.6×10^3 fps.

This value is clearly significant in comparison with the most probable velocity of certain components (see Table I). The effective value, insofar as possible measurements are concerned, is of course dependent upon the location of the plane of the orbit, and thus is maximum and nearly constant for passage near the equator, and variable in magnitude and direction for polar orbits.

In any case it can be considered as a definable perturbation on the body velocity.

V. SENSOR SENSITIVITY

To employ fruitfully the above-mentioned devices, it is of course necessary to arrange a sensor that will ideally have adequate sensitivity and time characteristics to permit detection and transmission to the recording device of signals corresponding to the least plentiful components.

The major constituents one expects to detect at the higher altitudes are O, N₂, and N, both in neutral and ion states. Speculation leads to consideration of hydrogen and helium as well, but in very small quantities. Detection of the various gases is dependent primarily upon their absolute abundance, and upon the sensitivity of the current-measuring instrument employed. Different elements in like concentration effect different responses in an ionization gage. In general, the apparent sensitivity is a function of the total number of electrons possessed by the molecule¹⁶ rather than its ionization potential or atomic weight, and thus the ratio of sensitivities of helium and nitrogen is less than their mass ratio. The overall result, then, insofar as ion-gage (or omegatron) sensitivity is concerned, is that the relative concentration is the more powerful factor.

Accordingly, in regard to gage sensitivity (S) to dissociated gases, one would expect the ion current resulting from dissociated or recombined gases to vary with the number of electrons per particle.

Ion-gage sensitivities can be considered in terms of the ratio of collected ion current to ionization current at a given density. A typical value for present thermionic gage capability is $S = 100 \mu\text{a}$ per microbar for an ionization current of 10 ma. Thus at this pressure each 100 electrons, on the average, emitted by the cathode produce one ion for collection. The density is such that it is unlikely that one electron can generate more than one ion pair during transit. Thus the sensitivity factor indicates that an electron has, on the average, one chance in a hundred of producing an ion pair at one microbar of pressure.

Further, as long as the electron has sufficient energy to overcome the particle ionization potential, the energy level for typical thermionically produced electrons is not of great consequence.

Similar sensitivity factors are appropriate for at least some radioactive ion gages such as a radioactive ionization gage using a tritium source.* However, the resulting ion currents are small compared with a thermionic gage

*See forthcoming final report on Contract AF19(604)-545.

because the number of ionizing source electrons is very small. This factor, a limit posed by practical considerations in the use of radioactive ionizing sources, probably precludes use of radioactive ion gages at very low densities. An additional factor that limits the usefulness of these devices is a dark current which overwhelms the ion current at densities that are relatively high in the sense of this report. The dark current is a manifestation of collector-electron emission resulting from x-ray emission caused by the high-energy radioactive source electrons, ions, or nuclei, as the case may be.

Phillips cold-cathode ionization gages are not useful at pressures less than 10^{-6} to 10^{-7} mb Hg for the reason that the gaseous conduction which underlies their operation cannot be sustained at the lower density values. Thus the thermionic ionization gage appears to be the only useful type at pressures to be encountered at very high altitudes.

At 300 km, and the assumed pressure of 2×10^{-8} mb, the ion current, assuming ground air composition, can be determined from:

$$i = 10^{-3} SP, \quad (5.1)$$

where

i = amp,

S = sensitivity in μ a per microbar, and

P = pressure in millibars.

Thus, for this example one should expect:

$$\begin{aligned} i &= 100 \times 2 \times 10^{-8} \times 10^{-3} \\ &= 2 \times 10^{-9} \text{ amp} \end{aligned}$$

Currents of this magnitude do not pose difficult problems for instrumentation design, for currents as low as 10^{-12} amp are presently being successfully measured in upper-atmosphere research program experiments.

The measurement of the ion current produced by an omegatron is somewhat more difficult but quite feasible. Although omegatron and ion-gage sensitivities are comparable, the ionizing currents that have been employed in omegatrons are considerably less, resulting in reduced ion currents. It appears, however, that the currents may be increased substantially, thus easing the measurement problem. In reference to an ion gage, it may be possible to

employ increased ionizing current, thus increasing the ion current by pulsing the accelerating voltage (grid voltage) rather than permitting ionization current to flow continuously. In this fashion a much greater (factor of 10 or perhaps 100) ion current can result, reducing the sensitivity required of the measuring instrument. This technique was employed in early V-2 work by the author for other purposes and was found to be satisfactory. A similar treatment of the omegatron may prove to be feasible.

Accordingly, for this and other reasons, it seems reasonable to conclude that the thermionic ion gage and the omegatron, with presently attainable current-measuring equipment, have adequate sensitivity for use in measurements of the type discussed in this report.

VI. SOME POSSIBLE DIFFICULTIES IN USE OF THERMIONIC ION GAGES AND OMEGATRONS

The problem of measuring very low pressures at very high altitudes is not different in principle from performing the measurements in the laboratory, with the major exception of unattended operation. Thus perturbations caused by outgassing of gage elements and the vehicle, pumping action by gage cathode, adsorption and absorption by gage elements, etc., must be considered. As the surfaces of the carrier become heated by solar radiation, they will produce quantities of gas which can perturb the measurement. An additional complication results from selective sorption of gases.

The hot cathode of an ion gage or omegatron is a chief offender in regard to absorption and particularly chemical reactions with the gas being measured. Thus steps to reduce cathode temperature and perhaps "on time" would be desirable from this standpoint. Specifically, it appears desirable to employ oxide-coated rather than pure tungsten cathodes to enable lower temperatures and to conserve power. Problems resulting from the potential assumed by the vehicle can likewise conceivably cause perturbations upon the assumed environment. Some studies bearing on this problem are being made in connection with current activities related to a bi-polar probe technique.

VII. CONCLUDING REMARKS

This report has presented a discussion of some measurement techniques which seem to the writer to offer a good possibility of very-high-altitude measurements of pressure, temperature, density, and composition. Considerable development work will of course be necessary prior to experimental use; however, the apparent promise of significant data should provide sufficient impetus for the development effort required.

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