

use of an orthonormal code.^{1,8} The code makes it possible to generate an orthonormal system of functions Q_j ($j = 1, \dots, m$) from the set of basis functions b_j . The functions Q_j have the following properties:

1) The inner product (Q_j, Q_k) is 0 or 1 over the set of n points \mathbf{X} :

$$(Q_j, Q_k) = \sum_{i=1}^n Q_j(\mathbf{X}_i) Q_k(\mathbf{X}_i) \quad (3)$$

$$= 0 \quad (j \neq k) \quad (4)$$

$$= 1 \quad (j = k) \quad (5)$$

2) A least-squares approximation of any function c_K over the set of n points \mathbf{X} , is given by

$$c_K \doteq \sum_{j=1}^m (Q_j, c_K) Q_j \quad (6)$$

In other words, if we let $b_j = Q_j$ in Eq. (1), then

$$\alpha_{Kj} = (Q_j, c_K) \quad (7)$$

Those Q_j 's, whose inner products with the function being approximated are small, are eliminated. This follows from Bessel's inequality

$$\sum_{i=1}^n c_K^2(\mathbf{X}_i) \geq \sum_{j=1}^s (c_K, Q_j)^2 \quad (s = 1, 2, \dots) \quad (8)$$

This allows the number of α_{Kj} 's stored in the memory of the guidance computer to be minimized and the memory elements used for other essential activity.

3) If the region R over which the control functions c_K are defined is known in a geometrical sense, then the approximation may be improved by defining the inner product as the multiple integral over this region:

$$(b_j, b_K) = \int_R \dots \int b_j(\mathbf{X}) b_K(\mathbf{X}) dX_1 dX_2 \dots dX_m \quad (9)$$

Usually this region R can be approximated geometrically as a series of truncated hypercones and hypercylinders.^{9,10} The integral (9) may then be evaluated numerically from formulas developed by Stroud.⁵ Additional information about the control functions such as values of partial derivatives may be incorporated into an approximation method⁷ using an orthonormal code to yield highly accurate approximants.

Wheeler,¹² through an analysis of the Euler-Lagrange equations, expressed the tangent of an optimal thrust angle as a rational function of the instantaneous state variables as in Eq. (2). This model requires the evaluation of forty undetermined coefficients, a nonlinear problem in numerical analysis. The author suggested several years ago⁸ that this type of problem can be handled by linear programming (L.P.). Moreover, L.P. allows one to specify the magnitudes of the errors of the approximant at predetermined points. Suzuki⁶ developed a special L.P. method for nonlinear problems of approximation which overcomes some of the difficulties of ordinary L.P. systems written for commercial applications. In that paper, a solution to the problem of determining the greatest error is given, i.e., determine ϵ such that

$$\epsilon = \min_{\alpha_j, \beta_j} \max_i \left| \left[\sum_{j=1}^M \alpha_j b_j(\mathbf{X}_i) \right] \left[\sum_{j=1}^M \beta_j b_j(\mathbf{X}_i) \right]^{-1} - c_K(\mathbf{X}_i) \right| \quad (10)$$

This is of importance since a large value of ϵ would indicate a change of the form of the approximant should be made. Other approaches to error analysis are given in the paper by Weinberger and Golomb.¹¹

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Magnetic Confinement of an Electric Arc in Transverse Supersonic Flow

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Introduction

A METHOD has been developed for the magnetic confinement of a d.c. electric arc in an unheated supersonic air stream directed normal to the electric field. (Here a confined arc is one restrained within the limits of the freestream, at a fixed station.) The arc column, when confined by this method, exhibits remarkable spatial stability. The absence of appreciable fluctuations in the length and geometry of the positive column makes meaningful measurement of column voltage gradient, average electrical conductivity, and electrode fall voltage possible. The time available for

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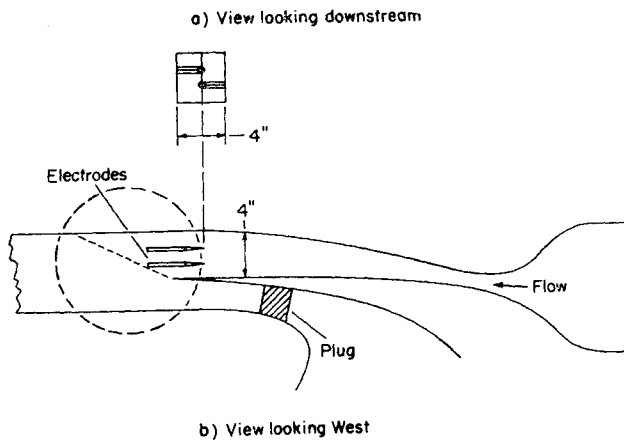


Fig. 1 Schematic drawing of test setup.

observation is of the order of seconds, indicating that detailed diagnostic studies of the structure and energy transfer processes of the high-speed convective arc column are feasible by this method. This note describes the experimental method and presents observations related to the confinement mechanism. Reference 1 presents further observations and quantitative data on the confined arc. Reference 2 gives a detailed presentation and analysis of results to date, along with a review of the literature on the convective arc and a discussion of the experimental techniques involved in arc establishment and stabilization by the present method.

Experimental Approach

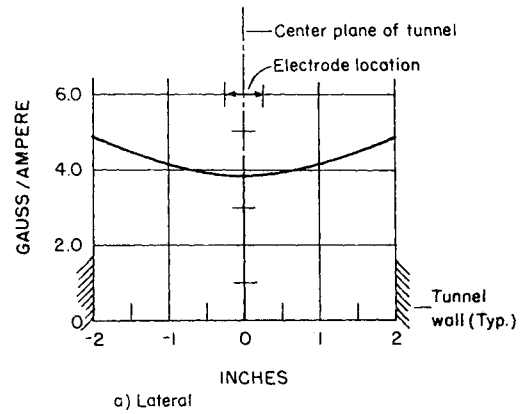
The method of confinement is based on the use of rail electrodes and a nonuniform externally applied magnetic field. The rail electrodes (two cone cylinders, 7.5° half-angle × 0.5 in. diam × 6 in. long) are mounted in a supersonic wind tunnel and oriented parallel to the freestream (Fig. 1). External coils provide a magnetic field that is normal to the electrode plane. Between the rails the electric field, magnetic field, and freestream velocity field are mutually perpendicular. The sense of the magnetic induction vector is chosen such that there will be a component of the magnetic Lorentz force on the arc in the upstream direction. The field coils, whose streamwise location is adjustable, are placed (see dotted circle in Fig. 1) such that there is a monotonic increase in magnetic flux density from the upstream to the downstream ends of the rails (Fig. 2).

The arc is initiated by the explosion of a wire between the upstream ends of the rails. The plasma column is then convected along the rails until a location and configuration are reached where aerodynamic and electromagnetic forces come into balance. The established arc remains here until the arc current is shut off.

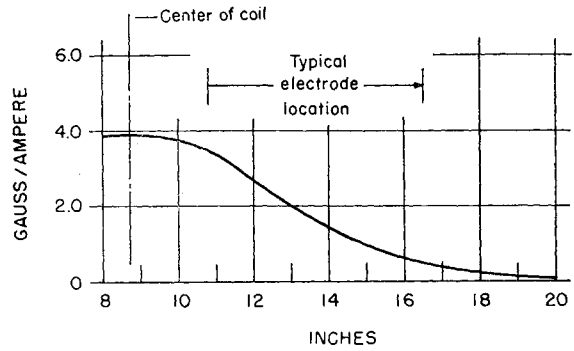
Rail electrodes and the monotonically increasing induction were used to increase the probability of force balance for a given run, by providing a range of values of external induction, and to provide a balance stable to streamwise perturbations in arc location.

Confined Arc

The arc streamwise location along the rails is determined by the freestream Mach number and pressure, the field-coil current, and the streamwise distance between the field-coil axis and the electrodes. When these conditions are such that both arc roots can fall at stations sufficiently remote from the upstream and downstream ends of the rails, the positive column exhibits high spatial stability, and the fluctuations in arc voltage are small (±4%). Figure 3 shows one frame from a high-speed (2500 frames/sec) motion picture of such a



a) Lateral



b) Longitudinal

Fig. 2 Calculated spatial variation in transverse component of magnetic induction.

stable arc. The camera was mounted to give a view looking downstream at a slight angle with the electrode plane.

In Fig. 3, it is seen that the anode root (bottom) is contracted for about 1/4 in. above the anode surface. The cathode root is also contracted, and is curved around the cathode so that the cathode spot is located on the top side of the cathode, away from the anode.

The positive column is clearly evident between the root constrictions of the arc in Fig. 3. From the high-speed motion picture, there were no observable spatial fluctuations of the column during this run. Thus the fluctuating loops and spirals, which characterize the electric arc moving along rails in still air,³⁻⁵ are properties of the convective arc column which do not necessarily carry over to the wind-tunnel experiment.

The slanting of the column (at about the Mach angle), obvious in Fig. 3, does not appear to result from a simple Hall effect.^{1, 2}

Fig. 3 The stable arc. The conditions for this run were as follows: arc current, 132 amp; arc voltage, 147 v; interelectrode spacing, 1.1 in.; electrode material, OFHC copper; Mach number, 2.5; freestream stagnation pressure, 20.2 in. Hg; freestream stagnation temperature about 530°R; an external magnetic induction at column, 2900 gauss.



The far-side location of the cathode root could be due to the streamwise component of magnetic induction; this component is of opposite sense on opposite sides of the electrode plane, and the Lorentz force on the cathode root would always be toward the far side of the cathode. A similar effect might also be expected from the self-magnetic field of the curved arc root.

The evidence is strong that magnetic confinement for the stable arc is determined by dynamic processes in the positive column: 1) streamwise arc location along the rails changes with field-coil location as well as with freestream conditions; 2) arc location is not affected significantly by sharp ridges cut into the cathode material, or by $\frac{1}{8}$ -in. flow baffles placed upstream of the cathode root; and 3) arc column location (and therefore stabilizing induction) is essentially the same for copper and carbon electrodes and for 0.6- and 1.1-in. interelectrode spacings.

There is an apparent disparity between these results and the results of rail-accelerator experiments in still air,^{6,7} since the rail-accelerator experiments indicate that the moving arc is dominated by cathode root phenomena rather than by processes in the column.⁷ This disparity is probably due simply to the fact that with the present setup there is no requirement for motion of the cathode spot over the cathode surface as there is with the rail accelerator.

In addition to the stable mode of arc confinement described previously, a fluctuating mode was observed whenever conditions were mismatched such that a root station for the stable arc would have fallen at or beyond the upstream or downstream end of the rail (cylinder). This mode is manifest by wild spatial fluctuations of the column and by concomitant fluctuations in arc voltage. These fluctuations evidently result not from any intrinsic instability in the column itself, but from root constraints that interfere with the stable column configuration.

The observation that root constraints may cause column fluctuation suggests that the fluctuation observed³⁻⁵ with the rail accelerator may have resulted from constraints imposed on column motion by the requirements for root motion, e.g., root motion in the stepping mode. If this were so, then one might expect that, with the removal of the requirement for root motion, a dramatic increase in column stability might occur, as is observed in the present experiment.

Conclusions

1) It is possible to magnetically confine within a supersonic airstream, a stable arc discharge sustained by an electric field essentially normal to the flow vector. Confinement results from the use of rail electrodes and a transverse, externally applied magnetic field with monotonic increase in flux density from electrode tip to electrode base.

2) When conditions are such that the arc is held between the rails at streamwise locations sufficiently remote from the rail ends, the positive column is characterized by high spatial stability.

(3) Confinement in the stable mode is determined by dynamic processes in the positive column and is independent of material or flow conditions at the surface of the cathode. In this mode, the arc column is confined between the rails in a region where the electric field is essentially two-dimensional. The streamwise location of the arc is determined by the streamwise location of the external field coils, by the field coil current, and by the freestream flow conditions.

4) The stably confined arc is characterized by a well-defined luminous column, with concentrated root marks on the electrodes, rather than by a discharge sheet extending in the streamwise direction.

5) In Mach 2.5 air flow with a freestream stagnation temperature of about 530°R, the average magnetic induction required for stable confinement of a 300-amp-d.c. arc varies with freestream stagnation pressure from about 1900 gauss at 10-in. Hg to about 3500 gauss at 25-in. Hg.

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Trajectory Extrapolation in a Central Force Field

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Introduction

THE path of an orbiting vehicle may be described analytically by its deviation from a nominal path. To establish a nominal path, we assume a central force field and extrapolate motion from the vehicle's current state. Our procedure yields a set of integration constants that aid extrapolation and lead to a simple description of path deviations. In this respect, we assume that the force field is indeed central; hence path deviations arise exclusively from small errors in the vehicle's current state. As in Ref. 1, a set of error coefficients is obtained by partial differentiation of the integrals of the equations of motion. If the force field is only approximately central, path deviations also arise from errors distributed along the path. The influence of distributed error can be approximated by integrating a system of linear differential equations, a technique not pursued here but described in Ref. 2.

State Extrapolation

The following variables describe the vehicle's current and terminal states:

- r, R = radial distances from the attractive center
- u, U = transverse velocities
- v, V = radial velocities
- T = time difference between the current and terminal states
- θ = centric range angle between the current and terminal states

where r , u , and v refer to current state and R , U , and V to terminal state. It is clear that both T and θ decrease monotonically to zero during the transformation from the current to the terminal state. Thus, either T or θ can serve as an

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