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Final Report

RESEARCH IN THE MEASUREMENT OF AMBIENT PRESSURE,
TEMPERATURE, AND DENSITY OF THE UPPER
ATMOSPHERE BY MEANS OF ROCKETS

Prepared by

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ABSTRACT

A summary discussion of research activities that took place under the subject contract is given. A listing of reports and papers prepared during the contract is included, as well as two appendices to formalize earlier informal letter reports.

1.0 INTRODUCTION

The Electrical Engineering Department of The University of Michigan has been, for many years, carrying out a program of upper-atmosphere research for, and in cooperation with, the Geophysics Research Directorate of the Air Force Cambridge Research Center. The program has been concerned with (a) the development of means, using rocket vehicles, for making suitable measurements of the ambient pressure, temperature, and density of the atmosphere above levels readily attainable by balloon; (b) the carrying out of measurements to establish the usefulness of the equipment developed; and (c) the determination of the stated properties of the atmosphere.

This program has been conducted under a series of contracts; this report summarizes the results of efforts put forth under Contract No. AF 19(604)-545. The contract covering the previous period of investigations was AF 19(122)-55, and the contract covering a subsequent program is numbered AF 19(604)-1948. The latter contract is in force at the time of writing this report.

The work statement of the last supplement to the contract for which this report is a final report follows.

"A. The Contractor shall, during the time specified herein, furnish the necessary personnel and facilities to conduct studies and experimental investigations by means of rocket-borne instrumentation at Holloman Air Development Center and Fort Churchill directed towards the determination of average values and seasonal variations of the pressure, temperature, density and wind of the upper atmosphere, primarily between the altitude of normal meteorological balloon ceiling and the maximum altitude of the available rocket. Atmospheric pressure, temperature, density, and winds will be determined from measurements of pressures that result from air flow around bodies of suitable geometry. In conducting this investigation the Contractor shall:

- (a) Investigate and develop temperature, pressure and density measuring devices such as the ionization pressure gauge and wind vane.
- (b) Investigate and develop means of determining angular orientation of rockets.
- (c) Investigate and develop means for determining air stream velocity along the surface of a cone or other geometric configuration.
- (d) Investigate and develop means for presenting data to transmission and/or recording systems.

- (e) Investigate the importance of boundary layer phenomena and determine such of its properties as are necessary for the suitable correction of measurements directed above.
- (f) Prepare a report containing the general engineering specifications of such a system specified in (a) above suitable for use in a small rocket of the Deacon type. This report is to contain detailed drawings of any unique items such as radioactive ionization pressure gage sub-system.
- (g) Process atmospheric pressure, temperature, and density data obtained in conjunction with test flights of the developed instrumentation, with the object of producing useful upper atmosphere data."

2.0 SUMMARY OF CONTRACT EFFORT

During the period of this contract, several related topics were pursued, most of which were in the area of equipment development. Two major developments were:

- (a) Prototype and first-flight models of a completely self-contained electronic system employing a radioactive ionization gage for the measurement of rocket-nose-cone surface pressure over the range of atmospheric to less than 10^{-3} millibars, with a precision of approximately 1 part in 50 or better.
- (b) A rocket-nose-cone surface flow-angle measurement device called a windvane. A method for determining Mach number using flow-angle information was developed, and the windvane equipment was devised to implement the experiment.

Several less extensive developments were carried out to supplement and complement (a) and (b) above and to assist in other areas of the contractual effort. These include:

- (c) Development of transistorized power supplies for flight use to provide:
 - (1) 24-volt d-c to 110-volt a-c 400 Ω supply for gyroscope power;
 - (2) 24-volt d-c to high-voltage d-c to power circuitry of windvane system;
 - (3) 6-volt d-c to high-voltage d-c to power radioactive ionization-gage system noted in (a) above.
- (d) Study of rocket attitude determination through the use of a single gyroscope mounted in several positions.
- (e) Development of flight camera using 35-mm film running continuously and associated strobe-light system for gyro-face data recording.
- (f) Adaptation of ionization-gage temperature-pressure measurement system to small (6-in. diameter) rocket, of Nike-Cajun type.
- (g) Modification of existing Air Force type J-8 attitude gyro to Aerobee rocket use.
- (h) Wind-tunnel study of windvane protecting cover and preliminary windvane circuitry.

In addition to equipment development, and theoretical studies, several reports were written, and several technical papers presented and published. These will be summarized in a later section of this report.

Several field tests were carried out during the course of the contract: (a) two Aerobee launchings in which the group was the major participant were instrumented; (b) two additional Aerobees carried minor equipment items furnished and serviced by project personnel; (c) prototype equipment was given a nonoperating physical test during 2 rocket sled tests at HADC; (d) a prototype Nike-Cajun firing (AM 6.30) employing radioactive ionization gages was carried out at WSPG preparatory to the IGY Fort Churchill program; and (e) a Nike-Cajun (AM 6.31) and an Aerobee (AM 2.21) were instrumented for pressure, temperature, and density measurement by the ionization-gage method. Although these instrumentations were prepared for firing during the term of this contract, the launchings were carried out at Fort Churchill during October of 1956 under a continuing contract.

3.0 SUMMARY DISCUSSION OF RESEARCH TOPICS

3.1 RADIOACTIVE-IONIZATION-GAGE PRESSURE-MEASUREMENT SYSTEM

The system developed and employed by this research group for the measurement, using rockets, of high-altitude pressure, temperature, and density is based upon the determination of nose-cone surface pressures. Measurement, for example, of the "impact" or "total head" pressure and the pressure at a point on the cone wall permit determination of the free-stream Mach number, by taking the ratio of these two pressures. Knowledge of the nose cone's velocity, taken with the Mach number, permits a temperature computation. Similarly, knowledge of the Mach number permits determination of the ambient pressure from the cone-wall pressure.

The experiment can be carried out for modest yaw angles and in altitude regions where the Reynolds number does not fall below several hundred. This means, in general, that the upper-altitude limit is reached in the region of 90-100 km.

The pressure transducer employed to enable measurement of the desired cone pressure has been developed through years of use. The latest version (at the time of writing this report) employs a radioactive ionization gage with a tritium (H^3) source, a unity feedback d-c amplifier, an automatic range-changing circuit, a transistorized power supply, and self-contained battery power source. The system is capable of measuring the pressure in the chamber of the ionization gage to a precision of at least 1 part in 50 over the range 10^{-3} to 10^{+3} millibars.

Much of the initial development of this system was carried out under this contract. Prototype models were constructed and test-flown on Nike-Cajun AM 6.30 as discussed in another section of this report. Figures 3.1 and 3.2 illustrate the prototype models.

Complete details of the pressure-measurement system are given in a separate report (see Section 4.2), and a detailed discussion of the temperature-measurement method is given in a technical paper (see Section 4.6).

3.2 WINDVANE EXPERIMENT

Consideration of the aerodynamic flow about a right circular cone moving with a modest angle of attack, at Mach numbers of the order of 1 to 5, led to studies of the possibility of making measurements of the Mach number by determining the angle of flow of the air over the cone surface. Laboratory test models were constructed and wind-tunnel tests were carried out to substantiate the theoretical results with measured values. Figure 3.3 shows the test model.

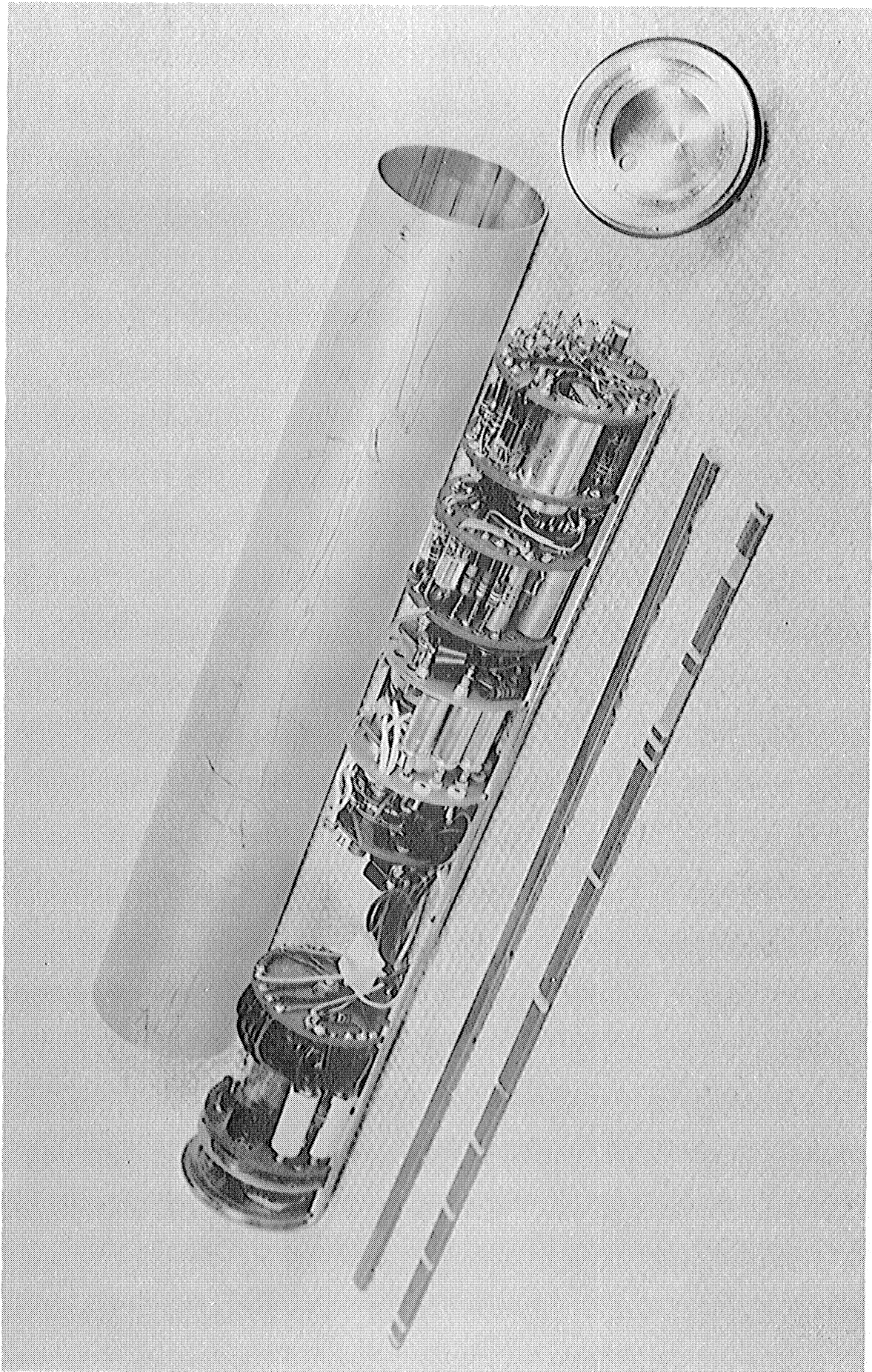


Fig. 3.1. Prototype of radioactive-ionization-gage pressure-measurement system, with cover removed.

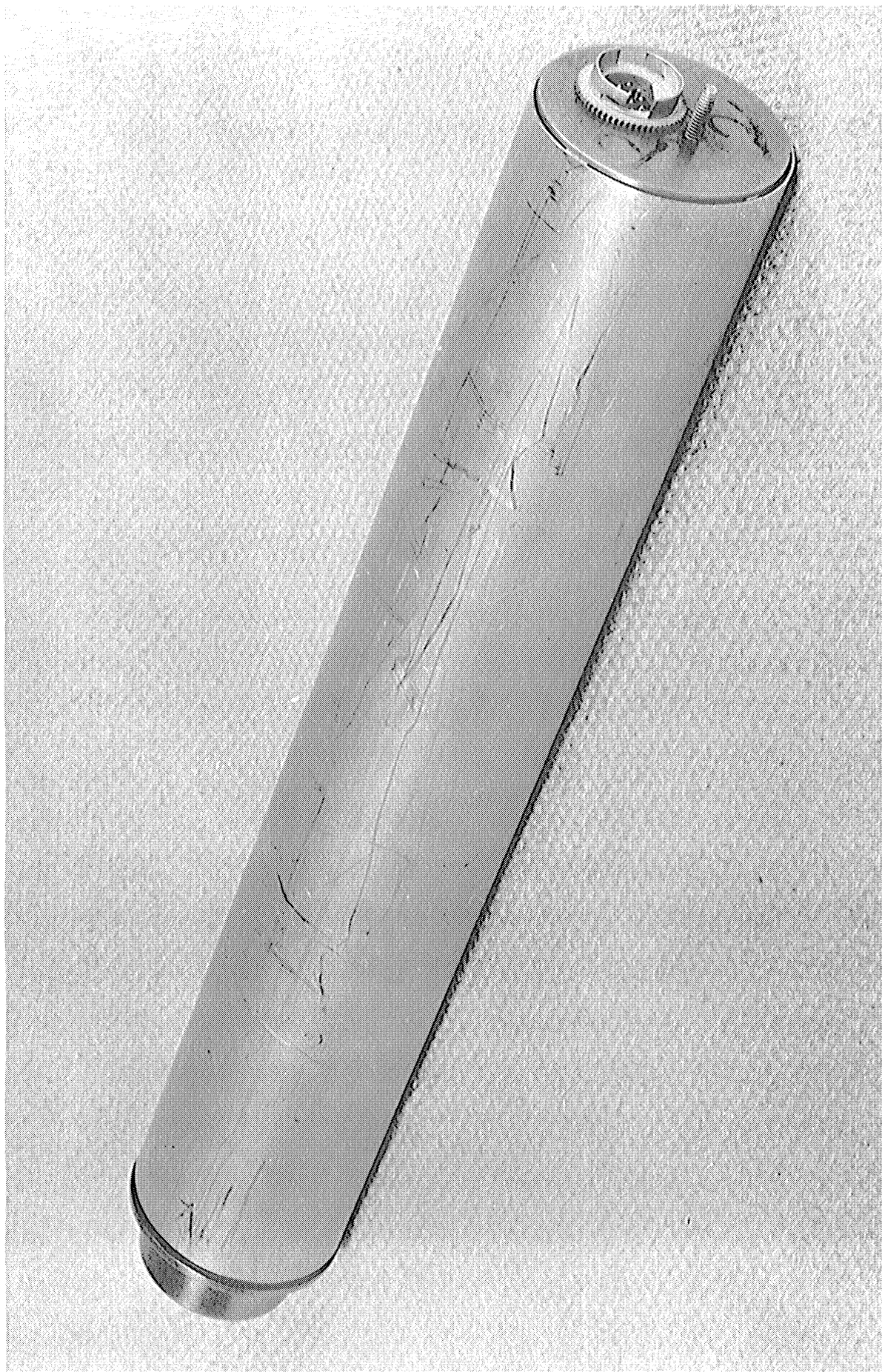


Fig. 3.2. View of prototype radioactive-ionization-gage pressure-measurement system, with cover on.

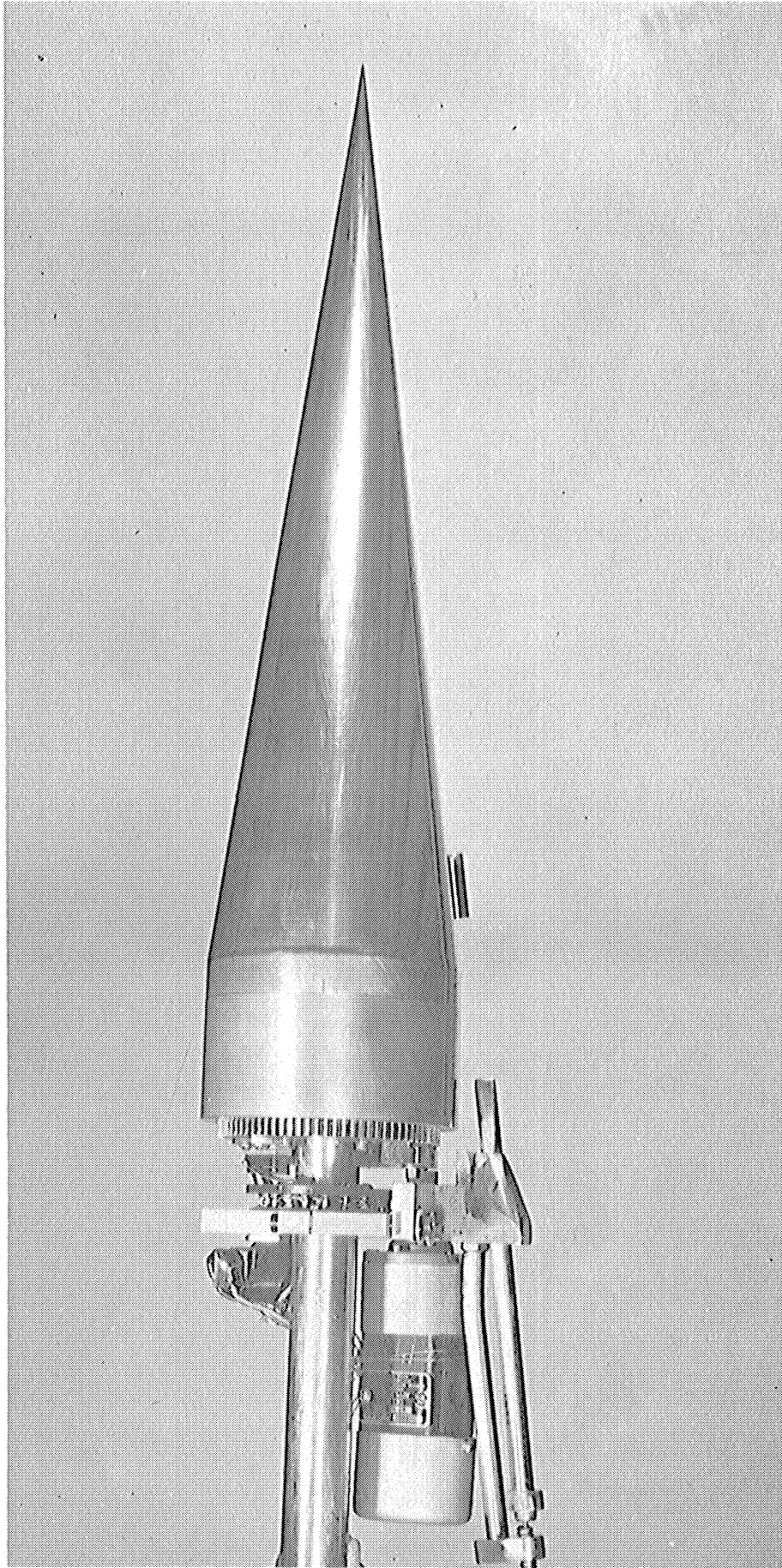


Fig. 3.3. Wind-tunnel test model of windvane equipment.

The flow angles in the wind tunnel were determined by measuring the angular position of a small 3/4-in. by 1/8-in. by 0.003-in. metal plate mounted perpendicular to the cone, the long dimension becoming aligned with the flow. The vane-supporting shaft activated a small capacitor which determined the frequency of a timed circuit. The circuit was supplied from a 10.7-mc stabilized oscillator, slope detection permitting determination of resonance variation and consequently of vane angle.

The data obtained from the wind-tunnel tests substantiated the theoretical study. A discussion of the tests and results appears in a scientific report (see Section 4.2).

In addition to work on these aspects of the problem, development of the equipment to permit a rocket-borne exploratory experiment was also pursued. It appeared, as a result of the wind-tunnel tests, that it would be worthwhile (during a rocket flight) to provide a small protective cover for the vanes until the flow pattern was well established at stream velocities above Mach 1, and to prevent excessive heating. Thus a small jettisonable cover was devised and checked, using high-speed photography, in the wind tunnel. Subsequent rocket launchings employed the protective cover, which, however, was later shown to be unnecessary. A "Fastex" photo during ejection in the wind tunnel is shown in Fig. 3.4, and a view of transducer and vane in Fig. 3.5.

Following the wind-tunnel tests, a flight instrumentation was prepared, based upon the results of the tests, and a flight test (AF 48) took place. Two vanes were installed at a point approximately 20 in. from an Aerobee nose-cone tip, spaced 120°. The vanes consisted of rectangular pieces of steel feeler gage stock 1/4 in. by 3/4 in. by 0.003 in., and were soldered with special high-temperature (600°) solder to the shafts of project-built transducers. Each transducer was essentially a three-plate condenser, carefully developed utilizing jeweled bearings and sapphire ball plate spacers. The objective in the design was to produce a device with minimum friction, as well as reasonably low mass, to permit precise alignment of the windvanes with the airstream at relatively low densities. It was determined in this regard that operation would be satisfactory to altitudes as great as 50 miles. To promote reproducibility of capacity as a function of angle, the tolerances to which the transducers were constructed were held as small as was feasible, considering reasonable machine-shop facilities. Thus the vane shafts were centerless ground, the diameter being chosen so that the clearance between the shafts and jewel bearings was about 0.0003 in. Shaft-end play was minimized by holding the pointed end of the shaft against a cap jewel through use of a small slug of Alnico. These provisions resulted in a device wherein the "play" was discernible only microscopically.

The transducer (capacitor) was connected in a parallel resonant circuit tuned slightly off resonance. Power at approximately 10 Mc was inductively coupled into the tuned circuit, across which was connected a crystal detector. Thus changes in capacitance of the transducer could produce changes in the

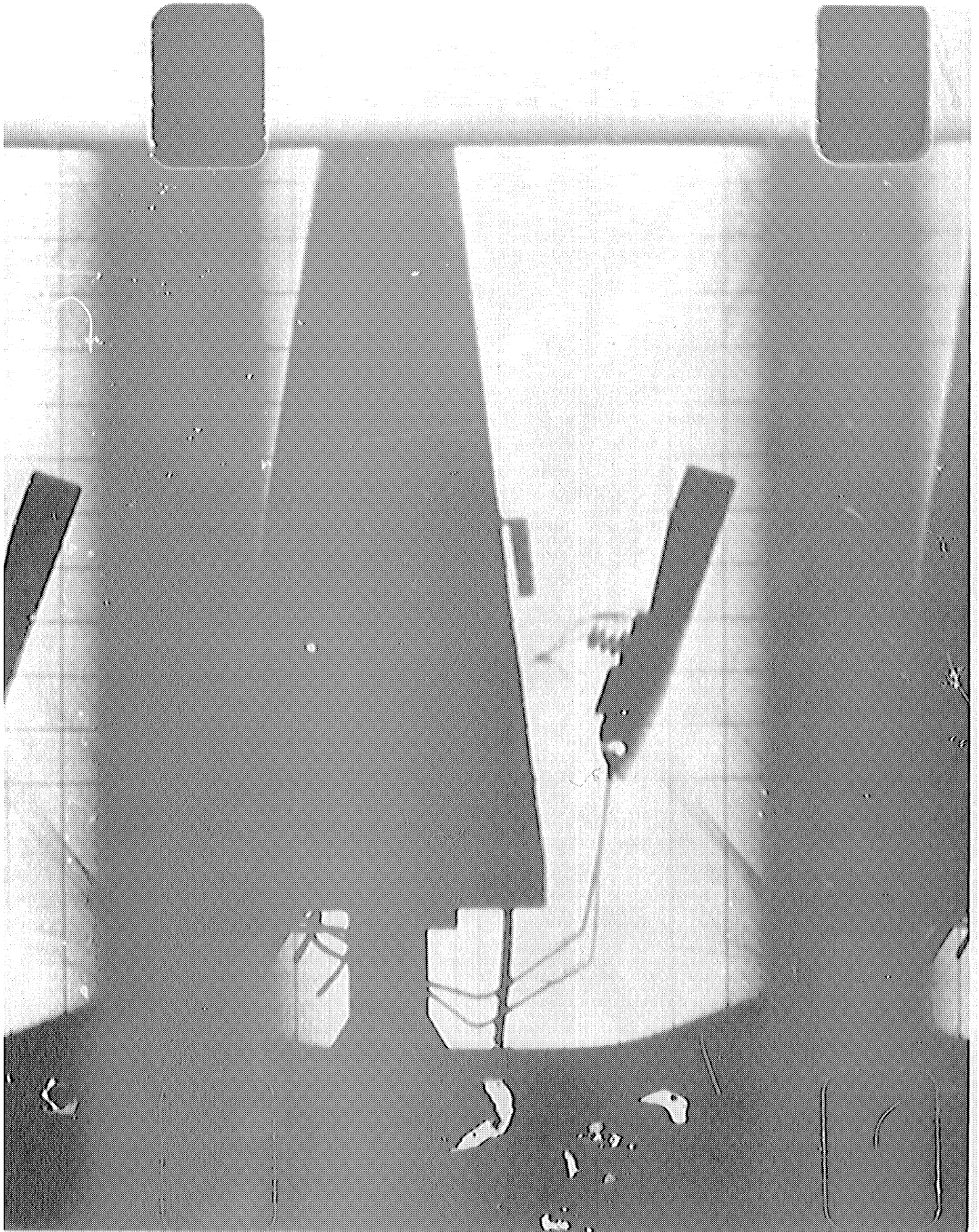


Fig. 3.4. Copy from "Fastex" camera film of windvane cover ejection at high Mach number in wind tunnel.

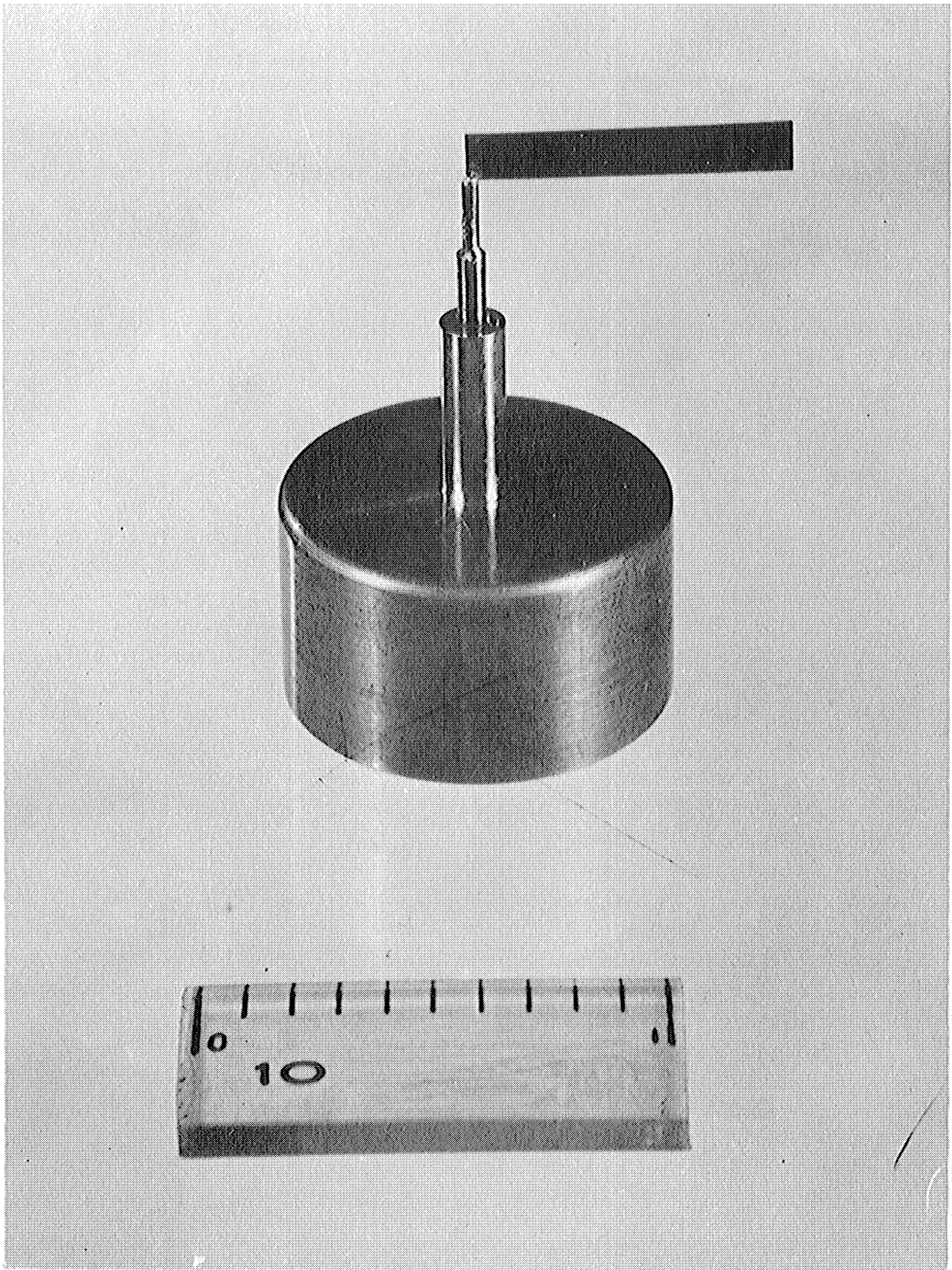


Fig. 3.5. Test windvane and associated transducer employed in wind-tunnel tests.

output voltage of the detector, and hence a measurement of angle change of the transducer. The power source of the tuned circuit employed a crystal oscillator and amplitude stabilized Class-C RF amplifier. In addition to this precaution for constant excitation of the resonant circuit, B+ power was obtained from a regulated power supply. Also the tuned circuit-detector system was temperature-controlled.

The output of the detector was applied to a cathode follower and thence to telemetering. To stabilize operation of the cathode follower, the heater supply was regulated by both Amperite current regulators and Thyrite regulators. B+ was obtained as above from the regulated power supply.

As a result of these precautions, the complete windvane system exhibited excellent stability in permitting the measurement of vane angles to an accuracy believed to be better than 0.1° . The goal in the development was reproducibility to 0.03° .

Instrumentation of this type, mounted on the outer surface of a missile nose cone, is of course subjected to rather high temperatures. This was considered in the design and accounts, e.g., for the use of high-temperature solder in the vane mounting, and to some extent for the clearance of shaft bearings. Further, it was deemed necessary to keep the vanes covered during the portion of the flight prior to missile burnout for protection from excessive heating during this critical heating period. Small aluminum wedge-shaped covers were accordingly designed. They were secured to the cone surface by piano wire which was spring-loaded. When removal was desired, it was then necessary only to apply power to the wire, melt, and thus part it, allowing the covers to be ejected by the springs. Cover alignment was maintained through use of integral keels which engaged appropriate slots in the cone surface. The design permitted a "clean" surface after ejection.

As mentioned above, two vanes were employed in this firing. The systems were completely independent except for the common regulated supply. In regard to results of this experiment, it was not firmly established how the vanes operated. A preliminary study of the telemetering record apparently indicated the following:

- (a) normal signals up to the point of cover ejection which occurs at about 30 seconds;
- (b) steady signals (apparently nearly constant) from cover ejection to 54 seconds where telemetering record ceases;
- (c) record blank until 90 seconds when apparently nearly constant signals again appear momentarily;
- (d) record returns for a few seconds at 118 seconds at which time recalibration occurs as planned;

- (e) record returns periodically until 182 seconds when it again fades and flag outputs are nearly constant whenever seen; and
- (f) record returns at 272 seconds where vane outputs show marked differences.

The above brief description of the early portion of the record gives some indication of the evidence available to determine operation of the system. The record up to 54 seconds appeared to indicate satisfactory operation. Although less constancy might be presumed during the period of flight following burnout, the Aerobee as studied in earlier University of Michigan flights generally does not experience much yaw, usually less than 1 to 2 degrees. This magnitude would produce vane voltage changes of the order of 0.1 volt.

The constancy of the outputs at about 90 seconds and later is, however, quite questionable, as greater angles than indicated are expected during that period. Had information from other equipment indicated that the missile did experience the expected angles, this constancy of vane output would perhaps have suggested that the vanes became separated from the shafts of the transducers some time between 54 and 90 seconds. However, there was angle-change evidence later in the flight and our opinion was and still is that the stream at this time lacked sufficient energy to cause vane failure. Likewise, the appearance of the transducer shafts after impact suggested that the vanes were wiped off at impact, rather than being lost due to excessive heating during the flight.

Thus it seemed more likely than not that the vanes were intact throughout the flight, but that they failed to respond as expected sometime after 54 seconds. An additional fact that perhaps supported this belief was that the circuitry associated with the vanes was later operated successfully, seeming to preclude failure during the flight.

Thus the first flight test of the vane system was unsuccessful. Further study of the results of the test did not clarify the situation. The ultimate conclusion reached was that the vanes were swept away very soon after cover ejection. Figure 3.6 illustrates the instrumentation.

The next step in the development involved an Aerobee experiment wherein an attempt was made to idealize the vane environment from the standpoint of theoretical requirements, and at the same time determine the vane angle and status photographically. Figures 3.7, 3.8, and 3.9 illustrate the vane installation and other instrumentation discussed below.

The forward right-circular nose-cone section employed on this missile differed considerably from those flown in the past. The regular ogive nose skin was truncated at a position of approximately 9 in. in diameter and a right-circular cone of 40° (included angle) was substituted. The increased angle caused greater excursion of the vane for a given yaw angle. The forward $6\text{-}1/4$ in. of the nose cone was machined from invar, followed by a fused-quartz cone approximately $1\text{-}1/4$ in. long and approximately $1/4$ in. thick. The interface between the quartz was a "rung" boundary between two optical flat surfaces.

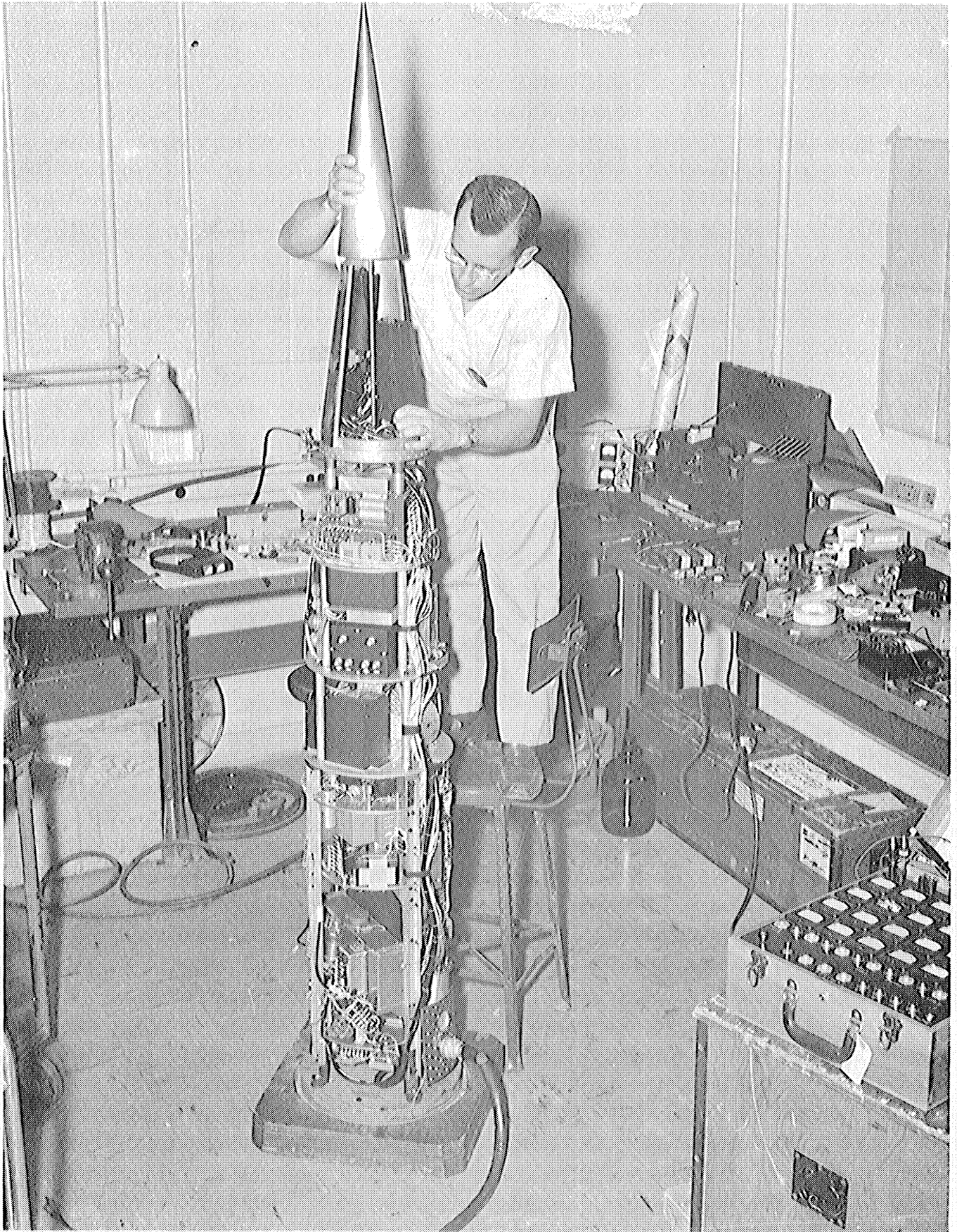


Fig. 3.6. Aerobee AF 48 instrumentation.

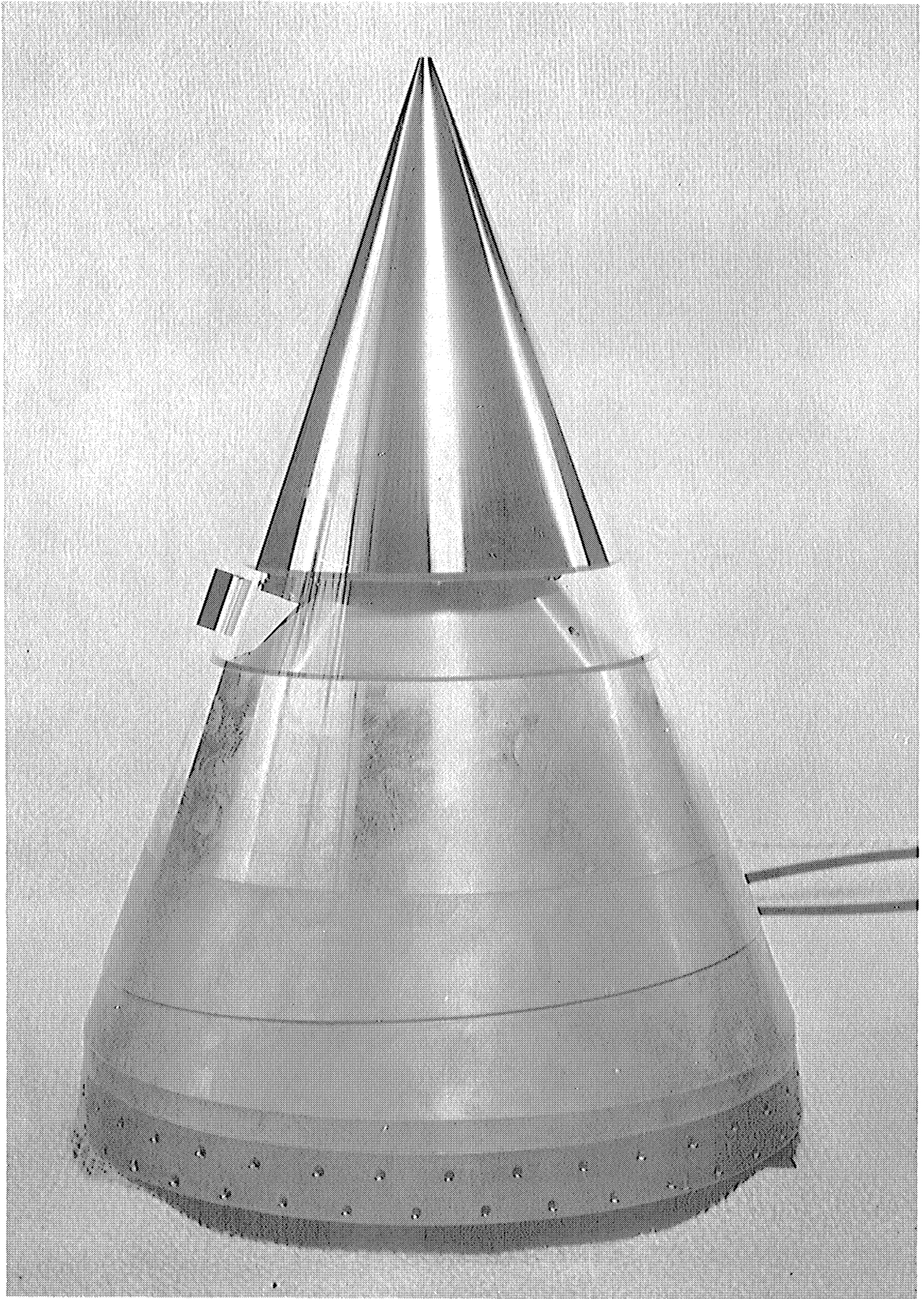


Fig. 3.7. Aerobee AF 58 nosepiece showing quartz ring and vane installation.

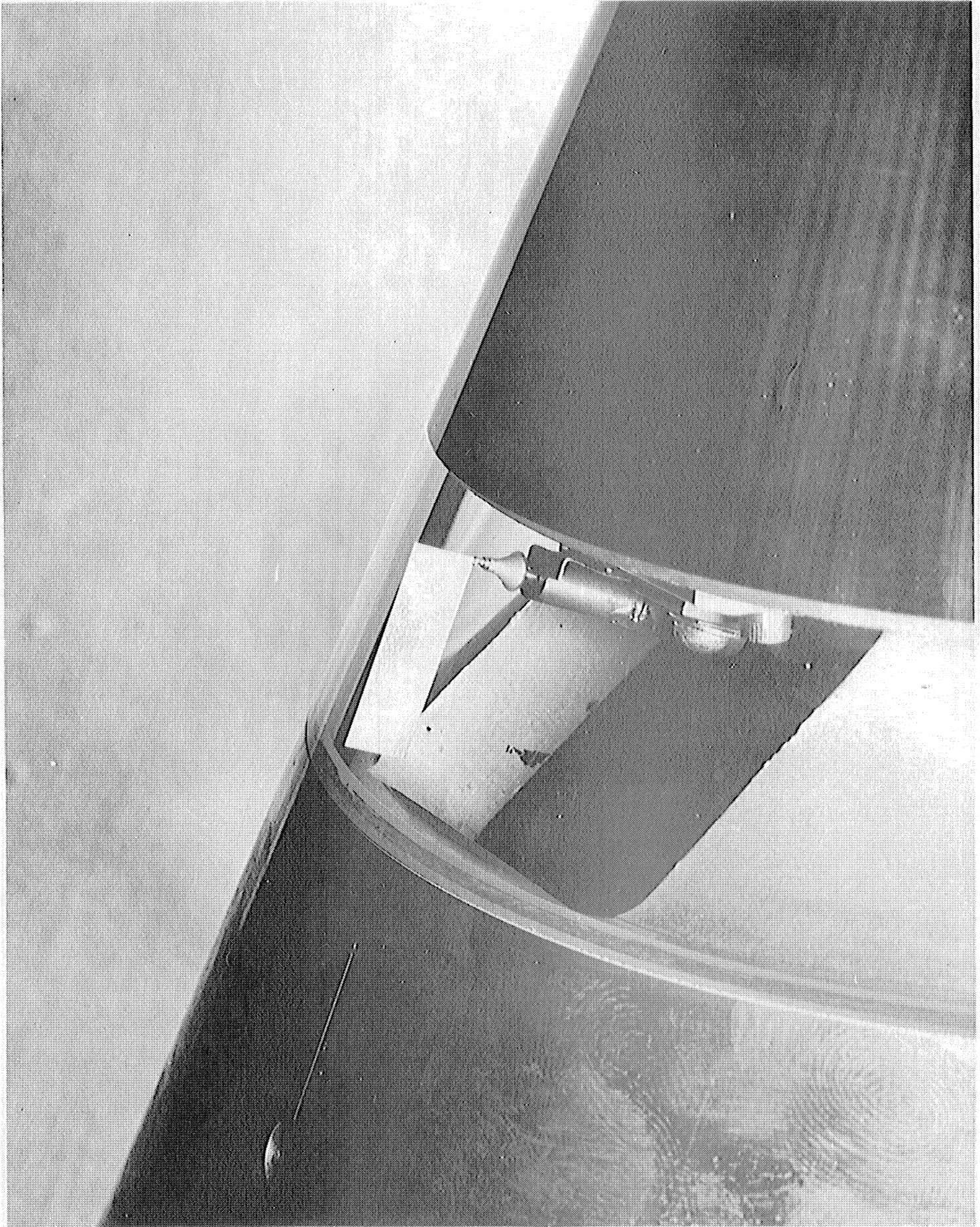


Fig. 3.8. Close-up view of vane in Fig. 3.7. Aerobee USAF 58.

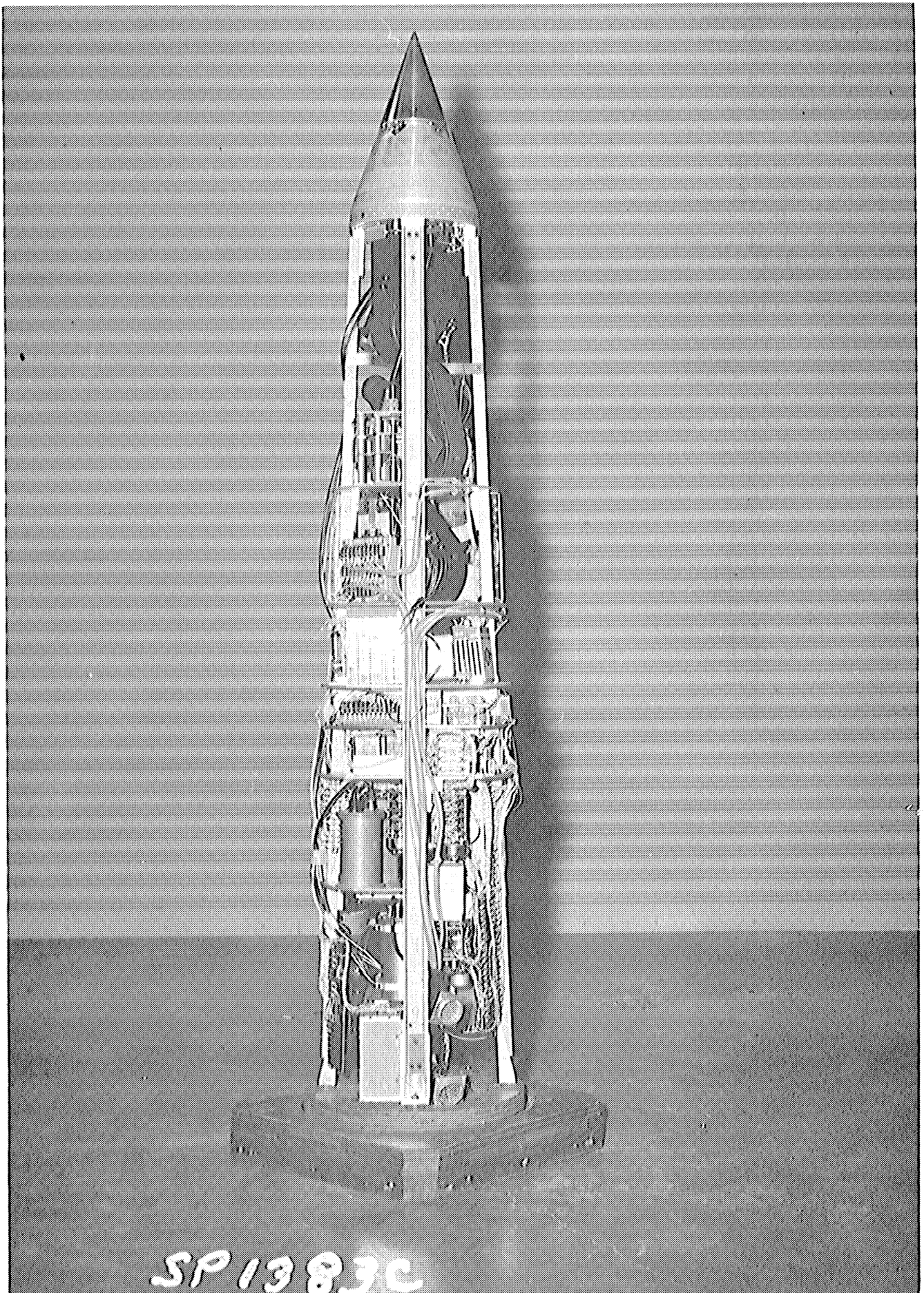


Fig. 3.9. View of complete instrumentation of Aerobee USAF 58. Nosepiece details shown in Figs. 3.7 and 3.8.

The quartz was backed up by an invar retaining ring, but the holding force was to be the interaction between the two optical flat surfaces. Below the quartz, a magnesium section approximately $4\frac{3}{4}$ in. long completed the cone. The magnesium section contained a canting mechanism which will be outlined in a following paragraph. In addition to the effect of forces due to molecular attraction, the quartz was held in place by a ring which was forced against the quartz by a number of small coil springs. The quartz itself did not support the nose section but was, in effect, an insert.

The windvane bearing assembly was affixed to the invar section and the windvane shaft projected through a small clearance hole in the upper portion of the fused quartz, thus being in a position suitable for photography by use of a mirror reflecting the image down along the missile axis.

It was desirable to provide a 6° cant of the forward section of the nose cone so that the vane would register a yaw angle even with the missile in a no-yaw attitude. This was accomplished by cutting the magnesium section at an angle of 3° with respect to a plane normal to the missile axis and mounting a bearing so that the forward section could be rotated. This provided a right-circular cone in the unrotated position and canted the cone off the missile axis by 6° when rotated 180° . The canting was accomplished during the period 34-40 seconds after takeoff.

An additional feature of the canting mechanism was oscillations of the nose tip through an excursion of 3° on either side of the 180° point. This was to program into the system a movement of the flag to demonstrate that the vane bearing assembly was not jammed.

The finishing of the fused-quartz ring was accomplished by a glass-specialty firm. The steps taken in the fabrication of the quartz will be outlined briefly. A fused-quartz blank was cut from a cylindrical billet. This was then cored and the inner surface was polished so that it would be parallel to the outside cone surface. The lower surface of the invar and the upper surface of the quartz were then made optical flats and the quartz was mounted in the nose cone. The outside was then finished to an optical surface, using the invar section as a guide. The clearance hole for the windvane shaft was then drilled with a supersonic drill. The entire assembly was then given a final polish to produce a continuous surface to prevent generation of undesirable shock waves ahead of the vane.

The nose-cone assembly was then returned to us and the light source and mirror were affixed to the magnesium section and, subsequently, the vane bearing assembly was positioned on the invar section. The bearing assembly used was primarily the same as used in the previous transducers: however, no transducing element was provided. The barrel assembly was considerably reduced in size to allow the vane to be photographed. An improved shaft-holding device for more positive axial retention of the shaft was made by cutting a groove in the vane shaft and allowing a piece of flat spring to ride freely in that groove.

The aforementioned holding method was in addition to the magnet holding and was not to be used unless the shaft became separated from the magnet.

The light source was comprised of two flash tubes synchronized with the shutter of a modified B-2 camera. The portion of the lighting system mounted in the nose cone consisted of the two flash tubes and their trigger transformers in a pressurized container. The energy was supplied through pressurized cables from power packs located on the lower instrument rack. The two flash tubes were of special configuration, roughly the shape of a block U. The light was emitted from the base of the U. An aluminizing process was used to increase the efficiency of the light source by providing a reflecting surface on the tube wall. A single front-surface mirror was employed to reflect the vane image downward through the canting mechanism to the camera which also was mounted on the fixed rack.

The camera used for the vane photography was modified as compared to the previous B-2 cameras employed by this group. These alterations were necessary because the camera was mounted in such a position that the lens faced forward. It was felt that the acceleration forces might cause jamming if no modifications were made. Since flash tubes were used for illumination, the film run was continuous; the shutter was left in place only to decrease the film exposure to sunlight. This change involved the removal of the film-advance claw and the addition of an idler on the take-up side of the film pressure plate. Stops were also added in such a way that the movement of the film pressure plate due to acceleration forces was limited. A small microswitch was mounted in such a position that it was actuated by a flat on the shaft originally used to drive the film-advance claw. This switch then initiated the trigger pulse, synchronizing the flashing light to the camera.

The results of this experiment were apparently the same as in the previous flight. There was no photographic evidence to indicate that the vane remained on the shaft after cover ejection.

There was thus a disparity between the flight results and wind-tunnel results. The wind-tunnel tests which reached even higher Mach number than the vanes had to sustain in flight did not cause vane failure except after many "runs" in the tunnel, whereas in flight the vane apparently failed immediately.

Two factors were considered: first, whether the cover in some manner damaged the vane, contrary to wind-tunnel tests (Fig. 3.4), and second, whether the 0.003-in. material used for the vane was not sufficiently rigid.

A new test was undertaken for which the above points were implemented. The cover was eliminated and in one case a vane of heavier material was employed. It was thus decided that a light vane (.003 x 1/4 x 3/4) of platinum iridium and a heavy vane (.010 x 1/4 x 3/4) of platinum would be prepared in the same jeweled bearing assembly as was used before. Neither vane was to be covered.

The instrumentation consisted simply of a circuit to check continuity through the vane. This was accomplished by fashioning a slip ring from a piece of Kovar tubing. A length of about 5/16 in. was cut from the tubing and then about 1/8 in. of this length was ground away until only a thin cross section along the element of the cylinder remained. This was then placed over the shaft in such a manner that it would form a slip ring with the thin section just touching the vane. The Kovar piece was positioned and fastened with a high-temperature cement. The projection that touched the vane was brazed to the vane at the same time that the vane was brazed to the shaft.

The presence or absence of the vane was then determined by allowing the continuity of the above device in shorting out a resistor which acted as a voltage divider in conjunction with the 250,000-ohm input resistance of the telemeter.

In addition to the vane continuity, it was decided that a check was desirable for determining whether or not the pivot became disengaged from the magnet during flight.

It should be pointed out that the construction of the bearing assembly was such that the shaft was positioned by two jeweled bearings and held in the bearings by the pull of an Alnico magnet. A positive stop was provided, but the low-friction position necessary for usable data required that the magnet retain the shaft. Thus the normal cap jewel was removed and continuity was measured through the shaft and the magnet.

The above instrumentation was mounted on the forward access door of Aero-bee (USAF-60) fired at Holloman Air Force Base on 13 October 1955. The results indicated that the vanes survived. Although the data became erratic in the final portions of the flight, indicating some disturbance of the experiment, the overall record appeared to indicate no separation of the vane from the shaft.

An additional later test followed this elementary experiment. It was more desirable to fly the instrumentation in its "normal" nose-cone location, and thus an experiment of like nature was prepared to be flown in January, 1956,* on Aerobee-Hi Test Round No. 3. The test was expanded to include a second light vane with a protective cover which was to be ejected just after burnout. The bearing assemblies of the previous experiment were replaced by low-friction, high-temperature potentiometers to give an indication of vane angle. While the potentiometers were not expected to give the resolution that was desired for the vane machmeter determination of temperature, they would make possible indication of erratic behavior of the vane.

This instrumentation was mounted on nose skin sections furnished by AFCRC. The two vanes mounted on the shafts of the potentiometers were mounted on one

*Launching of the rocket actually took place in May, 1956.

skin section approximately 3 by 5 in., while the covered flag was mounted on a section located on a 4- by 3-in. section which was taken from the nose cone at a position 90° from the 3- by 5-in. section. Figures 3.10, 3.11, and 3.12 illustrate the vane installation of this test.

The results of this experiment indicated that all vanes survived the flight. Positive results were obtained in the case of the heavier of the uncovered vanes; it was recovered still affixed to the shaft at the site of impact. The continuity circuit could not be rechecked since the vane was damaged during recovery operations. No similar results were available in the case of the other uncovered vane due to its orientation at impact. It was beneath the rocket and was therefore destroyed.

The cover remained on the third vane (inadvertent battery failure) during the entire flight and was recovered undamaged. It furnished definite proof that the vane cover does protect the vane. Indications of heating were found on all the instrumentation.

From these tests it was concluded that uncovered 0.005-in. vanes would sustain flight very well. Figures 3.13 and 3.14 show the recovered items.

During this test period, work progressed on an improved version of the circuit associated with the vane transducer. The previous circuit was adequate for laboratory and wind-tunnel purposes, but inadequate for flight use as it was sensitive to supply voltage, oscillator frequency, and other effects.

The new circuit developed was based upon phase rather than amplitude change as is generally customary with devices of this type. The circuit finally employed exhibited excellent stability, sensitivity (order of 1 volt per degree of vane angle), and independence of supply changes. Subsequent use during IGY flights of vanes showed proper operation of vanes and circuit.

Two reports devoted to the windvane problem have been prepared. They are discussed in a later section of this report.

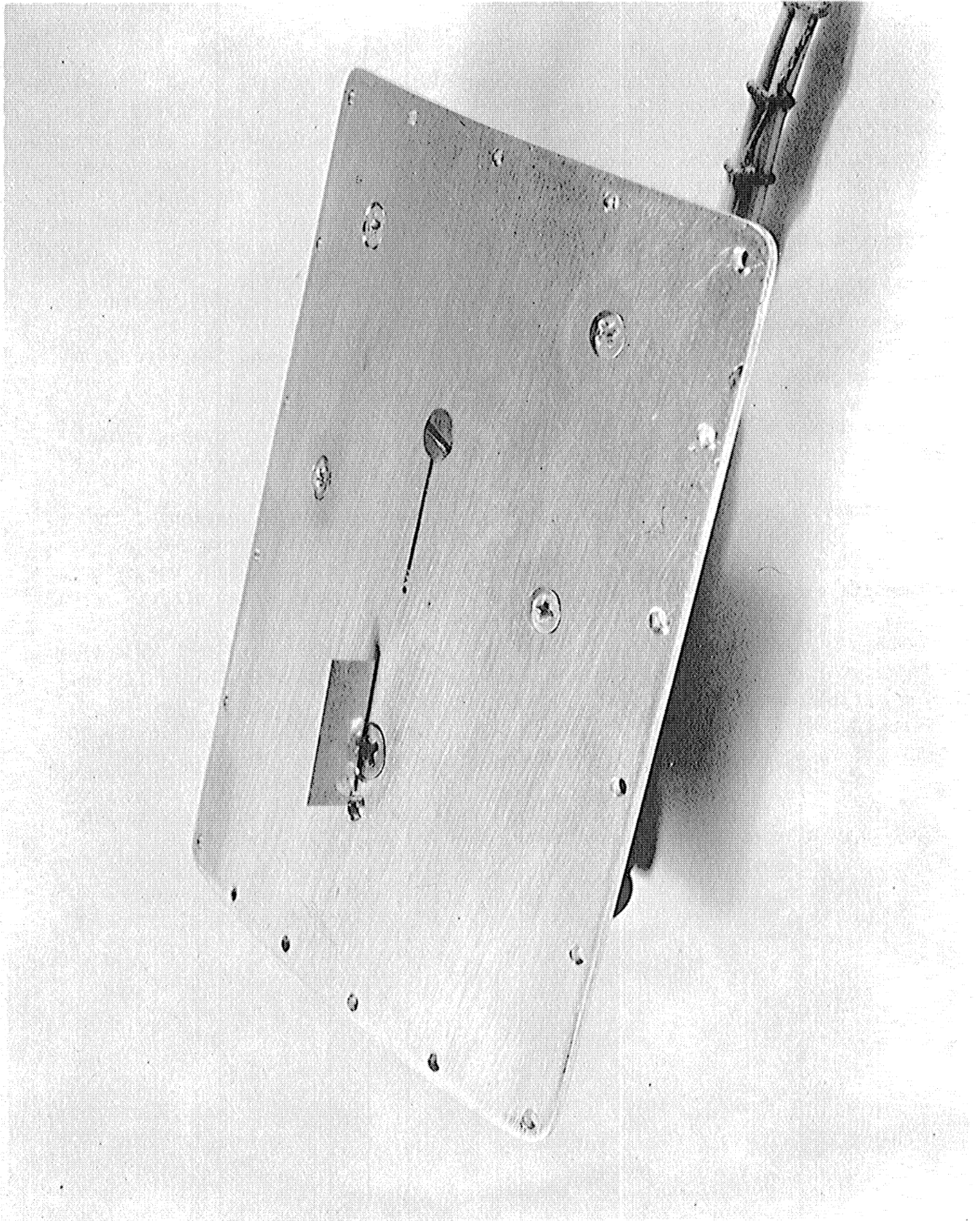


Fig. 3.10. Covered vane with cover removed.
Vane dimension $1/4$ in. by $3/4$ in. by $.005$ in.

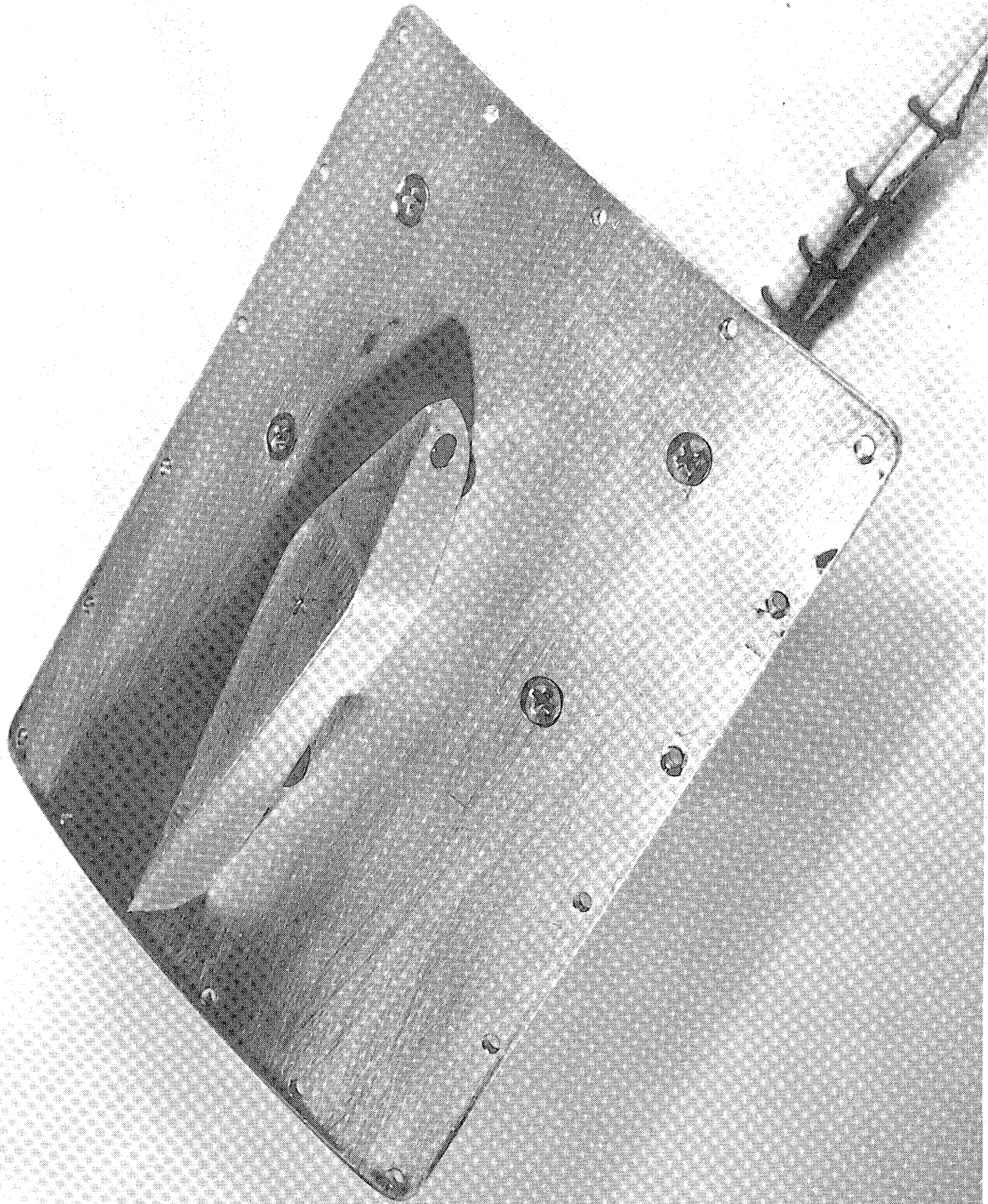


Fig. 3.11. Covered vane with cover in place.
Downstream end is end with small hole.

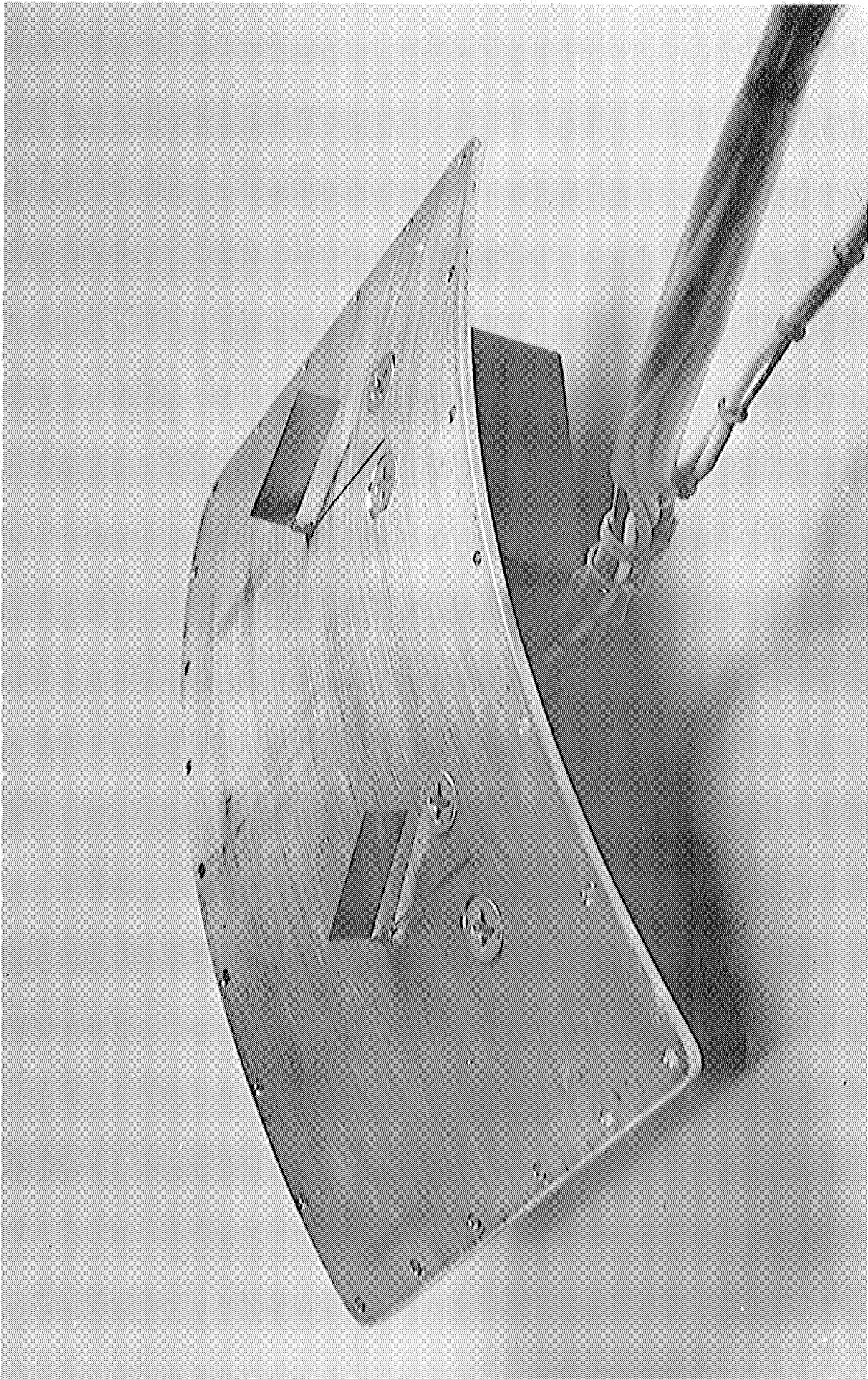


Fig. 3.12. Dual-vane plate. Vane dimensions are $1/4$ in. by $3/4$ in. by 0.010 in. and $1/4$ in. by $3/4$ in. by 0.003 in.

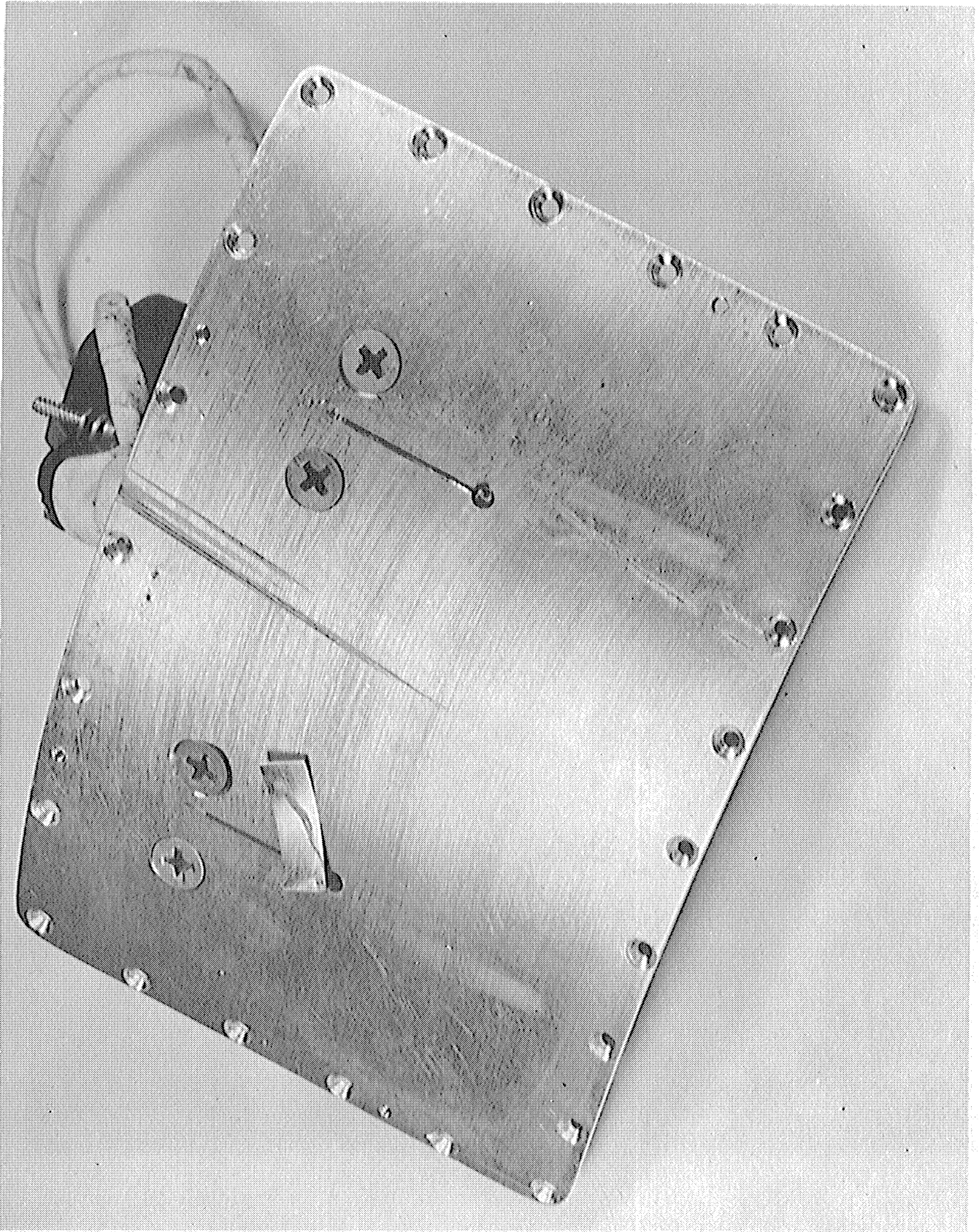


Fig. 3.13. Recovered plate with vane in place. Physical damage apparently occurred at impact.

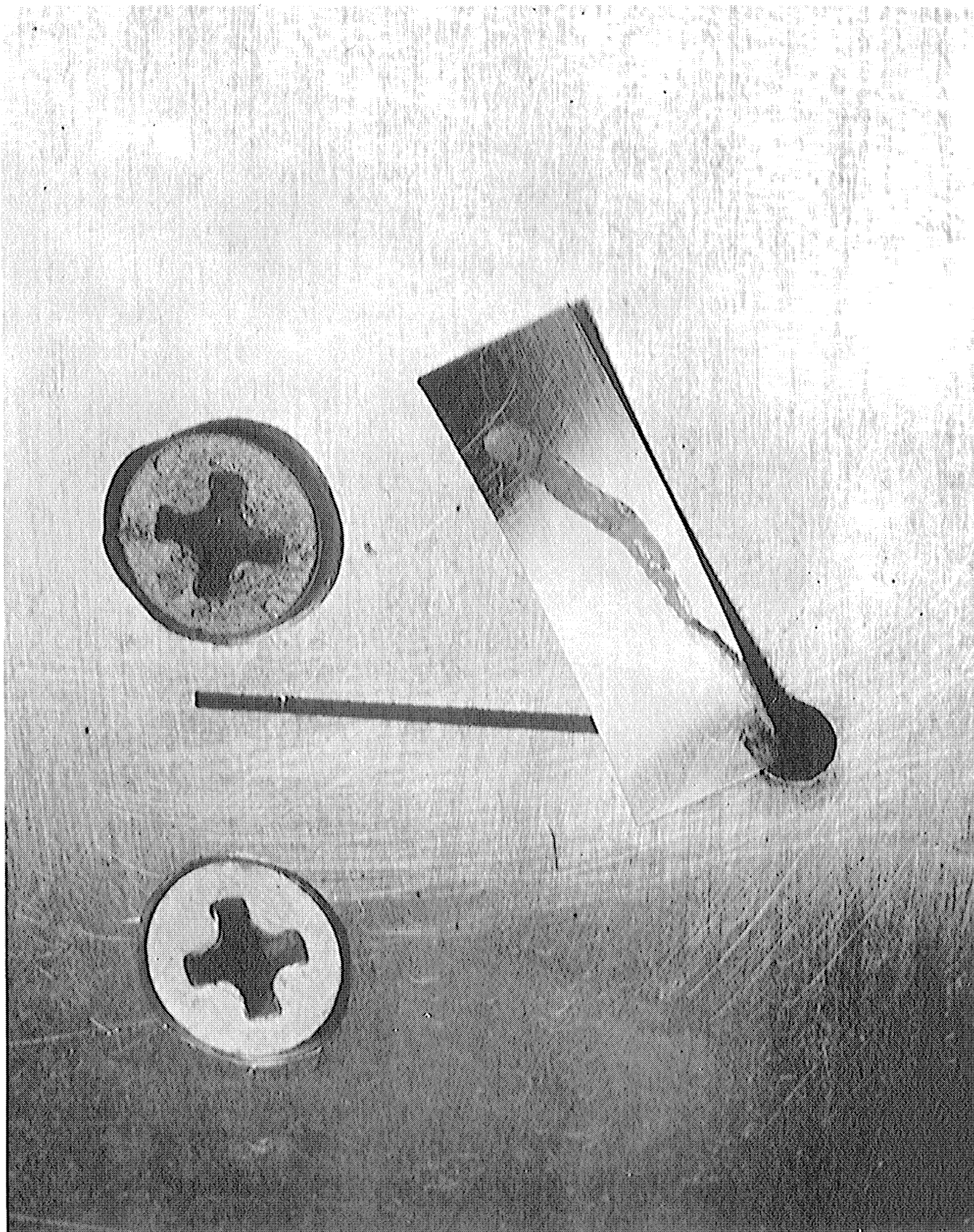


Fig. 3.14. Close-up of recovered vane. Welding flux employed in vane preparation melted and flowed along vane during flight, solidifying as can be seen.

3.3 AEROBEE USAF 31(T-DAY AEROBEE)

One of the initial activities under the contract was the flight preparation and launching of USAF 31, which had been constructed under the previous contract.

This rocket was scheduled as part of a "T-Day" program (T standing for temperature, T-day signifying multiple experiments for temperature measurement), which included a Signal Corps grenade Aerobee at W.S.P.G., a USAF Aerobee (No. 31) instrumented with "Alphatrons," and a "searchlight" experiment. The experiments were to be conducted as simultaneously as the state of the art permitted.

USAF 31 was instrumented with the following major items:

- (a) 5 alphatron pressure-measurement systems;
- (b) 1 gyroscope for altitude determination;
- (c) 2 cameras for gyroscope photography; and
- (d) an Air Force beacon-telemeter system.

The alphatron installation was carried out in conformity with the basic experiment conducted by this research group for the measurement of ambient pressure, temperature, and density, as detailed in a technical paper and discussed in previous project reports. An investigational aspect of the installation involved the employment of special valves in an experiment concerned with the general validity of use of pressure gages at high altitude that have been exposed to atmospheric pressure. Of the five alphatrons, two were mounted to indicate impact pressure and the remaining three, to indicate cone-wall pressure. One impact and two cone-wall units constituted the basic pressure-measuring devices necessary to satisfy the requirements of the temperature-measurement method. Each of the alphatron chambers of these three units was provided with a valve, whose function was to seal the unit and hence maintain a pre-established vacuum in the chamber until some predetermined point in the flight. That is, each chamber was pumped to and maintained as some low pressure (10^{-2} mm Hg) many hours before flight use, following which the valve was closed, thus sealing the chamber. The valves, which were motor-driven, were then activated at the desired point of the flight, exposing the gages to the atmosphere.

The two additional alphatrons that were employed without valves were used as controls for comparison with valved units. The data from these units were considered as supplementary to the data from the valved units.

The gyro installation was identical to that of earlier equipment. An additional camera was included to lend greater assurance that the gyro information would be recorded.

The rocket was launched successfully at 7:20 a.m. on 22 October 1952. Recovery of the nose-cone instrumentation was effected with minimum delay. All instrumentation appeared to function properly.

The flight results were analyzed and reduced, using an assumed vacuum trajectory. Subsequent arrival of actual trajectory data permitted slight adjustments. The analysis was recorded in an informal report entitled "Revised T-day Report," which is incorporated as Appendix I of this final report.

3.4 AEROBEE USAF 48

Aerobee 48, launched 14 July 1954, was instrumented to achieve several objectives. The first was to enable comparison of the University of Michigan gyro-aspect system used in previous instrumentations in connection with alpha-tron-temperature experiments, and the Air Force Cambridge Research Center sun-horizon-aspect system previously utilized in other Aerobee instrumentations. The gyro system had been used successfully for a number of rocket flights, as had the sun-horizon-camera system. Each system has, of course, its merits and drawbacks. The gyro system produces aspect data with relatively little data-reduction effort, whereas the sun-horizon system requires, relatively, a good deal. However, the latter system has the merit of being much simpler from the instrumentation standpoint, requiring only the proper installation of a special camera in a given instrumentation. The gyro is much more complicated in this respect, requiring careful preparation prior to flight, as well as a rather complicated installation, by comparison.

Operationally, the gyro system has the merit of being useful at any time of the day or night independent of weather or visibility conditions, whereas the sun-horizon system is useful only in the daytime and then only during sun-up periods. The sun-horizon system has the additional merit, however, of being an absolute system; that is, each data point it yields is independent of every other point.

The accuracy of the two systems was believed to be comparable and it was the purpose of this firing to establish the validity of this belief.

The gyroscope used in this system was a specially modified Bendix type J-8 instrument. Modifications included the installation of a motor-driven, remotely controlled erection system, the substitution of specially engraved scales, the use of bearings selected particularly for low friction, and procedures which permitted as nearly perfect balance of the gyrostat as may be achieved. This work was in general accomplished under controlled laboratory conditions utilizing a dust-free box to maintain conditions of cleanliness. It is believed that modification and preparation in this manner permits the attainment of data of accuracy better than 1° in an Aerobee rocket.

The data from the gyroscope were recorded in flight by a 16-mm motion-picture camera carrying the designation B-2. Like the sun-horizon system, the film must be recovered subsequent to flight.

The sun-horizon-aspect system likewise utilizes a B-2 camera, with a modified lens structure to permit wide-angle viewing. For the July, 1954, firing,

the installation of the aspect camera was the responsibility of the AFCRC, and thus will not be described here.

Both systems operated properly during the flight. The data from the gyro system has been reduced and reported in a special "Data Report" (see Section 4.4). Similarly, the data from the camera-aspect system, reduced by the New Mexico College of Agriculture and Mechanic Arts, have been presented in a report from that institution. During the reduction, close contact between the data-reducing groups was maintained and from this it was established that there was reasonable agreement between the two systems. To the knowledge of the writer, a critical comparison has not, however, been made.

The second objective was to conduct a preliminary windvane test, as discussed in Section 3.2.

The third was to test an alternative gyro mounting position and a new camera recording technique. A second gyro with associated camera and lighting system was utilized in the July Aerobee firing. The second gyro was prepared in a manner similar to the first; however, the installation was different in that an alternative mounting position was chosen. The new position, if proven successful, would permit a further reduction in the time required for data reduction, due to the elimination of certain steps in the computational procedure.

The camera utilized for recording the data from the second gyro was designed and built specially for this purpose. Whereas the B-2 camera used at that time for recording data was of the conventional shutter type, the new camera transported the film, 35 mm in this case, past the aperture at a constant velocity and did not use a shutter. To provide the effect of a shutter, a flashing-light system was developed and used with the camera. An electronic flash tube was employed and flashed at the appropriate interval, in this case 10 times per second; appropriate synchronism thus permits a frame rate of 10 frames per second. The use of the flashing light enabled, for instance, photography completely free from possible vibration. It likewise permitted, due to the readily attained high light intensity, the use of film with finer grain, and consequently, an improvement in image detail.

The second gyro and camera system operated satisfactorily. The camera was recovered and re-used on a later flight. Figure 3.15 is a photo of USAF 48 instrumentation. The continuous-run camera is the long rectangular box visible near the bottom of the rack. Figure 3.16 shows the full view of the instrumentation.

The fourth objective (responsibility of AFCRC) was to provide measurements of stream velocity on the nose cone. Equipment utilizing ion generators and receivers was designed, installed, and operated by personnel of the AFCRC. Since this experiment was not the responsibility of The University of Michigan, it will not be covered by this report. It is believed to have operated successfully.

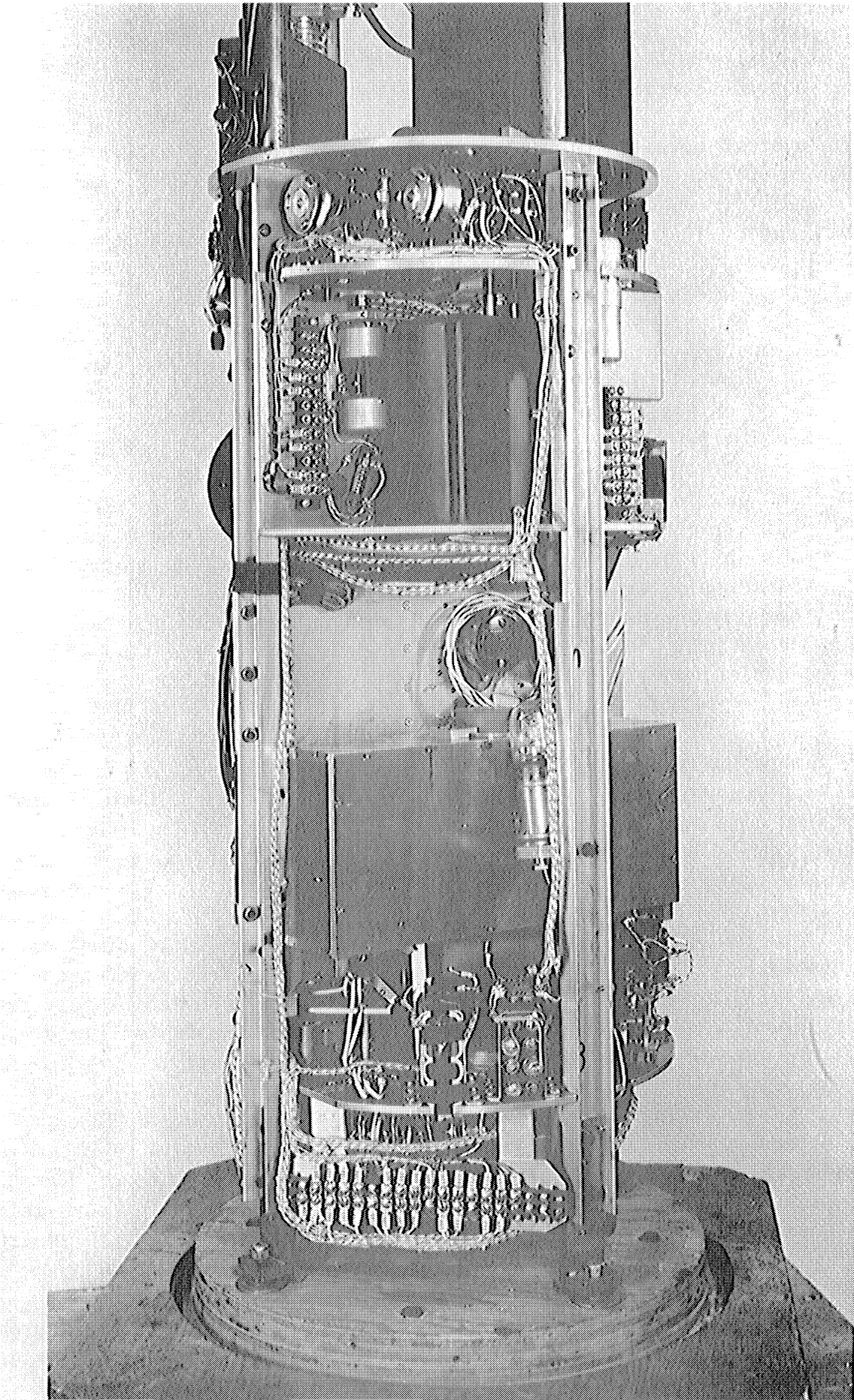


Fig. 3.15. Aerobee USAF 48 instrumentation.

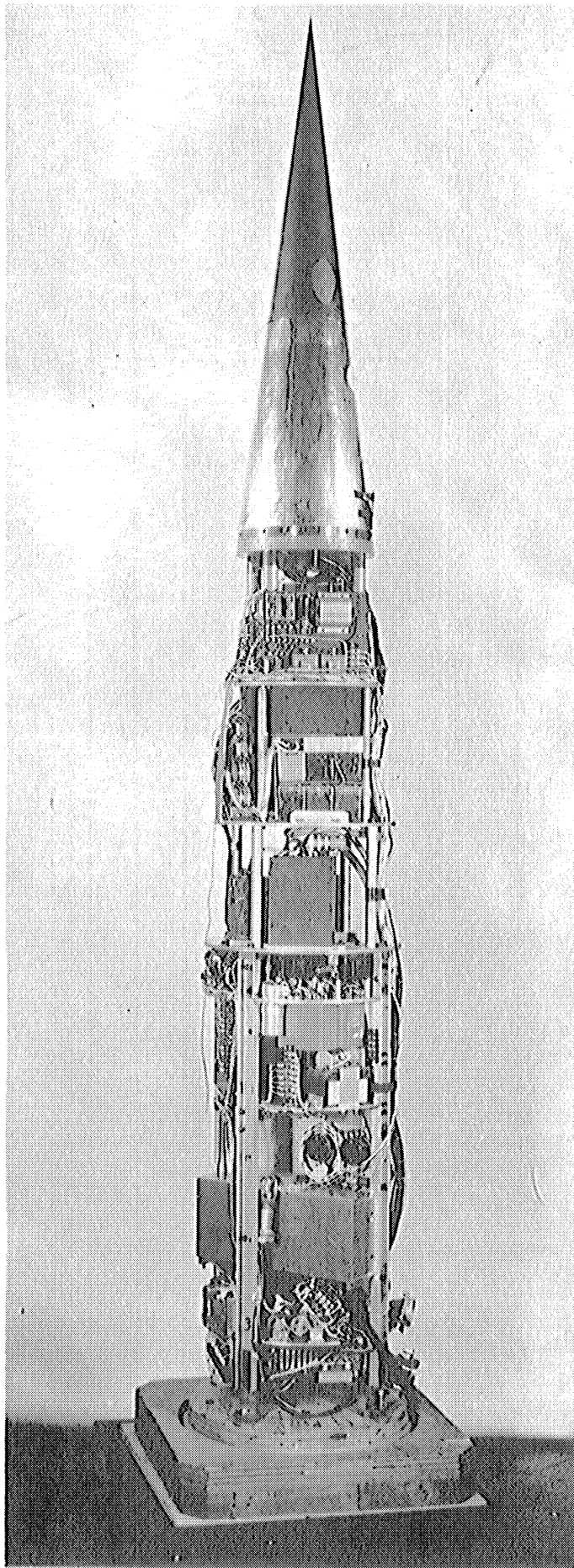


Fig. 3.16. Aerobee USAF 48.

In addition to the major portions of the instrumentation provided by The University of Michigan group for this firing, there were also accessory items necessary for operation. The power supply systems to power the gyroscopes and the flashing-light system fall in this category.

The gyros, light system, and regulated power supply require 110-volt, single-phase, 400-cycle power. Vibrator-type invertors operating from 24-volt lead acid cells provided the requisite power, three units being employed. One 30-watt unit supplied both gyroscopes, another 30-watt unit supplied the flashing-light system and a third, 15-watt unit powered the regulated power supply. To prevent "noise" generated by the invertors from possibly interfering with information signals fed to the telemeter, all invertors were heavily filtered on the primary side. In addition, phase-angle correction was applied to the invertor output.

Many vibrator-type invertors have been utilized in Aerobee rocket instrumentation by this project; failure of the units in flight has not been experienced nor was it during the July firing. However, for this flight, in view of the special importance of obtaining proper gyro operation, it was decided, to insure power for the gyros, to provide a switch-over system for the invertors that would operate in case of invertor failure. Thus a circuit was devised that would switch the gyros from the invertor which normally powered them to the other 30-watt invertor in the event that the output of the first invertor failed.

All equipment performed as desired, so changeover operation was not utilized. Details of the circuit appear in a special report (see Section 4.3, Technical Note No. CT-3).

The standard beacon-telemetry unit was employed for this rocket.

3.5 AEROBEE USAF 58

Aerobee USAF 58 was instrumented jointly by the Geophysics Research Directorate and The University of Michigan. Several experiments were carried out.

A photographically recorded windvane experiment was conducted. This portion of the instrumentation was discussed in detail in Section 3.2.

A photographically recorded gyroscope was incorporated for experimental attitude determination. In addition, an electrical take-off gyroscope obtained from G. M. Giannini and Co. (Model 3416-7 Free Gyro) was included to enable comparison of the two gyro systems and to permit evaluation of the electrical take-off gyro for possible use during the then forthcoming IGY program. The strobe-light-35-mm camera recovered from AF 48 was employed for recording during this flight. Both gyros operated satisfactorily during this flight, permitting the conclusion that the electrical data - take-off gyroscope would likely perform satisfactorily in future Aerobee launchings, such as those planned for IGY.

Two 16-mm cameras were included in the instrumentation (one with its optical axis perpendicular to the rocket axis, the other with its optical axis 30° to axis of rocket) to permit aspect determination by the camera method. This experiment was installed by and was the responsibility of GRD-AFCRC personnel. The purpose of using two cameras was to permit: (1) a cross check between the two units, and (2) extending data through the peak of the trajectory. A revised report of aspect data obtained by this method was issued in 1957.*

As in the case of USAF 48, an additional objective was correlation of the camera-aspect data with the gyro-aspect data.

A micrometeorite experiment was also included in the instrumentation. This too was prepared and installed by GRD-AFCRC personnel. The results of the experiment are not known to the writer.

Accessory instrumentation included the AFCRC-GRD beacon telemeter, a special FM-FM telemeter for the micrometeorite detection experiment, various power supplies, a strobe light system, and other minor control items.

3.6 NIKE-CAJUN AM 6.30

One of the tasks on this contract was the development of a small rocket version of the aerodynamic-ionization-gage system for pressure, temperature, and density measurement used by the group on Aerobee rockets. The Nike-Cajun rocket system was adopted for this experiment and thus indicated the volume and weight limitations under which the experiment would have to operate. This required redesign of the sensing elements as well as some simplification of the experiment.

Redevelopment of the radioactive-ionization-gage pressure-measurement system** into a compact, self-contained package made possible the inclusion of **three** systems in instrumentation for this rocket. One unit was mounted to measure ram (or impact or total head pressure), while the other two were mounted 180° apart on the cone wall to measure cone-wall pressure. No gyro was planned for this experiment due primarily to the desire to reduce the complexity and cost of the experiment. Actually, it is not certain that one was available which would produce the desired data under the high accelerations (30-40 g) experienced by equipment in this rocket. Corrections to data obtained under yaw were of course still necessary, and it was planned to effect these corrections by what amounts to an averaging process. This is to be discussed in detail in a forthcoming report on the firing results.

* Camera Aspect Data, U.S.A.F. Aerobee AF 58, 20 August 1957, by New Mexico College of Agriculture and Mechanic Arts, AFCRC, Cambridge, Massachusetts.

** See Section 3.1.

A special two-channel pulse-time L-band telemeter was developed by ARCRC-GRD personnel for use with this and similar small rocket instrumentations. Also a "sandwich" type slab antenna was developed for use with the telemeter, by the New Mexico College of Agriculture and Mechanic Arts.

Trajectory information, necessary to the experiment was to be obtained through (1) use of a "DOVAP" (Doppler Velocity and Position) beacon system designed and supplied by the Ballistics Research Laboratories of the Aberdeen Proving Ground, and (2) an S-band radar beacon (DPN-19) supplied by the U. S. Army Signal Corps.

These units comprised the equipment to be incorporated in the instrumentation, fourteen launchings of which were planned for the "Pre-IGY" and "IGY" periods. The first two rounds were considered prototype models, the first to provide a flight test of all the various equipment, and the second to provide a proof test of the rocket and instrumentation at Fort Churchill under IGY conditions.

The first rocket AM 6.30 was successfully launched at W.S.P.G. on 9 August 1956. The performance of the instrumentation was very unsatisfactory from the standpoint of obtaining useful upper-atmosphere data.

In general, all equipment with the exception of the telemeter failed in some respect during the flight. The power output of the doppler beacon dropped to zero seconds after launch, and the DPN-19 failed for an unknown reason a few seconds later.

Of the three pressure-measurement systems, one experienced an amplifier and range-changing circuit failure during boost phase, a second indicated improper output but performed properly insofar as range-changing was concerned, and the third operated satisfactorily to about 50-km altitude where the amplifier apparently failed.

Good quality telemeter signals were received throughout the flight.

Although the flight was outwardly a failure, it should be pointed out that it provided the first flight test for every instrument employed, and thus as a necessary developmental step, was very fruitful as it indicated weaknesses in the equipment.

As a consequence, modifications were effected in both the pressure-measurement units and the DOVAP beacon, preparatory to the next flight, AM 6.31, which was scheduled some two months later at Fort Churchill. That rocket was prepared under this contract but the launching took place under a subsequent contract. The launching, which was of the first Fort Churchill rocket to be fired, was successful and all equipment apparently operated properly throughout the flight with the exception of the DPN-19 S-band beacon which again failed. Details of that rocket and data reduction will be reported subsequently under the other contract.

The constructional technique employed for these rockets was chosen to permit relatively simple and economical construction and assembly. The structure was made up of sections: the nose cone, associated rack, the telemeter antenna, and an additional rack to accommodate the telemeter and DOVAP beacon. The racks were constructed of steel members commercially available and known as "Unistrut." Four longitudinal struts were hard-soldered to steel rings which in turn were bolted to the nose cone or the telemetering antenna (a structural member). Skins to cover these instrumentation sections were fashioned by rolling kitchen-variety stainless steel (21 gage) into semi-cylinders and securing the halves together with long stainless-steel hinges that were spot-welded in place. Thus assembly or disassembly of the skins was accomplished by wrapping or unwrapping the sections around the racks. They were retained in position axially by appropriate ledges and held together by merely inserting the hinge pin.

The DOVAP and telemetry units were cylindrical in form and fitted the aft instrument section. They were held in position, in rubber o-rings, by the struts.

The forward section was devoted to pressure instrumentation and the DPN-19 beacon. Each equipment group of the instrumentation was provided with its own pull-off lead cable and plug to minimize possible mutual interference and to simplify the general wiring. The location of the "slab" antenna between sections was arranged for the same purpose. The physical features of the structure and equipment installation are illustrated in Figs. 3.17 and 3.18.

3.7 SLED TESTS

Rocket-powered "sleds" at H.A.D.C. offer the opportunity to conduct tests of the physical stamina of various equipments. During this contract two minor tests were conducted for the sole purpose of determining the mechanical stability of certain items under conditions of high acceleration along three axes.

In one case a box containing Victoreen hi-meg resistors and a VX-41A electrometer tube (hot filament) were subjected to a sled run. No damage was observed and it was concluded that these components could be used satisfactorily on rockets having accelerations of 40-50 g.

In a second test, a complete prototype of the radioactive-ionization-gage pressure-measurement system was carried on a sled to check the mechanical integrity. The unit was not operated during the run. Again, no damage was detected.

These two experiments were not, of course, sufficiently important to warrant a run for the test alone, and were thus carried out during other major sled operations. We are indebted to the personnel operating the Holloman track for their assistance and efforts in accommodating and conducting these tests.

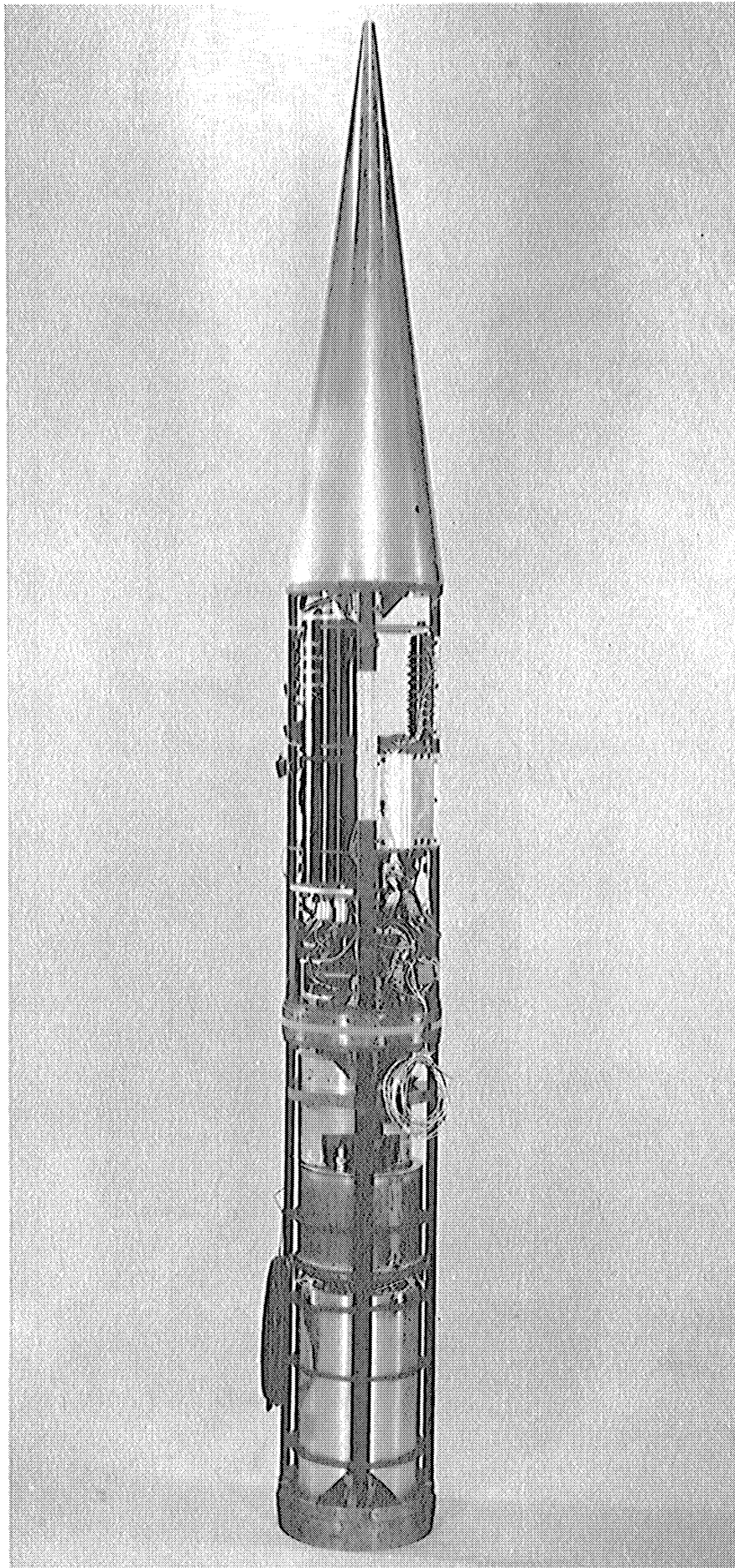


Fig. 3.17. Nike-Cajun AM 6.30 instrumentation without pressure-measurement systems and skin.

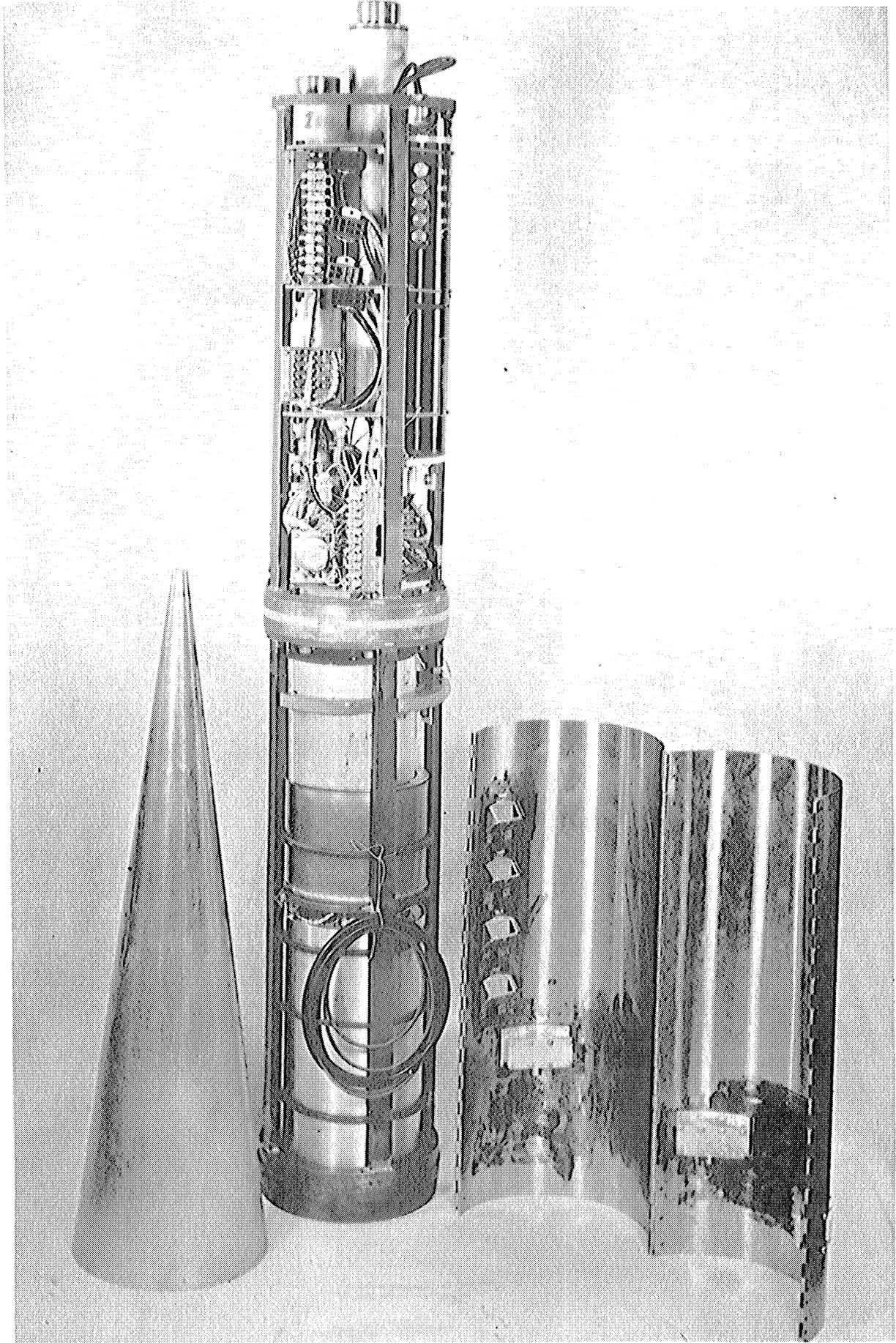


Fig. 3.18. AM 6.30 instrumentation showing disassembled skin and nose cone.

3.8 IGY ROCKETS AM 6.31 AND AM 2.21

As mentioned earlier in this report, the instrumentation for IGY rockets AM 6.31 and AM 2.21 was prepared during the period of this contract. Details of the instrumentation, a discussion of the launchings, and a report on the data obtained will be presented at a later date, as the launchings and data analysis took place under a continuing contract. But a brief summary of the equipment carried by these rockets follows.

AM 6.31 was a Nike-Cajun and was instrumented similar to AM 6.30 as discussed in the immediately previous section. Figure 3.19 shows the instrumentation.

AM 2.21 was an Aerobee model AJ 10-25 and carried the following instrumentation provided by the organizations noted:

- (a) 1 "ram" radioactive ionization gage. (Univ. of Mich.)
4 "cone wall" radioactive ionization gages. (Univ. of Mich.)
- (b) 2 windvanes and associated circuitry. (Univ. of Mich.)
- (c) 1 gyroscope, electrical data take-off. (Univ. of Mich.)
- (d) DOVAP beacon (BRL).
- (e) DPN-19 (WSSCA).
- (f) DRW-3 (WSSCA).
- (g) DKT-7 (GRD).
- (h) test transistor power supply (GRD).
- (i) miscellaneous associated items. (Univ. of Mich.)

This rocket was successfully launched 23 October 1956 as the first Fort Churchill Aerobee. The data are being reduced at the time of writing of this report. Good data were obtained.

Figure 3.20 illustrates the instrumentation.

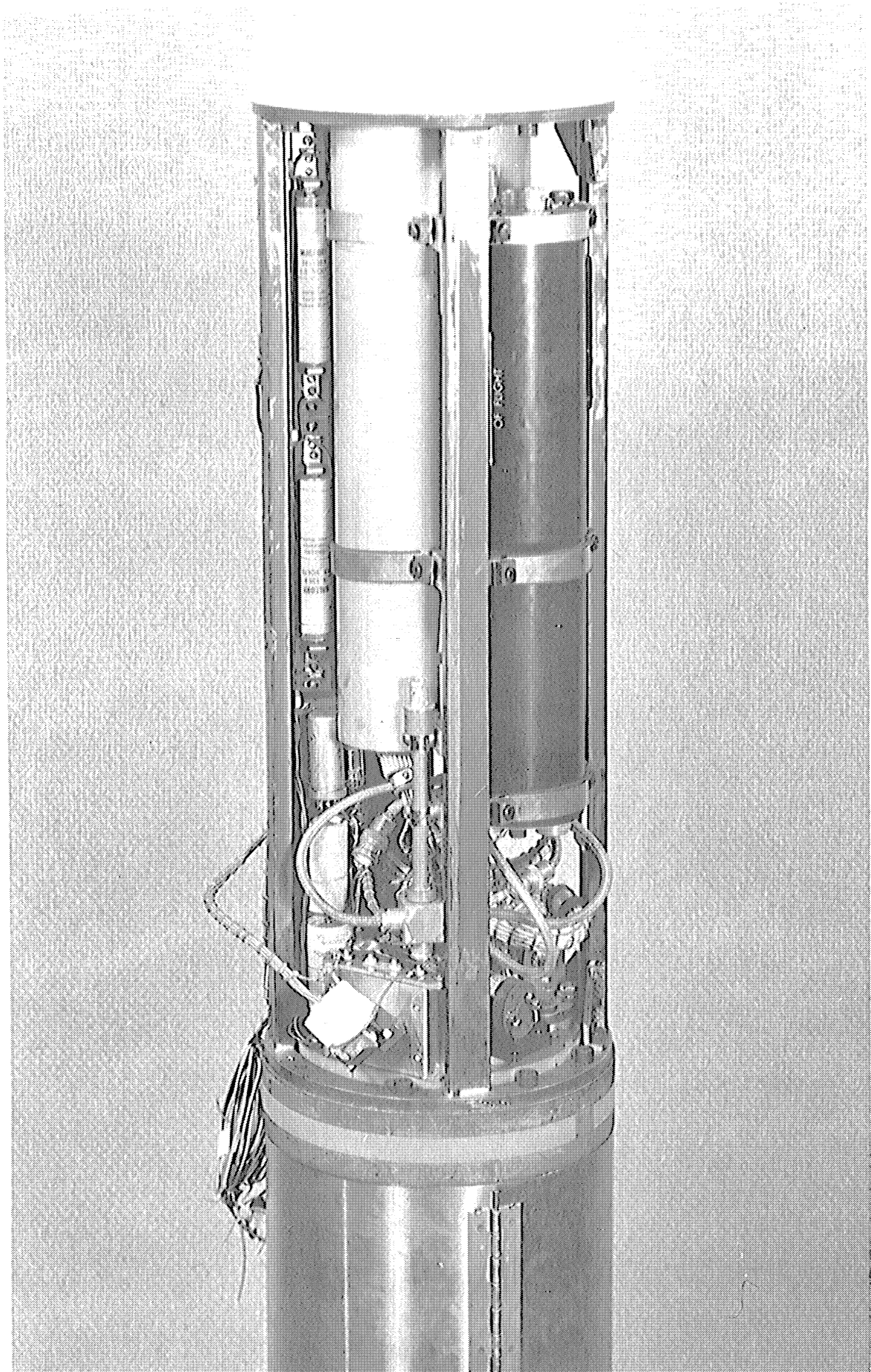


Fig. 3.19. Close-up of AM 6.31 showing DPN-19 and pressure-measurement systems in place.

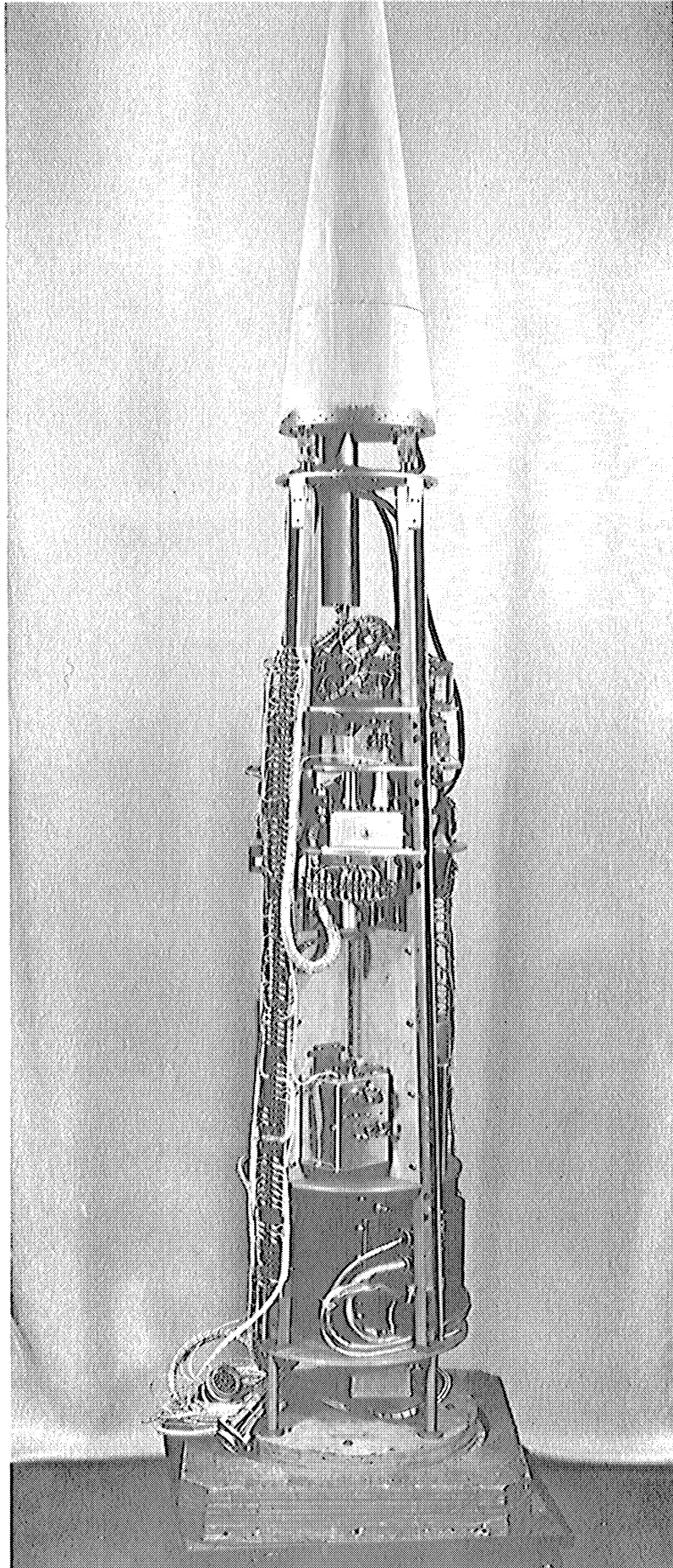


Fig. 3.20. Aerobee AM 2.21 instrumentation with one pressure-measurement system in place.

4.0 REPORTS, PAPERS, AND PUBLICATIONS

4.1 PROGRESS REPORTS

Quarterly progress reports were written during the course of the contract describing and summarizing the significant occurrences and efforts. Fifteen reports were written, numbered C1 - C15.

4.2 SCIENTIFIC REPORTS

Several scientific reports were written during the course of the contract. Titles and abstracts follow.

- CS-1. Table of Pressure Ratios P_0'/P_S and P_S/P_1 vs. Mach Number for a 15° Right Circular Supersonic Cone, H. S. Sicinski, A. A. Kirsons, and N. W. Spencer, September, 1953.

Abstract: This report presents a table of pressure ratios to be found on a 15° right circular cone tabulated versus the free-stream Mach number. The pressure ratios are given to five significant figures against 0.001 increments of the argument. The argument ranges from $M_1 = 1.048$ to $M_1 = 3.895$. Given are the ratio of total head pressure (P_0') to the cone-surface pressure (\bar{P}_S) and the ratio of the cone surface pressure (\bar{P}_S) to the free-stream static pressure (P_1). With these tables and two pressures on a right circular cone, it is possible to determine the free-stream Mach number and the free-stream static pressure. By measuring the free-stream velocity (or the velocity of the cone through the media if that is the situation) in addition, it is possible to compute free-stream static temperatures and densities.

After issuance, a minor error was detected in the computations of the ratios. The error at all Mach numbers is not greater than 1 to 2 parts per thousand. A revised and extended table will be issued at a later date.

- CS-2. This report is a copy of a published paper listed under Section 4.6.
- CS-3. This report deals with an experiment conducted under an earlier contract.

A Rocket Measurement of Upper Atmosphere Density by Paschen's Law, H. L. Smith, H. C. Early, and N. W. Spencer, May, 1955.

Abstract: The air density in the region adjacent to the surface of a supersonic cone in the upper atmosphere has been measured by means of a spark-breakdown technique based on Paschen's law. A spark gap was incorporated into the nose of an Aerobee rocket and consisted of alternate metal and dielectric laminations. The break-

down path was essentially parallel to the surface of the cone and provided for a measurement without changing the air flow past the missile. The spark-breakdown technique was evaluated by measurements taken in a large vacuum chamber, and investigations were made into the statistical deviation from the mean calibration curve, and of the consequences of initial ionization. The effects of the high air velocity upon the breakdown were also considered. The Paschen's law density measurements have been compared with data derived from alphanon pressure gages included in the rocket instrumentation and were found to have a consistently lower value. This discrepancy is believed to be due largely to the influence of the boundary layer upon the location of the discharge path.

- CS-4. Mach Number and Yaw Angle Determination for Conical Flow Regimes Using Two Surface-Flow Angle Indicators, H. S. Sicinski, and H. F. Schulte, January, 1955.

Abstract: In the search for principles applicable to measuring upper-air environments (i.e., temperature, density, wind vectors, etc.) from supersonic missiles in the regions above 30-km altitude, it became evident that knowledge of the angle of surface flow on a right circular cone coupled with body-coordinate rotational parameters would permit calculation of the free-stream Mach number and the yaw angle. An instrumentation was developed for measuring the surface-flow angle utilizing a rectangular, metal plate, 3/4 in. by 1/8 in. by 0.003 in., driving a capacitive transducer. The electronic version of the instrument requires a minimum force of 60 dynes at the plate's center of pressure for accurate alignment with the stream. Correlation of the theory was provided in a series of wind-tunnel experiments with the subsequent realization that the vane angle as measured by an external optical system would provide a reliable Mach meter for wind-tunnel instrumentations, particularly low-density tunnels where conventional Pitot tubes become complicated by having to measure very low pressures. Such an optical transducer for the vane angle would require a lower actuating force than the electronic version of the instrument. The discussion concerns the governing relations, the solution of the resulting transcendental equations, experimental results, and the possible applications.

- CS-5. Pressure and Density Measurements Through Partial Pressures of Atmospheric Components at Minimum Satellite Altitudes, H. S. Sicinski, N. W. Spencer, and R. L. Boggess, March, 1956.

Abstract: Pressure and density measurements made from a satellite at altitudes of from 400 to 900 km are expected to range from 10^{-10} to 10^{-8} mm Hg and 10^6 to 10^8 particles per cubic centimeter. These measurements are initially complicated by the lack of knowledge con-

cerning the gas composition at these altitudes. Data from ionization-gage-type pressure gages would provide ambiguous results since the responses of these devices vary with the nature of the gas. Until the composition is better defined, a measure of the partial pressure of the components will be a more useful approach to defining this region. Knowledge of the composition, density, and pressure can be had simultaneously through the use of a device which in principle is a modified "Omegatron or Synchrometer" as described by Sommer, Thomas, and Hipple. This device in its later forms is simple in structure and operation, being capable of high sensitivity similar to the conventional ionization gages. Unlike the conventional ion gage, which ceases to function as a pressure gage below 10^{-8} mm Hg, the modified "Synchrometer" continues operation into the range of 10^{-10} mm Hg. The principle of operation is similar to a cyclotron. A small beam of ionizing particles is passed parallel to a magnetic field, causing local ionization along the beam. These ions are then accelerated by the alternating potential between two parallel plates. As in the cyclotron, when the r-f frequency is equal to eH/M , the ions of mass M and charge e are accelerated in orbits of increasing size and eventually strike an ion collector. If the ionizing beam is kept constant in value along with the electric and magnetic field, the ion collector current is a measure of the partial pressure of the gas of this charge-to-mass ratio. A lightweight instrumentation is possible through the use of "tuned gages" having permanent magnets for their magnetic field. This approach reduces the complexity of scanning and its associated field measurements and provides partial pressure for one component. Additional weight reduction follows from the replacement of the filament with a photosensitive source of ionizing radiation. This feature with its dependence on sunlight should serve to separate the ionized population from the un-ionized, in addition to providing the opportunity to study the earth's atmosphere in the presence and absence of sunlight.

This paper was delivered as part of the program of the meeting of the Upper Atmosphere Rocket Research Panel Symposium on the Scientific Uses of Earth Satellites on January 26 and 27, 1956, at Ann Arbor, Michigan.

ES-1. This report was prepared under a continuing contract. Since the early work was performed under the contract for which this final report is submitted, it is listed here also.

A Radioactive Ionization Gage Pressure-Measurement System, N. W. Spencer, R. L. Boggess, L. R. Brace, and M. A. El-Moslimany, May, 1958.

Abstract: An air-pressure-measurement system employing a radioactive ionization gage has been developed for use in rockets for the determination of surface pressures which can be interpreted in terms of the ambient pressure, density, and temperature of the atmosphere. The system has proven useful for the measurement of pressures from atmospheric to at least 10^{-3} mb.

4.3 TECHNICAL NOTES

- CT-1. Telemetry Limiter for AFCRC Aerobee Beacon System, D. G. Dow, December, 1953.

This report discusses in detail the problem of providing voltage limiting at a telemeter input. Certain solutions are suggested.

- CT-2. Modification of Air Force Type J-8 Attitude Horizon Indicator for Aerobee Aspect Instrumentation, J. A. Foster, H. S. Sicinski, and H. F. Schulte, July, 1955.

Abstract: This report presents the procedures necessary for the modification of a standard Air Force type J-8 attitude horizon indicator for Aerobee aspect instrumentation. The gyro as manufactured is a "vertical gyro" which, as such, is not satisfactory for missile use. The modification procedure converts the instrument to a "free gyro" which is suitable for Aerobee missile use. Detailed drawings, photographs, and a description of special equipment and test procedures are included so that a well-trained technician or an engineer can achieve the desired result.

- CT-3. Power Supplies and a Remote-Control System Suitable for Aerobee Rocket Instrumentation, H. F. Schulte, December, 1955.

Abstract: A general discussion of the operating characteristics and experience with the Airpax 400-cycle, d-c to a-c power inverter is presented. Performance curves for the inverter and circuit diagrams of an electronically regulated power supply and a flexible remote-control system suitable for rocket-borne use are included. The important problem of power-supply reliability has also received attention. A simple, yet effective, method of enhancing reliability by automatically switching to auxiliary or standby power is diagrammed.

- CT-4. Vane Mach Meter Feasibility with Application to Missile Upper-Atmosphere Temperature Measurements, H. S. Sicinski and N. W. Spencer, May, 1955.

Abstract: The principles underlying the vane Mach meter and the feasibility of a successful reduction to practice for the purpose

of developing an instrument suitable for upper-air temperature measurements are outlined, as well as experimental errors expected, and resolutions in measurements required to achieve a satisfactory probable error in Mach number and ambient temperatures. The analysis given represents the current opinion of the authors as to this procedure for implementing these principles in missile-borne equipment. The discussion does not represent the entire body of theoretical facts necessary to permit a detailed analysis of data from the proposed experiment. Physical principles are given without proofs or developments to derive engineering principles necessary.

4.4 DATA REPORTS

A special "Data Report" was also issued during the contract:

Gyroscope Aspect Air Force Aerobee AF-48, Launched: 14 July 1954, 0655 Hours, H. S. Sicinski and N. W. Spencer, February, 1955.

Abstract: The data presented in this report are the results of two instrumentations, each consisting of a single modified Bendix J-8 Attitude Gyroscope mounted in two trial positions. The application of the instrument to use in a missile requires an extensive modification and testing program, viz., the conventional "erector-mechanism" is removed, a suitable mass replaces it, and the entire instrument is balanced in such a way that the center of gravity coincides with the intersection of the two rotation axes. This erector mechanism, in the conventional use of the gyro in aircraft, maintains the axis of the gyrostator normal to the earth's surface. Of the two instrumentations, one considered by this agency as the conventional unit, is called "Beta-gyro" to connote the position and the scale markings. The other mounting position, called the "Psi-gyro," was an experimental one and was thought to be less favorable to the instrument but more favorable to the data analysis. In the design of the aspect experiment, the Beta-gyro was the primary measure serving to evaluate the performance of the Psi-gyro. Missile evaluation was considered necessary, since the results of simulation tests for gravity effects in the laboratory were inconclusive.

Revised T-Day Report.—During the contract a revised report on the results of an earlier Aerobee launching (AF-31, 22 October 1952) was issued informally. This report presented significant upper-atmosphere data. In order that it be preserved it is included here as Appendix I.

Skin Temperature Report.—During the course of the development of the "small rocket" experiment, questions arose regarding the temperature rise of

a nose cone and associated portions under high-velocity conditions. A theoretical study was carried out and was summarized in an informal report. The report, included here as Appendix II, is:

Skin Temperature Analysis for Various Materials, Terminal Velocities, and Altitudes, H. S. Sicinski, June, 1956.

4.5 PAPERS PRESENTED

Several papers were presented at technical meetings during the contract period:

- (a) Density Gage Methods for Measuring Upper Air Temperature, Pressure and Winds, N. W. Spencer and W. G. Dow, presented by W. G. Dow at a conference in Oxford, England, 24-26 August 1953.

Abstract: Rocket-instrumentation methods using air-density gages for determination of upper-air temperatures, pressures, and densities are described. Illustrative results are given of direct temperature determinations using alphanon gages and cone aerodynamics for Aerobee rockets, 1949 to 1952; temperatures derived from V-2 rocket pressure measurements using thermionic ionization gages, 1946 to 1949, are presented. Ionization-gage evidence is shown of the existence during one V-2 flight of a horizontal wind stratum, and a method is described for using cone aerodynamics to determine winds. A dc and a uhf voltage breakdown method for air-density determination are described. The engineering design reliability problem in rocket instrumentation is emphasized; parachute recovery of data and instruments is briefly discussed.

- (b) Exploration of the Ionosphere by Means of a Langmuir Probe Technique, W. G. Dow and G. Hok, presented by G. Hok in Oxford, England, 24-26 August, 1953.

These two papers were subsequently published in a book:

Rocket Exploration of the Upper Atmosphere, edited by R.L.F. Boyd and M.J. Seaton, Pergammon Press, London, 1954.

- (c) Mach Number and Yaw Angle Determination for Conical Flow Regimes Using Two Surface-Flow Angle Indicators, H. S. Sicinski and H. F. Schulte, presented by H. S. Sicinski at a meeting of the Division of Fluid Dynamics of the American Physical Society, Fort Monroe, Va., 22 November 1954.
- (d) Pressure and Density Measurements Through Partial Pressures of Atmospheric Components at Minimum Satellite Altitudes, H. S. Sicinski,

N. W. Spencer, and R. L. Boggess, presented by H. S. Sicinski at Satellite Symposium, Ann Arbor, Michigan, January, 1956.

This paper was presented to the sponsor as report CS-5, already discussed.

- (e) Temperature and Electron-Density Measurements in the Ionosphere by a Langmuir Probe, G. Hok, H. S. Sicinski, and N. W. Spencer, presented by G. Hok at Satellite Symposium, Ann Arbor, Michigan, January, 1956.

4.6 PAPERS PUBLISHED

- (a) Dynamic Probe Measurements in the Ionosphere, G. Hok, N. W. Spencer, and W. G. Dow, Journal of Geophysical Research, 58, No. 2, June, 1953.

Abstract: Preliminary, rather successful attempts to determine the ionization in the E-layer by means of a probe technique are described. The probe current showed an extremely rapid rise between 90 and 105 km altitude. The result indicates a positive-ion density about ten times larger than the electron density. Further measurements with improved equipment are recommended.

- (b) Rocket Measurements of Upper Atmosphere Ambient Temperature and Pressure in the 30-75 km Region, H. S. Sicinski, N. W. Spencer, and W. G. Dow, Journal of Applied Physics, 25, No. 2, February, 1954.

Abstract: A method for determining ambient temperature and ambient pressure in the upper atmosphere is described, using the properties of a supersonic flow field surrounding a right circular cone. The underlying fundamentals stem from basic aerodynamic principles as combined with the developments of the aerodynamics of supersonic cones by G. I. Taylor, J. W. Maccoll, and A. H. Stone. The experiment provides the necessary cone pressures, velocities, and Eulerian angles, such that a Mach number characterizing the ambient space conditions may be computed. A description is given of the requisite experimental equipment and related techniques. Experimental data from two rocket-borne equipments are presented with the resulting calculated pressures and temperatures as experienced over New Mexico to approximately 70 km.

- (c) Rocket Instrumentation for Reliable Upper Atmosphere Temperature Determination, N. W. Spencer, H. F. Schulte, and H. S. Sicinski, Procedures of the I.R.E., 42, No. 7, July, 1954.

Abstract: This paper describes briefly rocket-borne electronic equipment which has been developed and utilized in the determination of ambient atmospheric pressure and temperature. In the design of the equipment, emphasis has been placed on reliability of operation, and

on the ability to produce data of significant accuracy. Typical resulting curves of ambient pressure and temperature are presented.

- (d) Items (d) and (e) under Section 4.5 were published in Scientific Uses of Earth Satellites, edited by J. A. Van Allen, published by University of Michigan Press, Ann Arbor, 1956.
- (e) An abstract of the paper noted in item (c), Section 4.5, was published in the Physical Review, 98, Second Series, No. 4, 15 May 1955.

5.0 CONCLUSIONS

This report, although entitled a final report, actually describes only the effort devoted to a portion of a continuing program. Thus only specific and limited conclusions can be drawn. These have already been presented in the various sections of the report. For example, at the termination of the period, the development of the windvane system was not complete but appeared to be nearing that state. Similarly, only prototype ionization-gage systems had been flight-tested.

Subsequent reports to be prepared under related contracts in force will present information resulting from the continuing program.

6.0 PERSONNEL EMPLOYED DURING WHOLE OF CONTRACT PERIOD

R. L. Boggess	Engineer; Project Engineer	Part-time (student)
L. H. Brace	Technician	Part-time
K. W. Canestra	Technician	Part-time (student)
J. A. Cornell	Technician	
H. V. Dada	Data Analyst	Part-time (student)
R. G. DeLosh	Technician	Part-time (student)
D. G. Dow	Assistant in Research	
W. G. Dow	Project Director; Consultant	
M. A. El-Moslimany	Engineer	Part-time (student)
P. D. Engelder	Technician	Part-time (student)
F. Etiz	Technician	Part-time (student)
J. A. Foster	Research Assistant; Engineer	Part-time (student)
H. V. Green	Technician	Part-time (student)
P. A. Hogan	Technician; Data Analyst	Part-time (student)
G. Hok	Faculty Advisor; Consultant	Part-time
R. B. Jones	Technician	Part-time
W. G. Kartlick	Research Technician	
M. R. Kestenbaum	Engineer	Part-time (student)
A. A. Kirsons	Technician; Data Analyst	Part-time (student)
D. L. McCormick	Machinist	Part-time
G. A. McPhillips	Technician	Part-time (student)
T. Muller	Technician	Part-time (student)
T. Pattinson	Research Technician	Part-time (student)
J. Pua	Technician	
D. R. Rush	Technician	Part-time (student)
D. K. Scharmack	Technician	Part-time (student)
H. F. Schulte	Research Engineer; Project Engineer	Part-time
R. E. Schwartz	Technician	Part-time (student)
P. M. Shaler	Technician	Part-time (student)
S. Shaw	Data Analyst	Part-time (student)
H. S. Sicinski	Research Physicist; Project Physicist	
R. H. Smith	Technician	Part-time (student)
N. W. Spencer	Project Engineer; Project Supervisor	Full time; later part time
J. Zoerner	Data Analyst	

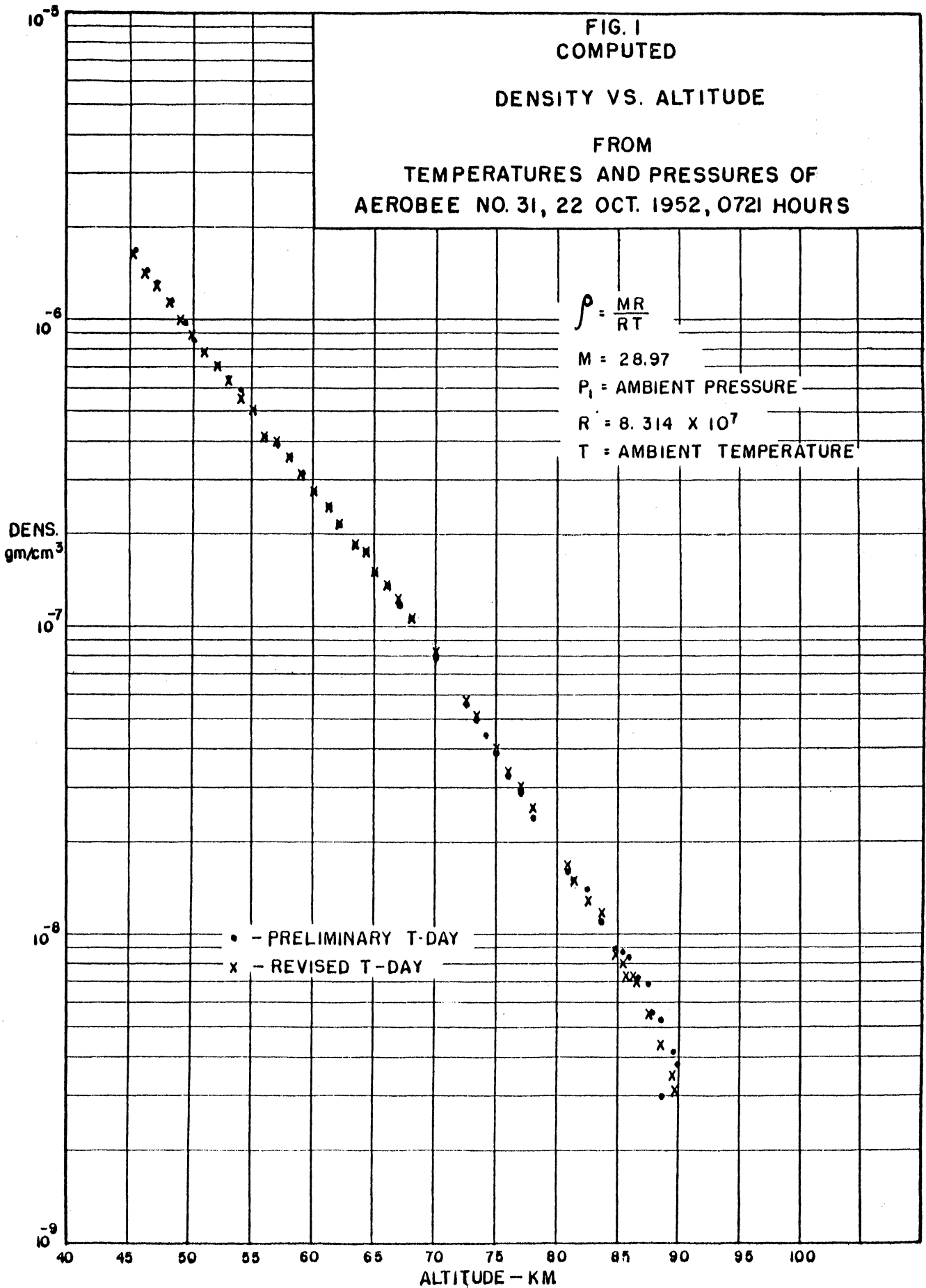
APPENDICES

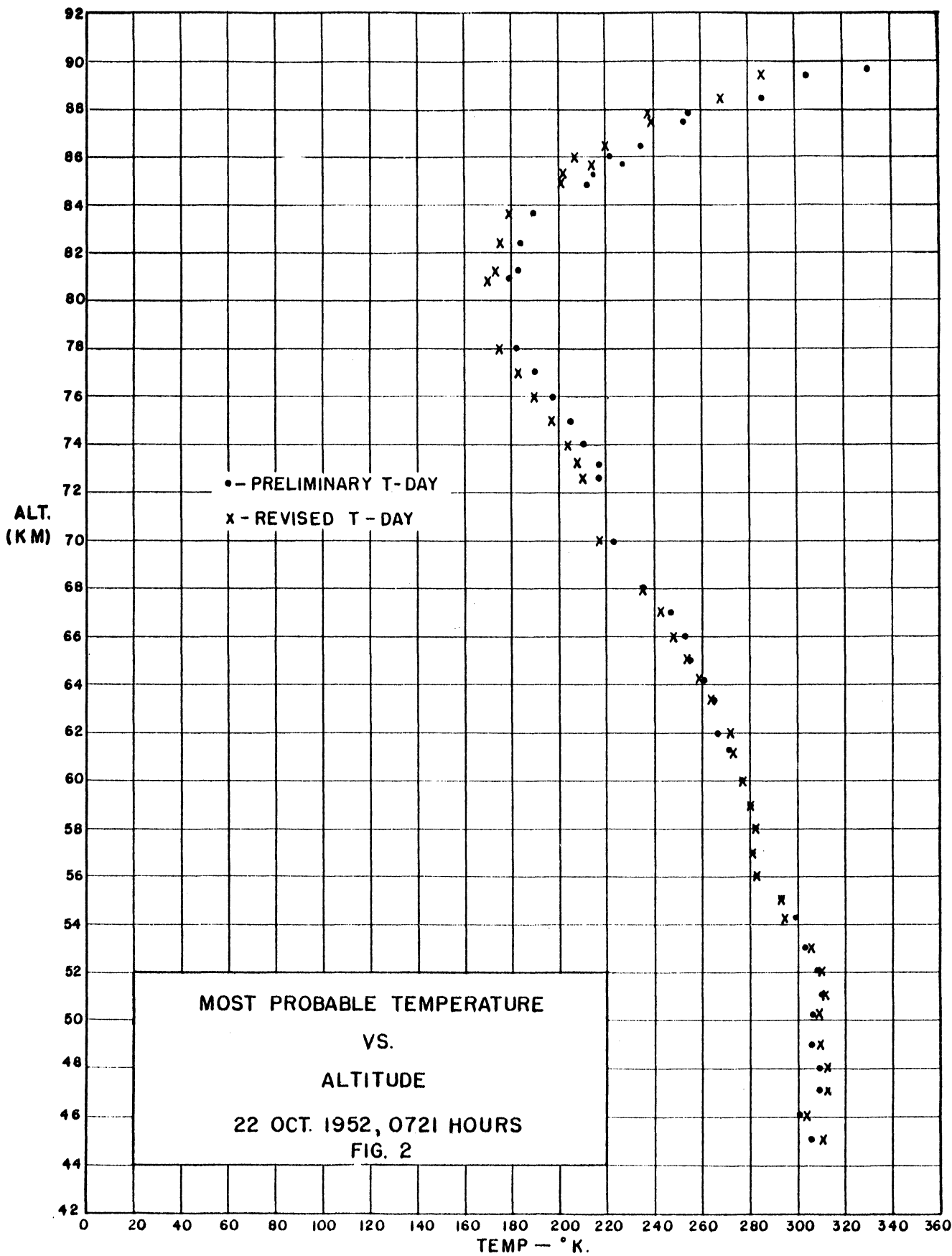
APPENDIX I

REVISED T-DAY REPORT

In a preliminary report issued in July, 1953, experimental data emanating from upper-air rocket number 31 (22 October 1952) were presented as derived from an assumed set of trajectory data for this missile. The assumed data were based on a smooth curve fitted to the discontinuous data provided by Beacon triangulation equipment. As a result of the unsatisfactory trajectory data, a discussion with the Flight Determination Laboratory personnel was held at Holloman Development Center where it was agreed that a re-evaluation of the experimental data would be undertaken. The procedure involved fitting first-through-fifth-degree polynomials to each of x, y, and z components of the position data, then differentiating these to the V_x , V_y and V_z components of the velocity. From the results of an analysis of variation of the fitted values versus the experimental points, a suitable polynomial for each of the x, y, and z components was chosen for use in the total vector velocity (V_s). The new velocity data are given in the final tabulation at the end of the report along with other revised quantities effected by the change in the velocity data. The revised velocity information (V_s) is very nearly the same as assumed curve differing by 64 fps at its maximum deviation. Although these differences may appear small, the reader is to be reminded that at some portions of the trajectory an error of 64 fps would produce an 18°K perturbation in the ambient temperature. The V_x and V_y velocity components make their effect most noticeable in the incident angle calculated from the spatial trajectory tangent and the missile attitude. A change in this angle is in turn propagated into the Mach number. The final values for V_x and V_y compare very well with the assumed values of V_x and V_y so that no changes in the original Mach numbers were necessary in this report. By carrying along the same Mach number from the original report, the pressure profile has not undergone a revision; however, the change in the velocity has also required a change in the temperatures and densities reported earlier.

The revised temperatures are slightly higher below 54 km and slightly lower above 61 km. The relative minimum at 81 km is 169°K , some 9° lower than the original report. The slope of the curve above 81 km remains unchanged, still quite flat compared to adopted rocket panel curve. Temperatures from radio-sonde launched one hour prior to the rocket launching are shown below 30 km which, along with the rocket data, provide a temperature profile from ground level to approximately 90 km. Similarly, the revised densities are lower below 54 km and slightly higher above 61 km. See Figs. 1 and 2.





VACUUM TRAJECTORY

One of the questions which arose in the evaluation of this missile before an experimental missile trajectory was available concerned the applicability of vacuum trajectories in satisfying the analysis. Such a procedure would greatly reduce the delay in obtaining the necessary trajectory. To answer the question of applicability, consider the manner in which the trajectory information enters into the analysis: first, as position (s, y, z) versus time; second, velocity components (V_x, V_y, V_z) versus time; and third, speed versus time. With a little more detail, an understanding of where precision in the measurements is emphasized may be had. The vacuum trajectory's limitation under the requirements will be discussed in a later paragraph.

1. POSITION VERSUS TIME

These data provide the common element for displaying the information obtained from the experiment, e.g., pressure versus altitude. Precision in this measurement is dictated when the results are to be correlated with concurrent experiments not carried on the specific missile (e.g., balloon data).

Errors in the position data are propagated to the velocity derived through differentiation of the position data.

2. VELOCITY COMPONENTS VERSUS TIME

By fitting a suitable polynomial to the position data, it is possible to obtain equations which, when differentiated with respect to the time, provide equations of the velocity components. In the experiment the assumption is made that the missile experiences an air stream tangent to the space trajectory at all times having a magnitude equal and opposite to the missile velocity. It is in defining this spatial position that the V_x and V_y components become important even though they are usually very much smaller than V_z . By the correlation of the spatial tangent with a missile-borne gyroscope, an angle of incidence (ϵ) for a particular time, in addition to rotational position (ψ) with respect to the air stream, is determined for use in interpreting the surface pressures on the body.

3. SPEED VERSUS TIME

Having the quantities ϵ and ψ in addition to the pressure information, a Mach number is defined. The speed squared divided by the Mach number squared multiplied by a suitable constant leads to the static temperature which in turn defines the static density.

A rigorous definition of the error propagated to the static temperature or pressure by deviations from the "true" speed would require an algebraic formulation involving the Mach number, the total head pressure, the cone surface pressure, and the velocity, each of which is also a function of time. For the purposes of this qualitative discussion, it suffices to say that for a specific set of these parameters, where the velocity is in the vicinity of 3000 fps, an error of ± 30 fps in the speed will propagate an error of $\pm 5^\circ\text{K}$ in the static temperature.

THEORETICAL CONSIDERATIONS OF VACUUM TRAJECTORY CALCULATIONS

The term "vacuum trajectory" for the purposes of this discussion shall refer to the changes or rates of change in the body orbit as produced by gravitational forces varying with an inverse square variation with altitude, centrifugal forces, and those produced by the Coriolis acceleration forces, each acting on a body beginning at an altitude Z_1 where the propulsion forces cease. Effects due to air resistance, winds, and the curvature of the earth's surface are not to be accounted for. The analysis will proceed from a knowledge of the terminal velocity V_1 , at an altitude z_1 , the radius of the earth at the launching site r_0 , and also the local acceleration due to gravity at the site, g_0 .

It is of interest to consider the order of the various forces acting on the body to eliminate terms in the equation of motion. The variation of the gravitation acceleration with altitude is considered significant by the author, so that if other forces are to be considered in the discussion they will have to be of similar order. For this purpose consider: at a distance r from the center of the earth the acceleration due to gravity is, $g_0 r_0^2/r^2$. Since r_0 is approximately equal to r in the variation, the change in acceleration due to altitude is approximately $(-2g_0 dr/r)$.

While the Coriolis force does not explicitly depend on altitude, it experiences a change due to the altitude dependence of the velocity. The force due to the Coriolis acceleration is $2uv$, where u is the angular velocity of the earth and v is the velocity of the body. Finally the centrifugal force is of the order u^2r with an altitude variation of the order u^2dr . Two cases are computed for all the forces involved to demonstrate extreme values.

Gravity

$$dr = 25 \text{ km}$$

$$dr = 100 \text{ km}$$

$$r_0 = 6,400,000$$

$$r_0 = 6,400,000$$

then

$$dg = \frac{2 \times 10 \times 25000}{6,400,000}$$

$$dg = 0.078$$

$$dg = 0.314$$

Coriolis

$$\text{at 25 km } v = 1250 \text{ m/sec}$$

$$\text{at 100 km } v = 230 \text{ m/sec}$$

then

$$da_{\text{cor}} = \frac{2 \times 2\pi \times 1250}{3600 \times 24}$$

$$da_{\text{cor}} = 0.182$$

$$da_{\text{cor}} = 0.0335$$

Centrifugal

$$dr = 25 \text{ km}$$

$$u = 7.28 \times 10^{-5} \text{ rad/sec}$$

$$da_n = (7.28 \times 10^{-5})^2 \times 25,000$$

$$da_n = 1.32 \times 10^{-4}$$

$$dr = 100 \text{ km}$$

$$da_n = 5.29 \times 10^{-4}$$

These data may then be evaluated by forming the following ratios for comparison: the ratio of Coriolis force to the acceleration of gravity and the ratio of centrifugal force to the acceleration of gravity.

$$\begin{array}{l} 25 \text{ km} \\ \frac{da_{\text{cor}}}{dg} = 2.33 \end{array}$$

$$\frac{da_n}{dg} = 10^{-3}$$

$$\begin{array}{l} 100 \text{ km} \\ \frac{da_{\text{cor}}}{dg} = 0.107 \end{array}$$

$$\frac{da_n}{dg} = 10^{-3}$$

This table demonstrates that at the low altitudes the acceleration of Coriolis is considerable in proportion to the variation of the gravitational acceleration with the altitude, while diminishing in importance with respect to the gravity variation with increasing altitude. The centrifugal forces are of second-order importance compared to the gravity variation at both extremes of altitude, so that the equation of motion need contain only effects due to gravitational forces and Coriolis forces. The differential equation governing the body takes the form:

$$\ddot{\vec{r}} = -g_0 \frac{r_0^2}{r^2} + 2(\vec{u} \times \dot{\vec{r}}) \quad (1)$$

where:

dots refer to differentiation with respect to time,
 g_0 = acceleration of gravity at earth's surface,
 \bar{r}_0 = radius of earth,
 \bar{u} = earth angular velocity, and
 \bar{r} = distance from earth's center.

Rewriting as:

$$\ddot{\bar{r}} = -g_0 \frac{r_0^2}{\bar{r}^2} + 2 \begin{vmatrix} \bar{i} & \bar{j} & \bar{b} \\ V_x & V_y & V_z \\ u_x & u_y & u_z \end{vmatrix} \cdot \quad (2)$$

Or written as components:

$$\begin{aligned} \ddot{x} &= 2(V_y u_z - V_z u_y) \\ \ddot{y} &= 2(V_z u_x - V_x u_z) \\ \ddot{z} &= -g_0 \frac{z_0^2}{z^2} + 2(V_x u_y - V_y u_x) \end{aligned} \quad (3)$$

Now, if the geographical location of the launching site is allowed to be generalized, then

$$u_x = 0 \qquad u_y = u \cos \phi \qquad u_z = u \sin \phi ,$$

while

$$V_x = x \qquad V_y = y \qquad V_z = z \quad (4)$$

Hence

$$\begin{aligned} \ddot{x} &= 2(\dot{y} u \sin \phi - \dot{z} u \cos \phi) \\ \ddot{y} &= 2(-\dot{x} u \sin \phi) \\ \ddot{z} &= 2 \dot{x} u \cos \phi - g_0 \frac{z_0^2}{z^2} \end{aligned} \quad (5)$$

Equation (1) is a second-order nonlinear differential equation with constant coefficients. In the attempts made to solve the equation, it was not possible to find a transformation to reduce the equation to an integrable linear form. The simplest method available for numerical solution is in the direct use of a Taylor's series. In principle the following steps were taken. For a second-order equation,

$$\ddot{\bar{r}} = f(\bar{r}, \dot{\bar{r}}, t) , \quad (6)$$

given that $\bar{r} = \bar{r}_0$, $\dot{\bar{r}} = \dot{\bar{r}}_1$ when $t = 0$, we can calculate $\ddot{\bar{r}}$ for $t = 0$ directly from the differential equation. Differentiating the equation, we have

$$\ddot{\bar{r}} = \frac{\partial f}{\partial t} + \frac{\partial f}{\partial r} \dot{\bar{r}} + \frac{\partial f}{\partial \dot{\bar{r}}} \ddot{\bar{r}} , \quad (7)$$

in which we can substitute the value of \bar{r} just found, and so determine $\ddot{\bar{r}}$. Differentiating again, we determine $\bar{r}^{(4)}$ and so on to any order desired. The results are substituted in Taylor's series for both \bar{r} and $\dot{\bar{r}}$, as follows:

$$\bar{r} = \bar{r}_0 + \bar{r}_1 t + \frac{\ddot{\bar{r}}(0)}{2!} t^2 + \frac{\ddot{\bar{r}}(0)}{3!} t^3 + \dots \quad (8)$$

and

$$\dot{\bar{r}} = \dot{\bar{r}}_1 + \ddot{\bar{r}}(0)t + \frac{\ddot{\bar{r}}(0)}{2!} t^2 + \frac{\bar{r}^{(4)}(0)}{3!} t^3 + \dots \quad (9)$$

These are used to calculate \bar{r} and $\dot{\bar{r}}$ up to such a value of t that the terms neglected do not affect the last figure retained. Let this be h . Then for $t=h$, we have \bar{r} and $\dot{\bar{r}}$; again using the relation found by differentiation we determine $\ddot{\bar{r}}(h)$, $\ddot{\bar{r}}(h)$, $\bar{r}^{(4)}(h)$, \dots and form new Taylor series in $t-h$. These are used to find values up to $t = 2h$. An important check is obtained by summing the odd and even powers in the series separately. If we have them for $t-h = \xi$, their sum gives r for $t = h + \xi$ but their difference gives r for $t = h - \xi$, which is among the values already calculated, and the two calculations for $h - \xi$ should agree. If they do, they check the whole of the formation of the derivative and the Taylor expansion about h . By repetition we can proceed to any desired value of t .

Carrying out the above procedure, we obtain the expression for r and V as a function of time. Terms higher than the third order have been neglected after investigation proved them negligible compared to the lower-order terms. The orbit of the body after burnout and including zenith is given by

$$r = 209.93029 \times 10^5 + 3975 t - 15.546256 t^2 + 0.000786075 t^3 + \dots, \quad (10)$$

while the velocity is given by

$$V = 3975 - 31.292512 t + 0.00235823 t^2 + \dots \quad (11)$$

(V is fps, r is in ft, and t is in sec.)

The zenith may be calculated by setting $V = 0$ in Eq. (11) and solving for the time and adding 42.9 sec. [The 42.9 sec account for the time necessary to reach the terminal velocity where Eqs. (10) and (11) reckon $t = 0$.]

Thus we obtain for the zenith time the value 171.16 sec, with a corresponding altitude of 337,030 ft.

While experimental values for this missile give zenith at approximately 166.8 sec with an altitude of 329,540 ft, the neglect of air resistance is the largest source of difficulties in obtaining agreement between the theoretical and practical applications of the derived equation. Air resistance could be accounted for, provided some estimate of the functional dependence of the drag forces on velocity, composition, temperature, etc., is made explicitly or perhaps as an empirical relationship establishing the drag force as a function of time and for a specific missile and geographical location.

ERROR PROPAGATION

In an earlier paragraph, the three uses made of the trajectory data were discussed deferring application of the vacuum trajectory. The possible application of the vacuum trajectory to these needs is considered in the following paragraphs. In all cases, the desired precision and manner in which errors in the specific quantity are propagated through the calculation are discussed by comparison between the experimental and theoretical values. The attempt made in this comparison is an investigation of the possibility of replacing a complicated experimental procedure with an analytical one based on a few select experimental boundary conditions. It is not to be construed that the failure of the equations chosen by the author to satisfy his specific needs precludes the possibility of finding an analytical solution satisfactory to some users of trajectory data.

POSITION VERSUS TIME

Using the derived expression for the solution of the equation of motion, values for the altitude were computed for comparison with experimental values of the same time. Results are shown in the following table and in Fig. 3.

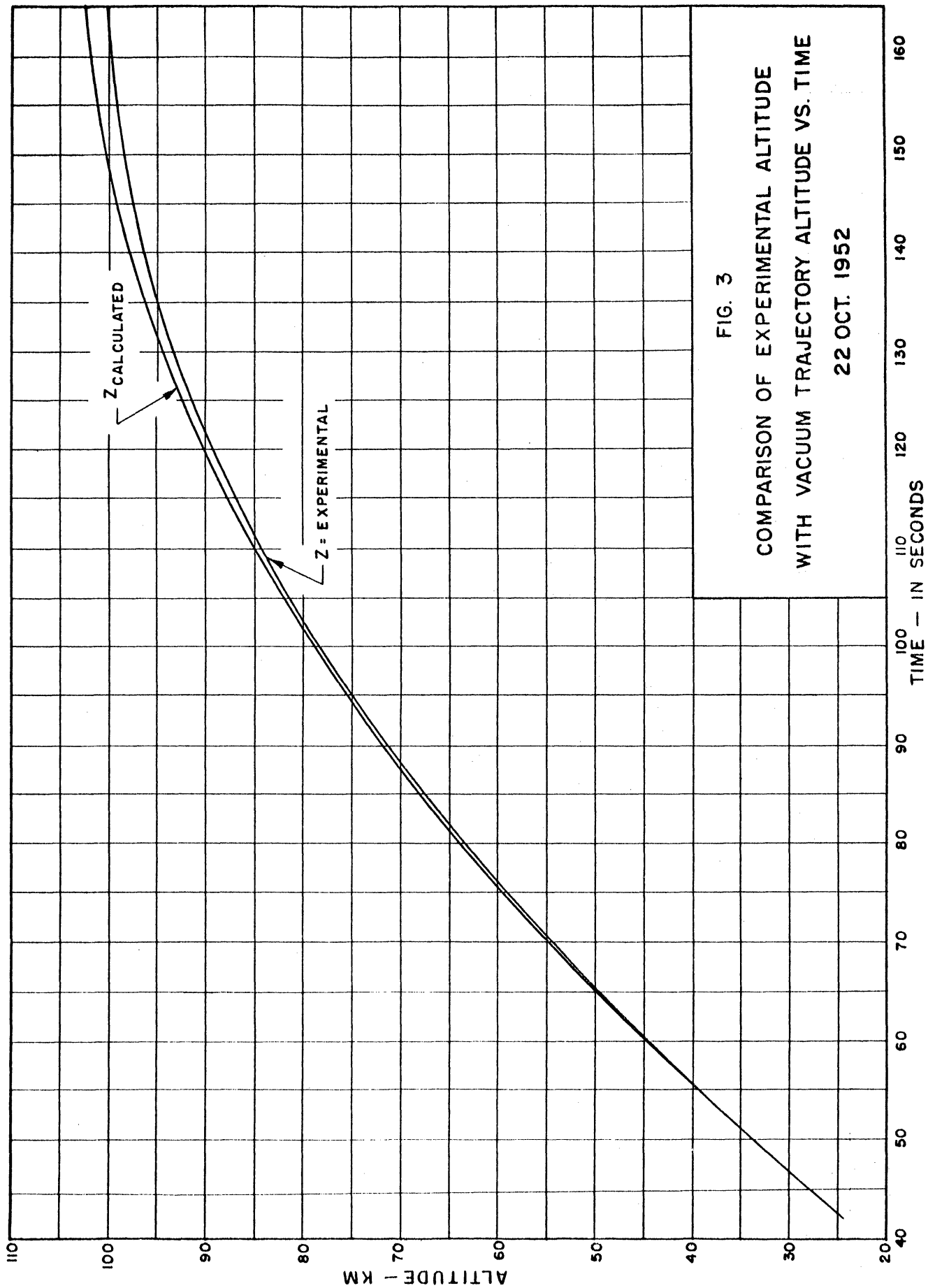


FIG. 3
 COMPARISON OF EXPERIMENTAL ALTITUDE
 WITH VACUUM TRAJECTORY ALTITUDE VS. TIME
 22 OCT. 1952

ALTITUDE VERSUS TIME

<u>Time</u> sec	<u>Z Experimental</u> feet	<u>Z Theoretical</u> feet
42.9	82,929	82,929
55.3	129,480	129,947
67.1	169,568	170,062
80.4	209,070	210,246
96.3	249,506	250,585
116.5	288,105	291,106
139.9	317,328	321,978
166.8	329,550 (Zenith)	336,693
171.2	329,300	337,030 (Zenith)

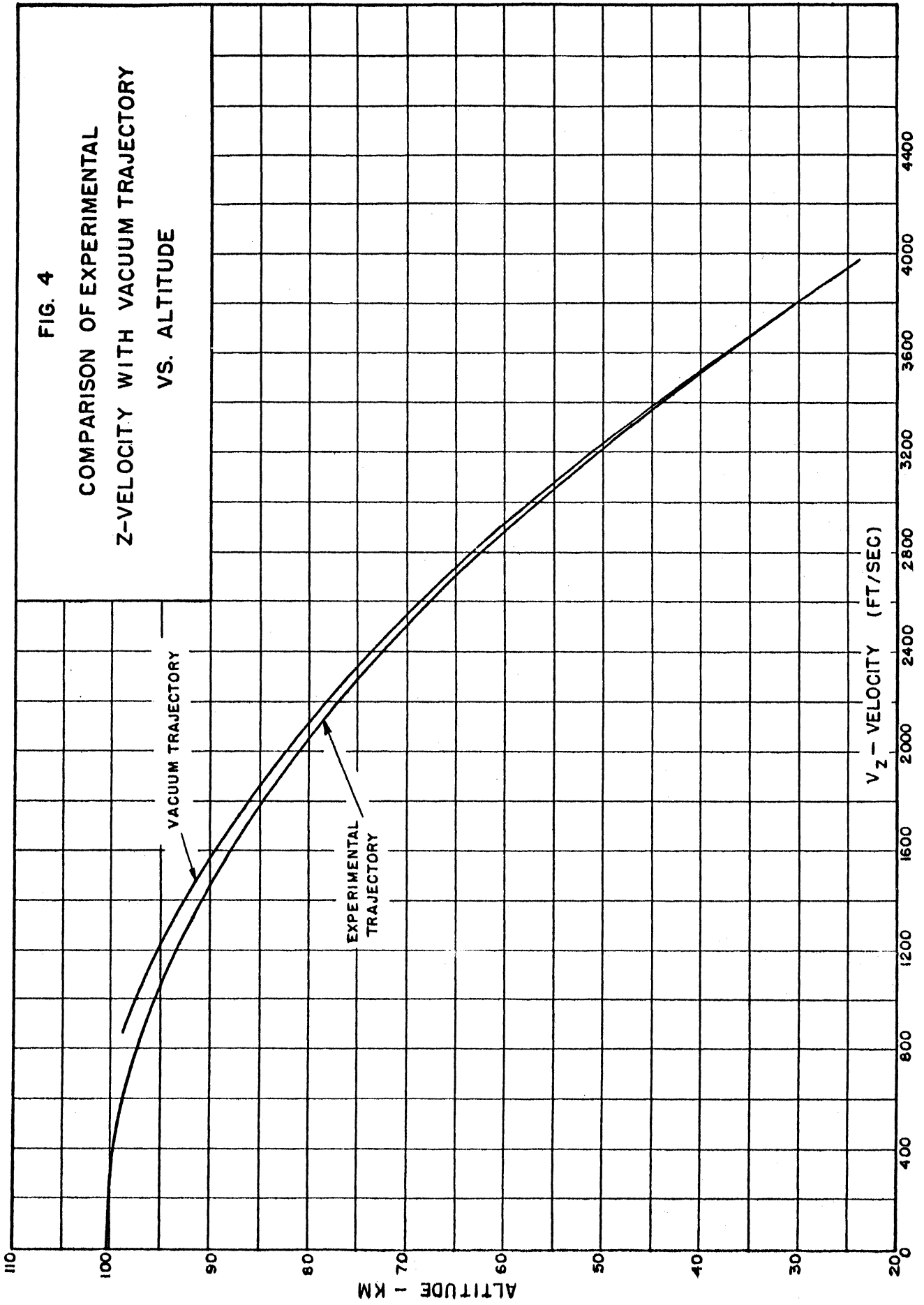
Within the bounds of an individual experiment the differences in the values would not produce any difficulty; however, comparing independent concurrent experiments such as a T-day series or balloon data would lead to annoying discontinuities. As a case in point, such a problem arose with this missile when the preliminary data reported were compared with a balloon run by members of a reviewing committee. This comparison produced a discontinuity between the extrapolation of balloon-pressure and rocket-pressure values. The defect was said to be due to the appearance of the first time-code marker on the trajectory film some 0.8 sec after the true zero time with the effect that altitude information taken from this film would place the missile at higher altitudes earlier than the corresponding pressure data. The difference of 0.8 sec in altitude position would in fact have corrected the discontinuity. Use of vacuum trajectories would produce "time" discrepancies ranging to 6 sec in the specific case of this missile. Perhaps these defects are peculiar to this missile since the beacon triangulation system became intermittent in operation after 72 sec, producing a larger scatter in the experimental points than normal. The larger domain works both ways: allowing experimental curves to agree better or disagree with the vacuum trajectory. Normal beacon data, being less scattered, would allow less choice in the polynomial chosen to represent the data, so that the conclusions drawn might be different from these.

SCALAR VELOCITY

Similarly, a table of velocities is constructed comparing experimental and predicted values. (See also Fig. 4 for graphical representation of the velocities at various altitudes.)

FIG. 4

COMPARISON OF EXPERIMENTAL
Z-VELOCITY WITH VACUUM TRAJECTORY
VS. ALTITUDE



V_z VERSUS TIME

Time sec.	V _z Experimental feet/sec	V _z Theoretical feet/sec
55.3	3553	3607
67.1	3180	3247
80.4	2760	2843
96.3	2258	2365
116.5	1619	1765
139.9	679	1120
166.8	0	131
171.2	-	0

V_x and V_y are not discussed since the neglect of air resistance in the equation of motion removes the only force capable of effecting a change in V_x and V_y.

Experimentally, both quantities showed increases with altitude. The amount of increase would have been difficult to predict if V_x and V_y were specifically required in an experiment where the velocity was derived from this analytical procedure. The assumption that V_s = V_z even in the case of the scalar velocity is not a good one; for example, in the present case at 45.11 km, V_z = 3388, V_x = 634, V_y = 77, and V_s = 3445. Using V_s = V_z produces a temperature of 300°K as compared to 311°K for the experimental V_s. Similarly, in the case of the vacuum trajectory, if the assumption is made that the V_y and V_x at burnout are approximately constant to zenith, then by calculating V_s from its components we obtain a value of 3472 with a corresponding temperature of 316°K. Comparing this with 311°K in the light of the experimental errors, the whole process would appear to be producing reasonable data. But the 5°K difference has occurred only 3 km from burnout where the initial boundary value for the velocity is given experimentally, so that the deviation from the experimental curve is necessarily small in the vicinity of the initial given point. More important is the behavior of the function when it is more distant and near the end of the range at zenith. Figure 4 demonstrates that the deviation grows with departure from the initial point and has its extreme deviation at zenith.

VELOCITY COMPONENTS VERSUS TIME

Since no provision was made for air resistance, V_x and V_y could not be satisfactorily predicted under this procedure.

REMARKS

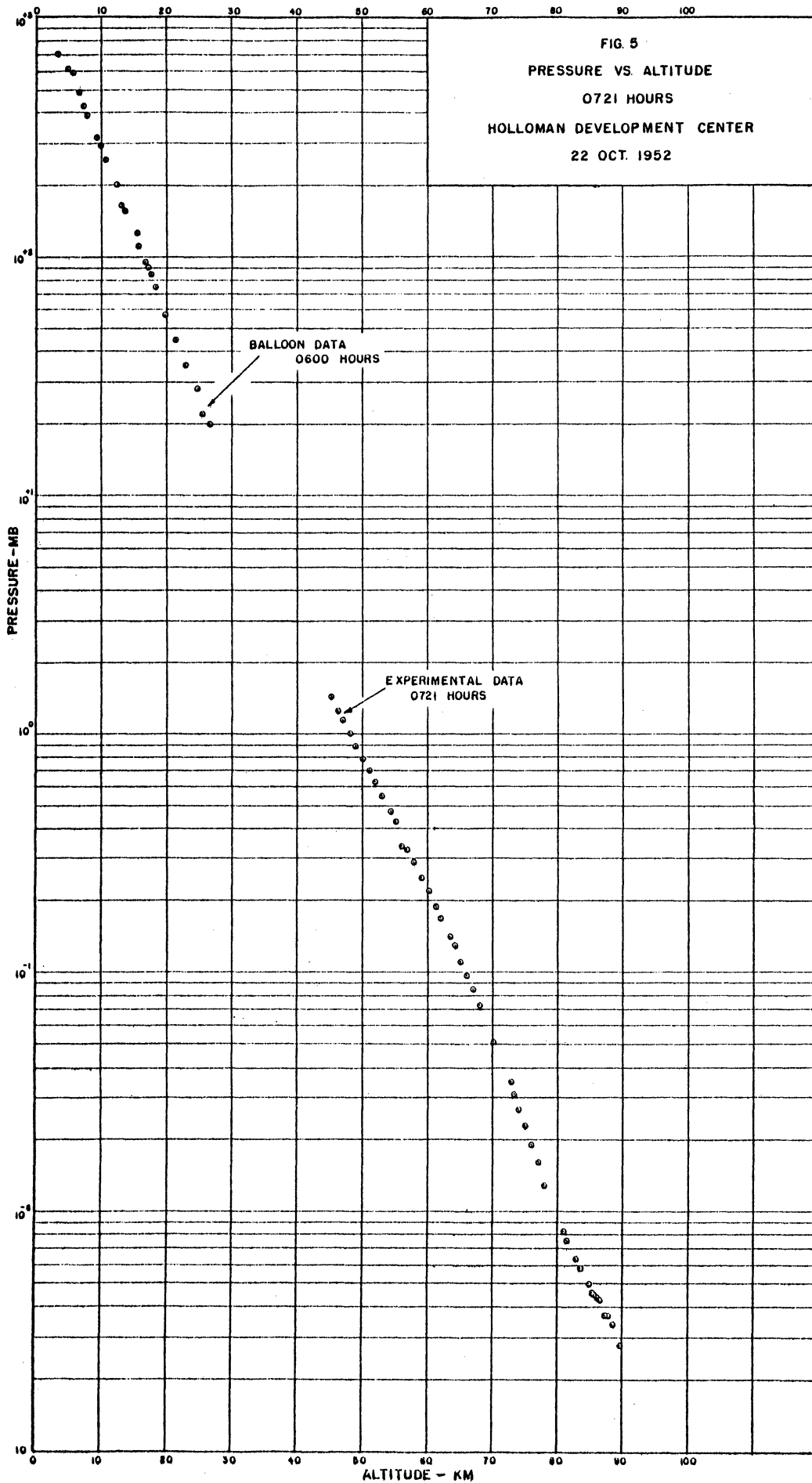
The alleged discrepancy between the extrapolation of the balloon-pressure and subsequent rocket-pressure values was alluded to earlier. The author was convinced the discrepancy existed as represented by the committee at the time they stated their argument. The time error did not really exist since the Flight Determination Laboratory corrected for it at the time they reduced the data; hence if the pressure defect exists, it must stem from another source. Upon further examination of the evidence, the author could not detect a similar discrepancy between the data reported by radio-sonde equipment launched approximately one hour prior to zero time and the data reported by the missile-borne equipment (see Fig. 5). This suggests that possibly the comparison made utilized balloon data taken at a different time than the one available to the author.

Probable errors for the ambient values are calculated as less than 6°K in temperature below 65-km altitude and 15°K above this altitude, while the probable error for the ambient pressure for these same altitude limits would be 4% of the pressure value given below 65 km and 11% at the other extreme.

Since the density is computed through an equation of state involving the pressures and temperature, its probable error is related to the probable error of these functions. The probable error for the density is approximately 4% of the value below 65 km and 13% above.

Data are provided when available in approximately 1-km intervals both in tabular and graphic form for the following parameters:

- a) total velocity (V_s),
- b) cone surface pressure (P_{σ_c}) in millibars,
- c) total head pressure (P_0') in mb,
- d) Mach number (M),
- e) ambient pressure (P_1), mb,
- f) ambient temperature (T), $^{\circ}\text{K}$, and
- g) ambient density (ρ_0), gm/cm^3 .



Altitude, km	°K	P'_0 (mb)	$P\sigma_c$ (mb)	P_1 (mb)	M	V_s	σ (gm/cm ³)
3.04	280.3			710			
4.21	272.5			615			
4.57	271.2			590			
6.10	258.6			486			
7.00	253.7			430			
7.62	248.2			393			
9.14	237.0			318			
9.65	233.0			296			
10.7	224.0			254			
12.2	211.0			200			
12.9	209.0			167			
13.7	210.0			157			
15.2	209.2			127			
15.6	206.0			112			
16.8	205.0			95			
17.1	203.5			90			
17.5	206.7			84			
18.3	206.4			75			
19.8	210.3			57			
21.3	213.4			45			
22.9	218.5			35			
24.4	222.0			28			
25.9	224.3			22			
26.5	225.0			20			
45.11	311	16.6	1.99	1.44	2.965	3445	1.62×10^{-6}
46.12	304	14.5	1.60	1.25	2.968	3410	1.44
47.03	312	12.7	1.57	1.15	2.903	3380	1.29
48.07	313	10.9	1.37	1.00	2.878	3355	1.92
49.04	310	9.51	1.22	0.89	2.868	3325	1.00
50.11	309	8.20	1.08	0.79	2.837	3286	0.890
51.03	312	7.26	0.844	0.70	2.800	3256	0.783
52.09	310	6.30	0.820	0.63	2.780	3225	0.709
53.00	305	5.62	0.734	0.55	2.777	3196	0.629
54.28	294	4.87	0.577	0.47	2.798	3160	0.558
55.05	293	4.37	0.538	0.43	2.774	3130	0.512
56.02	283	3.84	0.491	0.34	2.800	3100	0.419
57.00	281	3.35	0.404	0.33	2.779	3065	0.410
58.00	282	2.92	0.357	0.29	2.743	3034	0.359
59.00	280	2.49	0.357	0.25	2.722	3000	0.312
60.01	277	2.11	0.271	0.22	2.705	2965	0.277
61.20	272	1.84	0.263	0.19	2.699	2930	0.244
62.00	272	1.62	0.219	0.17	2.695	2926	0.218
63.40	263	1.35	0.166	0.14	2.668	2850	0.186
64.25	258	1.21	0.140	0.13	2.665	2820	0.176
65.00	253	1.05	0.138	0.11	2.665	2790	0.152

Altitude, km	°K	P _O ' (mb)	Pσ _c (mb)	P ₁ (mb)	M	V _S	ρ (gm/cm ³)
66.00	248	0.910	0.120	0.097	2.653	2750	0.137x10 ⁻⁶
67.00	242	0.789	0.108	0.085	2.643	2710	0.123
68.00	235	0.679	0.107	0.072	2.671	2670	0.107
70.00	216	0.485	0.0569	0.051	2.674	2590	0.082
72.60	210	0.311	0.0405	0.035	2.600	2480	0.058
73.25	207	0.279	0.0308	0.031	2.592	2455	0.052
74.00	203	0.241	0.0362	0.027	2.580	2420	0.046
75.00	196	0.201	0.0257	0.023	2.579	2375	0.041
76.00	189	0.168	0.0304	0.019	2.579	2335	0.035
77.00	182	0.143	0.0218	0.016	2.579	2290	0.031
78.00	174	0.113	0.0130	0.013	2.588	2250	0.026
80.95	169	0.066	0.0100	0.0083	2.468	2110	0.017
81.30	172	0.059	0.0074	0.0076	2.420	2090	0.015
82.45	174	0.043	0.0058	0.0064	2.342	2035	0.013
83.75	178	0.036	0.0059	0.0058	2.238	1965	0.012
84.92	200	0.028	0.0040	0.0050	2.044	1905	0.0087
85.31	201	0.026	0.0047	0.0046	2.009	1875	0.0080
85.71	213	0.024	0.0047	0.0045	1.929	1855	0.0073
86.01	206	0.023	0.0040	0.0044	1.936	1830	0.0073
86.44	219	0.021	0.0040	0.0043	1.855	1810	0.0070
87.42	238	0.017	0.0029	0.0037	1.733	1760	0.0056
87.81	237	0.016	0.0033	0.0037	1.707	1730	0.0056
88.51	268	0.011	0.0029	0.0034	1.570	1692	0.0045
89.46	285	0.0095	0.0023	0.0028	1.470	1635	0.0035
89.73	309	0.0094	0.0023	0.0028	1.399	1620	0.0031

APPENDIX II*

SKIN-TEMPERATURE ANALYSIS FOR VARIOUS MATERIALS, TERMINAL VELOCITIES, AND ALTITUDES

AERODYNAMIC HEATING AND HEAT TRANSFER

In any general approach to analyzing the skin temperature of a high-speed missile, the following factors should be given consideration:

- (1) heat received through the boundary layer,
- (2) heat lost by radiation to space,
- (3) heat received by radiation from the sun,
- (4) heat received by radiation from the earth,
- (5) heat transmitted to the interior of the missile, and
- (6) heat received from the combustion chamber.

The relative contribution of each factor is given as follows: factor (1) averages over 5 Btu/ft² sec; factor (3) is of the order of 0.1 Btu/ft² sec; factor (4) is approximately 0.02 Btu/ft² sec; while factors (5) and (6) are considered negligible.

The skin temperature will stabilize at the value for which the heat absorbed from the boundary layer just equals the heat radiated to space. For temperatures less than 4000° Rankine, the amount of heat absorbed is large compared to the amount radiated. The skin temperature therefore rises to the boundary-layer temperature asymptotically.

HEAT RADIATED TO SPACE

$$\left. \frac{\partial Q}{\partial t} \right]_{\text{absorbed}} = \sigma \epsilon / 3600 ST^4 \text{ Btu/ft}^2 \text{ sec}, \quad (1)$$

where

- S = surface of missile in feet,
- $\sigma = 17.3 \times 10^{-10}$,
- T = temperature in Rankine, and
- ϵ = emissivity factor.

*Prepared by H. S. Sicinski.

HEAT ABSORBED

For a cone of length L , vertex angle V at a velocity V , the heat transferred per unit area per second per degree temperature difference is

$$0.22 \lambda/L [VL/\nu]^{0.8} B^{1/3} \text{ Btu/ft}^2 \text{ sec R}, \quad (2)$$

where

- λ = thermal conductivity,
- ν = kinematic viscosity, and
- V = velocity, fps.

Assuming that the Reynolds number is raised to the first power, the terms in the above equation may be arranged as follows:

$$0.022 [\lambda/c_p\mu] c_p gB^{1/3} V \quad (3)$$

where

- c_p = specific heat in Btu/lb R
- μ = viscosity coefficient.

To simplify further, we use the fact that the Prandtl number is approximately constant between 1000°R and 3000°R and has the value 0.65. Coupled with the average value of $C_p = 0.27$, the expression becomes:

$$0.3\rho VB^{1/3} \quad (4)$$

where

- ρ_{sk} = skin density
- ρ = air density.

The bodies of the proposed missile will probably have a fineness ratio between 10 and 15. We find that the vertex angle of a cone of such ratio will lie between 8 and 11 degrees.

The formula for heat transfer becomes

$$\left. \frac{\partial Q}{\partial t} \right]_{\text{absorbed}} = (0.16 g\rho V)(S)(T_{BL}-T) \quad (5)$$

EQUILIBRIUM

At equilibrium, Eq. (1) equals Eq. (5).

The boundary-layer temperature is available from the expression

$$T_{BL} = T_a [1 + 0.18 M^2] \quad (6)$$

T_a = ambient temperature
 M = Mach number.

After substituting the value of T_{BL} and rearranging, we have

$$T^4 + 3 \times 10^{14} \left(\frac{\rho V}{\epsilon}\right) T - 3 \times 10^{11} \left(\frac{\rho V}{\epsilon}\right) (T_a + 0.18 M^2 T_a) = 0, \quad (7)$$

where it is evident that, up to temperatures of 4000°R, the first term is small compared with the second, and the body temperature is controlled by the boundary layer temperature. Therefore the highest possible skin temperature is given by

$$T = T_a(1 + 0.18M^2) \quad (8)$$

This is sometimes known as the adiabatic skin temperature (the result of using a zero-thickness skin or zero-specific-heat material).

TEMPERATURE OF SKIN VERSUS TIME

The heat per second transferred to the skin by the boundary layer is

$$0.16\rho VS(T_{BL} - T) \quad (9)$$

If $\rho_{sk}g$ is the weight of skin per ft³,
 t is the time in sec, ft,
 r is the skin thickness in ft,
 ρ is the air density in slugs/ft³, and
 $C_{p_{sk}}$ is the specific heat of the skin,

we can write

$$\log (T_{BL}-T) = - \frac{.16\rho V}{C_{p_{sk}} \rho_{sk} g r} t \quad (10)$$

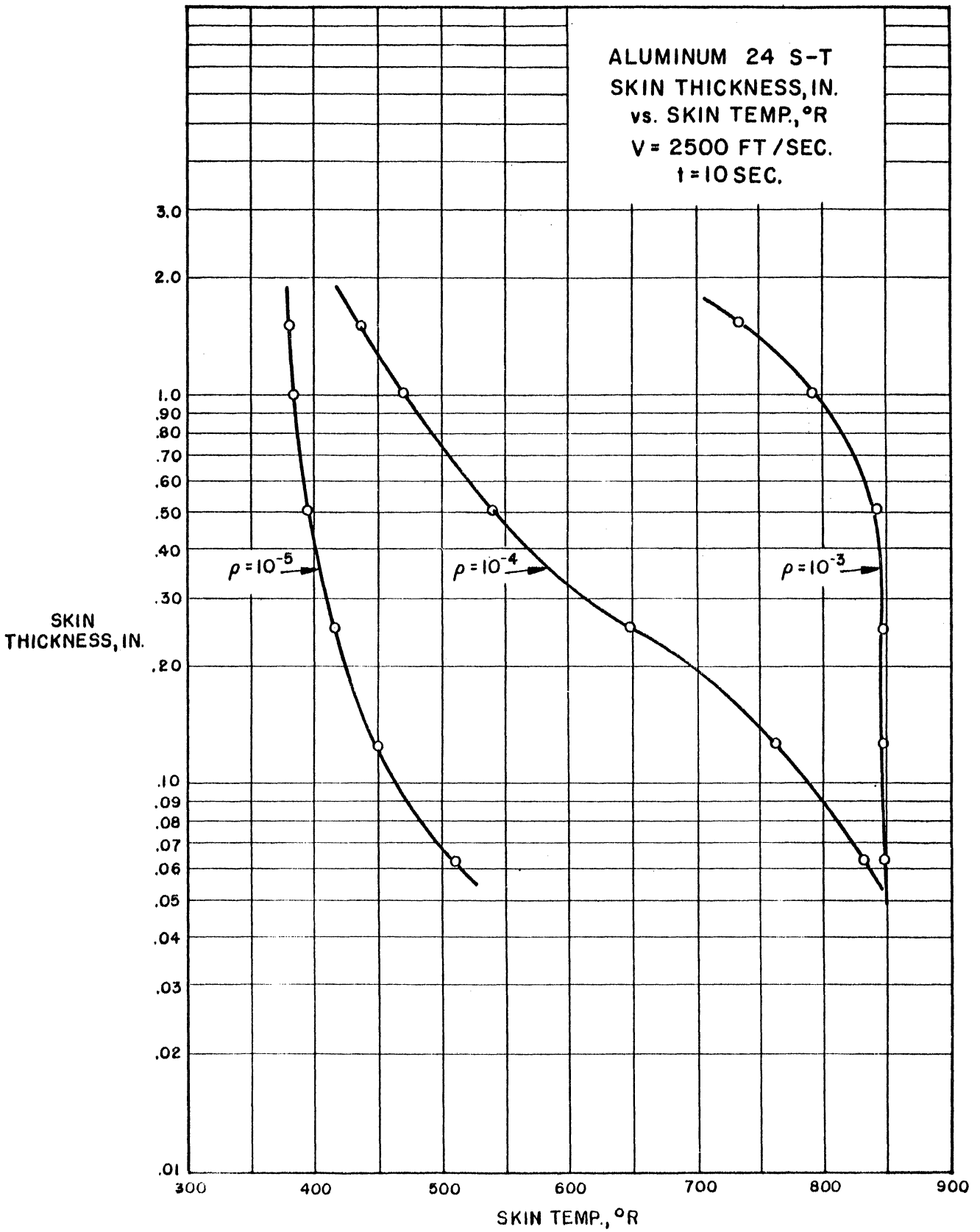
For the accessory conditions in evaluating the constants, we note that when the time is zero, the temperature of the body equals the temperature of the atmosphere T_a . This provides the equation

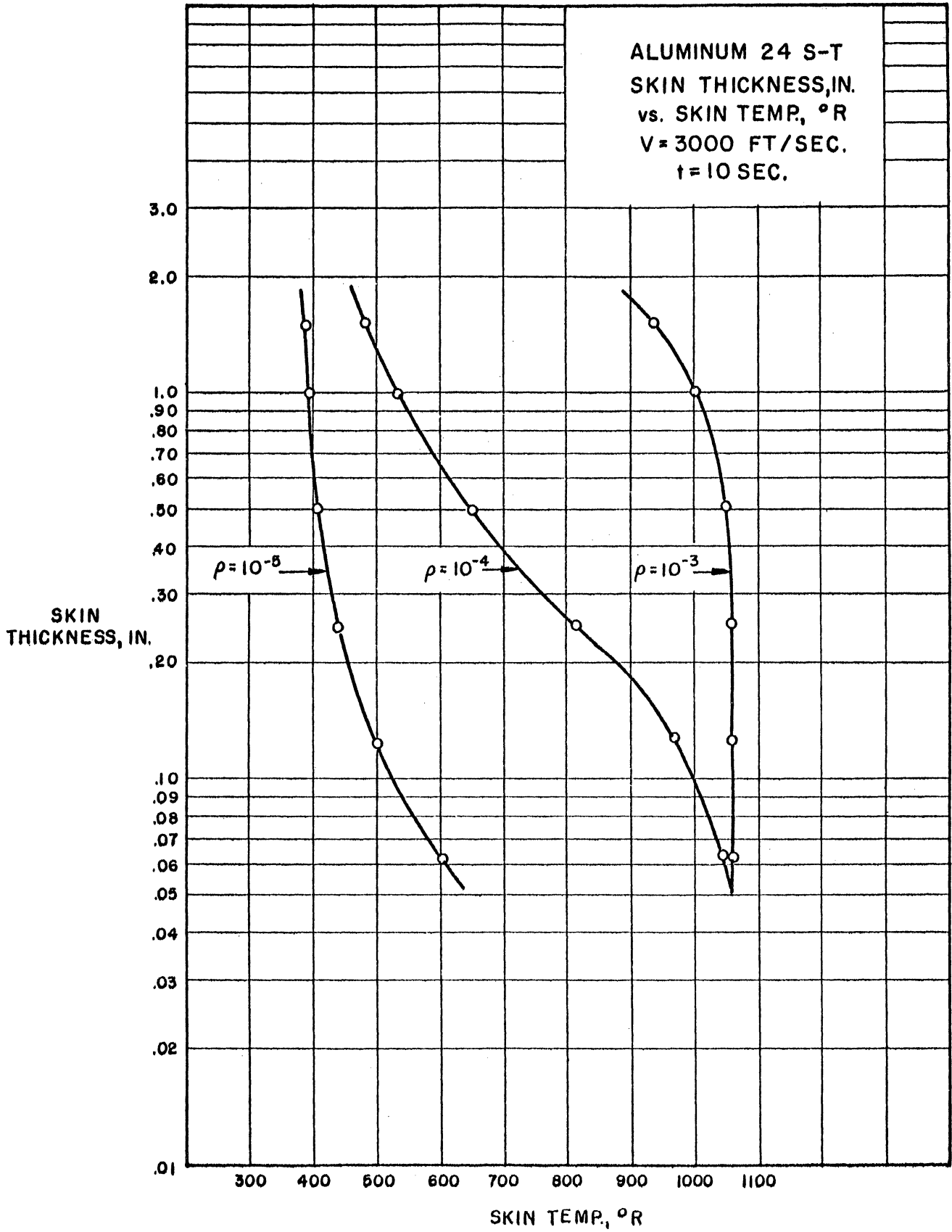
$$T_{\text{skin}} = T_{\text{BL}} - (T_{\text{BL}} - T_a) \exp\left(-\frac{.159\rho V}{C_{\text{psk}} r \rho_{\text{sk}} g}\right) . \quad (11)$$

Using this equation, the accompanying curves have been calculated for various materials, missile velocities, and various terminal altitudes. The altitudes are characterized by the air densities given on the plots. These correspond to the Rocket Panel data converted to slugs/ft³. The temperatures are plotted vs. the skin thickness and are those temperatures available in the bodies after being exposed to the boundary layer for a period of 10 sec. They may be interpreted as the temperature of a solid cone cylinder model having the given temperature at a depth below the skin (r).

The densities given on the plots correspond to approximately the following altitudes in feet:

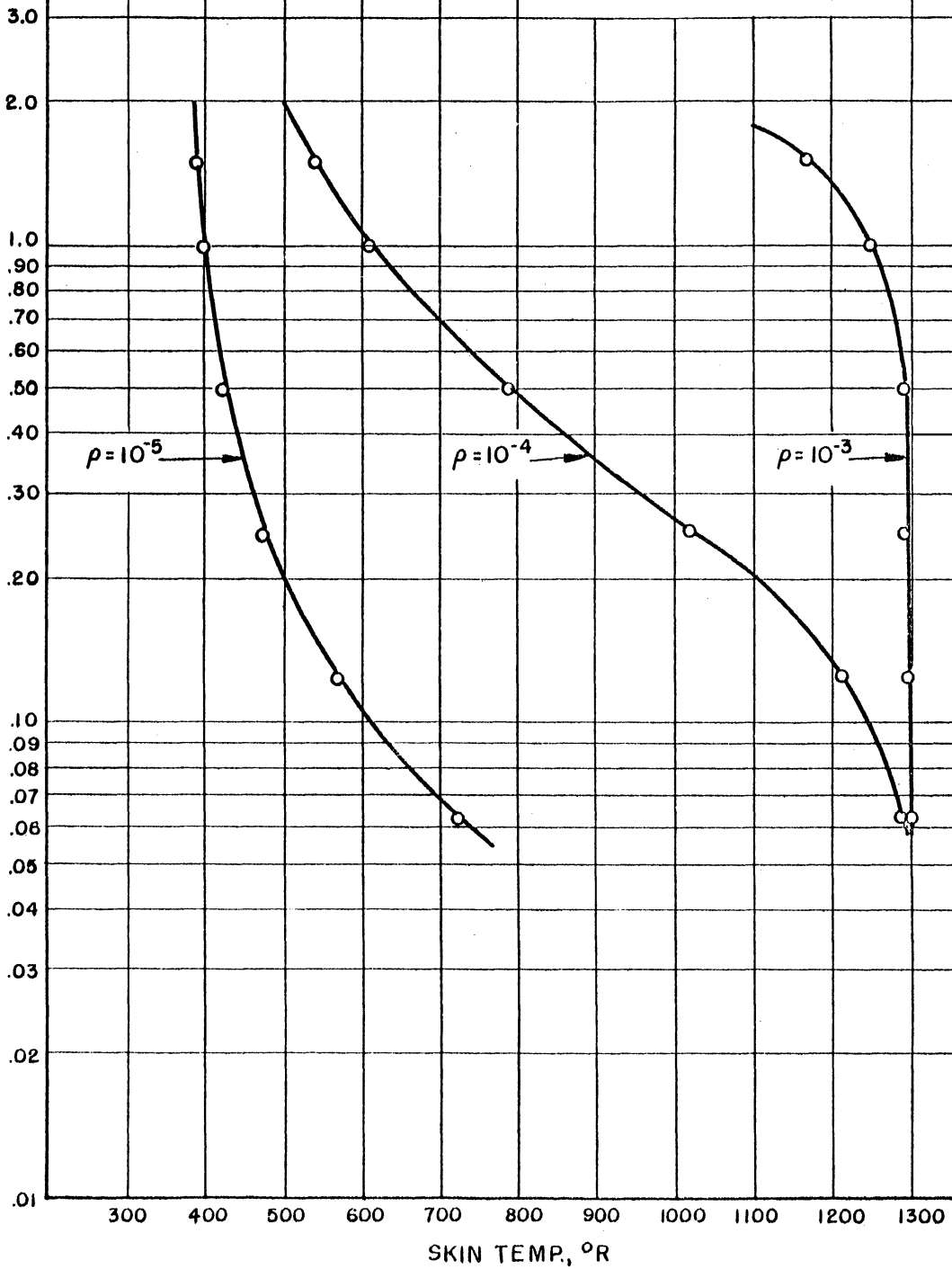
$$\begin{aligned} 10^{-3} &= 6,500 \\ 10^{-4} &= 66,000 \\ 10^{-5} &= 118,000 . \end{aligned}$$

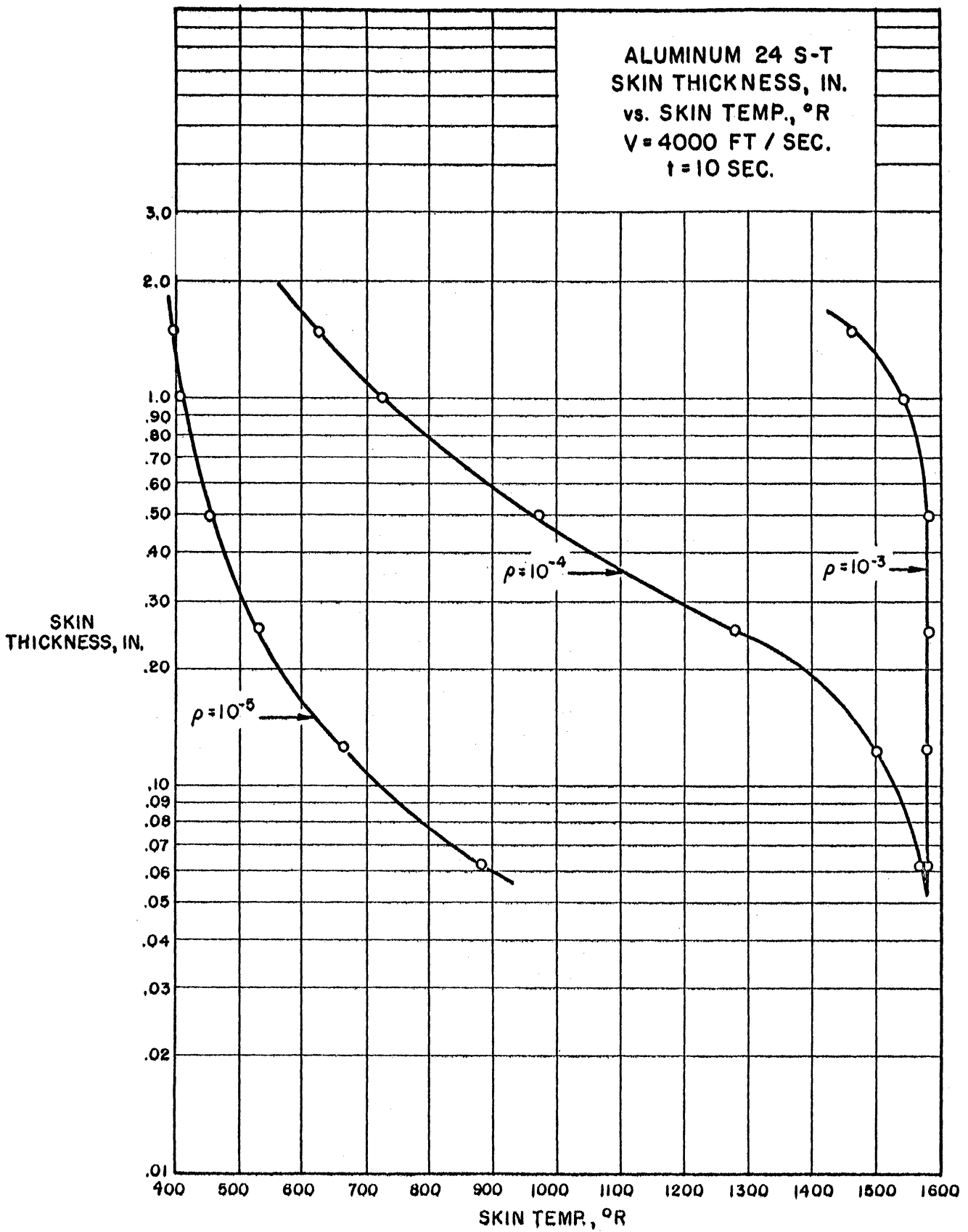


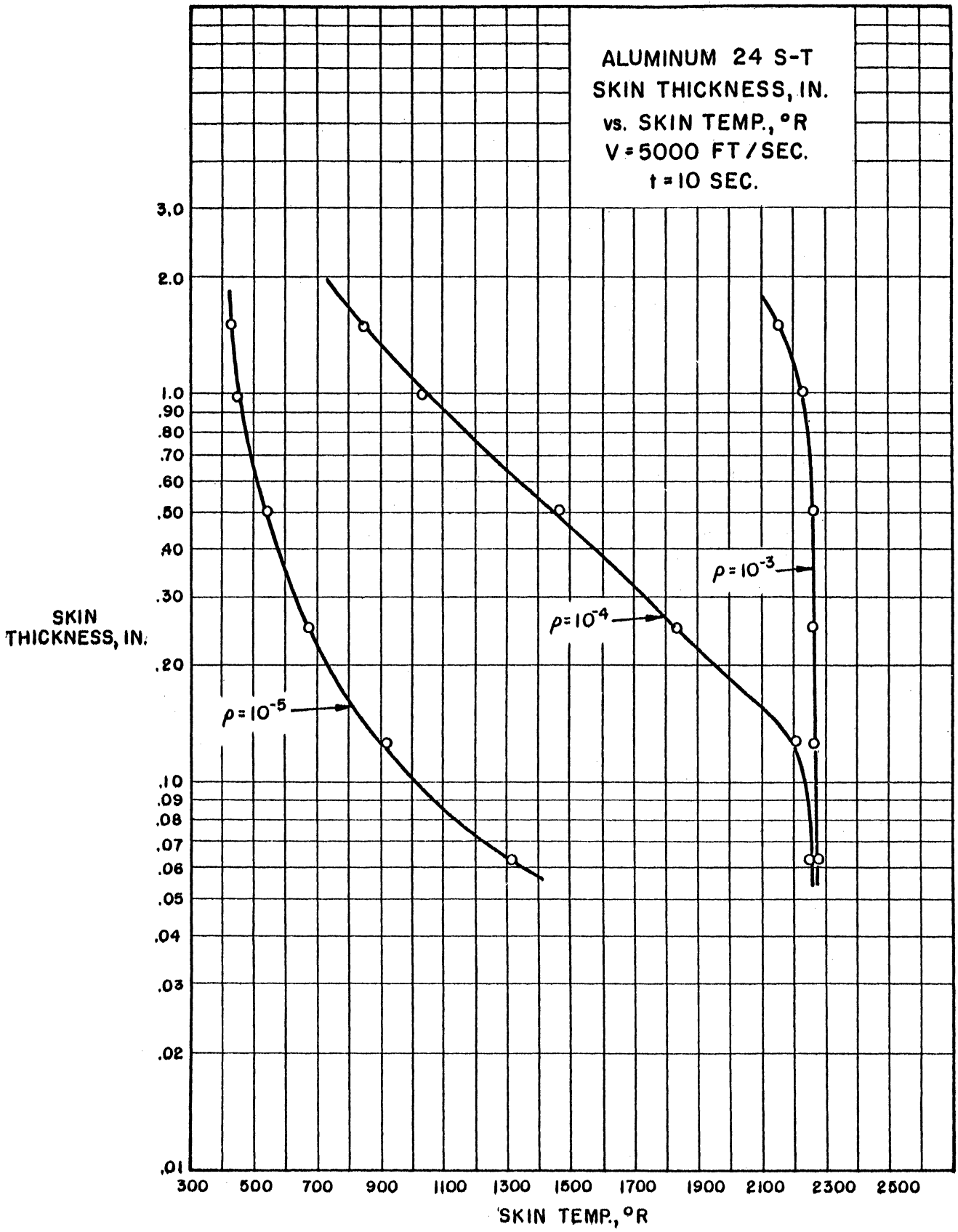


ALUMINUM 24 S-T
 SKIN THICKNESS, IN.
 vs. SKIN TEMP., °R
 V = 3500 FT / SEC.
 t = 10 SEC.

SKIN
 THICKNESS, IN.

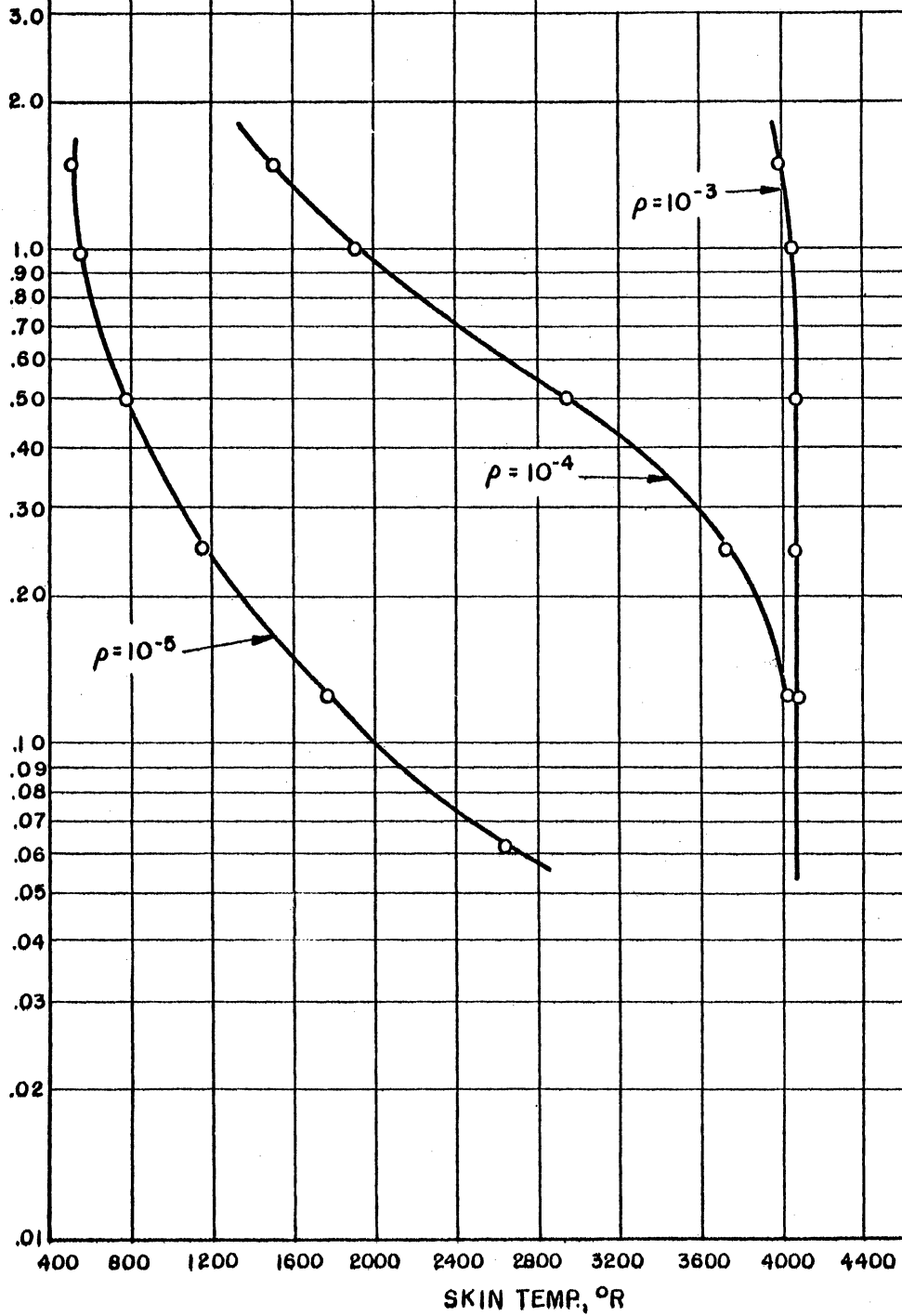


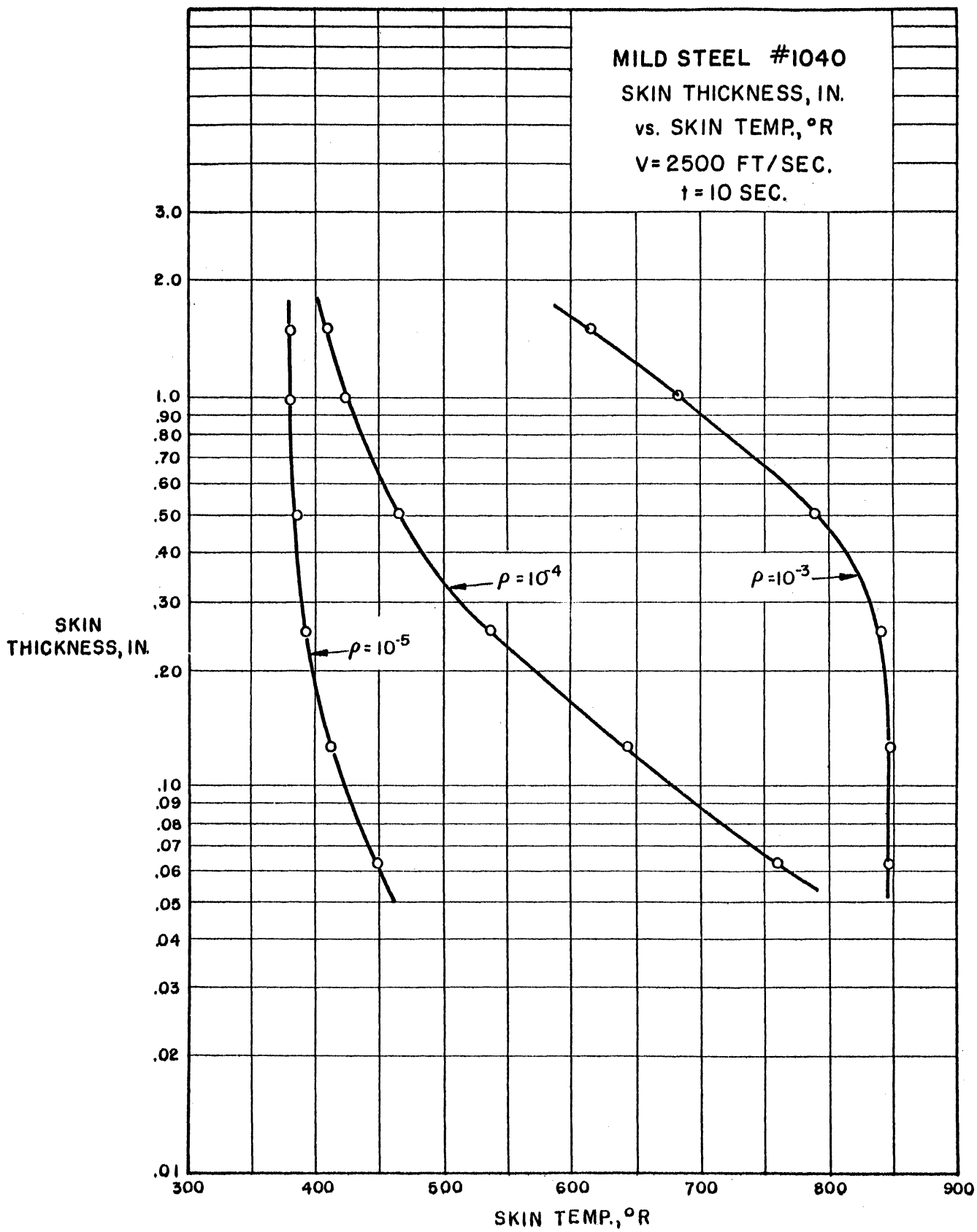


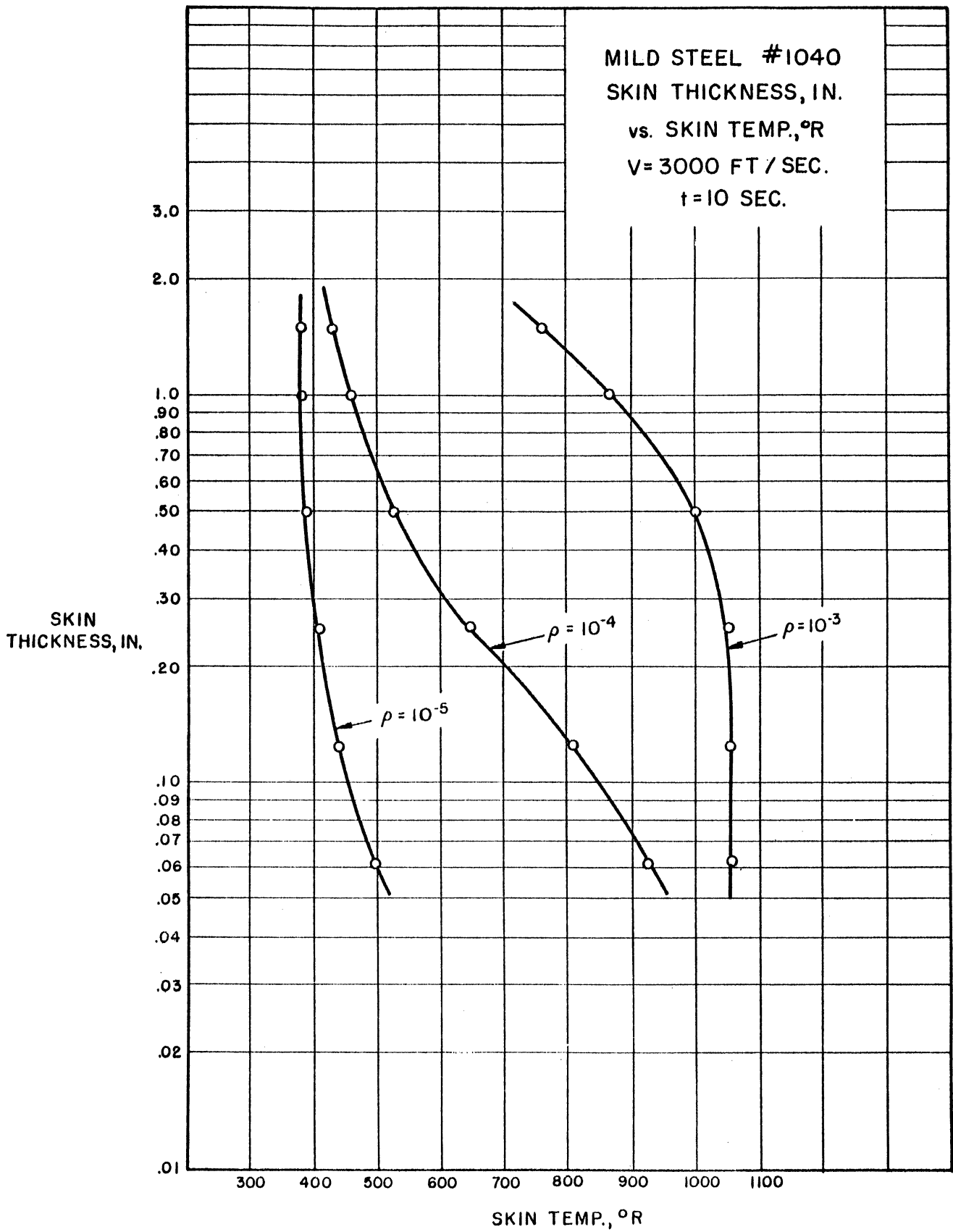


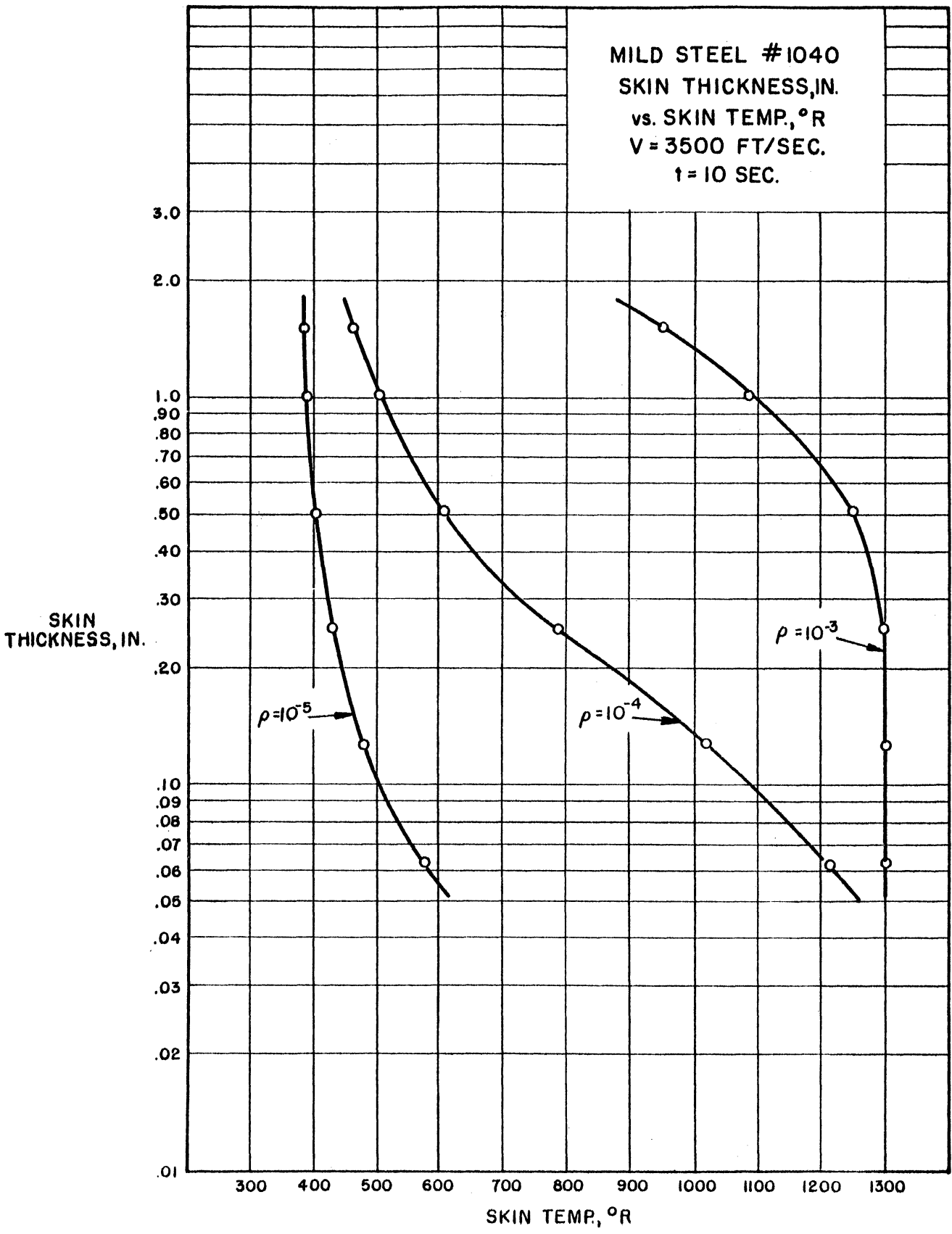
ALUMINUM 24 T-S
SKIN THICKNESS, IN.
vs. SKIN TEMP, °R
V = 7000 FT/SEC.
t = 10 SEC.

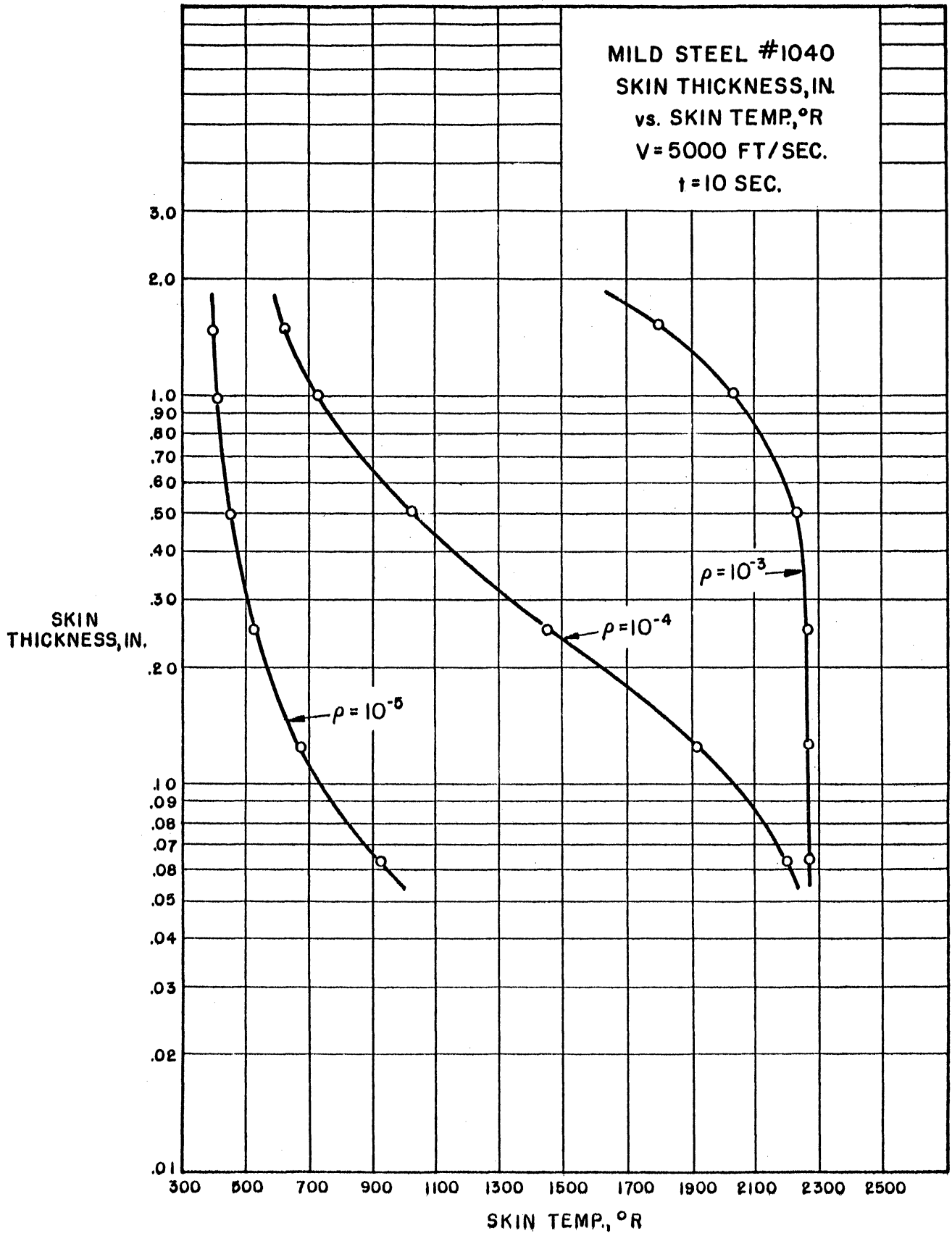
SKIN
THICKNESS, IN.

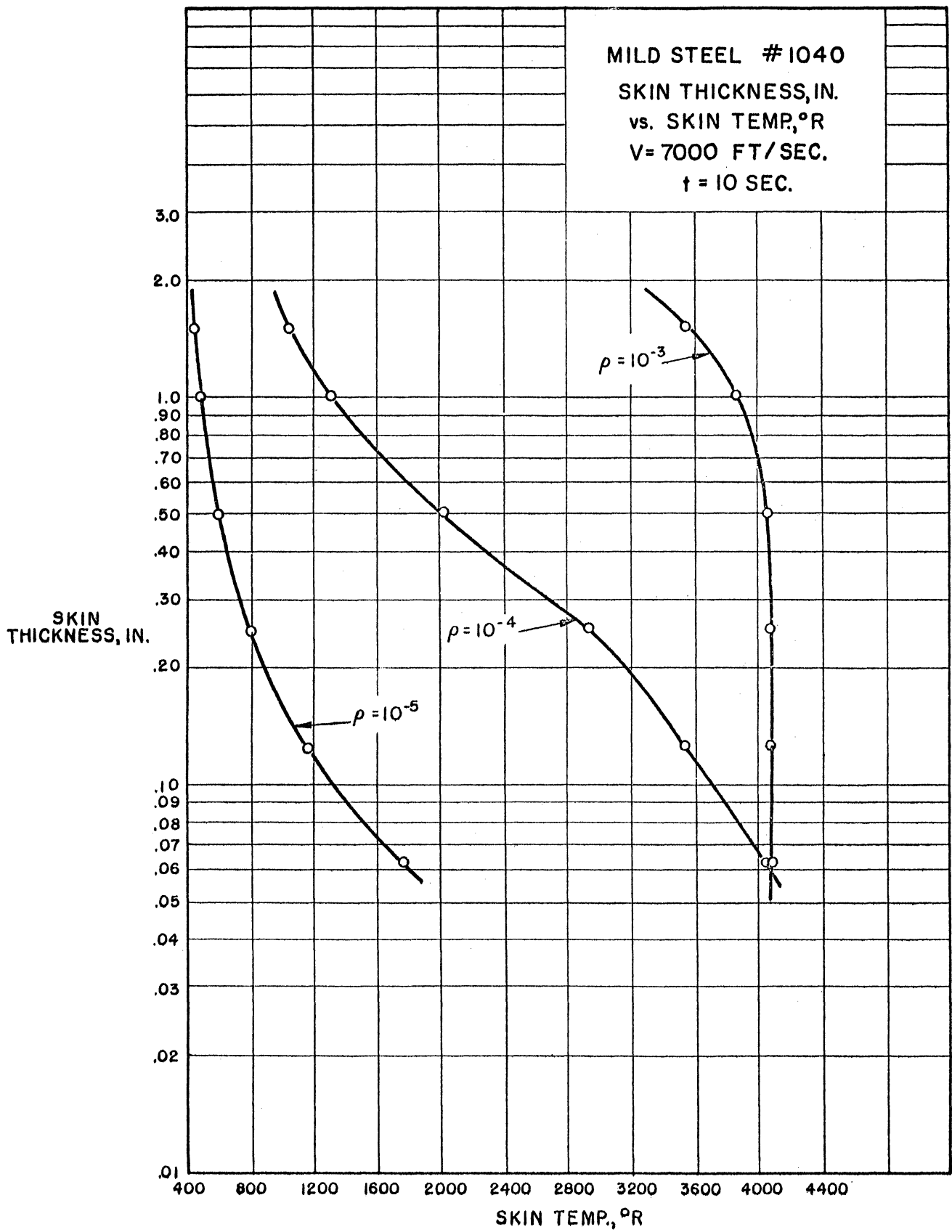






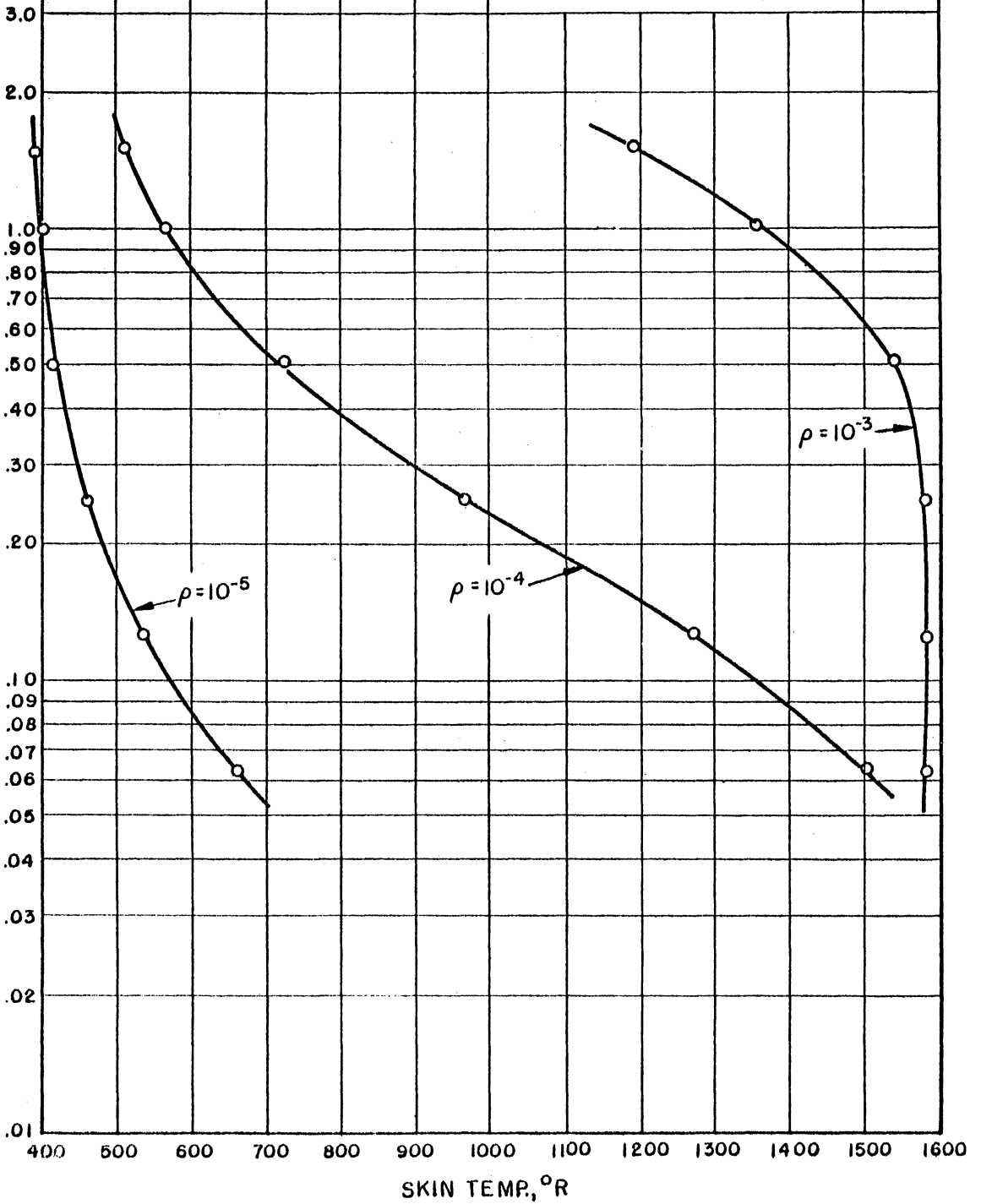


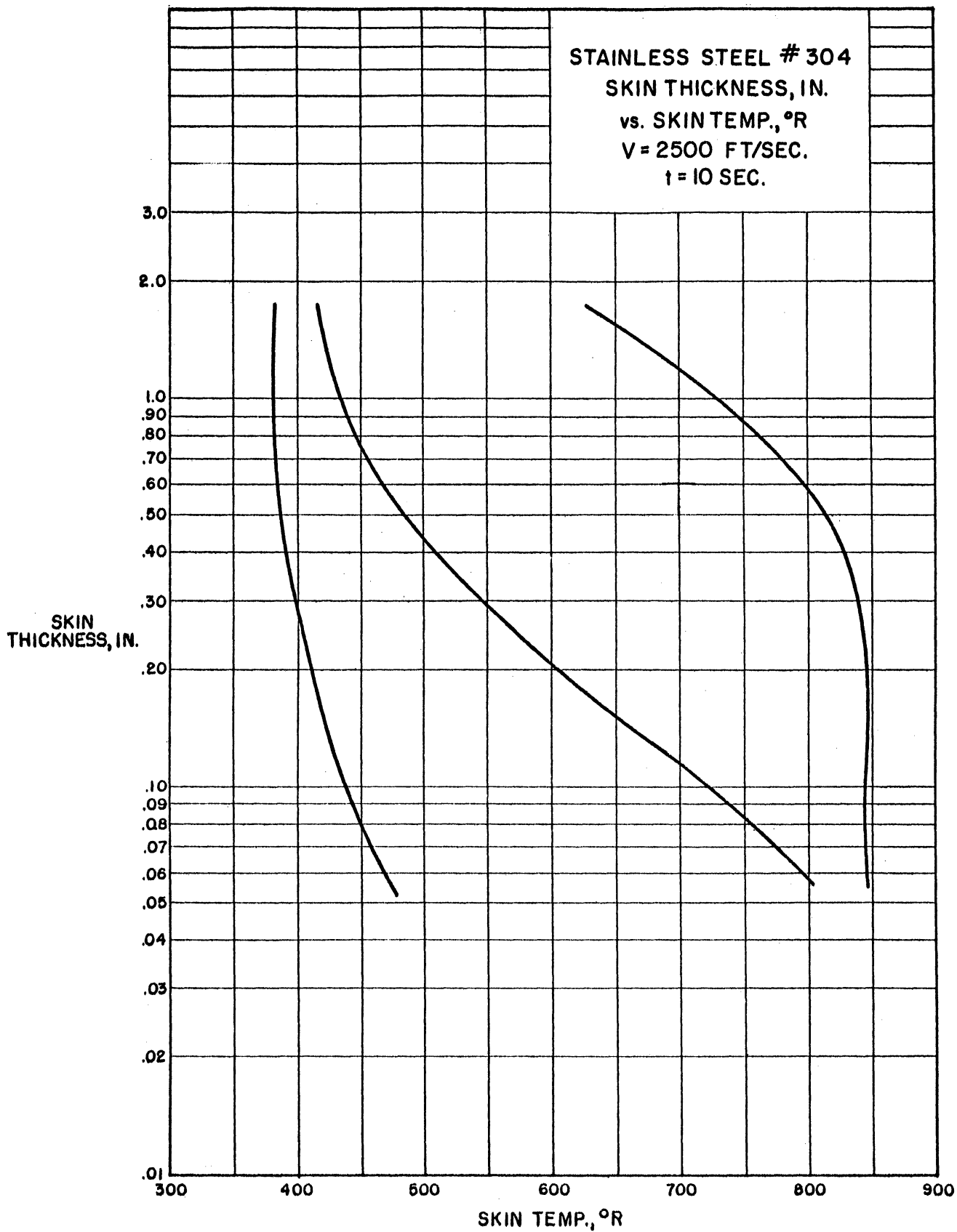


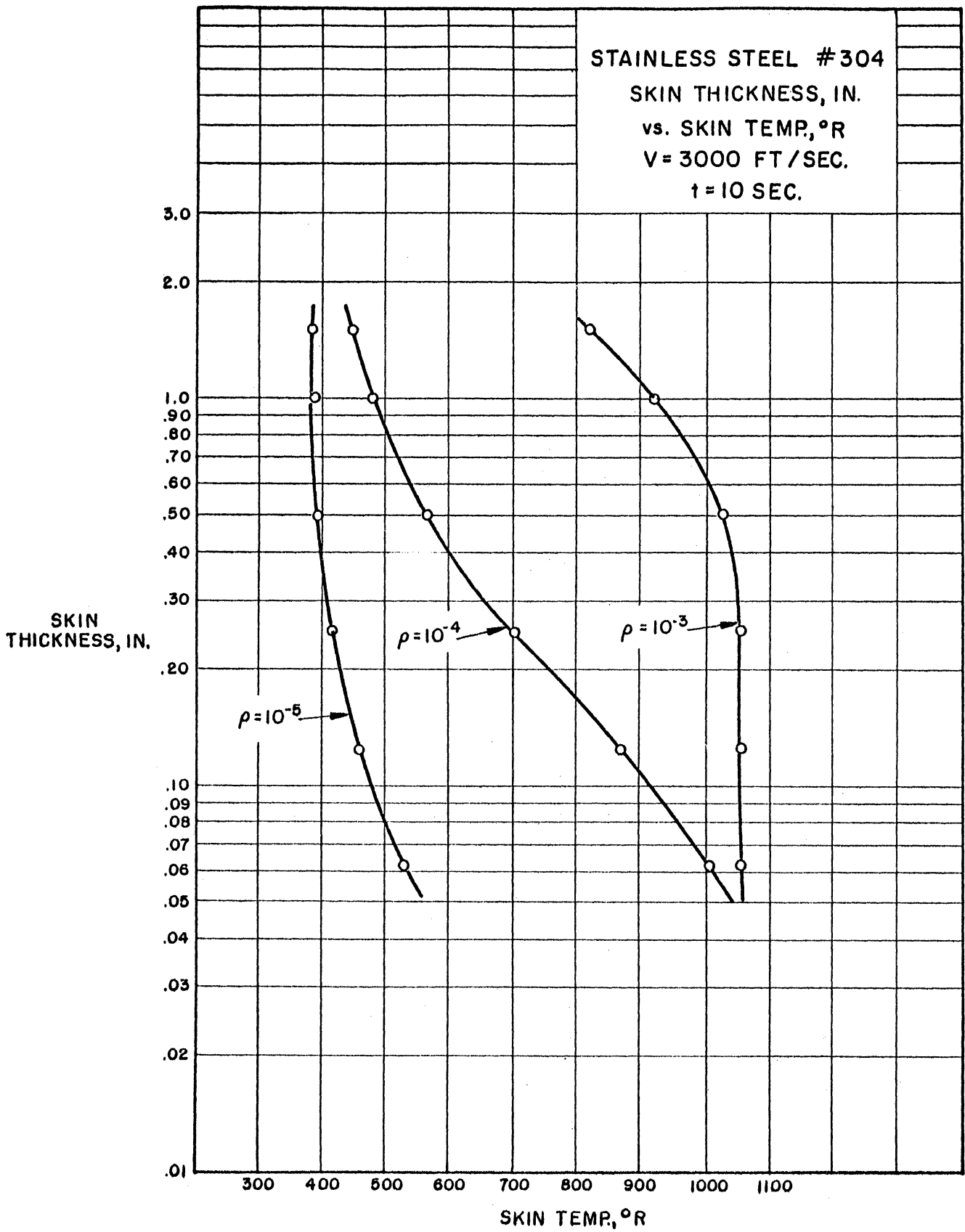


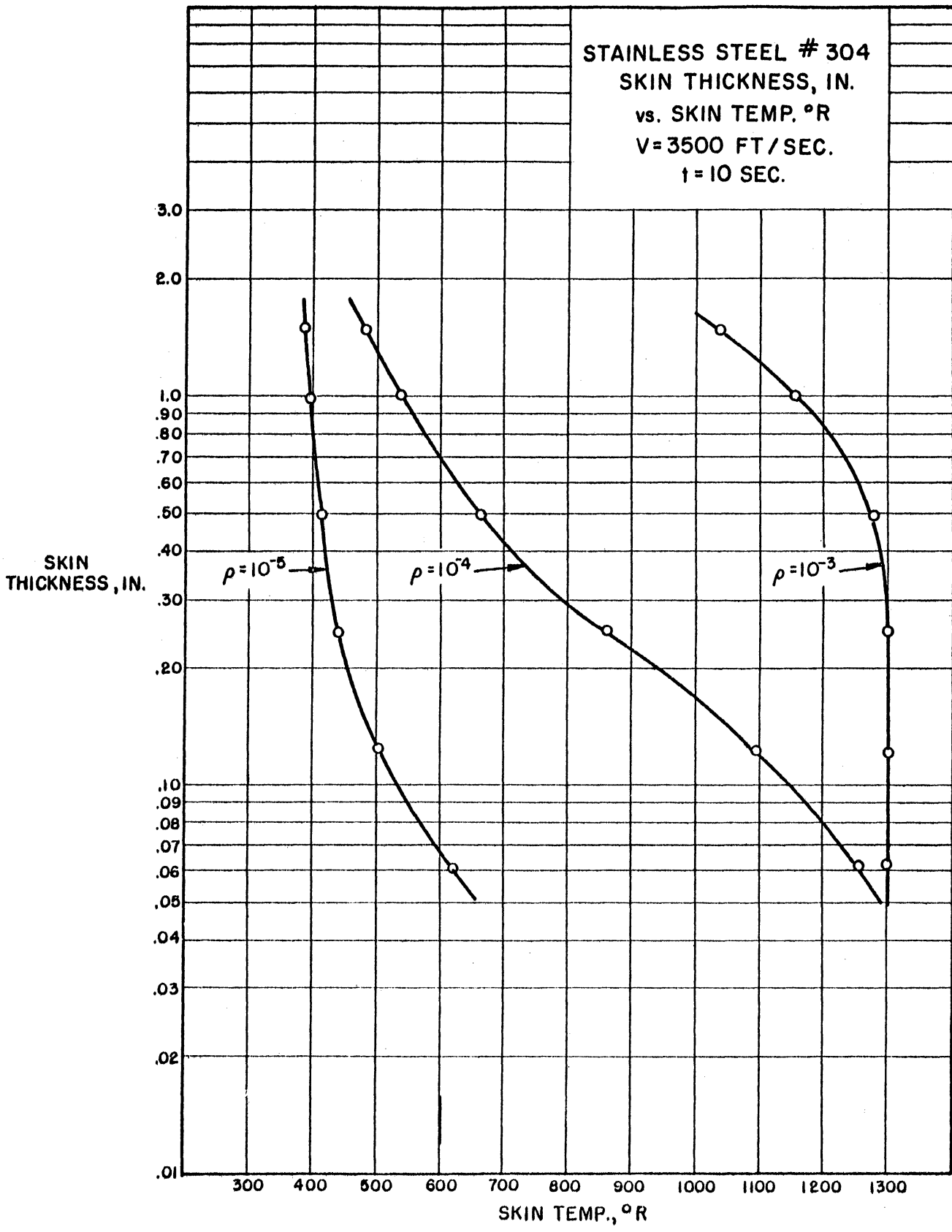
MILD STEEL #1040
 SKIN THICKNESS, IN.
 vs. SKIN TEMP., °R
 V = 9000 FT/SEC.
 t = 10 SEC.

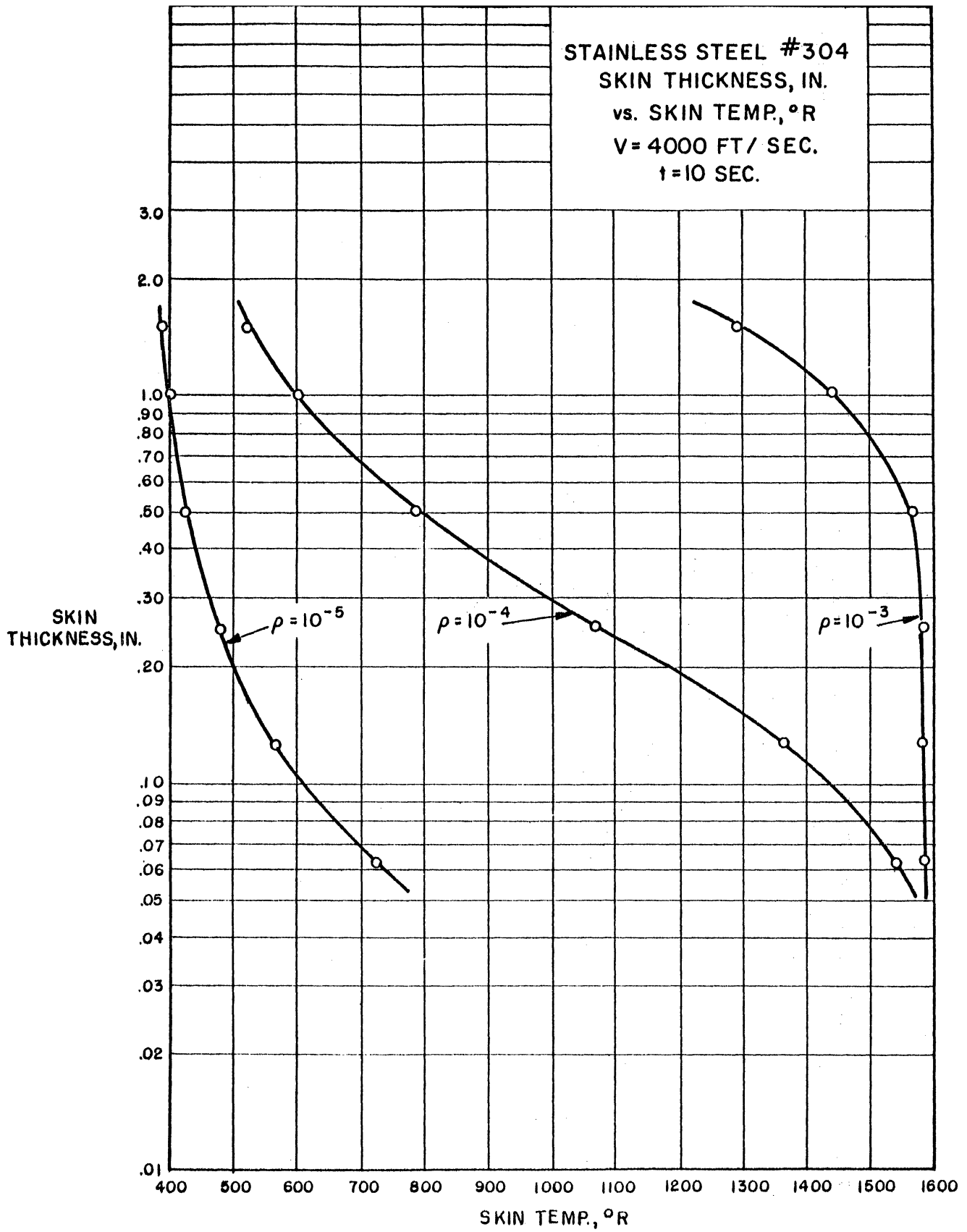
SKIN THICKNESS, IN.

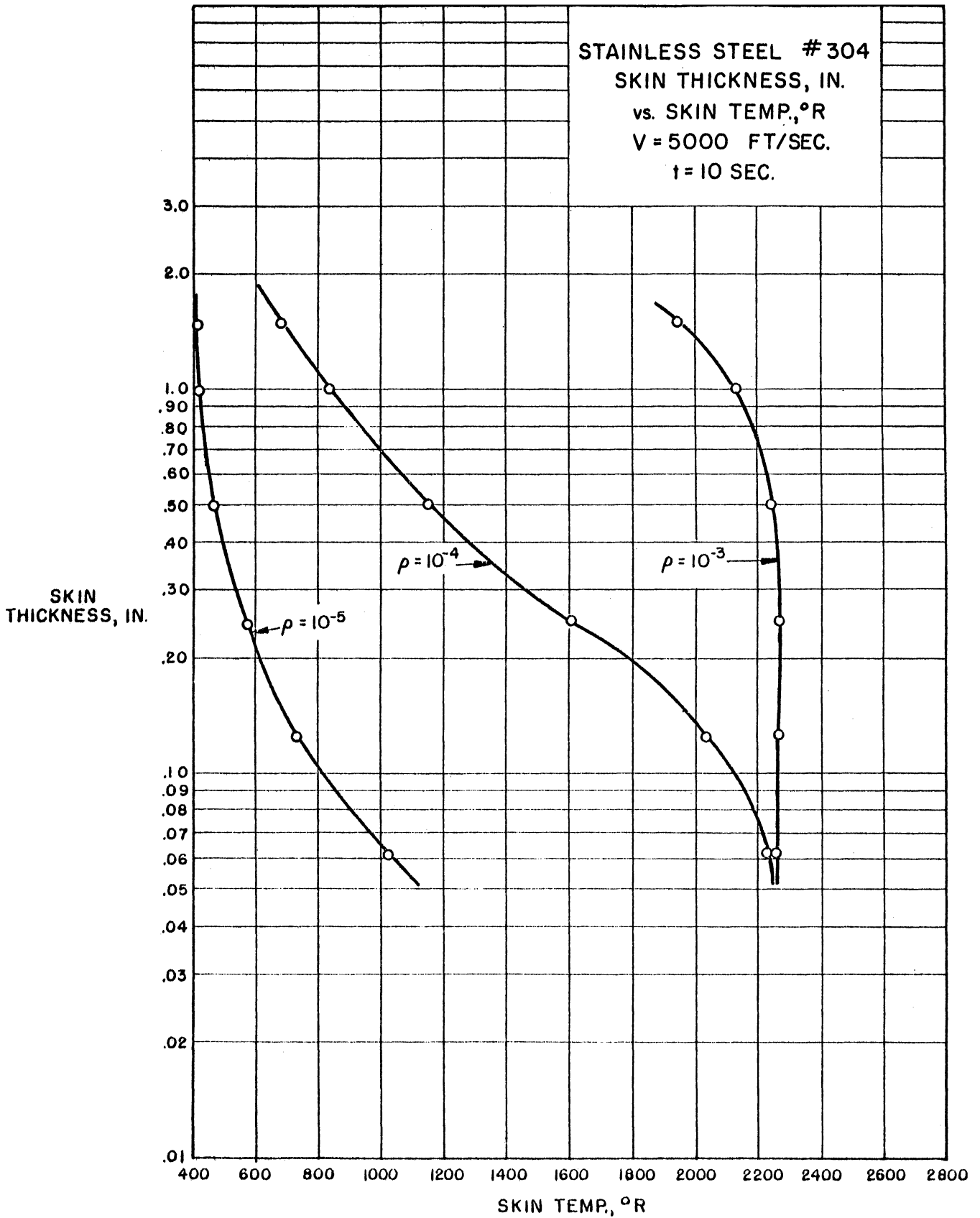


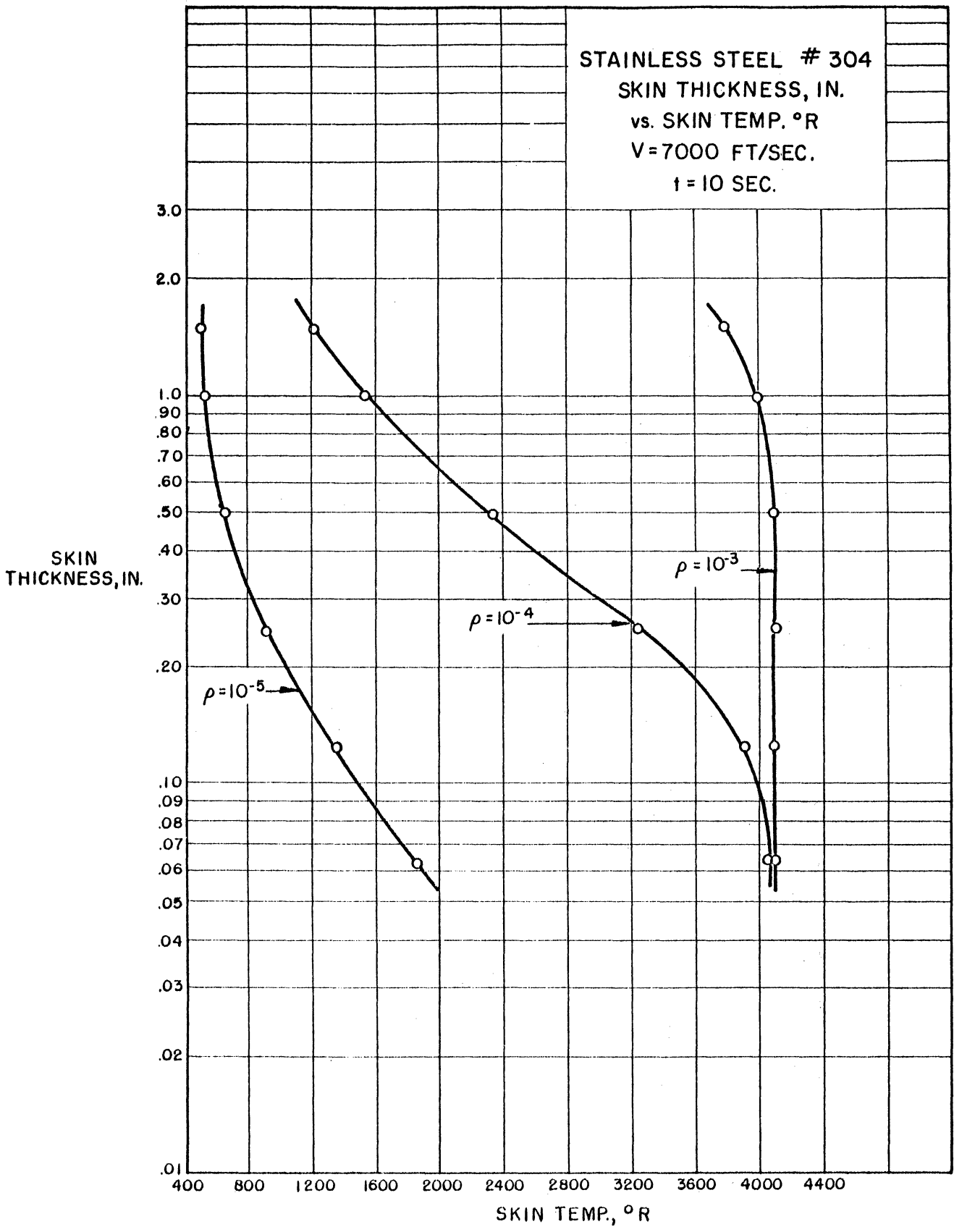












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