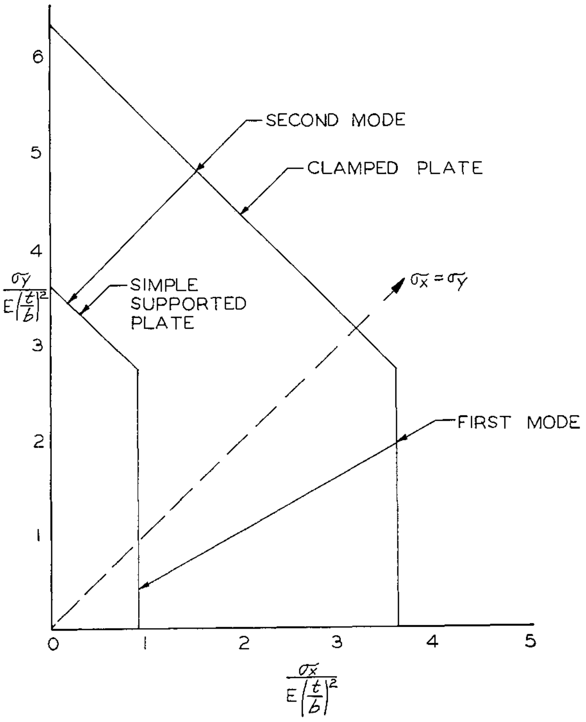

Fig 3 Buckling of flat plates under combined stress

buckles into the first mode when the stress in the x direction, which is larger than $\frac{1}{3}$ the stress in the y direction, reaches the following value:

$$\sigma_x = 0\ 9E(t/b)^2 \tag{3}$$

and the deflection form is

$$w = w_0 \sin(\pi x/b) \tag{4}$$

so that the unit axial shortening in the buckle mode is

$$\frac{\delta}{b} = \left(\frac{1}{2b}\right)\int_0^b \left(\frac{dw}{dx}\right)^2 dx = \frac{w_0{}^2\pi^2}{4b^2} \tag{5}$$

The condition of compatibility in the x direction gives

$$\alpha\Delta T = \sigma_x B_x/E - \nu\sigma_y/E + \delta/b \tag{6}$$

Substituting Eqs (3) and (5) in Eq (6) and solving for w_0 yields

$$w_0 = (2b/\pi)[\alpha\Delta T - 0\ 9(t/b)^2 B_x + \nu\sigma_y/E]^{1/2} \tag{7}$$

The critical stress condition for the curved plate is calculated by utilizing the following criterion:

$$(\sigma_x + \sigma_y)\ _r = 3\ 6E(t/b)^2 + 0\ 3Et/R \tag{8}$$

which was arrived at by adding the buckling stress of the cylinder to that of a square plate The radius R in the previous equation is taken as the constant radius which gives the same deflection as the sine wave,

$$R = b^2/8w_0 \tag{9}$$

The equation for compatibility in the y direction reduces to

$$\sigma_y/E = (1/B_y)[0\ 9\nu(t/b)^2 + \alpha\Delta T] \tag{10}$$

Substituting Eqs (7, 9, and 10) in Eq (8) and setting ν = 0 30 yields

$$\alpha\Delta T(b/t)^2 = 1\ 17B_y{}^2 + 3\ 05B_y - 0\ 27 \pm [1\ 36B_y{}^4 + 7\ 12B_y{}^3 + 1\ 38B_y{}^2 - 2\ 10B_xB_y{}^2]^{1/2} \tag{11}$$

As can be seen from Eq (11), the value of the parameter $\alpha\Delta T(b/t)^2$ at which the second mode occurs depends, in a rather involved way, on the values of the in-plane restraint parameters B_x and B_y A set of values of $\alpha\Delta T(b/t)^2$ for various restraints, including completely restrained $B_x = B_y = 1$, and lightly framed panels $B_x = B_y = 3$, is given in Table 1

Table 1 Values of $\alpha\Delta T(b/t)^2$, for appearance of the second mode

		B_x		
		1	2	3
B_y	1	6 74	6 33	5 84
	2	19 2	18 7	18 2
	3	36 6	36 1	35 5

Concluding Remarks

All the values of $\alpha\Delta T(b/t)^2$ are significantly higher than the value 2 43, which is the lowest value corresponding to initial buckling in the second mode with complete inplane restraint of the heated panel in the y direction The value of B_x is of minor importance as long as the panel buckles initially in the first mode In the application of the preceding results it should be noted that the assumptions of simple support and uniform compression may lead to overoptimism where these conditions are not closely approximated

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Detonation of Hydrogen-Oxygen at Low Temperature and High Pressure

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THE detonation velocity of hydrogen-oxygen mixtures is of fundamental interest in the study of combustion instability in conventional rocket motors as well as in the research on unconventional rocket motors utilizing detonative combustion [1] This note presents the experimentally obtained detonation velocity of gaseous hydrogen-oxygen mixtures at initial temperatures from room temperature to the vicinity of the oxygen vapor saturation point (~110°K) and initial pressures of 1–15 atm Stoichiometric and hydrogen-rich mixtures were considered of primary interest The previously existing experimental data (e g , Refs 2 and 3) thus have been extended

The tests were conducted with a stainless steel tube, 0 25-in i d , 0 50-in o d , 20 ft long and coiled in a 10-in diam (The curvature of the tube was shown to have a negligible effect on the detonation velocity) The velocity of the

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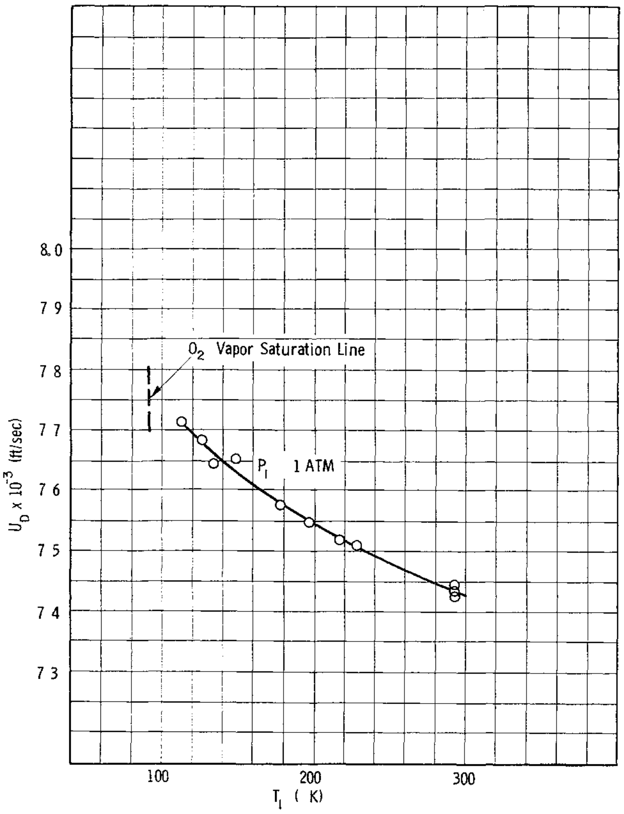

Fig 1 Experimental detonation velocity U_D of hydrogen-oxygen detonations as a function of the initial temperature T_1 for mole fraction of hydrogen X_{H_2} = 0 500 ±0 005

detonation wave was measured electronically by utilizing the ionized gases behind the detonation wave to trigger a time interval counter This was accomplished by means of three ionization probes spaced 8 ft apart as input to a thyratron circuit The hydrogen-oxygen mixtures were cooled to low temperatures by immersing the detonation coil in a bath of isopentane, which had been cooled by bubbling liquid nitrogen through it The determination of the actual mixture ratio of H_2-O_2 used for a run was accomplished by measuring the detonation velocity of a given mixture in a straight detonation tube at 1 atm and 20°C, and comparing the result to the extensive existing data and to partial pressure measurements

It was found that the combustion products (water) froze on the walls of the detonation coil and that, unless the ice was removed, erroneous results were obtained on the next run Several methods were tried to eliminate the ice without removing the coil from the bath including a helium shock tube driver (gaseous piston), but due to the extremely rapid freezing process and the minute vapor pressure of ice at low temperatures, these attempts were not successful Thus, the detonation coil had to be removed from the bath after each run, warmed to room temperature, dried, and evacuated before making the next run

The results of detonation velocities for fully developed (Chapman-Jouguet) waves vs initial temperature of the mixtures for initial pressures of 1, 5, 10, 15 atm and 0 500, 0 667, 0 730, 0 800 mole-fraction of hydrogen are presented in Figs 1–4 The test results indicate that for a given initial pressure the detonation velocity increases at a slightly greater than linear rate as the initial temperature is lowered down to the saturation point of oxygen The results for stoichiometric H_2-O_2 mixtures are compared with the theoretical results of Zeleznik and Gordon[4] and with the previous data of Moyle[2] and Gealer[3] in Fig 5

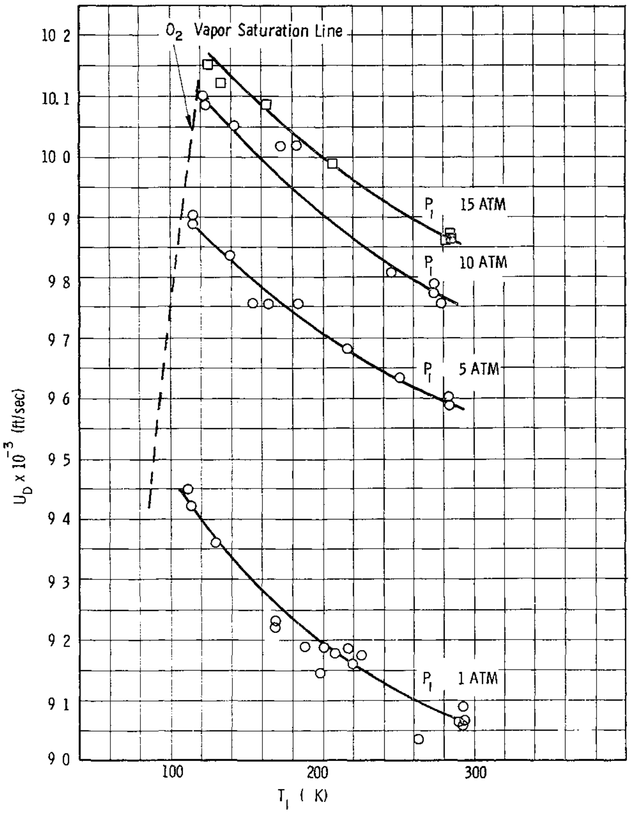

Fig 2 Experimental detonation velocity U_D of hydrogen-oxygen detonations as a function of the initial temperatures T_1 for X_{H_2} = 0 667 ± 0 0025

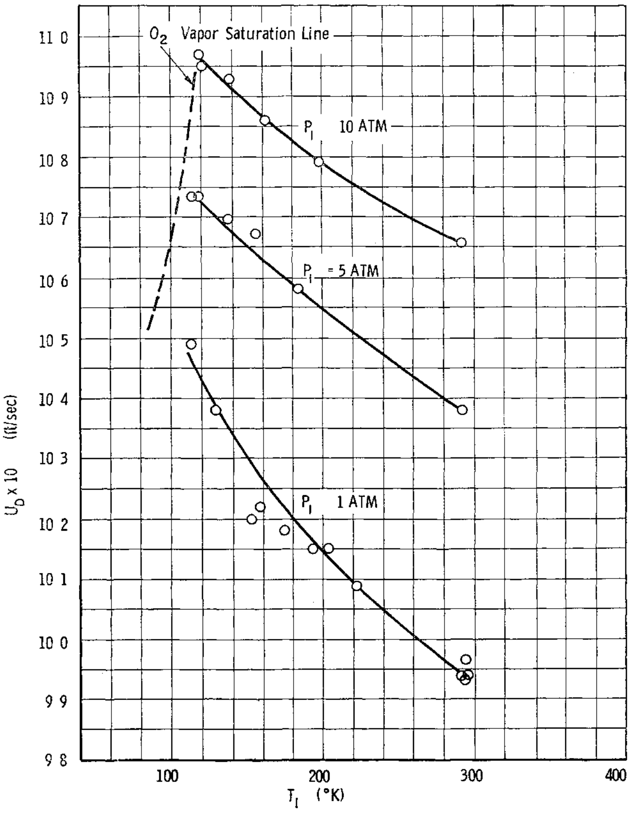

Fig 3 Experimental detonation velocity U_D of hydrogen-oxygen detonations as a function of the initial temperature T_1 for X_H = 0 730 ± 0 005

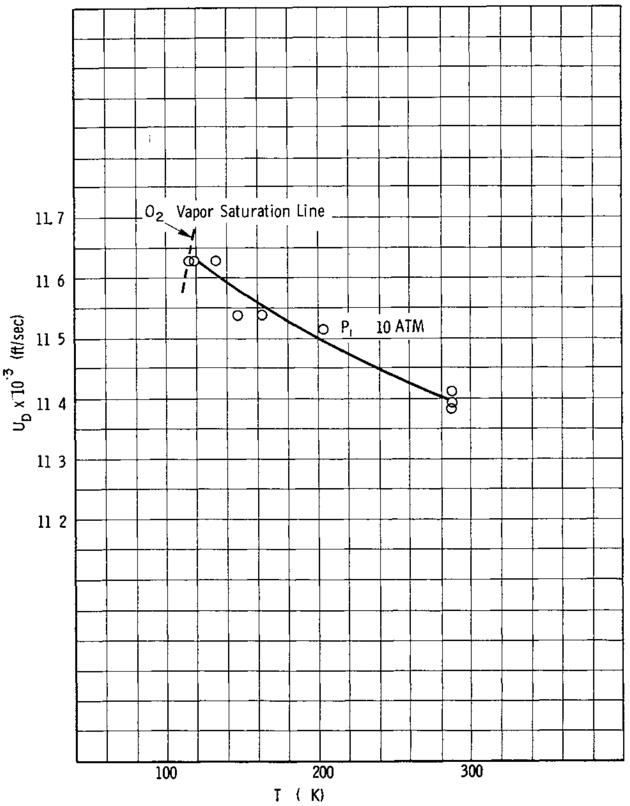

Fig 4 **Experimental detonation velocity U_D of hydrogen-oxygen detonation as a function of the initial temperature T_1 for $X_{H_2} = 0\ 80 \pm 0\ 01$**

In comparing theory with experiment it should be noted that the size of the detonation tube has a significant effect on the velocity so that the measured velocity is less than that predicted by the Chapman-Jouguet plane-wave theory Fay[5] has proposed that this velocity deficit is caused by a viscous boundary layer on the tube wall within the reaction zone On the basis of a two-dimensional analysis, Fay obtains the following expression for the velocity deficit ΔU_1:

$$\Delta U_1/U_1 = 2\ 1\sigma^*/D$$

where

$$\sigma^* = 0\ 22(t)^{0\ 8}(\mu_e/\rho_1 U_1)^{0\ 2}$$

and where

D = diameter of tube in centimeters
σ^* = boundary-layer displacement thickness
U_1 = propagation velocity of the detonation wave
t = thickness of reaction zone
μ_e = viscosity of the gas in the combustion zone at outer edge of boundary layer
ρ_1 = initial (upstream) density

For the stoichiometric hydrogen-oxygen reaction at 1 atm pressure and room temperature, Fay suggests the values t = 0 35 cm and μ_e = 12 3 × 10^{-4} g-cm^{-1}-sec^{-1} For application to this experiment we assume that the thickness of the reaction zone (t) is primarily determined by a recombination reaction so that t is inversely proportional to the square of the initial pressure Also, we assume that μ_e does not vary significantly with initial pressure and temperature

The results of the velocity deficit calculations are shown in Table 1 With this correction, good agreement between theory and experiment is obtained at low pressures At higher pressures a significant variation between theory and experiment is apparent which cannot be accounted for by the tube size effect It is possible that imperfect gas effects (not considered in the theoretical calculations of Ref 4) can be the major cause of this discrepancy

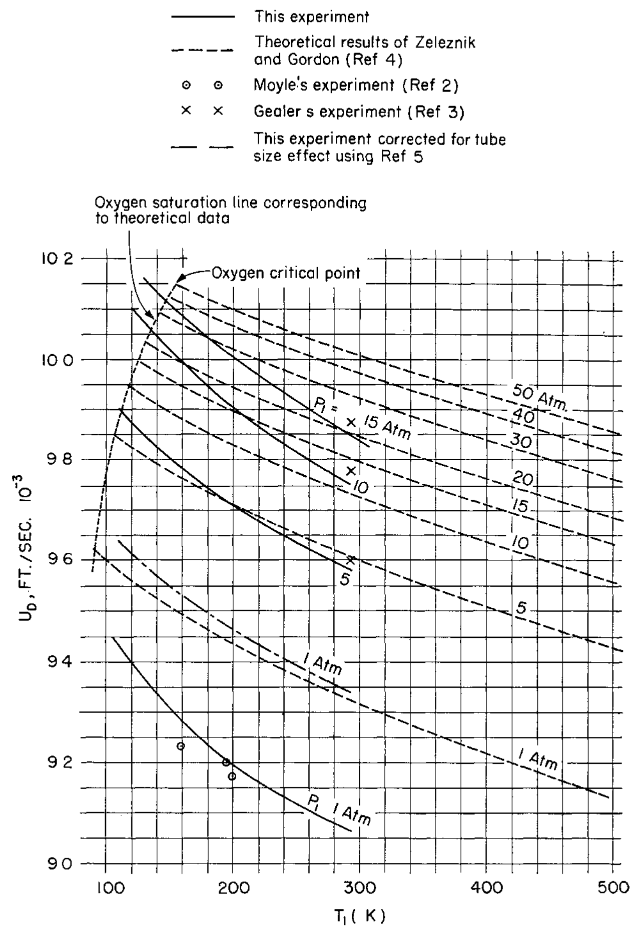

Fig 5 **Comparison of experimental detonation velocity U_D as a function of initial temperature T_1 at various initial pressures P_1 with results of other investigators for stoichiometric (X_{H_2} = 0 667) $H_2 - O_2$ mixtures**

Table 1 **Results of velocity deficit calculations using theory of Ref 5**

P_1, atm	T_1, °K	$\Delta U_1/U_1$, %	ΔU_1, fps
1	293	3 1	280
1	200	2 9	265
1	110	2 6	240
5	293	0 17	17
5	200	0 16	16
5	110	0 14	14
10	293	0 049	5
10	200	0 046	5
10	110	0 040	4
15	293	0 024	2
15	200	0 022	2
15	110	0 019	2

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