[7] Mangler, W., M.A.P. Volkenrode, AVA Monographs, Reports and Translations No. 1001, p. 65, 1948.
[8] Schlichting, H., *Grenzschicht-Theorie*, G. Braun, 1951.
[9] Hannah, Miss, Rep. Aero. Res. Council, R. & M. No. 2772, 1947.
[10] Cooke, J. C., *Note on the Boundary Layer near a Stagnation Point*. Rep. Aero. Res. Council, No. 15,016, 1952.

Errata—"Quadruple Velocity Correlations and Pressure Fluctuations in Isotropic Turbulence"

M. S. Uberoi
Department of Aeronautical Engineering, University of Michigan, Ann Arbor, Mich.
November 23, 1953

Some mistakes were overlooked in the original manuscript of my paper.[1] The correct formulas are:

In the "Introduction,"

$$\frac{1}{\rho^2}\overline{\left(\frac{\partial p}{\partial x_1}\right)^2} \cong \overline{U_1^2}\,\overline{\left(\frac{\partial U_1}{\partial x_1}\right)^2}\Big/(0.13)^2 R_\lambda$$

Below Eq. (20),

$$\overline{(\Delta V)^2} = \frac{35}{3}\left[\frac{\partial^4 R_l{}^l(r)}{\partial r^4}\right]_{\lambda=0}$$

Eq. (21) should be

$$\overline{(\Delta V)^2} = \frac{35}{3}\frac{V'^2}{\lambda^4}\left(\frac{30}{7} + 0.2R_\lambda\right)$$

Eq. (22) should be

$$(\lambda^2/V'^4)\,(dV'/dt)^2 = 25/R_\lambda{}^2$$

Eq. (23) should be

$$\frac{\rho^2 V'^4}{\lambda^2\overline{(\partial p/\partial y)^2}} = \left(\frac{2\lambda^2}{\lambda_\eta{}^2} - \frac{25}{R_\lambda{}^2} - \frac{2.3}{R_\lambda}\right)^{-1}$$

Changes in Eq. (23) have negligible effect on the final calculations.

Reference

[1] Uberoi, Mahinder S., *Quadruple Velocity Correlations and Pressure Fluctuations in Isotropic Turbulence*, Journal of the Aeronautical Sciences, Vol. 20, No. 3, p. 197, March, 1953.

Further Remarks Concerning Integral Transforms of the Wave Equation

Max. A. Heaslet and Harvard Lomax
Aeronautical Research Scientists, Ames Aeronautical Laboratory, NACA, Moffett Field, Calif.
November 19, 1953

Fraenkel[1] has investigated auxiliary terms that appear in an operational form of the potential equation of supersonic flow. The purpose of the present note is to supply a physical meaning to these terms and to indicate a similar case that has occurred previously.

Consider the normal form of the linearized equation of steady-state supersonic flow

$$(\partial^2\varphi/\partial x^2) - (\partial^2\varphi/\partial y^2) - (\partial^2\varphi/\partial z^2) = 0 \qquad (1)$$

where $\varphi(x, y, z)$ is the perturbation velocity potential and the x axis is aligned with the direction of the free stream. The Laplace transform of any function $F(x, y, z)$ is

$$\bar{F}(s; y, z) = \int_{f_0(y,z)}^{\infty} e^{-sx} F(x, y, z)\, dx \qquad (2)$$

The transform of Eq. (1) is, after straightforward calculation,

$$s^2\bar{\varphi} - \frac{\partial^2\bar{\varphi}}{\partial y^2} - \frac{\partial^2\bar{\varphi}}{\partial z^2} = \sum_{i=0}^{n} e^{-sf_i}\left[\Delta\frac{\partial\varphi}{\partial x} + \frac{\partial f_i}{\partial y}\Delta\frac{\partial\varphi}{\partial y} + \frac{\partial f_i}{\partial z}\Delta\frac{\partial\varphi}{\partial z}\right]_{x=f_i} \qquad (3)$$

where discontinuity surfaces $x = f_i(y, z)$ $(i = 0, 1, \ldots, n)$ exist in the flow field and the delta notation indicates the jump in the value of the function at a discontinuity surface, the value $\Delta\varphi$ being assumed equal to zero. Considering, in particular, surfaces of discontinuity which are envelopes of Mach waves or characteristics of Eq. (1), it follows that the bracketed terms in the right member of Eq. (3) are directional derivatives of $\Delta\varphi(x, y, z)$ along these surfaces. This direction, usually termed the conormal, lies in a plane determined by the free-stream velocity vector and the normal to the surface. The directional derivatives represent increments in tangential velocity and vanish from physical considerations. The same condition follows formally in linear theory from the assumed continuity of φ, and the right-hand side of Eq. (3) can thus be suppressed.

We originally encountered this effect in the study[2] of unsteady motion based upon the equation

$$\frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2} + \frac{\partial^2\varphi}{\partial z^2} - \frac{\partial^2\varphi}{\partial t^2} = 0 \qquad (4)$$

Lagerstrom had suggested in his lectures at the California Institute of Technology that certain simplifications could be achieved in the study of wing plan forms with supersonic edges through the introduction of a transformed variable $\bar{\varphi}$, where

$$\bar{\varphi}(x, z, t) = \int_{-\infty}^{\infty} \varphi(x, y, z, t)\, dy \qquad (5)$$

Let $y = g_1(x, z, t)$ and $y = g_2(x, z, t)$ be envelopes of the foremost discontinuity surfaces induced to the left and right sides of the wing. Since φ is continuous across these surfaces, Eq. (4) becomes

$$\frac{\partial^2\bar{\varphi}}{\partial x^2} + \frac{\partial^2\bar{\varphi}}{\partial z^2} - \frac{\partial^2\bar{\varphi}}{\partial t^2} = \sum_{i=1}^{2}(-1)^i\left[\Delta\frac{\partial\varphi}{\partial x}\frac{\partial g_i}{\partial x} - \Delta\frac{\partial\varphi}{\partial y} + \Delta\frac{\partial\varphi}{\partial z}\frac{\partial g_i}{\partial z} - \Delta\frac{\partial\varphi}{\partial t}\frac{\partial g_i}{\partial t}\right]_{y=g_i} \qquad (6)$$

The bracketed terms again are directional derivatives along the conormals to characteristic surfaces in four dimensions, and the continuity of φ on these surfaces leads to the fact that the right side of the equation is zero.

References

[1] Fraenkel, L. E., *On the Operational Form of the Linearized Equation of Supersonic Flow*, Readers' Forum, Journal of the Aeronautical Sciences, Vol. 20, No. 9, pp. 647–648, September, 1953.
[2] Lomax, Harvard, Heaslet, Max. A., and Fuller, F. B., *Three-Dimensional Unsteady-Lift Problems in High-Speed Flight*, NACA Rep. No. 1077, 1952.

Approximate Solutions for Heat Transfer with Convection Flows

M. Herbeck
Max Planck-Institut fur Strömungsforschung, Göttingen, Germany
November 19, 1953

Notation

T = temperature
λ = thermal conductivity