THE USE OF LOGIC IN SOLVING ENGINEERING PROBLEMS

Report of Study on Computer Project
Supported by The Ford Foundation in

The University of Michigan
College of Engineering

by

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I. INTRODUCTION

During the summer of 1962 the Project on the Use of Computers in Engineering Education at The University of Michigan sponsored a Study on the Use of Logic in Solving Engineering Problems. A conference held during the first week of this study was devoted to an intensive discussion of alternative approaches to developing improved methods of teaching students how to solve problems. At the end of the first week an initial project statement was written outlining the proposed objectives of the study, and the remainder of the summer was devoted to implementing those plans by preparing typical sections of a programmed text in problem solving.

II. INITIAL CONFERENCE

During the first week of June, 1962, a conference was held in which the following persons participated:

NAME	RANK AND DEPARTMENT
B. Carnahan	Assistant Director, Project on Computers in Engineering Education
I. M. Copi	Professor of Philosophy
D. L. Katz	Professor of Chemical Engineering and Director, Project on Computers in Engineering Education
H. A. Luther	Professor of Mathematics, Texas A. and M. College
S. O. Navarro	Associate Professor of Electrical Engineering and Director, Computing Center, University of Kentucky
H. F. Rase	Professor of Chemical Engineering, The University of Texas
R. G. Squires	Assistant Professor of Chemical Engineering, Purdue University
F. H. Westervelt	Assistant Professor of Mechanical Engineering
J. O. Wilkes	Instructor in Chemical Engineering
D. H. Wilson	Lecturer in Industrial Engineering
R. C. Wilson	Assistant Professor of Industrial Engineering

In discussing the ingredients required for successful problem solving it was found useful to distinguish between scientific principles and problem solving techniques. Here, by scientific principles are meant such laws as that of conservation of mass, the gas laws, Ohm's law, Hooke's Law, the laws of thermodynamics, and so on. Problem solving techniques include the use of mathematical models, both algebraic and graphical, symbolic logic, flow diagrams, trial and error solution techniques, computer programs, and so on.

Scientific principles were divided into three main types: first, Conservation Principles, such as mass conservation, energy conservation, momentum conservation, flux conservation, current conservation, and force balances, which relate quantities of the same type; second, Transformation Principles, such as the gas laws, Ohm's Law, and Hooke's Law, which interrelate different kinds of quantities and thus permit determination of one kind given another, as volume given temperature and pressure, or strain given stress; and third, Restriction Principles, such as the second law of thermodynamics, which set limits to the ranges of some variables and thus lead to the use of idealized models or postulated ideal states.

It was noted that different scientific principles of a single type were appealed to in various branches of engineering, and that the same problem solving techniques were often applicable in different branches of engineering. It was decided to prepare problems in different branches of engineering that appeal to the same type of scientific principle and can be solved by the same problem solving technique. This similarity would be indicated to the student (after he has worked through the problems independently) by displaying a single flow diagram illustrating their common method of solution, and a computer program to implement that method. It is hoped that this procedure will bring home to the student the unity of the engineering approach to problem solving and the utility of the abstract or symbolic method of solving problems.

During the initial conference a selective bibliography of relevant books was prepared, and finally, an initial project statement was written. (See Appendix A, page 11 .)

III. DEVELOPMENT OF TEACHING MATERIALS FOR A PROBLEM SOLVING COURSE

There are three main types of subject matter that college students are taught. Although most college courses include all three types, one type is usually emphasized more heavily. The first type consists of items of information to be mastered by being committed to memory, as typically contained in courses in geography or history. The second type is intended to deepen the students' comprehension and/or appreciation of material already somewhat familiar, as typically contained in courses in philosophy or literature. The third type is intended to develop or improve the students' skills in performing certain activities, such as writing, speaking, using mathematics or foreign languages, or problem solving.

Different types of instruction are appropriate to these different kinds of course objectives. Lectures and assigned readings in both ordinary and linear programmed textbooks are appropriate methods of teaching items of information. Discussion or seminar groups are appropriate ways of stimulating students to reflect upon what they already know. But to help students develop skills, they must be led to practice those skills in ways that put increasing demands upon them. The tutorial method is obviously most appropriate here, where the student must continually use skills, with his mistakes corrected as soon as they are made and his correct steps reinforced in a continuous fashion.

Every Engineering College department (at The University of Michigan) offers a problem solving course as an early professional course, usually at the sophomore level, since the student's freshman year is usually devoted to acquiring information in the basic sciences and skills in language and mathematics. Such problem solving courses are expensive to teach because ideally they should approximate the tutorial situation with a very low student-teacher ratio. They are also difficult to teach because students tend to progress at very different rates. A given problem that is discussed in class might bore the quickest students and, at the same time, bewilder rather than instruct the slowest students.

It was therefore decided that a programmed textbook would be very useful as instructional material in a problem solving course. Among the various advantages of the programmed text can be listed the following:

1. It will serve as tutor to the student, correcting his mistakes and reinforcing his insights.
2. It will decrease the student's dependence upon contact with the instructor, thus permitting an increase in the student-teacher ratio.
3. It will permit the student to progress at his own rate without penalizing him with boredom or bewilderment in the classroom.
4. It will improve the student's ability to solve problems of the sort presented simply as a result of practice.
5. It will point out analogies and generalizations so that the student may apply his knowledge of problem solving to new problems and unique situations which are not explicitly presented in the text.

Because there are usually several different methods available for solving the typical engineering problem, it was decided that a branched programmed text would be most useful, in that it would permit each student to travel the path most congenial to him in moving towards the correct solution to each problem. By directing his attention to an optimal method of solution after he had achieved his own solution, the text would stimulate him to adopt increasingly efficient methods of problem solving without imposing any penalty for using sub-optimal methods at first.

It was agreed that the programmed text should contain a minimal amount of substantive information, but that it should contain some, together with a list of references for those students who require more than the minimal amount supplied by the text. It could not be decided a priori how much, if any, information about subject matter and/or problem solving techniques should be given the student before he is led into a problem solving situation. Hence two different starts were made in constructing a programmed textbook in solving engineering problems. In the first, prepared by I. M. Copi, R. G. Squires, and F. H. Westervelt, items of information are introduced in the context of primarily Chemical Engineering problems that require the information for their solution. It appears as Appendix B, page 19 . In the second, prepared by S. O. Navarro, information is presented prior to giving primarily Electrical Engineering problems that require the information for their solution. It appears as Appendix C, page 65. The application of symbolic logic in solving engineering problems is discussed in Appendix D, page 85.

During the course of preparing the first text materials, several problems which follow were considered. Of these problems, the first four are included in the text (Appendix B), with problems 1 and 3 being programmed in scrambled form. Problems 5, 6, and 7, being slightly more difficult, will be the next to be included when the text is expanded.

Our programmed text is, in effect, a method of programming the student himself to solve problems of the sort considered, in the sense that a computing machine can be programmed to solve problems of a given type. Since one of our aims is to familiarize the student with the abstract, flow diagram approach to solving problems, we have included a typical flow diagram solution for Problem 7, together with its actual computer solution (expressed in the computer language, MAD).

Problem 1:

Water is fed into a storage tank by two inlet pipes. The first of these delivers 10 pounds of water per hour. The single exit from the tank removes water at the rate of 25 # water/hour. (In this text we use the symbol # to represent pounds.)

At noon the tank contains 500# water and at 2:00 p. m. it contains 600# water.

If all the flow rates are constant, at what rate does water enter the tank through the second inlet pipe?

Problem 2:

A man puts $250 into his bank account on the second of each month. Both the man and his wife may withdraw money. The bank balance on June 1 was $500 and on August 1 it was $514. A semiannual interest payment was made on June 15 equal to 2% of the June 1 balance. If the man withdrew $78 during June, and $86 during July, what is the average monthly withdrawal made by his wife during this 2 month period?

Problem 3:

4567#/hr. of wet laundry, 39.6% H_2O by weight, is fed into a dryer. If the dried laundry contains 4% H_2O by weight, determine the number of pounds of H_2O removed from the laundry per minute.

Problem 4:

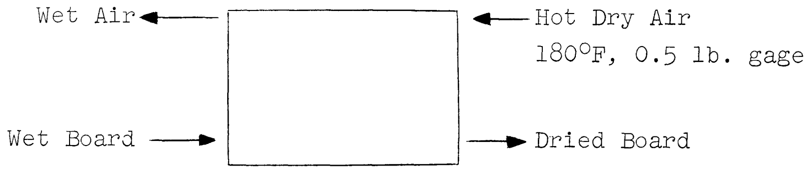
An air purification unit for use in submarines is designed to absorb CO, CO_2, and H_2O from the air. In order to test this absorber, an analysis of a test air stream, with a high percentage of CO and CO_2, was made at both the inlet and exit of the absorber. The results are given below

Component	% By Weight In	% By Weight Out
Air (i.e., N_2 and O_2, argon, etc.)	55.9%	84.6%
CO_2	21.8%	11.8%
CO	19.2%	3.4%
H_2O	3.1%	0.2%

Determine the efficiency (% material absorbed compared to input material) of the absorber for CO_2, CO and H_2O.

Problem 5:

Paper board is being dried in a single stage drier by means of hot air.



Wet Board: 28% dry solids
Dry Board: 90% dry solids
Dry Air: 1% water vapor by volume
Wet Air: 5% water vapor by volume
Molecular weight of air = 29

Five tons of dried board (10% water) is produced per hour.

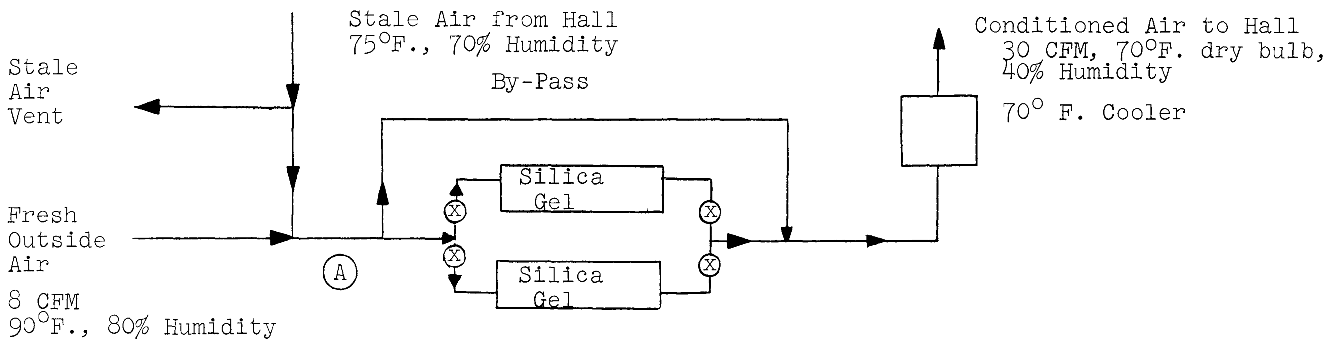
Compute:
a) Tons of water vaporized per hour.
b) Tons of dry (zero percent water) air used per hour.
c) Standard cubic feet of entering air per minute (60°F, 1 atm.)
d) Actual cubic feet of entering air per minute (Bar. = 29.6 inches Hg)

Problem 6:

The air conditioning of a convention hall has been proposed, and the following are the design specifications:

Inside Hall	75° dry bulb,	70% Humidity
Outside Fresh Air (Extreme Summer Conditions)	90° dry bulb,	80% Humidity
Capacity	6000 adults	
Air Requirements	8 cu.ft./min. of outside air per person 30 cu.ft./min. of conditioned air per person	

The air is to be dried by two silica gel dehumidifiers, one of which is kept on stream, while the other is being regenerated. The air leaving the dehumidifiers has negligable H_2O content.



For the extreme summer conditions, and for maximum capacity (6000 adults), what % of the air at (A) by-passes the dehumidifier?

Problem 7:

In the Casale process for making ammonia, nitrogen and hydrogen are fed to a catalytic reactor at 600 atm. and 930°F. In the reactor, a 15% conversion to NH_3 can be expected based on available N_2 and H_2 in the reactor. To prevent loss of valuable raw materials, the NH_3 produced is condensed out of the gas stream by using a surface condenser. A portion of the non-condensed gas containing 7% Argon, is recirculated, and the remainder is vented. The surface condenser operates at 500 atm. and a liquid NH_3 temperature of 32°F. If 75 tons/day of NH_3 are produced, calculate

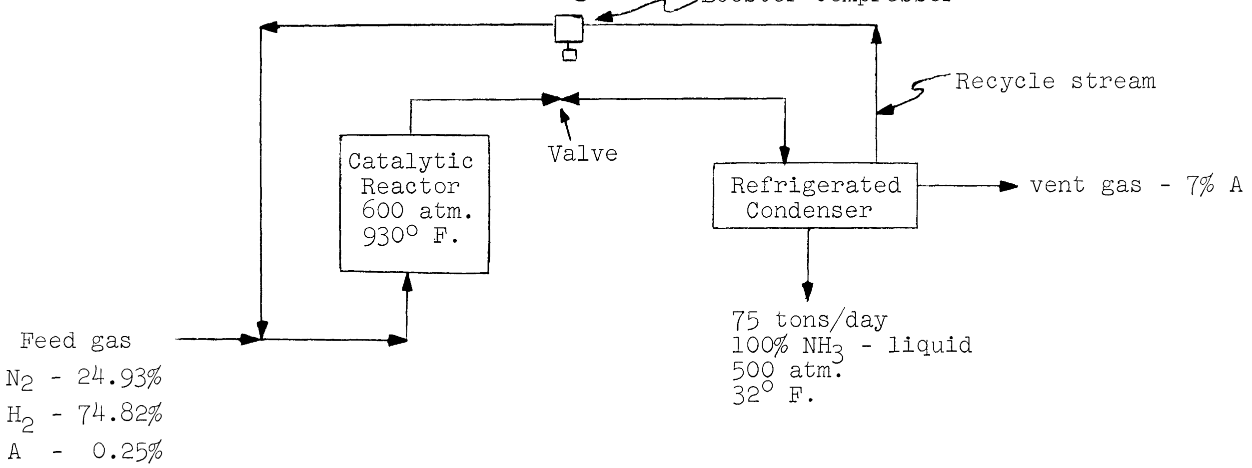
a. concentration of NH_3 in vent gas stream,

b. pounds/day of NH_3 lost,

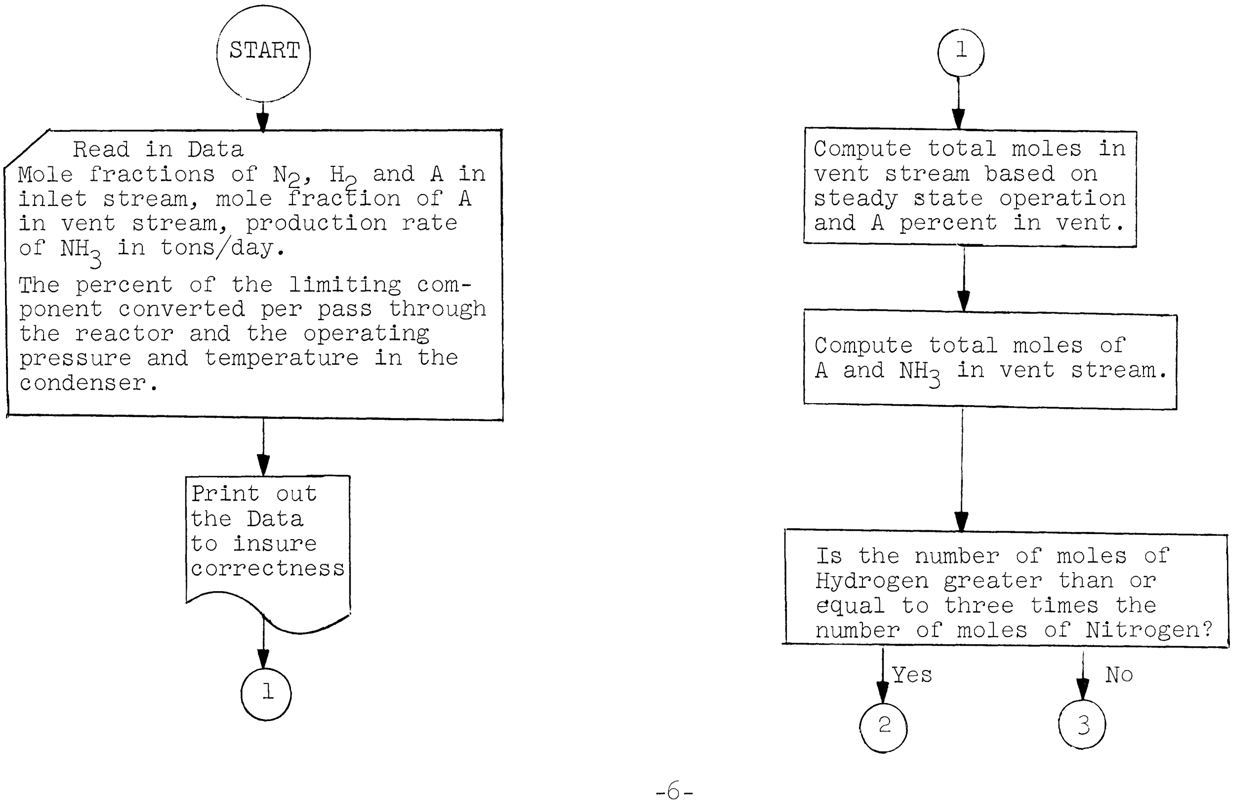
c. per cent conversion of N_2 to NH_3,

d. recycle ratio (ratio of recycle to feed stream on mol basis),

e. standard cubic feet of feed gas.



Flow Diagram for Problem 7:



Flow Diagram for Problem 7, continued

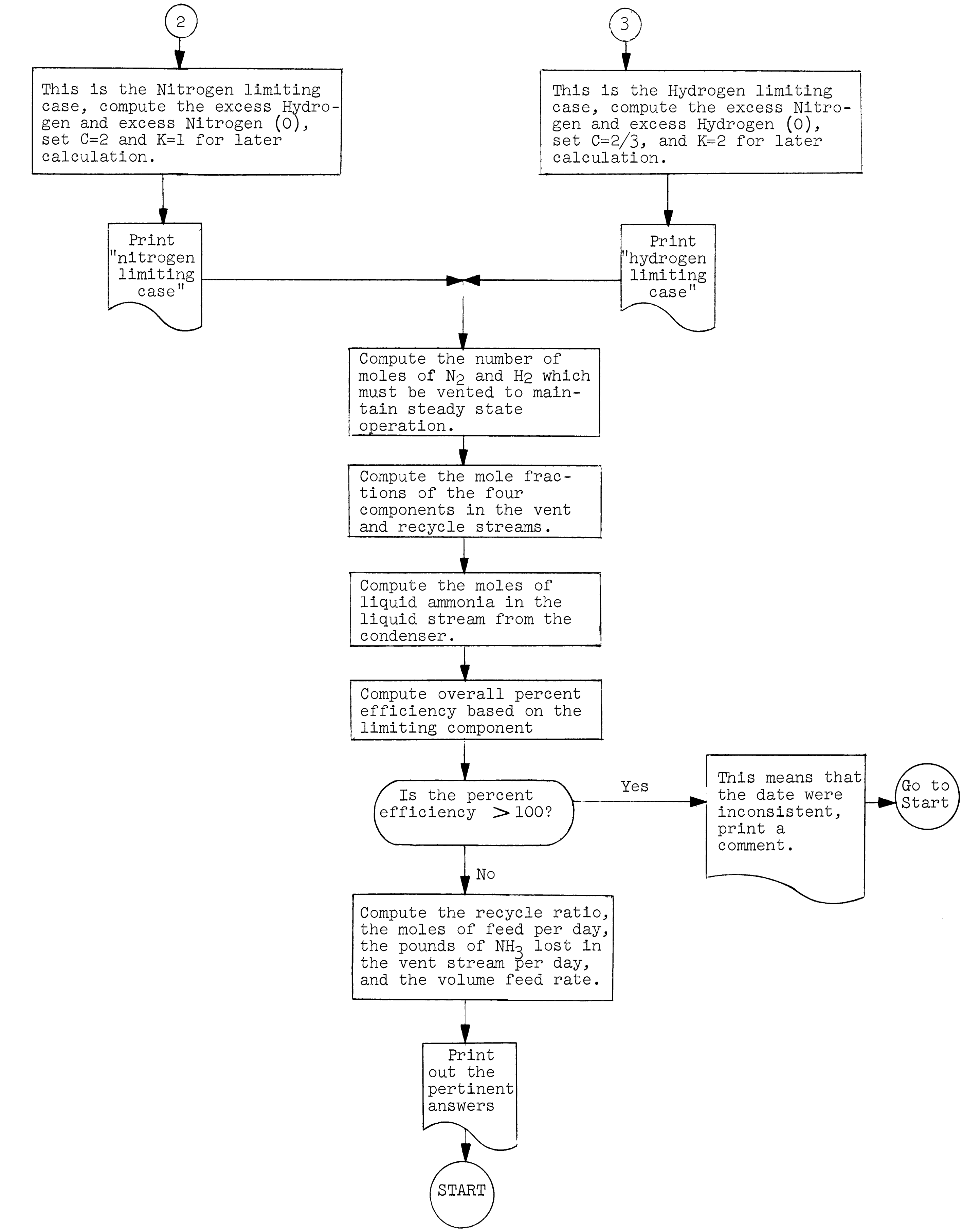


Table of Symbols for Problem 7:

C	A constant equal to 2.0 for the Nitrogen limiting case and 2/3 for the Hydrogen limiting case.
CONV	Fraction of N_2 converted to ammonia per pass in the reactor.
FEEDM	Moles of feed per day.
I	An indexing variable, used for subscription.
K*	Subscript (1 or 2) of the limiting component.
LOST	Pounds of ammonia lost per day in the vent stream.
MOLIN(1)...MOLIN(3)*	Moles of individual components per mole in feed stream.
MOLLIQ	Moles of liquid ammonia removed from the condenser per mole of feed.
PCEFF	Percent efficiency in conversion of nitrogen to ammonia for the overall process.
PC	Operating pressure in the condenser (psi).
RRATIO	Recycle ratio, moles of recycle per mole of feed.
STOICH	Moles of nitrogen and hydrogen present in stoich. Ratio in the feed per mole of feed.
TC	Operating temperature in the condenser (^{o}F.).
TONNH3	Rate of ammonia removal from the condenser (as liquid product) in tons/day.
VENTFR(1)...VENTFR(4)*	Mole (volume) fraction of individual components per mole in the vent and recycle streams.
VENTM	Total number of moles in the vent stream per mole in the feed stream.
VENTM(1)...VENTM(4)*	Moles of individual components in the vent stream per mole in the feed stream.
VFEED	Volume rate of the feed stream (SCF/day).
VP.	Subroutine which returns value of the vapor pressure of ammonia (in psi) given the temperature in ^{o}F as the argument.
XSHYD	Moles of excess hydrogen (above stoich. amount) in the feed per mole of feed.
XSNIT	Moles of excess nitrogen (above stoich. amount) in the feed per mole of feed.

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* The order of components is as follows:

Subscript	Component
1	Nitrogen
2	Hydrogen
3	Argon
4	Ammonia

MAD Program for Problem 7:

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$ COMPILE MAD, EXECUTE, DUMP, PRINT OBJECT
START       READ DATA MOLIN(1)...MOLIN(3), VENTFR(3), TONNH3, CONV,PC,TC
            PRINT  RESULTS MOLIN(1)...MOLIN(3),VENTFR(3),TONNH3,CONV,PC,T
           1C
            VENTM=MOLIN(3)/VENTFR(3)
            VENTM(3)=MOLIN(3)
            VENTM(4)=VENTM*VP.(TC)/(14.7*PC)
            WHENEVER MOLIN(2).GE.3.*MOLIN(1)
               PRINT COMMENT $ NITROGEN LIMITING $
               K = 1
               C = 2.
                XSHYD=1.-MOLIN(1)*4.-MOLIN(3)
                XSNIT=0.
            OTHERWISE
               PRINT COMMENT $ HYDROGEN LIMITING $
               K = 2
               C = 0.666667
                XSNIT=1.-MOLIN(2)*4./3.-MOLIN(3)
                XSHYD=0.
            END OF CONDITIONAL
            STOICH=VENTM-VENTM(3)-VENTM(4)-XSNIT-XSHYD
            VENTM(1)=STOICH/4.
            VENTM(2)=STOICH*0.75
            THROUGH NEXT, FOR I=1,1,I.G.4
NEXT        VENTFR(I)=VENTM(I)/VENTM
            VENTFR(1)=VENTFR(1)+XSNIT/VENTM
            VENTFR(2)=VENTFR(2)+XSHYD/VENTM
                MOLLIQ= C*(MOLIN(K)-VENTM(K))-VENTM(4)
                PCEFF=(MOLIN(K)-VENTM(K))/MOLIN(K)*100.
            WHENEVER PCEFF.G.100.
               PRINT COMMENT $ INCONSISTENT DATA $
               TRANSFER TO START
            END OF CONDITIONAL
               RATIO=(MOLLIQ+VENTM(4)-MOLIN(K)*CONV*C)/(C*CONV*VENTFR(K))
            FEEDM = 2000.*TONNH3/(17.*MOLLIQ)
            LOST=VENTM(4)*FEEDM*17.
            VFEED=379.*FEEDM
            PRINT RESULTS VENTFR(4),LOST,PCEFF,RATIO,VFEED
            TRANSFER TO START
            DIMENSION MOLIN(4), VENTM(4),VENTFR(4)
            INTEGER I, K
           R..... THE FOLLOWING INTERNAL FUNCTION IS FOR TEST PURPOSES
           R ONLY .....
             INTERNAL FUNCTION VP.(XYZ) = 62.4
            END OF PROGRAM
$ DATA
MOLIN(1)=0.2493,0.7482,0.0025, VENTFR(3)=0.07, TONNH3=75.,
CONV=0.15,PC=500., TC=32.*
MOLIN(1)=0.2443,0.7532,0.0025, VENTFR(3)=0.07, TONNH3=75.,
CONV=0.15,PC=500., TC=32.*
MOLIN(1)=0.2593,0.7382,0.0025, VENTFR(3)=0.07, TONNH3=75.,
CONV=0.15,PC=500., TC=32.*
MOLIN(1)=0.2393,0.7582,0.0025, VENTFR(3)=0.07, TONNH3=75.,
CONV=0.15,PC=500., TC=32=*
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IV. RECOMMENDATIONS FOR FURTHER WORK

We recommend that further work be done in the following four areas:

First, the programmed text in solving engineering problems should be expanded to include more and increasingly difficult problems of the sort actually taught in courses in the Engineering College; it should be further expanded to include flow diagrams and computer solutions for whole blocks of its problems; and it should be altered in the direction of utilizing the computer notion of "subroutines" in its programs.

Second, tests should be conducted to decide the relative advantages of the two types of programmed textbooks illustrated in Appendices B and C (pages 19 and 65).

Third, tests should be conducted with enlarged versions of the programmed text to determine its utility in college level courses in problem solving.

Fourth, further investigation should be conducted to determine the extent, if any, to which the study of symbolic logic will improve the student's ability to solve engineering problems.

APPENDIX A

Initial Project Statement on The Use of Logic in Solving Engineering Problems*

The purpose of this study is to prepare procedures and materials for teaching students to solve problems better than by the procedures currently being used in engineering schools.

Specifically, it is proposed that a master list of principles used by engineers to solve problems be assembled and subdivided with the hope that this list, when completed and developed, could become the basis for the teaching of an engineering problem or design course. It is assumed that the student would be taught science and engineering science through other courses.

A second objective is to choose a portion of the master list of principles and to prepare material which might be utilized for teaching students the use of the selected principles. In doing this a certain amount of knowledge would have to be presented to the student either for the first time or as review material to refresh his memory. It is hoped that example problems can be prepared which illustrate the similar applications of these principles in the various engineering disciplines. The hope is that a single solution procedure (such as a computer program or flow diagram) might evolve for solving problems in the various disciplines when based on the same principle. In teaching the student to solve these problems, specific tools would have to be presented and illustrated, including many topics related to mathematics and logic.

The material prepared should emphasize the enhancement of knowledge and development of intellectual skills involved in problem solving. A categorization of facts and skills in increasing order of complexity or abstraction follows.**

Knowledge
Comprehension
Application
Analysis
Synthesis
Evaluation

All of these ought to be developed by the engineering student for all are need in solving problems.

Sometimes the solution of problems involves a series of steps such as described by Polya[1], Ver Planck and Teare[2], and Rase[3]. Such a sequence of problem-solving steps might be:

1. Understand the statement of the problem.
2. Determine the relevant facts.
3. Classify and describe the facts.
4. Determine the applicable laws, principles, or theories.
5. Analyze the problem to be solved in the basis of these principles.
6. Determine whether or not additional facts are required.
7. Solve parts of the problem and possibly the whole problem.
8. Check the results to see if they are consistent.
9. Classify the results and see what new information might be synthesized from them.

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* This statement was dictated on June 8, 1962, following one week of intensive discussion.
** Bloom, Benjamin, ed., Taxonomy of Educational Objectives. David McKay Co., 1956.

This total problem-solving process may be considered as three basic operations: decomposition (breaking the problem into its component parts), analysis (analyzing the individual parts), and composition (synthesizing the overall results from the analyzed parts). In solving any problem, the ideas outlined above should be kept in mind.

Some scientific principles are used by engineers and scientists in a dual capacity. These principles may be used to classify knowledge and to describe the physical behavior of materials. The same principles may also be applied to solve engineering problems. In teaching students, we desire to have them learn to apply these principles. In so doing, we know that the student will have to draw upon the fund of knowledge available. When he looks to this fund he will find it already classified in certain ways. He must therefore understand the classification system.

A general classification of these principles might be:

1. Accounting
2. Exchanging
3. Limiting

An alternate for these three terms could be:

1. Conservation
2. Transformation
3. Restriction

The latter three might be termed principles of science, and the first three might be termed the methods used in solving problems involving the principles. It is important to distinguish between understanding a principle and applying it to solve a problem. It takes tools as well as knowledge and a comprehension of the overall problem situation to produce a solution. The comprehension of a principle and its application involve a higher level of effort than a mere knowledge of fact.

Some conservation principles which might be listed are as follows: Mass Conservation, Current Conservation, Force Balances, Energy Conservation, Momentum Conservation, Flux Conservation, and so forth. By conservation principle, we mean the relationship between quantities of like units, an accounting of the same kind of thing at various places or times.

Next, we consider the transformation principles. Sometimes one variable is available, whereas another is needed to solve a problem. By using a transformation relationship or principle it is possible to obtain the desired variable. Methods of representing data in terms of variables through mathematical relationships such as equations of state are examples of exchanging or transforming information. Graphical and tabular presentations are frequently used methods of representing information as well as equations. Ohm's Law and Hooke's Law are examples of transformations used in describing materials and their properties in terms of measured variables.

Often one cannot solve the problem as stated but can arrive at a limitation or restriction as to the maximum, minimum, or bounding condition. Such procedures are very valuable in limiting the decisions which need to be made. For example, the Second Law of Thermodynamics yields the maximum amount of work which may be obtained in a given process. The use of restrictions or limiting methods often involves idealized models or postulated ideal states. The restriction principles include the exclusion principle, uncertainty principle, quantum relationships, etc.

Tools and Methods in Problem Solving

It should be recognized that certain mathematical and logical tools are used in solving problems. Just as with the scientific principles, these tools are also used in correlating the physical behavior of substances. Accordingly these tools need to be understood from two standpoints, one with reference to their use in solving specific problems and the other, from the standpoint of examining the fund of knowledge available about the universe and selecting the appropriate data in the form of a correlation. Some of the tools and methods which might be identified are the following:

- Mathematics
- Symbolic logic
- Mathematical models
- Graphical procedures
- Flow charting concepts
- Iteration
- Trial and error solution techniques.

Some topics which need consideration at this point are loops, meshes, branches, topology (especially as used by the electrical engineer), linearity and nonlinearity, duality and equivalence.

Development of Model Material for Text to Teach Engineers

In preparing material for the engineering student, it is essential to present problems to be solved. To prepare the student to solve a particular problem, information must be given over and above that which he would be expected to bring with him from elementary chemistry, physics, and mathematics. In presenting this knowledge, one may use some of the principles and tools or methods of correlation by which the information was stored and made ready for his use. Likewise, one probably would itemize the principles under consideration and describe their utility for solving problems. Following such a presentation the student would be confronted with the problem and there would be a discussion of the classification of information and identification of the principle which applies. He then proceeds to apply the principle and the knowledge available to him to arrive at the required conclusion. It would be hoped that example problems could be given, along with their solution. If possible, it would seem useful if some of the problems could be solved in the form of flow diagrams. At this stage we would expect the computer solution to be shown for at least some of the problems.

One of the things discussed was that for a given principle, it would be possible to take problems from the various disciplines of engineering (electrical, mechanical, chemical, and material engineering) which illustrate that principle, for which the flow diagram would be similar or even identical. One could thus offer the student a choice of problems from which he could select those for which he has motivation and background information. It is also hoped that at the conclusion of a problem of this kind the student would be shown how to solve the problem symbolically to illustrate the power of the general solution. It is believed to be essential to emphasize the importance of the symbolic solution and to give the students an enthusiasm for this type of approach, as compared with one based on a specific situation with which he is more familiar. We often use "situation" problems because the student has a motivation to solve them numerically. We recognize this need but to some extent this keeps the teacher from using the more general symbolic solution which is in many cases preferable.

The solutions of problems should be presented in detail and discussed with special emphasis on points where decisions are made. The ultimate text might simulate the tutorial method so that the student would learn his own procedures for attacking and solving problems.

In preparing the report, we might prepare a group of problems based on the same general principle and then see how many of them are essentially identical in their solution. Those that are not, might be removed from the batch and used later. It is suggested that we not set our sights too high in terms of the number of principles which could be included in the report; possibly two or three would be all that can be handled in the time available during the summer. A selective bibliography is appended of engineering books which are used to introduce students to problem-solving, books on the philosophy and logic of solving problems, and books on the learning process, and testing.

It is expected that one outcome of the study would be a proposal for support for a two-year study to prepare a complete textbook of the kind discussed.

Bibliography

A. Texts presenting the general method of solving problems:

1. Kogan, Z., Essentials in Problem Solving, Arco Publishing Company, Inc., New York, 1956.
A short description of general methods which may be used in problem solving.

2. Polya, G., How To Solve It, Princeton University Press, Princeton, New Jersey, 1948.
An analysis of a general approach to problem solving with special attention to the requirements of students and teachers of mathematics. Presents a series of general questions which a student may ask himself when he becomes stymied while solving a problem. Emphasizes the importance of inductive reasoning and analogy.

3. Polya, G., Mathematics and Plausible Reasoning: Vol. I - Induction and Analogy in Mathematics, Princeton University Press, Princeton, New Jersey, 1954.
Differentiates between demonstrative reasoning and plausible reasoning. Indicates the type of reasoning and evidence that makes a hypothesis more plausible without actually proving it.

4. Polya, G., Mathematics and Plausible Reasoning: Vol. II - Patterns of Plausible Inference, Princeton University Press, Princeton, New Jersey, 1954.
Presents certain patterns of plausible reasoning based largely on inductive and analogical reasoning, and investigates their relation to probability calculus.

B. Texts in specific fields whose main emphasis is on techniques of problem solving:

1. Johnson, W. C., Mathematics and Physical Principles of Engineering Analysis, McGraw-Hill Book Company, New York, 1944.
A junior level text presenting the method of analysis and approach to fairly complex engineering problems. Methods of attack, physical interpretation, procedures for setting up equations and the use of approximations and assumptions are emphasized.

2. Pearson, D. S., Creativeness for Engineers, DPP, State College, Pennsylvania, 1961.
Presents an outline of an organized method of solving problems with chapters on problem recognition, definition, evaluation, synthesis, analysis, and interpretation.

3. Rase, H. F., The Philosophy and Logic of Chemical Engineering, Gulf Publishing Company, Houston, Texas, 1961.
A presentation of a philosophy of the chemical engineering profession, including its historical development, its goals, its relationship to other fields, and its unique characteristics. Two example problems are presented and the logical approach to their solutions demonstrates various reasoning techniques available to the engineer. The moral and economic responsibilities are emphasized.

4. Ver Planck, D. W. and B. R. Teare, Jr., Engineering Analysis - An Introduction to Professional Method, John Wiley and Sons, New York, 1954.
A junior level text on methods of problem solving. Emphasizes the translation of engineering situations into mathematical language and the analysis of the solution of the mathematical problem. Indicates the type of thinking processes involved in typical sample problems.

C. Freshman level texts designed to teach systematic methods of approaching problems and presenting problem solutions:

1. Brown, R. Q., Introduction to Engineering Problems, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1948.

2. Crawford, C. W., Introductory Problems in Engineering, Ronald Press, New York, 1949.

3. Dana, F. C., and L. P. Hillyard , Engineering Problems Manual, McGraw-Hill Book Company, New York, 1947.

4. Fogel, C. M., Introduction to Engineering Computations, International Textbook Company, Scranton, Pennsylvania, 1960.

5. Johnson, L. H., Engineering: Principles and Problems, McGraw-Hill Book Company, New York, 1960.

6. Leach, H. W., and G. C. Beakley, Elementary Problems in Engineering, Macmillan Company, New York, 1951.

7. Spicer, W. M., Taylor, W. S., and J. D. Clary, General Chemistry Problems, John Wiley and Sons, New York, 1943.

D. Sophomore level texts:

1. Anderson, H. V., Chemical Calculations, McGraw-Hill Book Company, New York, 1955.

2. Benson, S. W., Chemical Calculations, John Wiley and Sons, New York, 1952.

3. Corcoran, W. H., and W. N. Lacey, Introduction to Chemical Engineering Problems, McGraw-Hill Book Company, New York, 1960.

An introduction to chemical engineering problems to be used before the student has taken physical chemistry, industrial chemistry and industrial stoichiometry. Includes an introduction to chemical equilibria and chemical kinetics.

4. Henley, E. J., and H. Bieber, Chemical Engineering Calculations - Mass and Energy Balances, McGraw-Hill Book Company, New York, 1959.

A material and energy balance text which includes an introduction to thermodynamics, phase equilibrium, and chemical reaction equilibrium. Both steady and unsteady state systems are discussed.

5. Himmelblau, D. M., Basic Principles and Calculations in Chemical Engineering, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1962.

An introductory text on material and energy balances, including numerous practice problems and an introduction to unsteady state processes.

6. Hougen, O. A., Watson, K. M., and R. A. Ragatz, Chemical Process Principles, Part I, Material and Energy Balances, John Wiley and Sons, New York, 1954.

A material and energy balance text emphasizing the use of generalized procedures for estimating data and presenting graphical methods of attacking problems.

7. Kowalke, O. L., Fundamentals in Chemical Process Calculations, Macmillan Company, New York, 1947.

8. Lewis, W. K., Radasch, A. H., and H. C. Lewis, Industrial Stoichiometry, Chemical Calculations of Manufacturing Processes, McGraw-Hill Book Company, New York, 1954.

An introductory text in material and energy balances with emphasis on the applications of the inorganic chemicals industries.

9. List, H. L., and A. X. Schmidt, Material and Energy Balances, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1962.

An introductory text on material and energy balances. Material balances of physical separations, including one, two, and three component systems, of chemical processes and energy balances on physical and chemical processes are presented.

10. Thatcher, C. M., Fundamentals of Chemical Engineering, Charles E. Merrill Books, Inc., Columbus, Ohio, 1962.

A comprehensive introduction to chemical engineering with emphasis on thermodynamics.

11. Tracy , S. J., How to Solve Problems in Elementary Mechanical Engineering, Flushing, New York, 1954.

12. Williams, E. T., and R. C. Johnson, Stoichiometry for Chemical Engineers, McGraw-Hill Book Company, New York, 1958.

A material and energy balance text, which includes an introduction to unit operations.

E. Junior or senior level texts:

1. Johnson, W. C., Mathematical and Physical Principles of Engineering Analysis, McGraw-Hill Book Company, New York, 1944.
(See B1)

2. Li, Wen-hsiung, Engineering Analysis, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1960.
An undergraduate text in applied mathematics, emphasizing mathematical techniques used in solving engineering problems.

3. Ryder, F. L., Creative Engineering Analysis, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1961.

4. Sutherland, R. L., Engineering Systems Analysis, Addison-Wesley Publishing Co., Inc., Reading, Massachusetts, 1958.
A text emphasizing the analyses between different engineering fields, i.e., mechanical, electrical and acoustical. Sections on dimensional analysis, feedback, and computing machines are included.

5. Ver Planck, D. W., and Teare, B. R., Jr., Engineering Analysis - An Introduction to Professional Method, John Wiley and Sons, New York, 1954.
(See B4)

F. Texts on mathematical logic:

1. Church, A., Introduction to Mathematical Logic, Princeton University Press, Princeton, New Jersey, 1956.

2. Copi, I. M., Introduction to Logic, Macmillan Company, New York, 1961.

3. Copi, I. M., Symbolic Logic, Macmillan Company, New York, 1954.

4. Exner, R. M., and M. F. Rosskopf, Logic in Elementary Mathematics, McGraw-Hill Book Company, New York, 1959.

5. Quine, W. V. O., Mathematical Logic, Harvard University Press, Cambridge, Mass., 1951.

6. Rosenbloom, P. C., The Elements of Mathematical Logic, Dover Publications, New York, 1950.

7. Rosser, J. B., Logic for Mathematicians, McGraw-Hill Book Company, New York, 1953.

8. Suppes, P. C., Introduction to Logic, D. Van Nostrand Company, Inc., Princeton, New Jersey, 1957.

G. Miscellaneous mathematics texts:

1. Fisher, Sir R. A., Statistical Methods and Scientific Inference, Oliver and Boyd, London, 1956.
Discusses various statistical methods which can be used to express the nature and amount of undertainty involved in expressing scientific inferences from experimental data.

2. Jaynes, E. T., Probability Theory on Science and Engineering, McGraw-Hill Book Company. (In Press)

3. Jeffreys, Sir H., Scientific Inference, Cambridge University Press, Cambridge, 1957.
Discusses the nature of inference from empirical data to predict future phenomena, including the application of probability, error estimates, and sampling techniques. Presents the development of Newtonian dynamics, light and relativity, and mensuration as examples of scientific inductive reasoning.

H. Programmed instruction:

1. Automatic Teaching: The State of the Art, ed. by E. C. Galanter, John Wiley and Sons, New York, 1959.

2. Cram, D., Explaining Teaching Machines and Programming, Fearon Publishers, San Francisco, 1961.

3. Lumsdaine, A. A., and R. Glaser, Teaching Machines and Programmed Learning, National Education Association, Department of Audiovisual Instruction, 1960.

4. Mager, R. F., Preparing Objectives for Programmed Instruction, Fearon Publishers, San Francisco, 1962.

5. Programmed Learning, ed. by J. P. Lysought, The Foundation for Research on Human Behavior, 1961.

6. Williams, E. M., Some Experiments with Programmed Materials in an Undergraduate Engineering Program, An address given at the Pacific Southwest Meeting of the ASEE, December 28, 1961.

I. Miscellaneous

1. Asimow, M., Theory and Principles of Engineering Design, Course 106B, Engineering Design, Department of Engineering, The University of California, Los Angeles, to be published in Introduction to Design, by M. Asimow, Prentice-Hall.

2. Asimow, M., Introduction to Design, (see above) to be published in Elements of Engineering Design, by Woodson, T. T., and M. Asimow.

3. Dewey, J., How We Think, D. C. Heath and Company, Boston, 1933.

4. Dewey, J., Logic, The Theory of Inquiry, Henry Holt and Company, New York, 1938.

5. Polanyi, M., Personal Knowledge, Roulledge and Kegan Paul Ltd., London, 1958.

6. Polanyi, M., The Study of Man, Roulledge and Kegan Paul Ltd., London, 1959.

7. "Proceedings on the First Conference on Engineering Design Education," Case Institute of Technology, September 8-9, 1960, Published by the Office of Special Programs, Case Institute of Technology, Cleveland 6, Ohio, 1960.

8. Westervelt, F. H., Automatic System Simulated Programming, Doctoral Thesis, The University of Michigan, Ann Arbor, November, 1960.

APPENDIX B

Example Problems Illustrating the Use of a Scrambled Text in Solving Chemical Engineering Problems

The following pages are part of a text designed to help you learn to solve some engineering problems. Your help is needed in order to make the text more effective. You can help us in the following ways:

1. Even though you may have already seen the kind of problem presented here, please do the work requested of you as you go through the text.
2. Record each page, before you turn to it, on your work sheets. It is most important to record your mistakes as well as your correct moves.
3. Do not write your name on the work sheets. You are not to be graded on your work. We only want your help and we hope this text may be helpful to you in return.

Your help is greatly appreciated. Now turn to page 20 and begin your work.

Problem solving is an activity that is familiar to almost everyone. People of all ages derive pleasure from solving problems, puzzles and brain-teasers of all kinds. Our everyday lives present an unending succession of problematic situations.

Some people are able to develop an ability to solve difficult technical problems very effectively. These people usually spend a large part of their professional lives exercising this ability.

In view of the foregoing, you may be surprised to discover that very little is known about how people actually solve problems. The psychological aspects of problem solving present some fascinating and very difficult questions that have yet to be answered. Nevertheless, you already realize that you ought to develop your problem solving talents as fully as you can.

The text you are about to begin may be able to help you develop these talents. In other words, an objective of this text is to change your behavior in problem solving activities. You are therefore entitled to know how this text is intended to do this, what is expected of you, and what you should expect to be able to do as a result of your work.

This text is organized as a "scrambled text" of problems of increasing difficulty designed to develop your problem solving ability. The problems are presented in tutorial fashion. Some of the exercises are presented in small steps to help emphasize key concepts. You will have the opportunity to work at your own pace with continual monitoring of your progress. Your errors will be quickly detected and explained, and your successful efforts will be immediately verified.

This sounds fine but you must realize that your progress using this text depends upon your <u>active participation</u> in the work. Some problems, especially the early ones, are very easy. They are included so that you can have the chance to think about the <u>solution process</u> while producing the solution itself.

As a result of your work you may expect to:

1. Acquire a technical vocabulary that is useful in problem solving.
2. Acquire some concepts and tools that are useful in problem solving.
3. Recognize the class of problems to which (1) and (2) may be applied.
4. Organize a procedure for solving problems of this general type.
5. Recognize solutions and be able to show that a proposed solution is really a solution.

Just a brief word about using this text and we will get started. First, get and <u>use</u> an $8\frac{1}{2} \times 11$ inch pad of worksheets.

Second, follow the page directions carefully. The path through the pages does not, in general, follow the usual page ordering. (This is why we call it a "scrambled text.") Each page will tell you what page you should turn to next. You can expect to jump back and forth in a manner that may seem to be quite random. Therefore, it is a good idea to get a bookmark to keep your place in the text. Some pages may direct you to return to the page you just came from, in which case your bookmark will keep you from getting lost in the text or from having to return to the very beginning in such cases. We strongly recommend that you list on your worksheets the sequence of pages which you follow in solving the problems. This will enable you to review your reasoning at a later date and will also aid your instructor in correcting your reasoning if you are unable to solve the problem yourself. If you are ready to get started get your bookmark and your pad of worksheets and turn to page 23a.

There is no page anywhere in this text that directs you to turn to this page.

Either you turned here by accident or you did not read the page directions carefully enough. If you are just beginning, turn back to page 20 and try again to follow the directions.

Problem 1:

Water is fed into a storage tank by two inlet pipes. The first of these delivers 10 pounds of water per hour. The single exit from the tank removes water at the rate of 25# water/hr. (In this text we use the symbol # to represent pounds.)

At noon the tank contains 500 # water and at 2:00 p. m. it contains 600 # water.

If all the flow rates are constant, at what rate does water enter the tank through the second inlet pipe?

To make sure you understand the problem, draw on your worksheet a picture that represents the situation described in this problem.

When you feel you have done an adequate job of representing the problem situation by means of a picture, turn to page 39d.

Basis: 100 # B. D. Laundry entering

Bone Dry Laundry Balance:

Let X be # B. D. Laundry leaving

Input - Output = Accumulation

100 # B. D. Laundry - X = 0

This is a very simple balance and you might have done it in your head. But it is important to realize what was happening. First of all, you note that since the amount of material in the dryer is constant, the accumulation is zero. Such a process is called a "steady flow" process. For a steady flow process

Input = Output

For your choice of material to balance, 100 # B. D. Laundry leaves for every 100 # entering. Moreover, the B. D. Laundry appears in only two streams, one entering and one leaving. Thus, this substance "ties" these streams together. The use of such a tie substance will usually result in simple (even trivial, as above) mass balances and hence simple solutions.

Now you must choose some other material to balance in order to complete your problem. Choose such a material and turn to page 64.

Basis: 100 # wet laundry

Water Balance:

X is # dried laundry

Y is # H_2O to drain

Input - Output = Accumulation

$(100)(.396)\ \#H_2O - (Y + .04X)\ \#H_2O = 0$

Note that since the amount of material in the dryer at any time is constant, the accumulation term is zero. Such a process is called a "steady flow" process. For a steady flow process, then,

Input = Output

Your choice of water balance does not allow you to solve directly for either X or Y. A more direct route does exist which you will be shown later. However, your choice is certainly a feasible one. You have one equation with 2 unknowns. In order to get another equation you must put another material to balance. So pick another material and turn to page 29c.

Basis: 100 # dried laundry

Total Mass Balance:

Let X be # wet laundry

Y be # H_2O

Input - Output = Accumulation

X # total mass in - (Y + 100) # total mass out = 0

Note that since the amount of material in the dryer at any time is constant, the accumulation term is zero. Such a process, called a "steady flow" process, can be described by

Input = Output

Your choice of total mass balance does not allow you to solve directly for X or Y. A more direct route does exist which you will be shown later. However, your choice is certainly a feasible one. Note that you now have one equation with two unknowns. In order to get another equation you must pick another material to balance. So pick another materian and turn to page 48b.

Write the appropriate balance equation and then turn to the page indicated.

	Page
H_2O	48c
Total Mass	32c
Hydrogen	62b
Oxygen	41c
Other	50b

You have fallen into a very common trap. You wrote:

$$(X + 10) \# H_2O - 25 \# H_2O = 100 \# H_2O$$

All of the quantities on the left were taken on the <u>basis</u> of 1 hour. The accumulation (on the right) was for two hours!

Before beginning a calculation you must select and use a common basis. Either one hour or two hours would have been all right, but you <u>must</u> be consistent.

(Return to page 30a and try again.)

You have chosen the material you wish to balance. Write the corresponding material balance equation. Indicate the numerical quantities and units of the terms of your equation and define all unknown quantities symbolically.

Then, turn to the page obtained from the following table.

Material to Balance First	Turn to Page
Water	24a
Bone Dry Laundry	62a
Total Mass	40a
Oxygen	41c
Hydrogen	62b
Other	50b

Basis: 100 $\#H_2O$ entering

Total Mass Balance:

Let X be # dried laundry leaving

Y be $\#H_2O$ to drain

Input - Output = Accumulation

$$100 \left(\frac{100}{.395}\right) \text{ total mass in} - (Y + X) \text{ total mass out} = 0$$

Note that since the amount of material in the dryer at any time is constant, the accumulation term is zero. Such a process is called a "steady flow" process. For steady flow processes:

Input = Output

Choosing total mass to balance first does not let you solve directly for either X or Y. A more direct route does exist and it will be pointed out to you later. Now, however, you should realize that the present approach is feasible but requires another equation.

Pick another material to balance and turn to page 53a.

Basis: 100 # wet laundry

Total Mass Balance:

Let X be # dried laundry

Y be # H_2O to drain

Input = Output

(100) # total mass in = (X + Y) # total mass out

H_2O Balance (from previous page):

$39.6 \ \#H_2O = .04X \ \#H_2O + Y \ \#H_2O$

You now have two equations and two unknowns. Solve for Y, which is the $\#H_2O$ to drain (based on 100 # wet laundry). Convert Y to $\#H_2O$ to drain per minute and compare your answer with that on page 41a.

Basis: 100 # dried laundry

B. D. Laundry Balance:

Let X be # wet laundry entering

Input = Output

.604 X # B.D.L. = (.96)(100) # B.D.L.

and from the previous balance (water) you have

(Y is $\#H_2O$ to drain)

$.396 \ X \ \#H_2O = (Y + .04 * 100)$

And now you have two equations and two unknowns. Note, however, that if you had selected B. D. Laundry as the material to balance first, you would have been able to solve each equation directly.

The choice of B. D. Laundry as the first material to balance is recommended because this material was present in only two streams, one entering and one leaving. B. D. Laundry, therefore, "ties" these streams together. The use of a tie substance usually leads to simpler, more easily solved mass balances.

Now solve for Y (the $\#H_2O$ to drain) and turn to page 40c.

Write the corresponding material balance equation and turn to the page indicated.

Material to Balance	Turn to Page
Water	50c
Oxygen	62b
Hydrogen	41c
Total Mass	30b
Other	50b

Basis: 100 # B. D. Laundry entering

Total Mass Balance:

Let Y be # H_2O to drain

X be # B. D. Laundry leaving

Input = Output

$100 * (\frac{100}{60.4})$ # total mass in = $(Y + (\frac{100}{96}) X)$ # total mass out

but X is known from the previous balance (X=100). Thus, Y is found directly:

Y = 61.3 # H_2O to drain

This is based on 100 # of B. D. Laundry entering. You must now express Y in the units of # H_2O/minute to satisfy the requirements of the problem. Convert your answer to these terms and turn to page 58a.

Now that you have chosen the material you want to balance, write down the appropriate material balance equation. Indicate both the numerical quantities and units of the terms of your equation. Define all unknown quantities symbolically. Turn to the page indicated in the following table.

Material to Balance	Turn to Page
Water	35b
B. D. Laundry	23b
Total Mass	52a
Hydrogen	41c
Oxygen	62b
Other	50b

Basis: 100 # B. D. Laundry

Total Mass Balance:

Let X be # dried laundry

Y be # H_2O to drain

Input = Output

$(100)(\frac{100}{60.4})$ # total mass in = (X + Y) # total mass out

and from the previous water balance

$(100)(\frac{39.6}{60.4})$ # H_2O = (Y + .04X) # H_2O

You now have two equations and two unknowns. Now solve these equations for Y and turn to page 34b.

Basis: 1 month

Let X = wife's average monthly withdrawal

Man's average monthly withdrawal = $\frac{\$78 + \$86}{2} = \$82$

Bank's average monthly interest payment = 2%/2 = 1%

Input - Output = Accumulation

Money Balance:

$$\$250 + (.01)(500) - \$82 + X = \frac{514-500}{2}$$

$$250 + 5 - 82 - X = 7$$

$$X = \$166/\text{month}$$

When you have completed this problem, turn to page 55c.

The common name for the type of equation you were asked to write is a mass balance. Since, in this particular problem, you are balancing the masses of water, it will be called a water balance. Although water is the only mass which you need to balance in this problem, in more complex problems you may have to balance several different types of materials. It is, therefore, desirable to label each equation as to the material being balanced.

Compare your equations with those listed here and turn to the page indicated. (Compare numbers and units!)

(X + 10) # H_2O - 25 # H_2O = 100 # H_2O	(page 25a)
(2X + 20) # H_2O/2 hr. - 50 # H_2O/2 hr. = 100 # H_2O/2 hr.	(page 33a)
(X + 10) # H_2O - 25 # H_2O = 50 # H_2O	(page 32b)
X + 10 - 25 = 50	(page 46a)
(X + 10) # H_2O/hr. - 25 # H_2O/hr. = 50 # H_2O/hr.	(page 39b)
(X + 20) # H_2O - 50 # H_2O = 100 # H_2O	(page 42a)
Other solutions.	(page 47a)

Problem 2:

A man puts \$250 into his bank account on the second of each month. Both the man and his wife may withdraw money. The bank balance on June 1 was \$500 and on August 1 was \$514. A semiannual interest payment was made on June 15 equal to 2% of the June 1 balance. If the man withdrew \$78 during June, and \$86 during July, what is the average monthly withdrawal made by his wife during this two-month period?

Draw the diagram for this problem, and indicate your basis of calculation. Write and solve the applicable mathematical equations. Compare your solution to the sample solution given on page 34a.

Basis: 100 # wet laundry

Bone Dry Laundry Balance:

Let X be # dried laundry leaving

Input - Output

(100)(.604) # B.D.L. = X(.96) # B.D.L.

Total Balance (from previous page):

Let Y be # H_2O to drain

(100 #) total stream in = (Y + X) # total stream out

It seems obvious now that you should have taken the B. D. Laundry balance first since you could then solve directly for X. Knowing X, the total mass may be solved directly for Y.

The choice of B.D.L. for the first balance is recommended because it is contained in only two streams, one entering and one leaving. B.D.L. may therefore be used to "tie" these streams together. Such a material, which is present in only two streams, is called a "tie substance." The use of tie substances will usually lead to simple, easily solved, mass balances such as the one above.

Solve for Y, which is the # H_2O to drain based on 100 # wet laundry. Convert Y to # water to drain per minute and compare your answer to that on page 42b.

Write the corresponding material balance equation and turn to the page indicated.

Material to Balance	Turn to Page
B. D. Laundry	59b
Total Mass Balance	26a
Hydrogen	41c
Oxygen	62b
Other	50b

Now let's try to formulate a statement that expresses a relation between the quantities in our problem.

We might say: The quantities of water that flow into the tank minus the quantity of water that flows out during any period is equal to the water that accumulates in the tank during that period.

This statement is certainly true for our problem but it is a bit wordy. The same idea can be formulated as an equation:

Input (of water) - Output (of water) = Accumulation (of water)

____________ - ____________ = ________________

If we insert the data of our problem in the above equation form we shall produce an equation whose solution will solve our problem. Write the equation, including the unknown x, on your worksheet, being careful to include the units being used.

Now turn to page 28b.

Basis: 100 # dried laundry

Total Mass Balance:

Let Y be # H_2O to drain

X be # wet laundry entering

Input = Output

X # total mass in = (Y + 100) # total mass out

but you already know X (from previous balance) and so you now have

Y = 59 # H_2O to drain

This is still not quite the solution desired. You must get the drain flow in # H_2O/minute. Convert your value of Y into these units and turn to page 53c.

Now label your diagram if you have not already done so. Transfer the numerical values and the units in which these values are given for all the known quantities. Include the units as a part of the labeling. Use symbolic labels (say, x, y) for all unknown quantities.

When this has been done, turn to page 44b.

You chose 100 # dried laundry as your basis of calculation. This is all right. You can certainly work the problem this way.

The main criticism of this choice is that the information for the total production (that is, 4567 # wet laundry/hr.) is stated for the wet laundry stream. Your final result must reflect the appropriate total flow. But this certainly can be done. So let's go!

You want to try using 100 # dried laundry as the basis of calculation. Let's see what we know:

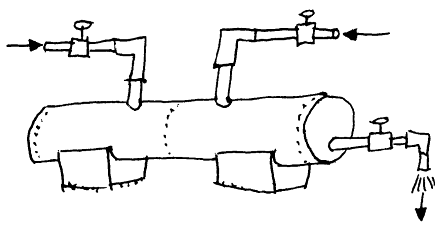
1. The basis of calculation is an arbitrary device designed to simplify your calculations. The choice of 100 # is much more convenient than some other possible choices.
2. Using the dried laundry as the material, the percentages given allow the immediate expression of the water and fabric amounts in the exit stream.

You are now ready to try to express the mathematical relationships between the streams. The form of this relationship is:

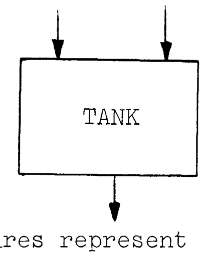
Input - Output = Accumulation.

Since this problem deals with more than one kind of material, you must ask yourself, "Input of what?", "Output of what?", "Accumulation of what?" In other words, which material do you wish to account for (or "balance")?

Choose a material and turn to page 60b.



page 61c.



page 48a.

Both of these pictures represent the problem statement. Your drawing must be correct in the number of inlet and exit streams. Check your drawing and correct it if necessary.

When your drawing agrees with the above in number of inlets and outlets, choose that drawing more nearly like yours and turn to the page indicated beside it.

Basis: 100 # dried laundry

B. D. Laundry Balance:

$$X = \frac{(100)(.96)}{.604} = 159 \text{ \# wet laundry}$$

Total Balance:

$$Y = 159\text{-}100 = 59 \text{ \# } H_2O \text{ to drain}$$

You have now solved for Y, based on 100 # of dried laundry. Now, convert Y to # H_2O to drain per minute and compare your answer to that on page 53c.

Good! You wrote:

$$(X + 10) \text{ \# } H_2O - 25 \text{ \# } H_2O = 50 \text{ \# } H_2O$$

You made a step, perhaps unconsciously, that needs to be expressed. Without specifically stating it you have selected 1 hour as the basis for your calculations. You should form the habit of writing down your basis. The usual method is to write down the basis of your calculation, then express the equation using this basis:

Basis: 1 hour

Water Balance: $(X + 10) \text{ \# } H_2O - 25 \text{ \# } H_2O = 50 \text{ \# } H_2O$

Let X be # H_2O/hr. entering in second pipe

Now solve your equation for the unknown flow and turn to page 40b.

Total Mass Balance:

Let Y be # H_2O to drain

Input = Output

$$100 \text{ \# total mass} = (60.4 \text{ \# B.D.L.}) \frac{(100 \text{ \# dried laundry})}{(96 \text{ \# B.D.L.})} + Y \quad \text{\# total mass out}$$

$$100 = 62.9 + Y$$

$$Y = 37.1 \text{ \# } H_2O \text{ to drain}$$

Now compute the # H_2O/minute to drain and compare your answer to that on page 42b.

This is fine! You wrote:

Water Balance: Let X be # H_2O/hr. entering in second pipe

$$(2X + 20) \ \# \ H_2O/2 \text{ hr.} - 50 \ \# \ H_2O/2 \text{ hr.} = 100 \ \# \ H_2O/2 \text{ hr.}$$

In so doing you selected # H_2O/2 hr. as the kind of unit and 2 hours as the basis for the calculation.

It is important to choose and make use of a consistent basis. The usual method is to indicate your choice of basis and then express the numerical equation in terms of this basis, i.e.,

Basis: 2 hours

Water Balance:

$$(2X + 20) \ \# \ H_2O + 50 \ \# \ H_2O = 100 \ \# \ H_2O$$

Now solve for the unknown flow and turn to page 40b.

Eureka!

We hope we didn't cheer too soon. If your answer for the unknown flow was

$$X = 65 \ \# \ H_2O/\text{hr.}$$

or an equivalent flow rate in different units, i.e., 1.085 # H_2O/min., etc., you have solved the problem. If not, check carefully for numerical mistakes and return here to check your answer.

Now turn to page 56a and compare your solution to the two sample solutions to problem 1 that are given there.

Basis: 100 # dried laundry

Water Balance:

Let X be # wet laundry

Y be # water to drain

Input = Output

$$X(.396) \ \# \ H_2O = Y + (.04)X \ \# \ H_2O$$

Total Balance (from previous page):

$$X \ \# \text{ total mass in} = (Y + 100) \ \# \text{ total mass out}$$

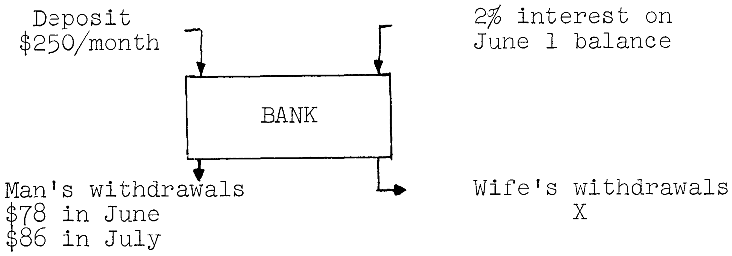
You now have two equations and two unknowns. Solve the equation for Y, which is # H_2O to drain based on 100 # dried laundry and compare your answer to that on page 55b.

Solution for Problem 2:

Balance:

June 1 = \$500

Aug. 1 = \$514



Basis: 2 months (For solution on 1 month basis turn to page 28a.)

Let X = wife's total withdrawal in 2 months

Input - Output = Accumulation

(\$250/month)(2 months) + (.02)(\$500) - (\$78 + \$86 + \$X) = \$514 - \$500

500 + 10 - 164 - X = 14

X = \$332

Therefore, the wife withdrew \$332 in 2 months. Average monthly withdrawal = \$166/month.

When you have completed this problem turn to page 55c.

Basis: 100 # B. D. Laundry

Y = 61.3 # H_2O to drain

Therefore 61.3 # H_2O does down the drain for every 100 # B. D. Laundry entering. Since there is 4567 # wet laundry/hour or (4567)(.604) # B. D. Laundry/hour processed we may calculate the # H_2O to drain/minute by

$$\frac{(61.3 \text{ \# } H_2O \text{ to drain})}{(100 \text{ \# B. D. Laundry})} \cdot \left(\frac{(4567)(.604) \text{ \# B. D. Laundry}}{\text{Hour}}\right) \left(\frac{\text{Hour}}{60 \text{ min.}}\right) =$$

28.3 # H_2O to drain/minute

You should be aware of the fact that you did not need to solve simultaneous equations to answer this problem. If you had selected B. D. Laundry as the first material to balance and either H_2O or total mass as the second balance, the solution is direct.

Return then to page 27b and redo the problem using the balance directions suggested above.

NOTE: Do not skip this step! There is important material that you need to know which is discussed along the correct solution path. So dig in and find out about it.

You have chosen an inconvenient unit of time as your basis. In view of the previous problems, we can understand your choice. In Problem 1, 1 hour was equivalent to 10 # H_2O entering in the first stream, 25 # H_2O leaving or 50 # H_2O accumulated. (To refer to problem 1, you may turn to page 23a but be sure to return to this page.) These numbers, 10, 25, and 50 are all reasonably convenient numbers for purposes of calculation.

In this problem, however, 1 hour corresponds to 4567 # wet laundry and 1 minute corresponds to $\frac{4567}{60}$ = 76.116+ # wet laundry. You must agree that these are hardly convenient numbers to work with.

In problems of this kind the recommended method of choosing a basis, is to pick a convenient weight such as 100 # of some material. Note that in any flow problem, picking 100 # of a material (e.g., water, wet laundry, B. D. Laundry, dried laundry, etc.) is equivalent to choosing a unit of time. For instance, since 4567 # wet laundry are processed every hour, 100 # wet laundry corresponds to a time basis of:

$$\left(\frac{100 \text{ \# wet laundry}}{4567 \text{ \# wet laundry/hr.}}\right)\left(\frac{60 \text{ min.}}{\text{hr.}}\right) = 1.31 \text{ minutes}$$

The answer, in terms of your basis, must be converted to the actual flow after your calculations have been completed.

Choose another basis and return to page 57b.

Basis: 100 # B. D. Laundry

Water Balance:

Let X be # dried laundry

Y be # H_2O to drain

Input - Output = Accumulation

$$(100)\left(\frac{39.6}{60.4}\right) \text{ \# } H_2O - (Y + .04X) \text{ \# } H_2O = 0$$

Note that since the amount of material in the dryer at any time is constant, the accumulation term is zero. Such a process is called a <u>steady flow process</u> and can be described by:

Input = Output

Your choice of a water balance first does not allow you to solve directly for either X or Y. A more direct route does exist and you will be shown it later. Now, however, your choice is certainly feasible, but since you have two unknowns and only one equation, you need another equation.

So pick another material to balance and turn to page 63b.

Based on 100 # dried laundry, you calculated that X = 159 # wet laundry.

However, the problem required 4567 #/hour of wet laundry to be processed. Therefore you must multiply your answer by the ratio of $\frac{4567}{159}$ in order to solve for the actual # H_2O to drain per hour. Of course, this must be divided by 60 min./hr. since you were asked to solve for # H_2O/min. to drain.

$$\frac{59 \text{ \# } H_2O \text{ to drain}}{100 \text{ \# dried laundry}} \cdot \frac{100 \text{ \# dried laundry}}{159 \text{ \# wet laundry}} \cdot \frac{4567 \text{ \# wet laundry}}{\text{Hr.}} \cdot \frac{\text{Hr.}}{60 \text{ min.}} =$$

28.3 # H_2O/minute to drain.

As previously pointed out, if you had chosen 100 # wet laundry as your basis, the second conversion factor, involving 159 # wet laundry, X, would not have been needed.

You should be made aware of the fact that you did not <u>need</u> to solve simultaneous equations to answer this problem. If you had selected Bone Dry Laundry as the first material to balance and then either H_2O or total mass, the solution was direct.

You should now either

1. Return to page 60a and try the problem with the same basis (100 # dried laundry) but choosing B. D. Laundry as your first material to balance and then either H_2O or total mass for the second balance.
2. Return to page 57b and try the problem using 100 # wet laundry as your basis, choosing B. D. Laundry as your first material to balance and either H_2O or total mass for the second.

NOTE: Do not skip this step!! There is important material that you need to know which is discussed along the correct solution path. So dig in and find out about it.

Basis: 100 # dried laundry

Water Balance:

Let X be # wet laundry entering

Y be # H_2O to drain

Input - Output = Accumulation

$.396 X \text{ \# } H_2O - (Y + .04 * 100) \text{ \# } H_2O = 0$

Note that since the amount of material in the dryer at any time is constant, the accumulation term is zero. Such a process is called a "steady flow" process. For a steady flow process,

Input = Output

Your choice of a water balance does not allow you to solve directly for either X or Y. A more direct route does exist and you will be shown it later. Your choice is certainly feasible, but since you have two unknowns and only one equation, you need another equation.

Pick another material to balance and turn to page 61a.

Since every 4567 # of wet laundry entering is equivalent to the passage of one hour in time, in effect then you have chosen one hour as the basis for your calculation.

Turn to page 35a.

Good for you!

Basis: 100 # dried laundry

Bone Dry Laundry Balance:

Let X be # wet laundry entering

Input - Output = Accumulation

(.604 X) # B.D.L. - (.96 * 100) # B.D.L. = 0

Note that since the amount of material in the dryer at any time is constant, the accumulation term is zero. Such a process is called a "steady flow" process. In a steady flow process:

Input = Output

.604 X = .96 * 100

Your choice of a B. D. Laundry balance first was good because you can now find X directly:

X = 159 # wet laundry

This happened because B. D. Laundry appears in only two streams, one entering and one leaving. This material "<u>ties</u>" these streams together. The use of a <u>tie substance</u> usually results in simpler, more easily solved mass balances, as you have already begun to see.

Now you must still find the amount of H_2O removed. You need another balance. Pick a second material to balance and turn to page 26c.

Write the appropriate balance equations and turn to the page indicated.

Material to Balance	Turn to Page
Water	57a
Oxygen	62b
Hydrogen	41c
B. D. Laundry	43a
Other	50b

Now that you have chosen the material you wish to balance, write down the appropriate material balance equation. Indicate both the numerical quantities and the units of the terms of your equation. Define all unknown quantities symbolically. Then turn to the page indicated.

Material to Balance	Turn to Page
Total Mass	25c
B. D. Laundry	59a
Water	41b
Hydrogen	41c
Oxygen	62b
Other	50b

You have chosen 100 # Bone Dry Laundry as your basis. This is all right. You can certainly work the problem this way.

The basis of calculation is an arbitrary device designed to simplify your calculations. The choice of 100 # as an amount to work with is more convenient than the number of # actually flowing.

In order to express the percentages of the different materials in the inlet stream, you must multiply your basis by a ratio of percentages. If you had chosen 100 # wet laundry as your basis, the amounts of H_2O and B. D. Laundry entering would be immediately apparent without resort to calculation.

In order to convert your final answer (which will be based on 100 # B. D. Laundry) to the units of # H_2O/ minute to drain, you will make use of the total flow 4567 # wet laundry/hour. Therefore, your choice of B. D. Laundry as a basis forces you to make the conversion from # B. D. Laundry to # wet laundry.

However, your basis will work, and you are now ready to try to express the mathematical relationships between the streams. These relationships take the form

Input - Output = Accumulation.

In problems of this kind you must ask yourself, "Input of what?", "Output of what?", "Accumulation of what?" In other words, which materials do you wish to balance?

Choose a material to balance and turn to page 27b.

You are now ready to try a problem on your own. Pattern your work after the problem you just did. Don't get careless or sloppy.

Turn to page 29a and begin problem 2.

This is fine! You wrote:

Water Balance:

Let X be # H_2O entering in second pipe

(X + 10) # H_2O/hr. - 25 # H_2O/hr. = 50 # H_2O/hr.

In so doing you selected # H_2O/hr. as the kind of unit and one hour as the <u>basis</u> of calculation.

It is important to choose and make use of a consistent <u>basis</u>. The usual method is to indicate your choice of <u>basis</u> and express your numerical equation using this basis, i.e.:

Basis: <u>1 hour</u>

Water Balance:

(X + 10) # H_2O - 25 # H_2O = 50 # H_2O

Now solve for the unknown flow and turn to page 40b.

Basis: 100 # water entering

Total Mass Balance:

Let X be # B. D. Laundry leaving

Y be # H_2O to drain

Input = Output

$100 \left(\frac{1.00}{.396}\right) = Y + \left(\frac{1.00}{.96}\right) X$

But X is already known from the previous balance (X = 152.5 # B. D. Laundry leaving) so you may solve directly for Y.

Solve for Y and convert to # H_2O to drain/minute and then turn to page 46c.

Have you really drawn the picture as requested on page 23a?

If you have not, let's not kid ourselves. Unless you ACTIVELY PARTICIPATE in using this text it will do you almost no good. Solving problems is not a spectator sport. You <u>must</u> play the game!

When, and only when, you have finished drawing a picture of the situation described in the statement of problem 1, page 23a, turn to page 31b.

Basis: 100 # wet laundry

Total Mass Balance:

Let X be # dried laundry

Y be # H_2O to drain

Input - Output = Accumulation

100 # total stream - Y + X) # total stream = 0

Note that since the amount of material in the dryer at any time is constant, the accumulation term is zero. Such a process is called a "steady flow" process. For a steady flow process, then,

Input = Output

Your choice of total mass balance does not allow you to solve directly for either X or Y. A more direct route does exist which you will be shown later. However, your choice is certainly a feasible one. You have one equation with two unknowns. In order to get another equation, pick another material to balance and turn to page 58b.

Is your result reasonable in terms of the problem statement? Can you think of any way to verify your result?

Check to see if the units of your answer are the units of a flow quantity.

Always check your work. If you can use a different method to check your work, this is probably best.

When you are satisfied with your answer, turn to page 33b.

Basis: 100 # dried laundry

From the B. D. Laundry Balance:

$$X = \frac{.96 * 100}{.604} = 159 \text{ \# wet laundry entering}$$

From the Water Balance:

$$Y = .04 * 100. + .396 * 159 = 59 \text{ \# } H_2O \text{ to drain}$$

You have now obtained a value for Y on the basis of 100 # dried laundry leaving. This is still not quite the answer desired since we must know the drain flow per minute.

Convert your answer into # H_2O to drain per minute and turn to page 53c.

You should have found that

$Y = 37.1$ # H_2O to drain

Now you must convert this to the actual conditions. Your calculations were based on 100 # wet laundry, while 4567 # wet laundry enter each hour.

Therefore

$$\frac{37.1 \text{ \# } H_2O \text{ removed}}{100 \text{ \# wet laundry}} \Bigg| \frac{4567 \text{ \# wet laundry}}{\text{hr.}} \Bigg| \frac{\text{hr.}}{60 \text{ min.}} = 28.3 \frac{\text{\# } H_2O \text{ removed}}{\text{min.}}$$

$$\text{Answer} = 28.3 \frac{\text{\# } H_2O \text{ removed}}{\text{min.}}$$

You should be made aware of the fact that you did not <u>need</u> to solve simultaneous equations to answer this problem. If you had selected B. D. Laundry as the first material to balance and then either H_2O or Total Mass, the solution was direct. Return to page 52b and try the problem this way.

(We're not being stuffy about this point! There is important material that you <u>need</u> to know which is discussed along the correct path. So dig in and find out about it.)

Basis: 100 # water entering

Water Balance:

Let X be # H_2O leaving in dried laundry

Y be # H_2O to drain

Input - Output = Accumulation

$100 \text{ \# } H_2O - (Y + X) \text{ \# } H_2O = 0$

Note that since the amount of material in the dryer at any time is constant, the accumulation term is zero. Such a process is called a "steady flow" process. For a steady flow process:

Input = Output

Your choice of material to balance does not allow you to determine either X or Y directly. A more direct route does exist and you will be shown it later. Now, however, you have two unknowns and only one equation. You need another equation.

Choose another material to balance and turn to page 56b.

Turn to page 62b.

Good! You wrote:

Water Balance:

Let X be # H_2O entering in two hours

(X + 20) # H_2O - 50 # H_2O = 100 # H_2O

You made a step, perhaps unconsciously, that needs to be expressed. Without specifically stating it, you have selected two hours as a basis for your calculation. You should form the habit of writing down your basis. The usual method is towrite down the basis of your calculation, then express the equations using this basis.

Basis: 2 hours

Water Balance:

(2X + 20) # H_2O - 50 = 100 # H_2O

Now solve your equation for the unknown flow and turn to page 40b.

You have now converted Y = 37.1 # H_2O to drain back to the actual conditions. This is necessary since Y is based on 100 # wet laundry (your basis of calculation), whereas the actual amount is 4567 # wet laundry / hour.

$$\therefore \quad \frac{37.1 \text{ \# } H_2O \text{ removed}}{100 \text{ \# wet laundry}} \left| \frac{4567 \text{ \# wet laundry}}{\text{Hr.}} \right| \frac{\text{Hr.}}{60 \text{ min.}} = 28.3 \; \frac{\text{\# } H_2O \text{ removed}}{\text{min.}}$$

$$\text{Answer} = \frac{28.3 \text{ \# } H_2O \text{ removed}}{\text{min.}}$$

Turn to page 55a.

Basis: 100 # dried laundry

Eliminating X between the two simultaneous equations you should have found that

Y = 59 # H_2O to drain.

This is not the desired answer yet because this is in terms of 100 # of dried laundry and you must find the drain flow in # H_2O/minute. Now you must convert your result to the correct units. Do this and turn to page 55b.

Basis: 100 # B. D. Laundry

B. D. Laundry Balance:

X = # dried laundry

Input = Output

100 # B.D.L. = .96(X) # B.D.L.

Total Balance (from previous page):

$100\ (\frac{100}{60.4})$ # total mass in = (X + Y) # total mass out

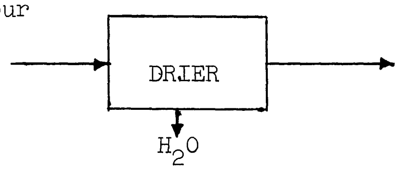
It seems obvious now that you should have taken B. D. laundry for your first balance since you can solve it directly for X. Knowing X you may then solve the total balance directly for Y.

The choice of B.D.L. for the first balance is recommended because it is contained in only two streams, one entering and one leaving. B.D.L. may therefore be used to "tie" the streams together. Such a material, present in only two streams, is called a tie substance. The use of tie substances usually leads to simple, easily solved, mass balances such as the one above.

Solve for Y, the # H_2O to drain, based on 100 # B.D.L. Convert this to # H_2O to drain/min. and compare your answer to that on page 53b.

Sample Solution to Problem 3:

4567 # wet laundry/hour
39.6% H_2O
60.4% B.D.L.



dried laundry
4% H_2O
96% B.D.L.

Basis: 100 # wet laundry

B.D.L. Balance:

Let X be # B.D.L. leaving

60.4 # B.D.L. in = X (this balance is probably done mentally rather than on paper)

H_2O Balance:

Let Y be # H_2O to drain

$39.6 = Y + (60.4)(\frac{4}{96})$

Y = 37.1 # H_2O in drain

$$\frac{37.1\ \#\ H_2O\ \text{to drain}}{100\ \#\ \text{wet laundry}}\left|\frac{4567\ \#\ \text{wet laundry}}{\text{hour}}\right|\frac{\text{hour}}{60\ \text{min.}} = 28.3\ \frac{\#\ H_2O\ \text{to drain}}{\text{min.}}$$

Now turn to page 44a and solve problem 4 by yourself, using the principles you have learned thus far.

Problem 4:

An air purification unit for use in submarines is designed to absorb CO, CO_2, and H_2O from the air. In order to test this absorber, an analysis of a test air stream, with a high percentage of CO and CO_2, was made at both the inlet and exit of the absorber. The results are given below

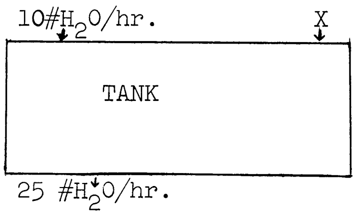
Component	Percent by Weight In	Percent by Weight Out
Air (i.e., N_2 and O_2, argon, etc.)	55.9%	84.6%
CO_2	21.8%	11.8%
CO	19.2%	3.4%
H_2O	3.1%	0.2%

Determine the efficiency (% material absorbed compared to input material) of the absorber for CO_2, CO and H_2O.

Solve this problem using the techniques you have just been developing.

When you have completed the solution, turn to page 45a to check the results.

Compare your diagram carefully with the sample on this page. Be sure that your units include the name of the material, (10#/hr. is not as good as 10#(H_2O)/hr. Don't let the simplicity of this problem prevent you from acquiring good work habits!)



	Time	Weight in Tank
2 hrs.	Noon	500 # (H_2O)
	2:00 p.m.	600 # (H_2O)

Correct your work, if necessary, and turn to page 30a.

Solution to Problem 4:

Stale Air In			Fresh Air Out
B.D.* Air	55.9		84.6
CO_2	21.8	→ ABSORBER →	11.8
CO	19.2		3.4
H_2O	3.1		0.2

Basis: 100 # Stale Air in

Tie Substance B.D. Air

Input = Output (since B.D. Air doesn't accumulate)

55.9 # B.D.A. = 55.9 B.D.A. (you can do this balance mentally)

NOTE:

$$\text{Absorber Efficiency} = \frac{\#\text{ material "A" in} - \#\text{ material "A" out}}{\#\text{ material "A" in}}(100)$$

$$= \frac{\#\text{ accumulation of "A"}}{\#\text{ of "A" in}}(100)$$

CO_2 Absorber Efficiency = $\dfrac{21.8 - 11.8\,(55.9/84.6)}{21.8}(100) = 64.3\%$
(this corresponds to CO_2 balance)

CO Absorber Efficiency = $\dfrac{19.2 - 3.4\,(55.9/84.6)}{19.2}(100) = 87.8\%$
(CO balance)

H_2O Absorber Efficiency = $\dfrac{3.1 - 0.2\,(55.9/84.6)}{3.1}(100) = 95.8\%$

Turn to page 63c.

* B.D. in this case implies no CO_2 or CO, as well as no H_2O.

Basis: 100 # dried laundry

The value of Y is

Y = 59 # H_2O to drain

This is not quite the correct solution yet since the problem asked for the # H_2O/minute. Convert your answer into these units and turn to page 53c.

You are just the guy we want to see! This problem is simple (admittedly) but you must not let this cause you to ignore the units for the terms in the equation. If you think that you can afford to wait until the problems are hard before you write in the units, you will find that it will be too late then.

Form the right habits now! Go back to page 30a and try again.

You have chosen 100 # of water entering as the basis for your calculation.

This is all right. The problem can certainly be based on this material.

The basis of calculation is an arbitrary device that should be chosen to simplify your calculations. The choice of 100 # as an amount to work with is more convenient than the actual amount flowing.

In order to express the percentages of the different materials in the inlet stream you will have to use ratios of percentages since the amounts are specified in terms of percentages of the wet laundry and dried laundry. If you had chosen 100 # of wet laundry as your basis the amounts of water and B. D. Laundry would have been apparent without any calculation.

In order to convert your answer, which will be based on 100 # of water entering, into the units required by the problem (# H_2O/min. to drain), you will need to use the total flow of 4567 # wet laundry/hour. So your choice of basis will require you to make a final conversion to express # wet laundry in terms of # H_2O entering.

However, all of this does work! So let's get started. You are ready to try to express the mathematical relationships between the various streams. The form of these relations is:

Input - Output = Accumulation

Now, however, you must ask yourself, "Input of what?", "Output of what?", "Accumulation of what?" In other words, which <u>material</u> to you wish to <u>balance</u>?

Choose a material to balance and turn to page 38a.

Based on 100 # of water entering you have found that 93.7 # H_2O go to the drain. Now you must convert this into # H_2O to drain/minute.

Since 39.6% of the wet laundry entering each hour is water, you need this rate of water entering to determine the final answer. You should have written something like

$$\frac{93.7\ \#\ H_2O\ \text{to drain}}{100\ \#\ H_2O\ \text{entering}} \cdot \frac{(.396*4567)\#H_2O\ \text{entering}}{1\ \text{hour}} \cdot \frac{1\ \text{hour}}{60\ \text{min.}} = 28.3\ \#H_2O\ \text{to drain/minute}$$

Turn to page ~~66b~~. 55a

You have written an equation not listed on page 28b. Let's examine some possible differences between your work and the equations given on page 28b.

1. Is the difference merely that of a common factor? (If you can multiply every term in your equation by the same constant to produce one of the equations on page 28b, then reduce your equation to the equivalent form on page 28b and go on from there.)
2. Look for an error in signs. (This is a common difficulty. But you must not mix up input and output quantities without proper signs. If this is your difficulty, reread the problem, fix up the signs and go on from page 28b.)
3. Perhaps you didn't understand the term "accumulation." Accumulation is the change in the amount of water in the tank during a specified period. (Turn to page 30a and try again.)
4. Look for different units among the terms in the equation. (Every term in an equation must represent the same kind of thing. Return to page 30a and try again.)

If none of these suggestions help, see your instructor.

Basis: 100 # B. D. Laundry

B. D. Laundry Balance:

Let X be # dried laundry

Input = Output

100 # B.D.L. = (.96)(X) # B.D.L.

and from the previous water balance you have

Y is # H_2O to drain

$(100)(\frac{39.6}{60.4})$ # H_2O = $(Y + .04X)$ # H_2O

Now you have two equations and two unknowns. Note that if you had selected B. D. Laundry as the material to balance first you could have solved each equation directly.

The choice of B. D. Laundry as the first material to balance is recommended because this material is present in only two streams, one entering and one leaving. B. D. Laundry therefore "ties" these streams together. The use of a tie substance usually leads to simpler, more easily solved, mass balances.

Now solve for Y, convert to # H_2O to drain/minute, and turn to page 58a.

Very good. You did not waste time in making elaborate drawings that are not needed. In most cases, a schematic flow diagram is all you need for problem solving. Of course, your schematic flow diagram must be correct. Now turn to page 30c.

Write the corresponding balance equation and turn to the page indicated.

Material to Balance	Turn to Page
Water	33c
Bone Dry Laundry	54a
Hydrogen	62b
Oxygen	41c
Other	50b

Basis: 100 # wet laundry

Water Balance:

Let Y be # H_2O to drain

Input = Output

$$100\ (.396)\ \#\ H_2O = (60.4\ \#\ B.D.L.)\left(\frac{4\ \#\ H_2O}{96\ \#\ B.D.L.}\right) + Y\ \#\ H_2O$$

Therefore $39.6 = 2.50 + Y$

$\therefore\ Y = 37.1$ # H_2O to drain

Now compute the # H_2O/minute removed and compare your answer to that on page 42b.

Turn to page 35a.

Basis: 100 # water entering

Bone Dry Laundry Balance:

Let X be # H_2O in dried laundry

Input = Output

$$100 * \left(\frac{.604}{.396}\right) = X \left(\frac{.96}{.04}\right)$$

and from the previous balance

$$100 = Y + X$$

You have two equations and two unknowns. Note that if you had selected the B. D. Laundry balance first, the water balance may be directly solved (since X is directly solved for first).

The choice of B. D. Laundry as the first material to balance is recommended because this material is present in only two streams, one entering and one leaving. The B. D. Laundry thus "ties" these streams together. The use of a tie substance usually leads to simpler, more easily solved balance equations.

Now solve for Y and turn to page 46c.

Basis: 100 # water entering

B. D. Laundry Balance:

Let X be # dried laundry leaving

Input = Output

$$100 \left(\frac{.604}{.396}\right) \text{ \# B.D.L.} = \left(\frac{.96}{1.00}\right) X \text{ \# B. D. Laundry}$$

Total Balance (done just before this):

$$100 \left(\frac{1.00}{.396}\right) \text{ \# total mass in} = Y + X \text{ \# total mass out}$$

It should occur to you that the B. D. Laundry balance should have been done first, since it can be solved directly. Then it would be possible to solve directly for Y, knowing X.

The B. D. Laundry for the first balance is recommended because it appears in only two streams, one entering and one leaving. Thus, it may be used to "tie" these streams to each other. Such a material, present in only two streams, is called a "tie substance." The use of tie substances usually results in simpler, more directly solved balances.

Now solve for Y, convert the answer into # H_2O to drain/minute and turn to page ~~59b~~. 46c

Basis: 100 # B. D. Laundry entering

Water Balance:

Let Y be # H_2O to drain

X be # B. D. Laundry leaving

Input = Output

$$100 * \left(\frac{39.6}{60.4}\right) \# H_2O = \left(Y + \frac{.04}{.96} * X\right) \# H_2O$$

but X is known from the previous balance (X = 100). Thus

Y = 61.3 # H_2O to drain.

This is based on 100 # B. D. Laundry entering. You must now express Y in the units of # H_2O to drain/minute. Convert your answer to these terms and turn to page 58a.

You want to try something else as the material to balance. We hope you didn't come here in desperation! Seriously, what else would you like to try? There are certainly some other possible choices. We admit it!

But you might consider whether their use will justify your extra work. Don't take our word for it. Try some of them out if you are still not convinced.

If you are convinced, choose another material to balance and return to the page you came from.

Basis: 100 # dried laundry

Water Balance:

Let Y be # H_2O to drain

X be # wet laundry

Input = Output

$$.396\ X \# H_2O = (Y + .04 * 100) \# H_2O$$

but you already know X (from the previous balance)

$$.396 * 159 = Y + .04 * 100$$

Solve for Y and turn to page 45b.

X = 152.5 # B. D. Laundry leaving (based on 100 # H_2O entering). Now write the appropriate balance equation and turn to the page indicated.

Material to Balance	Turn to Page
Water	53b
Hydrogen	41c
Oxygen	62b
Total Mass	39c
Other	50b

Basis: 100 # water entering

Y = 93.7 # H_2O to drain

based on 100 # water entering. Now you must find the number of pounds of water entering per minute in order to convert to the final desired answer.

Since 39.6% of the 4567 # wet laundry entering each hour is water, you should be able to calculate the # H_2O to drain by

$$\frac{93.7\ \#\ H_2O\ \text{to drain}}{100\ \#\ \text{water entering}} \left| \frac{(4567*.396)\#\ \text{water entering}}{1\ \text{hour}} \right| \frac{1\ \text{hour}}{60\ \text{min.}} = 28.3\ \#H_2O\ \text{to drain/min.}$$

You should be aware of the fact that you did not need to solve simultaneous equations to answer this problem. If you had selected B. D. Laundry as the first material to balance and either water or total mass as the second balance, the solution is direct.

Return to page 38a and rework the problem using the suggestions given above.

DO NOT TRY TO SKIP THIS EFFORT!

There is important material discussed in obtaining the solution along the better path that you need to know. So dig in and find out about it.

Basis: 100 # B. D. Laundry

Let X be # dried laundry

Y be # H_2O to drain

Total Mass Balance:

Input - Output = Accumulation

$(100)(\frac{100}{60.4})$ # total mass in - (Y + X) # total mass out = 0

Note that since the amount of material in the dryer at any time is constant, the accumulation term is zero. Such a process is called a "steady flow" process and may be described by

Input = Output

Your choice of total mass as the first balance does not allow you to solve directly for either X or Y. A more direct route does exist which you will be shown later. However, your choice is certainly feasible. You now have one equation and two unknowns. In order to get another equation, pick another material to balance and turn to page 37c.

-52b-

Good for you!

You have chosen 100 # of wet laundry as your basis. This is an excellent choice for the following reasons:

1. The basis of calculation is arbitrary. Your choice of 100 # wet laundry as opposed to 4567 # wet laundry, will simplify the subsequent calculations.
2. By choosing 100 # of wet laundry you have made more of the given information immediately applicable. (i.e., # H_2O and # B. D. Laundry entering are obvious from the corresponding given percentages. Note that no such advantage would be gained had you chosen 100 # H_2O leaving by the drain.)

You are now ready to formulate the mathematical equation which relates the masses in the different streams. Once again these relations are of the form

Input - Output = Accumulation

In problems such as this you must immediately ask yourself "Input of what?", "Output of what?", "Accumulation of what?" In other words, you must decide what material you wish to balance.

Choose a material to balance and turn to page 25b.

Write the appropriate balance equation for the material you selected and turn to the page indicated.

Material to Balance	Turn to Page
Oxygen	62b
B. D. Laundry	49b
Hydrogen	41c
Water	60a
Other	50b

Basis: 100 # water entering

Water Balance:

Let X be # B. D. Laundry leaving

Y be # H_2O to drain

Input = Output

$$100 = Y + \frac{.04}{.96} X$$

but X is already known from the previous balance (X = 152.5 # B. D. Laundry leaving) so you may solve directly for Y.

Solve for Y and convert your answer to # H_2O to drain/minute.

Then turn to page 46c.

Based on 100 # dried laundry, you calculated that X = 159 # of wet laundry. However, the problem required 4567 #/hour of wet laundry to be processed. Therefore you must multiply your answer by the ratio of $\frac{4567}{159}$ in order to solve for the actual # H_2O to drain per hour. Of course, this must be divided by 60 min/hr. since you were asked to solve for # H_2O to drain/minute.

$$\left(\frac{59 \text{ \# } H_2O \text{ to drain}}{100 \text{ \# dried laundry}}\right)\left(\frac{100 \text{ \# dried laundry}}{159 \text{ \# wet laundry}}\right)\left(\frac{4567 \text{ \# wet laundry}}{\text{hour}}\right)\left(\frac{\text{hour}}{60 \text{ min.}}\right) =$$

Answer = 28.3 # H_2O to drain/minute.

Note that if you had chosen 100 # wet laundry as your basis, your answer would have been in the units of

$$\frac{\text{\# } H_2O \text{ to drain}}{100 \text{ \# wet laundry}}$$

and the second conversion factor in the above calculation $\left(\frac{\text{\# dried laundry}}{\text{\# wet laundry}}\right)$ would not be necessary.

Turn to page 55a.

Basis: 100 # dried laundry

B. D. Laundry Balance:

Let X be # wet laundry

Input = Output

X(.604) # B.D.L. = (100)(.96) # B.D.L.

Total Balance (from previous page):

(Y is # H_2O to drain)

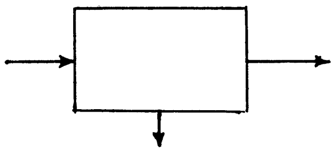
X # total stream in = (Y + 100) # total stream out

It seems obvious now that you should have taken the B. D. Laundry balance first since you could then solve directly for X. Knowing X, the total mass balance may be solved directly for Y.

The choice of B.D.L. for the first balance is recommended because it is contained in only two streams, one entering and one leaving. B.D.L. may therefore be used to "tie" these two streams together. Such a material, present in only two streams, is called a tie substance. The use of tie substances usually leads to simple, easily solved, mass balances such as the one above.

Now solve for Y, which is the # H_2O to drain based on 100 # dried laundry, and compare your answer to that on page 32a.

4567 # wet laundry/hour
39.6% H_2O
60.4% Bone Dry* Laundry



Dried laundry
4% H_2O
96% B. D. Laundry

H_2O to drain

You have learned from problem 1 that a more elaborate flow diagram is not necessary.

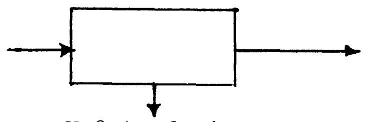
Check your diagram with the one given above. Be sure that you have indicated both the numerical quantities and the units in your labels.

Now, select a basis for your calculation and write it on your worksheet.

Turn to page 57b.

* By "Bone Dry" we mean completely dry (i.e., 0% H_2O). This term is frequently abbreviated B.D.

-55a-



Let's look more closely at the problem you just solved. There are two components entering the dryer, H_2O and B. D. Laundry. Therefore, we can take three material balances:

a. H_2O
b. B. D. Laundry
c. Total Mass

In your solution of the problem you must have chosen two of these equations. Let's write down all three equations based on 100 # wet laundry entering.

Let X = # dried laundry

Y = # H_2O to drain

Input = Output

a.	H_2O Balance	$(100)(.396) = (.04)X + Y$
b.	B. D. L. Balance	$(100)(.604) = (.96)X$
c.	Total Mass Balance	$100 = X + Y$

Note that the total mass balance is the sum of the H_2O balance and the B.D.L. balance. In other words, only two of the three balances are independent and there is nothing to be gained by writing the third balance after you have written the other two. In later, more complicated, problems you may be tempted to write equations which are not independent. You must be aware of this trap and avoid it.

Turn to page 43b to compare your solution to a sample solution of problem 3.

-55b-

Basis: 100 # dried laundry

Y = 59 # H_2O to drain

In order to convert this answer to # H_2O/minute to drain, you must make use of the given wet laundry feed rate, 4567 # wet laundry/hour. You must therefore calculate the # wet laundry (based on 100 # dried laundry) and then apply the ratio of $\frac{\text{actual \# wet laundry per hour}}{\text{\#wet laundry based on 100\# dried laundry}}$.

Therefore you must go back and solve your simultaneous equations for X (X = # wet laundry based on 100 # dried laundry). Note that, if you had taken 100 # wet laundry as your basis, you could simply have multiplied your answer by $\frac{\text{4567 \# wet laundry per hour}}{\text{100 \# wet laundry (basis)}}$ and therefore would not have had to solve for X.

After you solve for X, convert Y to #H_2O to drain/minute and compare your answer to page 36a.

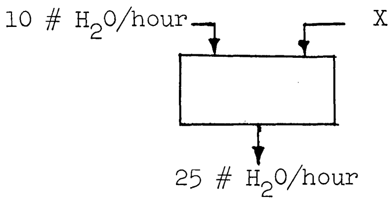
-55c-

Problem 3:

4567 #/hour of wet laundry, 39.6% H_2O by weight, is fed into a dryer. If the dried laundry contains 4% H_2O by weight, determine the number of pounds of H_2O removed from the laundry per minute.

Draw, on your worksheet, a labeled flow diagram representing the situation described above. When you have completed this drawing, turn to page 54b.

Solution to Problem 1:



Time	Water in Tank
12:00	500 # H_2O
2:00	600 # H_2O

Basis: 1 hour

Let X be # H_2O

Input - Output = Accumulation

Water Balance:

$$(10 + X) \# H_2O - 25 \# H_2O = \frac{600-500}{2} \# H_2O$$

$$10 + X - 25 = 50$$

$$X = 65 \# H_2O$$

Answer: 65 # H_2O/hour

Basis: 2 hours

Let X be # H_2O

Input - Output = Accumulation

Water Balance:

$$(20 + X) \# H_2O - 50 \# H_2O = 600\text{-}500 \# H_2O$$

$$20 + X - 50 = 100$$

$$X = 130 \# H_2O \quad (\text{ in 2 hours})$$

Answer: 130/2 # H_2O/2 hours = 65 # H_2O/hour

Turn to page 39a.

Write the appropriate balance equation and turn to the page indicated.

Material to Balance	Turn to Page
Hydrogen	41c
B. D. Laundry	49a
Oxygen	62b
Total Mass	63a
Other	50b

Basis: 100 # B. D. Laundry

Water Balance:

Let X be # dried laundry

Y be # H_2O to drain

Input = Output

$$(100)(\frac{.396}{.604})\ \#\ H_2O = Y\ \#\ H_2O + .04\ X\ \#\ H_2O$$

Total Balance (from previous page):

$$(100)(\frac{100}{60.4})\ \#\ \text{total mass in} = Y\ \#\ \text{total mass} + X\ \#\ \text{total mass}$$

You have two equations and two unknowns. Solve for Y, which is # H_2O to drain based on 100 # wet laundry. Convert Y to # H_2O to drain per minute and compare your solution to that on page 34b.

Find the <u>basis</u> you have chosen and turn to the corresponding page.

Basis	Turn to Page
100 # dried laundry	31a
1 hour	35a
100 # Bone Dry Laundry (entering or leaving)	38b
4567 # wet laundry	37a
100 # water entering	46b
1 minute	48d
100 # wet laundry	52b
Other	58c

Basis: 100 # wet laundry

Water Balance:

Let X be # dried laundry

Y be # water to drain

Input = Output

$$(100)(.396)\ \#\ H_2O = Y\ \#\ H_2O + (.04)(X)\ \#\ H_2O$$

Total Balance (from previous page):

$$100\ \#\ \text{total mass in} = (Y + X)\ \#\ \text{total mass out}$$

You now have two equations and two unknowns. Solve for Y, which is the # H_2O to drain, based on 100 # wet laundry. Convert Y to # water to drain per minute and compare your answer to that on page 41a.

Based on 100 # B. D. Laundry you have found that 61.3 # H_2O go to the drain. Now you must convert this into # H_2O to drain/minute. Since the problem specified that 4567 # wet laundry enter each hour, you need the fraction of this that is Bone Dry Laundry. This is clearly 100 - 39.6 = 60.4%, so finally you should have written:

$$\frac{61.3 \text{ \# } H_2O \text{ to drain}}{100 \text{ \# B.D.L. entering}} \left| \frac{(.604 * 4567) \text{ \# B.D.L. entering}}{1 \text{ hour}} \right| \frac{10 \text{ hr.}}{60 \text{ min.}} = 28.3 \text{ \# } H_2O \text{ to drain/min.}$$

Turn to page 55a.

Write the appropriate balance equations and turn to the page indicated.

Material to Balance	Turn to Page
Water	57c
Hydrogen	41c
Oxygen	62b
Bone Dry Laundry	29b
Other	50b

If you have chosen one of the materials mentioned in the table but have selected a different amount, your basis will work. However, in order to simplify the calculations, it is recommended that you choose a basis such as 1, 10, or 100 # of the material.

Throughout this book, wherever a quantity of material is chosen as a basis to simplify numerical calculations, the book's choice will be 100 units. Certainly any other multiple of 10 will work just as well. You must agree that using, for instance, 62 # as a basis, is no better than using the 4567 # that is mentioned in the problem.

If you have made some other choice of material or time, it is doubtful that you understand the problem. Reread the problem and pick a new basis in the light of the preceding discussion.

Then turn to page 57b.

Basis: 100 # H_2O entering

B. D. Laundry Balance:

Let X be # B. D. Laundry leaving

Input - Output = Accumulation

$$(100)(\frac{.604}{.396}) - X = 0$$

Since the amount of material in the dryer at any time is constant, the accumulation is zero. Such a process is called a "steady flow" process. For steady flow processes:

Input = Output

Now you can solve directly for X. Solve for X.

Since B. D. Laundry appears in only two streams, one entering and one leaving, this material "ties" these streams together. The use of <u>tie substances</u> usually leads to simple, easily solved balance equations (as the one above).

Now you must choose some other material to balance in order to solve the problem, namely the rate of water going to the drain. Choose another material to balance and turn to page 51a.

Basis: 100 # wet laundry

Bone Dry Laundry Balance:

Let X be # dried laundry leaving

Input = Output

$$(100 * .604) \# \text{B.D.L.} = (.96)(X) \# \text{B.D.L.}$$

and from the previous balance (water) you have (Y is # H_2O to drain)

$$(100 * .396) \# H_2O - (Y + .04X) \# H_2O = 0$$

and you have two equations and two unknowns. But it should be obvious that you should have taken the B. D. Laundry balance first and solved directly for X. Knowing X, the water balance would have given Y directly.

The choice of B. D. Laundry for the first balance is recommended because it is contained in only two streams, one entering and one leaving. B. D. Laundry may therefore be used to "tie" these streams together. Such a material (if present in only two streams), is called a <u>tie substance</u>. The use of a <u>tie substance</u> will usually lead to simple, easily solved, mass balances.

Now solve for Y (the # H_2O to drain) based on 100 # wet laundry. Convert Y to # H_2O to drain per minute and compare your result with the answer on page 42b.

Basis: 100 # water entering

Water Balance:

Let Y = # H_2O to drain

X = # dried laundry leaving

Input = Output

100 = Y + .04X

and from the previous total mass balance

$$100 \left(\frac{1.00}{.396}\right) = Y + X$$

You now have two equations and two unknowns. Solve these equations for Y and turn to page ~~58c~~.
51 b

Now that you have chosen the material you wish to balance, write down the appropriate material balance equation. Indicate both the numerical quantities and units of the terms in your equation. Define all unknown quantities symbolically. Turn to the page indicated in the following table.

Material to Balance First	Turn to Page
Water	36b
Bone Dry Laundry	37b
Hydrogen	41c
Total Mass	24b
Oxygen	62b
Other	50b

Write the corresponding material balance equation and turn to the page indicated.

Material to Balance	Turn to Page
B. D. Laundry	26b
Total Mass	61b
Oxygen	41c
Hydrogen	62b
Other	50b

Basis: 100 # dried laundry

Total Mass Balance:

Let X be # wet laundry entering

Y be # H_2O to drain

Input = Output

X # total mass in = (Y + 100) # total mass out

and from the previous (water) balance

.396 X # H_2O = Y # H_2O + .04 * 100 # H_2O

and now you have two equations and two unknowns. Now solve these equations for Y and turn to page 42c.

You made an artistic pictorial drawing for problem 1. This certainly represents the problem beautifully.

But wait a minute!.....Our objective here is to solve problem 1 and you have already spent a great deal of time in just drawing the problem situation.

You should realize that you must match your efforts to the problem you are trying to solve. A very simple problem should not require a very elaborate sketch. The time spent should be justified by the progress you make in solving the problem.

You should never draw a more elaborate picture than you need to solve the problem. In most cases a schematic flow diagram is entirely sufficient.

Now make a schematic diagram for problem 1 and, when you have finished, turn to page 30c.

Very Good Choice!

Basis: 100 # wet laundry

Bone Dry Laundry Balance:

Let X be # B. D. Laundry leaving

Input - Output = Accumulation

(100)(.604) # B.D.L. - X # B.D.L. = 0

X = 60.4 # B.D.L.

Note that since the amount of material in the dryer at any time is constant, the accumulation term is zero. Such a process is called a "steady flow process." For a steady flow process, then,

Input = Output

Your choice of B. D. Laundry for the first balance was good because B. D. Laundry is contained in only two streams, one entering and one leaving. B. D. Laundry may therefore be used to "tie" these streams together. Such a material, which is present in only two streams is called a <u>tie substance</u>. The use of <u>tie substances</u> will usually lead to simple, easily solved mass balances, such as the one above.

You still have not solved for the amount of H_2O removed. You need to take another choice.

Choose a second material to balance and turn to page 24c.

-62b-

This choice of material to balance is poor for this problem. You picked an element as the material to balance. In a process such as this, not involving chemical reactions, there is no advantage in balancing the amount of an element rather than a compound or a mixture in which the ratio of the elements is constant. However, in problems involving chemical reactions, which we will consider later, balancing an element is often advantageous.

Note also that the bone dry laundry may contain some hydrogen or oxygen in its chemical makeup. The % of these elements in the B. D. Laundry is, of course, not given.

Return to the page on which you selected the element to balance, and make another choice.

Basis: 100 # water entering

Total Mass Balance:

Let X be # H_2O leaving in dried laundry

Y be # H_2O to drain

Input = Output

$$(100)(\frac{100}{39.6}) = (Y + (\frac{100}{4})\ X)$$

and from the previous balance

$$100 = Y + X$$

You now have two equations and two unknowns. Solve these equations for Y and turn to page 51b.

Write the material balance equation for the material you chose and turn to the page indicated below.

Material to Balance	Turn to Page
Bone Dry Laundry	47b
Hydrogen	41c
Oxygen	62b
Total Mass	27c
Other	50b

Please write the present time on your worksheet.

This is the end of the present text. We hope that you have all "made it" here.

While the problems have been rather easy thus far, we hope that while working with this text as a kind of tutor, you have been helped to discover some ways of solving problems by yourself.

In order to help us extend this work further and to make the present text more useful, please write your suggestions and criticisms on your worksheet and then turn your worksheets and this text in to your instructor.

Thank you for your help.

Write the appropriate balance equation and turn to the page indicated.

Material to Balance	Turn to Page
Water	50a
Total Mass	27a
Hydrogen	41c
Oxygen	62b
Other	50b

APPENDIX C

Example Problems Illustrating the Use of a Scrambled Text in Solving Electrical Engineering Problems

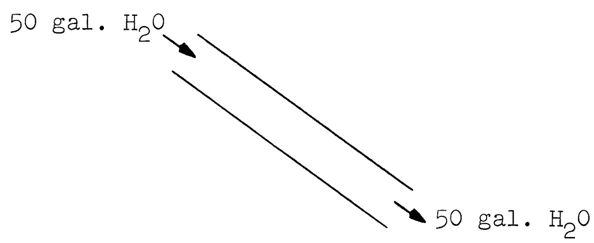
Lesson No. 1 - Conservation Principles

In the solution of problems there are many procedures which are based on conservation principles. One of the most important of these is the "law of conservation of mass." This principle states that mass cannot be created or destroyed, but only transformed.

Although the law of conservation of mass does not hold in certain atomic energy situations, it is very useful in many engineering and science fields.

Let us apply this principle to the flow of substances through pipes.

For example, suppose that 50 gallons of water are poured into the top end of a water pipe as shown below:



Since water cannot be generated or destroyed inside the pipe, we can expect the same water, 50 gallons of it, to come out at the bottom.

Of course, in practice some water will accumulate or stick to the walls of the pipe so that the water emerging at the bottom will not be exactly 50 gallons. In this case we can write:

(Water into the pipe) - (Water out of the pipe) = (Water accumulated)

In terms of what we have said, we would like you to answer the following question: If, instead of 50 gallons of water, we pour 100 gallons/hour into the top of the pipe, how many gallons will come out at the bottom in two hours? Assume that there is no accumulation of water inside the pipe.

Answer:

a. 100 gal./hr. Turn to page 69a.

b. 200 gallons Turn to page 68a.

Your answer is:

200 gal. (oil) = 50 gal. (oil) + 25 gal. (oil) + Y gal. (oil)

3000 gal. (gas) =150 gal. (gas) + X gal. (gas)

This is correct! You probably realized that the law of conservation of mass applies equally for each separate substance because oil cannot be changed into gasoline or vice versa in this system. Therefore, we can write a separate input-output equation for each component entering the boundaries of the system. Each equation uses consistent units throughout so that you are not adding gallons of oil to gallons of gasoline. Each equation has a single unknown so that now you should be able to solve for the numerical values of X and Y.

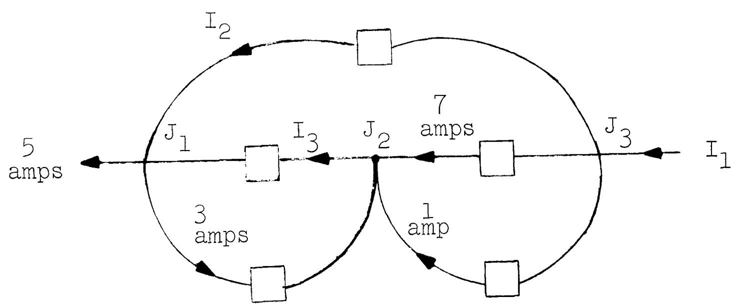
Solve for X and Y and turn to page 82d to check your results.

Your answer is: 40 gallons

This is almost correct, except that the units are wrong. Notice that the problem was concerned with rates of flow in gallons/hour rather than with the actual quantity in gallons.

Go back to page 68a and pick another answer.

You should now be ready to solve a multiple junction problem which involves more than one unknown. One such problem is given in the figure below. Copy this figure so that you will not have to come back to this page to work with it.



In this figure the boxes represent electrical appliances - heaters, lamps, etc. The value of the current is the same on both sides of the box.

Draw a system boundary so that I_1 is the only unknown current which cuts the boundary. Then solve for I_1.

Go to page 81b to check your results.

Your answer is I_2 = -3 amperes

This is correct! Apparently you were not confused by the fact that the answer came out negative! A negative answer means that the direction of I_2 is opposite to that shown in the diagram. It is recommended, however, that the diagram be left as drawn rather than changing the direction of I_2. The negative sign of the answer is all that is needed to establish the true direction.

Now draw a system boundary so that I_3 is the only unknown current across it. Then solve for I_3. Check your result by considering some other boundary.

Answer:

a.	I cannot find a boundary as indicated above.	Go to page 77c.
b.	I_3 = 5 amps	Go to page 82a.
c.	I_3 = 11 amps	Go to page 83b.
d.	I_3 = -5 amps	Go to page 68c.

Your answer is 10 gal./hr.

It seems that you subtracted 15 gal./hr. from 25 gal./hr. to obtain your answer, or else you were trying to guess.

Notice that what comes out at the bottom has to be put in at the top. The total rate of flow at the bottom is certainly more than 10 gal./hr. since the flow in each one of the lower pipes exceeds 10 gal./hr.

Go back to page 68a and try again.

Your answer is 200 gallons of water per hour.

You are violating the law of conservation of mass. You are putting oil into the system at the left and recovering water at the right.

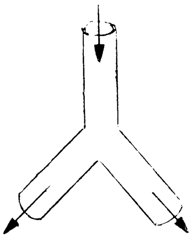
Go back to page 74c and try again.

Your answer is 200 gallons.

This is correct. Your reasoning probably went as follows: If I pour 100 gallons each hour (100 gal./hr.) the rate of flow at the bottom is also 100 gal./hr. Now, if I change the rate of flow (gal./hr.) to actual flow by multiplying the rate times the time, 2 hours, I obtain:

$$100 \frac{\text{gal.}}{\cancel{\text{hr}}.} \cdot 2\ \cancel{\text{hr}}. = 200 \text{ gal.}$$

Now suppose that you are given the system in the figure below.



Can you find the rate of flow in the pipe at the top?

Answer:

a. 10 gal./hr.	Go to page 67b.
b. 40 gal./hr.	Go to page 74c.
c. 10 gal.	Go to page 71b.
d. 40 gal.	Go to page 66b.

Your answer is 3 amperes.

You probably made an error in sign. The "current balance equation" about the proposed boundary is

$$I_1 + 9 = 5 + 1$$

Go back to page 81b and choose the correct answer.

Your answer is $I_3 = -5$ amps.

You must have made an arithmetic error. Check your results and pick the right answer from page 67a.

Your answer is 700.

Read the comment on page 77d.

Your answer is 100 gallons/hour.

Your answer is given as a rate of flow rather than as the actual flow in gallons. You will notice that we asked for the actual number of gallons emerging at the bottom in one hour.

Go back to page 65 and answer the question again.

-69b-

Your answer is 200 gallons of oil.

Your units are wrong. You cannot add apples to apples and get pears. You added gallons/hour to gallons/hour and gave your answer in gallons. This is a very common mistake when you are a beginner at working problems. Units are very important in engineering, so you must be very careful in handling them.

Go back to page 74c and try again.

-69c-

Your answer is:

$$200 \text{ gal.(oil)} + 300 \text{ gal.(gas)} = 150 \text{ gal.(gas)} + 50 \text{ gal.(oil)} + X \text{ gal.(gas)} + Y \text{ gal.(oil)} + 25 \text{ gal.(oil)}$$

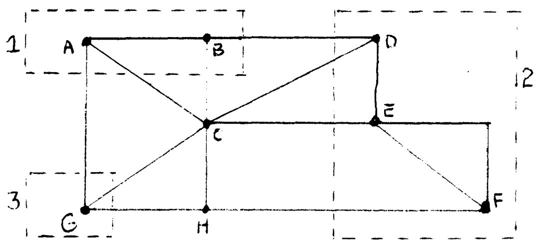
If you remove the words gas and oil from this equation your answer will be correct. In such a case you are adding gallons to gallons so that the equation would express the conservation of gallons. However, you should not add gallons of oil to gallons of gasoline unless you are purposely generating a mixture of the two substances. Go back to page 73b.

Let us generalize the law of conservation of mass in physical systems.

We all have an intuitive notion of what a "system" is. A useful definition of a system is:

"A system is a collection of components (parts or things) which perform a function which cannot be performed by any of the parts by themselves."

In practice systems are finite and have a boundary. Every part of the system is within the system boundary. The system boundary is usually indicated in a diagram by a dotted line forming a closed loop enclosing the elements of the system. For instance



The system consisting of junctions A and B is enclosed by system boundary number 1. Similarly the system of junctions D E F is enclosed by boundary number 2. Junction G is in system boundary number 3.

We may now state the law of conservation of mass within a system:

"The sum of all the masses entering a system less the sum of all the masses leaving the system is equal to the mass accumulated within the system boundary."

In equation form we may write:

Input - Output = Accumulation

In all of the problems which we have solved up to now, we assumed that there was no accumulation of mass within the system. The equation which we applied was

Input - Output = 0

or Input = Output

Turn to page 73b.

-71a-

Your answer is:

$$(200 + 3000) \text{ gal.} = (150 + 50 + X + Y + 25) \text{ gal.}$$

This answer is correct because it expresses the law of conservation of total mass. The equation expresses the <u>total mass balance</u>. However, this turned out to be an equation with two unknowns, X and Y. It is not a good equation for finding the values of X or Y. It is perfectly good for finding the value of (X + Y).

$$(X + Y) = 200 + 3000 - 150 - 50 - 25 = 2975 \text{ gal.}$$

Go back to page 73b and pick an answer which would permit solving for X gal.(gas) and Y gal.(oil) separately.

-71b-

Your answer is 10 gallons.

It seems that you subtracted 15 gal./hr. from 25 gal./hr. to obtain your answer. Then you were careless with the units and gave your answer in gallons rather than gal./hr.

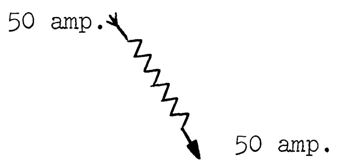
Notice that what comes out at the bottom has to be put in at the top. The total rate of flow at the top must exceed the individual rates of flow in each pipe at the bottom.

Go back to page 68a and try again.

-71c-

Your answer is: I am confused with this diagram.

You are probably confused with the symbol for a resistance ( ). This symbol may represent any device which dissipates electrical energy when a current passes through it. However, the current itself does not change in value when it goes through the resistance. Only the energy carried by the current changes in value. In fact, even a good conductor has a certain amount of resistance. The current behavior of a resistance is

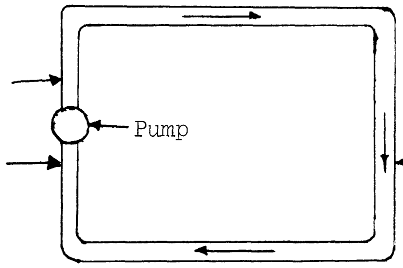


This can be explained by the law of conservation of mass, since the particles (electrons) entering the resistance must leave the resistance if mass is not created inside the resistance.

Go to page 77b and try to work the problem.

Lesson 2 - The Principle of Conservation of Mass in Electrical Systems

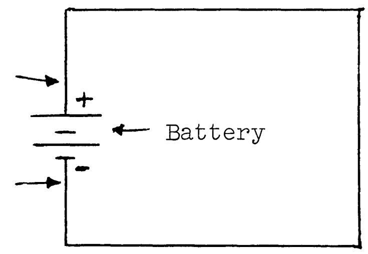
Consider the hydraulic system of the figure below. This system consists of a water pump and a water pipe. Let us assume that this system is in the horizontal plane, that is, there are no differences in elevation between any parts of the system. In this case water cannot flow unless the pump is operating and creating a pressure inside the pipe. Water flows from points of high pressure to points of low pressure. The pump must create differences in pressure in order to transport water from one point to another.



There is a very close analogy between this hydraulic system and an electrical system. In place of the water you may think of the electrons as flowing from one point to another. In place of the water pump you may think of an electrical pump called a battery or a generator. (We are all familiar with the batteries and generators in the motors of automobiles.)

Water flows from points of high pressure to points of low pressure.

Electricity flows from points of high electrical pressure or "electric potential" to points of low electric potential.



A number of molecules of water which occupy a certain volume is called a gallon.

A number of electrons which may be thought to occupy a certain volume is called a coulomb. A coulomb is equivalent to more than 624 million electrons.

The rate at which water flows may be expressed in gallons/min., gallons/hr., etc.

(Continued on page 73a.)

-73a-

The rate at which electricity flows may be expressed in units of coulomb/min, coulomb/hr, etc.

In actual practice, the most common unit for measuring the rate of flow of electricity is the coulomb/sec or ampere. The name ampere is just a convenient name for the coulomb/sec.

The rate of flow of electricity is usually called electric current.

Answer the following question: If 240 coulombs of charge pass through point P of a conductor every minute, what is the value of the current at point P?

Answer:

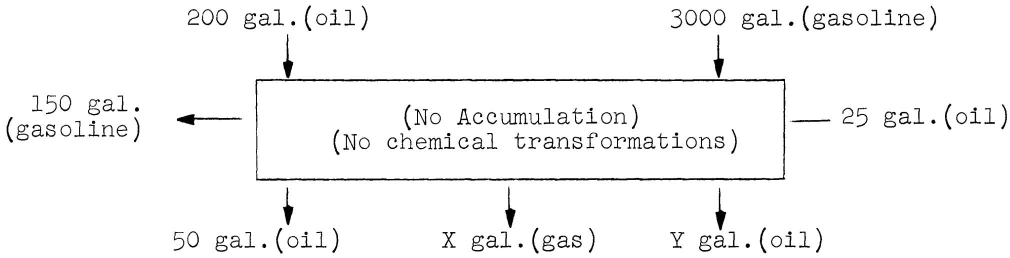
a. 240 amperes Go to page 77a.

b. 240 coulombs/min. Go to page 80a.

c. 4 amperes Go to page 78b.

-73b-

How would you express the conservation principle for the system below:



Answer:

a. 200 gal.(oil) + 3000 gal.(gas) = 150 gal.(gas) + 50 gal.(oil) + X gal.(gas) + Y gal.(oil) + 25 gal.(oil)

Go to page 69c.

b. (200 + 300) gal. = (150 + 50 + X + Y + 25) gal.

Go to page 71a.

c. $\begin{cases} \text{200 gal.(oil) = 50 gal.(oil) + 25 gal.(oil) + Y gal.(oil)} \\ \text{3000 gal.(gas) =150 gal.(gas) + X gal.(gas)} \end{cases}$

Go to page 66a.

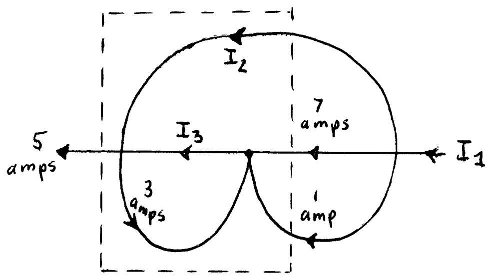
Your answer is I = 13 amperes.

It seems that you added the two known currents to obtain the unknown current. This cannot be done because the two known currents are not in the same direction when they cross the system boundary.

Go back to page 77b and try again.

Your answer is: I cannot find a boundary which is cut by I_2 as the only unknown current.

A boundary which will cut I_2 as the only unknown current is shown in the linear flow graph below.

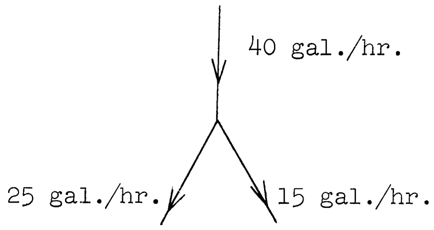


Solve for I_2 and pick an answer from page 81b.

Your answer is 40 gal./hr.

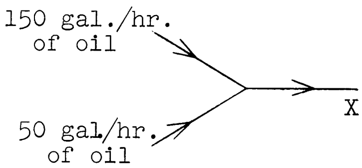
This is correct! The sum of the rates of flow at the bottom is equal to the rate of flow at the top.

Now let us try to develop a diagramming scheme which will help us in solving problems. For example, we may indicate the water pipe system in the previous problem as follows:



This is a "line diagram", a "linear flow graph" or a "flow diagram" in which the direction of flow is indicated by the arrows.

You should be able to work the water flow problem whose linear flow graph is given below.



Answer: a. X = 200 gal./hr. of water — Go to page 67c.
b. X = 200 gal. of oil — Go to page 69b.
c. X = 200 gal./hr. of oil — Go to page 75c.

-75a-

Your answer is 5 amperes.

This is in error. You probably used the equation

$$5 + 3 = 1 + I + 2$$

which disregards the directions in which the currents enter the system. Notice that the 5 ampere current enters the junction but the 3 ampere current leaves the junction.

Go back to page 79 and choose another answer.

-75b-

Your answer is: Cannot be solved with the information given.

If we had asked you to solve for all the currents flowing in the system, your answer would have been correct. However, we are only asking you for one of the currents entering the "system." By properly defining what the system is, it is possible to solve the problem.

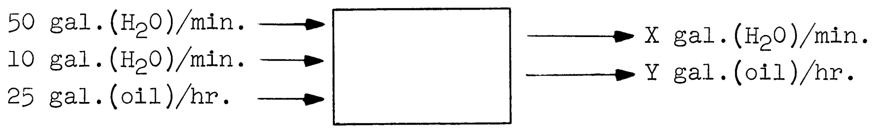
Go back to page 77b and find a system whose boundaries cut the two given currents (10a. and 3a.) and the unknown current (I) and no other current!

-75c-

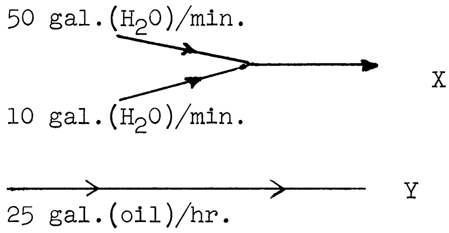
Your answer is X = 200 gal./hr. of oil.

This is the correct answer. Now you should be able to work with systems involving the flow of more than one kind of substance.

For example, suppose that we have a system as shown below:



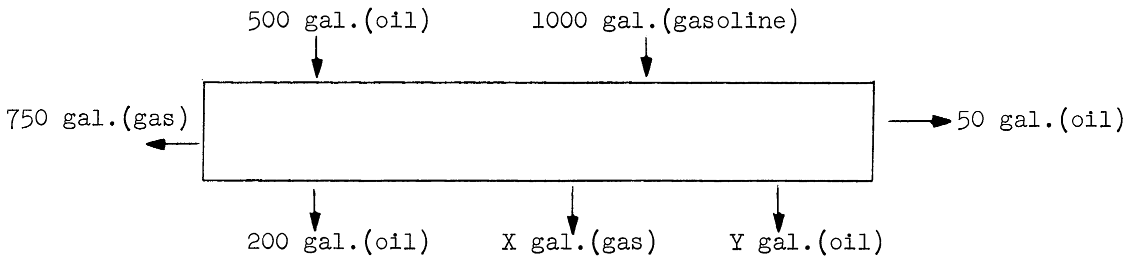
Assuming that the water and the oil never come in contact and that there is no accumulation of either substance inside the box, the linear flow graph for the system is:



$\therefore$ X = 60 gal.(H_2O)/min.

Y = 25 gal.(oil)/hr.

Now solve the system below. Go to page 82c to check the results.

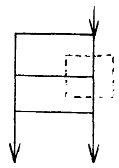


<u>Answer to the problem on page 82b</u>:

This problem is solved in five steps shown below. The water balance equation and the system boundary for which it was written are shown.

<u>Step 1</u>. $F_2 = 400 + 1350$

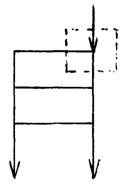
$= \underline{1750 \ \# \ H_2O/hr.}$



<u>Step 2</u>. $300 = F_1 + F_2$

$= F_1 + 1750$

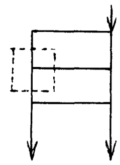
$F_1 = \underline{1250 \ \# \ H_2O/hr.}$



<u>Step 3</u>. $400 + F_1 = F_3$

$400 + 1250 = F_3$

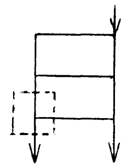
$F_3 = \underline{1650 \ \# \ H_2O/hr.}$



<u>Step 4</u>. $F_3 = F_4 + 1800$

$1650 = F_4 + 1800$

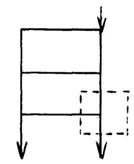
$F_4 = \underline{-150 \ \# \ H_2O/hr.}$



<u>Step 5</u>. $F_4 + 1350 = F_5$

$-150 + 1350 = F_5$

$F_5 = \underline{1200 \ \# \ H_2O/hr}.$

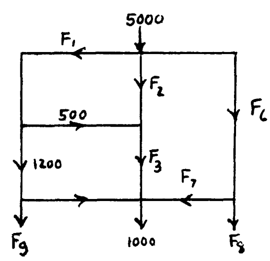


NOTE: By choosing the corrct system boundaries, every unknown stream may be solved for directly with just one balance. Try to do this as an exercise.

Go to page 81a.

Your answer is 1700.

This is the correct answer! You must have chosen the proper junction and solved for F_1. You should also have noticed that this was the only junction with a single unknown flow. Now that F_1 is known, find another boundary which is cut by only one unknown. Then solve for the unknown.



<u>Answer</u>:

a. I cannot find such a boundary. Go to page 80c.

b. 3300 Go to page 84a.

c. 300 Go to page 83c.

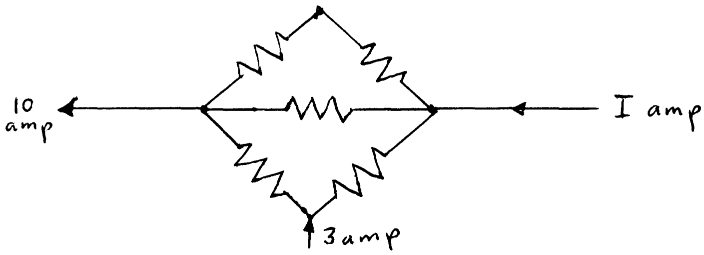
Your answer is 240 amperes.

This would be correct if the coulomb and the ampere were identical. Remember, however, that the ampere is the coulomb/second.

Go back to page 73a and try again.

Your answer is 1 ampere.

This is the right answer! Now solve the following problem in which you should be very careful when defining the system boundaries.



Answer:

a. I = ? I am confused with this diagram. Go to page 71c.

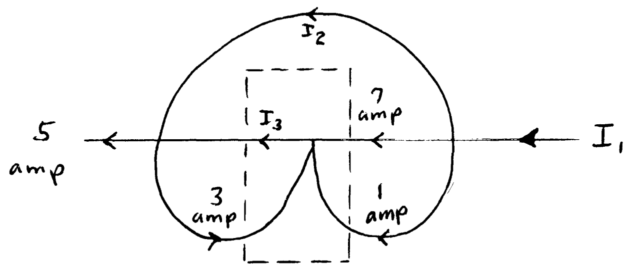
b. I = 7 amp Go to page 78a.

c. I = 13 amp Go to page 74a.

d. Cannot be solved with information given. Go to page 75b.

Your answer is: I cannot find a boundary as indicated above.

A boundary which is cut by I_3 as the only unknown current is shown in the linear flow graph below.



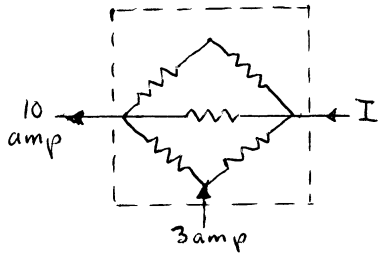
Find the value of I_3 by using this boundary, then go back to page 81b.

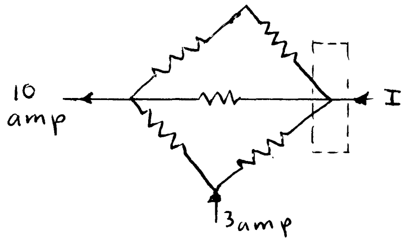
Your answer is -700.

Comment: You probably have an error in the direction of one of the flows. Check your error and return to the bottom of page 81a for another answer.

Your answer is: I = 7 amperes

Very good! You found the right answer. You followed the hint about the importance of defining the boundaries of the system. In this case you should define the system in such a way that only one of the currents which cuts the system boundary is an unknown (below, left).





If the boundary is defined as shown above (right), it would cut other conductors, and I cannot be solved for directly. With the boundaries defined as shown in the figure on the left, you probably wrote the following equation:

$$I + 3 = 10$$

and then you solved for I.

Go to page 66c.

Your answer is 4 amperes.

This is the correct answer. You obtained it by remembering that the practical unit of current is the ampere, and that an ampere is equal to a coulomb/sec. Since 240 coulombs pass point P in one minute (60 seconds), the current is

$$\frac{240\text{ coulombs}}{\text{min.}} \cdot \frac{1\text{ min.}}{60\text{ sec.}} = 4\text{ amperes}$$

Now we may continue our discussion of how to apply conservation principles to problems in electricity.

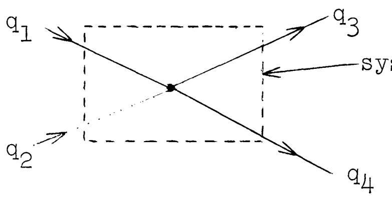
One of the most successful electrical theories assumes that the electron is a particle with a finite mass. This notion leads to the solution of many problems in electricity by the law of conservation of mass.

Go to page 79.

For example, in the figure below is shown an electrical junction, that is, a point at which two or more current-carrying conductors are joined together to make electrical contact.

Charges which are moving under the influence of an electric potential cannot be accumulated or stored in the junction. Therefore, we may state the following form of the principle of conservation:

"The sum of the charges entering a junction in any prescribed period of time is equal to the sum of the charges leaving the junction during the same interval of time."

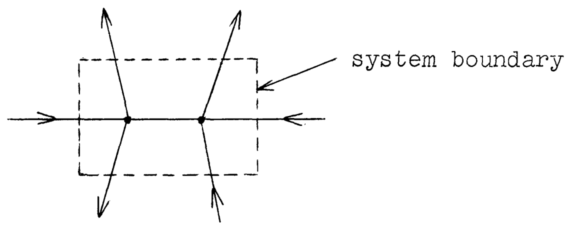


If, in the figure above, any three of the four charges are known, the fourth charge may be found by using the charge conservation principle.

Since an ampere is equal to a coulomb/sec., the law of conservation of charge may be rewritten as the law of conservation of current:

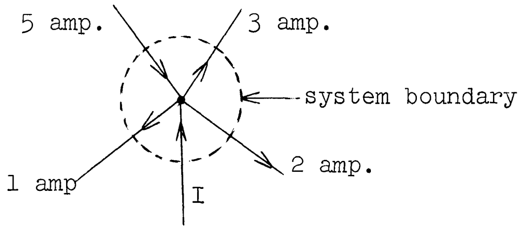
"The sum of the currents entering the boundaries of a system is equal to the sum of the currents leaving the boundaries of the system."

For example, in the figure below the system boundary encloses two junctions and is cut by six currents.



This rule is a version of what is known by the name of Kirchhoff's Current Law.

You should be ready for a problem. In the figure below compute the current labeled I.



Answer:

a. I = 5 amperes Go to page 75a.

b. I = 11 amperes Go to page 80b.

c. I = 1 ampere Go to page 77b.

Your answer is 240 coulombs/minute.

This answer is basically correct, but the units do not conform with current practical units. The practical unit of current is the ampere rather than many of the other units of rate such as the coulomb/min., coulomb/hr., etc.

Go back to page 73a and try again.

Your answer is 11 amperes.

This is not correct, and it is possible that you may have forgotten to take into account the directions of the currents. You probably wrote the equation

$$I = 5 + 3 + 1 + 2$$

Some of the currents on the right side of the equation are entering and some are leaving the system.

Go back to page 79 and try again.

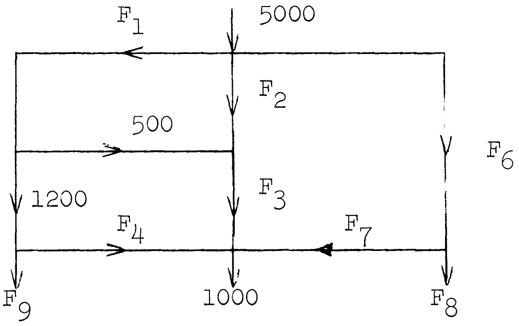
Your answer is: I cannot find such a boundary.

You are quite correct! Such a boundary does not exist. As a matter of fact, this problem is such that it cannot be solved by moving from junction to junction as was done in the previous examples.

Whenever you find yourself in this predicament, you must solve the problem by using simultaneous equations.

The solution of problems of this type by the use of simultaneous equations is discussed in Lesson No. 3, page 84b.

Suppose that we add a few more pipes to the problem which you have just solved and that we change some of the flows. This gives us the figure below:



Find a boundary which will cut only one of the unknown flows. Then solve for the unknown flow.

Answer:

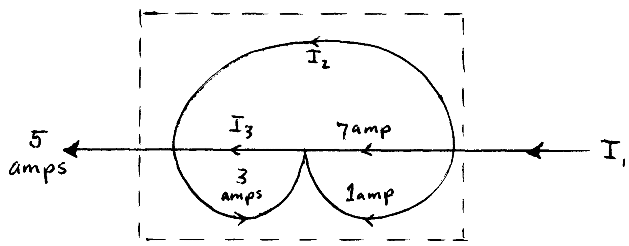
a. I cannot find such a boundary. Go to page 82b.

b. -700 Go to page 77d.

c. 1700 Go to page 76b.

d. 700 Go to page 68d.

A system boundary which is cut by I_1 as the only unknown current is shown in the linear flow graph below:



From this graph we obtain I_1 = 5a.

Now draw a boundary which is cut by I_2 as the only unknown current and find the value of I_2 for this system. Do not assume that I_1 is known!

After you have found the value of I_2 find another boundary which is cut by both I_1 and I_2. Using the value alreacy found for I_1 check the result of the other calculation of I_2.

Answer:

a. I cannot find a boundary which is cut by I_2 ad the only unknown current. Go to page 74b.

b. I_2 = -3 amp Go to page 67a.

c. I_2 = 3 amp Go to page 68b.

-82a-

Your answer is $I_3 = 5$ amperes.

You have probably used the wrong sign for one of the currents. If this is the case, checking your results with another boundary should have caught the error.

Correct your mistake and pick another answer from page 67a.

-82b-

Your answer: I cannot find such a boundary.

Inspect each junction, one at a time, and see if at least one junction has only one unknown flow entering or leaving it.

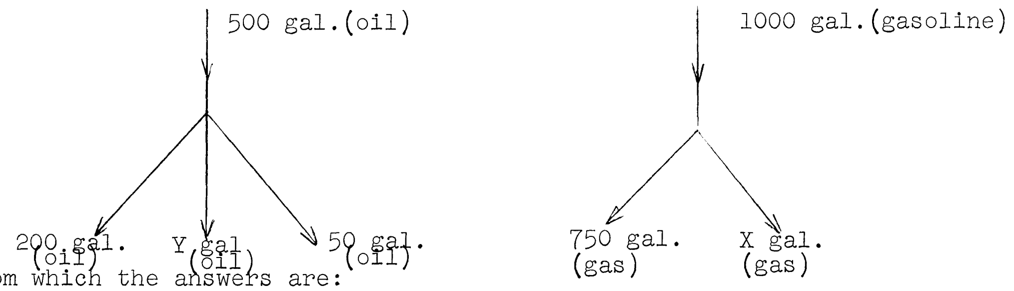
If you cannot find one such junction, take junctions two at a time, then three at a time, and so on.

Go back to page 81a and try again. If you still cannot find the boundary, go to page 83a.

-82c-

Answer to the problem on page 75c.

The linear flow graph of the system is



from which the answers are:

$X = 250$ gal. of gasoline

$Y = 250$ gal. of oil

If your answers are wrong, correct your errors. In any case turn to page 70.

-82d-

Answers to the problem on page 66a.

$X = (3000-150)$ gal.(gas) $= 2850$ gal. of gas

$Y = (200-50-25)$ gal.(oil) $= 125$ gal. of oil

Correct your errors, if any. Then turn to page 72.

[An elaboration of how to find a boundary which is cut by only one unknown will be included here.]

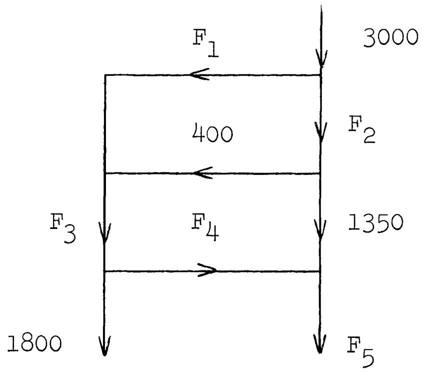
Go back to page 81a.

Your answer is I_3 = 11 amps.

This is very good. You have completed a multi-junction problem!

By applying common sense in choosing the system boundaries you were able to solve each unknown current independently. As you will see later this is not always possible. In some cases you will be forced to solve for some of the unknowns by using the previously computed values of other unknowns. Still in other cases you will have to solve a number of equations simultaneously.

If, instead of having electrical currents, you had water flowing into pipes, would your basic procedure change? Answer this question to yourself by working the problem below. Check your answers on page 76a. HINT: You will have to solve for some flows in terms of previously computed flows.



Numbers are in # H_2O/hr.

Your answer is 300.

You are on the wrong track! Go back to page 76b, read this page again, and then choose a new answer.

[This section will be expanded.]

Your answer is 3300.

It seems that you wrote a balance equation for the top junction and that you forgot one of the flows. (For example: $5000 = F_1 + F_2$, $F_2 = 3300$)

Go back to page 76b and answer the question again.

Lesson No. 3 - Solution of Problems by use of Simultaneous Equations

a. Why are simultaneous equations needed? Simple example.

b. How to determine a set of independent equations.

c. Examples of increasing complexity in Chemical Engineering, Electrical Engineering; perhaps an inventory problem (simple) in Industrial Engineering, etc.

Lessons 4 to N

Other conservation problems involving:

Energy Conservation

Momentum Conservation

Voltage Conservation (Kirchoff's Second Law)

etc.

Lessons N+1 to M

Concepts of iteration

Trial and Error

Problems involving logic such as the analysis of questionnaires, assignment of students to sections, design of a switching circuit, etc.

The concept of randomness

Linear programming - The simplex method

etc.

APPENDIX D

The Use of Symbolic Logic in Solving Engineering Problems

Experience suggests that the application of any branch of pure science in the practice of engineering will have two results. It can be expected to stimulate the development of the branch of science applied as well as to enhance the power of the engineer. Since engineering is the art of applying scientific knowledge to achieve practical goals, an engineering problem is any problem that arises in the course of making such application. Thus an engineer may be forced to assume the role of the scientist if he is to solve an engineering problem for which the necessary scientific knowledge does not yet exist, for in such a case developing the relevant branch of science is an indispensable means to reaching the engineer's goal. The distinction between "scientist" and "engineer" is therefore not absolute, but only approximate and administratively convenient. And even if the engineer does not become a scientist and work on scientific problems himself, he may raise and formulate scientific problems because of his engineering need for their solutions, thus stimulating the scientist to perform scientific research that might otherwise be neglected.

Symbolic Logic (or Mathematical Logic) is a pure science with potential application to many fields of engineering. It is less widely known by engineers, and less often studied by engineering students, than most other sciences, perhaps because its development has been more recent, perhaps because its applications are less obvious. Logic itself originated in ancient Greece as an organon or instrument for distinguishing good arguments from bad ones. It developed from an art of disputation into a science concerned with studying the methods and principles used in distinguishing correct from incorrect modes of proof. The use of special symbols in the study of logic goes back to the time of Aristotle, who used letters as variables in representing subject and predicate terms in propositions. Modern symbolic logic makes use of many more special symbols, to as great an extent as mathematics itself.

Symbolic logic is related to mathematics in several ways. They are both highly abstract in conception and symbolic in formulation. Some of the same scientists have made original and important contributions to both fields. In addition there are historical connections between the two fields that go back to earliest times. The ancient Greeks developed the first scientific mathematics by organizing geometrical truths in the form of a deductive or axiomatic system. Culminating in Euclid's Elements, the essence of the procedure was to define some concepts in terms of others assumed to be understood, and to deduce some propositions (the theorems) from other propositions (the axioms or postulates) which were assumed without proof.

This axiomatic method has served as a model for all subsequent scientific thought. It has pervaded mathematics to the extent that nothing is acceptable as a mathematical result, conclusion, or theorem unless it is derivable by strict logic from clearly stated premises. Here the notion of logic emerges as a central ingredient in the very definition of mathematics.

For mathematics really to be well defined it becomes necessary to state explicitly not only what propositions are accepted as axioms, but also what logical principles can be appealed to in deriving theorems from those axioms. With the development of new logical symbols that permit adequate formulation of the logical principles used by mathematicians, the need for organization and systematization of logic itself becomes apparent. Clearly the method appropriate to the goal of introducing order into logic is the axiomatic method. An axiom system for logic itself is required, which will define all logical concepts in terms of a small group of primitive logical notions assumed to be understood, and will deduce all logical truths as theorems from a small group of logical axioms or postulates assumed without proof in the system. In an axiom system of logic itself the deduction of theorems from axioms must of course proceed strictly according to a small number of explicitly stated rules of inference. An axiomatic system of logic in this sense is called a logistic system. Logistic systems have been developed for various parts of logic: the first, for the most elementary part of logic, the propositional calculus, was constructed in 1879 by the great German logician Gottlob Frege. Frege continued to try to construct more and more comprehensive logistic systems, which should encompass more and more of the field of logic. He worked also to construct an axiomatic system for all of mathematics, in which all the logic used in the system would be stated explicitly, as rules either to be assumed or to be derived along with the mathematical theorems.

The axiomatizing of mathematics itself had been very largely accomplished already. The program of "arithmetizing analysis" had established that the basic concepts involved in classical analysis (Theory of Functions of Real and Complex Variables) can be defined in terms of the arithmetic of whole numbers. Then the Italian mathematician Giuseppe Peano showed how all of the concepts of arithmetic can be defined in terms of just three notions, those of zero, number, and successor, and that all the classical truths of arithmetic can be deduced from just five axioms. This reductionist drive reached its culmination in the final work of Frege and in the later but independent work of the English mathematicians and philosophers Alfred North Whitehead and Bertrand Russell. In Frege's work, and later more satisfactorily in the monumental Principia Mathematica (1910-1913) of Whitehead and Russell, it was shown that all of the concepts of mathematics can be defined in terms of a small group of purely logical notions, and that all of the truths of classical mathematics can be deduced as theorems from a small group of purely logical axioms, using as principles of inference a small group of explicitly stated rules. The view that mathematics can be thus reduced to logic has come to be called the logistic thesis. In this sense mathematics can be regarded as being just a part of logic.

On the other hand, a system of symbolic logic can be regarded as a little mathematical system in its own right. As an abstract system of objects and operations it can be viewed as just another branch of modern algebra, one among many others. In this sense symbolic logic can be regarded as being just a part of mathematics. There seems to be no good reason for urging one of these points of view against the other. The two views complement each other by providing two perspectives from which to observe the same complex situation. Regardless of one's preference for one point of view or the other, it cannot be denied that symbolic logic and mathematics are closely related parts of a single fast growing and important area of scientific knowledge.

There is no doubt about the usefulness of mathematics in solving engineering problems. The applicability of mathematics in engineering, as well as in such exact sciences as physics and chemistry, is well known and reasonably well understood. It will be helpful to review the theory underlying its application, because much the same considerations are relevant to the application of symbolic logic.

The notation of mathematics can be regarded as a language in its own right, governed by syntactical rules of varying levels of complexity. In the presence of these rules, some expressions in the language of mathematics can be certified to be mathematical truths simply on the basis of their notational features. For example, the well known rules governing the use of the familiar symbols '+', '-', '=', juxtaposition and exponentiation, enable us to certify either by inspection or by elementary proof, that '$(X^2-Y^2) = (X+Y)(X-Y)$' is a mathematical truth in ordinary algebra (the algebra of real numbers). It is not only equations, of course, that are so certifiable by the rules of mathematical syntax: there are also inequalities and more or less complicated conditionals, such as the theorem asserting that if a function is bounded and continuous over a closed interval then it is uniformly continuous over that interval. The art of mathematical calculation is based upon the recognition of the equivalence of expressions the statement of whose equality is a mathematical truth. More generally, it can be remarked that all mathematical inference is based upon the recognition of conditionals ('if-then' statements) whose truth can be formally certified by reference to syntactical rules that refer only to the notational features of those conditionals.

The preceding remarks were concerned with _pure_ mathematics. Now, how does the language of pure mathematics get applied so usefully to the phenomena dealt with by the empirical sciences of physics and chemistry? The answer to this question centers about the topic of _measurement_. There are two aspects to the process of measurement that are not always clearly enough distinguished. One aspect is the correlation of numerals with physical magnitudes such as length, mass, temperature, or hardness. This correlation can be regarded as a semantical interpretation of some of the vocabulary of the language of mathematics in terms of the physical magnitudes that are being measured. It permits a situation to be described in the language of mathematics by having some of its parts or features named in that language. And, because so many equivalences or identities are demonstrable in that language, there are many other expressions that can consequently be used in describing the situation. The other aspect of measurement is the correlation of mathematical operations (or symbols for mathematical operations) with physical operations such as laying lengths end to end or combining masses physically. This correlation is a semantical interpretation of other parts of the language of mathematics in terms of physical operations. Where both aspects of measurement are realized, a situation can be described in the language of mathematics in many ways, both by having some of its parts or features named in that language and by having combinations of its parts or features named by the result of mathematically combining the mathematical names of those separate parts or features. In addition, mathematical or numerical descriptions permit the precise formulations of natural laws, which state the correlations of values for some parts or features of a situation with the corresponding values for others. Thus, if numerical magnitudes for some volumes, temperatures, and pressures can be attained by measurement, their relations can be generalized from those of the specific values observed to algebraic or analytic equations from which new sets of values for some magnitudes can be calculated if values for others are assumed to be given.

Symbolic logic has already been given some applications in science and engineering, and others can easily be envisaged. Thus it has been applied to insurance[1], to psychology,[2] to biology[3], to describing temporal passage[4], to cryptography[5], to the theory of measurement[6], and to electrical engineering[7]. The last application named follows the classical pattern already described in discussing the application of mathematics. Here propositional variables and their negations (class variables and their complements) are interpreted as naming or describing wire states. This interpretation permits a situation (a complicated circuit) to be described in the language of logic by having some of its parts (the wires) named in that language. In addition, the <u>and</u> ($\wedge$) and <u>or</u> ($\vee$) operations (intersection $\times$, and union + operations) are correlated with physical operations on the wires, with series and parallel connections of those wires, respectively. Thus a situation such as a series parallel switching circuit can be described in the language of symbolic logic in a variety of ways, both by having its parts named in that language and by having combinations of its parts named by the result of logically combining the logical names for those separate parts. This application not only permits the logical

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2. Fitch, F. B., "An Application of Symbolic Logic to Behavioristic Psychology." <u>J. of Symbolic Logic</u>, Vol. 4 (1939), p. 39; and Woodger, J. G. "The Formulation of a Psychological Theory." <u>Erkenntnis</u>, Vol. 7 (1937), pp. 195-198.

3. Woodger, J. H., <u>The Axiomatic Method in Biology</u>. Cambridge, England, 1937.

4. Russell, Bertrand. "On Order in Time." <u>Proc. Camb. Phil. Soc.</u>, Vol. 32 (1936), pp. 216-228.

5. Stamm, Edward, "Zastosowanie algebry logiki do teorji szyfrów (Application of the Algebra of Logic to Cryptography)." <u>Rozprawy Polskiego Tow. Mat.</u>, Vol. 1 (1921), pp. 40-52.

6. Weiner, Norbert. "A New Theory of Measurement." <u>Proc. London Math. Soc.</u>, Vol. 19 Vol. 19 (1919-20), pp. 181-205.

7. Shannon, C. E., "A Symbolic Analysis of Relay and Switching Circuits." <u>Trans. Amer. Inst. of Electrical Engineers</u>, Vol. 57 (1938), pp. 713-723.

calculation of simplifications for series parallel switching circuits, but also enables the electrical engineer to design an optimal series parallel switching circuit given the specification of functions it is to perform. Here we describe the same situation in different ways, moving from function to structure and back again by logical calculation. This permits us not only to solve a design problem given a specified function to be fulfilled, but also to describe the functions served by an object having a specified structure or design. Still another way in which already existing parts of symbolic logic can be applied in solving engineering problems is through the use of the propositional calculus and elementary quantification theory in the case study method of studying engineering. Here the various conditions to be met can be symbolized using standard logical notation, and the solution - or the conclusion that no solution is possible - can be derived by standard logical procedures.

However, all of these applications of symbolic logic to science and to the solution of engineering problems are only the beginning of what the total application is likely to be. The words of the distinguished logician W. V. O. Quine deserve to be quoted in this connection:

> "Mathematical logic has been applied, but the most important applications are surely still to come. The usefulness of a theory is not to be measured solely in terms of the application of prefabricated techniques to preformulated problems; we must allow the applicational needs themselves, rather, to play their part in motivating further elaboration of theory. The history of mathematics has consisted to an important degree in such give and take between theory and application. Much of the promise of mathematical logic for science lies in its potentialities as a basis from which to construct subsidiary techniques of unforeseen kinds in response to special needs."[8]

The realization of this promise can best be achieved through the collaboration of experts whose special knowledge includes symbolic logic, physical science, and engineering. Serious efforts to apply symbolic logic to scientific and engineering problems should lead to the growth and development of all three fields.

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8. Quine, W. V. O., Mathematical Logic, Cambridge, Massachusetts, 1947, p. 8.