MODELING AND ANALYSIS OF PROCESS COMPLEXITY AND PERFORMANCE IN MIXED MODEL ASSEMBLY SYSTEMS

by

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Dedicated to my family, for all their love and support

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ABSTRACT

Increasing global competition demands that the manufacturing industry move from mass production into mass customization production in order to provide more varieties of products and thus satisfy customer demands. It has been shown that the increase of product variety has a negative impact on manufacturing system performance. Therefore, it is essential to understand how product variety *complicates* an assembly system, affecting its operation performance. Such knowledge, once validated, can be further used to improve manufacturing system design and operation.

The objective of this dissertation is to develop an enhanced general methodology for modeling and analyzing process complexity for mixed model assembly systems. The following fundamental research has been conducted:

- A set of complexity metrics are proposed for measuring the complexity of various elements in a manufacturing system. These metrics are proposed by constructing a linkage with the communication system framework. Unlike the existing complexity measures defined in the literature, this research is the first effort to include production quality into the measurement of how well a manufacturing system can handle the process complexity induced by the input demand variety.
- A systematic method is developed for efficiently and explicitly representing complex hybrid assembly system configurations by the use of algebraic expressions, which can

overcome drawbacks of two widely used representation methods: block diagrams and adjacency matrices. By further extending the algebraic configuration operators, the algebraic performance operators are defined for the first time for the systematic evaluation of system performance metrics; these metrics include quality conforming rates for individual product types at each station, process capability for handling complexity, and production cycle time for various product types. Therefore, when compared to other methods, the proposed algebraic expression modeling method also has a unique merit in providing computational capability for automatically evaluating various system performance metrics.

- An integrated model is introduced for the first time to describe the effect of operator's factors on the process operation performance. The model includes intrinsic factors such as the operators' thinking time and experience; and extrinsic factors such as the choice task complexity induced by the product variety in mixed model assembly systems.

CHAPTER 1
INTRODUCTION

1.1 Motivation

Increasing global competition demands that the manufacturing industry move from mass production into mass customization production in order to provide a greater variety of products to satisfy customer demand. Implementing a mass customization scheme, however, requires overcoming many new technological challenges (Da Silveira, Borenstein and Fogliatto, 2001). From a production operation point of view, the correct implementation of modular production (Sturgeon, 2002) is at the core of these challenges.

In modular production, various products stemming from a common product family are manufactured by sharing the operations and resources of a single production system. Modular assembly systems consist of basic operation stations and variant operation stations; in such a system, all types of products will be processed by operations performed at the basic stations, while variant stations are used for individual product types. In this way, a variety of products can be produced in one assembly system; this is referred to as a mixed model assembly system, or an MMAS Although MMASs can provide better flexibility to meet customer demands with short production lead time, system configurations at an MMAS become highly complex, because the number of production paths and the differences among their processing times increase with the number of product variants produced. Moreover, individual workers usually need to perform multiple tasks at each station; this also increases the operator's mental workload due to task complexity. Therefore, these challenges reveal the stringent requirements and research challenges in the design and operation of an MMAS in order to effectively handle the increased complexity.

1.2 Research background and literature review

Traditionally, a single task assembly system is designed to follow a simple serial configuration, as shown in Figure 1-1 (a). In such a configuration, units are sequentially processed at every station. With the introduction of new technologies such as automated guided vehicles (AGVs), reconfigurable machine tools (RMTs), and re-sequencing buffers, assembly systems started to adopt different configurations. These different configurations include parallel configurations, as well as a combination of serial and parallel configurations, known as the hybrid configuration. Parallel configurations, such as the one shown in Figure 1-1 (b), consist of several stations positioned in parallel. In these systems, raw material enters each station, and an assembly or subassembly product is produced at each station.

Hybrid configurations can be further divided into simple hybrid configurations and complex hybrid configurations. Three examples of typical simple hybrid configurations are shown in Figure 1-1 (c) through (e). We first consider Figure 1-1 (c), in which, in order to increase the total production of the assembly system, a duplicate assembly line is added in parallel to the original assembly line. One of the advantages of this kind of configuration is that operators at the system can be shared between two congruent stations. As a second example, Figure 1-1 (d) shows an attempt to reduce the differences of the throughputs among different stations (i.e., to balance the line) by duplicating stations where bottlenecks occur and adding equivalent stations in parallel to the original stations. In this way, if the number of duplicate stations equals, for example, the number of the bottleneck stations, the production cycle time at the related stations is cut by half. Figure 1-1 (e) illustrates the third example of simple hybrid configurations. One can see from these three examples that a simple hybrid configuration can be considered as a *symmetric* assembly system configuration in the sense that a line of symmetry can be traced vertically, horizontally, or in both directions across their block diagrams.

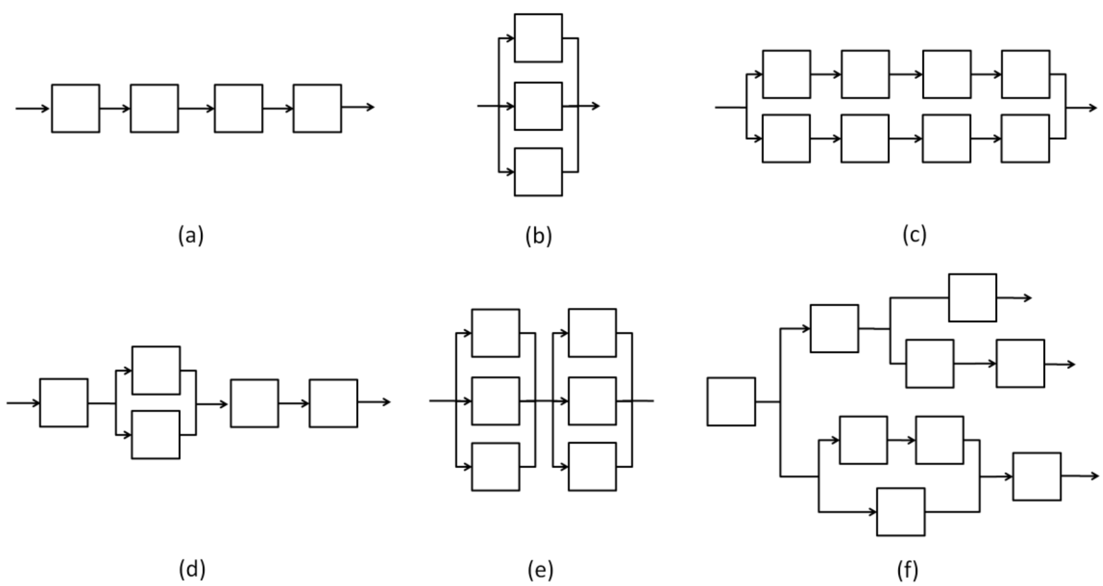

Figure 1-1 Examples of various assembly system configurations (adapted from Koren, Hu and Weber (1998))

Mixed model assembly systems used for modular production require complex asymmetric system configurations because different products are produced in the same assembly system by following different production paths. It is important to note that in modular production assembly systems, parallel stations are not necessarily duplicated stations, as in the case of single model assembly systems, since two stations in parallel can be completely different stations performing different assembly tasks. An example of the configuration of a complex asymmetric assembly system is provided in Figure 1-1 (f).

As the complexity of system configurations in an MMAS increases, the need for effectively modeling and analyzing system performance increases, especially for systematically improving the system's design and operation. In this spirit, the importance of system configuration was studied in Koren, Hu and Weber (1998), where a profound impact of manufacturing system configuration on performance was shown. Traditionally, assembly systems have been represented by the use of block diagrams, as the ones shown in Figure 1-1; this figure shows the advantages of the intuitive visual perception of the station layout configuration. Block diagrams themselves, however, do not have the computational capability to permit mathematical manipulation of system configurations or evaluation of system performance. The use of adjacency matrices representing block diagrams is another common practice found in the literature to represent assembly systems. The intuitive visual representation of the system configuration (serial or parallel relationship between stations) cannot, however, be directly perceived from adjacency matrices. Furthermore, adjacency matrices tend to result in largely sparse matrices, especially as the number of stations increases. As a consequence, adjacency matrices are not an effective or compact method of representing an assembly system.

To overcome the drawbacks of block diagrams and adjacency matrices methods, Webbink and Hu (2005) recently introduced a novel way of representing complex assembly systems, by using a string representation. The proposed method uses a string of characters to represent the stations, and parentheses to denote whether stations are related in a serial or a parallel fashion. Therefore, a compact way to represent complex assembly systems was achieved. Those string representations themselves, however, do not contain computational algorithms for evaluating MMASs performance. This research is motivated to further extend the string representation by using the algebraic expression method, which provides not only algebraic operators for compact representation of system configurations, but also allows the use of computational algebraic performance operators for systematically evaluating various MMASs performances.

In addition to the increased complexity of the system configuration in an MMAS, it has also been shown that the increase of product variety has a negative impact on the manufacturing system performance (Fisher, Jain and MacDuffie, 1995; MacDuffie, Sethuraman and Fisher, 1996; Niimi and Matsudaira, 1997). Therefore, it is essential to understand how product variety *complicates* an assembly system and affects its operation performance. Such knowledge, once validated, can be further used to improve manufacturing system design and operation.

There is increasing interest in recent years in the modeling and analysis of manufacturing system complexity. Deshmukh (1993) described the complexity of manufacturing systems in two aspects: the complexity induced by the demanded product varieties, and the complexity induced to the plant when the number of operations and stations is increased. Recently, some new entropic-related complexity measures have been proposed to describe the uncertainty induced by the rescheduling of the production (Huaccho Huatuco et al., 2009) or the decision making complexity for manufacturing organizations (Calinescu et al., 2001). Frizelle and Suhov (2008), through three case studies, also proposed methods to quantify the system's innate complexity while taking into consideration the data uncertainty that is due to measurement noises. Some advanced research has recently explored the defining of a measure of manufacturing complexity introduced by product variety, and the modeling of its propagation through the assembly system (Zhu et al., 2008; Hu et al., 2008). Those studies focused mainly on how to measure and model the complexity of product demands or the manufacturing system itself. Little research, however, has focused on how to describe manufacturing system performance in terms of whether it is capable to handle the product demand complexity. Therefore, this research is motivated to further define new quantitative metrics by incorporating production quality performance into the measure of the capability of the manufacturing system to handle the complexity induced by the variety of products.

Furthermore, it has also been recognized in the literature (MacDuffie, Sethuraman and Fisher, 1996) that the effect of product variety on manufacturing system performance impacts, not only the assembly system design and operation, but also the workers' performance at the plant. Therefore, in order to successfully implement a mass customization scheme, MMASs should be appropriately designed and operated considering the operators capabilities.

Modeling human operators' performance in an assembly system is a challenging research task, mainly because the factors involved are difficult to identify, define, and measure. Nevertheless, the inclusion of human operator's modeling is necessary for correct predictions of the system performance. In this spirit, Bernhard and Schilling (1997) asserted that inaccuracy on the simulation results is particularly apparent when modeling manufacturing systems with high proportion of manual operations. Baines et al.

(2004) further indicated that it is a common observation that the production rate of a simulation is usually greater than the actual rate of the real system and that, as a result, a greater accuracy in the simulation results could be achieved by including important factors affecting the worker's performance.

The factors that affect the performance of operators can be classified in two categories: intrinsic factors and extrinsic factors. Intrinsic factors include factors such as operator's age, working skills, experience, and so forth; while extrinsic factors include factors such as task difficulty and the number of choices in a given manufacturing task. Most models of human behavior and performance consider some of these factors.

Some research has been conducted to study the operator's effect in relation to process performance. For example, Fine (1986) developed a high-level model to describe the relationship of production cost with operator's learning skills and quality improvement efforts. In Baines et al. (2004), a model was proposed to study the effect of operator's age on his performance, which was measured as the amount of time required to finish a task. Furthermore, Wang and Hu (2010) considered the operator's fatigue due to product variety and studied its effect on system performance. None of these researches, however, has focused on how to adjust the production operations accordingly to the operator's performance, e.g., designing processes cycle time according to the operator's performance under various levels of task difficulty and subject's experience.

In MMASs, the number of product types and their mixed ratios varies among different operation stations. Therefore, the operator's performances will be affected by the choice task complexity and the amount of experience required at the specific station.

The allocation of a task cycle time is important to ensure that the operator will have sufficient mental deliberation thinking time to make the correct choice of parts, tools, and activities for task completion. In contrast, an excessive allocation of cycle time wastes production time and decreases the production throughput. Therefore, this research also develops an integrated model for characterizing the operator's performance and analyzing its effect on process quality and throughput in MMASs by considering intrinsic and extrinsic factors in the model. The proposed model will be justified based on the findings presented in the psychological literature. The effect of these operator's factors on process operation performance will also be investigated; these performance measures include process quality, throughput, as well as process capability on handling complexity induced by product variety in an MMAS. Two examples will be used to demonstrate potential applications of the proposed model.

1.3 Research objectives and tasks

The main objective of this dissertation is to further enhance the understanding of the effect of complexity, induced by the product variety, on MMAS performance. The specific tasks for achieving the proposed objective are as follows.

Task 1: Define complexity metrics for mixed model manufacturing systems. The focus of this study is on how to adopt a communication system framework based on information entropy, to effectively assess the capability of an MMAS to handle demand variety induced complexity.

Task 2: Develop a systematic approach for system configuration representation and operational performance analysis in MMASs. The focus of this study is on how to apply the algebraic expression method to achieve efficient and explicit representations of complex hybrid assembly system configurations. The algebraic configuration operators are further extended for new development of algebraic performance operators for systematically analyzing system operational performance.

Task 3: Model the quality conforming performance of operators in an MMAS. The focus of this study is on how to take into account the effect of cognitive factors affecting the performance of the operator. Furthermore, due to the inclusion of the operator's thinking time as one of the factors, the modeling of the throughput of the system is further studied.

1.4 Organization of this dissertation

This dissertation is presented in a multiple manuscript format. Chapters 2, 3, and 4 are written as individual research papers and include individual abstracts, main bodies, conclusions, and references. The structure of this dissertation and the relationship between its chapters are shown in Figure 1-2.

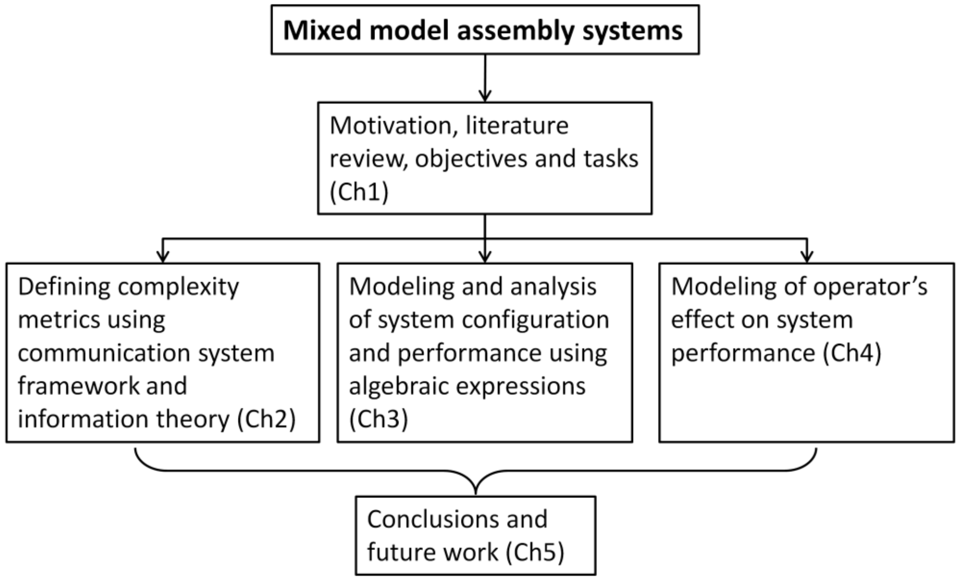

Figure 1-2 Organization of dissertation

Chapter 2 introduces a set of complexity metrics proposed for measuring the complexity of various elements in a manufacturing system. Those metrics were proposed by building a connection with the communication system framework. Unlike existing complexity measures defined in the literature, this research for the first time considers production quality in the measure of how well a manufacturing system can handle the process complexity that is introduced by the input demand variety. Examples are given to discuss various properties of the defined metrics and their potential applications.

Chapter 3 proposes a systematic method for efficiently and explicitly representing complex hybrid assembly system configurations through the use of algebraic expression approach, which can overcome the drawbacks of two popularly used representation methods, i.e., block diagrams and adjacency matrix. By further extending the algebraic configuration operators, the *algebraic performance operators* are defined for the first time for systematic evaluation of the system performance metrics. These metrics include quality conforming rates for individual product types at each station, process capability for handling complexity, and production cycle time for different product types. Therefore, compared to the recently developed string representation method, the proposed algebraic expression modeling method also has unique merit in its computational capability for automatically evaluating various system performance metrics. Furthermore, two examples are given to illustrate how the proposed algebraic representation can be effectively used in assisting the design and performance analysis for MMASs.

Chapter 4 introduces for the first time an integrated model to describe the effect of operator's factors on the process operation performance. The model includes not only intrinsic factors such as the operators' mental deliberation time and his or her experience, but also includes extrinsic factors such as the choice task complexity induced by the product variety in MMASs. Two illustrative examples are provided to show how the model is used in the production cycle time allocation by considering the operator's performance.

Finally, Chapter 5 concludes the dissertation and summarizes the dissertation contributions. Several future research topics are also suggested.

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CHAPTER 2

COMPLEXITY METRICS FOR MIXED MODEL MANUFACTURING SYSTEMS BASED ON INFORMATION ENTROPY

Abstract: Mixed model manufacturing systems are increasingly being used to meet global competition by providing a broad variety of products to customers. The increase of product variety adds more complexity to production processes, thus leading to a negative effect on the performance of production processes. Therefore, it is of great interest to effectively measure such complexity, and to quantify its effect on manufacturing system performance. In this paper, a set of complexity metrics are proposed for measuring the complexity of different elements in a manufacturing system; these metrics were achieved by constructing a linkage with communication system's framework. Different from existing complexity measures defined in the literature, this paper is the first to consider production quality into the measure of the process capability for handling the complexity induced by the input demand variety. Examples are given in the paper to discuss different properties of the defined metrics and their potential applications.

2.1 Introduction

The increasing global competition demands manufacturing industry to move from mass production into mass customization production in order to provide more varieties products to satisfy customer demands. For example, mixed model assembly systems (MMASs) are increasingly being used in many assembly plants, where different assemble operations are performed to produce different type of products in the same line. Although MMASs can provide better flexibility to meet customer demands with short production lead time, it brings out high requirements on handling the increased complexity of production operations. Therefore, at the design of an MMAS it is essential not only to know how to measure the operation complexity, but also to assess how well a manufacturing process can handle such an operational complexity to meet the variety of customer demands.

Recently, some theoretical measures of process complexity have been proposed for manufacturing systems under different contexts. For example, Nakazawa and Suh (1984) and Suh (1990) measured the process complexity in terms of the amount of efforts needed to complete a product, where the effort was quantified based on the product requirement on geometric precision and surface quality. From an assembly operation point of view, Fujimoto and Ahmed (2001) used the concept of entropy by measuring the amount of the uncertainty in key assembly operations, such as gripping, positioning and inserting parts. From a product point of view, Fujimoto et al. (2003) defined a measure of complexity based on the product structure and also showed that the impact of product variety could be reduced if the complexity of the product structure was reduced. Moreover, Deshmukh, Talavage and Barash (1992), Zhu et al. (2008), and Hu et al.

(2008) defined complexity to consider the demand variety based on the part mix ratio of the different products. Here, we refer to part mix ratio following the definition in Stecke (1992): "The part mix ratio are the relative numbers of parts of each type that will be produced in a cyclical manner during the next time period."

It has been shown that the increase of the product variety has a negative impact on the manufacturing system performance, e.g., Fisher, Jain and MacDuffie (1995), MacDuffie, Sethuraman and Fisher (1996), and Niimi and Matsudaira (1997). Deshmukh (1993) described the complexity of manufacturing systems by two aspects: the complexity induced by the demanded product varieties, and the complexity induced to the plant when the number of operations and/or stations is increased. Recently, some new entropic-related complexity measures have been proposed to describe the uncertainty induced by the rescheduling of the production (Huaccho Huatuco et al., 2009) or the decision making complexity for manufacturing organizations (Calinescu et al., 2001). Frizelle and Suhov (2008), based on three case studies, also proposed some form of quantification of the system's innate complexity while considering data uncertainty due to measurement noises. Those previous studies focused mainly on how to measure and model the complexity of the product demands or the manufacturing system itself. Differently, this paper focuses on how to describe the capability of a manufacturing process to handle the product demand complexity.

The objective of this paper is to further define a new quantitative metric, which can assess how well a manufacturing system can handle the complexity induced by the demanded input part mix ratio. The basic principle for defining such a metric is to assess whether the process can produce an output product mix ratio that meets the variety of the input part mix ratio as closely as possible. In practice, the output product mix ratio of a production process will not exactly match the input part mix ratio due to inevitable defective scraps produced by the production process. For example, Process A is designed to meet the demand consisting of two different product types with an input part mix ratio of 40% and 60% for type 1 and type 2, respectively. Assume that the production quality conforming rate is 98% for type 1 and 97% for type 2. In this case, the output product mix ratio becomes 39.2% for type 1 and 58.2% for type 2. Note that there are 2.6% scraps in the output product mix ratio. Based on the method in Zhu et al. (2008) and Hu et al. (2008), we can compute the input complexity as 0.9710. In this case, the proposed metric for describing the process capability for handling complexity, which is called the normalized process capability for complexity ($NPCC$) to be described in Section 2.3, can be calculated as 97.62%. It should be notified that such an $NPCC$ metric is related to production quality, but cannot be fully represented by production quality alone. For example, Process B is designed to produce a single type of products at a production quality conforming rate of 99%. Even though production quality of Process B is higher than that of Process A, the capability of Process B for handling demand complexity is not necessarily higher than that of Process A. In fact, in this special case, the amount of complexity handled by Process B is equal to zero because Process B has no demand input complexity. Therefore, the process capability metric for handling complexity should be defined to consider the input mix ratio and the process operation quality.

As shown in Figure 2-1, the input uncertainty of a manufacturing system is induced by the demanded product variety, which can be represented by the demand entropy $H(D)$ based on the demanded input mix ratio as defined in the existing literature

(Zhu et al., 2008; Hu et al., 2008). If the process can ideally produce the output products to exactly match the input demand variety, the output entropy $H(S)$ should be the same as the input entropy $H(D)$. However, due to the inevitable uncertainty of process operation quality, the output products may not exactly match the input demand. Therefore, the defined metrics should be able to describe the degree of matching between output entropy $H(S)$ and input entropy $H(D)$, which inspired us to employ the performance metrics used in communication channels.

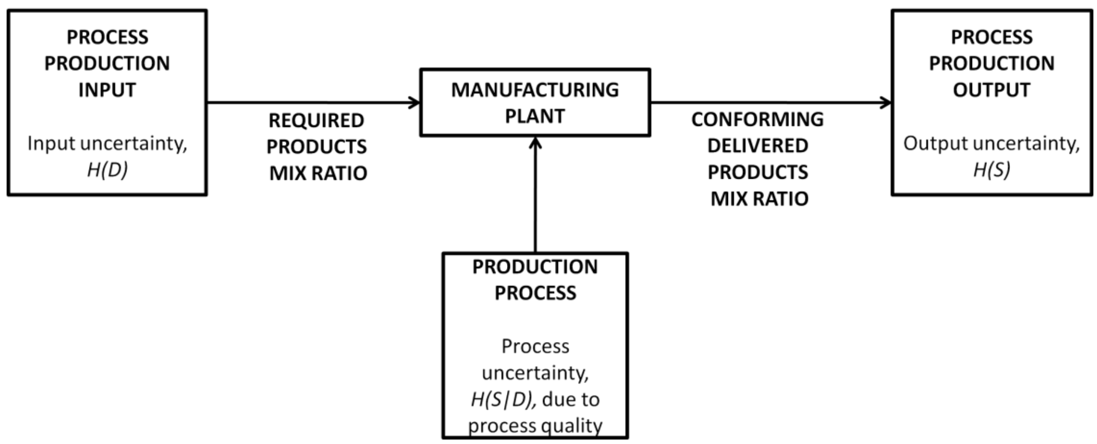

Figure 2-1 Manufacturing system representation

As it is known, the mutual information index $I(D,S)$ is defined to describe the dependency between random input codes D and output codes S for communication channel performance analysis (Shannon, 1948). In this paper, we will discuss in detail how this index can be used to assess the performance of a manufacturing process on handling the demand complexity. In this way, our work is an extension of the ideas in Zhu et al. (2008) and Hu et al. (2008) by considering the effect of process quality in the performance measures of a production process on handling the demand variety. In addition, all three elements of the complexity in a manufacturing system: input demand complexity, output product complexity, and process capability for handling input complexity are defined under a new integrated framework, based on the principles used to analyze communication systems. Therefore, the well developed communication channel analysis methods can be further applied in the future for optimizing manufacturing process capability in terms of handling demand complexity.

The rest of the paper is organized as follows. Section 2.2 presents the modeling of a manufacturing system in terms of the input/output relationship based on a communication system framework. Section 2.3 introduces the proposed metrics for measuring the performance of a manufacturing system; which are defined based on information theory used to analyze communication channels. In Section 2.4, two examples are used to illustrate different aspects in the use of the proposed metrics. A discussion on measuring complexity for a two-station production process is presented in Section 2.5. Furthermore, Section 2.6 is used to present an example of some potential uses of our proposed metrics. Particularly, the example illustrates the evaluation of a production process performance. Section 2.7 concludes the paper.

2.2 Modeling manufacturing systems using communication system framework

This section is at the first time to show the linkage of the input/output relationship between a manufacturing system and a communication system. As shown in Figure 2-2, a communication system consists of four major elements including information source, communication channel, destination, and noise source. These four elements in a communication system can also been founds in a manufacturing system based on Figure

2-1. The corresponding mapped elements of a manufacturing system are also given in Figure 2-2.

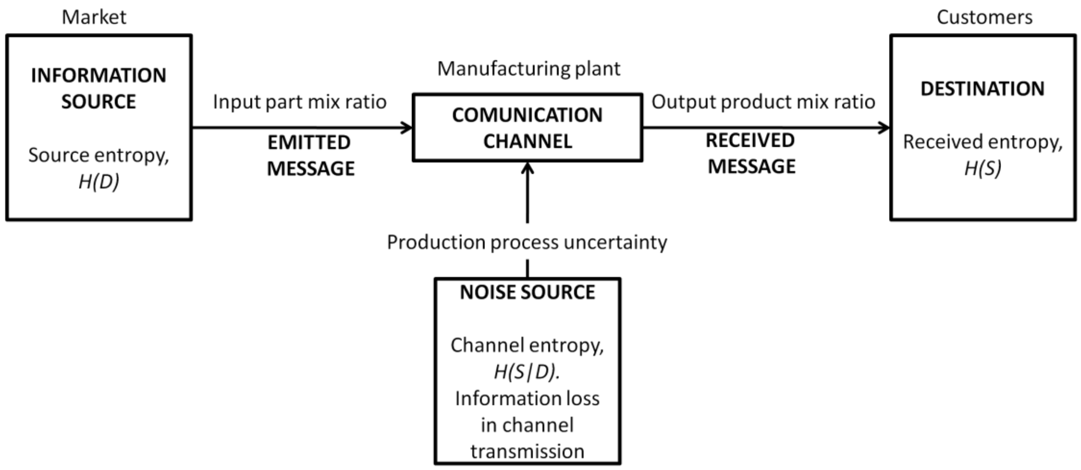

Figure 2-2 Mapping of elements from a communication system into a manufacturing system

Figure 2-2 shows how the key function of the channel is to reproduce the emitted message from the information source, as a received message at the destination. The emitted message (input of the channel) is a string of symbols from an alphabet created at the information source; while the received message (output of the channel) is the transmitted symbols received at the destination. The performance of the channel is affected by the noise source; which is evaluated by how well the channel output (received symbols) matches the channel input (emitted symbols).

For a mathematical representation of a (memoryless) discrete communication channel, a conditional probability matrix $\boldsymbol{P}_{(\boldsymbol{S}|\boldsymbol{D})}$ is often used. The matrix $\boldsymbol{P}_{(\boldsymbol{S}|\boldsymbol{D})}$ describes how each symbol is transmitted from the information source to the destination and it has the following structure

$$\boldsymbol{P}_{(\boldsymbol{S}|\boldsymbol{D})} = \begin{bmatrix} p_{00} & p_{01} & \cdots & p_{0,M-1} \\ p_{10} & p_{11} & \cdots & p_{1,M-1} \\ \vdots & \vdots & \ddots & \vdots \\ p_{N-1,0} & p_{N-1,1} & \cdots & p_{N-1,M-1} \end{bmatrix} \tag{2-1}$$

Here, p_{ij} corresponds to the probability that symbol i in the channel inputs (the input alphabet set) is transmitted as symbol j in the channel outputs (output alphabet set). N and M are the number of symbols in the input alphabet and the output alphabet, respectively. Note that N and M can be any number, thus allowing us to model a wide range of manufacturing processes. Equivalently, p_{ij} corresponds to the conditional probability defined as $p_{ij} = Prob\{Output = j|Input = i\}$ and satisfies the constraint $\sum_{\forall j} p_{ij} = 1$. The probability of p_{ii} reflects the transmission quality of the channel when transmitting symbol i. On the other hand, the value of p_{ij} for $i \neq j$ represents the transmission errors, and the total error rate for symbol i is equal to $q_{ii} = \sum_{\forall j \neq i} p_{ij} = 1 - p_{ii}$. To extend the concept of transmission quality in a communication system into that in a manufacturing system, p_{ii} is interpreted as the process quality rate corresponding to the fraction of conforming part i produced by the process, while $q_{ii} = 1 - p_{ii}$ is interpreted as the process error rate corresponding to the fraction of nonconforming part i produced by the process. The values of p_{ii} and q_{ii} can be estimated based on historical production data.

The advantage of adopting the communication systems framework is that the well developed metrics in communication systems (e.g., entropy, channel's capacity, transmission rate, etc.) can be further extended for analyzing the input/output mapping relationship in a manufacturing system. The linkages of these performance metrics between a communication system and a manufacturing system will be discussed in the following section.

2.3 Performance metrics for manufacturing systems using information theory

Suppose that the demanded product types are denoted by the random variable D, which is defined over the set Γ_D containing the labels of the different product types. In this way, the demand percentage of a product of type i can be represented as the frequency of producing the products of type i in the production process; which can be equivalently represented by the probability $P_i^D = Prob\{D = i\}$, where $D = i$ corresponds to the event that the next demanded unit at the manufacturing process is a product of type i. Similarly, we use the random variables S defined over the set Γ_S to represent the type of output product of the production process. In the same way, the probability of P_i^S represents the ratio of product type i in the outputs of the production process. Based on the concept of entropy defined in the information theory, a set of complexity measures in a manufacturing system will be defined as follows.

(1) Input demand complexity, measured by input entropy $H(D)$. Based on information theory, it is known that the entropy of D reflects the average uncertainty of demand D, defined as

$$H(D) = -\sum_{i=0}^{N-1} P_i^D \log P_i^D \tag{2-2}$$

Therefore, we can use the demand entropy of $H(D)$ to measure the input complexity induced by the demand variety of products.

(2) Output product complexity, measured by output entropy $H(S)$. Similar to Eq. (2-2), the output entropy of $H(S)$ can be calculated as

$$H(S) = -\sum_{j=0}^{M-1} P_j^S \log P_j^S \tag{2-3}$$

$H(S)$ is used to measure the output product complexity, reflecting the mixed ratio uncertainty of output products produced by the production process. For an ideal production process with no quality errors, the production mix ratio perfectly matches the demand mixed ratio, i.e., $P_i^S = P_i^D$, thus leading to $H(D) = H(S)$.

(3) Process capability for handling demand complexity (simply called process capability for complexity or PCC), measured by the mutual information $I(D, S)$. Based on information theory, the mutual information of two random variables D and S can be defined as

$$I(D, S) = \sum_{i=0}^{N-1} \sum_{j=0}^{M-1} P_{ij}^{DS} \log \frac{P_{ij}^{DS}}{P_i^D P_j^S} \tag{2-4}$$

where P_{ij}^{DS} is the joint probability distribution function of variables D and S, i.e., $P_{ij}^{DS} = Prob\{D = i, S = j\}$. Alternatively, $I(D, S)$ can also be represented as

$$I(D, S) = H(S) - H(S|D) \tag{2-5}$$

Here, $H(S|D)$ is the conditional entropy, which is defined as

$$H(S|D) = -\sum_{i=0}^{N-1} \sum_{j=0}^{M-1} P_{ij}^{DS} \log P_{j|i}^{S|D} \tag{2-6}$$

where $P_{j|i}^{S|D}$ corresponds to the conditional probability $Prob\{S = j | D = i\}$. Therefore, $H(S|D)$ is a measure of what the given input D does not predict about the output S. As a result, under an ideal perfect production process, the output product complexity exactly matches the input demand complexity, and thus $H(S) = H(D)$. In this case, we have $H(S|D) = 0$, which results in $I(D,S) = H(S) = H(D)$. Therefore, the process capability for handling demand complexity reaches its maximum. In contrast, under the worst (completely random) production process, output S and input D are independent of each other with $H(S|D) = H(S)$, which results in $I(D,S) = 0$. In this case, the process capability for handling demand complexity is zero. In a normal production process, $I(D,S)$ in Eq. (2-5) can be interpreted as the amount of information in output S predicted by input D, and thus reflecting a general dependency relationship between output S and input D. Therefore, we can use $I(D,S)$ to reflect the process capability for handling demand complexity, simply called process capability for complexity or PCC. In this way, mutual information $I(D,S)$ is defined in this paper as a metric to assess how well a production process can handle the demand variety of products in a manufacturing system.

(4) Process capacity for handling demand complexity (simply called process capacity for complexity or $MAX\text{-}PCC$), measured by the channel's capacity C. One of the most important concepts related to a communication channel is that of its capacity. The channel's capacity is a measure of how much information can reliably be transmitted through the channel. Formally, the channel capacity corresponds to the tightest upper bound of the rate at which a channel can reliably transmit information. Mathematically the capacity C of a channel is defined as

$$C = \max_{P_i^D\text{'s}} I(D,S) \qquad (2\text{-}7)$$

where the maximization is obtained by varying the input symbol probabilities P_i^D's exclusively. By extending this concept into a production process in a manufacturing system, we can also define C as the process capacity of handling demand complexity (simply called process capacity of complexity or $MAX\text{-}PCC$).

(5) Normalized process capability for handling demand complexity (simply called normalized process capability for complexity or $NPCC$), measured by the coefficient of constraint C_{SD}. Since the input demand complexity $H(D)$ can arbitrarily inflate the operation complexity $I(D,S)$, the coefficient of constraint C_{SD} proposed in Coombs, Dawes and Tversky (1970) is defined based on the normalized mutual information as

$$C_{SD} = I(D,S)/H(D) \tag{2-8}$$

The coefficient of constraint C_{SD} is an index defined over the normalized range [0,1], which is used to describe the normalized process capability for handling the demand complexity.

Although several entropic based metrics have been used in the literature for describing the manufacturing system complexity or demand complexity, this paper is at the first time to propose the metrics of PCC and $NPCC$ to describe how the manufacturing process can handle the input demand varieties. Specifically, PCC is used to evaluate different processes' performance under the same given demand varieties, while $NPCC$ is used when the demand varieties are different, especially to see how its capability will be changed over different demands. In this sense, $NPCC$ can be further used to evaluate the robustness of a process under various demand ratios.

(6) Total process quality, measured by total transmission rate Q. Based on the definition of matrix $\boldsymbol{P}_{(\boldsymbol{S}|\boldsymbol{D})}$, the total transmission rate Q of a communication channel is defined as

$$Q = \sum_{i=0}^{N-1} P_i^D p_{ii} \tag{2-9}$$

In a production process, Q can represent the total percentage of conforming products produced by the production process, which is a weighted sum of the probabilities of producing the conforming products by their corresponding input mix ratios. Therefore, Q can be used as a metric for measuring the quality performance of a production process.

2.4 Characteristics of proposed metrics

In this section we will use two examples to study the effect of different factors on the complexity of a manufacturing system. One example is used to study the effect of the total process quality on the $NPCC$. In this example, a positive relationship is observed between the process quality performance and the amount of complexity that the plant is capable of handling. The other example is used to study the effect of increasing the number of product types in the demand on the $NPCC$ of a station. We will be able to observe how the PCC of the plant increases with the number of product types, while the amount of $NPCC$ does not necessarily always increases.

2.4.1 Effect of process quality on PCC

Two scenarios are provided in this subsection to study the effect of process quality on the PCC, which correspond to two types of parts and three types of parts, produced by a single operation station.

(1) Two types of products are produced at a single operation station. In order to consider the process quality error, the performance of the station is modeled by an erasure symmetric channel (Cover and Thomas, 2006) as shown in Figure 2-3, where the output symbol ε corresponds to nonconforming products. Assume the station produces the same quality for both types of products, i.e., $p_{11} = p_{22} = p$, the station can be modeled by a systematic channel matrix as

$$\boldsymbol{P}_{(\boldsymbol{S}|\boldsymbol{D})} = \begin{bmatrix} p & 0 & q \\ 0 & p & q \end{bmatrix}.$$

where the last column of q's is added to show the nonconforming fraction of parts produced at this station.

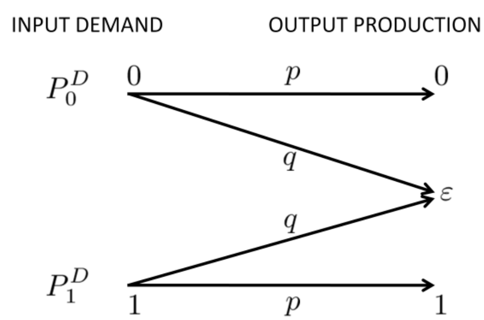

Figure 2-3 Representation of process quality for a station with a symmetric erasure channel

In order to study the effect of process quality, the input demand ratio will be set as $P_0 = 0.75$ and $P_1 = 0.25$, corresponding to part 1 and part 2 respectively. Also, process quality p is set within the range of $(0.5\ ,1)$ because the process quality level below 0.5 is unrealistic. Figure 2-4 shows the effect of the process quality Q ($Q = p$ for a symmetric channel) on the $NPCC$. We have also included the input demand complexity, and the output product complexity as references.

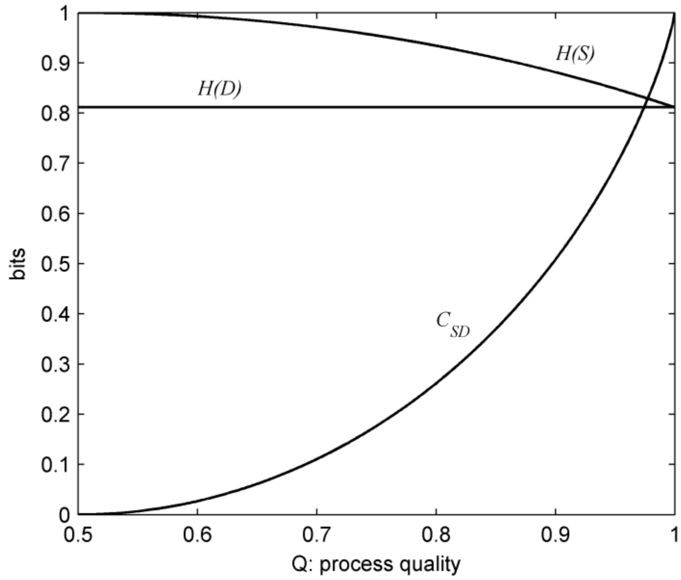

Figure 2-4 Input and output complexity, and $NPCC$ vs. process quality for two product types

(2) Three types of products are produced at a single operation station. Similarly, the station can be modeled by a symmetric erasure channel as

$$\boldsymbol{P}_{(\boldsymbol{S}|\boldsymbol{D})} = \begin{bmatrix} p & 0 & 0 & q \\ 0 & p & 0 & q \\ 0 & 0 & p & q \end{bmatrix}$$

The demand mix ratios of three types of products are set as $P_0 = 0.6$, $P_1 = 0.3$, and $P_2 = 0.1$. The total quality level Q will be set within the interval $(1/3\,, 1)$. Figure 2-5 shows the effect of process quality Q on the $NPCC$, in which the input demand complexity and the output production complexity are also included as references.

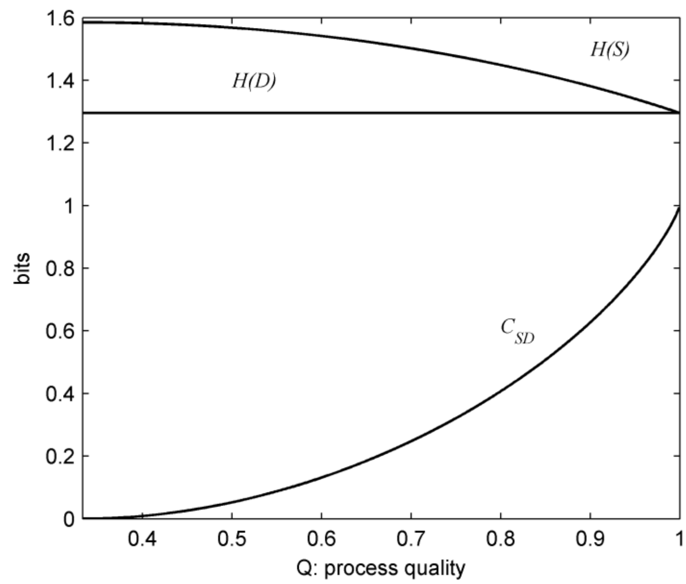

Figure 2-5 Input and output entropy, and $NPCC$ vs. process quality for three product types

From the above two-scenario analysis, it can be shown that the output product complexity monotonically decreases with the increase of the process quality, and approaches the input complexity when process quality Q is close to 1. This conclusion means that the minimum possible uncertainty of output products is the same as that of input demands. So, when the process quality $Q = 1$, the mix ratio of output parts exactly matches that of the input demand. The other conclusion is that the $NPCC$ (C_{SD}) is monotonically increasing with the increase of the process quality Q. This conclusion is consistent with our intuition that if the process has a high process quality level, it will have a high capability to handle the demand complexity.

2.4.2 Effect of adding product types on PCC and $NPCC$

Let us consider a single station currently producing two types of products denoted as type 0 and type 1, respectively. We will study the effect of adding one, two and three new product types into the demand. In the case of adding one product type, it is assumed that the input demand rations of P_1^D and P_2^D are equal. Similarly, when adding a fourth product type, it is assumed that $P_1^D = P_2^D = P_3^D$. The same assumption is made for the case with five product types.

Initially, the process quality with two product types is set as $p_i = 0.90$ for every product type i. Therefore, the total process quality of the plant is $Q = 0.90$. However, as the number of product types increases, the total process quality Q is expected to decrease exponentially, which we represent by the empirical rule

$$p_i = N^{-0.15} \tag{2-10}$$

where N is the number of product types in the demand. Figure 2-6 shows the plot of the process quality (percentage of conforming production) versus the number of choices.

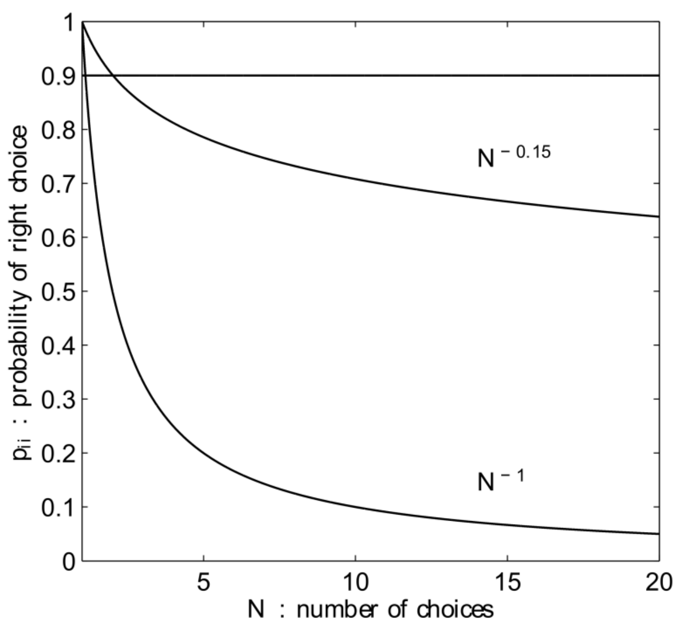

Figure 2-6 Number of choices vs. probability of producing conforming products

Figure 2-7 shows the positive effect of adding more product types to the demand on the PCC measured by the mutual information. From this figure, it can be seen that as the input demand ratio for product type 0, P_0^D approaches to 1, $I(D,S)$ under all four cases converges to zero. The reason for this is that as P_0^D gets closer to 1, the input complexity also approaches zero, and hence there is no information to be shared between the inputs and the outputs of the system.

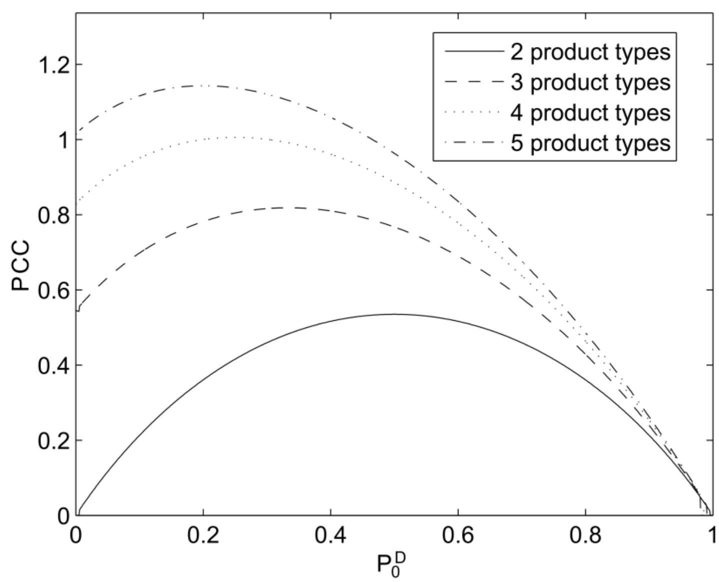

Figure 2-7 Effect of adding product types on the choice complexity

Furthermore, Figure 2-8 shows the effect of adding more product types on the $NPCC$. It can be seen that, approximately, when $P_0 < 0.26$, C_{SD} under the three-product-type case is greater than that under the two-product-type case. In contrast, approximately, when $P_0 > 0.26$, $NPCC$ under the two-product-type case is greater than that under the three-product-type case. Similar results can be observed for all other scenarios as well.

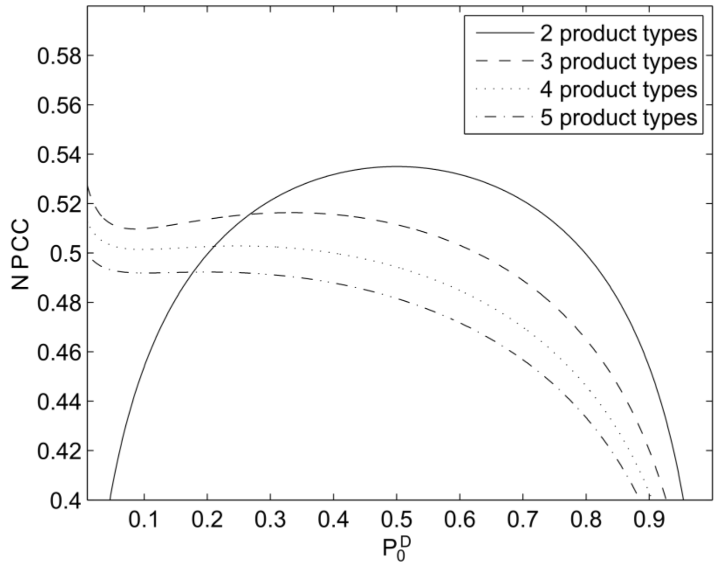

Figure 2-8 Effect of adding product types on $NPCC$

2.5 Performance metrics for two-station process

2.5.1 Process description

This section is used to illustrate how the metrics of PCC and $NPCC$ are affected by three different layouts of a production process, employed to produce two different parts used as two components in a final product. Each part (component) is designed to have two varieties, thus resulting in a total of four possible combinations of two parts to be used in final products. Each possible part combination has a different demand percentage represented by the following vector

$$P^D = [\pi_{00} \quad \pi_{01} \quad \pi_{10} \quad \pi_{11}] \tag{2-11}$$

where π_{ij} corresponds to the mix ratio of a product consisting of part 1 of type i and part 2 of type j. In our example, the mix ratio of the demand is set as

$$P^D = [0.40 \quad 0.30 \quad 0.20 \quad 0.10]$$

The input demand complexity can be computed based on Eq. (2-2), yielding $H(D) = 1.8464$.

Figure 2-9 shows a diagram of the three possible production process configurations considered in this example.

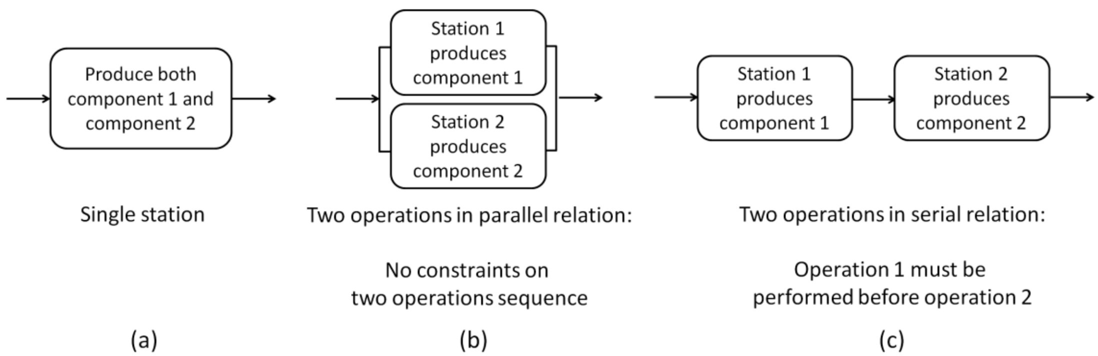

Figure 2-9 Three different production process configurations

In Layout (a), as shown in Figure 2-9, a single station is used to produce both part 1 and part 2 by using a single machine with the reconfigurable tooling capability. The conforming probabilities for each possible combination of parts can be assigned individually.

In Layout (b), as shown in Figure 2-9, two separate machines are used at two stations to produce part 1 and part 2 separately in a parallel configuration. In the paper, the parallel configuration means that there is no constraint on the operation sequence between operation 1 and operation 2, i.e., part 1 can be produced at station 1 either before or after part 2 is produced at station 2. Since a final product consists of both components (part 1 and part 2), the amount of final conforming products is corresponding to the minimum amount of each of two conforming components produced in this process. Therefore, the probability of the final product quality is equal to the minimum conforming fraction for producing part 1 and part 2.

In Layout (c), as shown in Figure 2-9, two separate machines are used at two stations to produce part 1 and part 2 separately but in a serial configuration. Differently from the parallel configuration in Layout (b), a serial configuration indicates a constraint on the operation sequence for producing part 1 and part 2, i.e., operation 1 for producing part 1 must be performed before operation 2 for producing part 2. For example, some features of part 1 may be used as the tooling fixture references at operation 2 for producing part 2. Therefore, the overall conforming fraction of a final product is the product of two conforming fractions associated with both parts.

It is worthwhile to clarify that the parallel and serial configurations, as shown in Figure 2-9, are used to distinguish whether the selected process has a constraint on the operations sequence for producing part 1 and part 2 when using different machines. Specifically, the parallel configuration means part 1 and part 2 can be produced separately by two independent operations at two different stations. In contrast, the serial configuration requires producing part 2 after producing conforming part 1. The selected two machines used in the parallel configuration may be different from those used in the serial configuration.

2.5.2 Process modeling

Each process layout yields a different output product complexity $H(S)$ and process capability for complexity PCC due to the constraints on the operation sequences that the selected production process must follow. In this example, all the stations will be modeled by a general erasure channel, discussed below.

<u>Layout (a) Model:</u>

Station 1 can be represented by the following matrix

$$\boldsymbol{P}_{(\boldsymbol{S}|\boldsymbol{D})} = \begin{bmatrix} p_{00}^{*} & 0 & 0 & 0 & q_{00}^{*} \\ 0 & p_{01}^{*} & 0 & 0 & q_{01}^{*} \\ 0 & 0 & p_{10}^{*} & 0 & q_{10}^{*} \\ 0 & 0 & 0 & p_{11}^{*} & q_{11}^{*} \end{bmatrix} \quad (2\text{-}12)$$

where p_{ij}^{*} corresponds to the probability of producing conforming products with part 1 of type i, and part 2 of type j. Since $\boldsymbol{P}_{(\boldsymbol{S}|\boldsymbol{D})}$ is a probability matrix, $q_{ij}^{*} = 1 - p_{ij}^{*}$. The output mix ratio is obtained by

$$P^{S} = [p_{00}^{*}\pi_{00} \quad p_{01}^{*}\pi_{01} \quad p_{10}^{*}\pi_{10} \quad p_{11}^{*}\pi_{11} \quad q_{00}^{*}\pi_{00} + q_{01}^{*}\pi_{01} + q_{10}^{*}\pi_{10} + q_{11}^{*}\pi_{11}] \quad (2\text{-}13)$$

<u>Layout (b) Model:</u>

The model structure of Layout (b) is represented by the diagram in Figure 2-10. The probability matrix representing station $i = 1,2$ is given by

$$\boldsymbol{P}_{\boldsymbol{i},(\boldsymbol{S}|\boldsymbol{D})} = \begin{bmatrix} p_{0}^{*i} & 0 & q_{0}^{*i} \\ 0 & p_{1}^{*i} & q_{1}^{*i} \end{bmatrix} \quad 2\text{-}14)$$

where p_{j}^{*i} corresponds to the probability of producing conforming products with part i of type j. It should be kept in mind that part i is only produced at station i. As before, q_{j}^{*i} corresponds to the probability of producing nonconforming products with part i of type j.

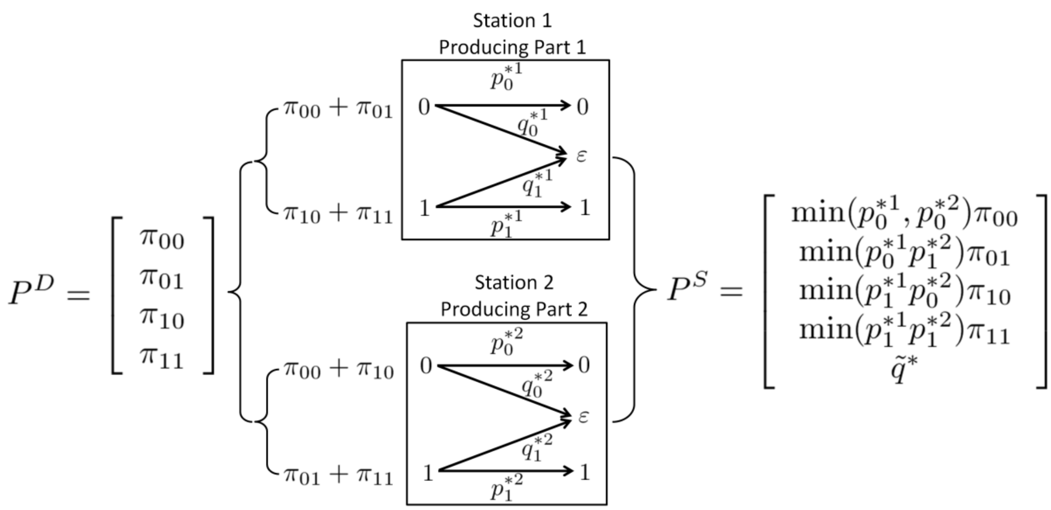

Figure 2-10 Layout (b) Model

The output mix ratio after station 1's operation is

$$P_1^S = [p_0^{*1}(\pi_{00} + \pi_{01}) \quad p_1^{*1}(\pi_{10} + \pi_{11}) \quad q_0^{*1}(\pi_{00} + \pi_{01}) + q_1^{*1}(\pi_{10} + \pi_{11})\,] \quad (2\text{-}15)$$

while the output mix ratio after station 2 is

$$P_2^S = [p_0^{*2}(\pi_{00} + \pi_{10}) \quad p_1^{*2}(\pi_{01} + \pi_{11}) \quad q_0^{*2}(\pi_{00} + \pi_{10}) + q_1^{*2}(\pi_{01} + \pi_{11})\,] \quad (2\text{-}16)$$

The production process formed by these two stations together can be modeled by the matrix

$$\boldsymbol{P}_{(\boldsymbol{S}|\boldsymbol{D})} =$$

$$\begin{bmatrix} \min(p_0^{*1}, p_0^{*2}) & 0 & 0 & 0 & \max(q_0^{*1}, q_0^{*2}) \\ 0 & \min(p_0^{*1}, p_1^{*2}) & 0 & 0 & \max(q_0^{*1}, q_1^{*2}) \\ 0 & 0 & \min(p_1^{*1}, p_0^{*2}) & 0 & \max(q_1^{*1}, q_0^{*2}) \\ 0 & 0 & 0 & \min(p_1^{*1}, p_1^{*2}) & \max(q_1^{*1}, q_1^{*2}) \end{bmatrix}$$

(2-17)

The output system mix ratio can be calculated by

$$P^S = [\min(p_0^{*1}, p_0^{*2})\pi_{00} \quad \min(p_0^{*1}, p_1^{*2})\pi_{01} \quad \min(p_1^{*1}, p_0^{*2})\pi_{10} \quad \min(p_1^{*1}, p_1^{*2})\pi_{11} \quad \tilde{q}^*] \tag{2-18}$$

where

$$\tilde{q}^* = \max(q_0^{*1}, q_0^{*2})\pi_{00} + \max(q_0^{*1}, q_1^{*2})\pi_{01} + \max(q_1^{*1}, q_0^{*2})\pi_{10} + \max(q_1^{*1}, q_1^{*2})\pi_{11}.$$

<u>*Layout (c) Model:*</u>

The model structure of Layout (c) is represented by the diagram in Figure 2-11. Layout (c) corresponds to the cascaded communication channels configuration (Silverman, 1955), and hence, the extensive literature dedicated to their study could be used in analyzing this layout.

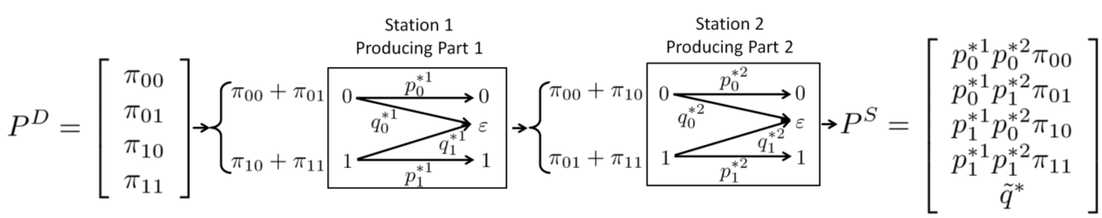

Figure 2-11 Layout (c) Model

The probability matrix representing station $i = 1{,}2$ is given by

$$\boldsymbol{P}_{\boldsymbol{i},(\boldsymbol{S}|\boldsymbol{D})} = \begin{bmatrix} p_0^{*i} & 0 & q_0^{*i} \\ 0 & p_1^{*i} & q_1^{*i} \end{bmatrix} \tag{2-19}$$

where p_j^{*i} corresponds to the probability of producing conforming products with part i of type j. The output mix ratio after station 1 is

$$P_1^S = [p_0^{*1}(\pi_{00} + \pi_{01}) \quad p_1^{*1}(\pi_{10} + \pi_{11}) \quad q_0^{*1}(\pi_{00} + \pi_{01}) + q_1^{*1}(\pi_{10} + \pi_{11})\,] \tag{2-20}$$

while the output mix ratio after station 2 is

$$P_2^S = [p_0^{*1}p_0^{*2}\pi_{00} + p_1^{*1}p_0^{*2}\pi_{10} \quad p_0^{*1}p_1^{*2}\pi_{01} + p_1^{*1}p_1^{*2}\pi_{11} \quad \tilde{q}_2^*] \tag{2-21}$$

Where $\tilde{q}_2^* = (1 - p_0^{*1}p_0^{*2})\pi_{00} + (1 - p_1^{*1}p_0^{*2})\pi_{10} + (1 - p_0^{*1}p_1^{*2})\pi_{01} + (1 - p_1^{*1}p_1^{*2})\pi_{11}$

The matrix representing the system as a whole is

$$\boldsymbol{P}_{(\boldsymbol{S}|\boldsymbol{D})} = \begin{bmatrix} p_0^{*1}p_0^{*2} & 0 & 0 & 0 & q_0^{*1} + p_0^{*1}q_0^{*2} \\ 0 & p_0^{*1}p_1^{*2} & 0 & 0 & q_0^{*1} + p_0^{*1}q_1^{*2} \\ 0 & 0 & p_1^{*1}p_0^{*2} & 0 & q_1^{*1} + p_1^{*1}q_0^{*2} \\ 0 & 0 & 0 & p_1^{*1}p_1^{*2} & q_1^{*1} + p_1^{*1}q_1^{*2} \end{bmatrix} \tag{2-22}$$

The output mix ratio of the system is

$$P^S = [p_0^{*1}p_0^{*2}\pi_{00} \quad p_0^{*1}p_1^{*2}\pi_{01} \quad p_1^{*1}p_0^{*2}\pi_{10} \quad p_1^{*1}p_1^{*2}\pi_{11} \quad \tilde{q}^*] \tag{2-23}$$

where

$\tilde{q}^* = (q_0^{*1} + p_0^{*1}q_0^{*2})\pi_{00} + (q_0^{*1} + p_0^{*1}q_1^{*2})\pi_{01} + (q_1^{*1} + p_1^{*1}q_0^{*2})\pi_{10} + (q_1^{*1} + p_1^{*1}q_1^{*2})\pi_{11}$.

2.5.3 Process conditions and analysis results

In Layout (a), a single station is used to produce both part 1 and part 2 by using a single machine with the reconfigurable tooling capability. It is assumed that the same conforming probability of $p_{ij}^* = 0.85$ is obtained for the 4 different final product combinations. Table 2-1 shows the detail demand mix ratio and the associated conforming probability.

Table 2-1 Demand and conforming rates for single station (Layout (a))

	Single Station			
	Part 1 Type 0 Part 2 Type 0	Part 1 Type 0 Part 2 Type 1	Part 1 Type 1 Part 2 Type 0	Part 1 Type 1 Part 2 Type 1
Demand	40%	30%	20%	10%
Conforming Percentage	0.8500	0.8500	0.8500	0.8500

In Layout (b) and Layout (c), the conforming probabilities of each station will be selected to reflect the dependency between the quality and the demand ratio for each specific combination of part i and type j. Specifically, for the highly demanded part/type, a machine with a higher conforming probability is used, as shown in Table 2-2.

Table 2-2 Demand and conforming rates for Station 1 and Station 2 (Layout (b) and Layout (c))

	Station 1 (Part 1)		Station 2 (Part 2)	
	Type 0	Type 1	Type 0	Type 1
Demand	70%	30%	60%	40%
Conforming Percentage	0.9750	0.8500	0.9500	0.9000

Table 2-3 shows the analysis results for each of above three possible layouts. It can be seen that the less restrictive Layout (b) with the parallel configuration has the highest *NPCC* among these three layouts. Additionally, the metric *NPCC* and the total quality Q are positively related as intuitively expected.

Table 2-3 Metrics under three process configurations

	Input Complexity $H(D)$	Output Complexity $H(S)$	Process Capability for Complexity $I(D,S)$	Normalized Process Capability for Complexity C_{SD}	Process Quality Q
Layout (a)	1.8464	2.1793	1.5695	0.8500	0.8500
Layout (b)	1.8464	2.1000	1.6618	0.9000	0.9050
Layout (c)	1.8464	2.1261	1.5933	0.8629	0.8718

2.6 Exemplary use of methodology

The following example is used to illustrate one of the possible applications of the proposed metrics PCC and $NPCC$. We will consider a single station that produces 3 different types of products. Based on historical data, we know that product type 0 is easier to be produced by the operator at the station than the other two types of products. The quality level of product type 0 has been estimated to $p_{00} = 0.90$. For the other products, the quality levels have been estimated by $p_{11} = 0.85$ and $p_{22} = 0.80$. Currently, the production line is used to produce twice as many products of type 0 as that of type 1 and type 2 together. In other words, the input demand ratio are estimated by $P_0^D = .67$ and $P_1^D = P_2^D = 0.165$. At these levels, the total process quality of the plant is estimated as $Q = 0.8752$. The $NPCC$ at such settings can be obtained as $C_{SD} = 0.5288$. The plant engineer is interested in achieving the maximum amount of $NPCC$ by adjusting the mix ratio of the demand. However, he is not going to accept a process quality below $Q' = 0.875$.

Figure 2-12 presents the contour plot of the process quality Q under all the possible combinations of the demand ratios P_0^D, P_1^D and P_2^D. In this plot, the unfeasible region corresponds to the combinations of the demand ratios where $P_0^D + P_1^D + P_2^D > 1$.

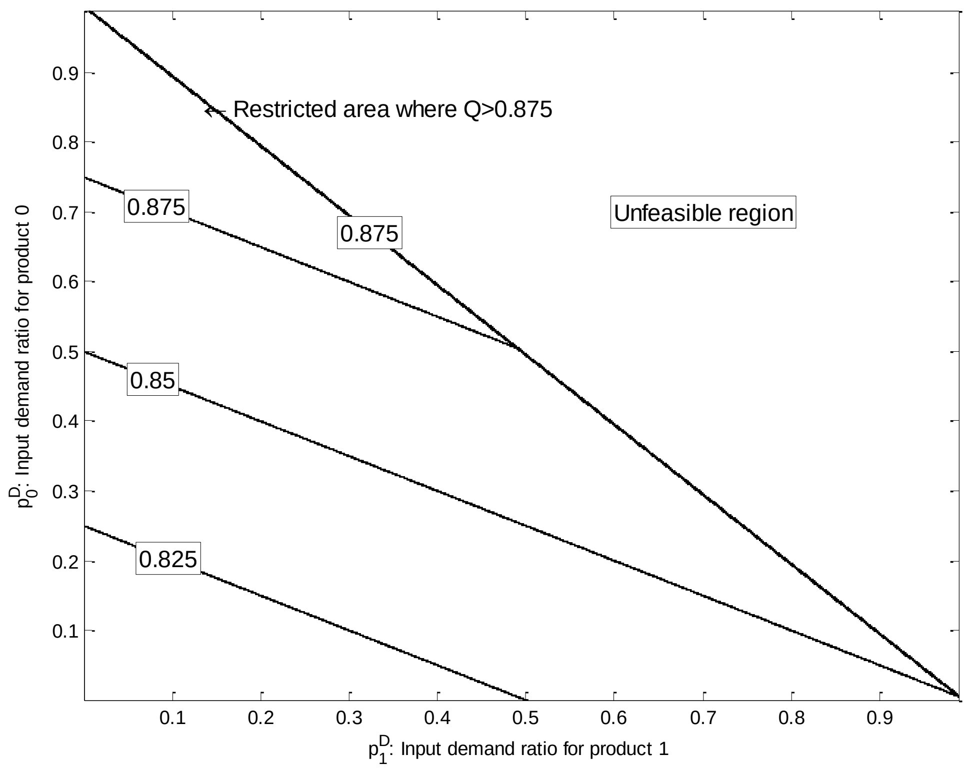

Figure 2-12 Total process quality Q

Table 2-4 summarizes all the analysis results under different criteria. The first column of Table 2-4 provides all indexes under the initial settings of the input demand ratios for three types of products. The second column contains the optimal input demand ratios that maximizes C_{SD} yielding the maximum value of $C_{SD}^{*} = 0.5934$. This result can also be seen based on the contour plot of C_{SD} as shown in Figure 2-13, in which the maximum point (the dot point) is achieved at the condition of $P_0^{D*} = 0.5381$, $P_1^{D*} = 0.4561$ and $P_2^{D*} = 0.0058$. Again, the unfeasible region corresponds to demand probabilities values where $P_0^D + P_1^D + P_2^D > 1$.

Table 2-4 Input probabilities with corresponding C_{SD} and Q

		Optimal criteria		
	Initial Setting	Max C_{SD}^*	Max Q^*	Max C_{SD}^* s.t.: $Q > 0.875$
P_0^D	0.670	0.5381	1	0.5381
P_1^D	0.165	0.4561	0	0.4561
P_2^D	0.165	0.0058	0	0.0058
C_{SD}	0.5288	0.5934	NA	0.5934
Q	0.8752	0.8766	0.9000	0.8766

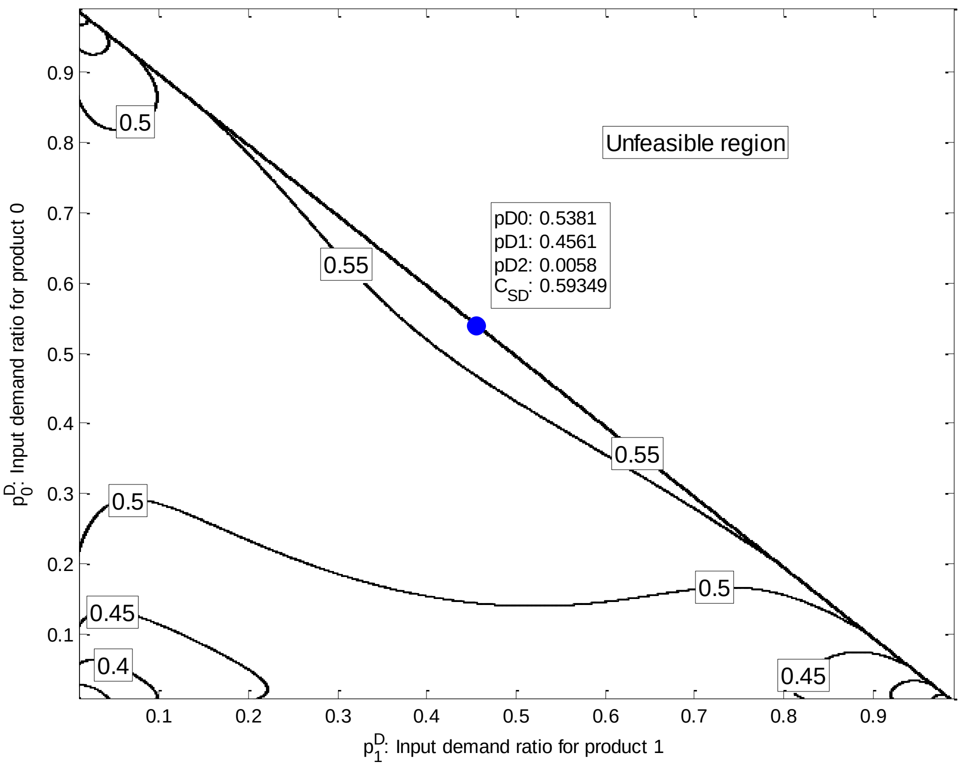

Figure 2-13 $NPCC$

In the third column of Table 2-4, it can be seen that the total process quality Q is maximized under the condition of $P_0^D = 1$. This solution is expected since the process quality Q surface shown in Figure 2-12 is linear in all its variables, thus the optimal value occurs at the upper left corner of Figure 2-12.

Finally, the fourth column results of Table 2-4 provides the optimal input demand ratio that maximizes C_{SD}, subjected to the process quality level $Q' \geq 87.5\%$. The restricted area defined by $Q > 87.5\%$ is also labeled in Figure 2-12. The value of C_{SD}, satisfying the constraint of $Q' \geq 87.5\%$ area is plotted in Figure 2-14, where the location of the maximum has been marked with a dot.

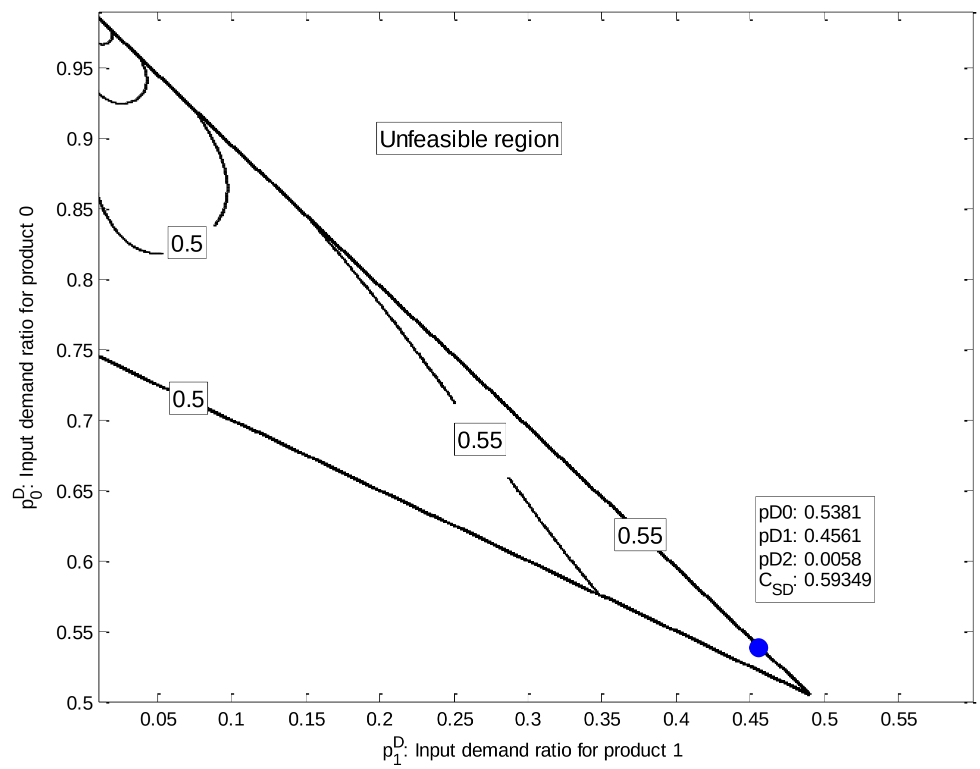

Figure 2-14 Restricted coefficient of constraint C_{SD}

From Table 2-4, it can be seen that the restricted maximum is the same as the unrestricted one since the total quality under the unrestricted case, and is above 87.5%. If only considering the process quality and the $NPCC$ as decision criteria, the solution suggests to completely eliminate product type 2 from the demand and recalibrate the mix ratio of the other two types of products to achieve maximum $NPCC$ at a satisfying process quality level. The reason for this is that the probability of producing conforming products of type 2 is the lowest among the three types of products. Consequently, the production of type 2 products decreased the total process quality at the plant, while not significantly increasing the $NPCC$ of the plant.

2.7 Conclusions and future work

This paper is at the first time to present a new modeling framework for measuring the complexity of a manufacturing system based on a communication system framework, in which the linkages of input/output mapping relationship between two systems are discussed. Based on this new framework, two entropic based metrics are used for measuring the performance of a manufacturing system. A new contribution of this paper is the consideration of the process quality into the complexity measures. In fact, when considering process quality, we see a divergence between what is demanded and what is produced. A measure of this divergence can be interpreted as a measure of the amount of complexity that the manufacturing process delivers. To measure the divergence between the complexity in the demand and the complexity in the delivered products, the metrics of PCC and $NPCC$ are defined to assess the process capability and normalized process capability for handling input complexity, respectively. Several examples are given in the paper to illustrate the different aspects in the use of the proposed metrics. Moreover, the effect of process quality and multiple stations' layout configuration are also investigated. Finally, an example in the use of the proposed methodology was illustrated to show how the defined metrics of the PCC and $NPCC$ are used to understand the maximum process capability under the constrained total process quality.

It should be clarified that this paper only focuses on defining complexity metrics for measuring process capability for complexity that are induced by demand mix ratio only. The future research will explore how to measure the process capability for handing operational complexity by further considering the frequency of change-over or the batch size of product varieties. For this purpose, the predictability of the product sequences will be further studied and included into the measure of input complexity in addition to the demand mix ratio. In addition, although a single symbol ε is used in the paper to represent the total nonconforming products, the model can be further extended by using ε_{ij} to represent different defects, which can also be handled by using the communication systems theory. In this way, we can further include other aspects of the manufacturing systems, such as different scrap costs, rewards, root causes, etc. into the model.

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CHAPTER 3

ALGEBRAIC EXPRESSION OF SYSTEM CONFIGURATIONS AND PERFORMANCE METRICS FOR MIXED MODEL ASSEMBLY SYSTEMS

Abstract

In recent years, mass customization has emerged as one of the leading production paradigms. Its main advantage is the ability to offer a wider range of products varieties than can be offered by mass production. From a production point of view, modular production enables the implementation of mass customization easily at a modular assembly system, leading to a mixed model assembly system (MMAS). One of the challenges in the design and operation of an MMAS is the high complexity of the station layout configuration for performing various tasks to produce different product variants. Therefore, it is desirable to have an effective way of representing complex system configurations and analyzing system performance. By overcoming the drawbacks of two widely used representation methods (block diagrams and adjacency matrix), this paper proposes to use algebraic expressions to represent the configuration of an MMAS. By further extending the algebraic configuration operators, the algebraic performance operators are defined for the first time for systematically evaluating the system performance metrics, such as quality conforming rates for individual product types at each station, process capability for handling complexity, and production cycle times for individual product types. Therefore, the benefits of using the proposed algebraic

representation are not only the effectiveness in achieving a compact storage of system configurations, but also its ability of systematically implementing computational algorithms for automatically evaluating various system performance metrics. Furthermore, two examples are given in the paper to illustrate how the proposed algebraic representation can be effectively used in assisting the design and performance analysis of an MMAS.

3.1 Introduction

In recent decades, market demand has changed from fairly homogenous and relatively stable to highly variable and rapidly changing. As a response, production systems have gone from a mass production paradigm to a mass customization model. Implementing a mass customization scheme, however, requires overcoming many technological challenges (Da Silveira, Borenstein and Fogliatto, 2001). From a production point of view, the correct implementation of modular production (Sturgeon, 2002) lies at the core of these challenges.

In a modular production assembly system, different product variants are produced in an assembly system, referred to as a mixed model assembly system or MMAS. The station configurations of an MMAS become highly complex because different products follow different production paths in the same assembly system. For example, the assembly system in Figure 3-1 is used to produce three different product types. It can be seen that the first two stations, S_1 and S_2, are basic operation stations, which means that every product of any type will be processed at stations S_1 and S_2. In contrast, Stations S_3, S_4, S_5, S_6, S_7, and S_8 are considered as variant operation stations, which implies that only certain types of products will be processed at these stations. Specifically, the first product type has a production path of $S_1 \rightarrow S_2 \rightarrow S_3 \rightarrow S_4 \rightarrow S_5$; the second product type follows the path of $S_1 \rightarrow S_2 \rightarrow S_3 \rightarrow S_6$; and the third product type is produced via the path of $S_1 \rightarrow S_2 \rightarrow S_7 \rightarrow S_8$.

It should also be noted that in an MMAS, parallel station configurations are not necessarily used for duplicated tasks as they are in single model assembly systems. As shown in Figure 3-1, the path of station 6 is used to produce product types that are different from the types produced in the parallel path of station 4 and station 5. This may also be true for two single parallel stations since each of two parallel stations may be designed to perform different tasks corresponding to different product component variants. Therefore, this variation leads to high complexity in evaluating system performance when considering the variety of product types at individual stations.

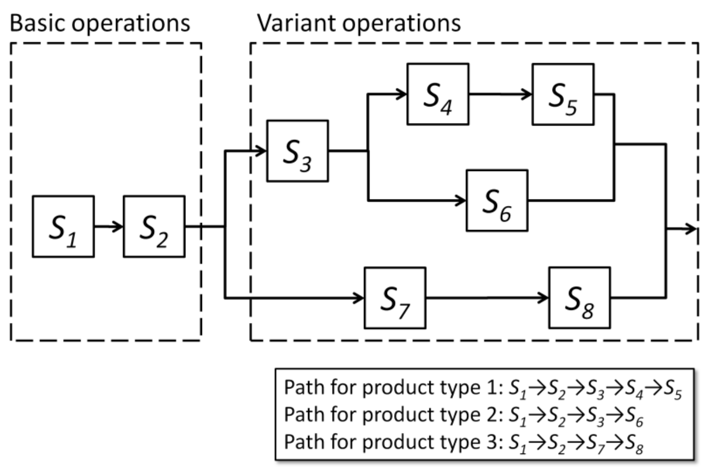


Figure 3-1 Basic operation stations and variant operation stations in delayed differentiation modular production

As the demand for MMASs increases, the need for effectively modeling and analyzing the operation performance of such a system increases, in order to enable systematic improvement of the system design and operation. In this spirit, the importance of system configuration was studied in Koren, Hu and Weber (1998), where a profound impact of manufacturing system configuration on performance was shown. Traditionally, assembly systems have been represented by the use of block diagrams, as shown in Figure 3-1; this type of diagram shows the advantages of the intuitive visual perception of the station layout configuration. Block diagrams themselves, however, do not have the computational capability to permit the mathematical manipulation of system configurations or evaluation of system performances. The use of adjacency matrices representing block diagrams is another common method of representing assembly systems, as noted in the literature. Adjacency matrices are mathematical structures that emerged in graph theory as a way to more easily manipulate and represent system configurations. The intuitive visual representation of the system configuration (serial or parallel relationships between stations) cannot, however, be clearly perceived. Furthermore, adjacency matrices tend to result in largely sparse matrices, especially as the number of stations increases. As a consequence, adjacency matrices are not an effective compact method of representing an assembly system.

To overcome the drawbacks of block diagrams and adjacency matrices methods, Webbink and Hu (2005) recently introduced a novel way of representing complex assembly systems by using a string representation. The proposed method uses characters to represent the stations, and uses parentheses to denote whether stations were related in a serial or a parallel fashion. Therefore, a compact way of representing complex assembly systems was achieved. The present paper further extends the string representation by applying a standard algebraic expression. This method not only provides algebraic operators for compact representation and storage of system configuration structures, but also can be easily extended as computational algebraic operators for systematically evaluating various MMASs performance metrics, including quality conforming rates, process capability for complexity (Abad and Jin, 2010), and cycle time analysis for individual product types at each station. Moreover, it also provides an effective way to systematically decompose a whole complex assembly system into different levels of sub-grouped systems, each of which are represented by their corresponding *equivalent* stations obtained using algebraic expressions. Therefore, the proposed algebraic expression representation permits a flexible aggregation or decomposition in representing the system configuration with a strong computational capability, which can further simplify the analysis and interpretation of the system operation characteristics for improving decision-making at various desired levels.

The use of binary operators to deal with graph problems has been proposed in the literature. Path algebras, also known as (max,+) algebras or dioid algebras, are mathematical structures used to solve a large number of path-finding and network problems in graph theory (Carre, 1971; Gondran and Minoux, 1983). The key idea is to use binary operators to rewrite, in a compact way, the algorithms used to solve these problems, thus achieving a pseudo-linear system of equations.

The application of path algebras to manufacturing systems was initiated in Cohen et al. (1985). Since then, it has been used to solve numerous kinds of problems in this domain, e.g., resource optimization (Gaubert, 1990), production planning and control (Yurdakul and Odrey, 2004; Xu and Xu, 1988), and modeling of discrete event systems (Cohen et al., 1989; Cofer and Garg, 1993). To the best of our knowledge, however, path algebra has not been directly used to represent complex assembly system configurations. Zissos and Duncan (1971) proposed the use of algebraic operators to represent logic circuits with the advantage that the symbols stand in a one-to-one correspondence with the physical elements of the system. Furthermore, a postfix or reverse polish notation of this algebraic representation was proposed in Duncan, Zissos and Walls (1975) with the advantage that postfix notation eases implementation in computer languages by inherently determining the order in which operations are to be resolved. Recently, Freiheit, Shpitalni and Hu (2004) proposed to use Boolean operators as a way to determine the system states (operative or not operative) in arbitrary station configurations. This was achieved by representing serial relations among stations by a disjunctive AND operator, and parallel relations among stations by a conjunctive OR operator. As a consequence, all the states where production is achieved are determined when a value of TRUE is achieved by the Boolean expression representing the system. This paper further extends previous work to achieve an intuitive representation of complex assembly system configurations using algebraic expressions, which can be consequently used for evaluating complex systems performance.

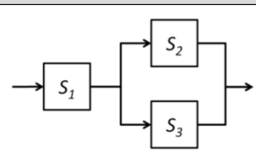
The rest of the paper is organized as follows. In Section 3.2, an algebraic representation of assembly system configurations is introduced. The transferring algorithms are provided for obtaining the algebraic representation from a traditionally used block diagrams or an adjacency matrix. Section 3.3 discusses how to extend the algebraic expressions of system configurations by defining the performance operators for evaluating various system performance metrics. Afterward, Section 3.4 introduces the concept of inverse operators and illustrates how to use the inverse operators to adjust individual station requirements to improve system performances. A case study is presented in Section 3.5 to show some potential applications of the proposed algebraic modeling method. Finally, conclusions and future work are provided in Section 3.6.

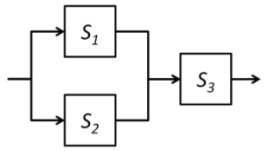
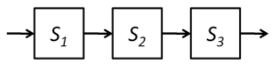
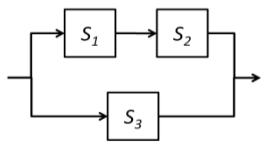
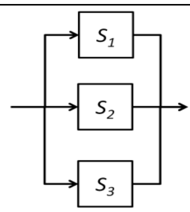
3.2 Algebraic representation of assembly system configurations

In this section, we describe how to effectively model the station configurations for a general assembly system with a hybrid configuration structure. Specifically, we propose the use of an algebraic expression with two binary operators, $\otimes$ and $\oplus$, to represent the serial and parallel relationship between two stations, respectively. The operands in these algebraic expressions are two associated stations. For example, $S_1 \otimes S_2$ is used to represent two stations with a serial configuration layout, while $S_1 \oplus S_2$ is used for two stations with a parallel configuration layout.

To enable comparison with the existing methods of block diagram and adjacency matrix, Table 3-1 shows these three equivalent ways for representing five simple assembly systems configurations consisting of three stations labeled as S_1, S_2 , and S_3. It can be seen that the proposed algebraic expression keeps the explicit representation of the serial/parallel configuration, like block diagrams, thus providing a better representation than the adjacency matrix method. Furthermore, mathematical computation algorithms can be easily added into these algebraic operators for evaluating system performance metrics. For example, $\otimes_Q$ and $\oplus_Q$ will be defined later for evaluating the quality conforming rate of the tasks performed at the given stations, and $\otimes_T$ and $\oplus_T$ will be defined for evaluating the production cycle time of the tasks performed at the given stations. Therefore, the proposed algebraic expression method also shows a better computational representation than the block diagram method.

Table 3-1 Three equivalent representations of system configuration

Block diagram	Adjacency matrix H	Algebraic expression
	$\boldsymbol{H} = \begin{array}{c c} & \begin{array}{ccc} S_1 & S_2 & S_3 \end{array} \\ \begin{array}{c} S_1 \\ S_2 \\ S_3 \end{array} & \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{array}$	$S_1 \otimes (S_2 \oplus S_3)$

	$$\boldsymbol{H} = \begin{array}{c} \\ S_1 \\ S_2 \\ S_3 \end{array} \begin{array}{c} \begin{array}{ccc} S_1 & S_2 & S_3 \end{array} \\ \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \end{array}$$	$(S_1 \oplus S_2) \otimes S_3$
	$$\boldsymbol{H} = \begin{array}{c} \\ S_1 \\ S_2 \\ S_3 \end{array} \begin{array}{c} \begin{array}{ccc} S_1 & S_2 & S_3 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \end{array}$$	$S_1 \otimes S_2 \otimes S_3$
	$$\boldsymbol{H} = \begin{array}{c} \\ S_1 \\ S_2 \\ S_3 \end{array} \begin{array}{c} \begin{array}{ccc} S_1 & S_2 & S_3 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{array}$$	$S_1 \oplus (S_2 \otimes S_3)$
	$$\boldsymbol{H} = \begin{array}{c} \\ S_1 \\ S_2 \\ S_3 \end{array} \begin{array}{c} \begin{array}{ccc} S_1 & S_2 & S_3 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{array}$$	$S_1 \oplus S_2 \oplus S_3$

Since the block diagram and adjacency matrix representations are commonly used in practice for representing system configurations, it will be practical to develop transferring algorithms for automatically obtaining an equivalent algebraic expression from either a block diagram or an adjacency matrix. The following two subsections will discuss these transferring algorithms.

3.2.1 Algebraic representation transferred from system block diagram

This subsection shows how an algebraic expression can be directly obtained from a block diagram representation. Let us consider an example of an assembly system with a system block diagram as shown in Figure 3-2. Figure 3-3 shows the detailed step-by-step transferring procedures. At each step, every pair of stations are grouped with a serial or parallel configuration by using operator $\otimes$ or $\oplus$, accordingly; this generates an equivalent station to represent the sub-grouped stations. For example, we first combine station S_2 with S_3 and generate the equivalent station $S_{2,3}$=$S_2 \otimes S_3$, as shown in Figure 3-3 (b). Next, we combine group $S_{2,3}$ with station S_4 and generate the equivalent station $S_{2,3,4}$=$(S_2 \otimes S_3) \oplus S_4$, as shown in Figure 3-3 (c). By performing all steps (a) through (e) shown in Figure 3-3, the algebraic expression of the whole system is

$$S_{1,2,3,4,5} = S_1 \otimes [(S_2 \otimes S_3) \oplus S_4] \otimes S_5 \tag{3-1}$$

The detail results of the transferred algebraic expressions at each step are summarized in Table 3-2.

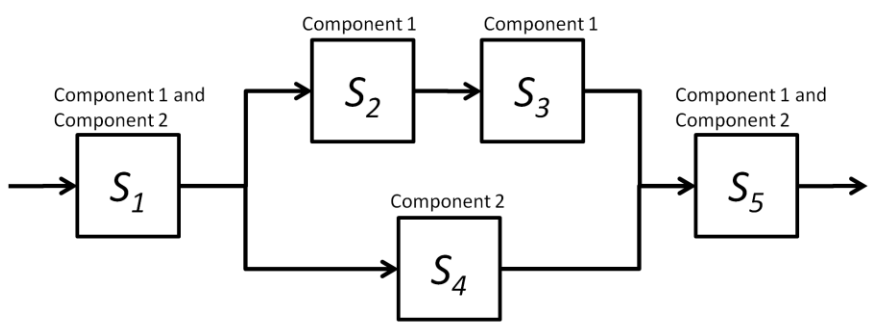


Figure 3-2 A modular assembly system configuration

Table 3-2 Obtaining algebraic expression of system configuration

	Station(s) included	Relationship	Operator used	Algebraic expression
Step 1	S_2 and S_3	Serial	$\otimes$	$S_{2,3} = S_2 \otimes S_3$
Step 2	S_4	Parallel	$\oplus$	$S_{2,3,4} = S_{2,3} \oplus S_4 = (S_2 \otimes S_3) \oplus S_4$
Step 3	S_1	Serial	$\otimes$	$S_{1,2,3,4} = S_1 \otimes S_{2,3,4} = S_1 \otimes [(S_2 \otimes S_3) \oplus S_4]$
Step 4	S_5	Serial	$\otimes$	$S_{1,2,3,4,5} = S_{1,2,3,4} \otimes S_5 = S_1 \otimes [(S_2 \otimes S_3) \oplus S_4] \otimes S_5$

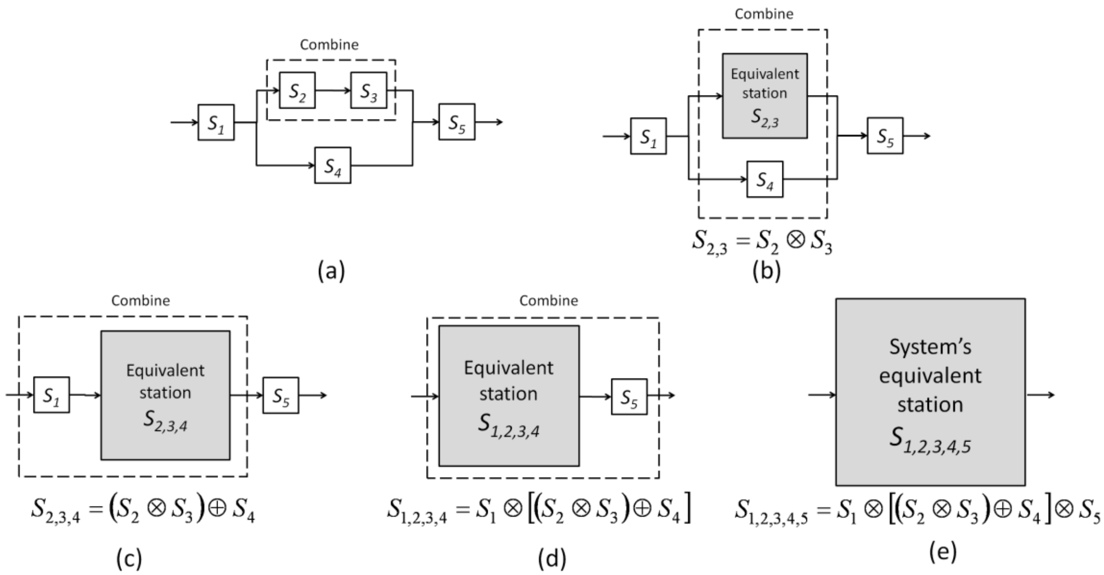


Figure 3-3 Equivalent station grouping

3.2.2 Algebraic representation transferred from adjacency matrix

In an adjacency matrix ***H*** representing the configuration of the stations in an assembly system, every column and every row of matrix ***H*** have been labeled according to the station they stand for. The transferring algorithm is proposed by iteratively combining two stations (i.e., simultaneously combining two rows and two columns of matrix ***H***) until all stations are combined into a single station (single entry in matrix ***H***). At each step, we group two stations (or equivalent stations) by the appropriate algebraic expression based on their configuration relationship (either serial or parallel). The transferring algorithm is shown by the flowchart in Figure 3-4. Table 3-3 illustrates an example of how the algorithm is used to transfer the adjacency matrix of the assembly system in Figure 3-2 into the algebraic expression representation.

While matrix ***H*** has more than one element **Do**
While there exist elements $h_{k,i} = h_{k,j} = 1$ AND $\exists l: h_{i,l} = h_{j,l} = 1$ for labels i, j, k and l **Do**
Create new row by combining row i and row j and label it $(i \oplus j)$
Set element $h_{k,(i\oplus j)} \leftarrow 1$ and $h_{r,(i\oplus j)} \leftarrow 0\ \forall r$
Eliminate rows i and j
Create new column by combining column i and column j and label it $(i \oplus j)$
Set element $h_{(i\oplus j),l} \leftarrow 1$ and $h_{(i\oplus j),r} \leftarrow 0\ \forall r$
Eliminate columns i and j
End While
While there exists elements $h_{i,j} = 1$, $h_{i,k} = 0$ for $k \neq j$ AND $h_{l,j} = 0$ for $l \neq i$ for labels i, j, k and l **Do**
Create new row by combining row i and row j and label it $(i \otimes j)$
Set element $h_{(i\otimes j),r} \leftarrow h_{j,r}\ \forall r$
Eliminate rows i and j
Create new column by combining column i and column j and label it $(i \otimes j)$
Set element $h_{r,(i\otimes j)} \leftarrow h_{r,i}\ \forall r$
Eliminate columns i and j
End While
End While

Figure 3-4 Algorithm for transferring adjacency matrix into algebraic expression

3.2.3 Algebraic representation of multiple tasks within a station

When multiple tasks are executed within a single station, as shown in Figure 3-5, we assume in the paper that those tasks are executed in a sequential order. Therefore, an equivalent serial configuration can be used for representing those tasks. For example, if two tasks A and B are executed within station k, then the algebraic expression of station k can be represented as $S_k = S_k^A \otimes S_k^B$, where S_k^i represents task i $(i = A, B)$ at station k.

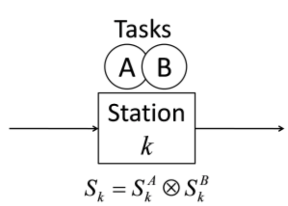


Figure 3-5 Algebraic representation of multiple tasks in a single station

Table 3-3 Transfer of an adjacency matrix into an algebraic expression

Adjacency matrix *H*	Step	Algorithm line #
$\begin{array}{cc} & \begin{array}{ccccc} S_1 & S_2 & S_3 & S_4 & S_5 \end{array} \\ \begin{array}{c} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \end{array} & \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$	Step 0 (initial)	
$\begin{array}{cc} & \begin{array}{ccccc} S_1 & \boldsymbol{S_2} & \boldsymbol{S_3} & S_4 & S_5 \end{array} \\ \begin{array}{c} S_1 \\ \boldsymbol{S_2} \\ \boldsymbol{S_3} \\ S_4 \\ S_5 \end{array} & \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & \mathbf{1} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \rightarrow \begin{array}{cc} & \begin{array}{cccc} S_1 & (S_2 \otimes S_3) & S_4 & S_5 \end{array} \\ \begin{array}{c} S_1 \\ (S_2 \otimes S_3) \\ S_4 \\ S_5 \end{array} & \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$	Step 1 $S_{2,3} = S_2 \otimes S_3$	**Lines:** 10-17 $i = S_2$ $j = S_3$ **Type:** serial **Operator:** $\otimes$ **New row/column label:** $S_2 \otimes S_3$
$\begin{array}{cc} & \begin{array}{cccc} S_1 & (\boldsymbol{S_2} \otimes \boldsymbol{S_3}) & \boldsymbol{S_4} & S_5 \end{array} \\ \begin{array}{c} S_1 \\ (\boldsymbol{S_2} \otimes \boldsymbol{S_3}) \\ \boldsymbol{S_4} \\ S_5 \end{array} & \begin{bmatrix} 0 & \mathbf{1} & \mathbf{1} & 0 \\ 0 & 0 & 0 & \mathbf{1} \\ 0 & 0 & 0 & \mathbf{1} \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \rightarrow$ $\begin{array}{cc} & \begin{array}{ccc} S_1 & \big((S_2 \otimes S_3) \oplus S_4\big) & S_5 \end{array} \\ \begin{array}{c} S_1 \\ \big((S_2 \otimes S_3) \oplus S_4\big) \\ S_5 \end{array} & \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \end{array}$	Step 2 $S_{2,3,4} = S_{2,3} \oplus S_4$	**Lines:** 2-9 $i = S_{2,3}$ $j = S_4$ $k = S_1$ **Type:** parallel **Operator:** $\oplus$ **New row/column label:** $(S_2 \otimes S_3) \oplus S_4$
$\begin{array}{cc} & \begin{array}{ccc} \boldsymbol{S_1} & \big((\boldsymbol{S_2} \otimes \boldsymbol{S_3}) \oplus \boldsymbol{S_4}\big) & S_5 \end{array} \\ \begin{array}{c} \boldsymbol{S_1} \\ \big((\boldsymbol{S_2} \otimes \boldsymbol{S_3}) \oplus \boldsymbol{S_4}\big) \\ S_5 \end{array} & \begin{bmatrix} 0 & \mathbf{1} & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \end{array} \rightarrow$ $\begin{array}{cc} & \begin{array}{cc} \Big(S_1 \otimes \big((S_2 \otimes S_3) \oplus S_4\big)\Big) & S_5 \end{array} \\ \begin{array}{c} \Big(S_1 \otimes \big((S_2 \otimes S_3) \oplus S_4\big)\Big) \\ S_5 \end{array} & \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \end{array}$	Step 3 $S_{1,2,3,4} = S_1 \otimes S_{2,3,4}$	**Lines:** 10-17 $i = S_1$ $j = S_{2,3,4}$ **Type:** parallel **Operator:** $\otimes$ **New row/column label:** $S_1 \otimes \big((S_2 \otimes S_3) \oplus S_4\big)$
$\begin{array}{cc} & \begin{array}{cc} \Big(\boldsymbol{S_1} \otimes \big((\boldsymbol{S_2} \otimes \boldsymbol{S_3}) \oplus \boldsymbol{S_4}\big)\Big) & \boldsymbol{S_5} \end{array} \\ \begin{array}{c} \Big(\boldsymbol{S_1} \otimes \big((\boldsymbol{S_2} \otimes \boldsymbol{S_3}) \oplus \boldsymbol{S_4}\big)\Big) \\ \boldsymbol{S_5} \end{array} & \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \end{array} \rightarrow$ $\begin{array}{cc} & \Big(S_1 \otimes \big((S_2 \otimes S_3) \oplus S_4\big)\Big) \otimes S_5 \\ \Big(S_1 \otimes \big((S_2 \otimes S_3) \oplus S_4\big)\Big) \otimes S_5 & [0] \end{array}$	Step 4 (final) $S_{1,2,3,4,5} = S_{1,2,3,4} \otimes S_5$	**Lines:** 10-17 $i = S_{1,2,3,4,5}$ $j = S_5$ **Type:** serial **Operator:** $\otimes$ **New row/column label:** $\big(S_1 \otimes$ *S2⨂S3⨁S4⨂S5*

3.3 Algebraic representation of system performance

It should be noted that the previously defined algebraic operators $\otimes$ and $\oplus$ are mainly used to represent the station layout configurations, which use stations as their operands and imply no mathematical operations. In this section, we will discuss how to assign specific mathematical operations to the operators $\otimes$ and $\oplus$ for evaluating assembly system performances. Since the process capability for complexity (PCC) is closely associated with the process quality metric (Abad and Jin, 2010), we will first discuss how to use the proposed algebraic expression to calculate quality conforming rates. Specifically, we will define quality performance operators by adding the subscript Q into the station configuration operators $\otimes$ and $\oplus$, i.e. $\otimes_Q$ and $\oplus_Q$, for evaluating the quality conforming rate of the tasks performed at the associated station. Therefore, $\otimes_Q$ and $\oplus_Q$ are called *quality performance operators* in this paper. In contrast to station configuration operators $\otimes$ and $\oplus$, quality performance operators $\otimes_Q$ and $\oplus_Q$ use the tasks assigned to the associated stations as their operands and convey mathematical operations for computing quality conforming rates of the corresponding tasks. Similarly, *cycle time performance operators* $\otimes_T$ and $\oplus_T$ are associated with mathematical operations for evaluating production cycle time for the tasks to produce specific product type at the associated stations. It should be noted that in a mixed model assembly process, the quality conforming rate and production cycle time should be analyzed for each product type throughout all related stations. The details of those performance metrics will be given in the following subsections.

3.3.1 Representation of quality metric for single station

Based on Abad and Jin (2010), if a mixed model production process is required to produce N types of different parts, the quality transformation matrix at station k ($k = 1, \dots, M,$ where M is the total number of stations) can be represented by a $(N+1) \times (N+1)$ square matrix $\boldsymbol{\Psi^k}$ as

$$\boldsymbol{\Psi^k} = \begin{bmatrix} \psi_{00}^k & 0 & \cdots & 0 & \psi_{0\varepsilon}^k \\ 0 & \psi_{11}^k & \cdots & 0 & \psi_{1\varepsilon}^k \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & \psi_{N-1,N-1}^k & \psi_{N-1,\varepsilon}^k \\ 0 & 0 & \cdots & 0 & 1 \end{bmatrix} \tag{3-2}$$

where $\psi_{ii}^k = Prob\{\text{Producing a conforming product type } i \text{ at station } k\}$ and $\psi_{i\varepsilon}^k = Prob\{\text{Producing a nonconforming product type } i \text{ at station } k\},$ thus, $\psi_{ii}^k = 1 - \psi_{i\varepsilon}^k$. Here, $\psi_{NN}^k \equiv 1$ stands for the fact that there is no rework or correction performed on nonconforming parts entering at station k. Also, for consistency with the matrix formulation of the model, if station k has no production operation on part type i, we will set $\psi_{ii}^k = 1$ and $\psi_{i\varepsilon}^k = 0$, which means no quality loss at station k for part type i.

The input of the demand mix ratio at each station is represented by vector $\boldsymbol{\pi^{IN,k}} = \left[\pi_0^{IN,k}, \dots, \pi_{N-1}^{IN,k}, \pi_\varepsilon^{IN,k}\right]^T$, where $\pi_i^{IN,k}$ is the input demand ratio of part type i at station k. The element $\pi_\varepsilon^{IN,k}$ corresponds to the portion of nonconforming products produced at the immediately previous station(s), which is considered as the pseudo input of station k for consistency of the model representation of the whole manufacturing system. Since $\boldsymbol{\pi^{IN,k}}$ is a vector containing the portions of the demand corresponding to every product type, it is constrained to satisfy $\sum_i \pi_i^{IN,k} + \pi_\varepsilon^{IN,k} = 1$.

Similarly, the output of the produced part mix ratio at station k can be represented as $\boldsymbol{\pi}^{\boldsymbol{OUT},\boldsymbol{k}} = \left[\pi_0^{OUT,k}, \dots, \pi_{N-1}^{OUT,k}, \pi_\varepsilon^{OUT,k}\right]^T$ with $\pi_i^{OUT,k}$ corresponding to the output mix ratio of part type i produced at station k. The following relationship between the input mix ratio $\boldsymbol{\pi}^{\boldsymbol{IN},\boldsymbol{k}}$ and the output mix ratio $\boldsymbol{\pi}^{\boldsymbol{OUT},\boldsymbol{k}}$ holds

$$\boldsymbol{\pi}^{\boldsymbol{OUT},\boldsymbol{k}} = \left\{\boldsymbol{\Psi}^{\boldsymbol{k}}\right\}^T \cdot \boldsymbol{\pi}^{\boldsymbol{IN},\boldsymbol{k}} \tag{3-3}$$

Therefore, $\boldsymbol{\Psi}^{\boldsymbol{k}}$ can be considered as a transfer function to represent the station k's quality performance as shown in Figure 3-6.

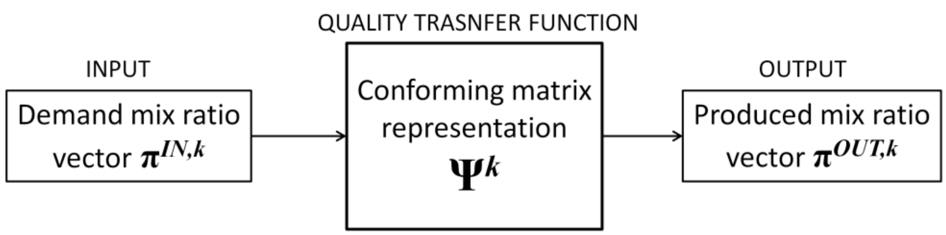


Figure 3-6 Quality transfer function

3.3.2 Algebraic operators for quality metric of multiple stations using equivalent station representation

The concept of the equivalent station is defined for iteratively calculating the quality transfer function when products are manufactured by assembly (sub) systems with multiple stations $i_1, i_2, \dots, i_n$; this is denoted by $E(i_1, i_2, \dots, i_n)$. By using such an equivalent station representation, the overall quality transfer function, represented by the conforming matrix $\boldsymbol{\Psi}^{\boldsymbol{E}(\boldsymbol{i_1},\boldsymbol{i_2},\dots,\boldsymbol{i_n})}$ is used to describe the output of the conforming rate after parts pass through multiple stations $i_1, i_2, \dots, i_n$. Similarly, the input demand mix ratio and the output-produced mix ratio for the equivalent station can be denoted as $\boldsymbol{\pi}^{\boldsymbol{IN},\boldsymbol{E}(\boldsymbol{i_1},\boldsymbol{i_2},\dots,\boldsymbol{i_n})}$ and $\boldsymbol{\pi}^{\boldsymbol{OUT},\boldsymbol{E}(\boldsymbol{i_1},\boldsymbol{i_2},\dots,\boldsymbol{i_n})}$, respectively. By extending the quality transfer function from a single station to an equivalent station including multiple stations, we can obtain the following relationship

$$\boldsymbol{\pi}^{\boldsymbol{OUT},\boldsymbol{E}(\boldsymbol{i}_1,\boldsymbol{i}_2,\dots,\boldsymbol{i}_n)} = \left\{\boldsymbol{\Psi}^{\boldsymbol{E}(\boldsymbol{i}_1,\boldsymbol{i}_2,\dots,\boldsymbol{i}_n)}\right\}^T \cdot \boldsymbol{\pi}^{\boldsymbol{IN},\boldsymbol{E}(\boldsymbol{i}_1,\boldsymbol{i}_2,\dots,\boldsymbol{i}_n)} \tag{3-4}$$

where $\boldsymbol{\Psi}^{\boldsymbol{E}(\boldsymbol{i}_1,\boldsymbol{i}_2,\dots,\boldsymbol{i}_n)}$ is defined in a similar way as matrix $\boldsymbol{\Psi}^{\boldsymbol{k}}$. Since $\boldsymbol{\Psi}^{\boldsymbol{E}(\boldsymbol{i}_1,\boldsymbol{i}_2,\dots,\boldsymbol{i}_n)}$ is a function of $\boldsymbol{\Psi}^{\boldsymbol{i}_1}, \boldsymbol{\Psi}^{\boldsymbol{i}_2}, \dots, \boldsymbol{\Psi}^{\boldsymbol{i}_n}$, and $\boldsymbol{\Psi}^{\boldsymbol{i}_k}$ is a diagonal station, the resultant equivalent station $\boldsymbol{\Psi}^{\boldsymbol{E}(\boldsymbol{i}_1,\boldsymbol{i}_2,\dots,\boldsymbol{i}_n)}$ is a diagonal station as well. The calculation of $\boldsymbol{\Psi}^{\boldsymbol{E}(\boldsymbol{i}_1,\boldsymbol{i}_2,\dots,\boldsymbol{i}_n)}$ will be conducted step-by-step by iteratively calculating the quality transfer function between two sub-grouped equivalent stations with either a serial or parallel configuration. The corresponding algebraic operators will be defined by the following Proposition 1.

Proposition 1: Let $\boldsymbol{\Psi}^{\boldsymbol{i}}$ and $\boldsymbol{\Psi}^{\boldsymbol{j}}$ be two quality conforming matrices corresponding to station i and station j, respectively. The quality conforming matrix, denoted as $\boldsymbol{\Psi}^{\boldsymbol{E}(\boldsymbol{i},\boldsymbol{j})}$, can be calculated by using the algebraic operators $\otimes_Q$ and $\oplus_Q$, which are defined as follows

(i) For a serial configuration between station i and j,

$$\boldsymbol{\Psi}^{\boldsymbol{E}(\boldsymbol{i},\boldsymbol{j})} = \boldsymbol{\Psi}^{\boldsymbol{i}} \otimes_Q \boldsymbol{\Psi}^{\boldsymbol{j}} = \left\{\sum_s \psi_{rs}^i \psi_{sv}^j\right\}_{rv} \tag{3-5}$$

(ii) For a parallel configuration between station i and j when the tasks at both stations are required to produce each product,

$$\boldsymbol{\Psi}^{\boldsymbol{E}(\boldsymbol{i},\boldsymbol{j})} = \boldsymbol{\Psi}^{\boldsymbol{i}} \oplus_Q \boldsymbol{\Psi}^{\boldsymbol{j}} = \begin{cases} \left\{\min\left(\psi_{rv}^i, \psi_{rv}^j\right)\right\}_{rv} & \text{If } v \neq N+1 \\ \left\{\max\left(\psi_{rv}^i, \psi_{rv}^j\right)\right\}_{rv} & \text{If } v = N+1 \end{cases} \tag{3-6}$$

(iii) For a parallel configuration between station i and j when the tasks at both stations are exact replications of each other and chosen with probability ω_i and ω_j, respectively,

$$\boldsymbol{\Psi}^{\boldsymbol{E(i,j)}} = \boldsymbol{\Psi}^{\boldsymbol{i}} \oplus_Q \boldsymbol{\Psi}^{\boldsymbol{j}} = \left\{\omega_i \psi_{rv}^{i} + \omega_j \psi_{rv}^{j}\right\}_{rv} \tag{3-7}$$

where $\{\cdot\}_{rv}$ is the r^{th} row and v^{th} column element of matrix $\boldsymbol{\Psi}^{\boldsymbol{E(i,j)}}$. Without losing generality of the algebraic representation, in the following discussion we will consider the parallel configuration described only by scenario *(ii).*

<u>Justification for Eq. (3-5) in Proposition 1:</u> Suppose that two stations, denoted by i and j, are in a serial configuration and that station i directly precedes station j. Based on Eq. (3-3) we have

$$\boldsymbol{\pi}^{\boldsymbol{OUT,i}} = \{\boldsymbol{\Psi}^{\boldsymbol{i}}\}^T \cdot \boldsymbol{\pi}^{\boldsymbol{IN,i}} \tag{3-8}$$

and

$$\boldsymbol{\pi}^{\boldsymbol{OUT,j}} = \{\boldsymbol{\Psi}^{\boldsymbol{j}}\}^T \cdot \boldsymbol{\pi}^{\boldsymbol{IN,j}}. \tag{3-9}$$

Since the output mix ratio of station i, denoted by $\boldsymbol{\pi}^{\boldsymbol{OUT,i}}$, is considered as the input demand mix ratio of station j, denoted by $\boldsymbol{\pi}^{\boldsymbol{IN,j}}$. Combining Eq. (3-8) and Eq. (3-9), we have

$$\boldsymbol{\pi}^{\boldsymbol{OUT,j}} = \{\boldsymbol{\Psi}^{\boldsymbol{j}}\}^T \cdot \{\boldsymbol{\Psi}^{\boldsymbol{i}}\}^T \cdot \boldsymbol{\pi}^{\boldsymbol{IN,i}}$$

$$\boldsymbol{\pi}^{\boldsymbol{OUT,j}} = \{\boldsymbol{\Psi}^{\boldsymbol{i}} \cdot \boldsymbol{\Psi}^{\boldsymbol{j}}\}^T \cdot \boldsymbol{\pi}^{\boldsymbol{IN,i}}.$$

Hence Eq. (3-5) in Proposition 1 is justified.

<u>Justification of Eq. (3-6) in Proposition 1:</u> Two stations, denoted by i and j, connected in a parallel configuration have the same demand mix ratio, i.e., $\boldsymbol{\pi}^{\boldsymbol{IN,E(i,j)}} = \boldsymbol{\pi}^{\boldsymbol{IN,i}} =$

$\boldsymbol{\pi}^{\boldsymbol{IN,j}}$, since the final product consists of two components required to be produced at station i and station j. Let us consider the proportion of products of type l produced at station i and station j, denoted by $\boldsymbol{\pi}_{\boldsymbol{l}}^{\boldsymbol{OUT,i}}$ and $\boldsymbol{\pi}_{\boldsymbol{l}}^{\boldsymbol{OUT,j}}$, respectively. The main assumption here is that each product type must pass through both stations. Thus, the effective proportions of conforming products of type l is the minimum of the individual proportions of conforming products of type l at station i and station j, i.e., $\min(\pi_l^{OUT,i}, \pi_l^{OUT,j})$.

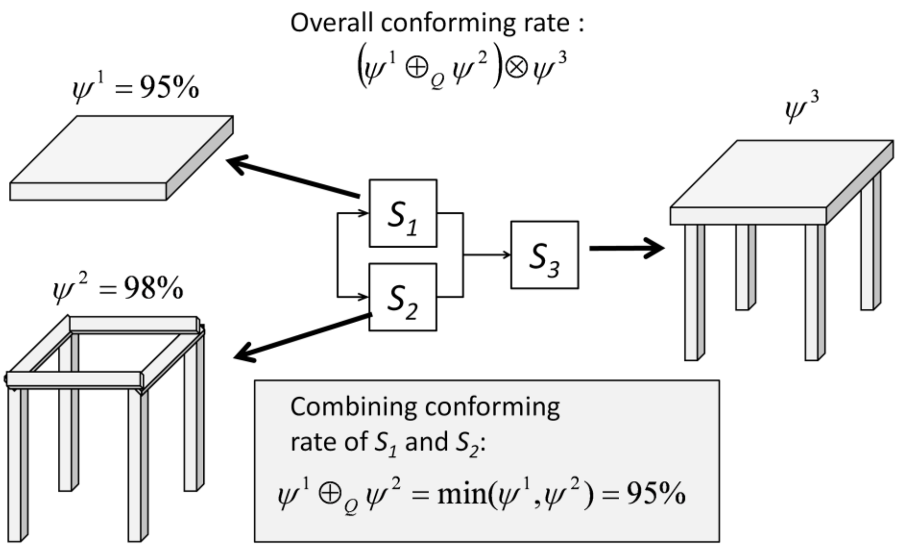


Figure 3-7 Example of parallel stations and algebraic operators for quality metric

For example, consider the process for producing a table, as shown in Figure 3-7, which consists of three stations as follows: station 1 produces the top of the table, station 2 produces a set of four legs, and station 3 assembles the top of the table with the four legs. Suppose that the proportion of conforming products produced at stations 1 and 2 are $\psi^1 = 95\%$ and $\psi^2 = 98\%$, respectively. As a consequence, the combined conforming rate between the top and the set of four legs is equal to $\min(\psi^1, \psi^2) = 95\%$, because we can only obtain 95% of the demanded number of conforming tables when entering station 3.

Since we only consider stations that can be represented by a matrix of the form presented in Eq. (3-2), by Eq. (3-3), we see that

$$\min(\pi_l^{OUT,i}, \pi_l^{OUT,j}) = \min\left(\psi_{ll}^i, \psi_{ll}^j\right) \cdot \pi_l^{IN,E(i,j)} \tag{3-10}$$

Based on a similar argument, we can calculate the proportion of nonconforming products of type l produced at station i and station j, denoted by $\pi_\varepsilon^{OUT,i}$ and $\pi_\varepsilon^{OUT,j}$, respectively. Since every product type l must be processed by both station i and station j, the resultant nonconforming rate of product type l produced by these two stations is equal to the maximum of the nonconforming rates of products of type l at these two stations, i.e., $\max(\pi_\varepsilon^{OUT,i}, \pi_\varepsilon^{OUT,j})$. Now, since we consider only diagonal stations, based on Eq. (3-3), it yields

$$\max(\pi_\varepsilon^{OUT,i}, \pi_\varepsilon^{OUT,j}) = \max\left(\psi_{l\varepsilon}^i, \psi_{l\varepsilon}^j\right) \cdot \pi_\varepsilon^{IN,E(i,j)}. \tag{3-11}$$

Based on Eq. (3-10) and Eq. (3-11), Eq. (3-6) in Proposition 1 is justified.

<u>*Illustrative example:*</u> An assembly system, such as the one shown in Figure 3-2, is used to produce a product consisting of two components, each with two different variants, for a total of four possible product types (four possible components combinations). Table 3-4 (a) gives the demand mix ratio of these four product types, $\pi_i^{IN,E(\cdot)}$ $(i = 1,2,3,4)$, where $E(\cdot)$ corresponds to an equivalent station containing every station in the system. For production planning, the demand mix ratio of all products having component 1 with

variant 1 and variant 2 can be calculated by $P_{11} = \pi_1^{IN,E(\cdot)} + \pi_2^{IN,E(\cdot)}$ and $P_{12} = \pi_3^{IN,E(\cdot)} + \pi_4^{IN,E(\cdot)}$, respectively. Similarly, the demand mix ratio of all products having component 2 with variants 1 and 2 can be calculated by $P_{21} = \pi_1^{IN,E(\cdot)} + \pi_3^{IN,E(\cdot)}$ and $P_{22} = \pi_2^{IN,E(\cdot)} + \pi_4^{IN,E(\cdot)}$, respectively. The results are given in Table 3-4 (b).

Table 3-4 (a) Demand mix ratio in terms of product types **(b)** Demand mix ratio in terms of component variants

(a)

	Variant on component 1	Variant on component 2	Demand mix ratio
Product type 1	1	1	$\pi_1^{IN,E(\cdot)} = 45\%$
Product type 2	1	2	$\pi_2^{IN,E(\cdot)} = 25\%$
Product type 3	2	1	$\pi_3^{IN,E(\cdot)} = 20\%$
Product type 4	2	2	$\pi_4^{IN,E(\cdot)} = 10\%$

(b)

	Demand mix ratio
Component 1 variant 1	$P_{11} = 70\%$
Component 1 variant 2	$P_{12} = 30\%$
Component 2 variant 1	$P_{21} = 65\%$
Component 2 variant 2	$P_{22} = 35\%$

Table 3-5 describes the required operations at each station and their corresponding quality conforming rates. As shown in Figure 3-2, stations 1 and 5 are basic operation stations, while stations 2, 3, and4 are variant operation stations.

Table 3-5 Task assignment and their corresponding conforming rates

		Conforming rates			
		Component 1		Component 2	
	Component processed	Variant 1	Variant 2	Variant 1	Variant 2
Station 1	1 and 2	92%	93%	94%	90%
Station 2	1	94%	94%	-	-
Station 3	1	96%	98%	-	-
Station 4	2	-	-	96%	92%
Station 5	1 and 2	93%	94%	93%	90%

Since station 1 and station 5 have more than one task, based on Figure 3-5, we can denote S_k^i as the individual task for producing component i at station k. In this way, stations 1 and 5 are represented in terms of their assigned tasks as $S_1 = S_1^1 \otimes S_1^2$ and $S_2 = S_5^1 \otimes S_5^2$, respectively. Furthermore, we can define $\boldsymbol{\Psi}^{\boldsymbol{k,i}}$ to represent the quality conforming matrix of individual task i at station k. Based on Table 3-5, matrices $\boldsymbol{\Psi}^{\mathbf{1,1}}$, $\boldsymbol{\Psi}^{\mathbf{1,2}}$, $\boldsymbol{\Psi}^{\mathbf{5,1}}$ and $\boldsymbol{\Psi}^{\mathbf{5,2}}$ are represented as

$$\boldsymbol{\Psi}^{\mathbf{1,1}} = \begin{bmatrix} 0.9200 & 0 & 0 & 0 & 0.0800 \\ 0 & 0.9200 & 0 & 0 & 0.0800 \\ 0 & 0 & 0.9300 & 0 & 0.0700 \\ 0 & 0 & 0 & 0.9300 & 0.0700 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\boldsymbol{\Psi}^{\mathbf{1,2}} = \begin{bmatrix} 0.9400 & 0 & 0 & 0 & 0.0600 \\ 0 & 0.9000 & 0 & 0 & 0.1000 \\ 0 & 0 & 0.9400 & 0 & 0.0600 \\ 0 & 0 & 0 & 0.9000 & 0.1000 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\boldsymbol{\Psi}^{\mathbf{5,1}} = \begin{bmatrix} 0.9300 & 0 & 0 & 0 & 0.0700 \\ 0 & 0.9300 & 0 & 0 & 0.0700 \\ 0 & 0 & 0.9400 & 0 & 0.0600 \\ 0 & 0 & 0 & 0.9400 & 0.0600 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\boldsymbol{\Psi}^{\mathbf{5,2}} = \begin{bmatrix} 0.9300 & 0 & 0 & 0 & 0.0700 \\ 0 & 0.9000 & 0 & 0 & 0.1000 \\ 0 & 0 & 0.9300 & 0 & 0.0700 \\ 0 & 0 & 0 & 0.9000 & 0.1000 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Therefore, the conforming matrix of stations 1 and 5 can be obtained by $\boldsymbol{\Psi}^{\mathbf{1}} = \boldsymbol{\Psi}^{\mathbf{1,1}} \otimes_Q \boldsymbol{\Psi}^{\mathbf{1,2}}$ and $\boldsymbol{\Psi}^{\mathbf{5}} = \boldsymbol{\Psi}^{\mathbf{5,1}} \otimes_Q \boldsymbol{\Psi}^{\mathbf{5,2}}$, respectively. For example, $\psi_{11}^{1} = \psi_{11}^{1,1} \times \psi_{11}^{1,2} = 0.92 \times 0.94 = 0.8648$. Therefore,

$$\mathbf{\Psi^1} = \begin{bmatrix} 0.8648 & 0 & 0 & 0 & 0.1352 \\ 0 & 0.8280 & 0 & 0 & 0.1720 \\ 0 & 0 & 0.8742 & 0 & 0.1258 \\ 0 & 0 & 0 & 0.8370 & 0.1630 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{\Psi^5} = \begin{bmatrix} 0.8649 & 0 & 0 & 0 & 0.1351 \\ 0 & 0.8370 & 0 & 0 & 0.1630 \\ 0 & 0 & 0.8742 & 0 & 0.1258 \\ 0 & 0 & 0 & 0.8460 & 0.1540 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

For stations 2, 3, and 4 with a single task, the corresponding conforming matrices $\mathbf{\Psi^2}$, $\mathbf{\Psi^3}$ and $\mathbf{\Psi^4}$ are directly obtained from Table 3-5 as

$$\mathbf{\Psi^2} = \begin{bmatrix} 0.9400 & 0 & 0 & 0 & 0.0600 \\ 0 & 0.9400 & 0 & 0 & 0.0600 \\ 0 & 0 & 0.9400 & 0 & 0.0600 \\ 0 & 0 & 0 & 0.9400 & 0.0600 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{\Psi^3} = \begin{bmatrix} 0.9600 & 0 & 0 & 0 & 0.0400 \\ 0 & 0.9600 & 0 & 0 & 0.0400 \\ 0 & 0 & 0.9800 & 0 & 0.0200 \\ 0 & 0 & 0 & 0.9800 & 0.0200 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{\Psi^4} = \begin{bmatrix} 0.9600 & 0 & 0 & 0 & 0.0400 \\ 0 & 0.9200 & 0 & 0 & 0.0800 \\ 0 & 0 & 0.9600 & 0 & 0.0400 \\ 0 & 0 & 0 & 0.9200 & 0.0800 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

The system's quality transfer function, based on the equivalent station $\mathbf{\Psi}^{\boldsymbol{E}(\mathbf{1,2,3,4,5})} = \mathbf{\Psi^1} \otimes_Q \left[\left(\mathbf{\Psi^2} \otimes_Q \mathbf{\Psi^3}\right) \oplus_Q \mathbf{\Psi^4}\right] \otimes_Q \mathbf{\Psi^5}$, can be calculated by iteratively applying the operators defined in Eq. (3-5) and Eq. (3-6) on two subgrouped stations as follows *(i)* $\boldsymbol{\Psi}^{\boldsymbol{E(2,3)}} = \boldsymbol{\Psi}^{\boldsymbol{2}} \otimes_Q \boldsymbol{\Psi}^{\boldsymbol{3}}$ (Eq. (3-5))

(ii) $\boldsymbol{\Psi}^{\boldsymbol{E(2,3,4)}} = \boldsymbol{\Psi}^{\boldsymbol{4}} \oplus_Q \boldsymbol{\Psi}^{\boldsymbol{E(2,3)}}$ (Eq. (3-6))

(iii) $\boldsymbol{\Psi}^{\boldsymbol{E(1,2,3,4)}} = \boldsymbol{\Psi}^{\boldsymbol{1}} \otimes_Q \boldsymbol{\Psi}^{\boldsymbol{E(2,3,4)}}$ (Eq. (3-5))

(iv) $\boldsymbol{\Psi}^{\boldsymbol{E(1,2,3,4,5)}} = \boldsymbol{\Psi}^{\boldsymbol{E(1,2,3,4)}} \otimes_Q \boldsymbol{\Psi}^{\boldsymbol{5}}$ (Eq. (3-5))

The final resultant quality conforming matrix is

$$\boldsymbol{\Psi}^{\boldsymbol{E(1,2,3,4,5)}} = \begin{bmatrix} 0.6750 & 0 & 0 & 0 & 0.3250 \\ 0 & 0.6254 & 0 & 0 & 0.3746 \\ 0 & 0 & 0.7040 & 0 & 0.2960 \\ 0 & 0 & 0 & 0.6515 & 0.3485 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{3-12}$$

3.3.3 Quality (Q) and process capability for complexity (PCC) for whole assembly system

Based on Eq. (3-4), the total quality conforming rate of all product types for the equivalent station $\boldsymbol{\Psi}^{\boldsymbol{E(i_1,i_2,\ldots,i_n)}}$, denoted as $Q^{E(i_1,i_2,\ldots,i_n)}$, can be calculated by

$$Q^{E(i_1,i_2,\ldots,i_n)} = \sum_{j<N} \pi_j^{IN} \psi_{jj}^{E(i_1,i_2,\ldots,i_n)} = 1 - \pi_\varepsilon^{OUT,E(i_1,i_2,\ldots,i_n)}. \tag{3-13}$$

Therefore, the quality conforming rate of the whole manufacturing system can be calculated by considering all stations S_1, S_2,…,S_M, as $Q^{E(1,2,\ldots,M)}$.

The process capability for complexity (PCC), defined in Abad and Jin (2010), is a performance metric that assesses how well a production process can handle the demand variety of products in a mixed model manufacturing process. PCC is calculated based on the mutual information index (Cover and Thomas, 2006), which is used to quantify the amount of information that two random variables share.

Assume that the input and output mix ratios are considered as the marginal probability distribution functions $\boldsymbol{\pi}^{\boldsymbol{IN},\boldsymbol{E}(\boldsymbol{i_1},\boldsymbol{i_2},\ldots,\boldsymbol{i_n})}$ and $\boldsymbol{\pi}^{\boldsymbol{OUT},\boldsymbol{E}(\boldsymbol{i_1},\boldsymbol{i_2},\ldots,\boldsymbol{i_n})}$ of two categorical random variables, respectively. If the joint probability matrix is denoted by $\boldsymbol{\pi}^{\boldsymbol{IN},\boldsymbol{OUT},\boldsymbol{E}(\boldsymbol{i_1},\boldsymbol{i_2},\ldots,\boldsymbol{i_n})}$, where the element in the i^{th} row and j^{th} column corresponds to $\pi_{ij}^{IN,OUT,E(i_1,i_2,\ldots,i_n)} = \psi_{ij}^{E(i_1,i_2,\ldots,i_n)}\pi_i^{IN,E(i_1,i_2,\ldots,i_n)}$. Thus, PCC can be calculated by the mutual information index as follows

$$PCC = \sum_{j,l} \pi_{j,l}^{IN,OUT,E(i_1,i_2,\ldots,i_n)} \log \frac{\pi_{j,l}^{IN,OUT,E(i_1,i_2,\ldots,i_n)}}{\pi_j^{IN,E(i_1,i_2,\ldots,i_n)}\pi_l^{OUT,E(i_1,i_2,\ldots,i_n)}} \tag{3-14}$$

A normalized value of PCC, ranging from 0 to 1, called $NPCC$, was also proposed in Abad and Jin (2010) based on the concept of coefficient of constraint (Coombs, Dawes and Tversky, 1970), that is

$$NPCC = PCC/H(D), \tag{3-15}$$

where $H(D)$ is the entropy of the input demand random variable D, given by

$$H(D) = -\sum_i \pi_i^{IN,E(i_1,i_2,\ldots,i_n)} \log \pi_i^{IN,E(i_1,i_2,\ldots,i_n)} \tag{3-16}$$

Illustrative example: We now continue the example in Section 3.3.2 to show how to calculate metrics $Q^{E(\cdot)}$, PCC, and $NPCC$ for the assembly system shown in Figure 3-2. Based on Table 3-4 (a), the input vector of demand mix ratio is represented as

$$\boldsymbol{\pi}^{\boldsymbol{IN},\boldsymbol{E}(\mathbf{1},\mathbf{2},\mathbf{3},\mathbf{4},\mathbf{5})} = [0.45 \quad 0.25 \quad 0.20 \quad 0.10]^T \tag{3-17}$$

By substituting Eq. (3-12) and Eq. (3-17) into Eq. (3-13), $Q^{E(1,2,3,4,5)}$ is calculated as 0.6661. Based on equation $\pi_{ij}^{IN,OUT,E(i_1,i_2,\ldots,i_n)} = \psi_{ij}^{E(i_1,i_2,\ldots,i_n)} \pi_i^{IN,E(i_1,i_2,\ldots,i_n)}$, $\boldsymbol{\pi}^{\boldsymbol{IN,OUT,E}(\mathbf{1,2,3,4,5})}$ is obtained as

$$\boldsymbol{\pi}^{\boldsymbol{IN,OUTE}(\mathbf{1,2,3,4,5})} = \begin{bmatrix} 0.3038 & 0 & 0 & 0 & 0.1462 \\ 0 & 0.1563 & 0 & 0 & 0.0937 \\ 0 & 0 & 0.1408 & 0 & 0.0592 \\ 0 & 0 & 0 & 0.0652 & 0.0349 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{3-18}$$

By using Eqs. (3-12), (3-17), and (3-18) in Eq. (3-14), PCC is obtained as 1.2076. Further, we obtain the results of $H(D) = 1.8150$ based on Eq. (3-16), and $NPCC = 0.6653$ based on Eq. (3-15).

3.3.4 Algebraic operators for cycle time metric of multiple stations using equivalent station representation

When mixed model production processes are considered, the production cycle time needs to be defined to consider each product type at each station. For N types of products, the cycle time vector $\boldsymbol{CT^k}$ at station k is denoted as $\boldsymbol{CT^k} = \left[CT_0^k, \ldots, CT_{N-1}^k\right]^T$, where $CT_i^k \geq 0$ indicates the amount of production time at station k required to produce each product of type i. In this work, CT_i^k is considered as either a deterministic value or the mean of a random cycle time. If product type i is not processed at station k, then $CT_i^k = 0$.

Based on a strategy similar to the one used in Section 3.3.2, we can define the algebraic operators $\otimes_T$ and $\oplus_T$ to compute the cycle time for equivalent stations, which will be given by the following proposition.

Proposition 2: Let $\boldsymbol{CT^i}$ and $\boldsymbol{CT^j}$ be two cycle time vectors corresponding to stations i and j, respectively. The cycle time vector of the equivalent station can be calculated by the following algebraic operators $\otimes_T$ and $\oplus_T$

(i) For a serial configuration between stations *i* and *j*,

$$\boldsymbol{CT^{E(i,j)}} = \boldsymbol{CT^i} \otimes_T \boldsymbol{CT^j} = \left\{CT_s^i + CT_s^j\right\}_s \tag{3-19}$$

(ii) For a parallel configuration between stations *i* and *j*,

$$\boldsymbol{CT^{E(i,j)}} = \boldsymbol{CT^i} \oplus_T \boldsymbol{CT^j} = \left\{\max(CT_s^i, CT_s^j)\right\}_s \tag{3-20}$$

where $\{\cdot\}_s$ corresponds to the s^{th} element of vector $\boldsymbol{CT^{E(i,j)}}$.

Justification of Proposition 2: If two stations are in a serial configuration, the total amount of production time required for a product to pass through these two stations is equal to the sum of the production times at individual stations. On the other hand, if two stations are in a parallel configuration, the total amount of production time required for the two parallel stations is equal to the longest cycle time of these two individual stations. Hence, the definition of operators $\otimes_T$ and $\oplus_T$ are justified.

For a station *k* performing M multiple tasks, as shown in Figure 3-7, the algebraic operator relationship among multiple tasks is considered as a serial configuration, i.e., $\boldsymbol{CT^k} = \boldsymbol{CT^{k,1}} \otimes_T \boldsymbol{CT^{k,2}} \otimes_T \ldots \otimes_T \boldsymbol{CT^{k,M}} = \sum_{\boldsymbol{j=1}}^{\boldsymbol{M}} \boldsymbol{CT^{k,j}}$, where $\boldsymbol{CT^{k,j}}$ indicates the cycle time of task *j* performed at station *k*.

Illustrative example: We continue the example given in Section 3.3.3; Table 3-6 gives the designed cycle time at each station corresponding to each component variant. Since stations 1 and 5 consist of two tasks, $\boldsymbol{CT^k} = \boldsymbol{CT^{k,1}} \otimes_T \boldsymbol{CT^{k,2}}$ $(k = 1,5)$. For example, $CT_1^1 = CT_1^{1,1} + CT_1^{1,2} = 7 + 5 = 12$ minutes. Based on Table 3-6, the cycle time vectors $\boldsymbol{CT^k}$ at each station can be represented as $\boldsymbol{CT^1} = [12,11,13,12]^T$, $\boldsymbol{CT^2} = [12,12,12,12]^T$, $\boldsymbol{CT^3} = [13,13,16,16]^T$, $\boldsymbol{CT^4} = [6,5,6,5]^T$, and $\boldsymbol{CT^5} = [12,13,9,10]^T$.

Table 3-6 Cycle time in terms of component variants

		Cycle time (min)			
		Component 1		Component 2	
	Component processed	Variant 1	Variant 2	Variant 1	Variant 2
Station 1	1 and 2	7	8	5	4
Station 2	1	12	12	-	-
Station 3	1	13	16	-	-
Station 4	2	-	-	6	5
Station 5	1 and 2	6	3	6	7

Furthermore, the total production cycle time for the equivalent system station, denoted as $\boldsymbol{CT^{E(1,2,3,4,5)}}$, can be obtained by iteratively applying the operators $\otimes_T$ and $\oplus_T$ as follows

(i) $\boldsymbol{CT^{E(2,3)}} = \boldsymbol{CT^2} \otimes_T \boldsymbol{CT^3}$ (Eq. (3-19))

(ii) $\boldsymbol{CT^{E(2,3,4)}} = \boldsymbol{CT^4} \oplus_T \boldsymbol{CT^{E(2,3)}}$ (Eq. (3-20))

(iii) $\boldsymbol{CT^{E(1,2,3,4)}} = \boldsymbol{CT^1} \otimes_T \boldsymbol{CT^{E(2,3,4)}}$ (Eq. (3-19))

(iv) $\boldsymbol{CT^{E(1,2,3,4,5)}} = \boldsymbol{CT^{E(1,2,3,4)}} \otimes_T \boldsymbol{CT^5}$ (Eq. (3-19))

The obtained total production cycle time vector for four product types is $\boldsymbol{CT^{E(1,2,3,4,5)}} = [49,49,50,50]^T$ minutes.

3.4 Inverse algebraic operators for improving system performance

This section is used to show how to define the inverse algebraic expressions to systematically analyze the effect of individual stations on the performance of an equivalent system station. Such results can be used to further improve the design of a manufacturing system to achieve a desired system performance or to identify the *weakest link* (such as the bottleneck when considering throughput) in an assembly system under a particular performance criterion. Based on the previously defined algebraic operators $\otimes_Q$, $\oplus_Q$, $\otimes_T$, and $\oplus_T$, the inverse operators $\otimes_Q^{-1}$, $\oplus_Q^{-1}$, $\otimes_T^{-1}$, and $\oplus_T^{-1}$, will be defined as follows.

3.4.1 Algebraic operators $\boldsymbol{\otimes_Q^{-1}}$ and $\boldsymbol{\oplus_Q^{-1}}$ for inverse computation of quality conforming rate

The following proposition will be used to describe the operations corresponding to $\otimes_Q^{-1}$ and $\oplus_Q^{-1}$.

Proposition 3: Let $\boldsymbol{\Psi^i}$ and $\boldsymbol{\Psi^j}$ be two quality conforming matrices corresponding to station i and station j, respectively; and let $\boldsymbol{\Psi^{E(i,j)}}$ be the equivalent station formed by stations i and j. The relationships between each quality operator and its corresponding inverse operator are defined as follows.

(i) For a serial configuration, $\boldsymbol{\Psi^{E(i,j)}} = \boldsymbol{\Psi^i} \otimes_Q \boldsymbol{\Psi^j}$, it yields

$$\boldsymbol{\Psi^i} = \boldsymbol{\Psi^{E(i,j)}} \overrightarrow{\otimes}_Q^{-1} \boldsymbol{\Psi^j} = \boldsymbol{\Psi^{E(i,j)}} \otimes_Q \{\boldsymbol{\Psi^j}\}^{\mathbf{-1}} \tag{3-21}$$

and

$$\boldsymbol{\Psi^j} = \boldsymbol{\Psi^i} \overleftarrow{\otimes}_Q^{-1} \boldsymbol{\Psi^{E(i,j)}} = \{\boldsymbol{\Psi^i}\}^{\mathbf{-1}} \otimes_Q \boldsymbol{\Psi^{E(i,j)}} \tag{3-22}$$

where the arrow $\rightarrow$ on the top of operator $\overrightarrow{\otimes}_Q^{-1}$ in Eq. (3-21) indicates that the operand $\boldsymbol{\Psi^j}$ on the right side of the operator $\overrightarrow{\otimes}_Q^{-1}$ should be computed by the inverse operator. Similar interpretation is given to $\overleftarrow{\otimes}_Q^{-1}$ in Eq. (3-22).

(ii) For a parallel configuration, $\boldsymbol{\Psi^{E(i,j)}} = \boldsymbol{\Psi^i} \oplus_Q \boldsymbol{\Psi^j}$, it yields

$$\boldsymbol{\Psi^i} = \boldsymbol{\Psi^{E(i,j)}} \oplus_Q^{-1} \boldsymbol{\Psi^j} = \begin{cases} \left\{\psi_{rv}^{E(i,j)}\right\}_{rv} & \text{If } \psi_{rv}^{E(i,j)} < \psi_{rv}^{j} \text{ AND } v \neq N+1 \\ \left\{\psi_{rv}^{i} \geq \psi_{rv}^{E(i,j)}\right\}_{rv} & \text{If } \psi_{rv}^{E(i,j)} = \psi_{rv}^{j} \text{ AND } v \neq N+1 \\ \left\{\psi_{rv}^{E(i,j)}\right\}_{rv} & \text{If } \psi_{rv}^{E(i,j)} > \psi_{rv}^{j} \text{ AND } v = N+1 \\ \left\{\psi_{rv}^{i} \geq 1 - \psi_{rv}^{E(i,j)}\right\}_{rv} & \text{If } \psi_{rv}^{E(i,j)} = \psi_{rv}^{j} \text{ AND } v = N+1 \\ \text{unfeasible} & \text{otherwise} \end{cases}$$

(3-23)

<u>Justification of Proposition 3:</u> Since operator $\otimes_Q$ corresponds to the conventional matrix multiplication operation, $\otimes_Q^{-1}$ should naturally correspond to the inverse matrix operation. Algebraic operators $\otimes_Q^{-1}$, and $\otimes_Q$ are not commutative, which is shown in Eq. (3-21) and (3-22). For operator $\oplus_Q^{-1}$, since $\psi_{rv}^{E(i,j)} = \psi_{rv}^{i} \oplus_Q \psi_{rv}^{j} = \min(\psi_{rv}^{i}, \psi_{rv}^{j})$ for $v \neq N+1$ and $\psi_{rv}^{E(i,j)} = \psi_{rv}^{i} \oplus_Q \psi_{rv}^{j} = \max(\psi_{rv}^{i}, \psi_{rv}^{j})$ for $v = N+1$; thus Eq. (3-23) is justified.

<u>Illustrative example:</u> We continue the example of the assembly system shown in Figure 3-2, in which the algebraic expression representation of the equivalent station is $S_{E(1,2,3,4,5)} = S_1 \otimes [(S_2 \otimes S_3) \oplus S_4] \otimes S_5$. Suppose that the plant is not satisfied with the current quality conforming performance as given in Eq. (3-12), and would like to increase the quality conforming rate of product type 1, since product type 1 has the

largest demand. For example, the decision is made to increase the quality rate from $\psi_{11}^{E(1,2,3,4,5)} = 0.675$ to $\psi_{11}^{E(1,2,3,4,5)} = 0.7$. Therefore, the new system's quality conforming rate is represented by $\boldsymbol{\Psi}^{\boldsymbol{E}(\mathbf{1},\mathbf{2},\mathbf{3},\mathbf{4},\mathbf{5}),\boldsymbol{NEW}}$ as

$$\boldsymbol{\Psi}^{\boldsymbol{E}(\mathbf{1},\mathbf{2},\mathbf{3},\mathbf{4},\mathbf{5}),\boldsymbol{NEW}} = \begin{bmatrix} \mathbf{0.7000} & 0 & 0 & 0 & 0.3000 \\ 0 & 0.6254 & 0 & 0 & 0.3746 \\ 0 & 0 & 0.7040 & 0 & 0.2960 \\ 0 & 0 & 0 & 0.6515 & 0.3485 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

It is assumed that it is feasible to improve only station 3 to meet the new system requirement of $\boldsymbol{\Psi}^{\boldsymbol{E}(\mathbf{1},\mathbf{2},\mathbf{3},\mathbf{4},\mathbf{5}),\boldsymbol{NEW}}$. The question that remains is what must be the new quality requirement at station 3 in order to achieve $\boldsymbol{\Psi}^{\boldsymbol{E}(\mathbf{1},\mathbf{2},\mathbf{3},\mathbf{4},\mathbf{5}),\boldsymbol{NEW}}$.

The step-by-step inverse operations are illustrated as follows

(i) $$S_{E(1,2,3,4,5)} = S_1 \otimes [(S_2 \otimes S_3) \oplus S_4] \otimes S_5$$

(ii) $$S_1 \otimes^{-1} S_{E(1,2,3,4,5)} = [(S_2 \otimes S_3) \oplus S_4] \otimes S_5$$

(iii) $$\left(S_1 \otimes^{-1} S_{E(1,2,3,4,5)}\right) \otimes^{-1} S_5 = (S_2 \otimes S_3) \oplus S_4$$

(iv) $$\left[\left(S_1 \otimes^{-1} S_{E(1,2,3,4,5)}\right) \otimes^{-1} S_5\right] \oplus^{-1} S_4 = S_2 \otimes S_3$$

(v) $$S_2 \otimes^{-1} \left[\left[\left(S_1 \otimes^{-1} S_{E(1,2,3,4,5)}\right) \otimes^{-1} S_5\right] \oplus^{-1} S_4\right] = S_3 \qquad (3\text{-}24)$$

Now, by replacing S_i for $\boldsymbol{\Psi}^{\boldsymbol{i}}$ and using the corresponding algebraic operators we have

$$\boldsymbol{\Psi}^{\mathbf{3},\boldsymbol{NEW}} = \boldsymbol{\Psi}^{\mathbf{2}} \overleftarrow{\otimes}_Q^{-1} \left[\left(\boldsymbol{\Psi}^{\mathbf{1}} \overleftarrow{\otimes}_Q^{-1} \boldsymbol{\Psi}^{\boldsymbol{E}(\mathbf{1},\mathbf{2},\mathbf{3},\mathbf{4},\mathbf{5}),\boldsymbol{NEW}} \overrightarrow{\otimes}_Q^{-1} \boldsymbol{\Psi}^{\mathbf{5}}\right) \oplus_Q^{-1} \boldsymbol{\Psi}^{\mathbf{4}}\right]. \qquad (3\text{-}25)$$

After substituting operators $\otimes_Q^{-1}$ and $\oplus_Q^{-1}$ in Eq. (3-21), (3-22), and (3-23), $\boldsymbol{\Psi}^{\mathbf{3},\boldsymbol{NEW}}$ can be obtained as

$$\boldsymbol{\Psi}^{\mathbf{3},\boldsymbol{NEW}} = \begin{bmatrix} \mathbf{0.9956} & 0 & 0 & 0 & 0.0044 \\ 0 & 0.9600 & 0 & 0 & 0.0400 \\ 0 & 0 & 0.9800 & 0 & 0.0200 \\ 0 & 0 & 0 & 0.9787 & 0.0213 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

By comparing $\boldsymbol{\Psi}^{\mathbf{3},\boldsymbol{NEW}}$ with $\boldsymbol{\Psi}^{\mathbf{3}}$, we can see that in order to achieve the new quality conforming rate of $\psi_{11}^{E(1,2,3,4,5)} = 0.7$, the quality conforming rate of product type 1 at station 3 should be improved from $\psi_{11}^{3} = 0.96$ to $\psi_{11}^{3,NEW} = 0.9956$.

3.4.2 Algebraic operators $\otimes_T^{-1}$ and $\oplus_T^{-1}$ for inverse computation of cycle time

As in Section 3.4.1, we can define inverse operators for $\otimes_T$ and $\oplus_T$ to solve algebraic expressions in terms of the cycle time of the individual product types.

Proposition 4: Let $\boldsymbol{CT^i}$ and $\boldsymbol{CT^j}$ be two cycle time vectors corresponding to station i and j respectively. The inverse algebraic operators $\otimes_T^{-1}$ and $\oplus_T^{-1}$, are defined by

(i) For a serial configuration, $\boldsymbol{CT^{E(i,j)}} = \boldsymbol{CT^i} \otimes_T \boldsymbol{CT^j}$, it yields

$$\boldsymbol{CT^i} = \boldsymbol{CT^{E(i,j)}} \otimes_T^{-1} \boldsymbol{CT^j} = \left\{CT_s^{E(i,j)} - CT_s^{j}\right\}_s \tag{3-26}$$

(ii) For a parallel configuration, $\boldsymbol{CT^{E(i,j)}} = \boldsymbol{CT^i} \oplus_T \boldsymbol{CT^j}$, it yields

$$\boldsymbol{CT^i} = \boldsymbol{CT^{E(i,j)}} \oplus_T^{-1} \boldsymbol{CT^j} = \begin{cases} \left\{CT_s^{E(i,j)}\right\}_s & \text{If } CT_s^{E(i,j)} > CT_s^{j} \\ \left\{CT_s^{i} \leq CT_s^{E(i,j)}\right\}_s & \text{If } CT_s^{E(i,j)} = CT_s^{j} \\ \text{unfeasible} & \text{otherwise} \end{cases} \tag{3-27}$$

Justification of Proposition 4: Since operator $\otimes_T$ corresponds to the conventional sum on vectors, $\otimes_T^{-1}$ will correspond to the conventional subtraction on vectors. Similarly, for operator $\oplus_T^{-1}$, since $CT_s^{E(i,j)} = CT_s^i \oplus_T CT_s^j = \max(CT_s^i, CT_s^j)$, then the definition on Eq. (3-27) is justified.

Illustrative example: We continue the example in Section 3.4.1 by illustrating how to redesign a station's cycle time to balance an assembly system. Assume it is desired to achieve the same production cycle time among the four product types, i.e., $\boldsymbol{CT^{E(1,2,3,4,5)}} = [49 \quad 49 \quad 49 \quad 49]^T$. Furthermore, suppose that only the cycle time of station 2 can be adjusted.

We can obtain a representation of $\boldsymbol{CT^2}$ by using the inverse operators $\otimes_T^{-1}$ and $\oplus_T^{-1}$ as

$$\left[\left(\boldsymbol{CT^{E(1,2,3,4,5),NEW}} \otimes_T^{-1} \boldsymbol{CT^1} \otimes_T^{-1} \boldsymbol{CT^5}\right) \oplus_T^{-1} \boldsymbol{CT^4}\right] \otimes_T^{-1} \boldsymbol{CT^3} = \boldsymbol{CT^{2,NEW}} \qquad \text{(3-28)}$$

Using the definitions of $\otimes_T^{-1}$ and $\oplus_T^{-1}$ in Proposition 4, we obtain vector $\boldsymbol{CT^{2,NEW}} = [12,12,11,11]^T$ minutes, that is, we need to reduce 1 minute of the production cycle time of product type 3 and product type 4 at station 2.

3.5 Case study

In this section, we consider a complex assembly system consisting of 20 stations for producing three product types (Ko and Hu, 2008, p. 4301-4306). The block diagram of the system configuration is shown in Figure 3-8. In no relevant order, stations are numbered from S_1 to S_{20} ($M = 20$).

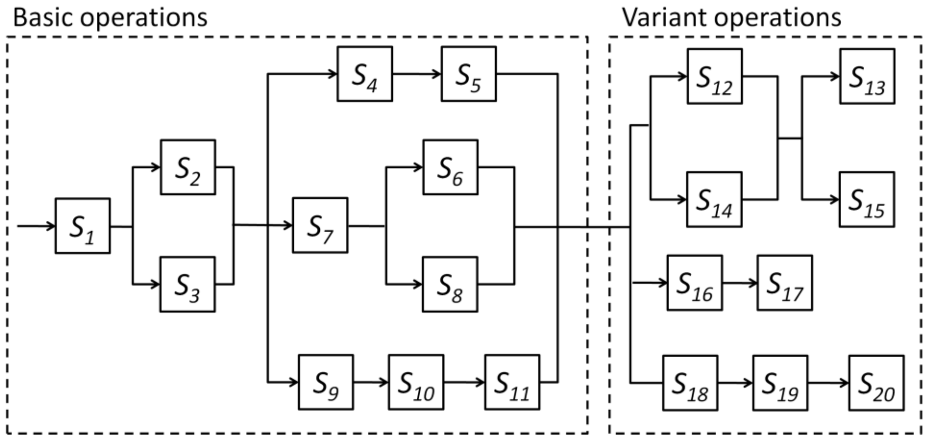


Figure 3-8 System block diagram for producing three product types

In order to efficiently obtain the conforming rate of each product type and their corresponding production cycle time, we further represent the assembly system by two sets: the assembly subsystem consisting of the basic operation stations denoted by S_B, and the assembly subsystem consisting of the variant operation stations denoted by S_V. In this example, assembly subsystem S_B contains stations S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, and S_{11}, while assembly subsystem S_V contains stations S_{12}, S_{13}, S_{14}, S_{15}, S_{16}, S_{17}, S_{18}, S_{19}, and S_{20}. Figure 3-8 shows the S_B configuration, while different S_V configurations for each product type are shown in Figure 3-9.

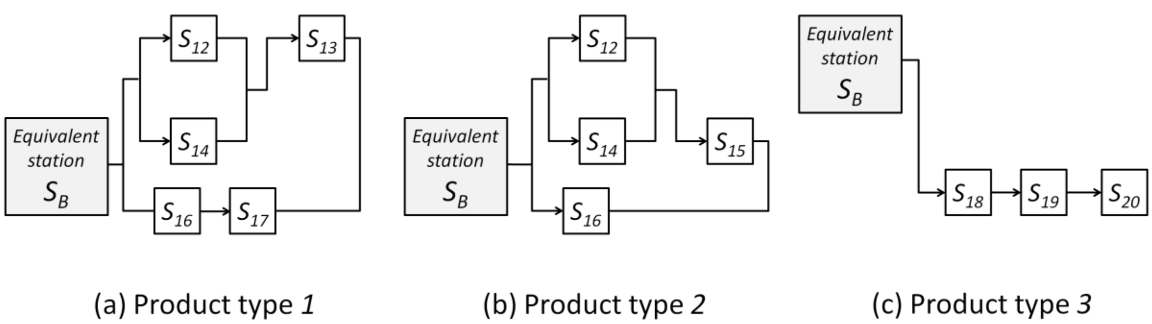


Figure 3-9 Block diagram for individual product types

By means of the proposed algebraic representation of the system configuration, we can further represent equivalent subsystems S_B and S_V as

$$S_B = S_1 \otimes (S_2 \otimes S_3) \otimes \Big[(S_4 \otimes S_5) \oplus \big[\big(S_7 \otimes (S_6 \oplus S_8)\big) \oplus (S_9 \otimes S_{10} \otimes S_{11})\big]\Big] \tag{3-29}$$

$$S_V = [(S_{12} \oplus S_{14}) \otimes (S_{13} \oplus S_{15})] \oplus [S_{16} \otimes S_{17}] \oplus [S_{18} \otimes S_{19} \otimes S_{20}] \tag{3-30}$$

The equivalent station of the whole assembly system, denoted by $S_{B,V}$, is given by

$$S_{B,V} = S_B \otimes S_V$$

The production throughput is designed to be five units per minute. Specifically, two units per minute are allocated for product type 1 and product type 3, and one unit per minute for product type 2. Thus, the input demand mix ratio is

$$\boldsymbol{\pi}^{\boldsymbol{IN,E}(\cdot)} = \left[\pi_1^{IN,E(\cdot)} = 0.40 \quad \pi_2^{IN,E(\cdot)} = 0.20 \quad \pi_3^{IN,E(\cdot)} = 0.40 \quad \pi_\varepsilon^{IN,E(\cdot)} = 0\ \right]^T$$

Based on Figure 3-9 (a), (b), and (c), Table 3-7 provides the task assignments to each station for each product type, their corresponding quality conforming rates and production cycle times.

Table 3-7 Conforming rate and cycle time for individual station

		Conforming rates			Processing times (sec)
	Product type processed	Product type 1	Product type 2	Product type 3	Product type 1, 2, and 3
Station 1	1, 2 and 3	99%	98%	99%	10
Station 2	1, 2 and 3	99%	98%	97%	19
Station 3	1, 2 and 3	97%	99%	98%	19
Station 4	1, 2 and 3	98%	98%	98%	33
Station 5	1, 2 and 3	99%	98%	99%	18
Station 6	1, 2 and 3	98%	99%	98%	33
Station 7	1, 2 and 3	98%	99%	99%	18
Station 8	1, 2 and 3	98%	97%	96%	36
Station 9	1, 2 and 3	99%	97%	98%	23

Station 10	1, 2 and 3	99%	99%	99%	31
Station 11	-	-	-	-	-
Station 12	1 and 2	97%	99%	-	52
Station 13	1	98%	-	-	54
Station 14	1 and 2	98%	99%	-	52
Station 15	2	-	99%	-	54
Station 16	1 and 2	99%	99%	-	52
Station 17	1	99%	-	-	54
Station 18	3	-	-	96%	14
Station 19	3	-	-	97%	19
Station 20	3	-	-	98%	18

It can be seen in Table 3-7 that station 11 is regarded as an excess station and no production task is assigned to it. Also, we assume that the station's cycle times are independent of the product type being processed.

3.5.1 Analysis of quality and cycle time for whole assembly system

We first compute the equivalent matrix $\boldsymbol{\Psi}^{\boldsymbol{E}(\boldsymbol{B})}$ corresponding to the basic operations subsystem S_B by replacing $\otimes$ and $\oplus$ in Eq. (3-29) with $\otimes_Q$ and $\oplus_Q$ respectively, thus yielding

$$\boldsymbol{\Psi}^{\boldsymbol{E}(\boldsymbol{B})} = \begin{bmatrix} 0.9130 & 0 & 0 & 0.0870 \\ 0 & 0.9039 & 0 & 0.0961 \\ 0 & 0 & 0.8944 & 0.1056 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Similarly, based on Eq. (3-30), $\boldsymbol{\Psi}^{\boldsymbol{E}(\boldsymbol{V})}$ corresponding to the variant operations subsystem S_V, is calculated as

$$\boldsymbol{\Psi}^{\boldsymbol{E}(\boldsymbol{V})} = \begin{bmatrix} 0.9506 & 0 & 0 & 0.0494 \\ 0 & 0.9801 & 0 & 0.0199 \\ 0 & 0 & 0.9126 & 0.0874 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Finally, based on $S_{B,V} = S_B \otimes S_V$, we can calculate $\boldsymbol{\Psi}^{\boldsymbol{E(B,V)}} = \boldsymbol{\Psi}^{\boldsymbol{E(B)}} \otimes_{\boldsymbol{Q}} \boldsymbol{\Psi}^{\boldsymbol{E(V)}}$, corresponding to the whole assembly system, as

$$\boldsymbol{\Psi}^{\boldsymbol{E(B,V)}} = \begin{bmatrix} 0.8679 & 0 & 0 & 0.1321 \\ 0 & 0.8859 & 0 & 0.1141 \\ 0 & 0 & 0.8162 & 0.1838 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Therefore, based on the proposed methodology, the final conforming rates of three product types can be obtained as $\psi_{11}^{E(B,V)} = 86.79\%$, $\psi_{22}^{E(B,V)} = 88.59\%$, and $\psi_{33}^{E(B,V)} = 81.62\%$.

The cycle time corresponding to each product type is calculated based on the algebraic expressions of equivalent stations S_B, S_V, and $S_{B,V}$ by using algebraic operators $\otimes_T$ and $\oplus_T$. The results obtained are $\boldsymbol{CT}^{\boldsymbol{E(B)}} = [59 \quad 53 \quad 60]^T$, $\boldsymbol{CT}^{\boldsymbol{E(V)}} = [24 \quad 24 \quad 20]^T$ and $\boldsymbol{CT}^{\boldsymbol{E(B,V)}} = [83 \quad 77 \quad 80]^T$. By means of vector $\boldsymbol{CT}^{\boldsymbol{E(B,V)}}$, we can see that the longest cycle time is $CT_1^{E(B,V)} = 83$ minutes, corresponding to product type 1, while the shortest cycle time is $CT_2^{E(B,V)} = 77$ minutes for product type 2.

3.5.2 Mathematical programming for optimizing PCC

We further illustrate an application of the proposed methodology for improving process operation performance. Assume we are interested in finding the input demand mix ratio values of vector $\boldsymbol{\pi}^{\boldsymbol{IN,E(B,V)}} = \left[\pi_1^{IN,E(B,V)} \quad \pi_2^{IN,E(B,V)} \quad \pi_3^{IN,E(B,V)} \quad \pi_\varepsilon^{IN,E(B,V)} = 0\right]$ to achieve an optimal process capability for complexity (PCC). Furthermore, the production throughput in such a mixed model assembly system can be obtained from the average cycle time per product type (ACT), defined as $ACT = \sum_{i=0}^{N-1} CT_i^{E(B,V)} \pi_i^{IN,E(B,V)}$. Therefore, the ACT quantifies the amount of time that it takes to convert raw material into a finished product, averaged over the different product types and their demand ratios.

The optimization problem can be mathematically formulated as

$$\max_{\boldsymbol{\pi}^{IN,E}} PCC$$

$$\text{s.t.: } 0 \leq \pi_i^{IN,E(B,V)} \leq 1$$

$$\sum_{i=1}^{3} \pi_i^{IN,E(B,V)} = 1$$

$$ACT \leq \eta$$

The key advantage of using our proposed method in solving such an optimization problem is the ease of mathematical representation achieved. In particular, both of the performance measures, PCC and ACT, can be represented based on similar algebraic expressions, as that of the assembly system configuration, via operators $\otimes$ and $\oplus$.

Figure 3-10 shows the contour plot of the PCC in terms of $\pi_1^{IN,E(B,V)}$ and $\pi_2^{IN,E(B,V)}$. Based on the constraint $\pi_3^{IN,E(B,V)} = 1 - \pi_1^{IN,E(B,V)} - \pi_2^{IN,E(B,V)}$, the unfeasible region is defined by $\pi_1^{IN,E(B,V)} + \pi_2^{IN,E(B,V)} + \pi_3^{IN,E(B,V)} > 1$, and the circled dot corresponds to the optimal values of $\pi_i^{IN,E(B,V)}$, where PCC is maximized.

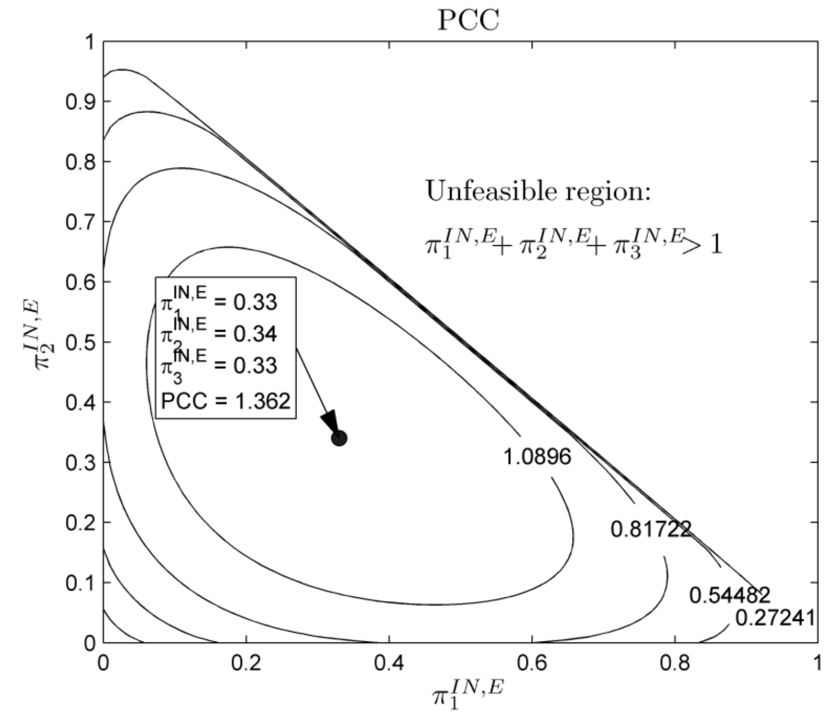


Figure 3-10 Contour plot of PCC

Figure 3-11 shows the contour plot of the ACT. In this example, $ACT = CT_1^{E(B,V)}\pi_1^{IN,E(B,V)} + CT_2^{E(B,V)}\pi_2^{IN,E(B,V)} + CT_3^{E(B,V)}\pi_3^{IN,E(B,V)}$, since $CT^{E(B,V)} = \begin{bmatrix} CT_1^{E(B,V)} & CT_2^{E(B,V)} & CT_3^{E(B,V)} \end{bmatrix}^T = [83 \quad 77 \quad 80]^T$, thus $ACT = 3\pi_1^{IN,E(B,V)} - 3\pi_2^{IN,E(B,V)} + 80$. Note that, since the ACT plot is shown as a plane, the minimum occurs at one of the corners of the solution space. In this case, the circled dot shows the values in the solution space for which the ACT is minimized.

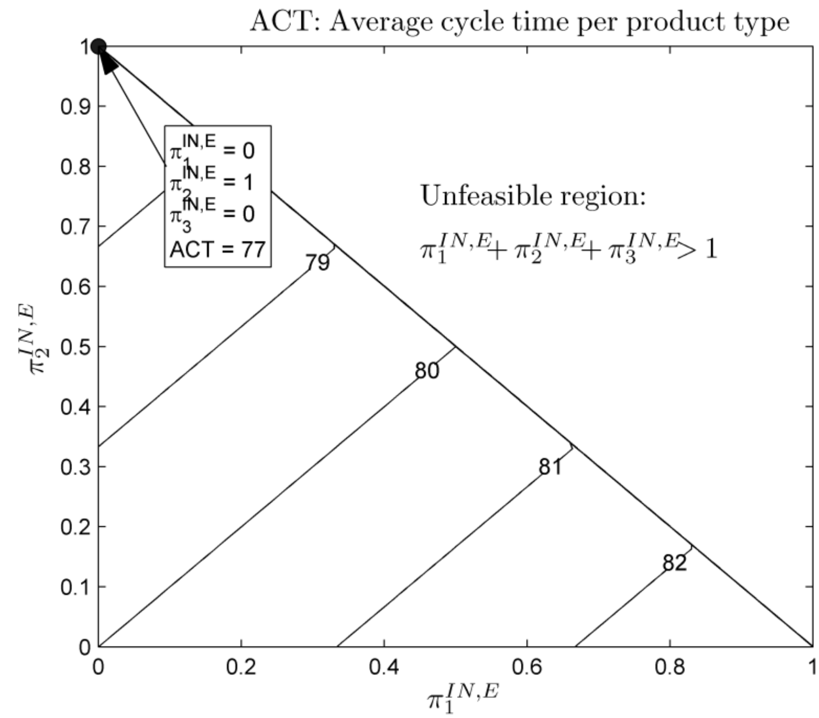


Figure 3-11 Contour plot of ACT

A combination of these two functions is shown in Figure 3-12, where the optimal points for the PCC, restricted by $ACT \leq \eta$ ($\eta =$ 77.5, 78, 78.5, 79, 79.5,80), are marked by a circled dot.

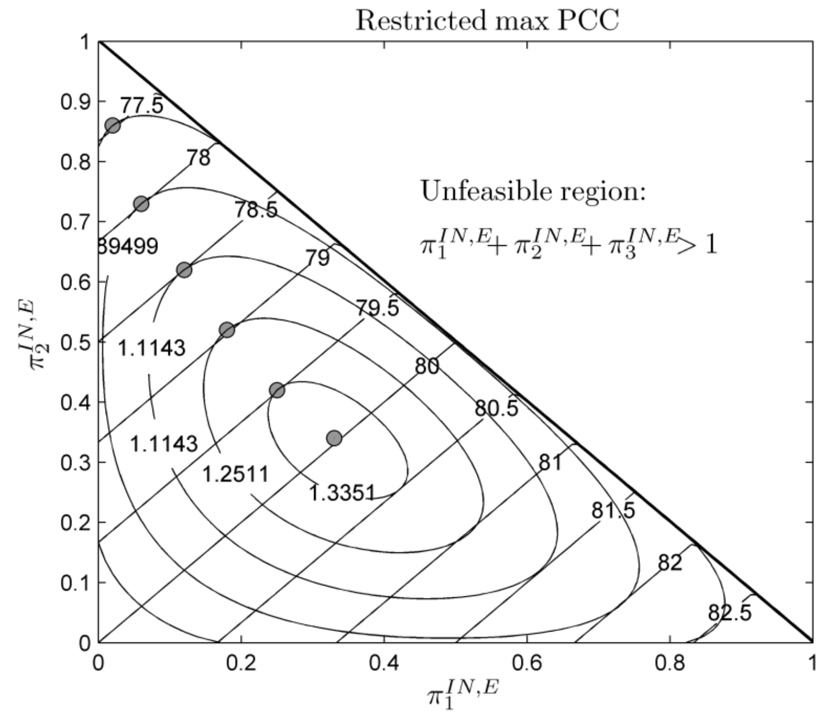


Figure 3-12 Maximization of PCC constraint by different levels of ACT

The first column of Table 3-8 shows the values of $\pi_i^{IN,E(B,V)}$, $(i = 1,2,3)$, for which the PCC gets maximized. The second column of Table 3-8 shows the values of $\pi_i^{IN,E(B,V)}$, $(i = 1,2,3)$ which maximizes the PCC subjected to $ACT \leq 79$.

Table 3-8 Solutions for un-restricted and restricted optimization of PCC

	Max PCC	Max PCC s.t. $ACT \leq 79$
$\pi_1^{IN,E(B,V)}$	0.33	0.18
$\pi_2^{IN,E(B,V)}$	0.34	0.52
$\pi_3^{IN,E(B,V)}$	0.33	0.3
PCC	1.362	1.2511
ACT	79.97	79

From Table 3-8, we can see that PCC, subjected to $ACT \leq 79$, is maximized at point $\boldsymbol{\pi}^{\boldsymbol{IN,E(B,V)}} = [0.18 \quad 0.52 \quad 0.3 \quad 0\,]$. This result is as expected because product type 2 has the highest conforming rate and lowest cycle time, therefore attaining the largest input mix ratio in the solution, that being $\pi_2^{IN,E(B,V)} = 0.52$.

3.5.3 Identification of weak stations via layer-by-layer tree decomposition

We now illustrate the use of the expression tree representation of the assembly system $S_{B,V}$ to determine *weak* sub-assembly systems with a low process quality Q. Consider Figure 3-13 (a), where the system $S_{B,V}$ has been expressed at its highest level of grouping as $S_{B,V} = S_B \otimes S_V$. Since we are interested in determining sub-systems that have a low Q performance, we further decompose, layer by layer, the equivalent station S_B, which is the equivalent station with the lowest process quality, given by $Q = 0.9038$. As shown in Figure 3-13 (b), the lowest process quality $Q = 0.9545$ within S_B corresponds to $S_{4\sim11}$. Continuing with a step-by-step decomposition, we further expand equivalent station $S_{4\sim11}$ into $S_{4,5}$, $S_{6,7,8}$ and $S_{9,10,11}$, as shown in Figure 3-13 (c). Based on this decomposition, we have that sub-system $S_{6,7,8}$ has the lowest performance at this level, given by $Q = 0.9564$, and thus further attention should be given to this sub-system to improve the overall process quality of the system. One should note that we have followed a greedy search approach, and hence we cannot claim that sub-system $S_{6,7,8}$ is a global minimum.

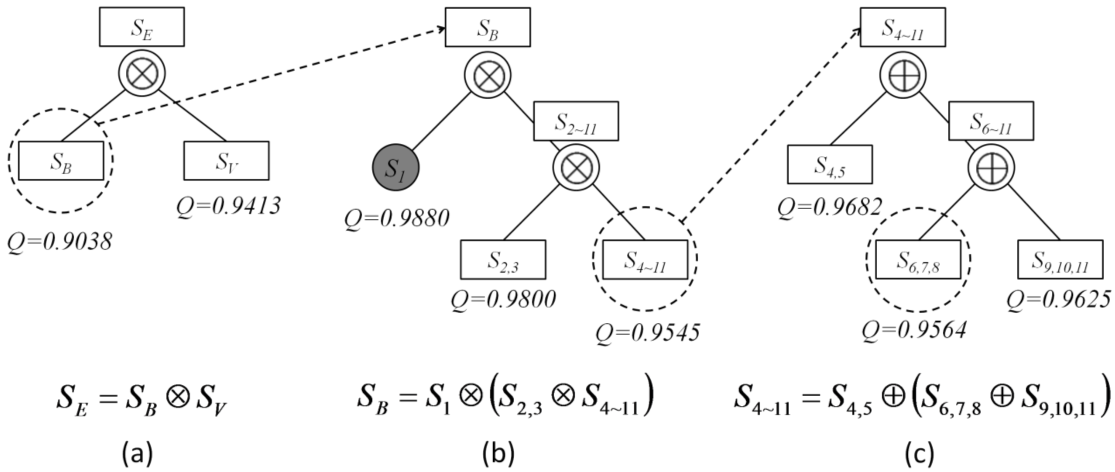


Figure 3-13 Layer-by-layer system decomposition

3.6 Conclusions and future work

This paper introduces an efficient way to represent complex mixed model assembly system configurations using algebraic expressions. The proposed methodology considers the serial and parallel relationships between stations. As a result, hybrid asymmetric system layout configurations are considered. Furthermore, a systematic method for obtaining commonly used system performance metrics, such as quality conforming rate and process cycle time, was shown. The proposed method consists of assigning specific mathematical operations to the operators in the algebraic expression. Examples are given for illustration of the methodology. Finally, the use of the algebraic expressions to help in the design of an assembly system layout configuration is discussed.

The research presented in this paper can be further extended by defining algebraic operators for obtaining other system performance metrics such as operational states of a system, reliability, scheduling, etc. Furthermore, the algebraic expressions may be mathematically formalized into interesting algebraic fields, and thus solving problems such as finding bottleneck stations; balancing a system; optimally combining two assembly systems through one or more stations, and so forth.

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CHAPTER 4

MODELING AND ANALYSIS OF OPERATORS EFFECT ON PROCESS QUALITY AND THROUGHPUT IN MIXED MODEL ASSEMBLY SYSTEMS

Abstract

With the increase of market fluctuation, assembly systems moved from a mass production scheme, to a mass customization scheme. Mixed model assembly systems have been recognized as enablers of mass customization manufacture. Effective implementation of mixed model assembly systems (MMASs) requires, however, among other things, a highly proactive and knowledgeable workforce. Hence, modeling the performance of human operators is critically important for effectively operating these manufacturing systems. Interesting cognitive factors have seldom been considered in modeling process quality of an MMAS. Thus, the objective of this paper is to introduce an integrated modeling framework by considering operator's factors in term of both intrinsic factors (e.g., operator's working experience and metal deliberation thinking time) and extrinsic factors (choice task complexity induced by product variety). The proposed model is justified based on the findings presented in the psychological literature. The effect of these operator's factors on process operation performance is also investigated; these performance measures include process quality, throughput, as well as process capability on handling complexity induced by product variety in an MMAS. Two examples are used to demonstrate potential applications of the proposed model.

4.1 Introduction

With the increase of market fluctuation, assembly systems moved from a mass production scheme to a mass customization scheme. The intent of mass customization is to provide customers with products that are closer to their specific demands by producing a high range of product varieties. At the same time, mass customization keeps costs low and thus, customized products remain affordable. Mixed model assembly systems (MMASs) have been recognized as enablers of mass customization manufacturing. However, it has been shown by both, empirical and simulation results (Fisher et al., 1995; MacDuffie et al., 1996; Fisher and Ittner, 1999), that increasing the variety of automobiles production has a significant negative impact on the performance of the MMAS on process quality and productivity. Such an impact affects not only the complexity of system configurations and operations, but also the requirements placed on the operators, who must develop skills for handling the multiple tasks needed to produce product variants. The role of operator's performance is especially critical in MMASs requiring intensive labor operations, e.g., automobile final general assembly (Lee and Vairaktarakis, 1997), and personalized product manufacturing. Therefore, in order to successfully implement a mass customization scheme, MMASs should be appropriately designed and operated by considering the operator's capability.

Modeling human operators' performance in an assembly system is a challenging research task, mainly because the factors involved are difficult to identify, define, and measure. Nevertheless, the inclusion of human operator's modeling is necessary for correct predictions of the system performance. In this spirit, Bernhard and Schilling (1997) asserted that inaccuracy on the simulation results is particularly apparent when modeling manufacturing systems with high proportion of manual operations. Baines et al. (2004) further indicated that it is a common observation that the production rate of a simulation is usually greater than the actual rate of the real system and that, as a result, a greater accuracy in the simulation results could be achieved by including important factors affecting the worker's performance.

The factors that affect the performance of operators can be classified in two categories: intrinsic factors and extrinsic factors. Intrinsic factors include factors such as operator's age, working skills, experience, and so forth; while extrinsic factors include factors such as task difficulty and the number of choices in a given manufacturing task. Most models of human behavior and performance consider some of these factors.

Some research has been conducted to study the operator's effect in relation to process performance. For example, Fine (1986) developed a high-level model to describe the relationship of production cost with operator's learning skills and quality improvement efforts. In Baines et al. (2004), a model was proposed to study the effect of operator's age on his or her performance, which was measured as the amount of time required to finish a task. Furthermore, Wang and Hu (2010) considered the operator's fatigue due to product variety and studied its effect on system performance. None of these researches, however, has focused on how to adjust the production operations accordingly to the operator's performance, e.g., designing task cycle times according to the operator's performance under different levels of task difficulty and working experience.

In MMASs, the number of product types and their mixed ratios varies among different operation stations. Therefore, the operator's performances will be affected by the choice task complexity and the amount of experience required at the specific station. The allocation of a task cycle time is important to ensure that the operator will have sufficient mental deliberation thinking time to make the correct choice of parts, tools, and activities for task completion. In contrast, an excessive allocation of cycle time wastes production time and decreases the production throughput. Therefore, the objective of this research is to develop an integrated model for effectively characterizing the operator's performance and analyzing its effect on process quality and throughput in MMASs; this will be accomplished by considering both intrinsic and extrinsic factors in the model. Specifically, two intrinsic factors, i.e., the operator's working experience and mental deliberation thinking time; and one extrinsic factor, i.e., choice task complexity due to the product variety, are considered.

The first intrinsic factor we consider is the operator's experience. In the literature, the operator's experience for performing a particular task refers to operator's autonomous learning ability due to repetitive execution of a task (Bohlen and Barany, 1976). While literature on learning curves abounds, e.g., Yelle (1979), and Argote and Epple (1990), there is limited research that addresses the effect of experience on quality conforming performance. Most of these existing models are based on the exponential learning curve, which was developed by Wright (1936). Later, Fine (1986) studied quality improvement and learning capabilities together, and analyzed their interactive effects on production costs. Furthermore, Kini (1994) extended such ideas by considering the impact of nonconforming units in the learning process. A model addressing quality improvement as a result of the learning effect was also proposed in Li and Rajagopalan (1998). In their model, the learning effect was caused by the accumulation of knowledge stemming from production and process improvements. In this research, we will focus only on the autonomous learning performance through the repetitive execution of a task in an MMAS. An exponential model will be used to describe the quality conforming performance in terms of the cumulative production cycles performing the task; this will be discussed in detail in Section 4.2.

The second intrinsic factor we will use is the operator's mental deliberation thinking time. Intuitively, the performance of a task is positively related to the time available to perform the task. In other words, there exists a trade-off between speed and accuracy in task performance, a relationship widely accepted in the literature (Plamondon and Alimi, 1997; Reed, 1973; Bootsma et al., 1994). There have been many experiments supporting these findings. For example, Schouten and Bekker (1967) studied how the available amount of time affected the correct rate of recognizing an auditory cue. In a second experiment, conducted by Pachella and Pew (1968), a subject was presented with various configurations of four lights and he or she had to respond with four of his or her fingers accordingly; as the task was performed, the response time was recorded. In these two experiments, the relationship between accuracy and response times were found to be well modeled by an exponential curve, as reported by Pew (1969). Additionally, more elegant attempts to model this tradeoff have been more recently proposed, all describing the exponential relationship between speed and accuracy. For example, in decision-making tasks, diffusion process models (Heath, 1992; Busemeyer and Townsend, 1993; Diederich, 1997) have gained increasing acceptance over the last few decades, mainly because of their ability to effectively describe the trade-off between speed and accuracy. In this research, we assume that an operator requires some mental deliberation thinking time to achieve a satisfying quality conforming rate. Specifically, the operator's mental thinking time is referred to as the time employed by the operator to accomplish cognitive tasks that are essential to add value to the process quality performance. In this context, value adding cognitive tasks may include recognizing what product variant to produce, selecting the right tools, performing mental evaluations, and acquiring production process environmental awareness. Based on existing research, we will model the effect of the mental thinking time on the process quality conforming rate by an exponential function; this will be discussed in detail in Section 4.2.

The third operator's factor considered in this research is the extrinsic factor of choice task complexity due to product variety in an MMAS. It is widely known that there is a strong connection between the theoretical properties of information theory's entropy (Shannon, 1948) and experimental findings in the cognitive science describing the mental workload imposed on subjects when making a choice among multiple alternatives. Based on a classical experiment conducted by Merkel in 1885 and later described by Woodworth and Schlosberg (1954), Hick (1952) discovered that, if all the choices are equally likely to be chosen, the performance of the operator is inversely proportional to the logarithmic of the number of choices. This finding was revolutionary to mathematical cognitive science. Later, Hyman (1953) extended this idea to the unequally likely choices scenario. The result was known as the Hick-Hyman Law, which describes a negative effect between the performance of an individual and the information entropy of the choices. Recently, in a mixed model assembly system environment, Zhu et al. (2008) and

Hu et al. (2008) proposed the use of information entropy as a measure of the choice task complexity due to the demand variety; this was called the operator choice complexity. This research will use such complexity index to quantify the effect of the task complexity on operator's performance in performing assembly operations in an MMAS. Since entropy is essentially a weighted average of the logarithmic of the probability of each outcome of an event, there is an exponential relationship between the event outcome proportions and its entropy. This result motivates us to consider an exponential model to describe the relationship between the process quality performance and choice task complexity; this will be discussed in detail in Section 4.2.

The organization of the rest of the paper is as follows. Section 4.2 describes the proposed mathematical model used to characterize the operator's performance in an MMAS. In Section 4.3, the proposed model is used to analyze the effect of the operator's factors on the throughput of an assembly system. The analysis is first discussed under a single station scenario and then under a multiple-station assembly line scenario. Two illustrative examples are given in Section 4.4 to show possible applications of the proposed model. Finally, Section 4.5 provides the conclusions and indicates possible directions for future research.

4.2 Modeling and analysis of operator's factors

4.2.1 Proposed integrated modeling framework and mathematical models

Figure 4-1 shows the proposed integrated modeling framework, which describes how operator's factors including intrinsic factors (operator working experience and mental deliberation thinking time) and extrinsic factor (choice task complexity due to part mix ratio), affect the process quality. Favorable levels of the operator's factors will improve the operator's cognitive performance and thus adding a value to improve the process quality conforming rate above its *worst quality level*. Here, the worst quality level refers to the lowest quality conforming rate that results from the most adversarial levels of the operator's factors scenario, such as an unskillful operator performing the most complex task with no deliberation thinking time. Moreover, Figure 4-1 also shows that the mental thinking time affects the production cycle time, while the throughput is affected by both the process quality and the production cycle time. Based on this proposed integrated framework, it is important to note that an increase in the operator's thinking time allocation will increase the process quality performance, while it will also increase the production cycle time, both affecting the production rate. Therefore, a mathematical model is needed to quantitatively describe the interactive effect of thinking time on process quality and production cycle time. This will be described in Section 4.3 in greater detail.

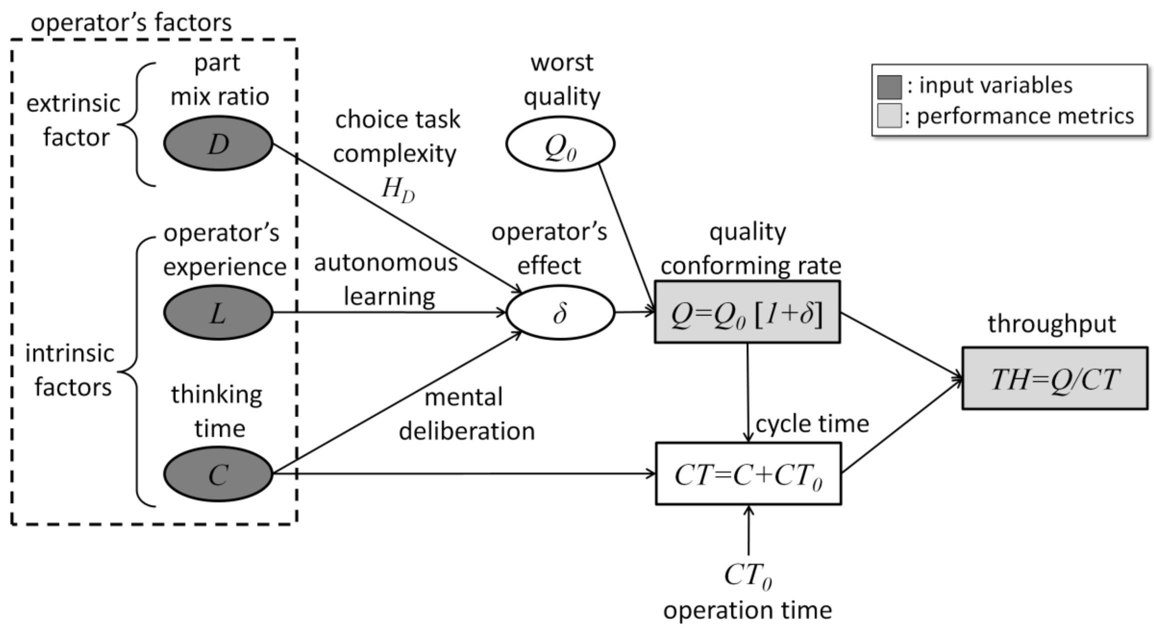

Figure 4-1 Proposed integrated modeling framework

Based on the literature, a mathematical model can be defined by an exponential function that is used to describe the effect of a single operator's factor F (e.g., D, L, C as shown in Figure 4-1) on the process quality conforming rate as

$$Q(F) = Q_0[1 + \delta(F)] \tag{4-1}$$

where Q_0 is the lowest process quality conforming rate, which corresponds to the worst quality performance that results from the most adversarial operator's factors scenario. $\delta(F)$ is the percentage of the improved quality conforming rate under a more favorable operator's factor scenario F than that of the worst scenario. Therefore, the larger that factor F is, the more quality improvement of $\delta(F)$ is expected. Based on general ideas found in the psychophysics literature (Dehaene 2003), where an exponential relationship is proposed to model the effect between the magnitude of a physical stimuli and its human's perception, the effect of the operator's factors will be characterized by an exponential relationship, thus the following exponential model is proposed for $\delta(F)$

$$\delta(F) = \Delta_M \cdot (1 - e^{-\beta F}) \tag{4-2}$$

where Δ_M is the upper limit of the quality conforming rate that can be maximally improved under the most favorable operator's factors scenario. The most favorable factor scenario corresponds to a very experienced operator performing the simplest task with sufficient mental deliberation thinking time. In this way, $\delta(F)$ is a convex increasing bounded function, which satisfies $\delta(0) = 0$ (corresponding to the worst scenario) and $\delta(F) \to \Delta_M$ as $\beta F \to \infty$ (corresponding to the most favorable scenario). Furthermore, the increasing rate of $\delta(F)$ is further attenuated as the value of F increases, which is a desired characteristic to reflect the limited operator's capability.

Furthermore, the model given in Eq. (4-1) can be generally extended to consider multiple operator's factors; this can be done by extending variable F as vector $\boldsymbol{F} = [F_1 \quad F_2 \quad \dots \quad F_r]^T$ to represent r factors. If we consider only additive effect of operator's factors $F_1, F_2, \dots, F_r$ (no interactions), the extended model can be represented as

$$\delta(\boldsymbol{F}) = \Delta_M \cdot (1 - e^{-\boldsymbol{\beta}^T \boldsymbol{F}}) \tag{4-3}$$

Here, parameter vector $\boldsymbol{\beta} = [\beta_1 \quad \beta_2 \quad \dots \quad \beta_r]^T$ is associated with factors $F_1, F_2, \dots, F_r$, to account for the relative sensitivity of each corresponding factor on the improved quality conforming rate.

The following discussion will further show how to implement the model by substituting the specific operator's factors of D, L, C into Eq. (4-3). For the purpose of developing a general model, these operator's factors will be normalized by their corresponding dimensionless factors, which will be discussed in detail as follows.

(i) Thinking time. The thinking time, denoted by C, is defined as the time for mental deliberation activities for executing a specific production task. We refer to task cycle time as the length of time needed at a station to complete its production task. Thinking time is used by the operator to perform cognitive activities with the aim of improving his quality conforming performance at the station. These cognitive activities include, but are not limited to, deliberating, comparing, remembering, and associating. If we denote the production task cycle time by CT; the minimum time required to complete a physical operation to produce a unit at the station by CT_0; and the metal deliberation thinking time by C; the production task cycle time at a given station CT satisfies $CT = CT_0 + C$. The normalized cycle time ρ_C is defined as

$$0 \leq \rho_C = \frac{CT - CT_0}{CT_M - CT_0} \leq 1 \tag{4-4}$$

where CT_M corresponds to the maximum value for cycle time, beyond which there is no further significant improvement in the process quality conforming rate even if operators were given more deliberation thinking time than $CT_M - CT_0$.

(ii) Choice task complexity induced by product variety. The demanded product variety increases the mental workload imposed on operators by complicating certain cognitive activities such as recognizing what part type to produce, what tool and fixtures to select, and the burdens placed on the operator by requiring him or her to alternate among multiple activities. As proposed by Boer (2000), the information entropy H_D will be used to measure the mental workload imposed on an operator due to the choice decision

among alternative tasks. Based on Zhu et al. (2008) and Hu et al. (2008), if P_i^D is used to denote the mix ratio that product type i (requiring performing a task i) is demanded at a station, the choice task complexity can be represented by

$$H_D = -\sum_i P_i^D \log P_i^D \quad (4\text{-}5)$$

Furthermore, the normalized complexity ρ_D is defined as

$$0 \leq \rho_D = \frac{H_{D,M} - H_D}{H_{D,M}} \leq 1 \quad (4\text{-}6)$$

where $H_{D,M}$ is the maximum task complexity represented by the maximum entropy value. It corresponds to the situation where all product types are demanded with the same frequency; this is represented by the entropy of a random variable with n equally likely outcomes (Cover and Thomas, 2006).

(ii) *Operator's experience.* It refers to the improvement in an operator's performance gained through autonomous learning by the repetitive completion of a task. In this paper we will measure the experience in terms of the number of cumulative units produced by operator. Similarly, the normalized complexity ρ_L is defined as

$$0 \leq \rho_L = \frac{L}{L_M} \leq 1 \quad (4\text{-}7)$$

where L_M corresponds to the effective learning period. The interpretation of L_M is that there is no further significant improvement in the process quality conforming rate after operators spent a sufficient amount of time executing a task, i.e., when $L > L_M$.

By substituting above three normalized operator's factors ρ_C, ρ_D, and ρ_L into Eq. (4-3), the proposed integrated model takes the form of

$$Q(\rho_C, \rho_D, \rho_L) = Q_0[1 + \delta(\rho_C, \rho_D, \rho_L)] \tag{4-8}$$

$$\delta(\rho_C, \rho_D, \rho_L) = \Delta_M \cdot \left(1 - e^{-\beta_C\rho_C - \beta_D\rho_D - \beta_L\rho_L}\right) = \Delta_M \cdot \left(1 - e^{-\boldsymbol{\beta}^T\boldsymbol{\rho}}\right) \tag{4-9}$$

where vector $\boldsymbol{\rho} = [\rho_C \quad \rho_D \quad \rho_L]^T$ and $\boldsymbol{\beta} = [\beta_C \quad \beta_D \quad \beta_L]^T$.

Figure 4-2 shows the effect trend of one of three factors ρ_C, ρ_D and ρ_L on the quality conforming rate when other two factors are fixed.

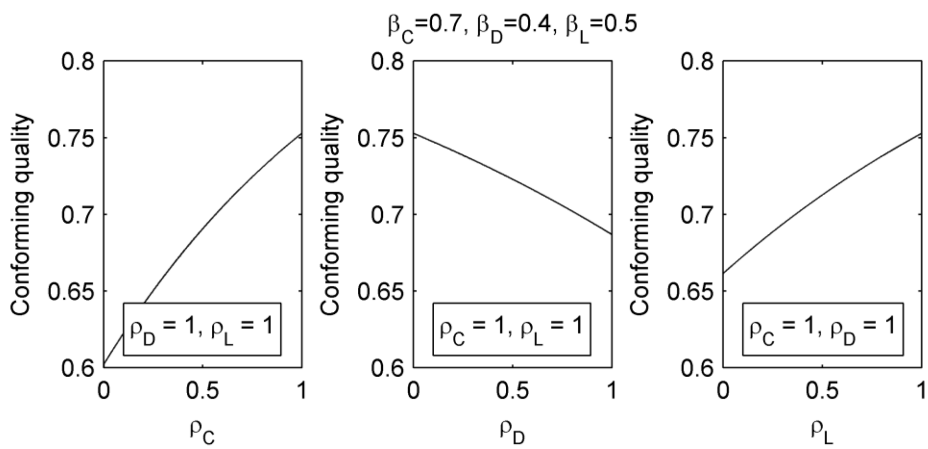

Figure 4-2 Effect of three operator's factors on process quality

4.2.2 Analysis of the effect of cycle time on production throughput at a single station

Modeling the throughput traditionally assumes a cycle time-independent quality conforming rate Q. Since our proposed model relates the production cycle time CT to the quality conforming rate Q, based on Little's Law (Little, 1961), the throughput under the proposed framework can be expressed as

$$TH = \frac{Q \cdot WIP}{CT} = \frac{[Q_0 + Q_0 \cdot \Delta_M \cdot (1 - e^{-\beta_C\rho_C - \beta_D\rho_D - \beta_L\rho_L})] \cdot WIP}{\rho_C(CT_M - CT_0) + CT_0} \tag{4-10}$$

where WIP is the work in process defined as the number of units within the manufacturing system. From Eq. (4-10) we can see that increasing the cycle time factor

ρ_C not only increases the denominator CT, but also increases the numerator Q, and hence, the overall effect on the throughput should be carefully studied.

For a fixed cycle time CT and WIP, the throughput can be trivially maximized at $\rho_D = 1$ and $\rho_L = 1$, by achieving the maximum Q. Since the effect of increasing factor ρ_C on the numerator (Q) is not the same as its effect on the denominator (CT), the resulting effect on TH varies with different ρ_C. Furthermore, since the rate of increment of the denominator is $CT_M - CT_0$ while the rate of increment of the numerator is $WIP \cdot Q_0\Delta_M\beta_C e^{-\beta_C\rho_C-\beta_D\rho_D-\beta_L\rho_L}$, then function TH is convex. Proposition 1 below provides a way of finding the optimal ρ_C^* that maximizes TH at a given station.

Proposition 1: For a given set of fixed parameters WIP, ρ_D and ρ_L, the maximum throughput occurs at ρ_C^* satisfying

$$TH^* = \frac{(Q_0\Delta_M\beta_C e^{-\beta_C\rho_C-\beta_D\rho_D-\beta_L\rho_L})\cdot WIP}{CT_M-CT_0} \tag{4-11}$$

<u>**Proof of Proposition 1:**</u> By taking the derivative on TH in Eq. (4-10) and substituting $TH = Q \cdot WIP/CT$, it yields

$$\frac{\partial TH}{\partial \rho_C} = \frac{\partial TH}{\partial CT}\cdot\frac{\partial CT}{\partial \rho_C} + \frac{\partial TH}{\partial Q}\cdot\frac{\partial Q}{\partial \rho_C}$$

$$= -\frac{Q\cdot WIP}{CT^2}(CT_M - CT_0) + \frac{WIP}{CT}(Q_0\Delta_M\beta_C e^{-\beta_C\rho_C-\beta_D\rho_D-\beta_L\rho_L})$$

$$= -\frac{1}{CT^2}[TH\cdot(CT_M - CT_0) + WIP\cdot(Q_0\Delta_M\beta_C e^{-\beta_C\rho_C-\beta_D\rho_D-\beta_L\rho_L})]$$

By setting $\frac{\partial TH}{\partial \rho_C} = 0$, we obtain Eq. (4-11). Thus, Proposition 1 is proofed.■

A useful application of Eq. (4-11) is to find the maximum throughput achieved under different levels of a factor ρ_C. Figure 4-3 (a) shows the throughput as a function of parameter β_C while the other two parameters are fixed. Figure 4-3 (b) corresponds to the optimal value of ρ_C, under which β_C maximizes the throughput.

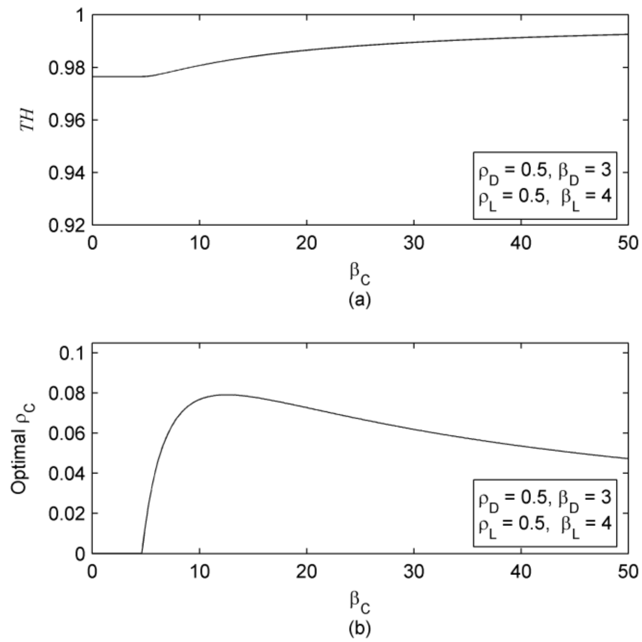

Figure 4-3 Maximum throughput as a function of β_C and optimal ρ_C

4.2.3 Analysis of throughput at an assembly line

When considering two or more stations in an assembly system, allocating process cycle times at each station constitutes an important issue in assembly line design; this is known in the literature as the line balancing problem (Thomopoulos, 1967; Merengo et al., 1999). The main objective of line balancing is to maximize the throughput of the system by reducing the difference of cycle times at individual stations. Since we assume that the task cycle time at each station has an effect on its quality conforming rate, the line balancing problem acquires a new dimension due to the fact that the output quality conforming rate and throughput are both affected by the cycle time assigned to each station. Moreover, the entropy of the input mix ratio transferred from upstream stations to downstream stations increases as the percentage of nonconforming units increases. This interaction effect can be analytically studied by considering the effect of β_i^k, which corresponds to factor i ($i = C, D, L$), at station k.

In this section we discuss these interrelated effects under the context of an assembly system with stations in a serial configuration. At this point, it should be clarified that the main assumption throughout this section is that there is a fixed total production cycle time $CT^{E(1,2,\dots,n)}$ corresponding to the cycle time of the system formed by station $1,2,\dots,n$ as

$$CT^{E(1,2,\dots,n)} = CT^1 + CT^2 + \cdots + CT^n \tag{4-12}$$

where CT^k is the cycle time at station k ($k = 1,2,\dots,n$). In other words, the total production cycle time corresponds to the time that elapses from the moment when a unit enters the line as raw material to the instant that it leaves the line as a finished product. The time spent in activities that are not directly relevant to production, such as transportation from one station to another, and time spent waiting in buffers, are not considered in the paper.

For an assembly line having n stations denoted by $S_{E(1,2,\dots,n)}$, the overall process quality conforming rate is defined as $Q^{E(1,2,\dots,n)} = Q^1 \cdot Q^2 \cdots Q^n$. Here, Q^k is the individual quality conforming rate at station k, which depends on the individual cycle time CT^k at station k, based on Eq. (4-8). Therefore, the analysis of the effect on process quality and throughput due to reducing individual CT^k at one or more stations needs to be carefully studied, which will be discussed in detail as follows.

Suppose Q^1 and Q^2 are used to characterize the quality conforming rate at two stations, stations 1 and 2, respectively, as follows

$$Q^1 = Q_0^1\left[\Delta_M^1\left(1 - e^{-\beta_C^1\rho_C^1 - \beta_D^1\rho_D^1 - \beta_L^1\rho_L^1}\right)\right]$$

$$Q^2 = Q_0^2[\Delta_M^2\left(1 - e^{-\beta_C^2\rho_C^2 - \beta_D^2\rho_D^2 - \beta_L^2\rho_L^2}\right)]$$

where Q_0^k and Δ_M^k are the minimal quality conforming rate and maximum possible percentage of quality improvement at station k ($k = 1{,}2$), respectively. Furthermore, factor ρ_i^k, correspond to the normalized operator's factor ρ_i at station k, for $i = C, D, L$.

(i) Station 1. An increase of CT^1 by Δ_C^1 will increase ρ_C^1 to

$$\rho_{C+\Delta_C}^1 = \rho_C^1 + \frac{\Delta_C^1}{CT_M^1 - CT_0^1} \tag{4-13}$$

thus, leading to an increase in Q^1 of ΔQ^1 given by

$$\Delta Q^1 = K^1 Q_0^1 \Delta_0^1\left(1 - e^{-\beta_C^1\delta_C^1}\right) \tag{4-14}$$

where $K^1 = e^{-\beta_C^1\rho_C^1 - \beta_D^1\rho_D^1 - \beta_L^1\rho_L^1}$ and $\delta_C^1 = \frac{\Delta_C^1}{CT_M^1 - CT_0^1}$.

(ii) Station 2. If $CT^1 + CT^2$ is fixed, an increment of CT^1 to $CT^1 + \Delta_C^1$ will decrease CT^2 to $CT^2 - \Delta_C^1$, which consequently decreases ρ_C^2 as

$$\rho_{C+\Delta_C}^2 = \rho_C^2 - \frac{\Delta_C^1}{CT_M^2 - CT_0^2} \tag{4-15}$$

The analysis of the effect of increasing CT^1 to $CT^1 + \Delta_C^1$ on Q^2 demands more consideration, since an increase of Q^1 by ΔQ^1 (Eq. (4-14)) also affects the propagated input entropy H_D^2 at station 2, which is obtained by

$$H_D^2 = Q^1 H_D^1 + H_{Q^1} \tag{4-16}$$

where $H_{Q^1} = -Q^1 \log Q^1 - (1 - Q^1) \log(1 - Q^1)$. Hence, an increase of ΔQ^1 at station 1 decreases the input entropy $H_D^2 = Q^1 H_D^1 + H_{Q^1}$ by

$$\Delta_H^2 = H_D^1 \Delta Q^1 + H_{Q^1 + \Delta Q^1} - H_{Q^1} \tag{4-17}$$

Overall, the decrease on Q^2 due to increasing CT^1 by Δ_C^1 can be computed as

$$\Delta Q^2 = K^2 Q_0^2 \Delta_0^2 \left(e^{\beta_C^2 \delta_C^2 - \beta_D^2 \delta_D^2} - 1 \right) \tag{4-18}$$

where $K^2 = e^{-\beta_C^2 \rho_C^2 - \beta_D^2 \rho_D^2 - \beta_L^2 \rho_L^2}$, $\delta_C^2 = \frac{\Delta_C^1}{CT_M^2 - CT_0^2}$ and $\delta_D^2 = \frac{H_D^1 \Delta Q^1 + H_{Q^1 + \Delta Q^1} - H_{Q^1}}{H_M}$.

Finally, the effect on the overall process quality $Q^{E(1,2)} = Q^1 \cdot Q^2$ after increasing the cycle time of station 1 from CT^1 to $CT^1 + \Delta_C^1$ is

$$\Delta Q^{E(1,2)} = \Delta Q^1 Q^2 - \Delta Q^1 \Delta Q^2 - Q^1 \Delta Q^2 \tag{4-19}$$

By using Eq. (4-19), we can maximize $Q^{E(1,2)}$ by finding the optimum CT^1, thus finding the optimum CT^2.

4.3 System performance analysis

In this section we will present two examples to illustrate the use of the proposed model. The first example illustrates how the process quality conforming rate and throughput is affected by allocating the production cycle times in a simple assembly line consisting of two serial stations. The second example further illustrates how to allocate the cycle time at individual stations to maximize the process capability for handling complexity in an assembly line with five stations.

4.3.1 Example 1: two stations in serial configuration

Analysis of quality conforming rate. We consider a simplified assembly process consisting of two sequential stations requiring intensive manual operations, as shown in Figure 4-4. S_k^i corresponds to task i at station k, for producing a product of type i.

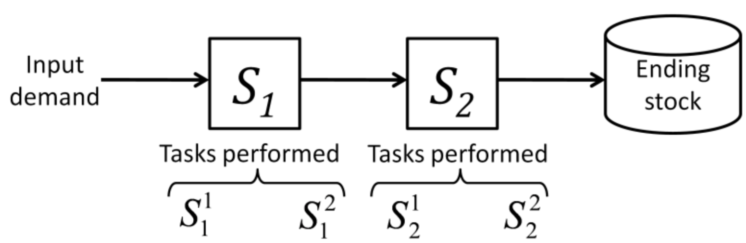

Figure 4-4 Two station manual assembly process

The process parameters at each station are given in Table 4-1 (a), (b) and (c).

Table 4-1 Process parameters for the proposed model

(a)

Station k	Tasks at station k	$CT_{i,0}^{k}$	$CT_{i,M}^{k}$	$Q_{i,0}^{k}$	$\Delta_{i,M}^{k}$
Station 1	S_1^1 and S_1^2	$CT_{1,0}^{1} = CT_{2,0}^{1} = 30$	$CT_{1,M}^{1} = CT_{2,M}^{1} = 190$	$Q_{1,0}^{1} = Q_{2,0}^{1} = 0.8$	$\Delta_{1,M}^{1} = \Delta_{2,M}^{1} = 0.25$
Station 2	S_2^1 and S_2^2	$CT_{1,0}^{2} = CT_{2,0}^{2} = 70$	$CT_{1,M}^{2} = CT_{2,M}^{2} = 95$	$Q_{1,0}^{2} = Q_{2,0}^{2}$ =0.8	$\Delta_{1,M}^{2} = \Delta_{2,M}^{2}$ =0.25

(b)

	Station 1
β_C^1	0.70568
β_D^1	0.37084
β_L^1	0.48344

(c)

	Station 2
β_C^2	1.0836
β_D^2	0.7889
β_L^2	0.9194

Table 4-2 shows the input demand mix ratio at each station corresponding to each product variety.

Table 4-2 Input mix ratio for two station scenario

	$Prob\{S_k^1\}$	$Prob\{S_k^2\}$
Station 1	$Prob\{S_1^1\} = 0.75$	$Prob\{S_1^2\} = 0.25$
Station 2	$Prob\{S_1^1\} = 0.40$	$Prob\{S_2^2\} = 0.60$

Based on Eq. (4-5), the entropy of the demand variety at station 1 is obtained as $H_D^1 = 0.5623$. The quality conforming rate at station 1, denoted as Q^1, affects the entropy of the input variety at station 2, since there is a proportion $1 - Q^1$ of nonconforming units entering station 2, which leads to further inspection efforts. This increases H_D^2 and thus complicating the process. Mathematically, the input mix ratio entering station 2 can be represented by $Prob\{S_1^1|Q^1\} = Q^1 S_1^1$ and $Prob\{S_1^2|Q^1\} = Q^1 S_1^2$, thus $Prob\{S_1^{\varepsilon}|Q^1\} = 1 - Q^1$, where $Prob\{S_1^{\varepsilon}|Q^1\}$ corresponds to the frequency of nonconforming products entering station 2. Furthermore, the input complexity at station 2 can be calculated by $H_D^2 = Q^1 H_D^1 + H_{Q^1}$, where $H_{Q^1} = -Q^1 \log Q^1 - (1 - Q^1) \log(1 - Q^1)$.

Assume that the combined total production cycle time of these two stations is constrained to being less than 200 seconds. A production engineer is required to split the total production cycle time into the individual cycle times of each station in order to achieve a desired optimal process performance, i.e., given $CT^{E(1,2)} = CT^1 + CT^2$, how to maximize the quality conforming rate or the throughput by determining CT^1. Figure 4-5 (a), (b) and (c) show the effect of varying CT^1 on Q^1, Q^2 and $Q^{E(1,2)}$, respectively. Note that $CT^{E(1,2)} = CT^1 + CT^2 = 200$ seconds.

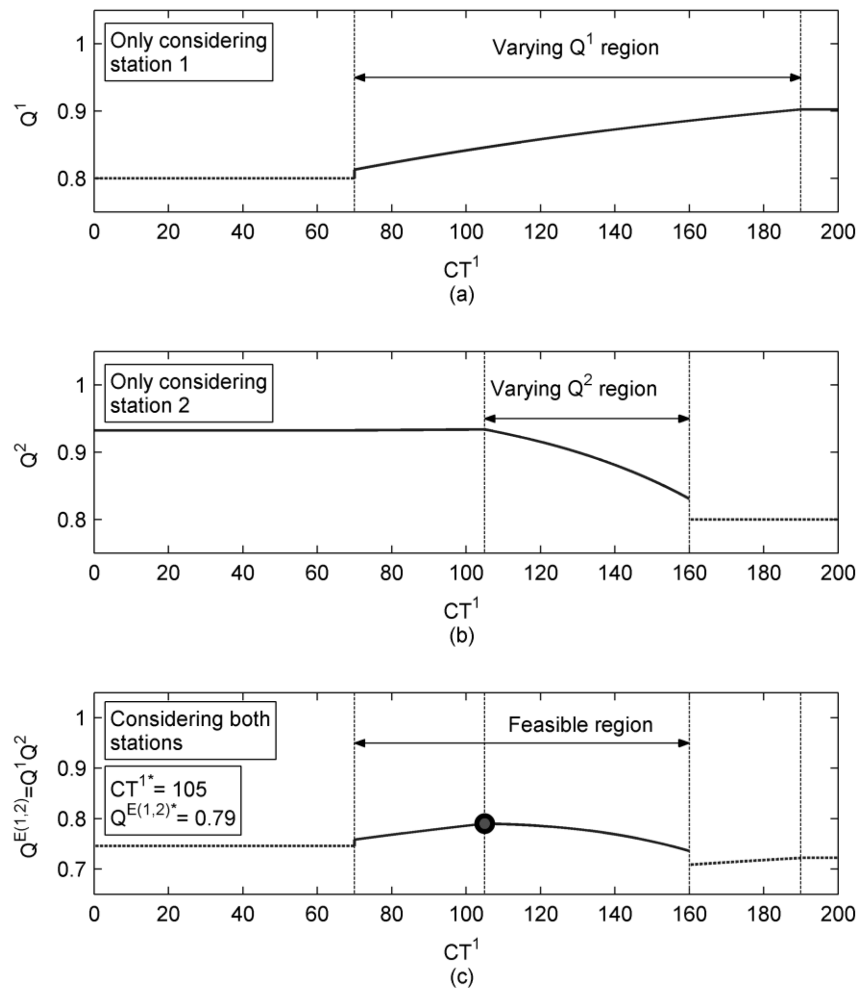

Figure 4-5 CT^1 vs. **(a)** Q^1, **(b)** Q^2, and **(c)** $Q^{E(1,2)}$

We can see in Figure 4-5 (c) that the maximum quality conforming rate for the production line is $Q^{E(1,2)*} = 0.79$, which is achieved when station 1's cycle time is allocated as $CT^{1*} = 105$, thus the optimal cycle time for station 2 is $CT^{2*} = CT^{E(1,2)} - CT^{1*} = 200 - 105 = 95$.

Table 4-3 Analysis results for two station scenario

	Independent optimal conforming rate Q^{k*}	Combined optimal conforming rate Q^{k*} to achieve $Q^{E(1,2)*}$	Optimal process time CT^{k*}	Throughput at individual stations TH^{k}
Station 1	0.9025	0.8458	105	0.0081
Station 2	0.9340	0.9340	95	0.0098
System	-	0.7900	200	-

Analysis of througput. Suppose that the goal is to maximize the effective throughput of the whole assembly line. For the system shown in Figure 4-4, the capacity of the bottleneck station corresponds to $r_b = \min(TH^1, TH^2)$, where TH^k denotes the throughput of station k. From Little's law, we know that under the assumption of deterministic cycle times, the WIP that achieves maximum throughput is $WIP = Q^{E(1,2)} r_b CT^{E(1,2)} = 0.7900 \times 0.0081 \times 200 = 1.2798$.

Figure 4-6 illustrates the effects of varying CT^1 on $TH^{E(1,2)}$. The maximum $TH^{E(1,2)*}$ by considreing $Q^{E(1,2)}$ occurs at $CT^1 = 93.84$, while the maximum $TH^{E(1,2)*}$ without considering $Q^{E(1,2)}$ occurs at $CT^1 = 100$. As we can see, there is an over optimistic prediction of $TH^{E(1,2)*}$ when not considering the operator's effect on $Q^{E(1,2)}$.

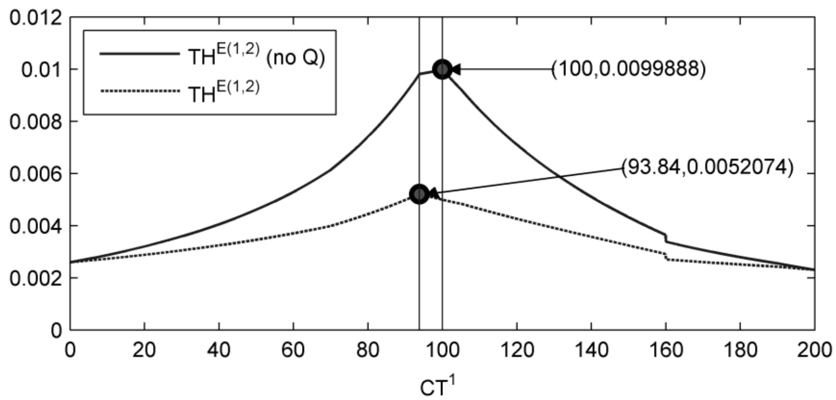

Figure 4-6 Effect of varying CT^1 on throughput

4.3.2 Example 2: five stations with hybrid configuration

The allocation of cycle times for individual stations is a critical issue in the design of a manufacturing system, especially for an MMAS with different product types. In this example, shown in Figure 4-7, an assembly system is used to produce a table, which consists of five stations producing three product types. We assume that every product must pass through all stations.

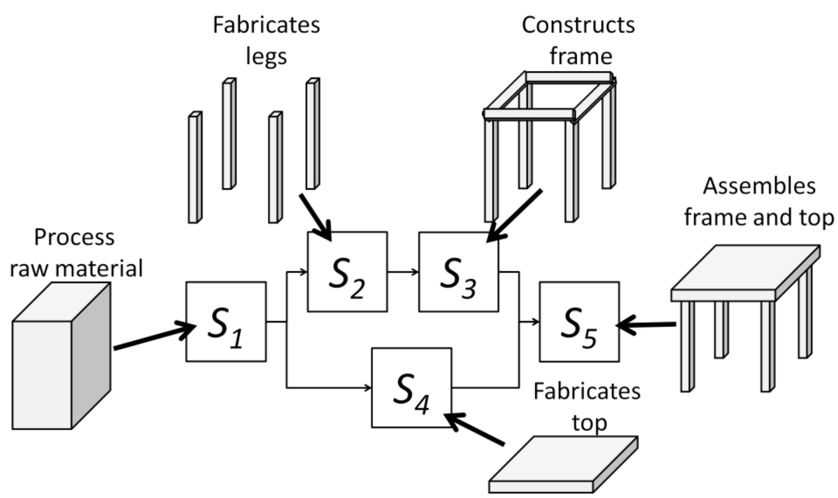

Figure 4-7 Production process for assembling a table

The objective is to maximize the process capability for handling complexity or PCC, which is defined in Abad and Jin (2010), by optimally allocating individual station's cycle time. Mathematically, the problem is formulated as

$$\max_{\boldsymbol{CT^k}} PCC$$

$$\text{s.t: } \boldsymbol{CT^{E(1,2,3,4,5)}} \leq 200 \text{ sec}$$

$$CT_i^k \geq CT_{i,0}^k, \text{ for } i = 1,2,3 \text{ and } k = 1,2,3,4,5$$

Based on Abad and Jin (2010), $PCC$is calculated by

$$PCC = \sum_{i,j} \pi_{ij}^{IN,OUT} \log \frac{\pi_{ij}^{IN,OUT}}{\pi_i^{IN} \pi_j^{OUT}} \tag{4-20}$$

where π_i^{IN} is the percentage of the input demand corresponding to the i^{th} product type, and π_j^{OUT} corresponds to the percentage of products of type j produced, including π_ε^{OUT} which is the percentage of nonconforming products produced by the system. Element $\pi_{ij}^{IN,OUT}$ is the percentage of demanding a product type i and producing a product type j. The relation $\pi_{ij}^{IN,OUT} = \psi_{ij}^{E(\cdot)} \pi_i^{IN}$ is held, where $\psi_{ij}^{E(1,2,3,4,5)}$ is the entry at the i^{th} row and j^{th} column of matrix $\boldsymbol{\Psi^{E(1,2,3,4,5)}}$, given by

$$\boldsymbol{\Psi^{E(1,2,3,4,5)}} = \begin{bmatrix} \psi_{00}^{E(1,2,3,4,5)} & 0 & \cdots & 0 & \psi_{0\varepsilon}^{E(1,2,3,4,5)} \\ 0 & \psi_{11}^{E(1,2,3,4,5)} & \cdots & 0 & \psi_{1\varepsilon}^{E(1,2,3,4,5)} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & \psi_{N-1,N-1}^{E(1,2,3,4,5)} & \psi_{N-1,\varepsilon}^{E(1,2,3,4,5)} \\ 0 & 0 & \cdots & 0 & 1 \end{bmatrix} \tag{4-21}$$

where

$\psi_{ii}^{E(1,2,3,4,5)} = Prob\{$Producing a conforming product type i at the assembly system$\}$

and $\psi_{i\varepsilon}^{E(1,2,3,4,5)} = Prob\{$Producing a defective product type i at the assembly system$\}$

Furthermore, based on the model proposed in this paper, the element ψ_{ii}^{k} is obtained by

$$\psi_{ii}^{k} = Q_{i,0}^{k}[1 + \Delta_{i,M}^{k}(1 - e^{-\beta_{i,C}^{k}\rho_{i,C}^{k}})] \tag{4-22}$$

where $Q_{i,0}^{k}$ is the minimal quality conforming rate at station k for product type i. Similarly, $\Delta_{i,M}^{k}$ is the upper limit of the possible improvement of the quality conforming rate at station k for product type i when the operator's factors are under the most favorable conditions. Furthermore, β_{C}^{k} and ρ_{C}^{k} are the cycle time parameter and the normalized cycle time factor, respectively at station k. For simplicity, we have considered ρ_{C}^{k} as the only operator's factor affecting the quality conforming performance, since the other factors (ρ_{D}^{k} and ρ_{L}^{k}) do not affect our optimization solution. The parameters for obtaining ψ_{ii}^{k} are given in Table 4-4, where $CT_{i,M}^{k}$ is the maximum cycle time at station k for product type i, after which there is no further quality conforming improvement.

Table 4-4 Model parameters for producing **(a)** product type 1, **(b)** product type 2, and **(c)** product type 3

(a)

	Product type 1				
	Parameters of proposed model			Processing times (sec)	
Station k	$Q_{1,0}^{k}$	$\Delta_{1,M}^{k}$	$\beta_{1,C}^{k}$	$CT_{1,0}^{k}$	$CT_{1,M}^{k}$
Station 1	$Q_{1,0}^{1} = 0.95$	$\Delta_{1,M}^{1} = 0.052$	$\beta_{1,C}^{1} = 0.80$	$CT_{1,0}^{1} = 15$	$CT_{1,M}^{1} = 150$
Station 2	$Q_{1,0}^{2} = 0.97$	$\Delta_{1,M}^{2} = 0.030$	$\beta_{1,C}^{2} = 0.70$	$CT_{1,0}^{2} = 25$	$CT_{1,M}^{2} = 100$
Station 3	$Q_{1,0}^{3} = 0.94$	$\Delta_{1,M}^{3} = 0.063$	$\beta_{1,C}^{3} = 0.60$	$CT_{1,0}^{3} = 20$	$CT_{1,M}^{3} = 90$
Station 4	$Q_{1,0}^{4} = 0.95$	$\Delta_{1,M}^{4} = 0.052$	$\beta_{1,C}^{4} = 0.80$	$CT_{1,0}^{4} = 15$	$CT_{1,M}^{4} = 100$
Station 5	$Q_{1,0}^{5} = 0.97$	$\Delta_{1,M}^{5} = 0.030$	$\beta_{1,C}^{5} = 0.90$	$CT_{1,0}^{5} = 12$	$CT_{1,M}^{5} = 70$

(b)

	Product type 2				
	Parameters of proposed model			Processing times (sec)	
Station k	$Q_{2,0}^k$	$\Delta_{2,M}^k$	$\beta_{2,C}^k$	$CT_{2,0}^k$	$CT_{2,M}^k$
Station 1	$Q_{2,0}^1 = 0.85$	$\Delta_{2,M}^1 = 0.176$	$\beta_{2,C}^1 = 0.90$	$CT_{2,0}^1 = 20$	$CT_{2,M}^1 = 120$
Station 2	$Q_{2,0}^2 = 0.95$	$\Delta_{2,M}^2 = 0.052$	$\beta_{2,C}^2 = 0.85$	$CT_{2,0}^2 = 10$	$CT_{2,M}^2 = 100$
Station 3	$Q_{2,0}^3 = 0.97$	$\Delta_{2,M}^3 = 0.030$	$\beta_{2,C}^3 = 0.85$	$CT_{2,0}^3 = 5$	$CT_{2,M}^3 = 120$
Station 4	$Q_{2,0}^4 = 0.92$	$\Delta_{2,M}^4 = 0.086$	$\beta_{2,C}^4 = 0.70$	$CT_{2,0}^4 = 7$	$CT_{2,M}^4 = 110$
Station 5	$Q_{2,0}^5 = 0.98$	$\Delta_{2,M}^5 = 0.020$	$\beta_{2,C}^5 = 0.65$	$CT_{2,0}^5 = 10$	$CT_{2,M}^5 = 90$

(c)

	Product type 3				
	Parameters of proposed model			Processing times (sec)	
Station k	$Q_{3,0}^k$	$\Delta_{3,M}^k$	$\beta_{3,C}^k$	$CT_{3,0}^k$	$CT_{3,M}^k$
Station 1	$Q_{3,0}^1 = 0.89$	$\Delta_{3,M}^1 = 0.123$	$\beta_{3,C}^1 = 0.70$	$CT_{3,0}^1 = 10$	$CT_{3,M}^1 = 90$
Station 2	$Q_{3,0}^2 = 0.98$	$\Delta_{3,M}^2 = 0.020$	$\beta_{3,C}^2 = 0.75$	$CT_{3,0}^2 = 15$	$CT_{3,M}^2 = 200$
Station 3	$Q_{3,0}^3 = 0.94$	$\Delta_{3,M}^3 = 0.063$	$\beta_{3,C}^3 = 0.85$	$CT_{3,0}^3 = 10$	$CT_{3,M}^3 = 180$
Station 4	$Q_{3,0}^4 = 0.95$	$\Delta_{3,M}^4 = 0.052$	$\beta_{3,C}^4 = 0.80$	$CT_{3,0}^4 = 15$	$CT_{3,M}^4 = 150$
Station 5	$Q_{3,0}^5 = 0.97$	$\Delta_{3,M}^5 = 0.030$	$\beta_{3,C}^5 = 0.80$	$CT_{3,0}^5 = 17$	$CT_{3,M}^5 = 140$

To obtain the overall process quality conforming matrix $\boldsymbol{\Psi}^{\boldsymbol{E}(\mathbf{1,2,3,4,5})}$ from the individual quality conforming matrix $\boldsymbol{\Psi}^{\boldsymbol{k}}$ at station k, the following algebraic quality expression is obtained based on Chapter 3.

$$\boldsymbol{\Psi}^{\boldsymbol{E}(\mathbf{1,2,3,4,5})} = \boldsymbol{\Psi}^{\mathbf{1}} \otimes_Q \left[\left(\boldsymbol{\Psi}^{\mathbf{2}} \otimes_Q \boldsymbol{\Psi}^{\mathbf{3}}\right) \oplus_Q \boldsymbol{\Psi}^{\mathbf{4}}\right] \otimes_Q \boldsymbol{\Psi}^{\mathbf{5}} \tag{4-23}$$

We assume that the total process cycle time $\boldsymbol{CT}^{\boldsymbol{E}(\mathbf{1,2,3,4,5})}$ cannot exceed 200 seconds, where vector $\boldsymbol{CT}^{\boldsymbol{E}(\mathbf{1,2,3,4,5})}$ can also be obtained based on the following algebraic cycle time expression

$$\boldsymbol{CT}^{\boldsymbol{E}(\mathbf{1,2,3,4,5})} = \boldsymbol{CT}^{\mathbf{1}} \otimes_T [(\boldsymbol{CT}^{\mathbf{2}} \otimes_T \boldsymbol{CT}^{\mathbf{3}}) \oplus_T \boldsymbol{CT}^{\mathbf{4}}] \otimes_T \boldsymbol{CT}^{\mathbf{5}} \tag{4-24}$$

In Eq. (4-24), $\boldsymbol{CT}^{\boldsymbol{k}} = [CT_1^k \quad CT_2^k \quad CT_3^k]$ is the cycle time vector at station k ($k = 1,2,3,4,5$), where the element CT_i^k is the cycle time for producing product type i at station k. For simplicity, we will assume that $CT_i^k = CT_j^k$, for $k = 1,2,3,4,5$.

Furthermore, $CT_{i,0}^{k}$ is the minimum cycle time required at station k to produce one product of type i.

We propose to use a genetic algorithm (GA) to solve this optimization problem because of its complicated structure; the main concepts of GA are defined as follows:

i. <u>Chromosomes</u>. The chromosomes are the potential solutions to the problem. In this example, the chromosomes are 5-element vectors of the form

$$\boldsymbol{CH} = [CT^1 \quad CT^2 \quad CT^3 \quad CT^4 \quad CT^5]^T \tag{4-25}$$

satisfying $\boldsymbol{CT^{E(1,2,3,4,5)}} \leq 200$ seconds and $CT_i^k \geq CT_{i,0}^k$, for $i = 1,2,3$ and $k = 1,2,3,4,5$. Here, for simplicity, we set $CT^k = CT_i^k$, i.e., the same cycle time is assigned for producing each product type i ($i = 1,2,3$) at station k.

ii. <u>Population</u>. The set of all chromosomes at each step of the GA.

iii. <u>Crossover</u>. Crossover is the act of combining two chromosomes, thus producing a new chromosome with characteristics similar to both of its parents. In this example, the crossover between chromosome $\boldsymbol{CH_i}$ and $\boldsymbol{CH_j}$ is obtained by

$$\boldsymbol{CH}_c = \left[(1+\lambda)CH_i + (1-\lambda)CH_j\right]/2 \tag{4-26}$$

for $-1 \leq \lambda \leq 1$. In this example we consider $\lambda = 0.5$.

iv. <u>Mutation</u>. When a chromosome mutates, it undergoes a small change, giving rise to a new chromosomes. Here, the mutation of chromosome $\boldsymbol{CH}$ to chromosome $\boldsymbol{CH_m}$ is defined as

$$\boldsymbol{CH_m} = \left\{\left(CT^k - \frac{\varepsilon}{5}\right) + \varepsilon\rho_k\right\}_k \tag{4-27}$$

where $\{\cdot\}_k$ is the k^{th} element of vector $\boldsymbol{CH_m}$, ε is a small quantity satisfying $\varepsilon > 0$. The quantity ρ_k is a random variable satisfying $0 \leq \rho_k \leq 1$ and $\sum \rho_k = 1$, for $k = 1,2,3,4,5$. Here, each ρ_k is equally distributed, and $\varepsilon = 30$.

In this example, the number of chromosomes at each population is equal to 100. Four methods are used for obtaining the chromosome at each population: chromosomes from the previous population with high performance; chromosomes obtained from mutation; chromosomes obtained from crossovers; and new chromosomes. Table 4-5 shows the percentages for each of these four methods used to generate new chromosomes in each population.

Table 4-5 Weights for generating new chromosomes in each population

	Percentage
Previous population	5%
Mutation	20%
Crossover	25%
Newly generated	50%

Figure 4-8 (a) shows the variability of the PCC within the initial population. Figure 4-8 (b) shows the optimal PCC attained during the first 100 populations.

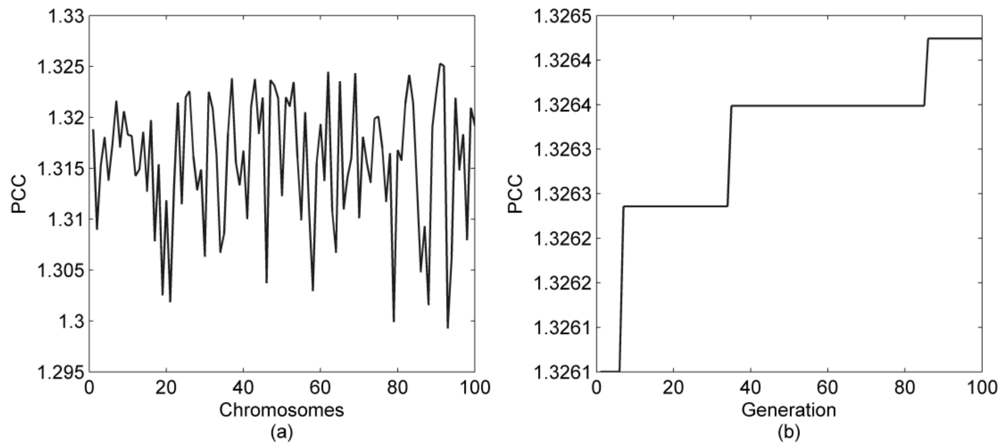

Figure 4-8 GA performance

Table 4-6 shows the details of the parameters used in the GA, as well as statistical characteristics of the solutions obtained through 1000 runs of the GA.

Table 4-6 Parameters and solutions from GA

	Values
Number of runs of GA	1000
Size of each population	100
Mean solution	1.3038
Std. dev. of solutions	0.0003

Finally, the average solution among 1000 runs of the GA is

$$\overline{\boldsymbol{CH}^*} = [77.12 \quad 27.60 \quad 25.36 \quad 50.80 \quad 19.10]$$

which provides the optimal cycle time allocation for individual stations under the objective of maximizing the PCC of the system. Furthermore, the standard deviation vector of the solutions is given by

$$\boldsymbol{\sigma}_{\boldsymbol{CH}^*} = [3.3414 \quad 2.5246 \quad 2.5996 \quad 0.9524 \quad 1.7541]$$

Note that the obtained standard deviations are relatively small compared with the corresponding mean values, and thus the obtained solutions converge well.

4.4 Conclusions and future work

This paper develops a general framework to model the effect of operator's factors on the process quality conforming rate and throughput in a mixed model assembly system. The proposed model considers both intrinsic factors (e.g., operator's mental thinking time and working experience) as well as extrinsic factors (e.g., product variety induced complexity) based on findings in the cognitive literature. We further use two examples to obtain some insights and implications of the proposed model; the examples describe how to allocate individual station's cycle time for achieving the desired optimal process performance such as, process quality, throughput, and process capability for handling complexity. Future research is needed for conducting further empirical validations of the proposed model through real world scenarios or experimental tests.

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CHAPTER 5
CONCLUSIONS AND FUTURE WORK

5.1 Conclusions and summary of original contributions

The implementation of a mass customization paradigm brings many new research challenges in the design and operation of mixed model assembly systems (MMASs) due to the inherent increase of complexity in the system configuration, production operation, and workers' performance in handling multiple tasks. Consequently effective mathematical tools are needed for appropriately modeling and analyzing these systems. This dissertation presents general methodologies for mathematically modeling and analyzing the process complexity induced by product variety in an MMAS. The major contributions of this dissertation are:

(i) Define a set of metrics for measuring a process (system) capability for handling the demand variety induced complexity in an MMAS.

A new modeling framework for measuring the complexity of a manufacturing system is introduced based on linkages with a communication system framework; the linkages between these two systems are explored in terms of their input/output mapping relationship, and their performance assessment metrics. Unlike existing complexity measures defined in the literature, the entropic based performance metrics are defined for the first time by taking the production quality factor into account when determining how well a manufacturing system can handle the operational complexity induced by the input demand variety. The rationales of defining those new metrics are based on the fact that the capability of an MMAS in handling the operation complexity should be assessed by the divergence between what is demanded and what is produced in the MMAS. A measure of this divergence can be interpreted as a measure of the amount of complexity that the manufacturing process delivers. To measure the divergence between the complexity in the demand and the complexity in the delivered products, the metric of process capability for handling complexity (PCC) is defined for the first time based on the *mutual information* metric, used for communication system's performance assessment based on information theory. Furthermore, some examples were provided to illustrate how the defined metric of the PCC is used to help understand and analyze the effect of the demand variety on system performance.

(ii) Develop a systematic approach for system configuration representation of MMASs using algebraic expressions.

This research proposes how to use algebraic expressions to efficiently and explicitly model assembly system configurations. The intent is to overcome the drawbacks of two widely used representation methods: block diagrams and adjacency matrices. The proposed methodology is based on the grouping corollary and considers hybrid serial and parallel relationships in a complex MMAS. Based on the proposed modeling method, a complex MMAS can be flexibly modeled by various levels of equivalent subsystems to match the different levels of decision-making strategies.

(iii) Develop essential algebraic performance operators for iteratively analyzing global performance of an MMAS with complex hybrid layout configurations.

By further extending the algebraic configuration operators, the algebraic performance operators are defined for the first time for systematically evaluating the system performance metrics, such as quality conforming rates for individual product types at each station, process capability for handling complexity, and production cycle time for different product types. Therefore, when compared with the recently developed string representation method, the proposed algebraic modeling method also has unique merit in its computational capability for automatically evaluating various system performance metrics. Two examples are given to illustrate how the proposed algebraic performance operators can be effectively used to assist in the design and performance analysis for mixed model assembly systems.

(iv) Develop an integrated model for modeling and analyzing the operator's factors and their effect on MMASs performance.

A quality conforming performance model is proposed by considering the effect of operator's factors on the process performance in an MMAS. The model considers intrinsic and extrinsic factors, such as mental deliberation thinking time, worker's experience, and choice task complexity. The proposed model structure is justified based on findings in the cognitive literature. A discussion on the implications of the proposed model on modeling the throughput of an assembly system is also presented, in which the cycle time allocation problem is investigated by considering the effect of the operator's factors. Finally, two examples are provided to show possible applications of the model.

5.2 Future work

The development of new mathematical tools for better understanding and more effectively analyzing the effect of the operational complexity in a mixed model assembly system is an emerging new research area, which requires abundant future research for achieving optimal design and operation performance. Some future research directions to extend this dissertation are suggested as follows.

(i) It would be useful to study how to measure process complexity through further consideration of the frequency of changeover or the batch size of product varieties. For this purpose, the predictability of the product sequences needs to be examined and included in the measure of input complexity, in addition to the demand mix ratio.

(ii) A single type of defect is used in this research to reflect the effect of production quality on the capability of a manufacturing system in handling process complexity. In order to consider the various costs of different types of scraps, the diagonal station representation currently used, which is based on a diagonal matrix of quality conforming rates, should be extended into a general matrix representation. For this extension, a general communication channel model should be used, and the further investigation is needed for expanding the associated performance metrics for the assessment of manufacturing system performance.

(iii) The proposed algebraic expressions can be further mathematically formalized by constructing an algebraic field that can be used for different interesting purposes. These purposes could include finding stations causing bottlenecks; balancing the stations at the system level; determining the optimal combination between two or more assembly systems (or subsystems) by sharing one or more stations with additional processes; and determining optimal scheduling strategies under complex assembly systems layout configurations.

(iv) The proposed metrics and analyses are all based on the assumption of independent inputs between stations. With the increasing application of radio frequency identification (RFID) systems and online computer monitoring systems, the information about the product types at the previous stations can be timely shared with later station operations. As a result, the complexity induced by product variety is not isolated among the various stations. In this case, a Bayesian network model may be included by using conditional probability to represent the dependent mix ratio of the input variety at each station, based on shared information about processed product types at the previous station.

(v) Further research directions for the proposed quality conforming model of the operator's performance should include an empirical validation of the model through real world scenarios or the design of experimental tests, and the subsequent work on data collection and validation analysis.