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PARALLEL NETWORK OSCILLATORS

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ABSTRACT

A brief review is given of the various methods for achieving electrical tuning of low level oscillators using conventional types of vacuum tubes. A class of oscillators is described which makes use of parallel signal paths between common input and output terminals with the relative signal gain in these two paths varied by means of a balanced control voltage resulting in frequency tuning. A general discussion is given of phase-shift-type parallel network oscillators. A specific example is given of a resonant type of oscillator which has a measured frequency coverage of 15 to 30 Mc/s with an oscillation amplitude relatively constant over the tuning range.

PARALLEL NETWORK OSCILLATORS

1. INTRODUCTION

Many techniques are available for realizing wide range electrical tuning of low level oscillators below microwave frequencies. The most important of these (where devices employing special tubes are not considered) are (1) reactance tuning, (2) resistance tuning, (3) ferrite tuning, (4) pulse circuit tuning, and (5) parallel network tuning.

The main limitation of reactance tuning is the small tuning range-- perhaps up to 20 or 30 percent. Resistance tuning (e.g., crystal diodes or variable triode plate resistance) is most effective in R-C oscillators at frequencies below a few Mc/s (limited upper frequency range) but does not favor constancy of output without special measures of amplitude control. Tuning with either ferroelectric or ferromagnetic materials can yield a large tuning range but suffers from annoying hystersis and temperature effects. Pulse circuit tuning (e.g., multivibrators) is limited to the low frequencies and does not give a good waveform.

Parallel network tuning has none of the disadvantages enumerated above. Unlike most devices, the tuning range is not dependent upon components but is ultimately limited by the gain bandwidth product of the tubes employed. Thus, very large tuning ratios can be achieved at the lower frequencies with ratios decreasing as the center frequency increases. In addition, problems of vacuum tube linearity can be avoided because tubes can be operated class C as well as class A with little change in basic behavior.

2. DESCRIPTION OF THE PARALLEL SYSTEM.

A parallel tuned system is one having two signal paths with the outputs of these two paths combined and fed back to a common input. Several signal paths also define a parallel tuned system; however, they do not appear as practical as systems having only two paths. Tuning is accomplished by varying the relative gains of the two signal paths. The basic block diagram is shown in Fig. 1. Let a small signal voltage (noise) be added in the feedback path and let the feedback path be opened at point "x" as described in Fig. 1. Then, the open loop transfer function becomes

$$\frac{e_0}{e_n} = \left[K_1 F_1(p) + K_2 F_2(p) \right] F_3(p) \tag{1}$$

in which $p = j\omega$. Let the relative gains of the two channels be varied symmetrically about the mean gain K_0 as

$$K_1 = K_0(1 + k) \qquad\qquad K_2 = K_0(1 - k) \tag{2}$$

where $\left| k \right| \leq 1$ and where k can be either positive or negative. Then,

$$\frac{e_0}{e_n} = K_0 \left\{ \left[F_1(p) + F_2(p) \right] + k \left[F_1(p) - F_2(p) \right] \right\} F_3(p) \tag{3}$$

Oscillation will take place at a frequency where the phase shift of the open loop function is a multiple of 360 degrees, providing that the open loop gain at that frequency is greater than unity. Clearly, varying k will have a profound effect in determining the oscillation frequency if suitable functions are used for F_1 and F_2 and to a lesser extent for F_3. Also, it is possible by careful selection of networks to make the magnitude of e_0/e_n fairly constant with k in which case the amplitude of oscillations will be fairly constant with frequency. For obvious reasons (which are represented by the negative sign
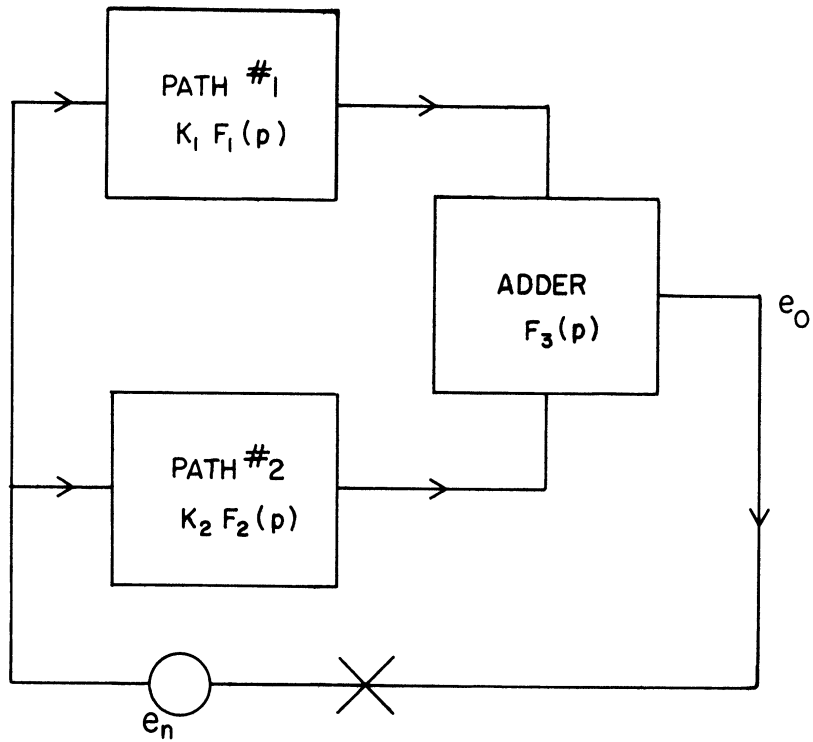
BLOCK DIAGRAM OF THE GENERAL
PARALLEL NETWORK OSCILLATOR

FIG. I

in equation 3), F_1 and F_2 must be considerably different functions of frequency if any tuning is to be achieved.

3. PHASE SHIFT PARALLEL NETWORK OSCILLATORS.

One can generalize a phase shift oscillator as one having some type of R-C or L-C transmission line or an actual transmission line as a critical element Ideally, these lines are lossless with a linear phase shift with frequency. Practically, they are approximated with lumped elements or with transmission line segments associated with shunting capacitances and other lumped elements. In order to study the idealized phase shift parallel system, it will be assumed that

$$F_1(p) = \exp(-pT) \quad F_2(p) = \exp(-paT) \quad F_3(p) = 1 \tag{4}$$

Then, (3) becomes

$$\frac{e_0}{e_n} = K_0 \left[(1 + k)\exp(-pT) + (1 - k)\exp(-paT) \right] \tag{5}$$

Using $p = j\omega$ and combining real and imaginary parts,

$$\frac{e_0}{e_n} = K_0 \left[(1 + k)\cos\omega T + (1 - k)\cos a \omega T \right]$$

$$- jK_0 \left[(1 + k)\sin\omega T + (1 - k)\sin a\omega T \right] \tag{6}$$

The frequency of oscillation is such as to cause the imaginary part of (6) to be zero. This frequency is defined by

$$(1 + k)\sin\omega T + (1 - k)\sin a\omega T = 0 \tag{7}$$

Of course, the open loop gain at the oscillation frequency must be greater that unity.

$$\left| K_0 \left[(1 + k)\cos\omega T + (1 - k)\cos a\omega T \right] \right| \geq 1 \tag{8}$$

If K_0 is positive, the phase shift of e_0/e_n at the oscillation frequency will be an even multiple of 180 degrees whereas if K_0 is negative, the phase shift will be an odd multiple of 180 degrees.

The limits of oscillation take place at frequencies ω_1 and ω_2 where $k = \pm 1$. The center frequency ω_m is given when $k = 0$. Thus,

$$\sin \omega_2 T = 0 \quad \text{at } k = +1$$

$$\sin a\, \omega_1 T = 0 \quad \text{at } k = -1$$

$$\sin \omega_m T + \sin a\, \omega_m T = 0 \quad \text{at } k = 0 \tag{9}$$

If we assume the special case where K_0 is negative and oscillation takes place at the lowest possible frequency, then $\omega_1 = \pi/aT$, $\omega_2 = \pi/T$, and $\omega_m = 2\omega_1\omega_2/(\omega_1 + \omega_2)$. Continuing with this most important case, the question of the magnitude of e_0/e_n becomes of interest. At the extreme oscillation frequencies ω_1 and ω_2, the cosine function is unity and the magnitude of e_0/e_n is $\left| 2K_0 \right|$. At the center frequency, it is

$$\left| K_0 \left[\cos \frac{2\pi}{a+1} + \cos \frac{2a\pi}{a+1} \right] \right| \tag{10}$$

which will be less than $\left| 2K_0 \right|$ except for the trivial case of $a = 1$ which yields a tuning range of zero. For values of a of one and two, the gain magnitude is $\left| 2K_0 \right|$ and $\left| (3)^{1/2} K_0 \right|$, respectively. For $a = 2$, the tuning ratio is two to one. For $a = 3$, the gain magnitude falls to zero at the center frequency—values of three or larger would not normally be used in order to avoid oscillation cessation near the center of the tuning range. Therefore, the phase shift type of parallel network oscillator is not too well suited for tuning ratios larger than three to one and more practically about two to one. It should be especially noted that for $a = 2$, the open loop gain varies only by a factor of $(3)^{1/2}$ to 2 over the entire tuning range and for $a < 2$, the variation will be even less.
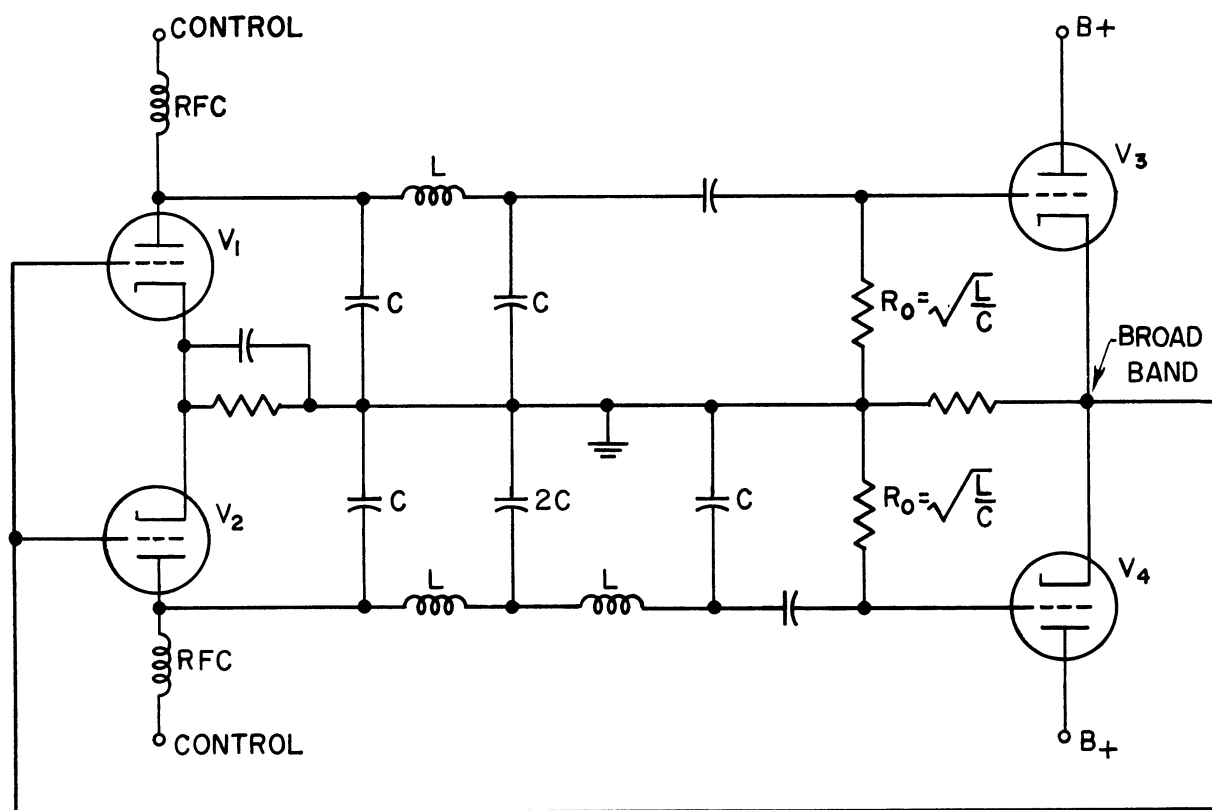
Therefore, if the tuning ratio is equal to or less than two to one, the amplitude of oscillations will be relatively constant with frequency.

Practically, it is difficult to construct an adding device having a transfer function of unity. If $F_3(p) = \exp(-pb)$ instead of unity, the preceeding relations become somewhat modified. Of most importance is the slight reduction in tuning range if b is somewhat smaller than T. If b and T have about the same value, the tuning range will be halved as compared to that when $b = 0$. The open loop gain magnitude as a function of frequency is not affected when F_3 is the exponential function.

## 4. EXAMPLE OF A PHASE SHIFT OSCILLATOR.

As a practical example of a transmission line type of parallel network oscillator, let $F_1(p)$ be given by a Pi network approximation to a transmission line and let $F_2(p)$ be given by two similar Pi sections in cascade. Then, $a \cong 2$ because the phase shift of the two sections at any given frequency is (ideally) twice that of one section. The result is the circuit of Fig. 2. Varying the relative gains of the two channels is achieved by applying push-pull modulation to the plates of V_1 and V_2. Of course, grid or cathode modulation (or possibly screen modulation if pentodes are used) can also be employed. It should be observed, however, that plate modulation for frequency variations in parallel network oscillators has the same advantages as plate modulation for amplitude variations in conventional oscillators.

It can be seen in Fig. 2 that the single Pi network has at its output a voltage similar to that at the center of the two section network. This leads to certain obvious simplifications of Fig. 2 resulting in the circuit of Fig. 3.

A PHASE SHIFT PARALLEL NETWORK OSCILLATOR

FIG. 2

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CONTROL

$R_L \gg R_0$

L L

C 2C C $R_0 = \sqrt{\dfrac{L}{C}}$

$R_L \gg R_0$

CONTROL
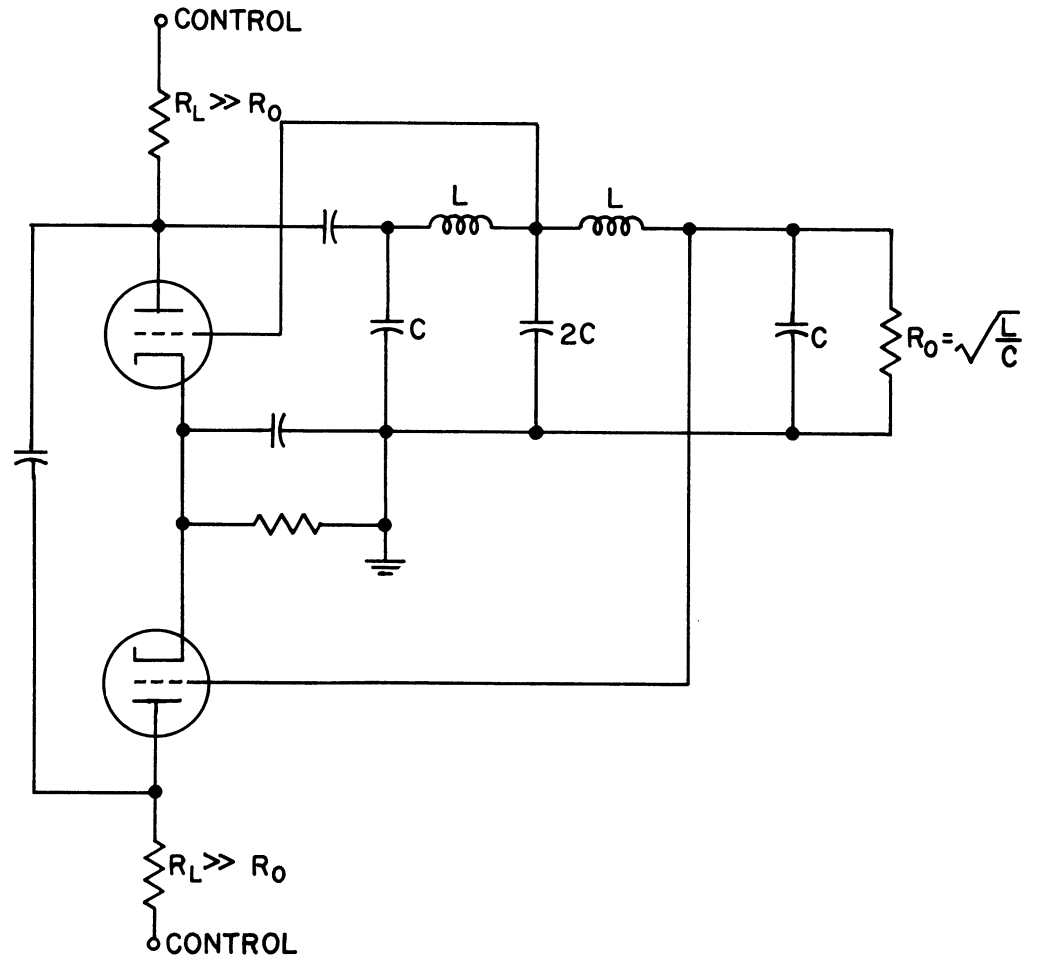
DISTRIBUTED PHASE SHIFT
PARALLEL NETWORK OSCILLATOR

FIG. 3

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The circuit is that of the usual constant-k filter. In a sense, time delay isolation has been used in the adding circuit resulting in advantages similar to those obtained in distributed amplifiers. In fact, a whole string of tubes can be placed along an artificial transmission line to give the distributed equivalent of a parallel network oscillator having several parallel signal paths. However, unlike the distributed amplifier, only a few tubes are active at any given frequency which poses severe problems regarding control voltages--e.g., if three tubes are used, means must be provided for shifting the plate voltage from tube to tube in a continuous fashion.

It is somewhat surprising that the circuit of Fig. 3 is that of a reactance modulated oscillator described by Dennis and Felch[1] having a tuning range of about 20 percent. However, the parallel concept indicates a quite different mode of operating the tubes which doubles the tuning range to something like 40 percent. The tuning range of the circuit of Fig. 3 can be made to approach two to one by using a more sophisticated network approximation to a transmission line.

5. A RESONANT TYPE PARALLEL NETWORK OSCILLATOR.

Oscillators fall into two general types. The first is the phase shift type as already described which makes use of low pass (or high pass) filters. The second type makes use of resonant circuits (i.e., bandpass filters) as is done in the great bulk of conventional oscillators.

If parallel resonant circuits are used for $F_1(p)$, $F_2(p)$, and $F_3(p)$, one has equations of the form

1. F. R. Dennis and E. P. Felch, "Reactance Tube Modulation of Phase Shift Oscillators", BSTJ, Vol. 28, No. 4, PP. 601-7, October, 1949.

$$K_0 \, F_1(p) = \frac{-pg_{ml}/C_1}{p^2 + pB_1 + \omega_1^2} \tag{11}$$

with similar expressions for the other functions. B_1 is the half-power band-width, ω_1 is the center radian frequency, C_1 is the shunt capacitance, and g_{ml} is the transconductance. The parallel system to which the following data and discussion applies is shown in Fig. 4. The plate loads of V_1 and V_2 are the functions $F_1(p)$ and $F_2(p)$, and the common plate load of V_3 and V_4 is the function $F_3(p)$. All components not lettered in Fig. 4 are either coupling or bypass elements. The tubes used will be assumed to be 12AT7 types.

This circuit with $C_1 = C_2$ and C_1, C_2, and C_3 furnished only by stray capacitances and with $R_1 = 300$ ohms, $R_2 = 560$ ohms, and $R_3 = 68$ ohms, and with $f_1 = 12$ Mc/s, $f_2 = 26$ Mc/s, and $f_3 = 18$ Mc/s has been constructed and tested. The experimental curve of frequency as a function of the plate voltage E_1 (where E_1 and E_2 are balanced with respect to the center voltage) is shown in Fig. 5. The output voltage as a function of frequency (measured at the plates of V_3 and V_4) is also shown in Fig. 5. In particular, it should be noted that the oscillation amplitude does not change markedly as the limits of the tuning range are approached.

The two-to-one frequency coverage displayed by Fig. 5 by no means represents the best that can be done--any reasonable 15 Mc/s increment can be tuned such as 1 to 16 or 100 to 115 Mc/s. Through careful construction, a 20 to 40 Mc/s tuning range can be obtained using the same tube, which is about one-third to one-half the practical gain bandwidth product of the 12AT7 tube. With some of the newer close-spaced triodes having very large gain bandwidth products, appreciable tuning ranges can be obtained up to center frequencies of 1000 Mc/s or more using either resonant or phase shift types of parallel network oscillators.
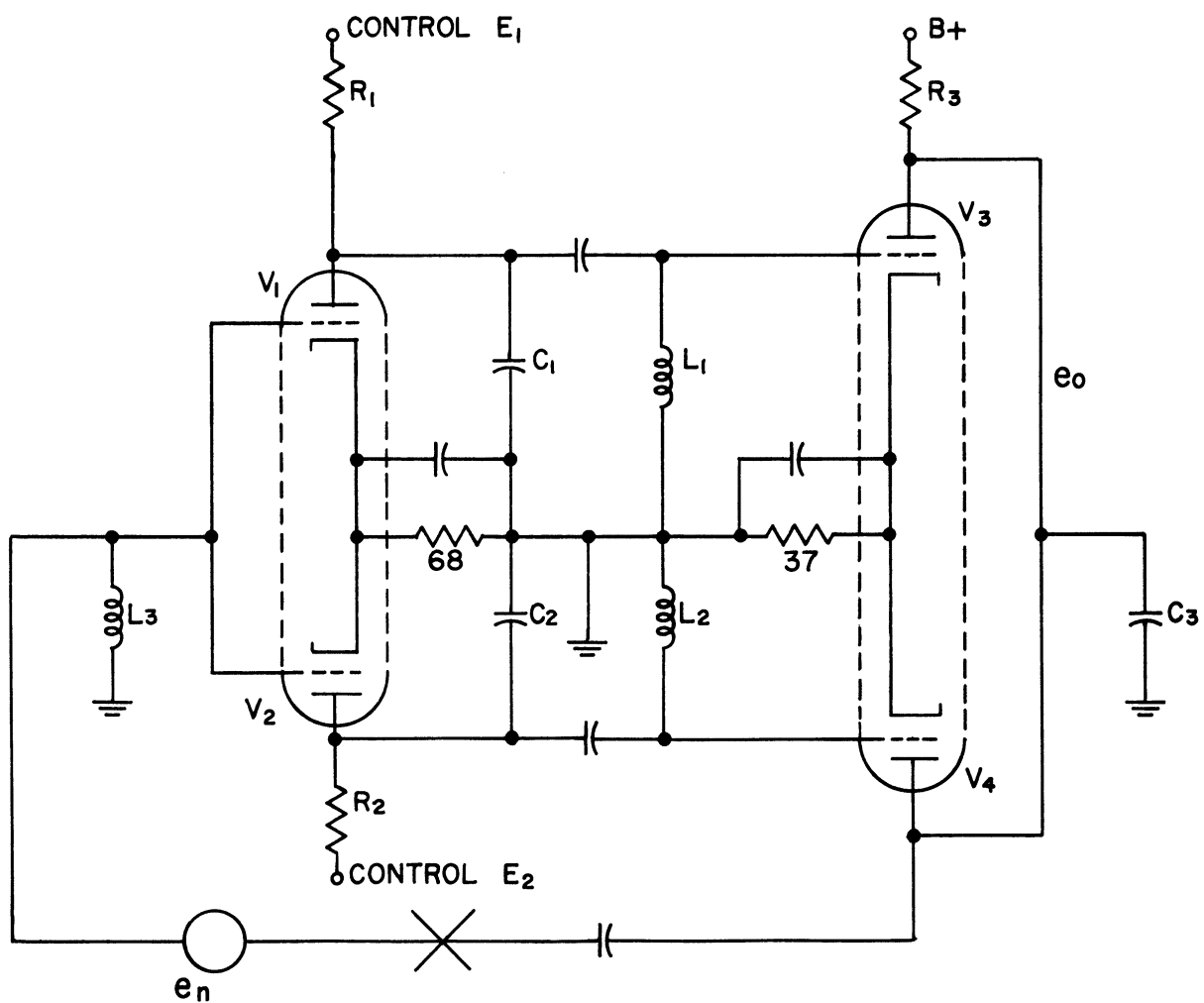
RESONANT PARALLEL NETWORK OSCILLATOR

FIG. 4

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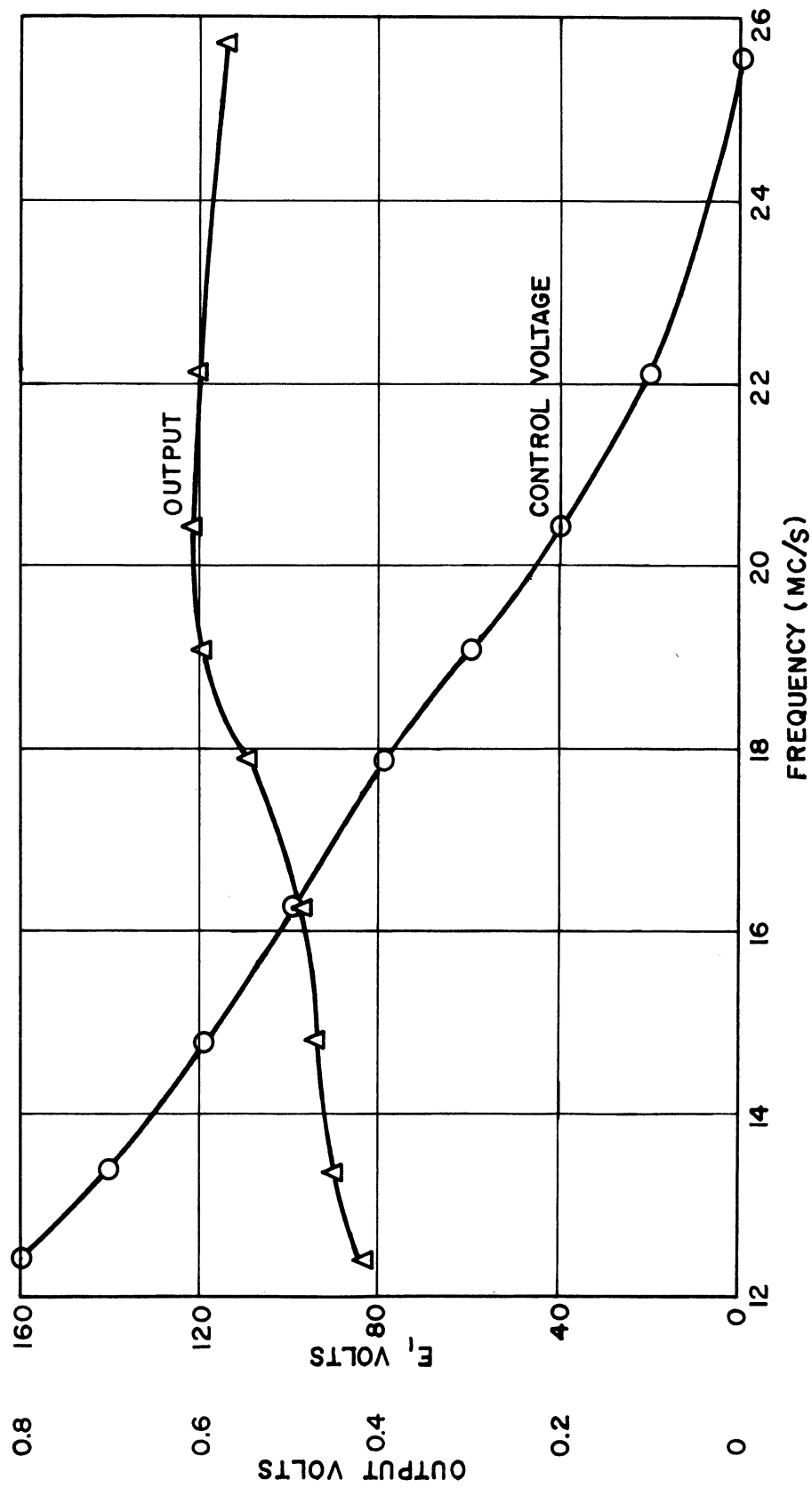
CHARACTERISTICS OF THE RESONANT PARALLEL NETWORK OSCILLATOR

FIG. 5

6. ANALYSIS OF A RESONANT TYPE OF OSCILLATOR.

Assume that all tubes of Fig. 4 are the same type and that the transconductances of V_1 and V_2 are equal to half the rated values at the center frequency. Also, plate load impedances will be assumed so small that plate resistances can be neglected. In addition, it will be assumed that $C_1 = C_2 = C$, $R_1 = R_2 = R$, $B_1 = B_2 = B$, $B_3 = B'$, $C_3 = C'$, $g_{m1} = (g_m/2)(1 + k)$, $g_{m2} = (g_m/2)$ $(1-k)$, and $g_{m3} = g_m$. Finally, let $\omega_3 = \omega_0$, be the center frequency. Then,

$$\frac{e_0}{e_n} = \frac{g_m^2}{2CC'}\left[\frac{(1+k)p}{p^2 + pB + \omega_1^2} + \frac{(1-k)p}{p^2 + pB + \omega_2^2}\right]\frac{p}{p^2 + pB' + \omega_0^2} \qquad (12)$$

In order to keep the analysis relatively simple, the narrow band approximation will be employed. Admittedly, this may not be very exact in many cases but it does have the advantage of simplicity. Therefore, if we assume that $\omega_2 - \omega_1 \ll \omega_0$, $B \ll \omega_0$, and $B' \ll \omega_0$, (12) becomes, upon translating the system centered around ω_0 to a center frequency of zero (band pass to low pass transformation),

$$\frac{e_0}{e_n} = \frac{g_m^2}{4CC'} \cdot \frac{p + B/2 - jk\omega_x}{(p + B/2 + j\omega_x)(p + B/2 - j\omega_x)(p + B'/2)} \qquad (13)$$

which has poles and zeros as indicated in Fig. 6. The zero moves along a vertical line a distance proportional to k. ω_x is the frequency difference $\omega_2 - \omega_0 = \omega_0 - \omega_1$. At the extremes of the tuning range, $k = \pm 1$ in which case the zero cancels a pole and the oscillator is a simple two stage single channel resonant device, one of the two parallel channels being cut off. The extreme frequencies are found from the requirement that the two remaining poles furnish a phase shift of zero. This gives the tuning range as

$$\Delta B = \frac{2\omega_x}{1 + B/B'} = \frac{2(\omega_2 - \omega_0)}{1 + B/B'} \qquad (14)$$
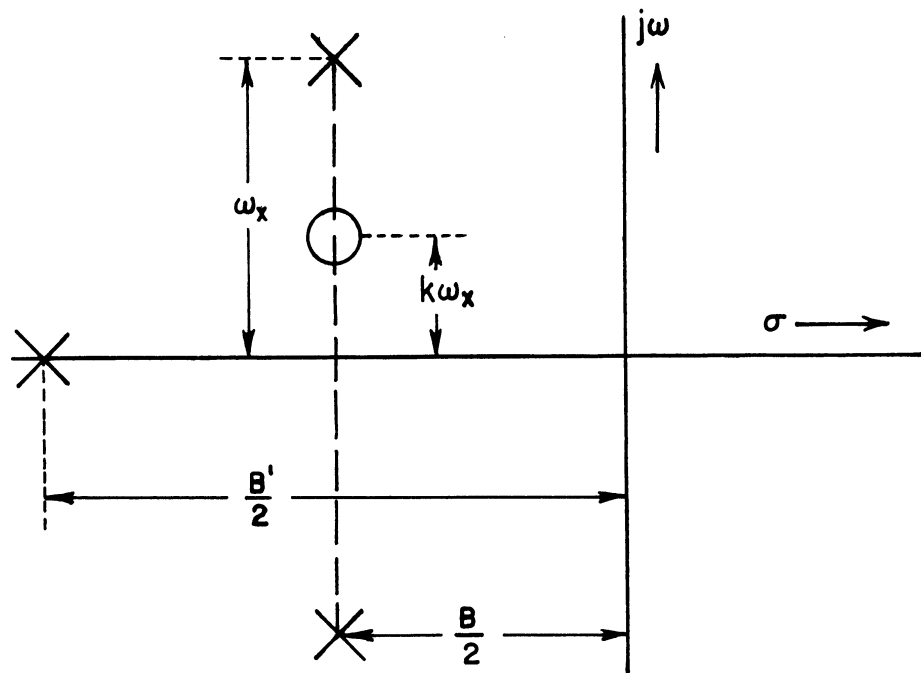
POLES AND ZERO OF THE RESONANT TYPE OSCILLATOR

FIG. 6

which is a maximum for $B' \gg B$ and is half the maximum possible value for $B' = B$. The open loop gain at the extreme frequencies is

$$\left|\frac{e_0}{e_n}\right|_{ex} = \frac{g_m^2}{4CC'}\left[\frac{4(B+B')^2}{BB'\left[4\omega_x^2 + (B+B')^2\right]}\right]$$ (15)

and that at the center frequency is

$$\left|\frac{e_0}{e_n}\right|_{ct} = \frac{g_m^2}{4CC'}\left[\frac{B/B'}{\omega_x^2 + B^2/4}\right]$$ (16)

The ratio of these two gains is

$$\frac{(e_0/e_n)_{ex}}{(e_0/e_n)_{ct}} = \frac{1 + (2\omega_x/B)^2}{1 + (2\omega_x)^2/(B+B')^2}$$ (17)

Using $p = j\omega$ in (13) and manipulating, we get the general transfer function as

$$\frac{e_0}{e_n} = \frac{g_m^2}{4CC'B'(\omega_x^2 + B^2/4)}\left[\frac{1}{1-\omega^2\left(\frac{1+2B/B'}{\omega_x^2 + B^2/4}\right)}\right].$$

$$\left[\frac{1 + j\frac{2(\omega + k\omega_x)}{B}}{1 + j\frac{\omega(\omega_x^2 + B^2/4 + BB'/2) - \omega^3}{(B'/2)(\omega_x^2 + B^2/4) - \omega^2(B + B'/2)}}\right]$$ (18)

The general condition of oscillation can be determined by setting the third bracketed quantity in (18) equal to unity. Then, at the oscillation frequency, the open loop gain is easily determined from (18) which shows how the open loop gain varies with frequency. Not much can be done to minimize this

variation except for the obvious procedure of making B and B' large. Clearly, ω_x is primarily determined by the desired tuning range.

The pole-zero plot of Fig. 6 is rather typical of parallel circuit oscillators. The poles are those of both of the two channels plus those common to the two channels. In addition to the fixed poles, zeros are introduced by the parallelling process. These zeros move in position as k is varied and it is their motion that causes changes in the frequency of oscillation.

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