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THE DESIGN OF A PILE OSCILLATOR FOR THE FORD NUCLEAR REACTOR

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A pile oscillator is a device for periodically varying the neutron population in a nuclear reactor. The population change may occur as a localized variation in neutron density, or the density of neutrons throughout the reactor may be caused to vary in a systematic manner. Such a variation may be effected by a periodic oscillation of the multiplication constant, k_{eff}, of the reactor. The required changes in k_{eff} have been achieved in reactor oscillators by mechanical oscillations of either absorbers or reflectors located in or near the reactor core.

The oscillator discussed in this paper produces the required change in k_{eff} by oscillation of an absorber placed near the reactor core. Since it is desirable that the variation in neutron population be sinusoidal in time, the absorber is designed accordingly.

In order to use a pile oscillator the amplitude of the sinusoidal changes in neutron population must be measured and the phase angle between neutron oscillations and oscillations of k_{eff} must be determined. Appropriate circuitry for making these measurements is also described in this paper.

Pile oscillators have been used for a variety of purposes. Quantities such as neutron absorption and scattering cross-sections, delayed neutron characteristics, neutron mean lifetime in the reactor, effective resonance integrals and the purity of uranium samples have been measured with pile oscillators. Pile oscillators have also been used to determine the dynamic stability of reactor systems and to measure power coefficients of reactivity.

The pile oscillator described in this paper has been designed for the University of Michigan's Ford Nuclear Reactor, a swimming pool type research reactor. The oscillator will be used both in connection with teaching laboratories and as a tool for studying the dynamic characteristics of the reactor system.

COMPUTATION OF A TRANSFER FUNCTION FOR THE FORD NUCLEAR REACTOR

A set of equations that can be used to describe the time behavior of the neutron population in a fission reactor operating near critical is given below:

$$\frac{dn}{dt} = \frac{\delta k - \beta}{\tau} n + \sum_i \lambda_i C_i$$

$$\frac{dC_i}{dt} = \frac{\beta_i}{\tau} n - \lambda_i C_i \qquad (1)$$

(See appendix for definition of symbols)

The second equation of the set describes the i_{th} delay group precursor population. Five such delay groups are considered in the analysis to be given. The resulting equations form a set of six linear differential equations.

A useful transfer function for purposes of analysis is one that relates changes in neutron population, δn, to corresponding changes in multiplication constant, δk. If δk is assumed to vary with time, then the first differential equation has a time-varying coefficient. In much the same way that one determines the small signal response of electronic amplifiers, we shall find the response of the reactor neutron population, δn, to small variations of δk. Let

$$\begin{aligned} n &= n_o + \delta n \\ C_i &= C_{i_o} + \delta C_i \end{aligned} \tag{2}$$

Here n_o is a steady state value of neutron population and δn is a small excursion of neutron population about n_o. C_{i_o} and δC_i are similar descriptions of precursor population.

Substituting these expressions for n and C_i into the set of differential equations (1), discarding second order terms, and using operational notation we have:

$$\frac{\delta n}{n_o\,\delta k} = \frac{1}{\tau s \quad 1 + \sum\limits_{i}^{5} \dfrac{\beta_i}{(s + \lambda_i)\tau}} \tag{3}$$

Equation (3) can be written in the following form:

$$\frac{\delta n}{n_o\,\delta k} = \frac{\prod\limits_i (s + \lambda_i)}{s\tau \prod\limits_i (s + \alpha_i)} \tag{4}$$

where α_i is the i_{th} root of the fifth order polynomial in the denominator that results from rationalizing the right-hand side of equation (3).

Using values of β_i and λ_i given in reference (1) and assuming that τ for the Ford Nuclear Reactor is 2×10^{-5} seconds, we have for the Ford Nuclear Reactor Transfer function:

$$\frac{\delta n}{n_o\,\delta k} = \frac{(s + 14.3)(s + 1.61)(s + .465)(s + .154)(s + .0315)}{2 \times 10^{-5}\, s\,(s + 364.5)(s + 14.21)(s + 1.425)(s + .336)(s + .0775)} \tag{5}$$

The amplitude and phase angle for (5) with $j\omega$ substituted for s are plotted in Fig. 1.

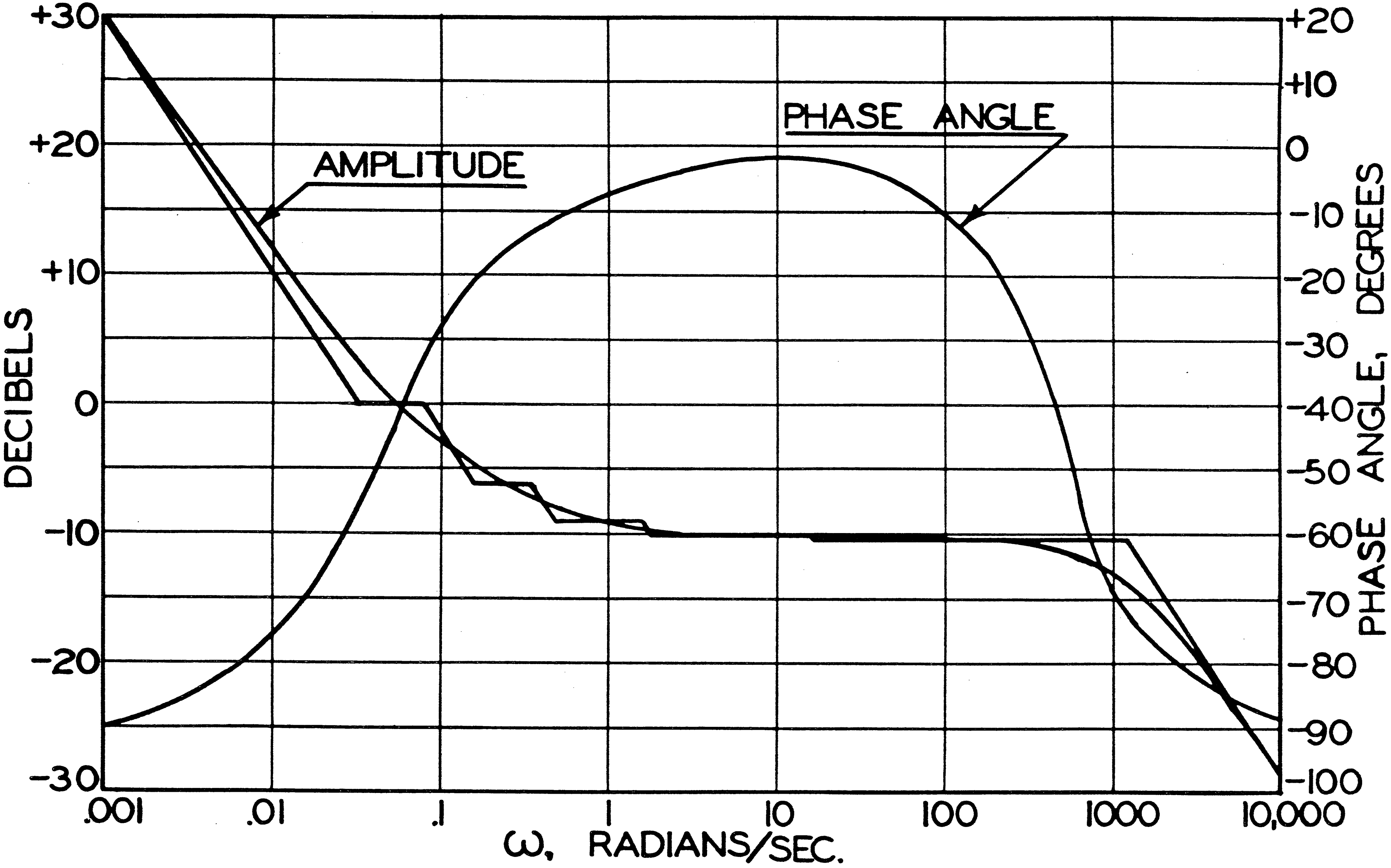

Fig. 1. Calculated Transfer Function Ford Nuclear Reactor

USE OF THE OSCILLATOR IN DETERMINING REACTOR STABILITY

As has been pointed out in the literature a reactor operating near rated power can be thought of as a feedback system.(2,3) Not only are there feedback mechanisms inherent in the reactor itself but an automatic control system connected to reactor control rods or to coolant pumps may form additional feedback loops.

As an example of an internal feedback mechanism, consider temperature effects. A small temperature change may vary the moderating and reflecting properties of the reactor core material as well as the geometrical shape. Such changes may introduce a change in k_{eff} which will in turn change n, the neutron population. Figure 2 is a block diagram of a reactor with internal feedback loops. The closed loop response of the system of Fig. 2 is

$$\frac{\delta n}{n_o\ \delta k} = \frac{R(s)}{1 + R(s)T(s)n_o} \tag{6}$$

where as indicated in Fig. 2 R(s) represents the reactor transfer function without feedback and T(s) is a feedback term.

For experimental investigations it is useful to invert equation (6). There results:

$$\frac{n_o\ \delta k(s)}{\delta n(s)} = \frac{1}{R(s)} + T(s)n_o \tag{7}$$

By methods described in reference (6) the quantities in equation (7) can be determined and the stability of the system investigated. The following procedure may be followed:

1) Operate the reactor at low power so that $T(s)n_o$ will be small compared to $1/R(s)$. Vary δk sinusoidally over a range of frequencies, from .001 to 100 cps. in the case of the Ford Nuclear Reactor, and measure δn with the circuitry described in the next section of this paper. If the power level is low enough $1/R(s)$ may be computed from these data. The experimental values may be checked with that given by equation (5).

2) Operate the reactor at a higher power level. After allowing the system to reach temperature equilibrium determine the reactor response over the same frequency spectrum as in step 1. As shown by equation (7) these data should allow computation of the quantity $1/R(s) + T(s)n_o$.

3) Instability will exist if for some frequency and some n_o the quantity $1/R(s) + T(s)\ n_o$ is equal to zero. This may be investigated by plotting the transfer function $(n_o\ \delta k)/\delta n$ as a complex number for the frequencies at which data are taken. At a given frequency the vector difference between the transfer function at very low power and at some higher power is the quantity $n_oT(s)$.

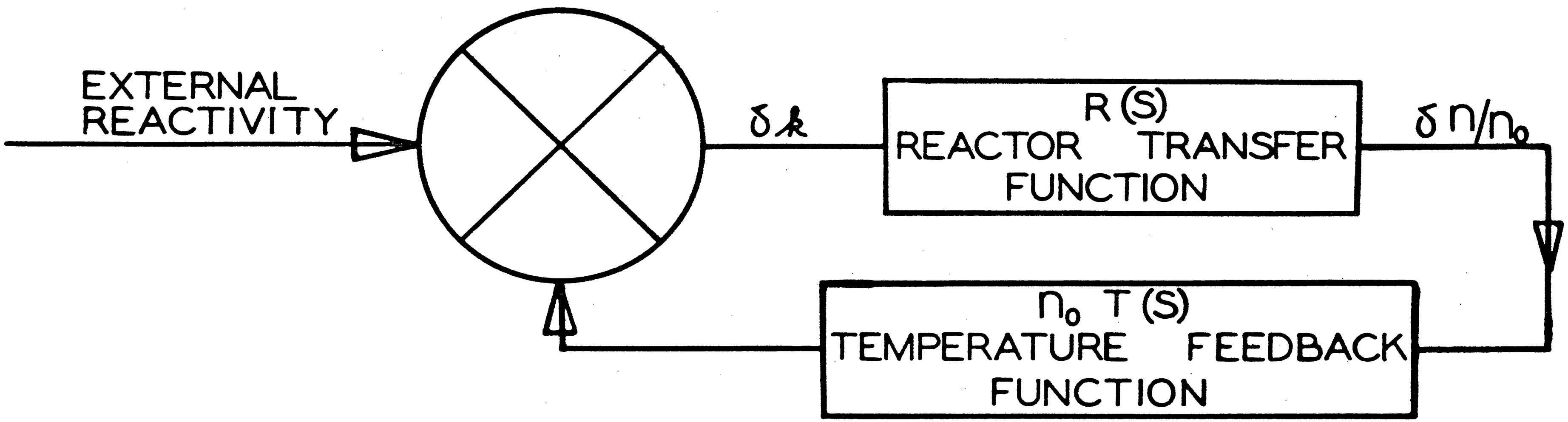

Fig. 2. Internal Reactor Feedback Loop

If this vector can be extended to go through the origin then there exists some power level at which the reactor system will be unstable. This follows from the fact that at a given frequency the phase angle of $n_oT(s)$ is independent of the reactor power level, n_o.

MECHANICAL AND ELECTRICAL DESIGN

The Ford Reactor uses BSF type fuel elements in a matrix surrounded by graphite reflecting elements. It was felt that it would be most convenient to insert the oscillating absorber into a position normally occupied by a reflector element.

The design of the oscillator involved computation of the δk introduced by the oscillation of an appropriately shaped absorber in the reactor. Calculations were made using two-group perturbation theory. These showed that about 0.5 gm of cadmium foil placed in the outer row of graphite reflectors would produce a change in k_{eff} of 10^{-5}.(4) Since derivation of the reactor transfer function assumes a small δn, the oscillator was designed with this restriction in mind. Other calculations showed that the δk produced by the moving absorber of the oscillator would not be much different from that produced by the same quantity of a stationary absorber.(4)

An eccentrically mounted circular disc covered with cadmium foil and rotated with constant angular velocity in and out of a fixed cadmium box was designed to vary k_{eff} sinusoidally. By choosing the eccentricity such that the distance between the center of the disc and drive shaft is 0.35 r, where r is the rotating disc radius, the exposed cadmium area, and thus k_{eff} will vary harmonically. Figure 3 shows the mechanical design of the absorber.

For detecting the changes in neutron population, a parallel-circular-plate boron-coated ionization chamber is used.

Both the random nature of the detection process used in the ionization chamber and the randomness of the multiplication process in the reactor introduce noise along with the desired signal. In order to separate our approximately sinusoidal signal from this noise we have made use of correlation techniques. The output signal of the ionization chamber, after amplification, can be represented as

$$f(t) = A \cos \left(\frac{2\pi t}{T} - \Theta\right) + \psi(t) \qquad (8)$$

where A is the signal amplitude
T the period of the true signal and of the oscillating δk
Θ the phase shift of the signal with respect to the oscillating δk
$\psi(t)$ is the noise signal.

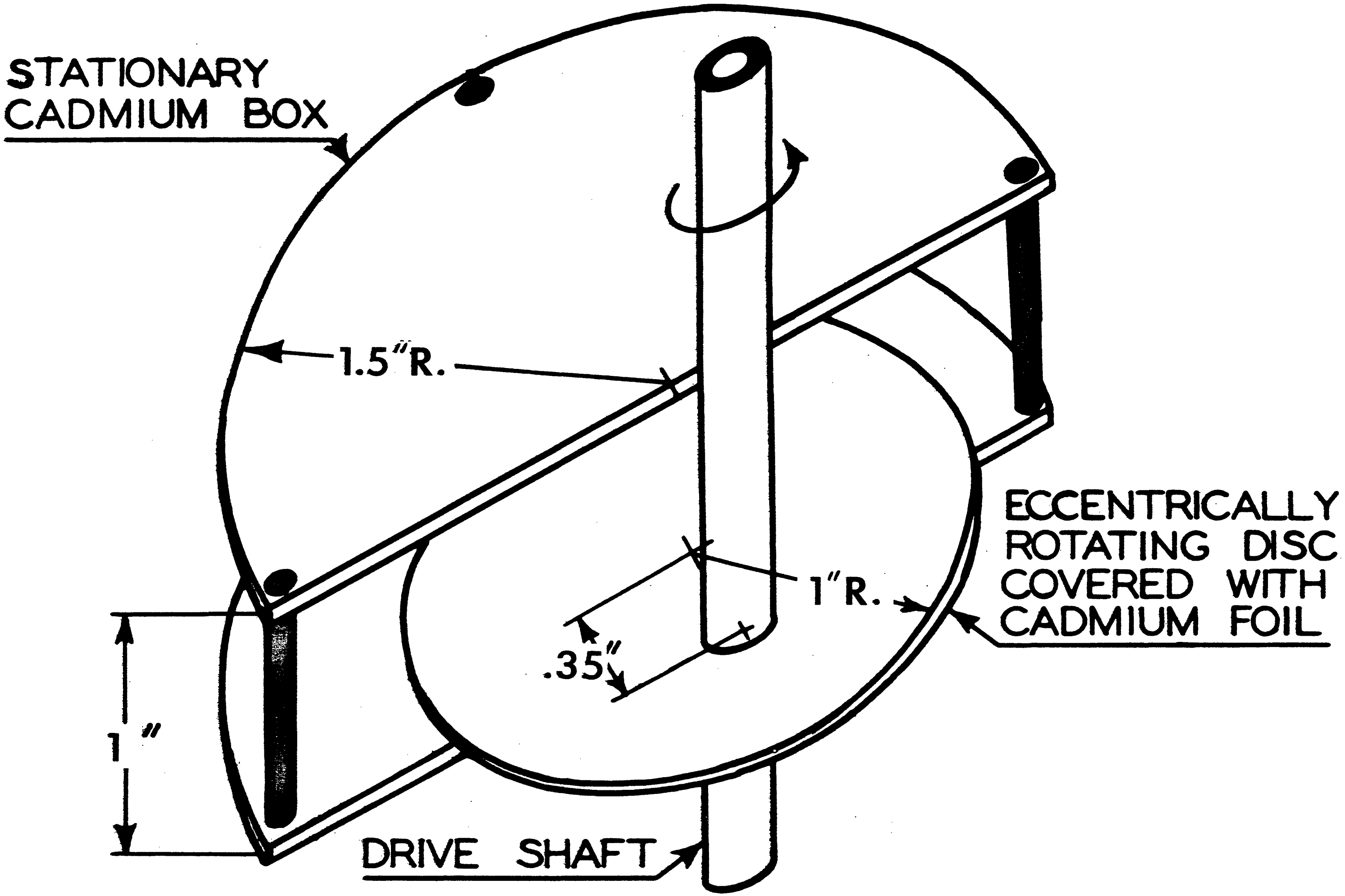

Fig. 3. Oscillating Mechanism

If this function is multiplied by sin $2\pi t/T$ or cos $2\pi t/T$ and integrated for n complete cycles there results:

$$I = \frac{2}{nT}\int_0^{nT} f(t)\cos\frac{2\pi t}{T}\,dt$$
$$J = \frac{2}{nT}\int_0^{nT} f(t)\sin\frac{2\pi t}{T}\,dt \qquad (9)$$

If f(t) is represented as

$$f(t) = A\left(\cos\frac{2\pi t}{T}\cos\Theta + \sin\frac{2\pi t}{T}\sin\Theta\right) + \psi(t) \qquad (10)$$

It can be shown that integrals I and J are now

$$I = A\cos\Theta + \frac{2}{nT}\int_0^{nT}\psi(t)\cos\frac{2\pi t}{T}\,dt$$
$$J = A\sin\Theta + \frac{2}{nT}\int_0^{nT}\psi(t)\sin\frac{2\pi t}{T}\,dt \qquad (11)$$

The integrals involving the noise terms in equation (11) are on the average zero since the noise is assumed to be nonperiodic. For each oscillation frequency and its corresponding period, T, we find a certain

amplitude, $A = \sqrt{I^2 + J^2}$ and a certain
angle, $\Theta = \tan^{-1}(J/I)$ (12)

The operations described by equations (9) are achieved by the use of sine and cosine potentiometers.

The integrator chosen is a type that has been used in pile oscillator instrumentation by Breton.(7) It consists of a dc split-field motor whose angular velocity is proportional to the applied voltage. The angle of rotation of the motor shaft over a certain time interval is thus proportional to the integral of the signal imposed on the field control winding. On the same shaft are connected a revolution counter and a tachometer. The output voltage of the tachometer is fed back to the motor as shown in Fig. 4. This type of integrator is used because of its excellent performance over the wide range of frequencies in which we are interested.(5) The entire detecting system is shown in block diagram in Fig. 5. In the experimental procedure, the signal f(t) is multiplied by the cosine function for 10 minutes and then by the sine function for a similar time. By noting the number of revolutions recorded on the revolution counter in the integrator, the phase and magnitude of the δn signal can be ascertained from equations (12). The frequency of the driving function δk can be changed both by changing gear ratios (not shown in Fig. 5) and driving motor speed. As can be seen from Fig. 5 the sine and cosine potentiometers as well as the mechanical oscillator all move at the same frequency.

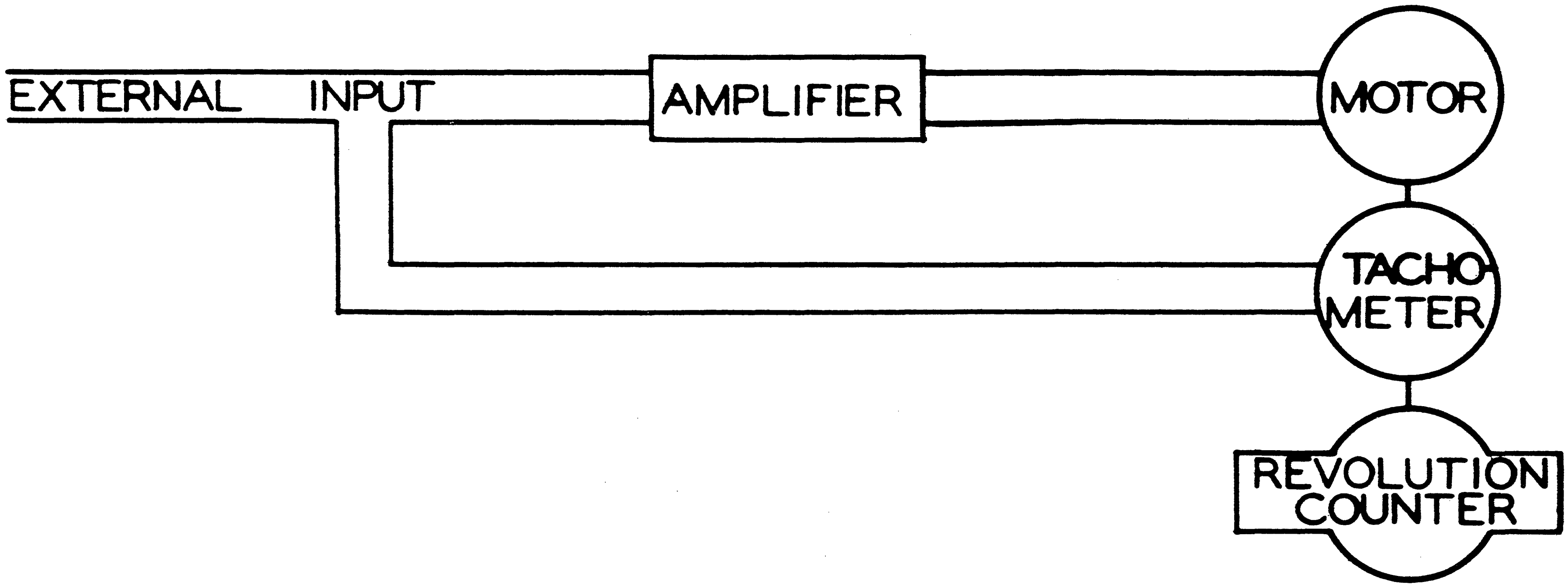

Fig. 4. Velodyne Integrator

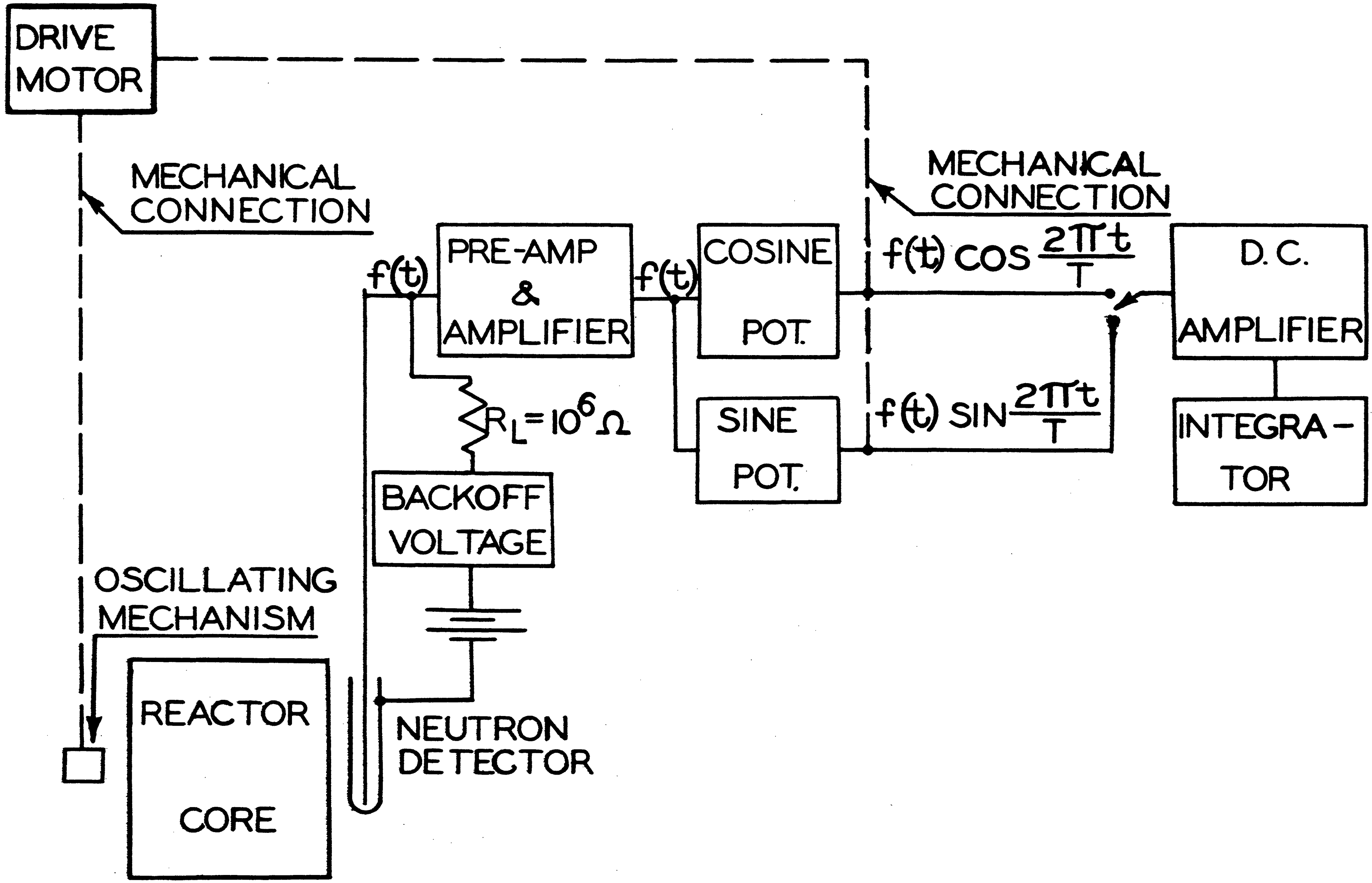

Fig. 5. Detecting Circuitry

APPENDIX

Key to Symbols Used

n = neutron population

k_{eff} = effective multiplication factor

δk = k_{eff-1}

β = fraction of neutrons produced that are delayed

τ = mean generation time of promt neutrons (2×10^{-5} sec for the Ford Nuclear Reactor)

λ_i = decay constant of the i_{th} group of delayed neutrons. Five delay groups are considered.

C_i = concentration of the i_{th} group of delayed neutrons

n_o = steady state value of neutron population

δn = perturbation of neutron population

C_{i_o} = steady state value of precursor population

δC_i = perturbation of precursor population

α_i = i_{th} root of the polynomial (See equation 4)

$R(s)$ = reactor transfer function without feedback

$T(s)$ = temperature feedback function

$f(t)$ = signal plus noise

$\psi(t)$ = noise signal

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