Running Head: Instructional Identities

Instructional Identities of Geometry Students

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Abstract

In this paper we inspect the hypothesis that geometry students may be oriented toward how the teacher will evaluate them as students or otherwise oriented to how their work will give them opportunities to do mathematics. The results reported here are based on a mixed-methods analysis of twenty-two interviews with high school geometry students. In these interviews students respond to three different tasks that presented students with an opportunity to do a proof. Students' responses are coded according to a scheme based on the hypothesis above. Interviews are also coded using a quantitative linguistic ratio that gauges how prominent the teacher was in the students' opinions about the viability of these proof tasks. These scores were used in a cluster analysis that yielded three student profiles that we characterize using composite profiles. These profiles highlight the different ways that students can experience proof in the geometry classroom.

Keywords: High school, geometry, student, identity

1. Introduction

This study offers a contribution to the study of students' identity in mathematics classes by zooming into the experience students have in a specific instructional situation—the situation of doing proofs in American high school geometry classrooms. The American high school geometry course, usually taken in 9th or 10th grade (when students are 14-15 years old) presents an important curricular context to inspect a main hypothesis of the study—that identity depends on context—since this course not only affords students opportunities to do proofs but also takes responsibility for their learning 'to do proofs.' We use that circumstance to inspect a key question for the study of students' mathematical identities—what kinds of students can one find in an instructional situation?

When American students come to take the high school geometry class, in 9th or 10th grade, they have been in school for many years. They have had many different teachers and been taught many different subjects. When students walk into the geometry classroom on the first day of class they have expectations about school in general and mathematics classes in particular. As the year proceeds they build up experiences with the content specific to the new class, the teacher, and their classmates. They come to have more specific expectations. They learn how to frame their experiences. They become part of the geometry class.

We extend the work on 'doing school' (Chazan, 2000; Eckert, 1989; Fried, 2005; Herbst & Brach, 2006; Jackson, 1968; Lave, 1997; 2001; Pope, 2001; Powell, Farrar, & Cohen, 1985) by showing the different ways that students "do school" in one particular context within the American high school geometry class ('doing proofs'). We propose and characterize a set of instructional identities that give a way of understanding what is meant by "doing school" in the particular context of doing proofs in high school geometry classrooms. Through these identities we understand what actions students see as available to them in the instructional situation of 'doing proofs' and what meanings they make of the environment of the high school geometry classroom in general and the tasks that are put before them in particular. The general notion of instructional situation, and 'doing proofs' as a particular example of it, are explicated below in the section "Interpretations of instruction in mathematics classrooms."

We are interested in the frames (in the sense of Goffman, 1974) that are available for high school geometry students to organize their experience. From our theory of instruction in classrooms we hypothesize that there are at least two ways that student could allocate value to their mathematical work. One frame for students' valuation of their work emphasizes how their work trades for claims on having fulfilled the didactical contract; the other emphasizes the value of their work as doing interesting mathematics[2]. In this paper we examine empirical data to ground this hypothesis.

The research presented here is an example of how studies of student identity can take context into account. We are simultaneously studying students and the environment that they are acting within. The three students profiles presented in the results section inform what we know both about students and about the doing of proofs in geometry, and the study itself informs a methodology of studying identity in context by looking at instructional situations.

This paper begins with an overview of the theory of identity and the theory of mathematics instruction on which we base the development of instructional identities. We then describe the data and methods used in the current study before turning to an account of three instructional identities that emerged from the data and how these direct student action in response to tasks. When we conceived of this study we anticipated only two profiles but the data analysis provided grounds to suspect there are three distinct student profiles with respect to work on 'doing proofs.' This third profile presents an opportunity to further develop our theory to account for all the data.

[2] This claim can be seen as closely related to claims in educational psychology about students' goal orientation (for an overview see Ames, 1992). In this paper we explore these frames in detail, building on a theory that the classroom stages a symbolic economy.

2. Theorizing Identity

We take instructional identities to result from the interaction between individuals' dispositions to act and the instructional context in which these dispositions are experienced. Two assumptions about the nature of identity are fundamental in developing this conception:

- Identities are dynamic and vary with context
- Identities are experienced in practice (Holland et al, 1998)

Taken together, these two assumptions mean that identities are seen in actions and reactions, they are not static characteristics or traits. And those actions and reactions are dependent on context. Below we expand on these two aspects of identity; in particular we conceptualize this context as it relates to instruction.

2.1 Identities are Dynamic and Vary with Context

Identities can be understood to be responsive to social context. Individuals take on different roles in different contexts. The geometry class offers a particular set of contexts for these different roles: the various instructional situations of a geometry class can provide context for different instructional identities to manifest.

The roles of the participants give meaning to the actions they undertake. For example, in a geometry class, the following exchange would have very different meanings depending on the activity that the participants are engaging in:

A: What do we know about these two lines?

B: They are parallel.

A: Really?

B: Yeah, see, if you measure the distance between them, it stays the same.

A: So we can say they're parallel?

If A and B are students exploring a geometric configuration, then the conversation could be one in which B is giving A grounds to believe in a conjecture. However, if the students are working on a proof then the conversation could be one in which A is skeptical of the validity of

B's empirical justification for the claim that the lines are parallel. The activity that the students are engaged in gives meanings to the utterances. In the case where A and B are exploring a geometric configuration, A's final question is aimed at understanding what it takes for two lines to be parallel. In the case where A and B are doing a proof A's final question is aimed at prompting B to think of what counts as justification in a proof.

A helpful construct for understanding dynamic identities is *figured worlds*. Holland et al (1998, p. 52) define figured worlds as: "A socially and culturally constructed realm of interpretation in which particular characters and actors are recognized, significance is assigned to certain acts, and particular outcomes are valued over others." People acting within figured worlds act 'as if' such-and-such a thing was true. For instance, in some classrooms, students and teachers act "as if" completing two-column proofs corresponded to the ability to prove geometric claims (see Herbst, 2002). Instructional situations, such as 'doing proofs,' 'exploring a figure,' or 'constructing a figure' can be thought of as distinct activities that make up the figured world of the geometry classroom. Students of this geometry class become familiar with each of these situations.

Within particular instructional situations, students may differentiate by taking on different identities with different stances toward the work required by or toward the stakes of the situation. For example, while doing a proof, students and teachers act as if it matters if one gathers information about a figure by measuring. Some students will not measure because they know it is against the rules, while other students will not measure because they know it will not get them further in the proof task, while still other students will measure, because they see measurement as a way to gain more information about the mathematical task at hand, though some of them realize they can only use measurement heuristically and will need something else to justify their claim. Because different students understand the figured world in different ways these students feel that different actions are appropriate when faced with a mathematical task.

The framework of figured worlds highlights the interplay between individual and social influences on behavior. Within a figured world, the social environment shapes the individual's views of what is possible and desirable; simultaneously the individuals that live inside it shape the figured world. The collective supports, constrains, and gives meaning to what goes on in the classroom. In the case of the current study, we showcase the relationship between the socio-technical[3] context of the classroom and the identity of geometry students.

2.2 Identities are Experienced in Practice

Identities are characterized as being experienced in practice in the ways that individuals participate in communities, and how they understand that participation (Holland, Lachicotte, Skinner, & Cain, 1998; Wenger, 1998). As individuals move through their lives they move through different communities and their actions within those communities simultaneously shape and characterize their identities. That is, identities are experienced as reactions to, and participation in, the environment. Conceptualizing identity in this way means that identities are not internal aspects of individuals, but rather identities are built up of an individual's actions within an environment. These actions are in line with the dispositions to act that are available to an individual in a situation.

Bourdieu (1990) shows how dispositions to act are important guiding forces for individual action; yet these dispositions are not individual traits as much as they are resources for individual action inscribed in individuals' perceptions of situations. Bourdieu defines habitus as "systems of durable, transposable dispositions, structured structures predisposed to function as structuring structures" (p. 53). Habitus provides a structure for understanding the environment based on past experience. While participants experience those dispositions as if they were created in the moment, habitus develops across a lifetime with the accumulation of experience. These dispositions are structured by past experience and actively structure new experiences. It is these dispositions that are experienced as the motivation to take up one action over another.

When students encounter a problem statement in a high school geometry class that could be solved by measuring, but does not include enough information to be proven without

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[3] We use this word to underscore that the context we refer to is not merely social but also includes objects of knowledge such as mathematical tools and inscriptions.

measurement, some of them may be inclined to stop work on the problem. We conjecture that this is not because they are disinclined to do work or because are incapable of deducing from assumptions or otherwise measure. Instead students have become disposed to abide by the norm that neither measurement nor assumption are appropriate actions when 'doing proofs' and because they wish to do well in class they are disposed not to act in inappropriate ways. We conjecture that this disposition is embedded in the situation in the sense that it is what students think they ought to do when they are 'doing proofs.' The disposition is internal to the student in the sense that it is their construction of the situation that leads to their judgment.

In this study we are interested in the dispositions to act, which geometry students perceive as relevant within the context of proving in the high school geometry classroom. We are interested in seeing which dispositions cluster together in students' profiles and how a theory of instruction can help explain these clusters of dispositions. By looking carefully at these clusters of dispositions and their connection to the didactical contract we are able to give a coherent account of the actions that students take in response to different tasks.

Identity is conceptualized in many different ways for many different reasons. By viewing identities as being experienced in practice and dynamic according to context we are able to honor the students' experience of making choices in the moment while engaged in classroom activity as well as the historical aspect of students' identity that is shaped over time in a variety of contexts. In the current study we show how differences in students dispositions manifest themselves as differences in actions in response to possible proof tasks.

3. Interpretations of Instruction in Mathematics Classrooms

In the following section we briefly describe a theory of instruction and apply it to the geometry classroom. We do not claim that it is the only explanation for the data we found. Rather the theory was at the source of the design of the interview protocols and we expect that the interaction between this theory and the data collected helps not only to interpret the data but also to ground and improve the theory.

3.1 The Symbolic Economy of the Classroom

The theory of instruction in classrooms that is we use as a basis for our analysis uses the notions of mathematical task, instructional situation, didactical contract, and economy of symbolic goods. The central notion of this theory is that of didactical contract: teacher, students, and mathematical content are bound by an unspoken contract whereby the teacher has to teach mathematics to the student and the student has to learn mathematics from the teacher within the temporal and social confines of a school classroom. The hypothesis that such contract exists justifies seeing a classroom as an economy of symbolic goods where transactions are made that concern knowledge claims. In this economy, the work that teachers and students do together moment by moment in mathematical tasks has to be exchanged into valuables, in particular, into the right to say that they have taught or learnt one or another object of study. The notion of instructional situation has been proposed to name each of the customary frames that are used in a given course (e.g. in high school geometry) to exchange some particular kind of work for a claim on particular kinds of knowledge at stake (Herbst, 2006). Thus, situations of 'exploration of a figure' allow the exchange of measurement or manipulation work for the claim that students' have discovered a key property; situations of 'doing proofs' enable the exchange of students' writing of statements and reasons for the claim that they know 'how to do proofs.' The norms of situations further specify the norms of the didactical contract—for example students are supposed not to measure when they 'do proofs' but may measure when they 'explore a figure.'

We propose that students' orientation toward the symbolic economy of an instructional situation can be one of recognition or misrecognition of the trade described above. This paper adds to the theory of classroom exchanges a grounded understanding of how students are disposed toward this economy of symbolic goods. Following the claim that instructional identities vary with context, the classroom environment and the interactive structures within which proof and reasoning occur place constraints on the possibilities of action and interpretation that are available to teachers and students. Negotiations of what will trade for what are often implicit, but the result of these negotiations can be understood to exert strong pressure on what students will do and how they expect their work to be valued by the teacher.

Bourdieu (1990, 1998) explains economies of symbolic goods through the example of gift exchange. Ordinarily the giving of a gift may be taken as a spontaneous act of good will from the gift giver. Sociologists such as Marcel Mauss (1922) have argued that the interaction is not complete with the acceptance of the gift, but a new cycle of giving has begun: The gift receiver is now obligated to take the role of the gift giver. To this Bourdieu adds that it is important for the reciprocal gift to appear not as a response to the initial gift, but as another spontaneous act of good will. The second gift would lose its value if it were seen as fulfilling an obligation.

This camouflaging of obligation is what Bourdieu refers to as "misrecognition." For an economy of symbolic goods to function, both parties must at one time recognize the exchange and assume the obligation, and at another "misrecognize" the exchange and acting as if each gift is an act of good will.

The didactical contract creates responsibilities for teachers and students vis-à-vis content. Teacher and students are obliged to trade with each other. The economy in which they trade involves classroom work on mathematical tasks and claims on the objects of teaching and learning (see Figure 1). We claim that teachers and students act in a way that is similar to the gift exchange example above. Students, in particular, may misrecognize[4] the exchange by acting as if they are doing mathematical work because of the intrinsic interest of those tasks instead of for the exchange value they have (see Figure 2). The students might come to class expecting to work on interesting tasks. On the other hand, teachers and students might recognize the

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[4] The terms "misrecognition" and "recognition" that we borrow from Bourdieu carry a connotation of illusion and truth that we do not mean to invoke. We simply mean that there are two frames for valuing classroom work within the symbolic economy. 'Mis'recognition is reading of the exchange between mathematical work and its value that does not take into account the fact that teachers and students are doing work together because of an obligation to the didactical contract. Instead of seeing work as being done because it will be evaluated by the teacher (as the recognizers do), misrecognizers see work as being done so that they can learn mathematics.

exchange by acting as if they were doing classroom work because of their obligation to their symbolic economy. The students do work with an eye towards how the teacher will evaluate that.

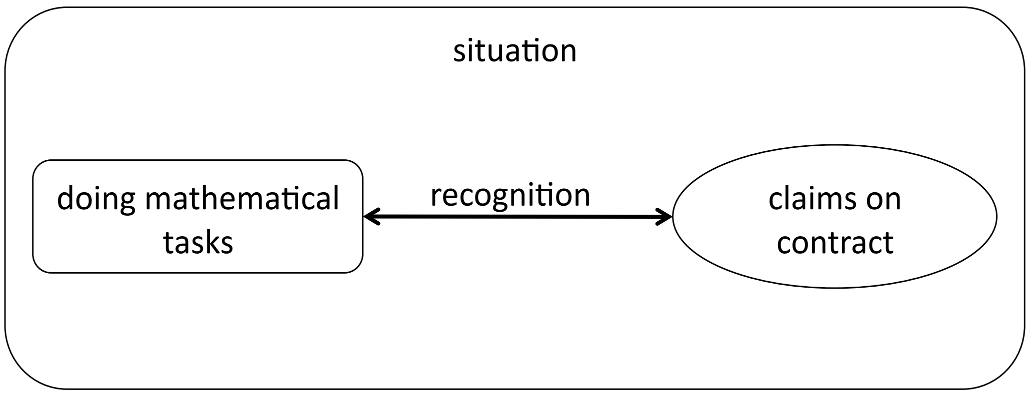

Figure 1: The trade of classroom work for claims on the didactical contract

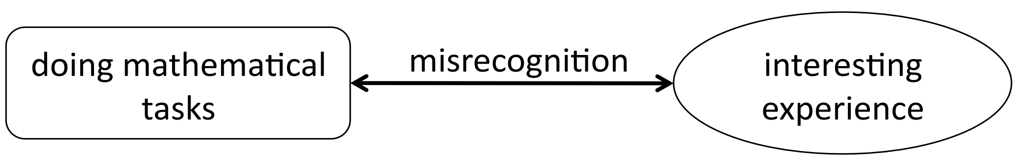

Figure 2: Intrinsic value of the task obscuring recognition of the trade

We propose that both teachers and students are continuously balancing the tension of misrecognition and recognition. Just as there is a need to misrecognize the obligations, there is a need to recognize the constraints that these obligations entail. This can also be seen in the gift exchange analogy. The actors in a gift exchange need to misrecognize the obligations involved so that they can appreciate the gesture of the gift. They need to also recognize that the gift giver has fulfilled his obligation and created a new obligation for the gift receiver. In the case of classroom work, it is instrumental for students to misrecognize the exchange and act as if they are doing their classroom work for the valuable mathematical experience that it supports. Students do this by not saying things like, "Just tell me what you want me to write" and the teacher does this by not overly prescribing the work that she expects students to do in response to tasks (for a breach of this consider the well known effect Topaze proposed by Brousseau, 1997, p. 25). However students also need to recognize their obligations to the exchange and act as if they are doing their classroom work so that they can lay claim on having fulfilled the didactical contract. Students do this by presenting their work in a way that is understood by the teacher (instead of, say, only working through problems in their head) and the teacher does this by being clear in her expectations (so that, say, students know that they are expected to present written proof of their work).

We hypothesize that, although all students must both simultaneously recognize and misrecognize the value of their work, some students are more attuned to the intrinsic value of their work in terms of doing geometry tasks, while other students are more attuned to the value of their work in terms of what those actions are worth in the teacher's evaluation. The data analyzed in this study lends credence to the notion that some students value their work in terms of the intrinsic mathematical work they do while others do so in terms of the claims that they can make on the didactical contract. The data analysis produced also a third group of students who made apparent the possibility that a student might not value their work in either of these ways.

3.2 Contract, Situation, Task

The theory of classroom exchanges is not complete without understanding more about the objects of the trade (classroom work and claims on the didactical contract) and the 'marketplaces' in which this trade occurs (the instructional situations). Figure 3 shows the model of the situation as a marketplace, as well as the exchanges that students use to value their work outside of the instructional situation. The two solid arrows show exchanges that students allocate value to: doing mathematical work can value insofar as laying claim on having fulfilled the didactical contract and insofar as having engaged with interesting tasks (recognition and misrecognition of the contract, respectively). The dotted arrow represents an exchange without value; mathematical tasks simply must be finished, but do not provide the student with anything of value (non-recognition of the contract). In conducting this study we expected to find evidence for the exchanges represented by the solid arrows, but the non-valued exchange was not anticipated.

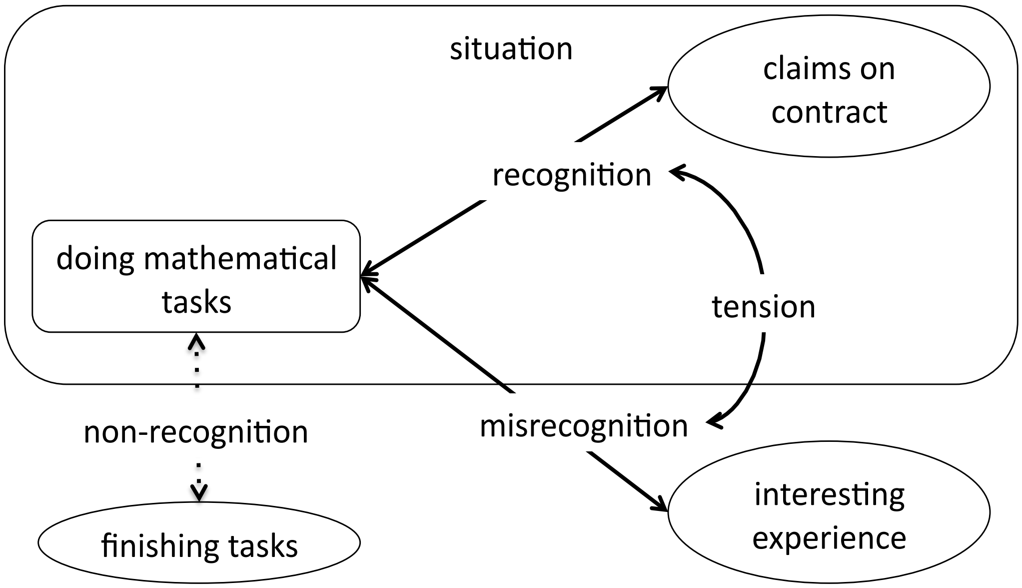

Figure 3: Symbolic Economy of the Classroom

Tasks, situations, and contract, as developed by Herbst (Herbst, 2003, 2006; Herbst & Brach, 2006) provide a frame for a three-tiered analysis of classroom interactions. A task is a segment of classroom work where students work on a problem or question: it can be modeled by identifying what the problem seeks (the product), the resources, material, technological, and symbolic that students may use and the operations (cognitive and behavioral) that students do to achieve that product (Doyle, 1988). The doing and completion of a task may have value in regards to the entitlement to claim that part of the contract has or has not been accomplished. To facilitate this exchange, tasks are framed by instructional situations, such as 'exploring a figure', 'doing a proof', 'calculating a measure', 'teaching a theorem', etc. These situations frame the exchange between the work that the teacher and students are engaged in and the valuables that can be claimed through that work. The situation provides an answer to the question, "what are we doing?" and provides a frame for participants to understand what the teacher and students are supposed to do and what they may lay claim on by doing it.

The existence of a didactical contract makes it possible for teachers and students to interact around tasks and to exchange that work within situations. Such contract obligates teachers to design tasks and support students' work on those tasks; it also obligates the teacher to account for how the work done in those tasks fulfills the contract. The contract also obligates students to engage with the tasks designed by the teacher on account of the promise that such tasks will help them learn mathematics.

Because we assume that students' identities are shaped by the context that they are working within, it is important to have a well-described model of instructional interactions. This model allows us to pay attention to the value that students ascribe to their mathematical work in the sense that we can hypothesize the values that we expect students to attribute to their work.

3.3 Example of a Situation: 'Doing Proofs'

In the geometry classroom, the instructional situation of 'doing proofs' is an example of how the didactical contract has developed a marketplace for students and teachers to engage in the activity of proving geometric claims and giving this activity a value. Here we describe the norms of this particular instructional situation. This is not an explicit agreement between an individual teacher and her students, but a tacit historically developed agreement that describes interaction in many geometry classrooms (Herbst, 2002). The situation of doing proofs is the marketplace in which work on proof tasks is exchanged for claims on having learnt or taught 'proof.' Herbst and Brach (2006) lay out the accountability (or division of labor) structure for the situation of doing proofs. This accountability structure is an example of how teachers and students divide the activities of teaching and learning 'proof.' The teacher is accountable for posing problems that call for a proof as part of the response, with clear statements of what shall be taken as given and what is the statement that is to be proved, as well as providing an accompanying diagram with all of the relevant geometric objects available for inspection. The student is responsible for marking known statements on the diagram through various markings and for laying out a sequence of "statements" and "reasons" in the form of a two-column proof (see also Herbst, Chen, Weiss, and González, 2009). These norms of the situation shape possibilities for student action. They support some actions and suppress others. For example, the format of two-column proofs supports students in supplying a reason for each statement in a proof (Weiss, Herbst & Chen, 2009).

In this section we have laid out a theory of instruction in mathematics classrooms. Since we view students' identities as dynamic with respect to context it is important to have an explicit model of that context. The symbolic economy of an instructional situation is the context in which students and teachers recognize and misrecognize their obligation to the didactical contract. Herbst and Brach's model of the situation of 'doing proofs' gives us the context in which students encounter proof tasks, make choices for action, and value their experiences. The student profiles detailed below show how different students hold different implicit conceptions of the contract and economy that lead to students enacting different instructional identities.

We now briefly return to the central research question; 'Who is the geometry student?' Taking identity to be expressed in action, this question can be transformed into 'How do students differ in regard to the actions that they see as possible in the geometry classroom, particularly in the situation of 'doing proofs'?' Figure 4 shows the connections between students' dispositions, the instructional situation, students' interpretation of the task, and the resulting action. Norms of the instructional situations (shown in the upper right of Figure 4) interact with student's dispositions that are activated by the context of the situation (shown in the upper left of Figure 4) and influence how students interpret the mathematical task (shown in the middle of Figure 4). Their interpretation of the task then, in turn, determines which actions they deem an appropriate response to the task (shown in the bottom of Figure 4).

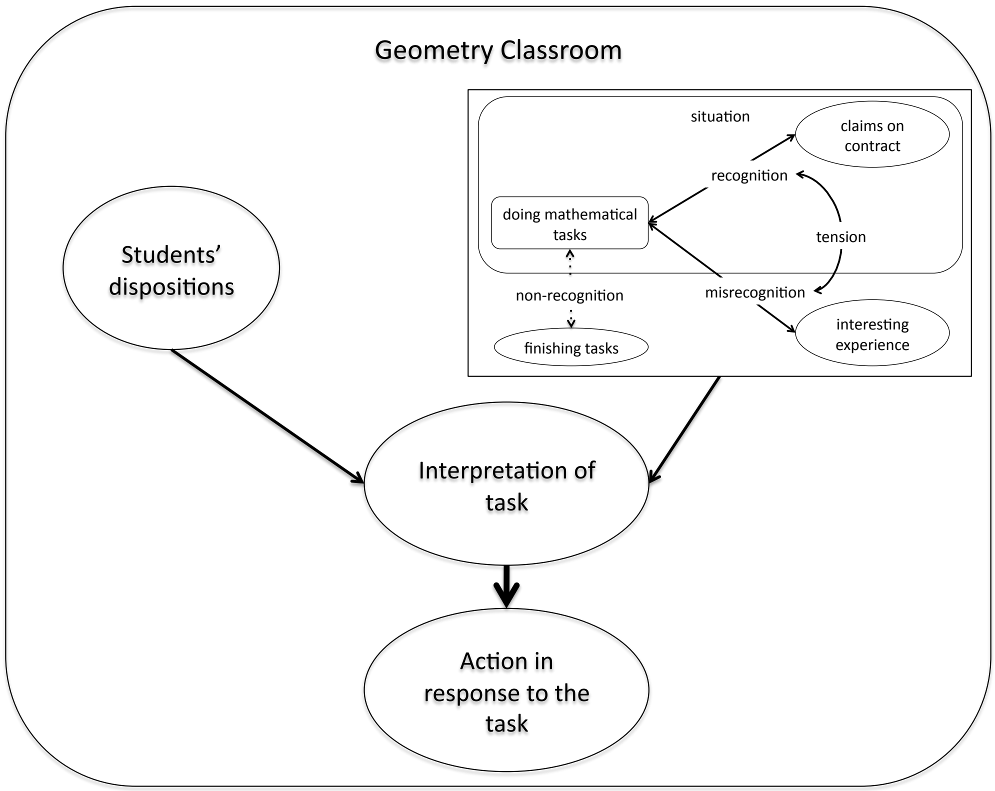

Figure 4: Interaction between students' dispositions and the classroom environment that influence students' actions in response to the task

In the following sections we discuss how we have gone about studying this research question through examining interviews with high school geometry students.

4. Data

The current analysis is a secondary analysis of data from interviews with students in three classes taught by three experienced teachers. The interviews were designed to gather students' views of doing proof in geometry classrooms—specifically about the tasks in which students could expect to be held accountable for producing a proof. These interviews are part of a larger data corpus that is comprised of over a hundred hours of video from geometry classrooms (both intact and experimental lessons), student interviews, student work, and interviews with several teachers. In this study we account for a subset of twenty-two individual interviews with students from three of those teachers—Cecilia, Lucille, and Megan.

4.1 Participants

The data set includes six interviews with students from Cecilia’s class, eight interviews with students from Megan’s class and eight interviews with students from Lucille’s class. These classes were taught in the same large and diverse comprehensive high school in a midsize city of the American Midwest. Cecilia and Megan taught an honors level geometry class and Lucille taught a regular geometry class; all three of these classes were proof-based courses and used the textbook by Boyd, Burrill, Cummins, Kanold, and Malloy, 1998). Each of these classes had between 25 and 35 students; only a subset of each class was interviewed. The subset of students was carefully chosen to give a cross-section of the students in the class in terms of attitude towards proving and course grade.

4.2 Interview Protocol

The interview protocol asked students some general questions (e.g., like why does the student think that proofs are done in geometry class and what they think the difference is between a good student and a good thinker) as well as required students to comment on three tasks, each of which could be construed as an opportunity to engage in proving but not all of which included that expectation in the statement. Each task targeted the same mathematical terrain, the ‘Medial Line Theorem’ (or midpoint connector theorem) which can be seen in Figure 6, but provided students with different resources for doing a proof. As can be seen in the statement of the tasks below, some of the tasks provided diagrams, some provided explicit statement of the claim to be proved and of the assumptions that could be used in the problem, some provided the larger conceptual ideas at play in the problem (“midpoint”, “triangle”, “parallel”, etc.), some provided a story context. A person who is constructing a proof could hypothetically use any and all of these resources, but not all of these resources are relevant according to the norms and dispositions of proving in high school geometry classrooms.

The first task was a word problem titled the "antwalk problem" (see Figure 5), the second task was a concise statement of a theorem (see Figure 6), and the third was a proof exercise similar in form to those in the students' textbook, including "given" and "prove" statements. The last task was not given with a figure, but if students remarked that a figure was missing they would be asked which of four figures was most likely to accompany the task (see Figure 7).

While each of those tasks was amenable to doing a proof, an important hypothesized difference among the tasks is that only one of these tasks would normally be encountered within the instructional situation of 'doing proofs', while the others might be encountered in different instructional situations in the high school geometry classroom. The antwalk problem would be most likely found in a situation of 'exploration,' where students would be expected to find the answer to a question by exploring a geometric object with any means at their disposal. The theorem task was one that is commonly found in geometry textbooks as a statement with an accompanying proof given by the text. Students might encounter statements like this when they participate in the instructional situation of 'installing a theorem' but would unlikely be held accountable for proving it. The final task was very similar to proof exercises found in the textbook and assigned by teachers as homework. These exercises always provide explicit 'given' and 'prove' statements and call on students to provide a proof, which is usually expected in two-column form. We hypothesized they would normally encounter that task in the situation of 'doing proofs.'

Despite their differences, these three problems all present students with an opportunity to do mathematical work that could include proving a claim. Differences among students become apparent when some students take up but others reject this opportunity. Differences among students are also manifest in the justifications that students give for their decision to prove or not to prove.

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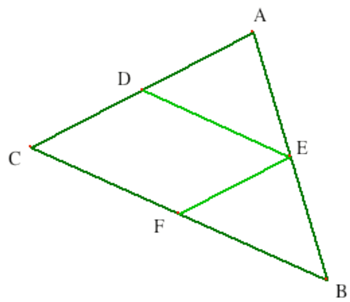
<td>
</td>
<td>Imagine two ants walking around this triangle.

Ant Jill goes AE, EF, FC, CD, DE, EB.
Ant Jack goes BC, CA, AB.
When they reach B, each of them argues to have walked more than the other one. Who is right and why?</td>
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Figure 5: Antwalk Problem

THEOREM
A LINE THROUGH THE MIDPOINT OF TWO SIDES OF A TRIANGLE IS PARALLEL TO THE THIRD SIDE AND HALF ITS LENGTH.

Figure 6: Medial Line Theorem

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<td>Given: ABC triangle, AE = EB, AD = DC

Prove: $\overline{BC} // \overline{DE}$ and BC = 2 DE</td>
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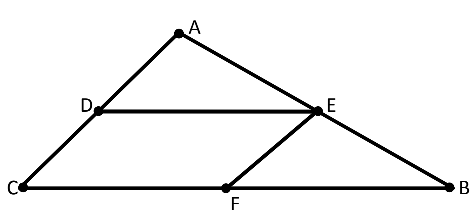
<td>
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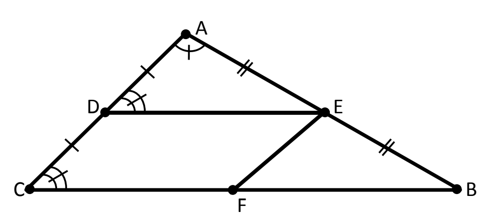
<td>
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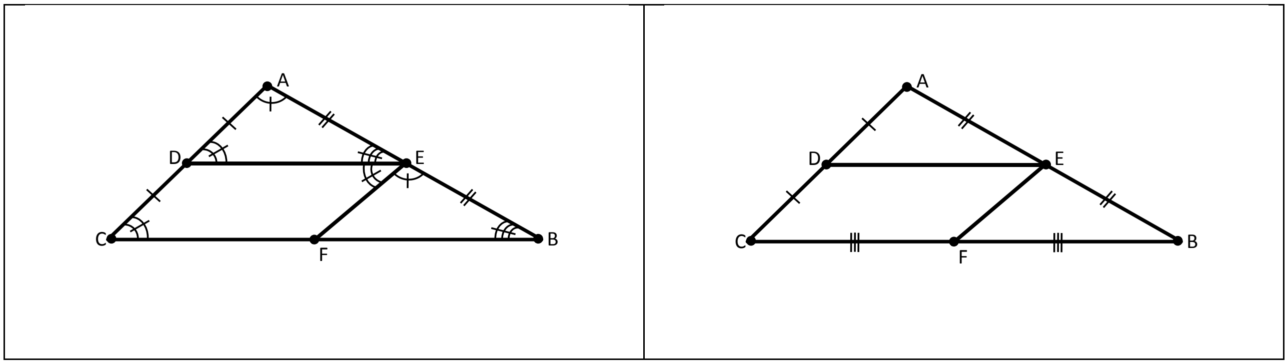

Figure 7: Proof Exercise and Possible Figures

Unlike other task-based interviews that ask students to complete tasks, this interview protocol asked students to report on whether and how they experience problems like those provided. For instance, after being shown a task the students were asked, "How likely is it that your teacher would assign this problem?" and "How likely is it that she would expect students to give a proof as part of their response?" These questions put students in the role of informant as to when and how proof work is done in their class and what the teacher expects from the students. Below we describe how these interviews were coded and the results of the analysis.

5. Methods and Results

Each of the tasks in the interview protocol had been designed to breach hypothesized norms of proof tasks. In particular we hypothesized that students would not accept responsibility to produce a proof in response to the antwalk problem and that they would not expect to have to prove something stated as a theorem. The interview protocol asked them to consider those tasks along with the possible expectation by the teacher that they would produce a proof in response. We coded the interviews according to how students "repaired" (i.e., noticed and perhaps amended) these breaches of the norm. Their responses were classified as to whether they reflected a recognition or misrecognition of the trade of work on the task for claims on the didactical contract. The language of each interview was also coded to see how prominent of a role the teacher played in the student's responses. Below we expand on these coding schemes and report the results of the analysis.

5.1 Coding Classification

To gauge the extent to which a given student "recognized" or "misrecognized" the instructional exchange, we applied two consecutive processes of data reduction. First, students' responses to each interview question were coded as corresponding with one of several response codes for the repairs that students could make to the tasks. Table 1 shows what codes we used for the responses to each of the tasks in the interview protocol.

The first three rows in Table 1 show the codes related to the Antwalk problem, the middle three rows are related to the Theorem Task, and the final three rows are related to the Proof Exercise. The columns of Table 1 show different aspects of each of the codes. The first column lists the name of each code. The letter in parentheses correspond the orientation inferred by the code (see the final column). The second column describes the breach in the problem that is noticed by the student. For several of the codes the breach that the student notices is the same but the codes differ in the third row, depending on the repair that the student offers in response to the breach. The final column lists the orientation to the didactical contract that is inferred from the students suggested repair to the breach. The suggested repairs are classified as mathematical, neutral, or evaluative. A mathematical orientation reflects the idea that work on mathematical tasks is valuable because it is engagement with interesting tasks; a neutral orientation reflects the idea that work on mathematical tasks has no value; an evaluative orientation reflects the idea that work on mathematical tasks is valuable because it allows for claims to be made on the didactical contract.

Insert Table 1 about here

Each response code repairs a breach in the normal way that proof tasks are assigned to students. All three of the tasks shown to students during this interview breach some norm of the instructional situation of 'doing proofs'. When confronted with the tasks students either repaired the breach or dismissed the task as not suitable for their class. Below we review the breaches in the tasks and explain the repairs.

As a proof task the antwalk problem breaches the norms that all the information needed to do the proof is contained in the problem and that the problem is stated in terms of geometric objects—to do a proof the student would have to add a hypothesis about the points D, E, and F, such as the hypothesis that they are midpoints and the student would also need to translate ant walks into segment lengths. The first two response codes represent repairs to these breaches. The first repair is made by filling in missing information to make a proof possible. The second repair is made by ignoring the fact that the problem is about ants and focusing on the mathematical act of comparing segment lengths. The third response code is a rejection of the problem.

The "theorem" task also breaches norms about how proof tasks is ordinarily stated. It does not directly ask for a proof and it also does not parse the statement into a "given" and a "prove". The first response code repairs the first breach by wondering at what a proof might be for the theorem. The second response code repairs the first breach by noting that the theorem should be proven but no interest is shown in the content of the proof. The third response code repairs the task by casting it as a theorem that would be given in class (by the teacher) and then used (by students) in homework exercises (no mention is made of proof).

In the proof exercise's case, the statement of the problem does not breach any norms, however the task is given without a diagram, which is a breach. The response codes correspond to repairs involving diagrams that contain extra information not given in the statement of the exercise, that contain appropriate information (e.g. the markings represent the information given in the statement of the problem) that will be helpful in the proof, or that does not contain any markings at all.

Each of these response codes reflects an orientation towards the exchange involved in the didactical contract. Responses that correspond to a mathematical orientation were given a code of "a." Responses that correspond to an evaluative orientation were given a code of "c." Neutral responses were given a code of "b." Repairs that primarily showed the student focusing on the mathematical argument at stake in the proof represent a mathematical orientation. Repairs that primarily showed the student focusing on the response to the task that the teacher expects represent an evaluative orientation. Repairs that primarily showed the student focusing on the most straightforward method of finishing the task represent a neutral orientation. After coding each interviewee's response to each task, we classified each interviewee founding the orientation whose code had been most common in the responses to the three tasks. We refer to this classification as the interviewee's orientation. Below, in the section on "Instructional Identities" we explain this orientation in further detail.

Table 2 shows the response codes and coding classification for each student. Missing codes (resulting from responses that did not fit the codes) are indicated with a '*'. In some cases (Andra, Craig, Yuri, and Lance) only two answers were coded and they were different. They included one non-neutral code and a neutral code (e.g., Lance has one answer coded b and another c, while Andra has one answer coded a and another b); we assigned to these students the non-neutral code.

Insert Table 2 about here

5.2 References to the Teacher

Another way in which the interviews differed was in relation to how much they referred to their teacher in the responses to the question. To gauge this, for each interview we counted the number of times that the student referred to the teacher (that is, we counted the words 'teacher,' 'Ms./ Mrs. X' and 'she,' her,' 'they,' when these pronouns pointed to the teacher) while the student was discussing the three tasks. The total number of words in the analyzed text was divided by this count. This ratio, which we call 'teacher token', is a quantitative, linguistic estimate of the extent of the teacher's influence on students' responses to the questions.

$$teacher\_token = \frac{total\_number\_of\_words}{number\_of\_teacher\_references}$$

Even though the interview questions were consistently asked in terms of the teacher (whether the teacher would expect students to do a proof for the task at hand), the number of times that different students chose to refer to the teacher varied. We take this variation to be meaningful in reconstructing how students view the figured world of the geometry classroom.

Figure 8 shows the teacher token for each interviewee. The number of occurrences of references to the teacher in all interviews ranged between 0 to 37 and the teacher token varied from 50 to 400. Because the teacher token increases very quickly when the number of occurrences approaches zero, we capped this variable at 400 (just above the highest non-infinite teacher token). The higher the ratio, the lower the number of times the student mentioned the teacher; the lower the ratio, the higher the number of times the student mentioned the teacher. So students listed near the top of the figure rarely mentioned the teacher (if at all), and the students listed near the bottom of the figure mentioned the teacher relatively frequently.

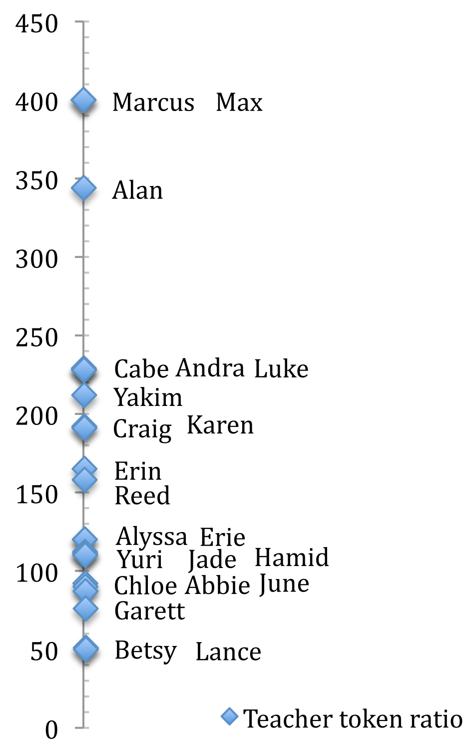

Figure 8: Teacher Token Ratios

5.3 How do students classified after different orientations compare vis-à-vis their references to the teacher?

We were interested in seeing how students' coding classifications are related to their teacher token ratio. Figure 9 shows a box and whisker plot of the 'teacher token' ratios grouped by coding classification (with 1, 2, and 3 corresponding respectively to "a," "b," "c"). On account of the apparent differences between groups we compared statistically the mean ranks of the students' 'teacher token' ratios between groups of students with different coding classifications. For this analysis 'teacher token' was treated as a continuous variable and coding classification was treated as a categorical variable.

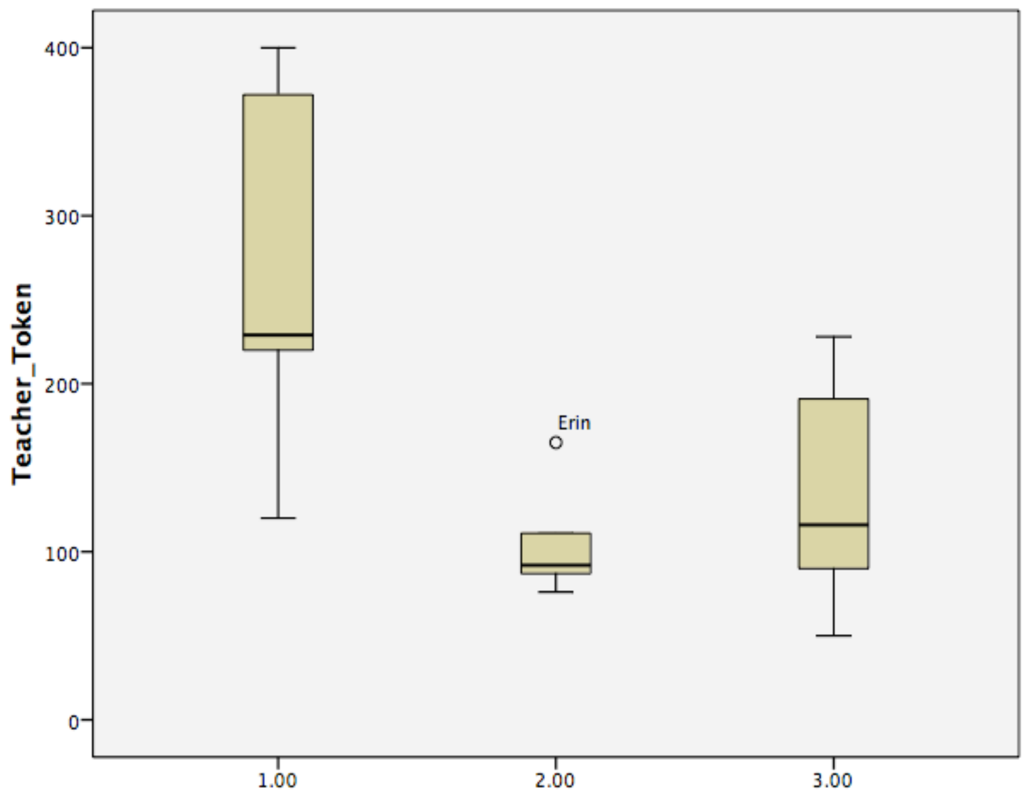

Figure 9: Box and Whisker Plot of students 'teacher token' ratios grouped by coding classification

Because the distribution of students' 'teacher token' ratios could not be assumed to be normal within each coding classification group, and because we have a relatively small sample size, we did this comparison using the Kruskal–Wallis technique for one-way analysis of variance by ranks. Table 3 shows the mean ranks for the three groups; 18.00 is the mean rank for the group of seven students with a coding classification of "a", 6.80 is the mean rank for the group of five students with a coding classification of "b", and 9.30 is the mean rank for the group of ten students with a coding classification of "c".

Insert Table 3 about here

Table 4 shows the results of the Kruskal-Wallis Test. The test yields a Chi-square statistic of 10.799, which is statistically significant. This test tells us that at least one of the mean ranks of the groups is different than the other two. To know which of the groups were different from each other we ran a post hoc Tamhane test. The results of this analysis can be seen in Table 5. From this table we can see that the mean of the 'teacher token' ratio of students with a coding classification of "a" is significantly different than the mean of the 'teacher token' ratio of students with a coding classification of either "b" or "c". The means of the 'teacher token' ratio of students with a coding classification of "b" and "c" are not significantly different from each other.

Insert Tables 4 and 5 about here

From this analysis we can see that students with a coding classification of "a" are more likely to have a high 'teacher token' ratio. Students with a coding classification of "b" or "c" are more likely to have a lower 'teacher token' ratio. This analysis does not reflect the difference between students that have coding classifications of "b" or "c." In other words, the less the student's talk refers to the teacher, the more students' response codes reflect a mathematics-centric view of classroom interaction. But his analysis does not show a difference in the language of students who we classified as evaluatively oriented from students who we classified as neutral to the value of their work. This analysis gives weight to the instructional profiles presented below by showing the teacher token as another way to ground differences among students in each profile.

5.4 Cluster Analysis

Based on the coding classification and teacher token variables, students were sorted into clusters using a k-means cluster analysis. The results of this analysis can be seen in Figure 10. There are three students in Cluster A, Marcus, Max, and Alan. Cluster B has eight students, Cabe, Andra, Yakim, Erin, Luke, Craig, Karen and Reed. Cluster C has eleven students, Alyssa, Jade, Chloe, June, Garett, Erie, Yuri, Hamid, Abbie, Betsy, and Lance.

Students who were oblivious to the impending evaluation from the teacher and only concerned with the mathematical content that was available to be learned populated the upper group (Cluster A), and students who only saw the potential evaluation of the teacher populated the lower group (Cluster C). The students in the middle (Cluster B) were less aware either of the teacher's evaluation or of the mathematical content. The middle group could be characterized as lacking recognition or misrecognition, rather than as a blend of the other two groups.

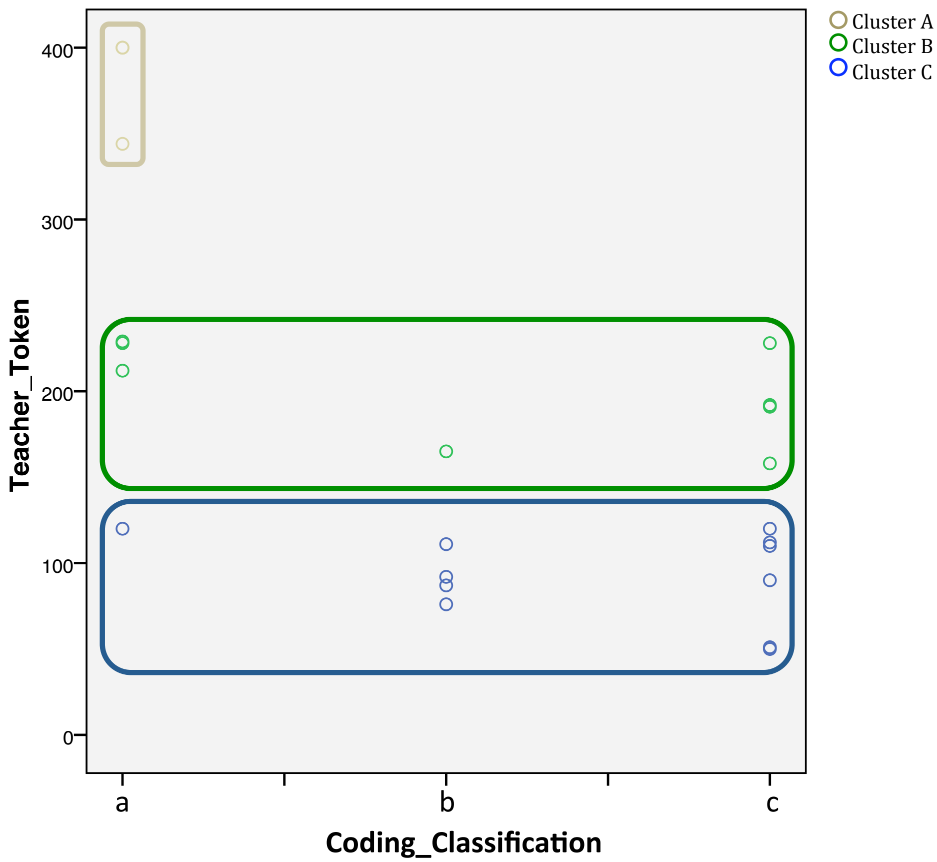

Figure 10: Cluster analysis

The analyses described above show two things. First, the language choices the students made in their interviews to discuss their response to the task correlates with our appraisal of their view of the work as either intrinsically valuable (misrecognizing the didactical contract) or contractually valuable (recognizing that their work is part of a didactical contract). Second, students fall into three clusters, those who are mathematically oriented, those who are evaluatively oriented, and those who do not seem concerned with the value of their work. These clusters allowed us to develop profiles that represent positions that students can take with respect to work done in the geometry classroom (Hoyles, 1997; Lambdin & Preston, 1995). These profiles are not descriptions of individuals but rather they are composites of students who share common dispositions.

In the following section we translate these clusters into qualitative profiles and explain how the orientation is apparent in the responses to the interview tasks.

6. Instructional Identities

In this section we discuss differences among profiles of students' instructional identities at play while 'doing proofs'. We do this by discussing the interviews in depth. We look at responses to each task for interviews in each cluster. And we propose student profiles to describe how each cluster corresponds to a way of being a student in the geometry classroom.

We make the argument that these differences are based on normative student positions with respect to the didactical contract within the instructional situation of 'doing proofs'. The student profiles below exemplify three operational stances to the contract as regards this instructional situation. As stated earlier, the didactical contract sets up an economy of symbolic goods, an economy that can be recognized or misrecognized by the student. The profiles discussed here represent clusters of dispositions; actual students' actions reflect a tendency toward or deviation from these clusters of dispositions. Agency of individuals is expressed in the ways that students improvise actions that depart from these identities; thus such agency is neither visible in these profiles nor ignored.

One profile embodies the position of 'recognizing' the didactical contract—that is recognizing that students are obliged to produce two-column proofs according to the norms of the situation of 'doing proofs' and this work will be evaluated by the teacher. Another profile embodies the position of 'misrecognizing' the didactical contract—that is acting as if the value of students' work lies on the engagement with interesting tasks where students can do what they deem sensible. Finally, the third profile embodies the position of 'non-recognition' that is, students take all the work done as simply something that must be completed, not something that has any value. Unlike students who recognize or misrecognize, these students act as if they are naïve of the exchange structure.

This "non recognition" profile points to the possibility that students do not engage in recognition or misrecognition. Students who are "non recognizers" do not second-guess the value of the work that they are doing. To extend the analogy with Bourdieu's gift exchange, these are unenlightened gift exchangers. They do not feel obligation after receiving a gift (as those who 'recognize' the exchange), and they don't particularly enjoy giving or receiving (as those who 'misrecognize' the exchange).

The first composite profile is that of the "recognizer". This student approaches proof tasks as if he is always trying to please the teacher in the sense that his highest priority when approaching a task is to complete it in the way that will receive highest recognition from the teacher. The second profile is of the "non-recognizer". This student approaches proof tasks as if she is oblivious to the possibility that her work has value beyond its completion. She is neither attuned to the trade value or mathematical value of her work. The final profile is that of the "misrecognizer". The misrecognizer sees the value of his work on proof tasks with regards to the mathematics that is available for him to do and oblivious to the value the teacher will give to the work done. While the recognizer is likely to 'recognize' his obligation to the teacher, the misrecognizer is likely to 'misrecognize' this obligation and focus on the mathematics.

To highlight the differences between the student profiles, below is a comparison of reactions to the three problems in the interview protocol. The reactions showcase how different stances toward the didactical contract within the situation of 'doing proofs' appear in student actions and interpretations. The lines of transcript reported below belong to individual interviews and have been collected to build a composite profile of an instructional identity.

6.1 Reactions to the Antwalk Problem

We refer to the first problem on the protocol as the "antwalk problem" (Herbst & Brach, 2006). In general, students agreed that this problem is not a problem that they would encounter in their geometry class. However, after this initial impression, the recognizer, the misrecognizer, and the non-recognizer responded in different ways to the possibility that their teacher would assign it. The misrecognizer looks for the mathematical relations that exist in the problem. He notices that if the points on the sides of the triangle were midpoints then he would be able to add the number of segments that each ant walked and compare the result. The following excerpt illustrates this.

> M:[5] I think a proof would probably work easiest to solve this problem
>
> I: How's that, excuse me?
>
> M: Because you could, you could say like, if, you could find out if like *EF* were to, were the median or like *F* was midpoint of *CB* and *D* was the midpoint of *CA* and *E* was the midpoint of *AB* so you could find out the distance each one walked, of each segment and then add them up to see who would walk the farthest.

The antwalk problem does not give enough information to answer the question that it poses; so the misrecognizer notes that if he made an assumption then he would be able to answer the question.

The recognizer's reaction is very different from the misrecognizer's. The recognizer also notices that the problem does not give enough information to solve the problem but instead of assuming the missing information or estimating he rejects the task as undoable. Mathematically, it is a plausible move to make an assumption and deduce inferences based on that assumption. However, the figured world of the geometry classroom does not allow for these actions while doing proofs: Students are expected to use only the information given (Herbst & Brach, 2006). In the following quote the recognizer rejects the possibility of making an assumption.

> R: Well, I'd probably think about knowing like, knowing that I can't guesstimate, or estimate at least like what these lengths are, like I'd think well I'd know that's approximately half but you don't know if it's perfectly made to match the answer so.

[5] M is a line spoken by a student whom we identify as a misrecognizer. The letter I corresponds to the interviewer. Likewise R designates students identified as recognizers and N students identified as non recognizers.

The recognizer first mentions that he cannot estimate the answer, even though he can approximate the relative lengths in the figure. The recognizer goes on to say that even if he did feel that he could estimate, it would not be prudent because his estimate might not match 'the answer,' presumably held by the teacher.

Students in the recognition category also reject this problem for another reason. The recognizer does not believe that his teacher would give a word problem about ants.

> R: Uh, I don't think the way it's presented I would see it cause the little, two ants thing I just don't I can't see it happening in like a high-school class, well in Ms. Keating's class I just don't think that's her style.
>
> R: I don't think she would use it cause she uses more geometry stuff, like she would probably use that but she would say more like AE+EF+FC+CD+DE+EB is greater than or less than BC+CA+AB, she would put it in more geometry form.

This view of the problem is not based on mathematics like the misrecognizer's reason, but based instead on an understanding of the teacher and the problems that the teacher chooses for the class. This could be seen as a superficial take on the problem, but instead the recognizer's reading of the problem comes from a position thinking that every problem that the teacher assigns must lead to work that will be evaluated. It is not clear to this student how statements about ants could be related to ideas that are valued by the teacher. They types of statements that are valued by the teacher are those about geometric objects.

The non-recognizer is much less sure of her answers to the interview questions than her peers. She is hesitant about whether or not the antwalk problem is one that she would be given. But, she says, if she were given the problem she would most likely be asked to produce a proof as an answer.

> I: Okay, how likely it is that if you would receive a problem like this, you would be expected that the answer would come in the form of a proof?
>
> N: Oh. Um…that's…ahh…I guess that's pretty likely actually if we were to get that.
>
> I: Okay. So even though it doesn't say do a proof you might be expected to do a proof.

N: Exactly, cause that's how we're used to figuring stuff out.

The non-recognizer's first response is very hesitant. She says that students would do a proof *if* they were given that problem. She doesn't explicitly say that she would not receive the antwalk problem, but she will not endorse it either. Her response to the second question seems to be free from the context of the antwalk problem; no matter what problems students are given, students do a proof. It is worth noting that she does not say that the teacher would expect a proof, but instead she says, "that's how *we're* used to figuring stuff out." She is expressing a student centric view, as opposed to the teacher centric view expressed by the recognizer.

6.2 Reactions to Theorem Task

The second problem on the interview protocol is the statement of the 'Medial Line Theorem.' In general, students agreed that they have encountered theorems like this in their geometry class. However, students disagreed on when they would see this kind of statement, who would be responsible for the proof, and what activity would follow the proof.

Similar to the response to the antwalk problem, the misrecognizer is most concerned with the mathematics presented in the task. When the misrecognizer is asked to pick a figure to accompany the theorem task he picks the figure that contains the elements that are needed to prove the theorem.

M: Well, since line's through the midpoint you can kind of like this is that but those two are congruent so you know it's the midpoint and the other two are congruent so you know that's the midpoint and you can tell that it goes through the points.

The misrecognizer picks the figure that has the hypothesis of the theorem marked (the segment has end points on the midpoints of the sides of the triangle). He bases the choice of figures on the mathematics at play.

In response to the theorem task the recognizer sees the teacher's routine as a warrant for the way tasks are given to students.

I: Would you get that as a homework problem?

R: Um, well we'd usually go over it in class as notes and then we'd have it on our homework like ask doing examples of it to show how to use it, that's usually what we do for homework.

Completely without consideration to the specific content being taught, the recognizer relates that theorems are usually presented by the teacher. Once a theorem has been introduced in class, through notes, students are asked to do problems where they apply the theorem so that they will understand the theorem better.

The non-recognizer, like the misrecognizer, expects figures accompanying proof tasks to have information in them that is related to the reasoning in the proof, but she is much less sophisticated in her observations.

I: Okay, is there anything about it that she might change [about the figure accompanying the theorem task] to make it more like something she'd give to you?

N: She might add congruent angles just to show, she might add more information to make it more provable but that's just me saying that since we haven't really gotten to something like that so…

The non-recognizer expresses that there should be more information to aid in proving, but she is not able to be specific. Since proofs usually involve proving triangles congruent, and that usually hinges on having congruent angles, the non-recognizer suggests that the figure could have congruent angles marked.

6.3 Reactions to Proof Exercise

The final problem on the interview was a proof exercise, presented with explicit "given" and "prove" statements. All students agreed that the last task, with a "given" and a "prove" was the most likely for them to see in their class. Even though they agreed on this point, they had different views on the figure that would accompany the task.

The first contrast is between the misrecognizer and the recognizer, where the misrecognizer expressed an opinion based on the relevant mathematics and the recognizer was concerned with ease of completion of the task. The misrecognizer picks a figure that includes all the elements that would be needed to write the proof. He is of the opinion that objects that were not included in the given, like congruent angles, that are essential to the argument showing the lines are parallel, should be included in the figure.

> I: This one? Why do you say that?
>
> M: Because you know that if you have congruent corresponding angles which would be those two right there, then you know the lines are parallel, so you know C is parallel to MN right away which would give part of the, which would be part of the something that you're trying to prove.

This opinion is in line with the misrecognizer's other mathematical warrants, and it is in contrast with the opinion expressed by the recognizer. The recognizer also picks based on the work that he expects to do with it, but his reasons are not mathematical.

> R: This one looks most useful because it looks more standard than like the really obtuse triangle...

He observes that figures that include objects that are not standard, like obtuse triangles, result in a more difficult proof and therefore he picks a figure with standard objects.

The second contrast is between the non-recognizer and the recognizer. The recognizer would like the figure to contain a small amount of information, but the non-recognizer would prefer a large amount of information. In this quote from the non-recognizer, she talks about the markings on the diagram. These markings convey information about the objects in the figure; in particular, they tell which objects are congruent to each other.

> N: I don't know I think this one has, these two don't have a lot of information but and that one looks like, I guess it could be either of those two [the two diagrams with the most markings.]

The non-recognizer sees these marks as helpful and would rather have more marks so it will be easier to complete the proof. The recognizer would rather have fewer marks on the figure that he is working with. For him more marks mean a more complicated proof.

> R: If it was possible to solve it with either of these, I'd prefer this one [diagram with few markings] because it doesn't have too much information so it doesn't seem like we'd have to do too many steps or over complicated, but it isn't something like this where I have no idea where to start from.

What is seen as better by the non-recognizer because it shows more options is seen as overwhelming by the recognizer. The recognizer is focused on completing the proof in a way that is identical to the way the teacher envisioned it, so the more information means that there are more required things to include and more opportunities for error. The non-recognizer is only focused on finishing the task, so more information means more possible paths to completion.

These three profiles of the recognizer, the non-recognizer, and the misrecognizer showcase three different ways that students can engage with proof tasks in the geometry classroom. For the three problems in the interview protocol this analysis shows three unique responses to the problems. Depending on how the student is disposed to interpret the economy of symbolic goods of the geometry classroom she will be more or less likely to honor the value in her work based on the evaluation of the teacher, or based on the mathematics that she sees as available to learn. We have illustrated three student profiles of instructional identities within the instructional situations of 'doing proofs' and the actions of students who make up the profile. Table 6 summarizes when students would respond with a proof.

Insert Table 6 about here

7. Conclusion

The study reported here shows three instructional identities of high school geometry students; these identities are recognizable responses to variations in 'doing proofs'. The three profiles are recognizers, misrecognizers, and non-recognizers. Recognizers and misrecognizers both see their mathematical work as having value. Recognizers value their work in terms of the evaluation that it will receive from the teacher and misrecognizers value their work in terms of the interesting nature of the problems that they are working on. Non-recognizers do not see any value in their work. They simply see their work as something that they must complete, but not as having any worth.

To arrive at these profiles we employed a theory of classroom interaction as a symbolic economy. In this economy students and teachers must simultaneously recognize and misrecognize the exchange of classroom work for claims on the didactical contract. We also employed a linguistic estimate of how often the students referenced the teacher in their interview. We showed that students who recognized the exchange of work for claims on the contract were more likely to reference the teacher in their interview.

The primary analysis (Herbst & Brach, 2006) of these interviews with geometry students treated the students as one homogenous group. While this was useful for building a model for the situations of doing proofs, it ignored the ways the individuals experience the geometry classroom. The analysis we offer here begins to reveal the texture in the student body of the geometry classroom within the instructional situation of 'doing proofs'.

From the analysis (see also Table 6) we can see that students from all three profiles would provide a proof in response to the proof exercise, but not for the antwalk problem or the theorem task. From the student's comments it appears that they might be more likely to provide a proof in response to the antwalk problem if they felt empowered to make assumptions or if the problem contained more information. It also appears that students would be more likely to provide a proof in response to the theorem task if they were provided a diagram with that included appropriate elements. Through understanding how students are inclined to respond to proof tasks we are better positioned to expand the number of problems to which students will respond with a proof.

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Table 1: Topics and Codes

	Response Code	***Breach Noticed***	***Repair Offered***	***Orientation Inferred***
Antwalk Problem	Fill in gaps (a)	The statement of the problem is missing information that would be needed in the proof	Fill in missing information; make a proof possible	Mathematical: The student sees that making assumptions is a mathematically valid move
	Make it work (b)	The problem is stated as a problem about distances walked by ants, not geometric objects	Focus on the mathematical act of comparison	Neutral: The student accepts this as a task and focuses on the mathematical operation of comparison
	Dismiss (c)	The problem is stated as a problem about distances walked by ants, not geometric objects	Rejecting the task	Evaluative: The student sees that the problem is not about geometric objects and therefore rejects the problem as not one that the teacher would value
Theorem Task	Wonder at proof (a)	The statement of the problem does not contain "given" and "prove" statements	Wonder at proof of theorem	Mathematical: The student sees that the proof of the statement is worth exploring
	Needs proof (b)	The statement of the problem does not contain "given" and "prove" statements	Note need of proof	Neutral: The student see the statement as similar to other statements that the class has proved in the past
	Notes and homework (c)	The statement of the problem does not contain "given" and "prove" statements	Cast theorem as one that would be given in class and then used in homework exercises	Evaluative: The student sees that statement as one that the teacher will expect the students to apply in the future
Proof Exercise	Just right (a)	The proof exercise is given without a diagram	Pick a diagram that has information marked that corresponds to the	Mathematical: The students expects the marks on the diagram to support the proof

		information needed for the proof	
More is more (b)	The proof exercise is given without a diagram	Pick a diagram that has much information	Neutral: The student expects that more information marked on the diagram will mean that the student will have to do less work to complete the task
Less is more (c)	The proof exercise is given without a diagram	Pick a diagram that has few objects marked	Evaluative: The student expects that less marks on the diagram will mean a shorter proof

Table 2: Codes Assigned to Interviewee's Responses to Tasks

Student	Antwalk Problem	Theorem Task	Proof Exercise	Coding Classification
Marcus	a	a	a	a
Max	a	b	a	a
Alan	a	*	a	a
Cabe	b	a	a	a
Yakim	c	a	a	a
Alyssa	a	a	b	a
Andra	b	*	a	a
Erin	b	b	b	b
Jade	b	b	b	b
Chloe	b	a	b	b
June	c	b	b	b
Garett	b	b	a	b
Craig	b	*	c	c
Yuri	c	b	*	c
Lance	b	*	c	c
Luke	c	c	c	c
Karen	c	c	a	c
Erie	c	c	c	c
Hamid	c	c	c	c
Abbie	c	c	b	c
Reed	c	c	c	c
Betsy	c	c	c	c

Table 3: Kruskal-Wallis Mean Ranks

	Coding_Classification	N	Mean Rank
Teacher_Token	1.00	7	18.00
	2.00	5	6.80
	3.00	10	9.30
	Total	22	

Table 4: Results of Kruskal-Wallis Test

	Teacher_Token
Chi-Square	10.799
Df	2
Asymp. Sig.	.005

Table 5: Results of Tamhane Post Hoc Analysis

	(I) Coding_Classification	(J) Coding_Classification	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
Tamhane	1.00	2.00	169.943*	43.299	.014	38.57	301.31
		3.00	145.943*	44.649	.030	14.50	277.38
	2.00	1.00	-169.943*	43.299	.014	-301.31	-38.57
		3.00	-24.000	24.799	.727	-92.30	44.30
	3.00	1.00	-145.943*	44.649	.030	-277.38	-14.50
		2.00	24.000	24.799	.727	-44.30	92.30

Table 6: Responses to Tasks

	Misrecognizers	Recognizers	Non-recognizers
Antwalk	Would prove by making an assumption	Would not prove because there is not enough information; problem about ants	Would prove because students are expected to do proofs
Theorem Task	Would prove with a figure that contains elements needed in proof	Would not prove; the teacher would prove this statement	Would prove with a that contains many elements
Proof Exercise	Would prove with a figure that contains elements needed in proof	Would prove with a figure that contains standard objects; also a figure that contains few elements	Would prove with a figure that contains many elements