### TUESDAY, JULY 17, 2007 Class #7<sup>1</sup>

#### To do before class:

set up student desks with materials	set up the whiteboard with date and warm-up problem
print lesson plan	make handouts
make charts	new giant number line on chart paper marked off in 24ths

### On-going goals:

- 1. To continue to learn about the students and to make connections with each person. Do this in the break room and in the interstices of the morning as well as during class.
- 2. To continue to learn about different aspects of the students' mathematical habits, skills, knowledge, and inclinations by assigning a task and observing how they approach, solve, and discuss problems.
- 3. To help each student learn to express and explain mathematics in class. (Try to make sure that these students get turns to talk or go to the board: Ariel, Autumn, Tosana, Pharoah, Mamadou, Dovan, Nathan, Honoré. This list is based on things I noticed or that happened yesterday.)
- 4. To continue to develop a sense of ourselves as a group that does mathematics together.
- 5. To make explicit a set of shared values, and expectations, and routines for how we work on mathematics together (a) that we are careful with mathematical ideas and interested in and curious about patterns, puzzles, and problems; (b) that we treat other people's ideas with respect and interest, and respond to and use them as appropriate; (c) that working on mathematics is a collective as well as individual endeavor that is, that knowledge must be established within a local community or group and, ultimately, connected to the wider mathematical community.
- 6. To continue to establish norms for mathematical work: paying careful attention to language, representing ideas and building correspondences among them, experimenting with possibilities, using structures for mathematical reasoning, analyzing errors and missteps; attending to others' ideas with interest and respect, asking questions, seeking sufficiently complete explanations; asking fundamental questions about mathematical problems (e.g., why is that true? how could that be explained? is there another way to do or represent that? do we have all the solutions? is there one way, more than one way, infinitely many ways to . . .?), proving.
- 7. To unpack and label explicitly different aspects of mathematics and mathematical work and mathematics learning for students' direct attention and learning.
- 8. To develop habits of making and using records, both collective and individual. These include interest in and care with notation, representation, structure and organization, legibility and usability, keeping track of changes and corrections; keeping track of other questions and observations.

<sup>&</sup>lt;sup>1</sup> The 2007 EML instructional materials are based on work by Deborah Ball, Laurie Sleep, and Meghan Shaughnessy

d. Infinite density on the line

# Mathematical territory of lab class (this is an evolving list):

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Mathematical concepts and procedures	Practices of mathematics	Mathematical vocabulary and language	Practices of learning mathematics	
Place value a. Counting by tens forwards and backwards b. Using place value to add and subtract by tens and multiples of ten (without using procedures) c. Procedures for computing multi-digit numbers d. Trading for equivalent values e. Placement of digits in written form determines value  Equivalence <sup>2</sup> f. An expression rewritten in an equivalent form maintains its value  Fractions g. Fractions as parts of a whole h. Equal partitioning i. Fractions as numbers j. Representations of fractions k. Infinitely many fractions  Number line a. Direction (larger to the right,	<ul> <li>a. Explaining ideas and solutions</li> <li>b. Representing mathematical ideas</li> <li>c. Defining terms</li> <li>d. Proving different kinds of mathematical claims</li> <li>e. Identifying and using conditions and constraints in a problem</li> <li>f. Analyzing solutions, both correct and incorrect, and figuring out how they work or where they went wrong</li> <li>g. Modeling procedures</li> <li>h. Rewriting expressions into equivalent forms: <ul> <li>In ways that are strategic for specific purposes (e.g., to see mathematical structure, to enable computation)</li> </ul> </li> </ul>	<ul> <li>a. "Conditions" of a problem</li> <li>b. "At most one"</li> <li>c. Permutation</li> <li>d. Exactly</li> <li>e. Only</li> <li>f. Prove</li> <li>g. Equivalent</li> <li>h. System, systematic</li> <li>i. Table</li> </ul>	<ul> <li>a. Recording and representing         <ul> <li>Writing explanations and reflections in words</li> <li>Recording one's own work (using words, symbols, pictures, and diagrams)</li> <li>Inventing methods of representation and recording</li> <li>Developing and using systems to keep track</li> </ul> </li> <li>b. Proving and knowing what is entailed in proving something         <ul> <li>Understanding the need to prove claim</li> <li>Knowing what it would take to prove certain kinds of claims</li> <li>Being able to structure a mathematical argument to prove a claim</li> </ul> </li> <li>c. Summarizing the results of work on a problem         <ul> <li>Determining what is</li> </ul> </li> </ul>	
smaller to the left) b. Equal partitions c. Order			mathematically important and what is not	

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<sup>&</sup>lt;sup>2</sup> We need some language to capture the notion of "equivalence." Is there another word, or phrase, or a way to get the larger notion of equivalence across contexts?

### **OVERVIEW OF CLASS**

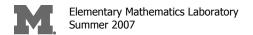
### Activities, time blocks, goals, and materials needed

Activity segment	Time	Goals	Materials needed
Warm-up problem	10	Practice attending to the two key elements we worked on yesterday: the whole, and equal parts	Chart of warm-up problem Small copies of warm-up problem
Opening of class	5		Homework check-in sheet
Discuss warm-up	10		
Fraction definition and representations <sup>3</sup>		<ul> <li>Focus attention on equal parts as core to definition of fractions</li> <li>Recognize how fractions are named (by two things: the number of parts of equal size <u>and</u> the size/division of the whole)</li> </ul>	Charts of partitioned rectangles Charts of 24 circles
	50	<ul> <li>Represent fractions on the line</li> <li>Numbers are ordered on the line (smaller to larger, left to right, between)</li> <li>Points on the line are marked by equal partitioning</li> <li>Begin to assemble elements for a working definition of fractions (on the way to developing a complete definition this week)</li> <li>Mapping across different representations of fractions</li> </ul>	Giant number line Small number lines
Break	10		
Other on-going mathematical work spanning the ten-day program	55	<ul><li>Skills practice</li><li>Train problem</li><li>Notebook writing</li><li>Wrap-up</li></ul>	Cuisenaire rods Jumbo rods Homework

<sup>&</sup>lt;sup>3</sup> The class activity featured in the Mamadou-Half-Rectangle video is highlighted in yellow on this lesson plan

## **DETAILED LESSON PLAN**

Time/format	Activity segment	Detail	Commentary: notes and anticipations
9:00 – 9:10	Warm up	Representations of fractions as shaded parts of rectangles, asking students to label the shaded parts	Greet and talk with students individually as they enter and start working.
		1) What fraction of the big rectangle is shaded red?	
		2) What fraction of the big rectangle is shaded blue?	
		What fraction of the big rectangle is shaded yellow?	
		4) What fraction of the big rectangle is shaded green?	
9:10 – 9:15 whole group	Opening of class	Make some comments about:	
9:15 – 9:25	Discuss warm-up	In eliciting their explanations, focus on:	Try to make these elements explicit and named.
9:25 – 9:40	Fraction definition and representations	Put up the following figure on chart paper:  What fraction of the big rectangle is the blue region? What fraction of the big rectangle is the green region?  As eliciting their explanations, continue to focus on:  identifying the whole equal parts	<ul> <li>Moving toward the following definition of fraction (not likely to be explicated today):</li> <li>Definition of fractions as part of any whole</li> <li>Divide the whole into d equal parts.</li> <li>Write 1/d for one of these equal parts; we call d the denominator.</li> <li>If you have d copies of 1/d, you have the whole</li> <li>If you have n copies of 1/d, you write n/d; we call n the numerator.</li> <li>d and n are whole numbers.</li> <li>d ≠ 0</li> </ul>



		<ul> <li>how many parts</li> <li>how much is shaded (first blue and green separately and then, if time, together)</li> <li>Generate a working definition of fraction (list of important considerations in identifying fractions, or components of fractions): "When we are identifying fractions, these are the important ideas"</li> </ul>				
9:40 - 10:05		(1) Identify $\frac{1}{8}$ in different contexts: $\begin{array}{c c} & & & & & & & \\ & & & & & & \\ & & & & $	A couple things to correct from Monday's class: making points or tick marks to indicate the location of a number on the line; correct notation for negative numbers (-1 instead of 1-)  Attend to the importance of understanding that "equal" depends on the context: counts, regions, lengths. Possible questions:  Why is 1/8 on the number line at a point that is between 0 and 1 but it is 3 when we talk about 1/8 of 24?  Why is that 7/8? (refer to working definition of fractions)			
10:05 – 10:15		Students place numbers from warm-up activity on the number line (give individual number lines): $\frac{1}{4} \ \frac{3}{6} \ \frac{2}{8} \ \frac{1}{2}$	0 1			
		Write: Which is greater — $\frac{1}{2}$ or $\frac{2}{8}$ ?  Are there any more fractions at the same point as $\frac{1}{4}$ or $\frac{2}{8}$ ?	Might talk about "unit fractions" if opportune.			
Break 10:15 – 10:25						
10:25 – 10:35 (if time allows)	Skills practice	(Skills practice with modeling subtraction problems using base-ten blocks)				
10:35 – 11:10	Continue work on the Train Problem	(A multi-part problem involving a proof that students worked on across the two weeks of the summer program)				
11:10 - 11:15 individual	Individual notebook writing	(Notebook question re: Train Problem)				
11:15 – 11: 20 whole group	Wrap up	Distribute homework and have students fill in the class' "first ideas for our working definition of fractions" from chart developed today.				