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# ‘Bayesian source detection and parameter estimation of a plume model based on sensor network measurements’ by C. Huang *et al.*: Rejoinder

We are grateful to the discussants for their valuable contributions and illuminating comments. Our thanks also go to both Editors, Nalini Ravishanker and Fabrizio Ruggeri, for organizing this discussion.

Source detection and parameter estimation in plume modeling remain interesting and challenging, as most discussants have pointed out. One of the consequences of the discussion is that these challenges are brought to the attentions of the broader statistical and engineering communities. In our rejoinder, if a specific point is not discussed, it is because we agree with it.

## RESPONSE TO FASSÒ AND FINAZZI

The review in Drs Fassò and Finazzi (henceforth FF) provides several references. These articles underscore both the importance and the challenges in plume modeling.

We agree with FF that the reversible jump MCMC (RJMCMC) can be an alternative solution in choosing the number of plume sources. FF also suggest a possible way to use parameter mappings among different models (forward and backward) in RJMCMC. Our concern with this approach is the computational cost and whether the procedure would converge. On the other hand, DIC can be calculated directly from individual MCMC runs. In addition, Laine and Tamminen [1] found evidence that the RJMCMC results agree reasonably well with DIC.

FF point out that early warning and detection is another important perspective in plume modeling, and they make several good suggestions. One anonymous reviewer also commented on Figure 2 of our article that most plume densities actually die down at the end time point 2. It is interesting to see whether the detection can be carried out at an earlier stage, say, time point 1. As this remains an important future direction for research, we tried a small-scale simulation. With the setup of $n_s = 9$ censors and 20 observations each, our procedure fails to converge. However, with a slight increment of sensor number to $n_s = 16$ and observations number to 30, the procedure is successful in detecting the spatial and temporal origins of both plume sources; see Table I.

Table I. Estimated posterior means of the parameters for $K = 2$.

| Parameter<br>Setting | $\tilde{a}_1 = 3$<br>$\tilde{a}_2 = 3$ | $\tilde{b}_1 = 1$<br>$\tilde{b}_2 = 1$ | $\tilde{c}_1 = 1$<br>$\tilde{c}_2 = 1$ | $(\tilde{x}_{01}, \tilde{y}_{01}) = (-0.2, 0)$<br>$(\tilde{x}_{02}, \tilde{y}_{02}) = (0.1, 0)$ | $\tilde{t}_{01} = 0.4$<br>$\tilde{t}_{02} = 0.5$ | $\tilde{v} = 0.6$<br>$\tilde{\sigma}$ |
|---|---|---|---|---|---|---|
| $n_s = 16$, $\tilde{\sigma} = 0.05$<br>Independent | 2.982<br>3.031 | 0.960<br>1.049 | 1.004<br>0.994 | $(-0.200, 0.002)$<br>$(0.098, -0.002)$ | 0.400<br>0.499 | 0.607<br>0.054 |

The data are observed at 30 equally spaced time points from 0 to 1, at each of the $n_s = 16$ sensor locations.

## RESPONSE TO HOLAN AND WIKLE

We agree with Drs Holan and Wikle (henceforth HW) that a more comprehensive simulation would help us better understand the problem, and allow the procedure to put to the test in real application. HW also suggest using Whittle formulation in approximating the likelihood, as a compromise between the exact likelihood and the quasi-likelihood. While Whittle's approximation can be more computationally efficient compared to the exact likelihood, Fuentes [2] comments that the grid size needs to be at least 100 to obtain a good approximation. In our article, the largest grid size is 25, which may not be sufficient to use the Whittle approximation.

In a series of articles by Wikle, his co-authors and others (e.g. see references in our article, and HW's discussion), the Bayesian hierarchical model framework associated with discretized PDEs has been developed and applied in practice with great success. The more general PDE (Equation (2) in HW) offers great flexibility in modeling various scenarios in practice. We are very grateful for the insightful comments by HW, which definitely point to the important future direction for research in this area. It remains an open question as to how such a procedure would perform in detecting plume sources both spatially and temporally. This was our goal in our article.

## RESPONSE TO STEINBACH

Dr Steinbach offers a thorough discussion of three approaches in sensor-network data analysis, and he gives the strengths and weaknesses of each approach. We agree with Dr Steinbach that in future research it would be important and interesting to explore/combine all three approaches and investigate the procedures through both simulation and real applications.

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