Moving Toward More Authentic Proof Practices in Geometry

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Abstract: Through the *Standards* documents, NCTM has called for changes related to Reasoning and Proof and Geometry. There is some evidence that these recommendations have been taken seriously by mathematics educators and textbook developers. However, if we are truly to realize the goals of the *Standards,* we must pose problems to our students that allow them to play a greater role in proving. We offer nine such problems and discuss how using multiple proof representations moves us toward more authentic proof practices in geometry.

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Key Words: proof, reasoning, geometry, representation

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Moving Toward More Authentic Proof Practices in Geometry

Through the introduction of the *Standards* documents (1989, 2000), NCTM put forth some significant recommendations related to the Reasoning & Proof and Geometry standards that have had the potential to impact the high school geometry curriculum. First, it has been recommended that reasoning and proof should not be taught solely in the geometry class. Rather, instructional programs in all grade bands should enable students to:

- recognize reasoning and proof as fundamental aspects of mathematics;
- make and investigate mathematical conjectures;
- develop and evaluate mathematical arguments and proofs; and
- select and use various types of reasoning and methods of proof. (NCTM 2000, p. 56)

Despite these recommendations, the high school geometry course continues to be the dominant place where formal reasoning and the deductive method are learned. One reason for this is practical: After students conjecture about the characteristics and relationships of geometric shapes and structures found in the real world, geometry offers a natural space for the development of reasoning and justification skills (NCTM 2000). However, even in the high school geometry course, students have typically not been provided with the kinds of experiences recommended in the Reasoning & Proof standard.

A second recommendation that has had the potential to impact the high school geometry curriculum is related to the modes of representation that are used to communicate mathematical proof. In the 1989 NCTM Geometry Standard, two-column proofs were put on the list of geometry topics that should receive “Decreased Attention” (p. 127). In the 2000 *Standards*, NCTM clarified its position, stating, “The focus should be on producing logical arguments and presenting them effectively with careful explanation of the reasoning rather than on the form of proof used (e.g., paragraph proof or two-column proof)" (p. 310).

Since these recommendations have been published, we have begun to see some changes to the written curriculum (i.e., textbooks). For example, many authors have addressed the proof form recommendation by promoting paragraph and flow proofs in their textbooks (see, e.g., Larson, et al. 2001). *Discovering Geometry* is another example of a curricular shift where Serra (2008) expanded the role of the students by asking them to discover and conjecture through investigations but saves the opportunity to write formal proofs for the final chapter of the textbook. Most recently, the CME Project's *Geometry* (Education Development Center (EDC) 2009) asks students to conjecture and analyze arguments, proposes a variety of ways to write and present proofs, and asks students to identify the hypotheses and conclusions of given statements.

While we do not necessarily endorse all of these changes, we see these curricular adjustments as evidence that mathematics educators and textbook developers are, in fact, rethinking the geometry course. Through our research, however, we have also noticed that even when it is their goal to do so; many teachers find it difficult to move away from the two-column proof form where students are provided with "Givens" and a statement to "Prove" (Cirillo 2008, Herbst 2002). In fact, the two-column form is so prominent that some research has shown that when proofs are written in other forms (e.g., paragraphs), high school students are unsure of their validity (McCrone and Martin 2009).

One reason that the two-column proof holds such a prominent position in the geometry course is historical and will be discussed shortly. A second reason is likely related to the "apprenticeship of observation" (Lortie 1975) where teachers tend to teach in ways that are similar to how they were taught as students. In this article we argue that this version of "doing proofs" does not do enough to involve students in the manifold aspects of proving that are found in the discipline of mathematics. This is important because unless we expand our vision of proving in school mathematics, we cannot fully realize the aforementioned goals of NCTM's Reasoning & Proof and Geometry Standards. The focus of this article is on the two recommendations discussed above. In particular, we focus on the Council's recommendation to expand the role of the student in the work of constructing and writing proofs and support this work through various proof representations. We became interested in this topic through our own experiences as former classroom teachers and current mathematics teacher educators who have research interests related to teaching proof in geometry.

In this article, we first provide some historical context that sheds light on the prominent position that the two-column proof form holds in the geometry course. We do this in order to show how the student's role in proving has been narrowed down over time. We then present a set of problems that are intended to expand the role of students by providing them with opportunities to make and investigate conjectures and develop and evaluate mathematical proofs. Finally, we discuss various proof forms as *representations* used to communicate mathematics. We conclude with a brief discussion of how these activities allow students to participate in more authentic proof practices in geometry.

Historical Context

A perusal of geometry textbooks covering the last 150 years reveals that problems where students are expected to produce a proof have changed quite a bit. As Herbst (2002) noted, this custom of proving developed gradually. Before the 20th century, students could be expected to prove statements that had been given to them in a conceptual register where geometric objects are referred to by their general names (e.g., triangle, angle) rather than by the labels for specific objects (e.g., △ABC, ∠ABC). Students also had the chance to use proof to determine what could be concluded, for example, in response to a question about a figure described in general. While less common, some "original" problems (those problems left for independent exploration) included finding the conditions or hypothesis (i.e., the "Given") on which basis one could claim a certain conclusion (i.e., the "Prove").

Over the 20th century the scope of labor for students in proving has substantially narrowed down. Perhaps there were reasons for that narrowing down. It is interesting though that this narrowing down occurred simultaneously with the standardization of the "two-column form" for writing proofs. If a goal for our students is simply to construct the statements and the reasons that prove a conclusion follows from the "Givens," then the two-column form offers a useful scaffold to assist students in this work. Were we to increase the share of labor that students do when proving, however, we might have to think of other types of problems and forms of representation to support and scaffold their work. In thinking about expanding the student's role in the proof process, two questions are important to consider: What kinds of problems might be posed to increase students' share of the labor? What kinds of support could be provided to students to do this work? We address these two questions in the sections that follow.

Expanding the Role of the Student through Alternative Problems

One reason that the two-column form has come under so much scrutiny in recent times is related to the belief that it is not an authentic form of mathematics. A critical piece that has been lost in our modern version of what doing proofs is like in school mathematics today is related to conjecturing and setting up the proof. This is important if you believe, as Lampert (1992) argued, that "conjecturing about…relationships is at the heart of mathematical practice" (p. 308). Related to this is the importance of determining the premises ("Givens") and statements to be proved:

> Many people think of geometry in terms of proofs, without stopping to consider the source of the statements that are to be proved….Insight can be developed most effectively by making such conjectures very freely and then testing them in reference to the postulates and previously proved theorems. (Meserve and Sobel 1962, p. 230)

Because we believe that students should play a larger role in the important work of setting up and carefully analyzing proofs, here we present problems that are reminiscent of the historical problems described above in that they do not simply provide students with the "Given" hypotheses and ask them to "Prove" particular statements. Rather, we propose nine different problems that provide students with opportunities to expand their role in the process of proving.

In Problems 1-3 (see Appendix A), students are asked to participate in setting up the proof by either providing the "Givens," the "Prove," and/or the diagram for the proof. In Problem 1, the student is provided with a conjecture (i.e., The diagonals of a rectangle are congruent) and a corresponding diagram and asked to write the "Given" and the "Prove" statements. In contrast, in Problem 2, the student is provided with the "Given" and the "Prove" statements but is asked to draw the diagram. Finally, in Problem 3, when provided with a particular theorem (in this case: If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram) the student is asked to do all three of these tasks (i.e., write the "Given," the "Prove," and draw the diagram).

Problem 4 is similar to the first three in that students are invited to determine the "Given," but this time, they are not provided with a conjecture or a theorem to be proved. Rather, here, students are asked to determine what would have been "Given" in order to construct the proof that is provided. They are then asked to condense those two "Givens" into a single, more concise statement. This exercise asks students to reflect on two different ways that the line segment bisector premise might be handled. Problem 4 is similar to the “fill in” type proofs that we have seen in some textbooks (e.g., Larson et al. 2001 and Serra 2008) except that rather than having students fill in the statements or reasons, they are filling in the “Given” premises.

Next, in Problem 5, students are asked to draw a conclusion or determine what could be proved when provided with particular “Given” conditions and a corresponding diagram. This type of problem can be a useful scaffold in that it isolates particular geometric ideas such as definitions or postulates of equality, for example. In Problem 6, students are asked to determine what auxiliary line might be drawn in order to prove that two angles are congruent. This is not a common problem posed to students because, typically, teachers either construct the auxiliary lines for their students or a hint is provided in the textbook that helps students determine where this line should be drawn (Herbst and Brach 2006). We view these first six problems as scaffolds that could eventually allow students to conjecture and set up a proof on their own.

Problem 7 is unique in the sense that the student is asked what could be proved, but the givens are ambiguous. It is expected that the student will consider two different cases corresponding to whether the quadrilateral is concave or convex. In both cases the student could argue that the remaining pair of sides are congruent to each other.

Finally, Problems 8 and 9 students have the opportunity to take part in analyzing proofs. In Problem 8, a paragraph proof is provided, and students are asked to find the error. In this proof, the corresponding parts that are proved to be congruent are two pairs of angles and one pair of sides. The student author determined that the triangles were congruent by ASA based on the order that these corresponding parts were proved congruent, rather than attending to how these parts are oriented in the triangles. In Problem 9, students are provided with a proof and asked to determine what theorem was proved.

In this section, we proposed nine problems that would allow teachers to increase their students' involvement in proving by having them: make reasoned mathematical conjectures; use conjectures to set up a proof; and evaluate mathematical proofs by looking for errors and determining what was proved. In the next section, we address the issue of supporting students in proving by commenting on multiple proof representations.

Proof Representations that Support *Constructing* and *Writing* Proofs

Representation is one of five Process Standards that highlights the ways in which students acquire and make use of content knowledge (NCTM 2000). Here, we are thinking about various proof forms as representations of geometric knowledge. Providing students with access to various proof representations is useful because: "Different representations support different ways of thinking about and manipulating mathematical objects" (NCTM 2000, p. 360). And while it is important to encourage students to represent their ideas in ways that make sense to them, it is also important that they learn conventional forms of representation to facilitate both their learning of mathematics and their communication of mathematical ideas (NCTM 2000). The purpose of this section is to highlight four different ways that proofs can be represented in geometry and discuss how these various representations have the potential to facilitate proving.

As pointed out by Anderson (1983), successful attempts at proof generation can be divided into two major episodes – "an episode in which a student attempts to find a plan for the proof and an episode in which the student translates that plan into an actual proof" (p. 193). Here, we refer to these two activities as *constructing* and *writing* a proof, respectively. The proof forms that we highlight include: proof tree, flow proof, two-column proof, and paragraph proof. Descriptions and examples of each representation can be found in Appendix 2. Here we briefly discuss the ways in which these proof representations can support students in proving.

Proof Trees. The proof tree is an outline for action, where the action is *writing* the proof. Anderson (1983) described the proof tree as follows:

> The student must either try to search forward from the givens trying to find some set of paths that converge satisfactorily on the statement to be proven, or [s/he] must try to search backward from the statement to be proven, trying to find some set of dependencies that lead back to the givens. (p. 194)

In other words, students might begin by asking themselves: What would I need to do in order to prove this statement? Using a proof tree to think through a proof could be a useful scaffold to support students in *constructing* a proof. The proof tree could also be a useful tool to scaffold the work of finding out what the givens are or what conclusion can be proved.

Flow Proof. A flow proof uses the same statements and reasons as a two-column proof, but the logical flow connecting the statements is indicated by arrows (Larson et al. 2001). The flow proof helps students to brainstorm, working through the most difficult parts of solving a proof: (1) understanding the working information – analyzing the given and the diagram, and (2) knowing what additional information is needed to solve the proof – analyzing what is being proved (Brandell 1994). A disadvantage to this proof form might be that students are not required to supply reasons that justify their statements in the way that the “Reasons” column of the two-column proof forces them to do so. For that reason, however, it allows students to focus on making the argument and thus could be particularly useful toward constructing a proof.

Two-column Proof. A two-column proof lists the numbered statements in the left column and a reason for each statement in the right column (Larson et al., 2001). The two-column form *requires* that students record the claims that make up their argument (in the statements column) as well as their justifications for these claims (in the reasons column). In this sense, the two-column form appears to be a rigid representation. This could be intimidating to students. However, students can be flexible when using this representation. For example, they might leave out a reason that they do not know but still move ahead with the rest of the proof; the incomplete form reminds them that they still have something to complete (Weiss, et al. 2009) However, the consecutively numbered steps of the proof may lead students to believe that the deductive process is more linear than it actually is. The deductive process, in general, hides the struggle and the adventure of doing proofs (Lakatos 1976).

Paragraph Proof. A paragraph proof describes the logical argument using sentences. This form is more conversational than the other proof forms (Larson et al. 2001). Paragraph proofs are more like ordinary writing and can be less intimidating (EDC 2009). For this reason, they look more like an actual explanation than a structured mathematical device (EDC 2009). While paragraph proofs may seem less intimidating, they are also less structured (EDC 2009). One of the teachers that we studied concluded that the paragraph form was not appropriate for high school students because students tended to "forget" to write the reasons that justified their statements. As a result, students would often come to invalid conclusions (Cirillo 2008). Yet, if a goal is to help students develop mathematical literacy, this proof form most closely resembles the representation that a mathematician would use to write up a proof. Another advantage of this form is that when writing a proof by contradiction, the paragraph form seems a more sensible choice than some of the other options (Lewis 1978).

Conclusion

Through their *Standards* documents, NCTM has called for changes related to Reasoning & Proof and Geometry. There is some evidence that these recommendations have been taken seriously by mathematics educators and textbook developers. In this paper, however, we argued that if we are truly to realize the goals of these standards, we must pose problems to our students that allow them to play a greater role in proving. We presented problems that asked students to write the premises ("Givens") and the statements to be proved as well as construct the diagrams. We suggested that students should be provided with opportunities to make reasoned conjectures and evaluate mathematical arguments and proofs. Last, we suggested that teachers promote and allow various types of reasoning and methods of proof. We believe that this is important because adherence to a specific proof format may elevate focus on form over function. A focus on form potentially obstructs the creative mix of reasoning habits and ultimately hinders students' chances of successfully understanding the mathematical consequences of the arguments.

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APPENDIX A: ALTERNATE PROBLEM TYPES

PROBLEM 1: Writing the "Given" and "Prove" from a conjecture

Suppose you conjectured that the diagonals of a rectangle are congruent and drew the diagram on the right. Write the "Given" and the "Prove" statements that you would need to use to prove your conjecture.

PROBLEM 2: Drawing a diagram when provided with the "Given" and the "Prove"

Draw a diagram that could be used to prove the following:

Given: Parallelogram PQRS where T is the midpoint of $\overline{PQ}$ *and V is the midpoint of* $\overline{SR}$.
Prove: $\overline{ST} \cong \overline{QV}$

PROBLEM 3: Setting up the "Given," the "Prove," and the diagram when provided with the theorem

Determine what you have been given and what you are being asked to prove in the theorem below. Mark a diagram that represents the theorem.

Theorem: If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.

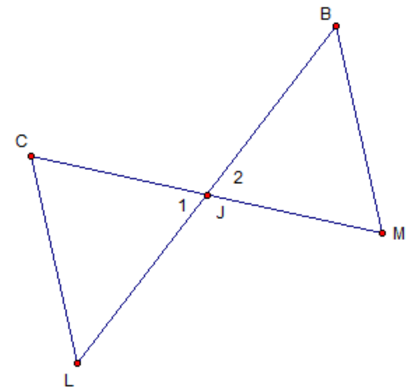
PROBLEM 4: Determining the "Given" from a Flow Proof

1. *Provide the two missing "Given" statements for this proof.*
2. *Write a single statement that could replace these two given statements.*

Given: ___________________

____

Prove: $\overline{CL} \cong \overline{MB}$



?
(Given)

?
(Given)

$\overline{CJ} \cong \overline{MJ}$
(Definition of Midpoint)

$\overline{JL} \cong \overline{JB}$
(Definition of Midpoint)

$\measuredangle 1 \cong \measuredangle 2$
(Intersecting lines form congruent vertical angles)

$\triangle CJL \cong \triangle$ MJB
(SAS $\cong$ SAS)

$\overline{CL} \cong \overline{MB}$
(CPCTC)

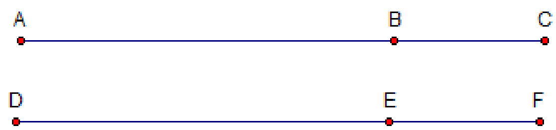
(Adapted from Serra, 2008, p. 239)

PROBLEM 5: Drawing Conclusions from the "Given"

What conclusions can be drawn from the given information?

Example A:

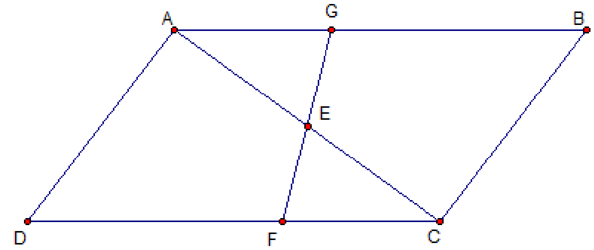
Given: $\overleftrightarrow{ABC}$, $\overleftrightarrow{DEF}$
$\overline{AB} \cong \overline{DE}$
$\overline{BC} \cong \overline{EF}$



(Adapted from Lewis, 1978, pp. 135 & 68)

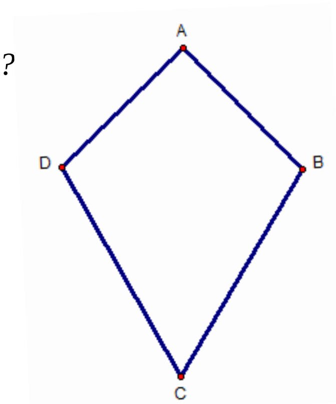
Example B:

Given: Quadrilateral *ABCD* where
$\overline{FG}$ is bisected by diagonal $\overline{AC}$



PROBLEM 6: Drawing an auxiliary line.

What auxiliary line might we draw in to construct this proof?
Is it possible to construct the proof without considering an auxiliary line?



Given: Kite ABCD with $\overline{AD} \cong \overline{AB}$ and $\overline{DC} \cong \overline{BC}$

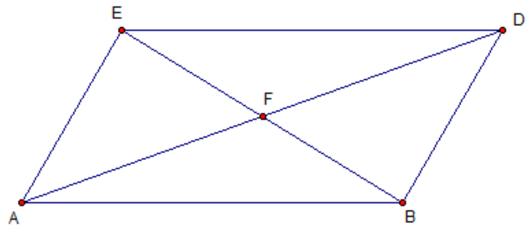
Prove: $\measuredangle$B $\cong$ $\measuredangle$ D

PROBLEM 7: Solving a problem that involves writing a conjecture
(i.e., deciding what to prove)

Consider a quadrilateral such that it has two congruent consecutive segments and two opposite angles congruent. The angle determined by the two congruent sides is not one of the congruent angles. What else could be true about that quadrilateral? What could you proved in this scenario? What are the "Given" statements?

PROBLEM 8: Finding the error in a proof.

In the figure at the right, $\overleftrightarrow{AB} \parallel \overleftrightarrow{ED}$ *and* $\overline{AB} \cong \overline{ED}$.
Luis uses this information to prove that $\triangle ABF \cong \triangle DEF$.
Explain why his paragraph proof is incorrect and give a reason why he may have made this error.

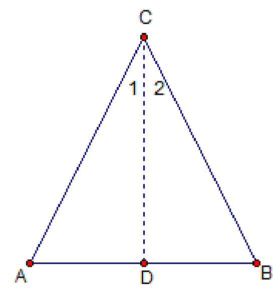


Proof:
It is given that $\overleftrightarrow{AB} \parallel \overleftrightarrow{ED}$ so $\angle DEB \cong \angle ABE$, because parallel lines form congruent alternate interior angles with a transversal. It is also given that $\overline{AB} \cong \overline{ED}$. And $\angle AFB \cong \angle DFE$ because they are vertical angles, and vertical angles are congruent. So $\triangle ABF \cong \triangle DEF$ by ASA.

(Adapted from EDC, Inc., 2009, p. 122)

PROBLEM 9: Determine the theorem that was proved by the given proof.

Write the theorem that was proved by the proof below.



Statements	*Reasons*
1. Let $\overline{CD}$ be the bisector of vertex $\measuredangle ACB$, D being the point at which the bisector intersects $\overline{AB}$.	1. Every angle has one and only one bisector.
2. $\measuredangle 1 \cong \measuredangle 2$	2. A bisector of an angle divides the angle into two congruent angles.
3. $\overline{CA} \cong \overline{CB}$	3. Given.
4. $\overline{CD} \cong \overline{CD}$	4. Reflexive property of congruence.
5. $\triangle ACD \cong \triangle BCD$	5. Side-Angle-Side ≅ Side-Angle-Side
6. $\measuredangle A \cong \measuredangle B$	6. Corresponding parts of congruent triangles are congruent.

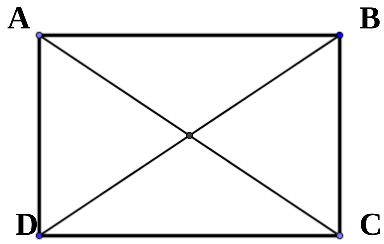
(Adapted from Keenan & Dressler, 1990, p. 172)

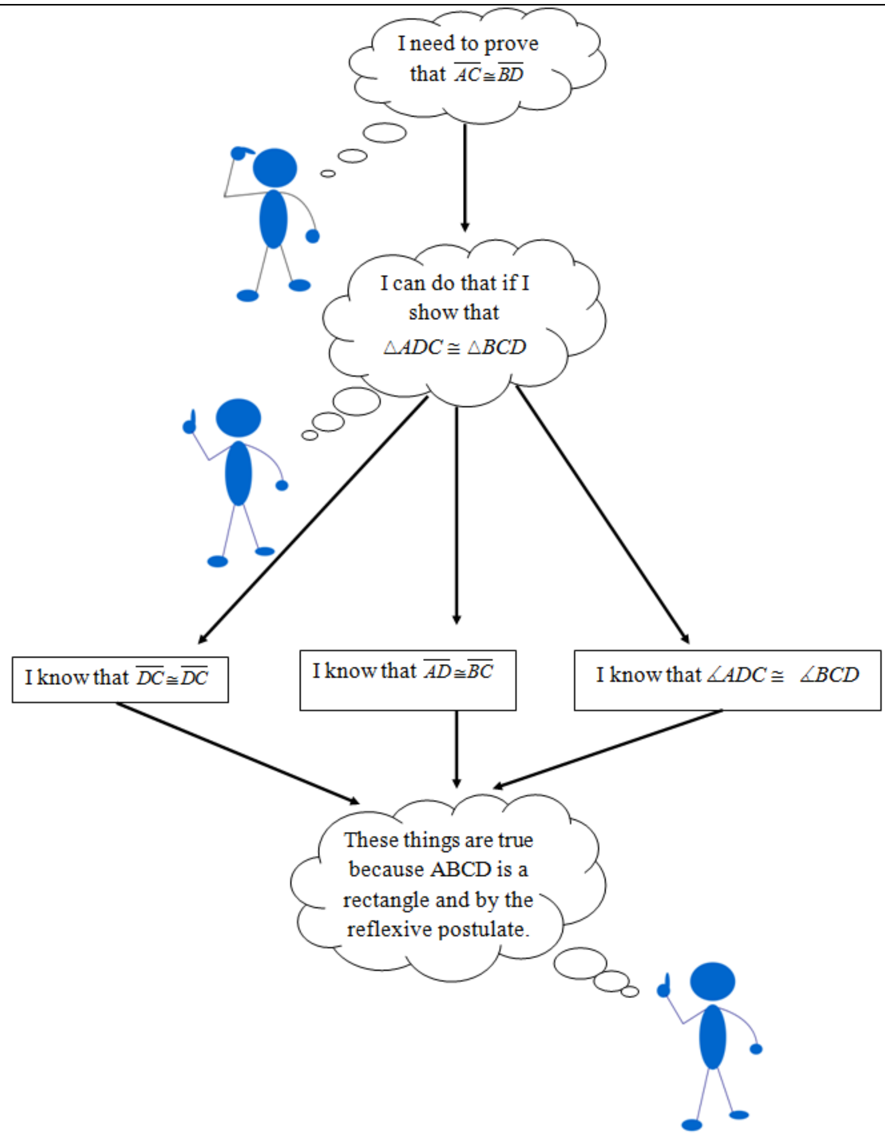
APPENDIX B: PROOF REPRESENTATIONS

THEOREM: If a parallelogram is a rectangle, then the diagonals are congruent.

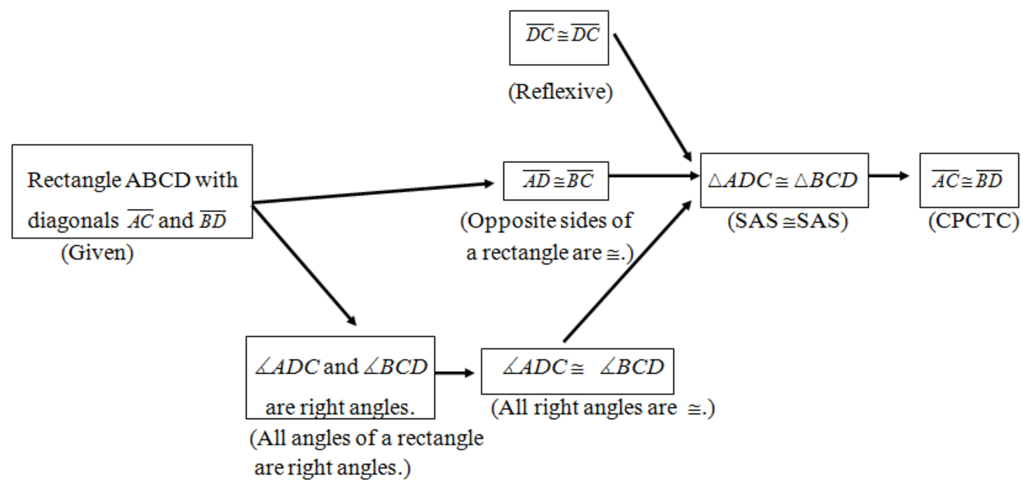
Given: Rectangle *ABCD* with diagonals $\overline{AC}$ and $\overline{BD}$

Prove: $\overline{AC} \cong \overline{BD}$



Proof Form	**Proof**
A **proof tree** is an outline or plan of action that specifies a set of geometric rules that allows students to get from the givens of the problem, through intermediate levels of statements, to the to-be-proven statement. (Adapted from Anderson, 1983)	

A **flow proof** uses the same statements and reasons as a two-column proof, but the logical flow connecting the statements is indicated by arrows. Depending on whether it is the plan or the proof itself, students may or may not choose to write the reasons beneath the statements.



A **two-column proof** lists the numbered statements in the left column and a reason for each statement in the right column.

Statements	**Reasons**
1. Rectangle ABCD with diagonals $\overline{AC}$ and $\overline{BD}$	1. Given
2. $\overline{AD} \cong \overline{BC}$	2. Opposite sides of a rectangle are congruent.
3. $\overline{DC} \cong \overline{DC}$	3. Reflexive Postulate
4. $\measuredangle ADC$ and $\measuredangle BCD$ are right angles.	4. All angles of a rectangle are right angles.
5. $\measuredangle ADC \cong \measuredangle BCD$	5. All right angles are congruent.
6. $\triangle ADC \cong \triangle BCD$	6. Side-Angle-Side ≅ Side-Angle-Side
7. $\overline{AC} \cong \overline{BD}$	7. Corresponding Parts of Congruent Triangles are Congruent (CPCTC)

A **paragraph proof** describes the logical argument with sentences. It is more conversational than a two-column proof.

Since ABCD is a rectangle with diagonals $\overline{AC}$ and $\overline{BD}$, then $\overline{AD} \cong \overline{BC}$ because opposite sides of a rectangle are congruent. By the reflexive postulate $\overline{DC} \cong \overline{DC}$. Since all angles in a rectangle are right angles, then $\measuredangle ADC$ and $\measuredangle BCD$ are right angles. Thus, $\measuredangle ADC \cong \measuredangle BCD$. By Side-Angle-Side, $\triangle ADC \cong \triangle BCD$. Thus, $\overline{AC} \cong \overline{BD}$.