ENGINEERING RESEARCH INSTITUTE
UNIVERSITY OF MICHIGAN
ANN ARBOR

SOME GENERAL PROPERTIES OF THE HEARING MECHANISM

Technical Report No. 30
Electronic Defense Group
Department of Electrical Engineering

By: W. P. Tanner, Jr.
J. A. Swets
D. M. Green

Approved by: A. B. Macnee

TASK ORDER NO. EDG-3
CONTRACT NO. DA-36-039 sc-63203
SIGNAL CORPS, DEPARTMENT OF THE ARMY
DEPARTMENT OF ARMY PROJECT NO. 3-99-04-042
SIGNAL CORPS PROJECT NO. 194B

March, 1956

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

ACKNOWLEDGEMENTS

During the preparation of this report the authors consulted many people, some of whom had a major influence on the thinking represented in it.

Mr. T. G. Birdsall, of the Electronic Defense Group, worked closely with the authors throughout this study. Without his assistance and advice, the report probably would not have been completed.

Professor Julian H. Bigelow of the Institute of Advanced Study, Princeton, New Jersey, read the report in rough form, and, in conference with the authors, furnished detailed criticism. Many improvements were incorporated in the report as a result of this discussion.

Mr. R. R. McPherson of the Electronic Defense Group, and Professor G. E. Peterson of the Speech Department of the University of Michigan displayed an active interest throughout this study. They were consulted frequently and made many valuable suggestions. A contribution was also made by the members of the Program Review Committee of the Electronic Defense Group who devoted several sessions to a discussion of this material.

Much, but not all, of the advice of these people was accepted. The authors therefore take full responsibility, although not full credit, for the viewpoints expressed in the report.

Mr. G. A. Roberts, with the assistance of Mr. J. A. Lauder, designed, constructed, and maintained most of the equipment used in the experiments. These services were very capably performed.

Finally, acknowledgement is due to the many observers for the dependability and quality of their services.

ABSTRACT

This report deals with the auditory processes of detection and frequency analysis. It presents a decision-making theory of detection and some data supporting the theory. Other data included are consistent with a theory of frequency analysis that characterizes the human observer as instantaneously sensitive to a limited range of frequencies, with a shifting of the frequency range of sensitivity requiring a measurable amount of time. Finally, the problem of speech perception is considered in terms of the mechanism inferred from these studies.

The results of this investigation are regarded as contributing to the methodology of psychophysical testing, to auditory theory, and to general behavior theory. The data indicate directly a need for certain innovations in conventional testing procedures by virtue of exhibiting some properties of the auditory mechanism not previously considered. The nature of both the detection and frequency-analysis properties discussed calls for a recognition of the influence of central processes on sensory processes, and demonstrates the feasibility of studying central processes by sensory experiments.

SOME GENERAL PROPERTIES OF THE HEARING MECHANISM

1. INTRODUCTION

This report summarizes the results of an experimental investigation directed toward identifying the parameters of interest in the study of the auditory processes of detection and frequency analysis. This research is not to be interpreted as a detailed investigation of the parameters; it is rather designed to establish the existence of the parameters and to illustrate the feasibility of submitting these parameters to experimental study. The nature of this research is such that each experiment reported herein is intended as sample study in an area which eventually should be investigated more completely. These sample studies have all employed frequencies around 1000 cycles and durations near .1 seconds. Interpretations of the experimental data consequently are restricted to these conditions.

The viewpoint is that only those propositions which cannot be tested for their validity should be accepted on faith alone as assumptions. Any other proposition is a theorem. The data are accepted on faith. Whenever the model leads to a theorem which contradicts the data, then the model must give way. At least one of the assumptions must be discarded. Thus, in Section 2 it is shown that data contradict the conventional threshold theorem[1], and consequently

1. Actually, the existence of a threshold has always been accepted as an assumption. According to the view presented here, its validity is testable, and consequently it should assume the role of a theorem.

assumptions leading to the theorem must be rejected. A different model, the theory of statistical decision, is tested as an alternate, and this model is shown to be consistent with data so far collected. The model suggests the substitution of a new experimental variable for signal power at threshold. This variable is described in Section 2, and some of the research necessary for its definition is outlined.

The data reported in Section 2 are from experiments in which the observers are detecting signals in noise. The observers' performance in these experiments compares favorably with the mathematical optimum for the signals and noise employed. The signal parameters of interest include frequency, duration, and signal power. The noise parameter of interest is the noise power per unit bandwidth. One way to approach the mathematical optimum would be to employ a device which decides on the basis of the integrated output of a narrow-band filter gated only during the observation interval. Two things would be true of such a device. Noise outside of the band of the filter would not interfere with the detection performance. That this is true of the auditory mechanism is suggested by the results upon which Fletcher (Ref. 7) based the "critical-band" concept. The second property of such a device is that it would not detect signals outside the filter band.

Since Fletcher's work was based on a threshold type measure, it is desirable that Fletcher's result be confirmed in an experiment designed to be consistent with the model proposed in Section 2. The modified "critical-band" experiment is reported in Section 3, confirming Fletcher's basic result. The data are, of course, interpreted according to the model proposed in Section 2. While this experiment is a sample study, investigating only the conditions for a signal at 1000 cycles and .1 seconds in duration, it should be emphasized that the sample called for a major experimental effort involving more than 6000 observations for each of three observers.

The second property, that a signal outside of the band of the filter would not be detected, has not previously been submitted to direct experimental investigation, although some data reported by Garner (Ref. 12) suggest that this is true. This problem also is submitted to a sample study showing that it is indeed a real problem. Only a limited range of frequencies around 1000 cps for durations .1 and .3 seconds are studied. The results are statistically significant at a high level of confidence, justifying the recommendation that the problem be submitted to extensive investigation.

It was possible to design both of the above studies without depending on exact definition of the new experimental variable (see Section 2). The results are in a form interpretable in terms of this variable when it is defined.

The results of the experiments outlined above raise some questions concerning the mechanisms involved. For example, the fact that the observer performed in a near-optimum way in the detection experiment reported in Section 2 leads to the question of coincidence. Was the experiment through accident designed so that the characteristics of the observer's mechanism were nearly those required for optimum performance? If this is true, and the mechanism is fixed, then the observer's performance should fall off considerably under different conditions. On the other hand, if the observer can control the parameters of the mechanism, that is, if he can somehow select parameters to match the task, then his performance may approximate the optimum over a wider range of conditions.

In order to handle this problem a distinction is made between those components of the mechanism which are fixed and those which are controlled by the observer. Consideration of the problem of control has lead to a conceptualization of the hearing mechanism as consisting of at least two stages.[1] The first stage is considered fixed (essentially a multiplexing system), serving only to transmit information. The second stage is regarded as one under control of the observer. At any instant in time this second stage can consider only a single observation. In other words, it can consider only the outputs of a narrow band of the multiplexed elements of the first stage, with these outputs combined to form a single observation. The center of the band (in terms either of frequency or of multiplexed elements), the width of the band, the duration of the observation interval, and the time at which the observation is made, are parameters of the second stage. The output of this second stage furnishes the basis for decisions, and it is only at this point that events are classified.

This description is a first approximation. Further study may lead to refinements of the model, postulating intermediate stages with some fixed and some controlled parameters. At this time little can be said about this possibility. It is mentioned now only because the comment aids in clarifying the present status of the model. So far, at least, the model deals only with relatively simple phenomena. Certain behavior consistent with the postulation of controlled parameters appears to occur. How this is achieved is not yet incorporated in the model.

Now, if one is able to observe at a predetermined frequency then the observation can be interpreted as an observation representing that frequency. To illustrate this point, consider a single trace oscilloscope (an A-scope) with a pulse in some position on the trace. If the position of the beam at any instant

1. This conceptualization is, of course, a model. As long as the model is consistent with the data, then it is satisfactory. Whether or not the model is a true representation of the mechanism is beside the point. Any other model which is functionally equivalent, is equally satisfactory.

in time is in one-to-one correspondence with antenna position, then the position of the pulse can be interpreted in terms of direction with reference to the observer. If the position of the beam at any instant in time represents a frequency with reference to receiver tuning then the position of the pulse can be interpreted as an indication of the frequency of the signal. If, in hearing, the observation is somehow correlated with the condition of the receiver (i.e., the center of the band of attention), then a mechanism for frequency identification exists.

Some experiments are reported below bearing on the control of attention. One, referred to above as showing that only one frequency can be attended to at any instant in time, is further consistent with the contention that the sharing of observations over time is related to the frequency separation between two frequencies under observation. For the durations studied it appears possible to make an observation at two frequencies separated by more than the bandwidth of attention, but nevertheless close together, whereas it appears to be impossible when the frequencies are separated by greater amounts. The decrease in detectability with separation is gradual. Whatever is responsible for this behavior is referred to below as a scanning or sweeping mechanism. The terms "scanning" and "sweeping" are defined in terms of this operation, a fact which should be remembered as the reader studies the experiments below.

Experiments demonstrating the method of study of frequency recognition suggested by the model are reported in other papers (References 34 and 36). In this report a section is devoted to the argument that it is possible to perceive speech with the mechanism suggested by the data in spite of the apparent limitations of the mechanism. The inclusion of this section in this report emphasizes the authors' conviction that the model should apply to the study of large signals in the presence of low noise levels as well as to the low signal-noise levels studied in the experiments described below.

Briefly, this report attempts to identify the significant parameters in an area of study. Methods of investigating these parameters are illustrated by sample experiments. Further directions of study are suggested in each of the sections.

2. SIGNAL DETECTION FOR THE CASE OF THE SIGNAL-KNOWN-EXACTLY

2.1 Introduction

This section deals directly with the validity of the conventional threshold concept, and suggests a theory of auditory detection to replace this concept. Attention is directed to the case of the signal known exactly; i.e., the experiment is designed so that the parameters of the signal are fixed. It is possible for a proper device to utilize frequency, phase, amplitude, duration, and starting-time information in signal detection. In the experiments reported here, the observer is told the *a priori* probability of signal existence, and the values assigned to the various events; i.e., detection of an existing signal, failure to detect an existing signal, false alarms, and correct rejections.

First, reasons for re-evaluating the threshold concept at this time will be advanced. Next, a precise statement of the threshold concept and its implications for theory and data analysis will be attempted. The alternative theory will then be presented. Experiments designed to compare the two theories will then be reported, and implications of the alternative theory for psychophysical testing will be discussed.

2.2 Events Leading to the Need for Re-examining the Threshold Concept

There are several reasons for re-examining the threshold concept at this time. First, some psychophysical results cannot be consistently predicted in terms of the concept. Also there have been several developments in the field of mathematics suggesting alternate methods of handling psychophysical data. Third, physiological observations seem more consistent with some of the suggested methods than with the threshold concept.

2.2.1 Events Not Predictable in Terms of Threshold Theory. Smith and Wilson (Ref. 33) have shown that, in auditory experiments, data do not exhibit the independence between false-alarm rate and thresholds that is required as an assumption in a threshold theory. Tanner and Swets (Ref. 38) have reported similar data collected in vision experiments.

Blackwell, (Ref. 2) in a thorough investigation of psychophysical methods in vision, reports several relations between level of threshold and psychophysical procedure which should not exist if one adheres strictly to the threshold concept. Examples of these relations are:

1) Threshold estimates obtained using the forced-choice[1] method of response are, in general, lower than thresholds obtained using the conventional yes-no method.
2) Forced-choice thresholds are a function of the number of alternatives from which the observer must choose.
3) Yes-no thresholds are raised if the observer is warned against false-alarms.

1. The forced-choice psychophysical technique is one in which the signal is presented in one of n temporal intervals. The observer's task is then to state in which of the n intervals he believes the signal occurred. The name "forced-choice" derives from the requirement that the observer must select one of the intervals.

Strict adherence to the threshold concept demands the conclusion that the threshold should be independent of psychophysical techniques. This conclusion is discussed in detail in Section 2.3.

2.2.2 Development of the Theory of Signal Detectability. Military problems arising as the result of the increased use of electronic devices led to an interest in the theory of detecting weak signals in noise. The work carried out during the war is reported in Lawson and Uhlenbeck (Ref. 19). Subsequent work by Peterson and Birdsall (Ref. 26), Fox (Ref. 8) and Van Meter and Middleton (Ref. 39) incorporates concepts of the theory of statistical decision into the theory of signal detectability. Careful analysis reveals the fact that the theory of signal detectability is essentially a theory of behavior which might well apply to human behavior in detecting signals. The theory of signal detectability is the model for the decision-making theory of detection which is proposed below as an alternative to the threshold theory.

2.2.3 Development of Communication Theory. Shannon's (Ref. 31) Mathematical Theory of Communication has not been as directly applied as has the theory of signal detectability. Until this application is determined, if indeed it ever is, the theory serves a useful purpose as a guide. For example, the definition of entropy, and the transmission of information as a reduction in receiver entropy, is consistent with the analysis of forced-choice experiments. A two-alternative, forced-choice experiment is one in which the source entropy is one bit per symbol, while the four-alternative experiment has a source entropy of two bits per symbol. Shannon's theorem on encoding and channel capacity predicts a smaller error (in the information sense) for the smaller code for a given energy per symbol. For the case of orthogonal message ensembles[1], such

1. An orthogonal message ensemble is one in which each message is equally-likely and the probability of detection is the same for each message. Further, the errors are evenly distributed. Thus for an orthogonal ensemble, $(i = 1, 2, \ldots\ldots n)$, $P(i) = 1/n$, $P_i(i) = a$ for all i and $P_i(j) = b$ for all i and j, $i \neq j$, where a and b are constant.

as those encountered in the simple forced-choice experiment, information error is directly related to detection error.

2.2.4 Physiological Evidence. Spontaneous activity in the adapted optic nerve has been reported by Granit (Ref. 13), Skoglund (Ref. 32), Kuffler (Ref. 18), and Fitzhugh (Ref. 6). This spontaneous activity is similar to noise. Examination of recordings indicate that the intervals between impulses are random, and that the effect of signals might easily be duplicated by the random process alone. Also, the effects of signals appear to be random, and a study of these effects show that even fairly strong signals may result in impulse activity frequently duplicated by the noise alone. Indications from nervous system activity certainly suggest that the problem has all the aspects of that of detecting signals in noise.

2.3 Definition of Threshold Concept

It appears impossible to define the threshold concept in the usual manner without leaving unstated many of its implications. The definition here is developed rather than stated with the hope that this approach will leave the reader with a clear understanding of the meaning of this concept to the authors.

2.3.1 Loss of Information in the Decision-Making Process. Woodward and Davies (Ref. 41) show that the optimum receiver is one which records *a posteriori* probability. A device which divides events into classes may discard this information. In other words, if the output of the receiver contains only those events which surpass some cutoff level, an event which is not in the output is shown only to be in a class which fails to reach the cutoff. How close the event came to exceeding cutoff is not known. The information distinguishing between the members of this class is lost. The same loss occurs if all events surpassing the cutoff are merely classified in the output as being members of a class.

2.3.2 Threshold Device as a Decision-Making Device. A threshold device is essentially a decision-making device which loses information. It merely groups those events which fail to reach the cutoff as members of a class. The only information contained at the output of the receiver is that no event reaching the cutoff had occurred. How close an event came to the cutoff is not known. All that is known about a posteriori probability is that it is to one side of some critical value. Any more precise information is discarded by the threshold device.

According to the threshold concept as it exists in sensory theory, distinction between members of the class of events on the supra-threshold side of the critical value is preserved. This is consistent with the observation that intensity discrimination is possible.

2.3.3 The Meaning of a Correction for Chance in Interpreting the Threshold. The application of a correction for chance successes in a psychophysical experiment, using as an estimate of the guessing factor the false-alarm rate in a yes-no experiment, implies first that the threshold level is seldom, if ever, exceeded unless a signal is present. Almost all of the false alarms are assumed to be guesses occurring at some random rate in the absence of information. The guess is as likely to occur when an event is far from threshold as it is when an event is just short of threshold. Guessing is completely independent of the level of a posteriori probability upon which it might have been based had this information not been discarded by the threshold device.

The same argument can be used concerning the application of the chance correction to forced-choice data, in which the number of alternatives is used to determine the guessing factor. Here the justification for the application of the correction depends on the validity of the following conditions:

1) Information upon which to base a response is present a certain percentage of the time. Almost always this information leads to a correct response.

2) Only rarely is the threshold exceeded when a signal is not present. Otherwise there would be some cases when these events (which would lead to yes-no false-alarms) would exceed the threshold response resulting from the presence of a signal.

If these conditions are not met, the implication is that the observer has information not indicated by his choice. If, for example, the threshold response due to signal is exceeded by some other event, he makes an information-determined incorrect response. In this case, according to information theory, he would know his probability of being correct. He would also know that if he were not correct some other choice would have a greater probability of being correct than the other alternatives not chosen. Thus, if the second choice in a forced-choice experiment with three or more alternatives can be shown to contain information, then the chance correction and the threshold conditions implied by it, must be considered invalid.

2.3.4 Distinction between Sub-Threshold and Supra-Threshold Events.

Some experiments use psychophysical techniques which imply that there is a clear distinction between events which are less than threshold and events which are greater than threshold. The techniques referred to are those which permit the observer the knowledge that the signal is always present. The decision the observer is asked to make in these experiments is the level of sensory response necessary for him to have confidence that the signal is present in an experiment in which he did not have knowledge that it was, in fact, present. Unless the distinction between events which do or do not reach threshold level holds, it is difficult to justify such a procedure. Perhaps this procedure can be justified if thoroughly experienced observers are employed. However, in view of the data reported below showing the role of expectancy (a priori probabilities) and values and costs, factors which do not necessarily enter at the conscious level, every precaution must be employed to eliminate the effect of the observer's knowledge of the theory to be tested and his position or belief with respect to the a priori validity of the theory.[1] In spite of all this, it is possible that experienced observers can select consistent criteria as conditions change.

2.3.5 Threshold Variability. The authors interpret the term threshold variability as used in the literature to refer to random variation in the threshold or the sensitivity of the observer as a function of time. Under this term come short-time variations which might lead to the psychophysical function, and longer-term variations which might result from physiological conditions of the observer. This latter category includes variations which are a function of the time of day, season of the year, drugs, etc. If the experiment is performed over a short period of time, too short for physiological conditions to change significantly, then only the short-time variation should exist. Under these conditions, the threshold can be expressed as a random variable with a mean and a variance.

If the mean and variance of the threshold level are merely functions of time, then the authors consider this a fixed threshold. If, on the other hand, the measured threshold can be shown to vary as a function of knowledge

1. This sentence is not intended to cast aspersions on the integrity of anyone making such measures. It is rather intended to show that, if he behaves in the same way as those he studies, he must perform an exceedingly difficult task in order to control for variables ordinarily crucial to the judgement.

parameters, particularly the *a priori* probability of signal occurrence and the values assigned to the four possible results of a yes-no decision, then the concept of a fixed threshold may be rejected. It is the authors' opinion that, in the past, most experiments have been conceived in terms of a threshold which is fixed according to the definition given above.

2.4 An Alternative Theory; the Decision-Making Theory of Detection

The bases for the alternative theory are the recent developments in the theory of statistical decision, or the theory of testing statistical hypotheses, and the mathematical theory of signal detection. For an adequate treatment of the mathematical developments the following are recommended: Peterson, Birdsall, and Fox (Ref. 26) for the theory of signal detectability, and Wald (Ref. 40) and Bross (Ref. 3) for statistical decision theory. For the application of these mathematics to the sensory detection problem the reader is referred to Tanner and Swets (Ref. 38) and to Swets, Tanner, and Birdsall (Ref. 35) who treat the visual case of the signal-known-exactly. The application to the auditory case is exactly the same as the application to the visual case. Because of the identity of the two cases, and because the visual application is available elsewhere, the treatment below attempts to present only the essentials.

2.4.1 Basic Assumptions. Certain of the assumptions involved in applying the mathematics of decision-making to the problem of sensory detection are of such direct relevance as to require explicit expression. These are listed below as postulates. Included among these are some assumptions that, although general to the problem of making decisions, are regarded as worthy of mention in this connection; these comprise the first three postulates.

Postulate 1: An individual, in making a decision, chooses one alternative from a finite set of alternatives.

Postulate 2: There is associated with each alternative in the set an *a priori* probability greater than zero.

Postulate 3: The sum of the *a priori* probabilities associated with the alternatives in the set is one.

In the interests of continuity of presentation of the sensory-detection problem, these more general postulates, applicable to decision making in any context, and the reasons for including them here, are elaborated in the appendix. The remaining postulates are specific to the sensory-detection problem; these are discussed along with their presentation.

In the sensory detection problem, the set of alternatives considered by the observer in the decision process is the set specified in the experimental design. The alternatives in this set are related to the existence and nature of signal events which may occur in the environment and impinge upon receptor organs. In terms of the experimental procedure, each of the alternatives is presented to the observer several times; each time the observer is asked to state which alternative occurred.

Postulate 4: Information concerning the end organ activity, that results from the alternative that occurred in any given instance, is transmitted over the sensory systems to higher centers, where it combines with *a priori* information to serve as the basis for the decision.

Postulate 5: The information transmitted by the sensory systems takes the form of a number or "measure", x, that is descriptive of the alternative that occurred. The members of the set of possible measures lie along a continuous axis.

This axis is called a decision axis, since the numbers on this axis constitute the sensory basis for making decisions. Although the sensory information transmitted as a result of the occurrence of any alternative is undoubtedly complex, it can be described by a single number. For no matter how complex the informational event, it can be represented by a point in an n-dimensional space. This space, in turn, can be mapped onto a line by the simple expedient of considering a single, but sufficiently descriptive property, of each informational event represented in the space, namely, the probability that the event will result from each of the possible alternatives. This conception is particularized in the next postulate.

> Postulate 6: There is an hypothesis associated with each alternative in the set. The hypothesis is the conditional probability density function that yields the probability that the measure x results when the alternative exists.

The theory of testing statistical hypotheses is a special case of the mathematics of decision-making. The predictive power that stems from applying these mathematics to sensory problems has its basis in the following assumption.

> Postulate 7: The human observer, in choosing one alternative from a set, or equivalently, in deciding which one of a set of statistical hypotheses to accept, uses an optimum process.

Several implications of this postulate have been supported in psychophysical tests involving fixed observation intervals. It has been demonstrated, for example, that the human observer attempting to detect visual signals is capable of performing in accordance with each of five different definitions of optimum (Ref. 35). Some of these definitions of optimum concerned decisions between the two hypotheses of a yes-no test; others applied to decisions among the four hypotheses involved in four-alternative, forced-choice test. Because of the relatively large body of evidence existing in the visual case, there has not been a comprehensive attempt to test implicates of Postulate 7 in the auditory case. An audition experiment reported in Section 2.6, however, shows the human observer to be capable of approaching the behavior specified by a single definition of optimum for the two-hypothesis test. In another paper, audition experiments are reported in which the observer's decision behavior compared favorably with that specified as optimum in two variations of the three-hypothesis test; in one of these the set of alternatives consisted of Signal 1, Signal 2 (of a different frequency) and No Signal, whereas in the other the observer decided among the three hypotheses associated with a three-alternative, forced-choice-in-time task (Ref. 34).

The next postulate is, in one sense, specific to the sensory decision problem in which the alternatives are distinguished only by differences in signal power.

> Postulate 8: The average value of x (the measure on the decision axis) for any hypothesis is a monotonic increasing function of the signal power of the associated alternative.

The word "average" is used in this postulate because it is believed that a constant value of the measure will not result from the occurrence of the "no-signal" alternative. Rather, variation in the measure as a function of time will be the rule. That is to say, the sensory channels are noisy; they contribute unwanted, random components to the sensory information that are not qualitatively distinguishable from the components resulting from environmental signals. And so, the next postulate:

> Postulate 9: The measure entering the decision when the "no-signal", or "noise-alone", alternative occurs is a random variable.

The nine postulates listed above are basic to the application of the mathematics of decision-making to sensory detection and recognition problems. This statement should be qualified to the extent that Postulates 8 and 9 are restricted to certain sensory problems. Postulate 8 does not apply exclusively when, for example, the decision is a choice among two signals differing in frequency, the case studied by Tanner (Ref. 36). It does, however, apply when in certain instances when the "no-signal" alternative is not in the observer's set. This last statement is not true of Postulate 9.

Certain other assumptions made are tentative; they are made because some assumptions of this type are necessary if calculations of aid in prediction are to be performed. Thus, Postulates 10 and 11 below are not in the specific form they take here, essential to the application of decision-making theory to sensory detection problems.

> Postulate 10: The conditional probability density function (the distribution of measures along the decision axis) of any signal alternative results from adding a constant to the density function of the "noise-alone" alternative.

The density function of the "noise-alone" alternative will be called the noise distribution, or simply, N; the density function associated with a signal will be called the signal-plus-noise distribution, SN. The important consequence of Postulate 10 is that N and SN are of equal variance. The size of the constant which is added to the noise distribution by a signal is determined by the functional relation of the measure and signal power. This relation is only specified as monotonic increasing (Postulate 8); it can be determined more precisely from the experimental data.

> Postulate 11: The distribution of measures of noise alone on the decision axis is a Gaussian distribution function.

This, with Postulate 10, implies that the hypothesis associated with any alternative is a Gaussian distribution function with a variance equal to noise variance. This concept is depicted in Figure 1. The conditions of this postulate are met if there exists a monotonic transform of the measured variable into a decision variable which meets the conditions of Postulate 11.

1970 A-63-119 JEM 4-19-54

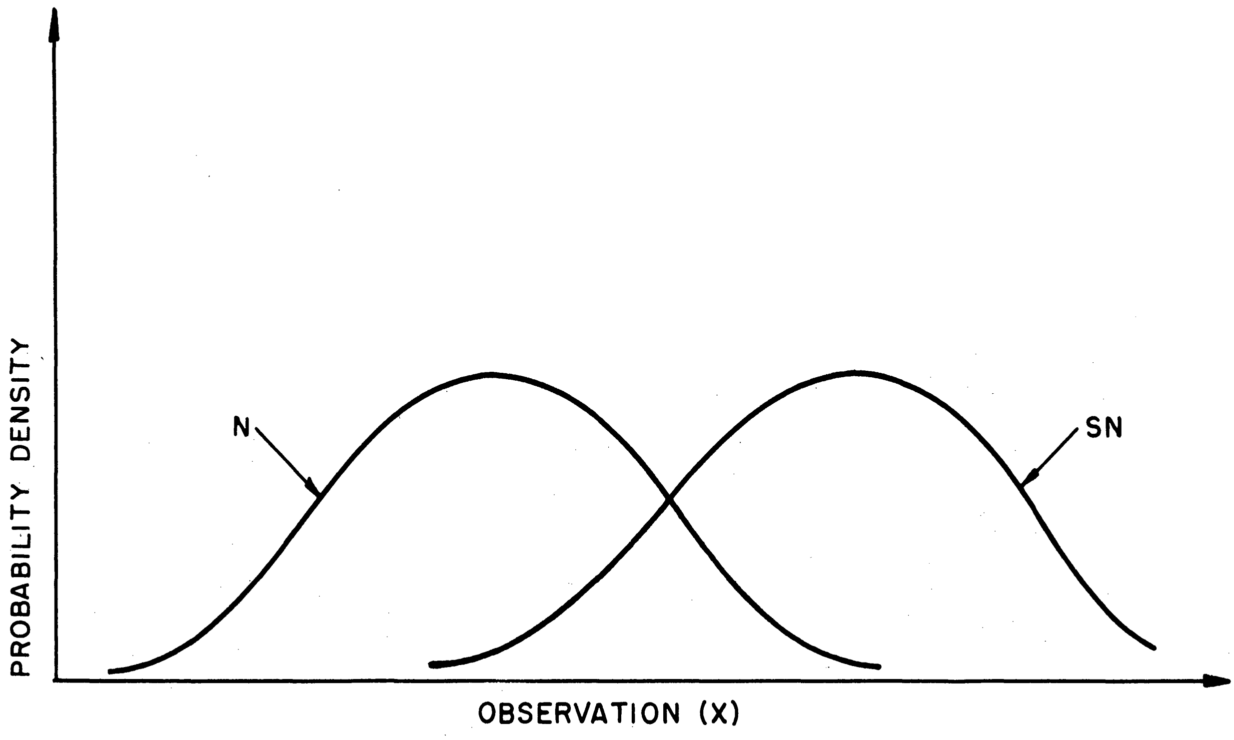

FIG. 1 DISTRIBUTIONS OF NOISE AND SIGNAL-PLUS-NOISE

In order to illustrate the restriction so placed, assume that the measured variable is a product-moment correlation bounded by the values -1 and +1. A transform of this axis is Fisher's Z. Experiments of the type reported below describe the decision axis as if the measures were in terms of Fisher's Z whether or not this is in fact the axis. The actual decision could be made on a correlation axis, or an energy or voltage term which would specify the correlation whether or not this correlation is determined. Thus, the assumptions of normal statistics and equal variance are good assumptions if any transform of the measured variable approximates the conditions.

2.4.2 Definition of d'. To summarize, we have assumed that the absence of a signal results in a distribution of values of the measure on the decision axis. The addition of a signal simply moves the mean of this distribution to some greater value of the measure. Let d' be defined as the difference between the means of the noise and the signal-plus-noise distributions, divided by the standard deviation of the noise distribution; that is,

$$d' = \frac{M_{SN} - M_N}{\sigma_N} \tag{1}$$

It may be noted that d' is a dimensionless quantity; it does not, for example, refer to any neural quantity, but is simply a number associated with each signal.

2.4.3 The Determination of a d' in a Forced-Choice Experiment. In order to determine a value for d' it is necessary to describe how the observer's behavior is conceptualized. In the forced-choice task the observer listens in n time intervals, only one of which contains a signal. The observer's task is to say in which of the n time intervals he believes the signal to have been presented. Conceptually, it is supposed that the observer takes a measure x, in each of the n intervals and selects the temporal interval in which the largest value occurs.

It is intuitively obvious that if the signal produces a large shift in the noise distribution, the probability that the largest value of x will be obtained in the interval that contains the signal is also large. In particular, the probability that the observer will choose the correct interval is a monotonic function of d'.

For the case of Gaussian distributions, with equal variance for all hypotheses, the probability of a correct choice in a four-alternative, forced-choice situation is given by Eq 2.

$$P(c) = \int_{-\infty}^{+\infty} \left[F(x)\right]^{3} g(x)\, dx \qquad (2)$$

where F(x) is the area of the noise distribution to the left of x, and g(x) is the density function due to signal-plus-noise. It may be noted that, since the signal-plus-noise density function is simply the noise density function shifted by d' (i.e., $g(x) = f(x-d')$) Equation 2 is actually a function of d'. Thus d' may be defined in a forced-choice experiment by simply determining a percent correct for some signal power and then using Equation 2 to determine d'. The graph of d' versus percent correct is shown in Figure 2.

2.4.4 The Determination of d' in a Yes-No Experiment. The determination of d' by a forced-choice methodology is not the only method available. The decision-making theory gives an explicit description of the inferred behavior of an observer in a yes-no experiment. In a yes-no situation, the observer presumably takes a measure x in the observation interval. His task is that of testing statistical hypotheses; he must decide whether x is more representative of signal-plus-noise or noise alone. In many respects, this is similar to the problem a researcher faces when he must decide whether a result is attributable to the manipulation of the independent variable rather than to chance. The first step is to determine a critical region, that is, a set

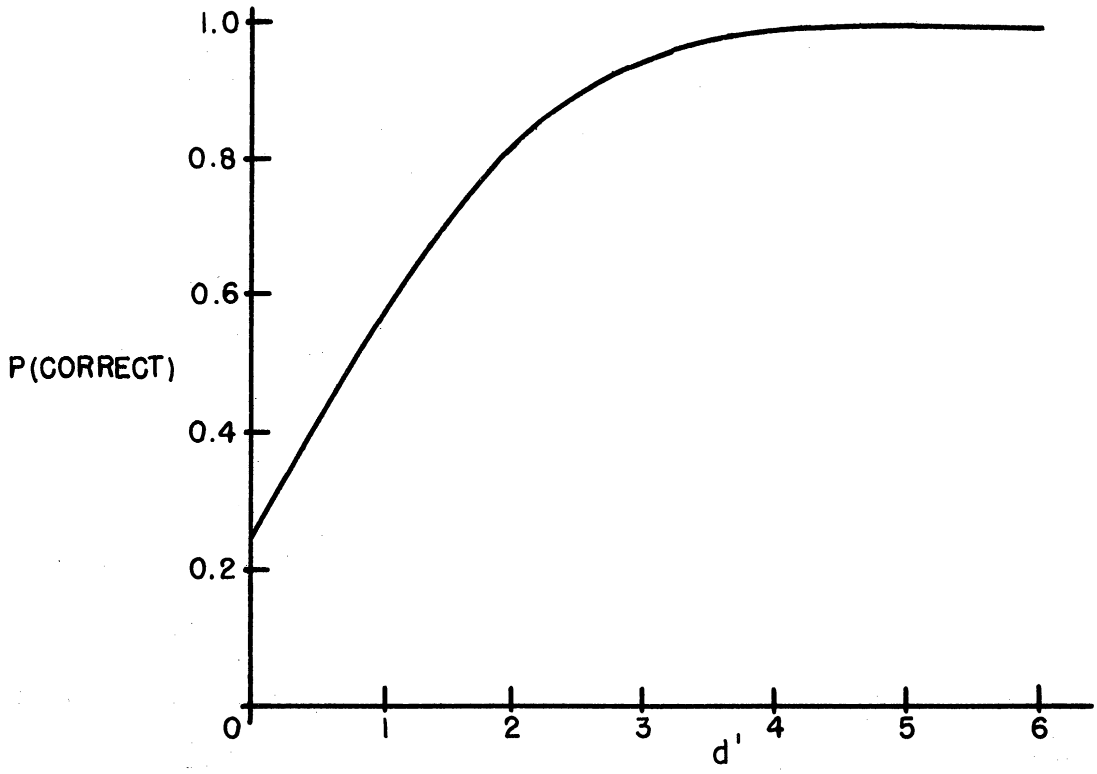

FIG. 2 P(C) AS A FUNCTION OF d'—A THEORETICAL CURVE

of values not likely to occur as a result of chance fluctuation. Analogously, according to the theory, the observer sets a cutoff level such that if a value of x exceeds this level he will report the presence of a signal, and if the value is less than this amount he will report that no signal exists. Consider any point on the decision axis at which the observer will accept the signal alternative. Then for every value on the decision axis greater than this point he should clearly accept the signal alternative, by the sixth postulate. Thus the determination is essentially the establishment of a cutoff level. The assumption, which might well have been a part of the postulate set, is made that the observer is rational; that is, if in the course of an experiment characterized by a single cutoff level, the same measure occurs more than once, the same behavior will result on each occurrence.

Where the cutoff point will be established depends upon the values and costs in the situation, the a priori probability of the signal and the magnitude of the signal. These factors and their determination of the cutoff will be discussed in the next section. First consider how a particular choice of the cutoff determines the conditional probability of saying "yes" when there was no signal, $P_N(A)$, and the conditional probability of responding affirmatively when the signal is in fact presented, $P_{SN}(A)$. The subscripts SN and N here refer to whether signal-plus-noise or noise alone was present.[1] The A may be read "Acceptance of the signal alternative." It is intuitively clear that for a very strong signal (i.e., a signal which yields a large d'), there exists a cutoff such that the associated $P_{SN}(A)$ is large and $P_N(A)$ is small.

The cutoff value determines a point on the decision axis such that all values above this point will be accepted. Thus

1. SN and N may conveniently, and without confusion, refer both to the distributions along the decision axis, as stated earlier, and to events comprising the occurrence of a signal or noise alone.

$$P_N(A) = \int_{x=c}^{+\infty} f(x)\, dx \qquad (3)$$

and

$$P_{SN}(A) = \int_{x=c}^{+\infty} g(x)\, dx \qquad (4)$$

where $f(x)$ is the Gaussian density function of the noise and $g(x) = f(x-d')$, or the density function of the signal-plus-noise, and where c is the cutoff value of x. It is instructive to consider the relation of $P_N(A)$ and $P_{SN}(A)$ for various values of the cutoff. These relations are plotted on Figure 3 with d' as the parameter. It may be seen from this curve that $P_N(A)$ and $P_{SN}(A)$ are not independent. Further, from the different values of d' plotted it may be seen how an increase in the signal power effects the two probabilities. This curve was recommended by Peterson and Birdsall (Ref. 26) and is called the Receiver Operating Characteristic (R.O.C.) curve. If the axes are plotted on probability scales the curve is a straight line. This is shown in Figure 4.

By inducing the observer to shift his cutoff level, several values of $P_N(A)$ and $P_{SN}(A)$ may be determined. A straight line through these determined values, when plotted on probability paper, then determines a value of d' associated with a given signal energy. If the experimentally determined points actually fall near a straight line, this indicates the reasonableness of the eleventh postulate. Further, a comparison of the values of d' determined in the forced-choice and yes-no methodology for a given signal power provides a method of checking the internal consistency of the theory.

2.4.5 The Determination of the Cutoff Point. In the previous sections it has been observed that for each value of signal power there is a corresponding d'. This d' in a yes-no situation determines a series of values of $P_N(A)$ and $P_{SN}(A)$. The observer, by establishing a certain cutoff point, selects from these many possible values a particular pair of values for $P_N(A)$ and $P_{SN}(A)$.

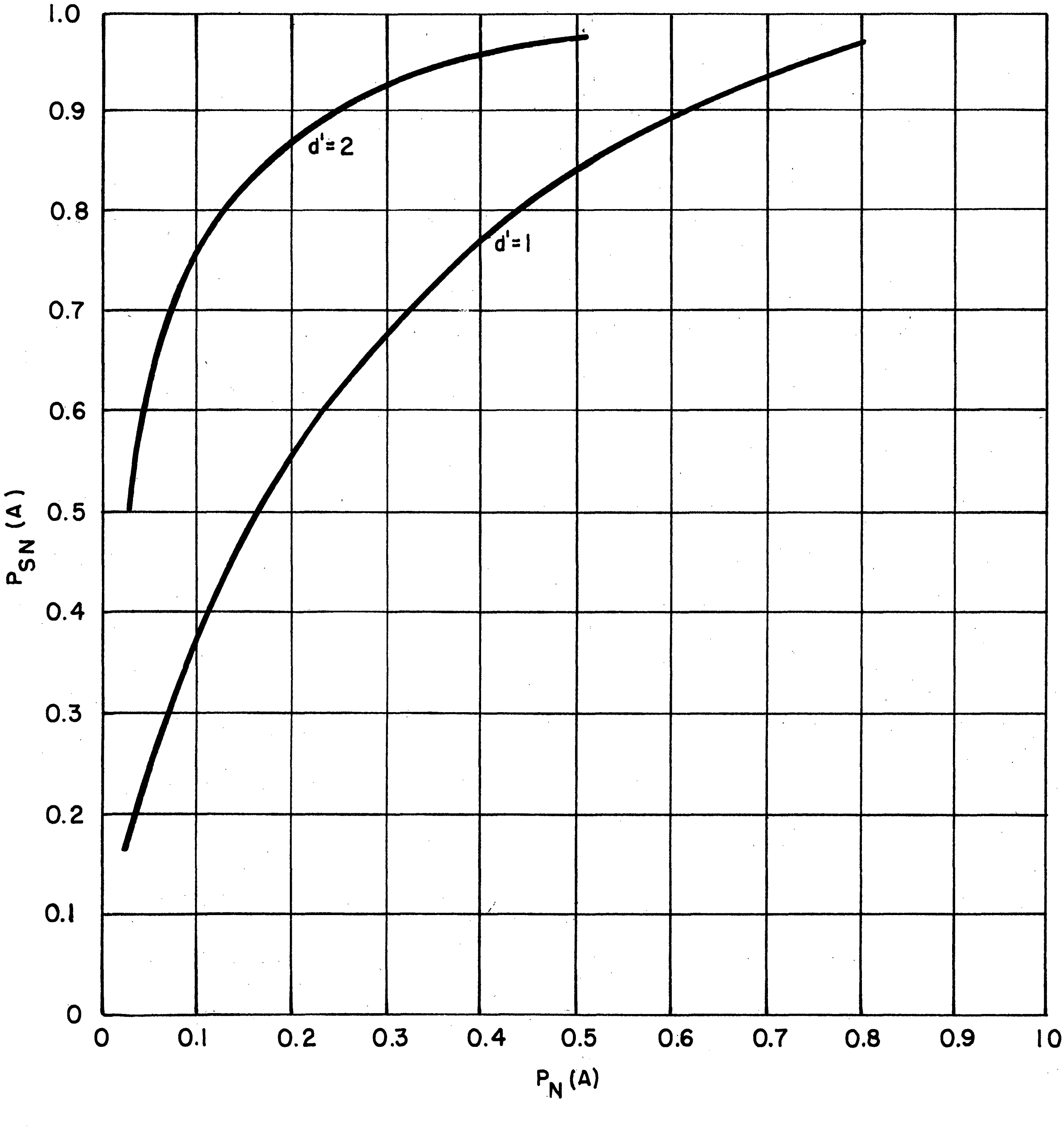

FIG. 3 $P_{SN}(A)$ VS. $P_N(A)$ WITH d' AS THE PARAMETER

2262 A-63-214 HJF 8-9-55

$\sqrt{d} = d'$

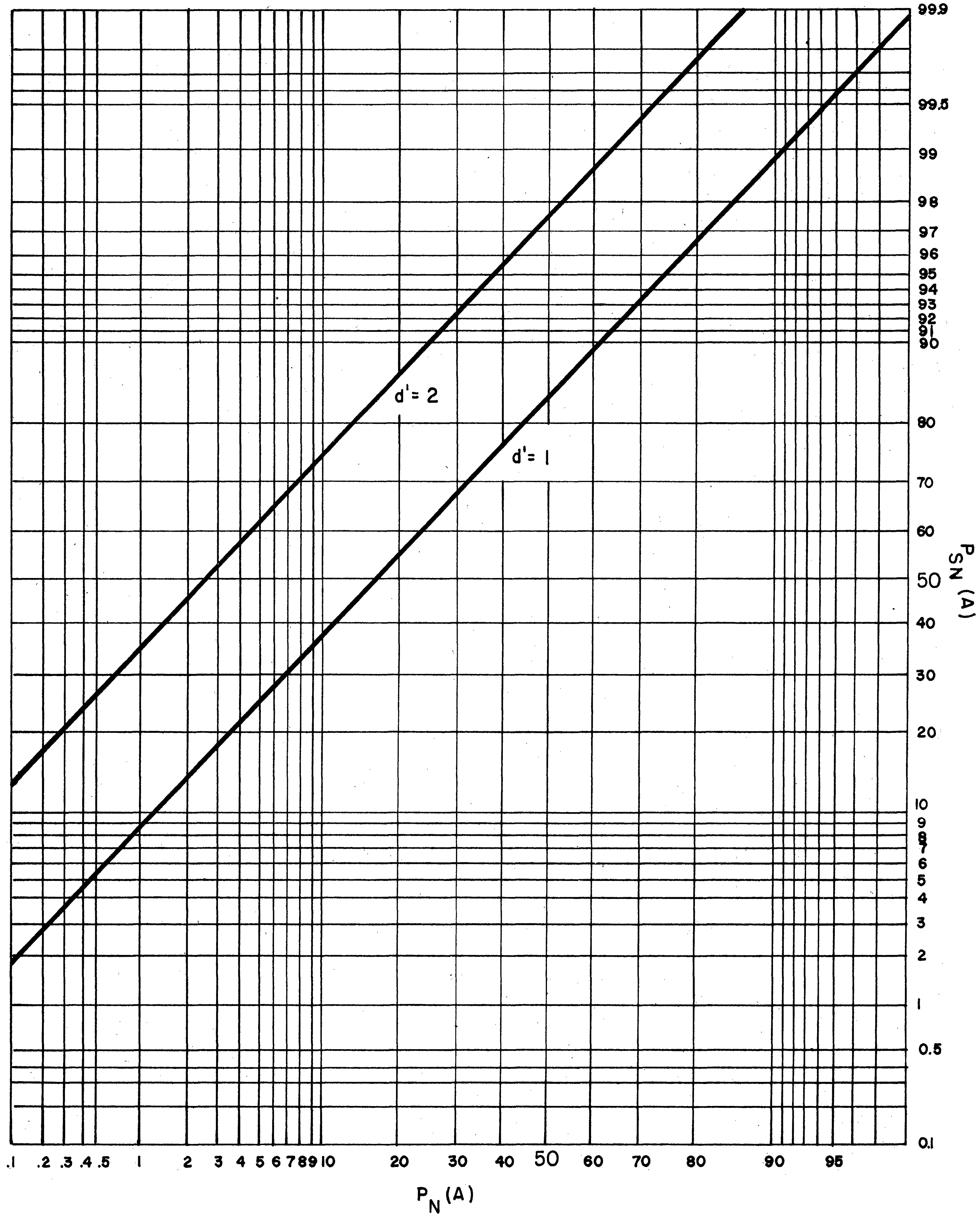

FIG 4. $P_{SN}(A)$ VS $P_N(A)$ ON PROBABILITY AXES

Given a definition of optimum, it is possible to specify what value of the cutoff is most advantageous for each situation. The type of optimum which the authors feel is most reasonable for an observer to adopt in this situation is a procedure which maximizes the expected value of the response, expected value being defined as the product of the probability of an event and the value associated with the event summed across the possible events. In a yes-no experiment there are four possible events or outcomes of the decision. Listed below is the notation used to specify the probability of the event and the value associated with each event.

Event:	Notation for Probability of the Event	Value or Cost Associated with the Event
If the signal is presented, the observer says "yes"	$P_{SN}(A)$	$V_{SN \cdot A}$
If the signal is not presented, the observer says "no"	$P_N(CA)$*	$V_{N \cdot CA}$
If the signal is presented, the observer says "no"	$P_{SN}(CA)$	$-V_{SN \cdot CA} = K_{SN \cdot CA}$
If the signal is not presented, the observer says "yes"	$P_N(A)$	$-V_{N \cdot A} = K_{N \cdot A}$

For an incorrect response, a negative value is assigned as indicated in the table. Thus, in any yes-no experiment the observer is presented with four numbers representing the values and costs of the various outcomes of a response. This is called the risk function. It is intuitively obvious that if the value associated with saying "yes" when the signal is presented ($V_{SN \cdot A}$) is very large and the other three values are equal, then it is advantageous to adopt a liberal cutoff point. This will achieve a large probability of saying "yes" when the signal is presented, $P_{SN}(A)$, but, of course, increases the probability of saying "yes" when no signal is presented, $P_N(A)$. This may be seen in Figure 3.

* CA denotes the converse of accepting, that is, saying "no".

Nevertheless, because of the large value associated with $P_{SN}(A)$, this would achieve the greatest overall gain.

Another important determinant in the adoption of the cutoff point is the *a priori* probability of the signal being presented. Consider a situation in which the four values in the risk function are all equal. If the observer is told that the signal will be presented in ninety percent of the trials it is clear that he should set a liberal cutoff point. This is true because although the values and the costs are the same for all responses, there will simply be more times when the signal is presented and hence more opportunities for the observer to say "yes" when the signal is presented. If, in this same situation, the observer is told that the *a priori* probability of the signal is one tenth, he should adopt a very conservative cutoff. Peterson and Birdsall (26) have derived a determination of the cutoff point so that the expected value of the response is maximized. Specifically, maximizing the expected value is equivalent to requiring that

$$P_{SN}(A) - \beta\, P_N(A) \text{ is a maximum} \tag{5}$$

where

$$\beta = \frac{1 - P(SN)}{P(SN)} \quad \frac{V_N \cdot CA + K_N \cdot A}{V_{SN} \cdot A + K_{SN} \cdot CA} \quad \text{and}$$

where $P(SN)$ is the *a priori* probability of the signal occurrence. β may be shown equal to the tangent of the ROC curve (Figure 3) at the point of optimum behavior; thus, the value of β can be used in a graphic determination of the $P_N(A)$ and $P_{SN}(A)$ the observer should maintain to maximize the expected value.

<u>2.4.6 Implications of the Decision-Making Theory.</u> One of the immediate implications of this theory is that the observer must have knowledge of the signal to perform optimally in a yes-no experiment. Consider a situation where the values and *a priori* probabilities have been assigned. These numbers

determine a β. However, as shown in Figure 3, the amplitude of the signal, together with the value of β, determines the optimum values of $P_{SN}(A)$ and $P_N(A)$. Thus, the observer must be well acquainted with the amplitude before he can be expected to behave optimally. Since this type of situation demands a complete knowledge of the physical parameters (i.e., frequency, amplitude, starting time and the duration of the signal), Peterson and Birdsall have termed it "signal-known-exactly". In this type of situation, Peterson and Birdsall have determined that a mathematically ideal receiver could, at best, obtain a $(d')^2$ equal to twice the signal energy divided by the noise power per unit bandwidth. That is,

$$(d'_{opt})^2 = \frac{2E}{N_o}$$

The observers' approach to the optimum d' in a yes-no experiment is discussed in Section 2.8 below.

2.4.7 <u>An Experimental Comparison of the Two Theories.</u> One of the most direct ways to compare the decision-making and threshold theories is to investigate the relation of $P_N(A)$ and $P_{SN}(A)$. The threshold theory assumes that a change in the false-alarm rate, $P_N(A)$, merely reflects a change in the tendency to guess. It may be remarked that the threshold theory assumes that $P_N(A)$ is usually a small number. Since these guesses are regarded as occurring at the same rate when a signal is presented and when no signal is presented, $P_N(A)$ may be taken as an index of the probability of a guess occurring.

Let the probability of a correct "yes" response be p', the probability of a true, "sensory-determined" "yes" response, p, and the probability of a correct guess, c. Then according to threshold theory, we may write the rational equation

$$p' = p + c(1-p); \tag{7}$$

the total probability of a "yes" response is equal to the probability of a sensory-determined positive response plus the probability of a correct guess, the latter occurring only on trials when the signal is not "heard" and therefore modified by (1-p). This equation may be expanded to assume the form

$$p' = p+c-cp. \tag{8}$$

According to one of the first rules of probability theory, the probability of the union of two events, in this case the probability of a correct "yes" response of either kind, (p'), is equal to the probability of one plus the probability of the other (p+c), minus the probability of the joint occurrence of the two events. The probability of the joint occurrence of a sensory-determined positive response and a correct guess is seen by Eq 8 to be equal to the product of their respective probabilities, and hence, by an elementary rule of probability, the correct guesses and "sensory-determined" responses are independent events. The validity of the use of the correction for chance or guesses depends upon this independence, since the correction formula

$$p = \frac{p'-c}{1-c}, \tag{9}$$

is simply a transformation of the rational equation, Eq 7.

The percentage of correct "yes" responses may be used as an estimate of p', or of $P_{SN}(A)$ in the decision-making theory notation, and the percentage of "yes" responses when no signal was presented may be used as an estimate of c, or $P_N(A)$. It can be seen from Eq 7 that, according to threshold theory, $P_{SN}(A)$ and $P_N(A)$ are linearly related. Figure 5 shows this relation graphically. The parameter of this graph is p, the probability of a "sensory-determined" positive response. This p should become larger as the signal intensity is increased. If the chance correction is applied to the points along any of the lines in Figure 5, the resultant line will be horizontal, demonstrating the

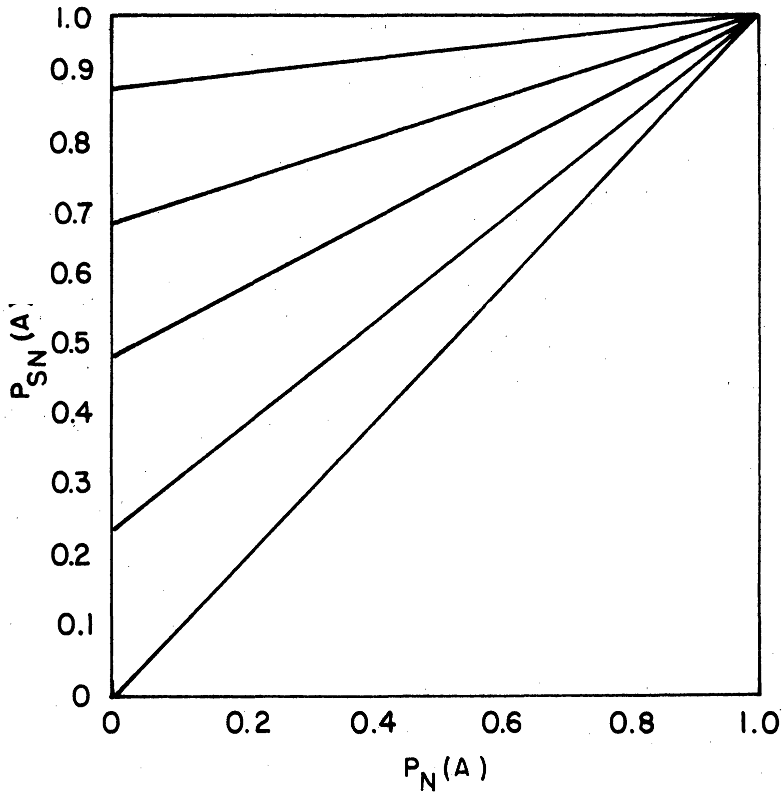

FIG. 5 $P_{SN}(A)$ VS $P_N(A)$ UNDER THE THRESHOLD ASSUMPTION

2262 A-G3-216 HJF 8-8-55

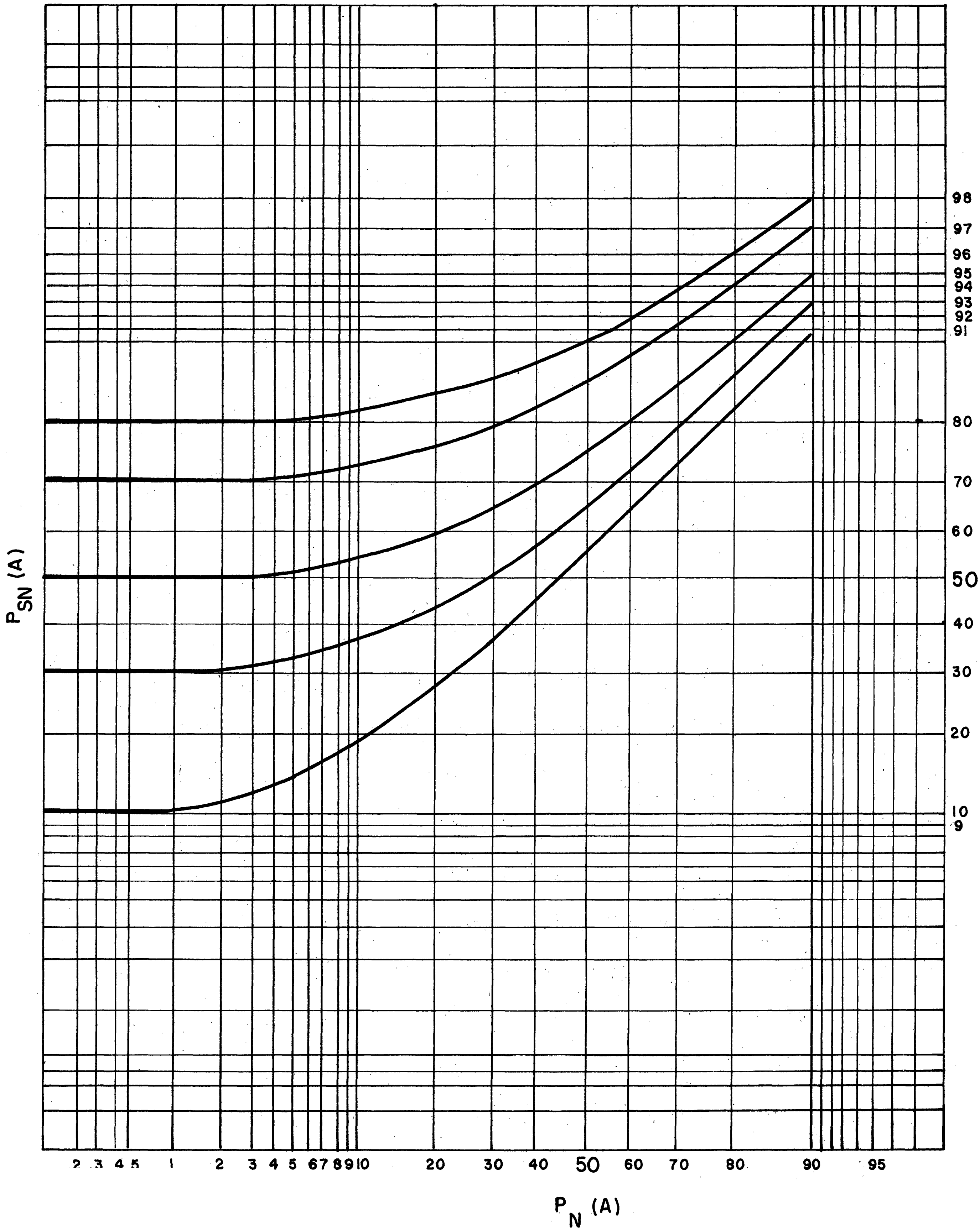

FIG. 6 THE FORM OF P_{SN}(A) VS P_N(A) ON PROBABILITY AXES ACCORDING TO THE THRESHOLD ASSUMPTION

assumed independence of $P_N(A)$ and $P_{SN}(A)$. Figure 6 shows the same graph as Figure 5, with probability axes. Figures 5 and 6 may be compared with Figures 3 and 4 to contrast the two theories. One test of the two theories is to induce the subject to vary the percentage of "yes" responses given when no signal is presented. The shape of the plot of $P_{SN}(A)$ versus $P_N(A)$ plot should distinguish between the theories.

A stronger test of the theories is obtained, however, by determining the correlation of c and p. As discussed above, the threshold theory assumes that the proportion of "sensory-determined" correct "yes" answers and the guessing factor, c, are independent. Thus, in the situation where the observer is induced to change the percentage of "yes" responses given when no signal is presented, the threshold theory must claim that no relation will exist between p and c. The decision-making theory does not recognize a "sensory-determined" correct answer since the theory does not recognize a threshold, as usually conceived.

One might take a curve such as Figure 3, the theoretical ROC curve for the decision-making theory, and use $P_N(A)$ and $P_{SN}(A)$ as values for c and p' respectively in Equation 7. By solving Equation 7 for p, one would see a correlation between p and c. That is, the decision-making theory does not hold that a false "yes" response reflects a guessing mechanism which works independently of the detecting mechanism. Thus, according to the threshold theory, c and p will show no relation, whereas according to the decision-making theory a relation should exist. The decision-making theory also predicts that the d' obtained in the yes-no experiment is the same as one estimated from the forced-choice experiment where the same signal power is employed. No attempt is made in this paper, however, to demonstrate that the estimates of d' from yes-no and forced-choice experiments are identical for all values of signal power. Rather the emphasis in this paper is placed on the demonstration that the observer can apparently shift the cutoff value to various values, and does not have a single sensory threshold in the usual sense of the word.

2.5 Experimental Procedure

Two types of experiment are employed. In the yes-no experiment the observation interval is defined for the observer by the flash of a light. The observer is then asked to state whether or not a signal existed. In the forced-choice experiment, the light flashes four times, marking four intervals. The signal is known to exist in one of four intervals, and it is the observer's task to state in which of the four intervals it occurs.

The experiments are programmed by N. P. Psytar (Ref. 28). The title of the apparatus is a condensation of the phrase Noise Programmed Psychological Tester and Recorder. The machine programs the experiment by sampling a noise source; if the voltage of the sample exceeds a specified value, a thyratron is triggered and a signal is presented. The probability of the voltage triggering the thyratron is controlled by regulating the bias level on the thyratron.

In the yes-no experiment, if the thyratron is triggered the signal is presented, if not, noise alone is presented. In the forced-choice experiment, where it is desired to have each of the intervals equally likely to contain the signal, the first interval is set at a probability of .25. If the thyratron triggers, it blocks the sampling of the remaining intervals. If not, the probability of the second interval triggering is .333. Again, triggering blanks the remaining intervals. The process is repeated, with the third interval set with the conditional probability equal to .500, and the fourth with the conditional probability equal to 1.00.

The presentation information is transferred to an answer circuit. The observer indicates his answer by pushing a button. If he pushes the correct button, the answer is recorded either as a correct "yes" or correct "no" answer in the yes-no experiment and he is informed of the correctness of his answer by the flash of a light. In the forced-choice experiment he again answers by pushing a button. If he is correct, it is merely recorded as a correct answer, and again he is informed by a flash of light.

The observer has before him a set of three lights which inform him of the progress of the experiment. The first light is a warning light. When this light blinks the observer knows a test cycle is beginning. The second light blinks in coincidence with the four observation intervals in a forced-choice experiment or the single interval in a yes-no experiment. The last light is an answer light. When this light is on, the observer is expected to record his answer by means of his answer buttons. The answer buttons record only during the time the answer light is on, and only the first button pushed during this time is recorded. In other words, once the observer pushes a button during the answer period, he has comitted himself to an answer, and the correctness information is fed to him.

2.5.1 Randomness of N. P. Psytar. One of the purposes of constructing N. P. Psytar in the manner described is to achieve an automatic programming device which offered the possibility of achieving trial cycles which are indeed independent. Proof of such an achievement is virtually impossible through the application of any reasonable statistical technique. The problem can be restated in this fashion: the null hypothesis is easier to reject than accept. Large numbers of trial cycles have been run, followed by correlation analysis. The resulting distributions look not unlike those which one would expect if independence exists. This observation, along with the logical argument that the programming is achieved by sampling a noise source, presents a strong argument for accepting the independence of the trial cycles. The logical argument is necessary to the total argument which leads to a conclusion amounting to the acceptance of the null hypothesis.

2.5.2 Signal and Noise Generation and Measurement. Throughout the experiments conducted, one of two signal sources is employed. These are Hewlett-Packard audio-oscillators, models 200-AB and 200-I. Two different noise sources were also employed, a General Radio 1390A random noise generator and a Scott 811A random noise generator. In any single experiment, however, neither the signal generator nor the noise generator is changed.

A block diagram of the signal and noise circuits is shown in Figure 7. The signal from the audio oscillator is fed into a gate circuit, the output of which is fed to an adder. The gated signal starts at zero voltage and contains an integral number of cycles. From the adder this is fed to a Williamson amplifier, and then to the observers' PDR-8 earphones. The noise free signal at the input to the observers' PDR-8 earphones is compared with the acoustic output using a 6 cubic centimeter coupler.

The noise is fed from the noise generator to the adder, to the Williamson amplifier, and then to the earphones. All measurements throughout the experiments are made at the input to the earphones. The signal is measured on a Hewlett-Packard 400B average reading voltmeter calibrated in RMS voltage, while the noise is measured both on the 400B, and on a 400D with a 160 cycle bandpass filter in the circuit leading to the 400D. The purpose of the dual measure on the noise is to permit detection of spectrum changes. A measure of the overall power of the noise varies with spectrum changes, and consequently is not a good indication of noise power per unit bandwidth (denoted hereafter

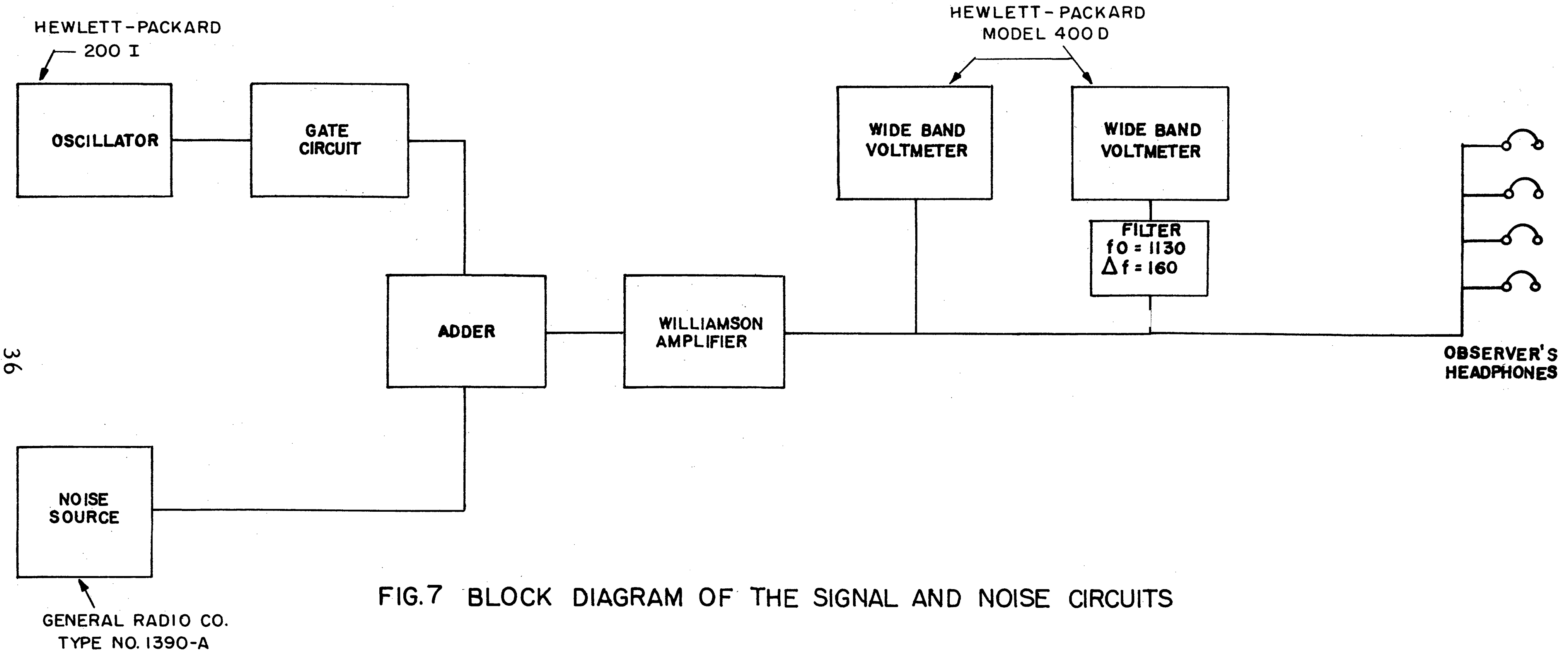

FIG. 7 BLOCK DIAGRAM OF THE SIGNAL AND NOISE CIRCUITS

as N_o). N_o is the more significant of the two measures. Actually, the only spectrum changes in the noise occurred between experiments using different noise sources. Thus, the measurement of N_o is of further importance in permitting comparison of results from different experiments.

2.6 The Experiments

2.6.1 The Relation Between $P_{SN}(A)$ and $P_N(A)$. Two observers were employed. Only one signal level and one noise level were used. Forced-choice experiments and yes-no experiments were conducted at this signal level. The forced-choice results are presented in Table I and the yes-no results in Table II.

Four-alternative, forced-choice experiments were used. Approximately one hundred observations were made without interruption. The obtained percentage correct was converted to d' by Figure 2. The last row is the average over the sessions and therefore represents the best estimate of d'. The observers were paid 1/2 cent for each correct response in the forced-choice experiments.

It should again be noted that there are four events which may occur in a yes-no experiment. (These are listed on page 26). The values and costs for the four events were held constant through the experiments, each at one cent. The *a priori* probability of the signal occurrence was announced before each session. The *a priori* probability for any session may be seen in Table II. Two sessions were conducted at each of five values of *a priori* probability, with 300 observations being made in each of the ten sessions. It can be seen that the observers changed the percentage of "yes" responses when no signal was presented in accordance with the announced *a priori* probability of the signal. As a matter of fact, the rank-order correlation between $P_N(A)$ and β (see Eq 5) is equal to unity in the case of each observer. If, in addition, c and p as defined in Eq 7 are found to be highly correlated, the observers' use of the optimum process in deciding between the two alternatives is demonstrated.

TABLE I

	No. of Observations	Observer 1 Proportion Correct	Observer 1 d'	Observer 2 Proportion Correct	Observer 2 d'
	99	.62	1.22	.58	1.09
	99	.55	1.00	.49	.80
	99	.64	1.28	.63	1.26
Totals	297	.60	1.16	.56	1.03

TABLE II

No. of Observations	A priori Probability of a Signal P(SN)	Observer 1		Observer 2	
		$P_N(A)$	$P_{SN}(A)$	$P_N(A)$	$P_{SN}(A)$
300	.10	.05	.47	.13	.37
300	.10	.02	.29	.05	.29
300	.30	.10	.58	.22	.53
300	.30	.16	.59	.19	.48
300	.50	.34	.75	.46	.82
300	.50	.37	.69	.34	.60
300	.70	.53	.83	.54	.78
300	.70	.47	.83	.50	.79
300	.90	.74	.95	.59	.88
300	.90	.85	.97	.79	.96

Figures 8 and 9 present the estimated $P_N(A)$ and $P_{SN}(A)$ for the yes-no experiments. Three hundred observations were used to determine each experimental point. On probability axes, the decision-making theory predicts the experimental points should fall along a straight line, having a slope of one if the assumption of equal variance of noise and signal-plus-noise distribution is valid. The interception of this line and the scale of d' along the ordinate represents the detectability of the signal. It should be remembered that the signal power was constant throughout the experiment; only the a priori probability of the signal was varied. The family of curved lines represent the expected form of the data assuming the threshold theory. One of the lines should theoretically fit all the data.

It is apparent from Figures 8 and 9 that the decision-making theory is more successful in representing the form of the yes-no data. In each case, the points are well fitted by a straight line. The points in Figure 8 are best fitted, however, by a line with a slope of less than one. Such a slope would be predicted by the decision-making theory if the variance of the signal-plus-noise distribution increased with increases in its mean. It may be noted that the estimates of d' from the yes-no data, assuming equal variance, are very similar to the estimates of d' from the forced-choice experiment; 1.16 compared with 1.16, and 0.92 compared with 1.03 for the two observers.

The final test of the two theories is to determine the correlation between c and p of Eq 7. The rank-order correlation was .91 for Observer 1 and .94 for Observer 2. Both correlations are highly significant, having associated probabilities of much less than .001. The magnitude of this correlation strongly suggests that the threshold theory is incorrect and supports the decision-making theory.

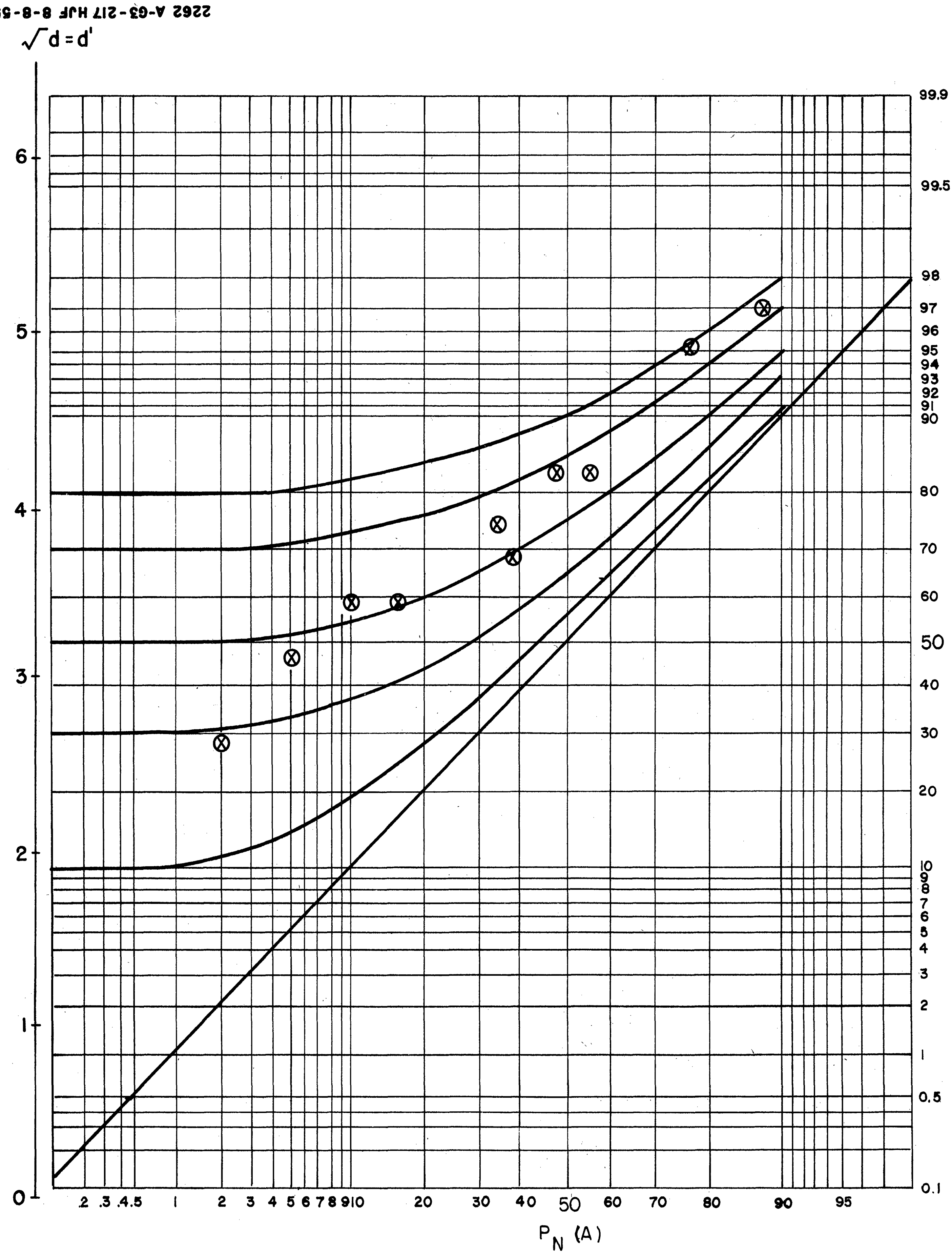

FIG. 8 P_{SN}(A) VS P_N(A) FOR OBSERVER I

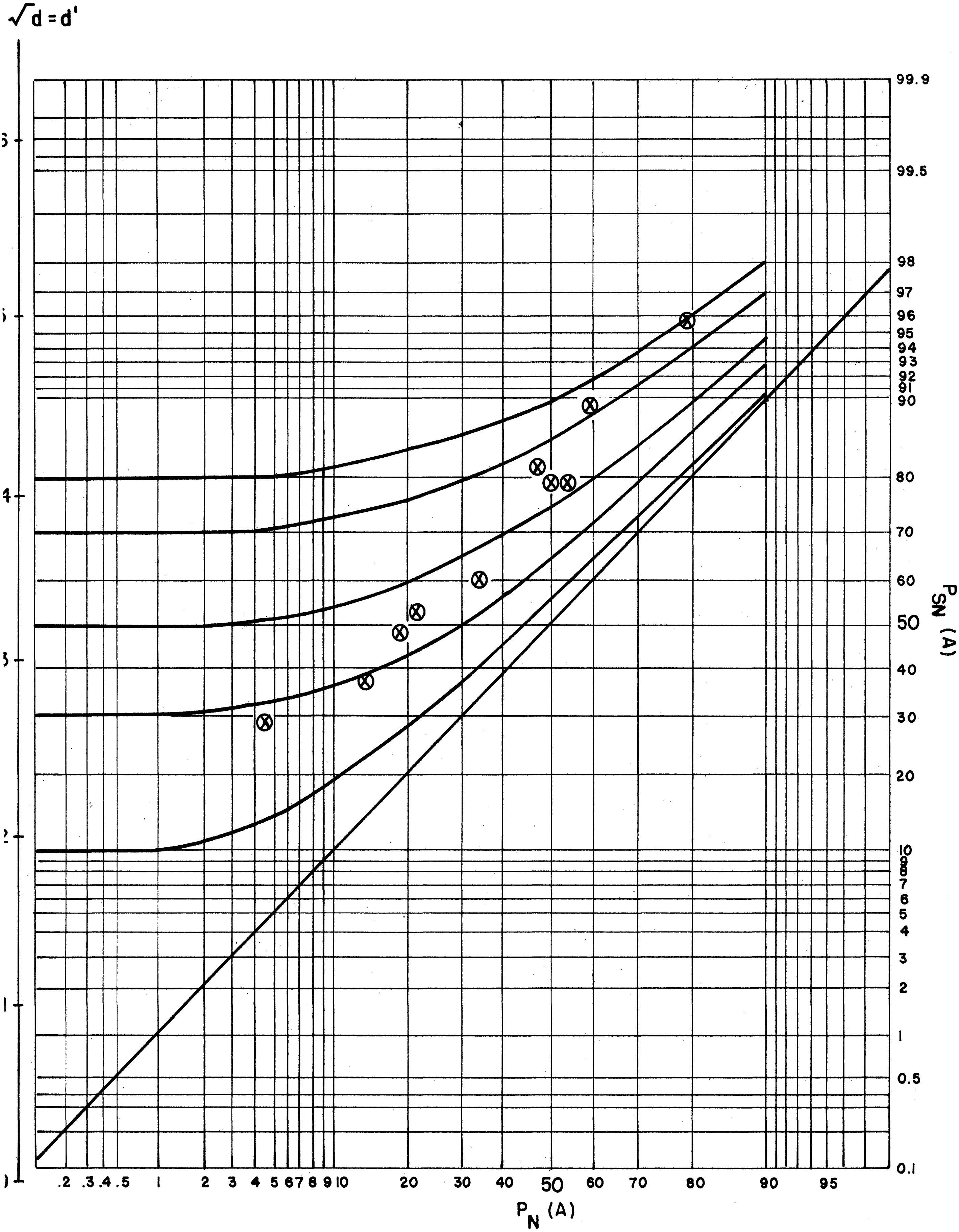

FIG. 9 $P_{SN}(A)$ VS $P_N(A)$ FOR OBSERVER 2

2.6.2 A Second Experiment. The results of the experiments do not demonstrate that a threshold does not exist. They merely demonstrate that, if a threshold exists, it is much lower in terms of the noise distribution than had been previously assumed. It is interesting to speculate on how low a threshold must be, to be consistent with the data reported above. An experiment employing the forced-choice method was performed to supply at least a partial answer to this question.

According to the decision-making theory, the observer in a four-alternative, forced-choice experiment orders the four measures taken during the four time intervals and selects the largest as most likely to represent the signal. Since we are attempting to determine how low the threshold must be, if it exists, a logical question to ask is what would happen if the observer is asked to pick an interval in which the signal did not occur. Thus, in this "try-to-be-wrong" task, the observer should select the temporal interval in which the lowest value of the measure occurs. Predicted performance can be determined under the assumption of a complete ordering of the values, that is, no threshold, or a threshold located at minus infinity with respect to the noise distribution.[1] The percentage correct in the "try-to-be-wrong" experiment is obviously a function of the detectability index (d') of the signal. The predicted behavior assuming complete ordering is shown by the curve in Figure 10 labeled number 3. Since the probability of a correct response in a "try-to-be-wrong" experiment is the probability that one drawing from the signal-plus-noise distribution is a smaller value than the smallest of three drawings from the noise distribution, curve 3 is simply a reflection of curve 1, the curve presented on linear axes in Figure 2.

1. The equal variance assumption is critical. In an earlier paper (Ref. 35) a different assumption led to results which fitted data better, and the same might be the case here.

2262 A-G3-219 HJF 8-9-53

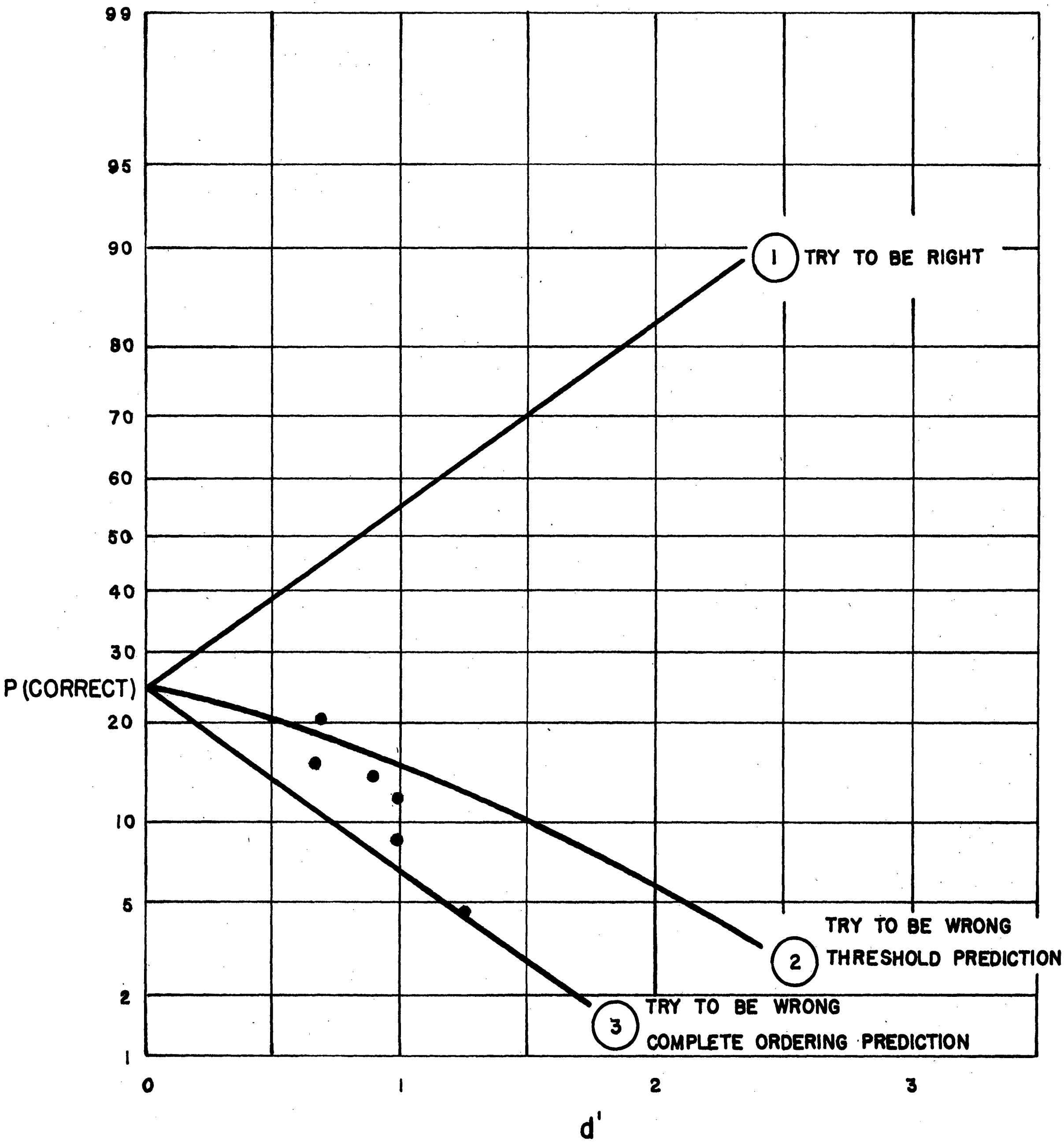

FIG. 10 RESULTS OF THE "TRY TO BE WRONG" EXPERIMENT

According to the conventional threshold assumption, that a threshold exists that is rarely exceeded by noise alone, the probability of being right in the "try-to-be-wrong" experiment may be obtained as follows. From Eq 7, the probability of a correct response in a four-alternative, "try-to-be-right" experiment is

$$P(c) = p' = p + \frac{1}{4}(1-p). \qquad (10)$$

Now, the observer can choose an interval in which the signal did not occur with at least probability p, the probability that the sensory response to the signal exceeds the threshold. The threshold is not exceeded on a given trial with probability (1-p); on such a trial the observer chooses randomly among the four intervals, and hence chooses an interval which did not contain the signal with probability 3/4 (1-p), and the interval which contained the signal with probability 1/4(1-p). This latter probability, the probability of a correct response in a "try-to-be-wrong" experiment under the threshold assumption, can be shown to be equal to 1/3(1-p') or 1/3(1-P(c)) by solving for p in Eq 10 and substituting this solution for p in 1/4(1-p). Thus, according to the threshold assumption, the probability of a correct response in a "try-to-be-wrong" experiment is one third the probability of an incorrect response in a "try-to-be-right" experiment employing the same signal and noise levels. The plot of this probability against signal strength is also shown in Fig. 10.

Using these two curves it is possible to compare the two theories by employing a signal that produces some determinate probability of a correct answer in an experiment where the observer is attempting to identify the interval in which the signal, in fact, occurred. Then, for each probability in this "try-to-be-right" experiment, a prediction can be made on the basis of each theory as to the expected percentage correct in the "try-to-be-wrong" experiment.

Six observers were used in this experiment. The signal was a 1000 cps tone .1 second in duration. Constant signal and noise powers were employed throughout the experiment. In the first part of the experiment, the observers were asked to try to identify in which interval the signal occurred. This permitted the determination of a d' so that the results of the "try-to-be-wrong" experiment could be evaluated. 100 observations were made on each trial session. These were run alternately, first a session in which the observer attempted to be right and then one in which he attempted to be wrong. The points on the graph are the results obtained from the observers in the "try-to-be-wrong" experiment. They are placed on the graph directly below the point on the top curve which indicates the corresponding percentage correct in the "try-to-be-right" experiment. Approximately 300 observations define each point.

One factor which would greatly effect the results is the distribution function of the signal and signal-plus-noise. If the distribution of noise alone were not Gaussian, this would change greatly the shape of the third curve. Secondly, the curve predicted by the complete ordering hypothesis is an ideal curve. That is, in order to attain the hypothesized curve the mechanism must be able to take four measures, one in each interval. It must store these measures in a perfect memory device and be able to order these stored measures perfectly no matter how small the differences in these measures are. Any variance introduced by extraneous factors, such as lack in attention or the like, will produce data points that fall higher than the lowest curve of Figure 10. When these facts are taken into consideration, the data indicate that something close to complete ordering is being accomplished, since five of the six points fall below the line predicted by the conventional threshold theory.

2.6.3 Summary. The two experiments reported above indicate that the assumption of a fixed threshold that is high enough to be exceeded only rarely by noise alone in invalid. These two experiments are considered sufficient to establish the need for a theoretical alternative to the threshold theory of auditory detection, and to support the particular alternative presented, since the alternative theory has been substantiated more thoroughly in another sensory modality, that of vision. Thus, the testing procedures and data analysis procedures employed to obtain and summarize the data reported below are those dictated by the decision-making theory of detection as discussed in this section.

2.7 Problem in Precise Definition of d'

In the visual studies (38), d' varies as the square of the increment in signal power. In the auditory work, d' varies as signal power raised to a power somewhat greater than one. This is demonstrated by a set of data shown in Fig. 11. In both the auditory and visual experiments the exponent should be 1/2 if the model for the signal-known-exactly applies. According to this model, that is, d' varies proportionally with signal voltage. If d' is properly defined in the psychophysical experiment (if the observer does, in fact, know the signal exactly), the observed d' should vary with voltage in the same fashion. Peterson and Birdsall treat a number of specific cases for signal detection, and in each case the removal of information (phase, frequency, time, etc.) results in a detectability index that is nonlinear with voltage for low signal strengths. In the auditory case, some work has been done indicating that it is likely that the assumption of known phase, and of no uncertainty in frequency, account for these deviations; that is, if d' is defined by a model assuming something less than exact information, the observed d' will vary in the same fashion with signal strength as the theoretical d'. These results will be reported in a future paper.

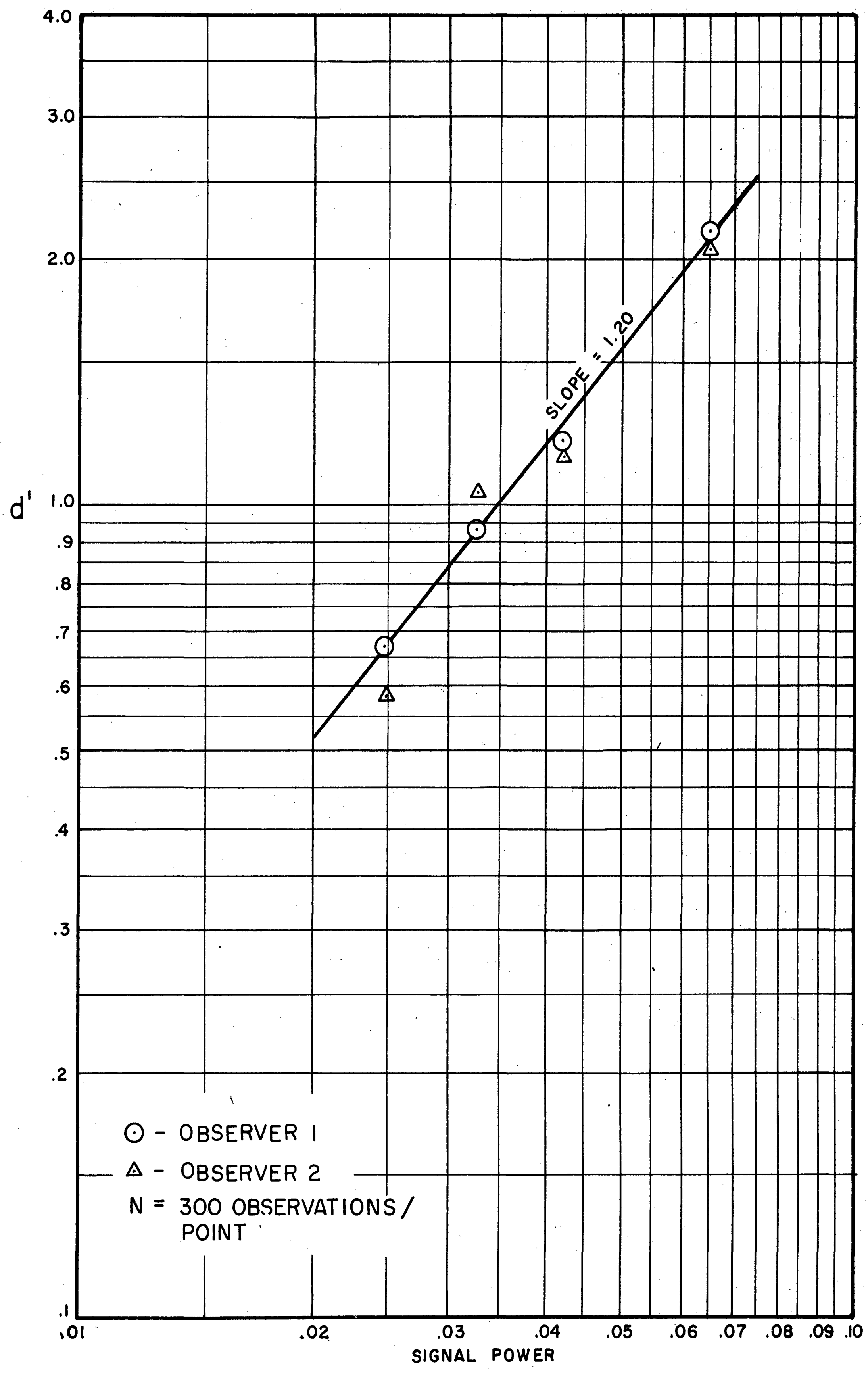

FIG 11. LOG d' AS A FUNCTION OF LOG SIGNAL POWER

2.8 Observer Efficiency

The ratio $\frac{2E}{N_o}$ (see page 28) in these experiments is approximately 9 based on measurements at the output of the earphones. The observer's d' can be interpreted as the $\sqrt{\frac{2E}{N_o}}$ necessary to lead to the observer's performance given a mathematically perfect device. The ratio of $(d')^2$ to $\frac{2E}{N_o}$ is thus a measure of observer efficiency. For these experiments, the ratios for the two observers are approximately .16 and .11 respectively. Roughly, this performance represents a decrease in effective value of $\frac{2E}{N_o}$ of approximately 8 db from the effective value for the mathematically perfect device.

The performance of the mathematically perfect device is predicted on the basis of a filter matched to pulse duration, in this case a filter 10 cps wide. If the filter were increased by approximately six times in width, the total noise power at the output of the filter is increased by approximately 8 db, a result which would match the observer's performance, and further compares favorably with Fletcher's estimate at 1000 cycles. However, the result that d' varies as signal power raised to a power suggests that at high signal levels the same type of calculation would lead to narrower estimates of bandwidth. Therefore, the problem is to arrive at a method for estimating bandwidth which appears independent of signal power. An attempted solution is presented in the next section.

3. THE NARROW-BAND PROPERTY OF THE AUDITORY MECHANISM

Three experiments were conducted in an attempt to determine whether there exists an auditory band of attention as suggested by the work of Fletcher (Ref. 7) and Garner (Ref. 12). The problem dealt with in these experiments is whether or not the human observer is able to listen to a certain small range of frequencies to the exclusion of other frequencies.

The statement of this problem does not preclude the possibility that the auditory mechanism has the properties of a wide-band device as well as the properties of a narrow-band device. Evidence from neurophysiological studies suggests strongly, as a matter of fact, that the auditory system is such a dual mechanism. Galambos and Davis (Ref. 11) found receptors in the cat that react to a single narrow range of the acoustic spectrum and others that are aroused by any part of the spectrum. Whether the auditory system can act as a wide-band device is left for consideration elsewhere.

3.1 A Preliminary Experiment

The first experiment performed represents a simple, direct attack upon the question of whether observers who expect a certain signal frequency fail to perceive other frequencies. A training period, during which the observers became acquainted with the experimental procedures and were practiced in listening for signals in noise, was conducted employing only a 1000 cps tone burst .143 second in duration. Throughout this period the four-alternative, forced-choice method of response was used. When the observers had progressed sufficiently so that no further learning effects were anticipated, the frequency of the signal was changed to 1300 cps, and presented at the same energy level and duration which yielded a proportion of correct choices of approximately .65 for the 1000 cps signal. The observers were not informed of the change. For the 1300 cps signal, the proportion of correct responses was not significantly different from .25, or chance success. Later tests showed that when a frequency of 1300 cps was expected, the proportion of correct choices at the same energy level of signal and noise, and the same duration, approximated .65. Apparently, the observers did not perceive the signal frequency of 1300 cps when expecting 1000 cps; they insisted, as a matter of fact, that the experimenter had failed to turn on the signal generator.

Unfortunately, the nature of this experiment is such that a systematic set of similar experiments, varying the frequency difference between the expected and unexpected signal frequencies in an attempt to determine the width of the band of attention, is impossible with a single set of observers. Because the observers in these experiments typically serve over a period of six months to a year, it is necessary to convince them after a single experiment involving deception that no further deception during their period of observation will be attempted. It is also impossible, due to the nature of the experiment, to accumulate a large number of observations with a single frequency difference between the two signals involved. The data reported above are based on fifty observations of the 1300 cps signal by each of four observers. Because of the small number of observations on which these data are based, they are not presented as decisive; the experiment is described here, in part, to illustrate the concept of the band of attention held by the authors. More confidence can be placed in the type of result reported, however, than is indicated by the specific data described. Since these data were collected, similar experiments have been performed by others, using as subjects the students in two undergraduate laboratory courses in psychology at the University of Michigan. These latter experiments, conducted by Karoly and Isaacson (Ref. 17), involved 54 observers each of whom made 150 observations of the expected frequency and 20 observations of the unexpected frequency at various levels of intensity; the results are very similar to those just described. The proportions of correct responses to an unexpected signal of 1500 cps in one experiment, and to an unexpected signal of 500 cps in another experiment, were significantly less than the proportion of correct responses to the expected signal of 1000 cps even when the energy level of the unexpected signals was greater than that of the expected signal.

In order to obtain equal properties of correct response for the expected and unexpected frequencies, the signal at the unexpected frequency had to be from 10 to 18 db greater than the signal at the expected frequency.

3.2 A Repetition of Fletcher's Experiment

Another experiment was performed to determine the existence of the band of attention, and, if found to exist, to obtain estimates of its width. The design of this experiment permitted the accumulation of a large number of observations by one group of subjects.

The general method was that first used by Fletcher in his study of critical bands (Ref. 7). The observer is asked to detect a pure tone in a background of noise. The frequency range of the noise is varied, with the center of the band of noise remaining at the frequency of the tone which is to be detected. As the bandwidth of the noise is decreased the tone becomes gradually more detectable. From a study of the manner in which detection is improved with decreasing noise bandwidth, the characteristics of the band of attention are inferred.

While the experiment reported below is conceptually very similar to Fletcher's, there are major points of difference. Fletcher's measurement assumed a threshold. In the second place Fletcher writes as if the band is such that noise outside of the band has no effect at all on the detectability. In other words, there are points not far from the center of the band where the attenuation of the noise is essentially infinite. According to Fletcher, these points define the "critical band" for this frequency tone. Fletcher estimated the critical band to be approximately 65 cps in an experiment in which the signal was a tone of 1000 cps, the signal frequency used in the experiment reported below. Fletcher also points out that the critical bands correspond to a 1/2 mm length on the basilar membrane for any frequency.

Within the critical band the following relationship is said to hold: "For these smaller bands and for this type of statistical noise, the intensity of the masked tone must be adjusted to be equal to the average intensity of the noise in the band in order for it to be perceived." (Ref. 7) Since Fletcher does not state the type of statistical noise employed, it is difficult to interpret exactly the implication for the shape of the critical band. If white noise were used, and it seems likely that it was, and if the filters used were essentially square, then the statement could hold for any shape of band. If the critical band is square, then the statement could hold for any noise spectrum, although strange detuning type effects may appear for certain types of noise. If neither the noise is white within the band nor the band is square, then the average must be based on a weighting function which is a function of the observer's critical band.

Schafer, Gales, Shewmaker, and Thompson (Ref. 29) realized the importance of using a square band of noise, and in an experiment similar to that of Fletcher, employed a band constructed by mixing pure tones. They conclude that the observer acts as a filter the Q of which is approximately 37 at 800 cycles. A single-tuned filter, having a Q of 37 is 22 cps in width, measured at the half-power points. Schafer, et. al., report that the equivalent square bandpass is slightly over 60 cps in width.

Both of the papers referred to above use the threshold measure as the dependent variable. It is felt that the results of an experiment based on a subjective report that a tone is above threshold may differ from an experiment in which detection is measured as it is in a forced-choice experiment. There is further the suggestion that certain parameters of the hearing mechanism may be under control of the observer, such that any set of measurements as those in Fletcher's experiment apply only to a special set of conditions. Signal duration, which in Fletcher's experiment was indeterminate, although certainly very long, may be a critical factor. A thorough study of the restricted band of attention may involve many experiments. Thus, it is necessary to devise a method which is conceptually clear. The method and experiments described below are an attempt at devising such a method.

3.2.1 An Alternative Approach. In this paper, no attempt is made to determine the bandwidth of the hearing mechanism by making a calculation of the intensity and frequency distribution of the sound at the basilar membrane. Rather, if the hearing mechanism acts as a narrow-band receiver, it is supposed that the information passes through this auditory filter to some high-order detector. Thus, in an experiment similar to that of Fletcher, the general scheme is conceptualized as a situation in which there are two filters in series: the external filter which the experimenter manipulates and the internal filter, or band of attention, associated with the observer's hearing mechanism. Thus, the important variable in this study is the ratio of the two filters rather than the value of the external bandpass alone. It should, perhaps, be emphasized that the concept of an internal "filter" is used solely as an analytical device; it is not regarded as a specification of the exact nature of the auditory mechanism.

The relevant components to be considered in an experiment which studies the bandwidth of the hearing mechanism are shown in block diagram form in Figure 12. In the diagram, it will be noted that the signal is delivered to the adder through a narrow-band filter. This filter is added to attenuate the high-frequency, transient responses of the gated signal. Thus, when external noise is filtered, the observer cannot identify the presence of the tone by listening for these transient responses. Since it is assumed that the observer's bandpass remains constant in width throughout the experiment reported in this

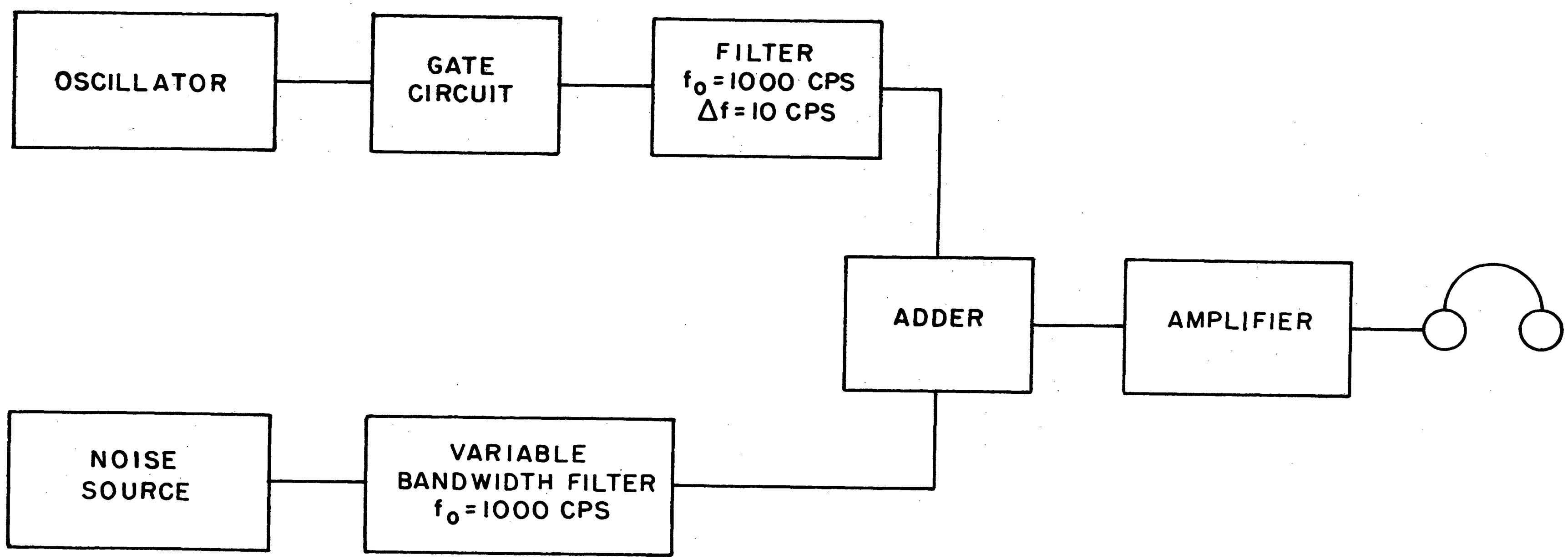

FIG. 12 AUDIO COMPONENTS IN STUDY OF OBSERVER'S BANDWIDTH

2262 A-S3-41 AEM 8-22-55

section (as signal duration and signal power are held constant), the signal power entering the measure on which the observer bases his decision is assumed constant through the experiment. The only variable which would then affect the detection of the signal is the power of the noise entering the observer's measure.

3.2.2 The Noise Power as a Function of the Ratio of External to Internal Bandwidths. The first question to be answered is how the noise entering the measure is influenced by the ratio of the two bandpasses. If this can be determined, and if it can be determined how this noise power varies as external bandpass is changed, then it is possible to solve these relations for the internal, or auditory, bandwidth. To determine how the noise power varies with the ratio of the bandwidths, it is necessary to define the bandpass characteristics. The bandpass characteristic of the external filter is, of course, a function of the components used in constructing the filter; in this experiment, a single-tuned filter is employed. The width of the internal filter, in the absence of advance knowledge, is calculated under several different assumptions about its bandpass characteristic. The criterion of congruence with other estimates of the width of the band of attention will be seen to eliminate, tentatively at least, all but one of these assumptions.

Thus, the assumption of a given bandpass characteristic of the internal filter permits the calculation of the noise power entering the observer's measure when the noise power per unit bandwidth is held constant at the input to the external filter and the ratio of the relevant bandwidths is varied. Figure 13 shows this relation graphically for the several types of internal bandpass considered. These are a single-tuned filter, a square filter, a Gaussian filter using the one sigma points for the estimate of width, and a Gaussian filter using half-power points for the estimate of width.

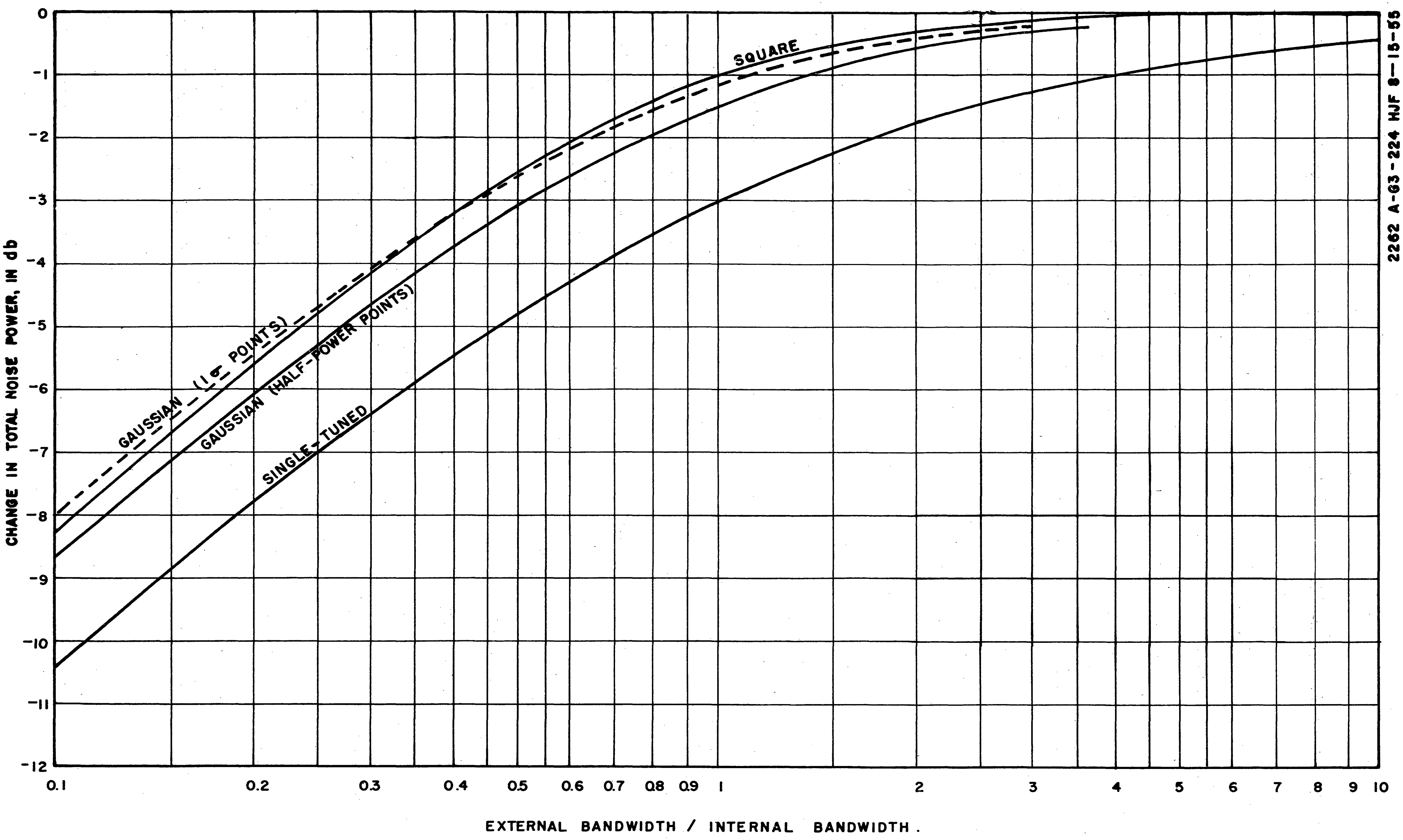

FIG. 13 NOISE POWER AT DETECTOR AS A FUNCTION OF THE RATIO OF EXTERNAL TO INTERNAL BANDWIDTH

3.2.3 General Description of the Experiment. From Figure 13 it may be seen that, if the internal bandpass is held constant, a change in the external bandpass will produce a change in the noise power entering the observer's measure. Specifically, if the external bandwidth is decreased for some constant internal bandwidth, the noise entering the measure is decreased. Thus, one would expect the detection of the signal to increase as the external bandpass is decreased. The problem now becomes one of inferring from the improvement in detection the actual change in this noise power.

In the first test, with the external bandwidth held constant, the observer is asked to detect the same signal which is later employed in the second test with variable external bandwidth. In this first test, the external bandpass is, in fact, removed from the circuit. The power of the masking noise is reduced, so that in the course of the experiment the detection probability increases. The improvement for the three observers in the detection variable d' as the noise power is reduced is shown in Figures 14, 15, and 16. The experimental procedure has been explained in Section 2. The four-alternative, forced-choice method is used to determine the d'. At least 400 observations define a point.

Since the external filter is held constant and the internal filter is assumed to be constant, Figures 14,15, and 16 show how a change in the noise power at the output influences the detection variable. These graphs, of course, also show how a change in detection reflects an equivalent change in the noise power.

The second test is conducted with the same basic components used in obtaining Figures 14, 15, and 16. Instead of changing the noise power at the output by reducing, from a reference level, the noise power input, an equivalent procedure is employed. This procedure is to reduce the bandwidth of the external filter in the manner shown by Figures 17, 18, and 19. The resultant change in the detection variable is shown by the ordinate of Figures 17, 18, and 19.

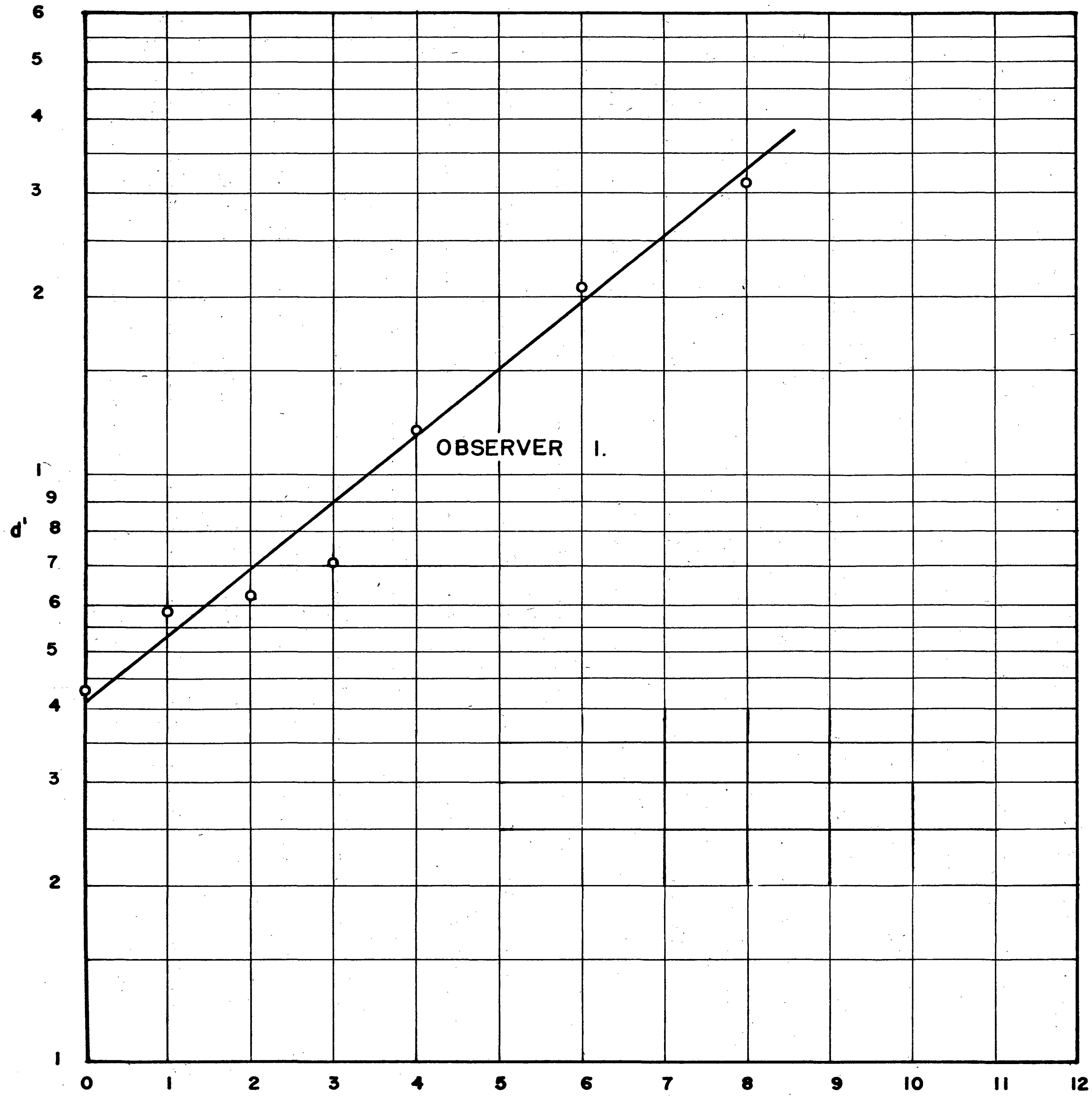

FIG. 14 d' AS A FUNCTION OF NOISE POWER FOR OBSERVER I

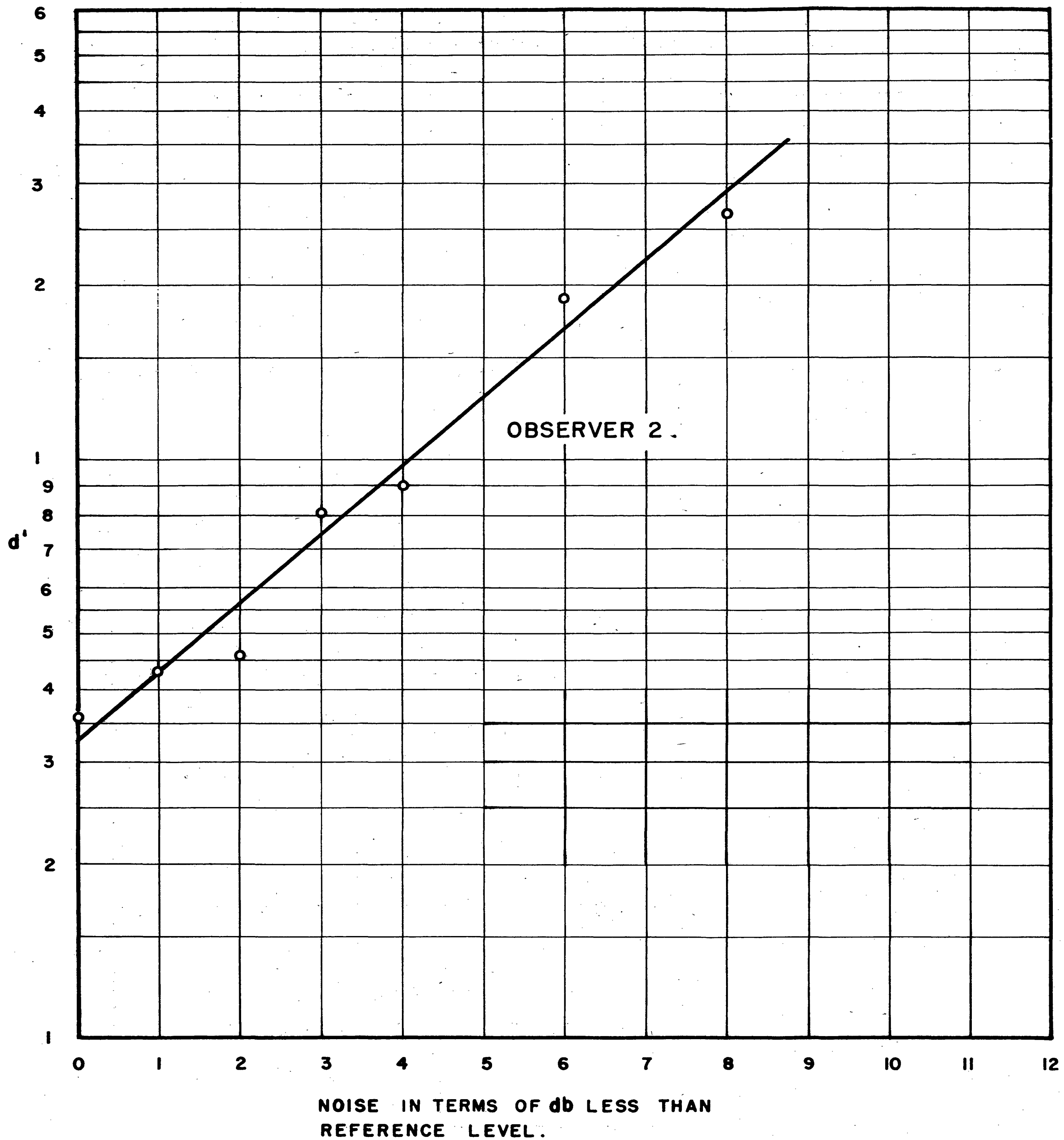

FIG.15 d' AS A FUNCTION OF NOISE POWER FOR OBSERVER 2

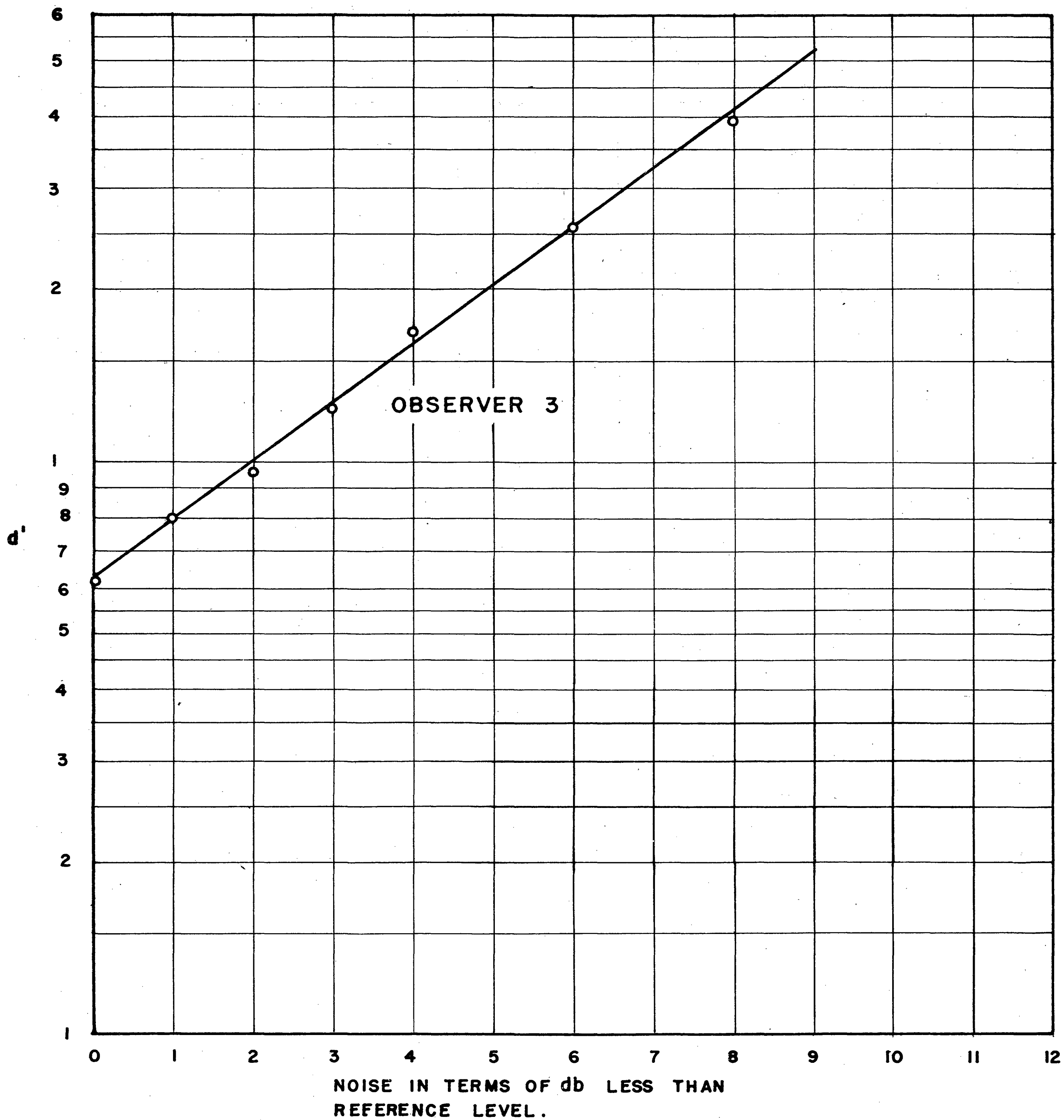

FIG. 16 d' AS A FUNCTION OF NOISE POWER FOR OBSERVER 3

2262 A-63—221 HJF 8-15-55

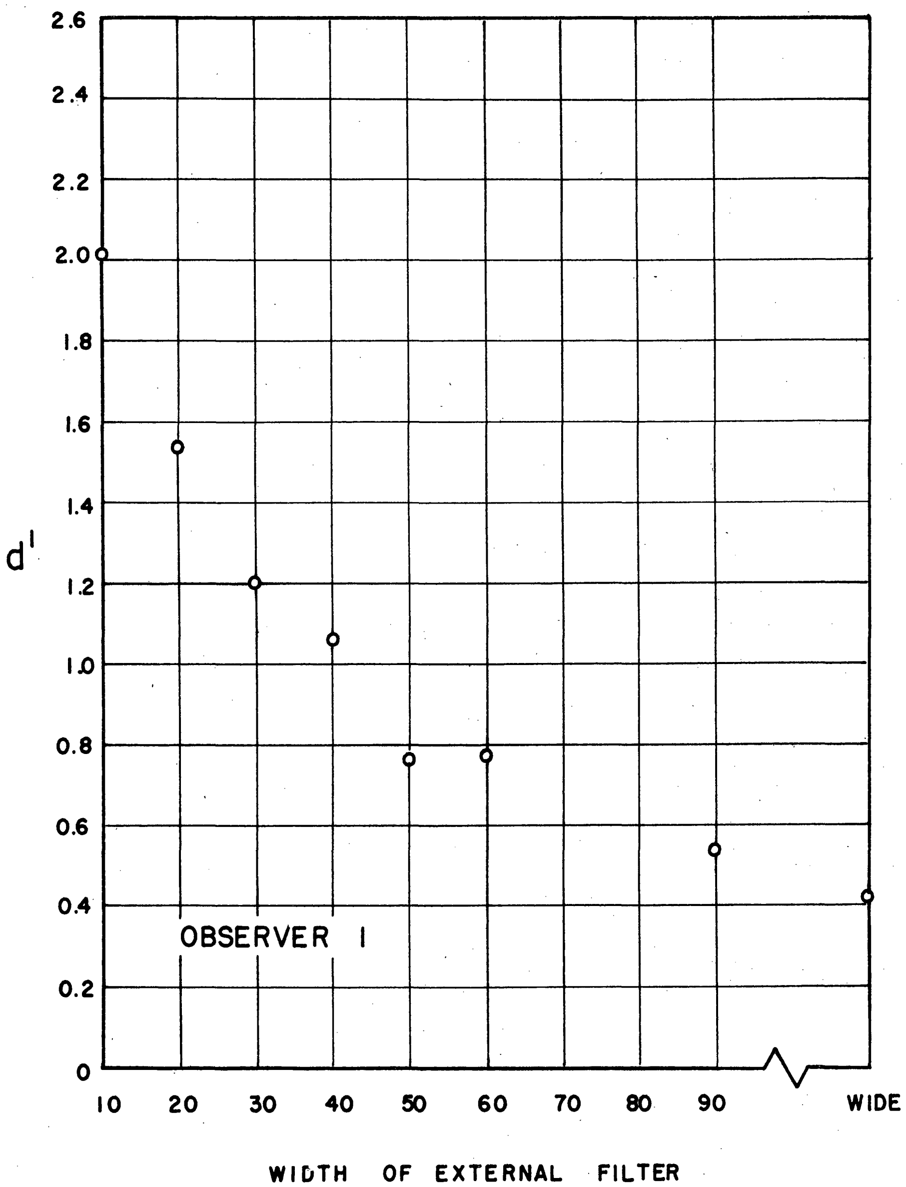

FIG. 17 d' AS A FUNCTION OF EXTERNAL BANDWIDTH FOR OBSERVER I

2262 A-63-226 HJF 8-15-55

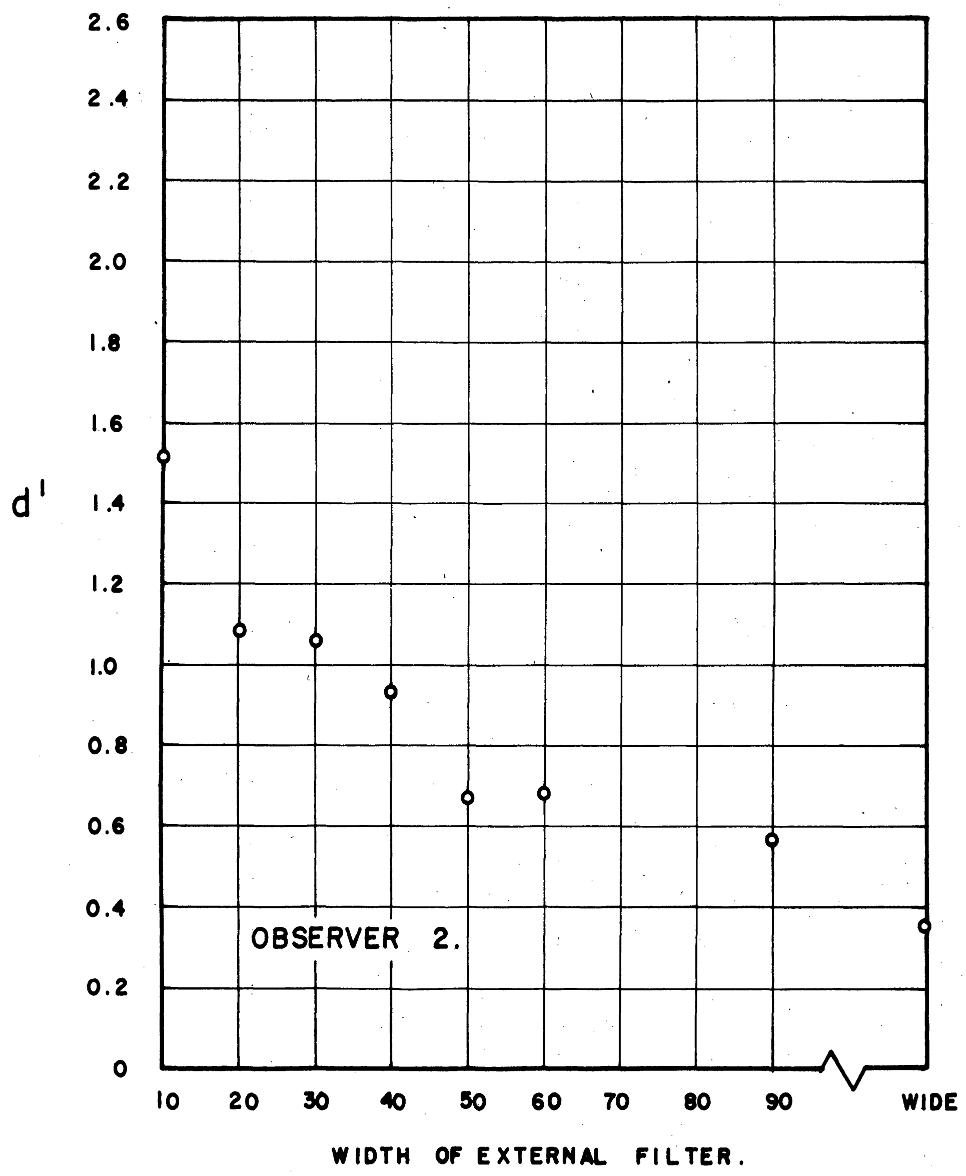

FIG. 18 d' AS A FUNCTION OF EXTERNAL BANDWIDTH FOR OBSERVER 2

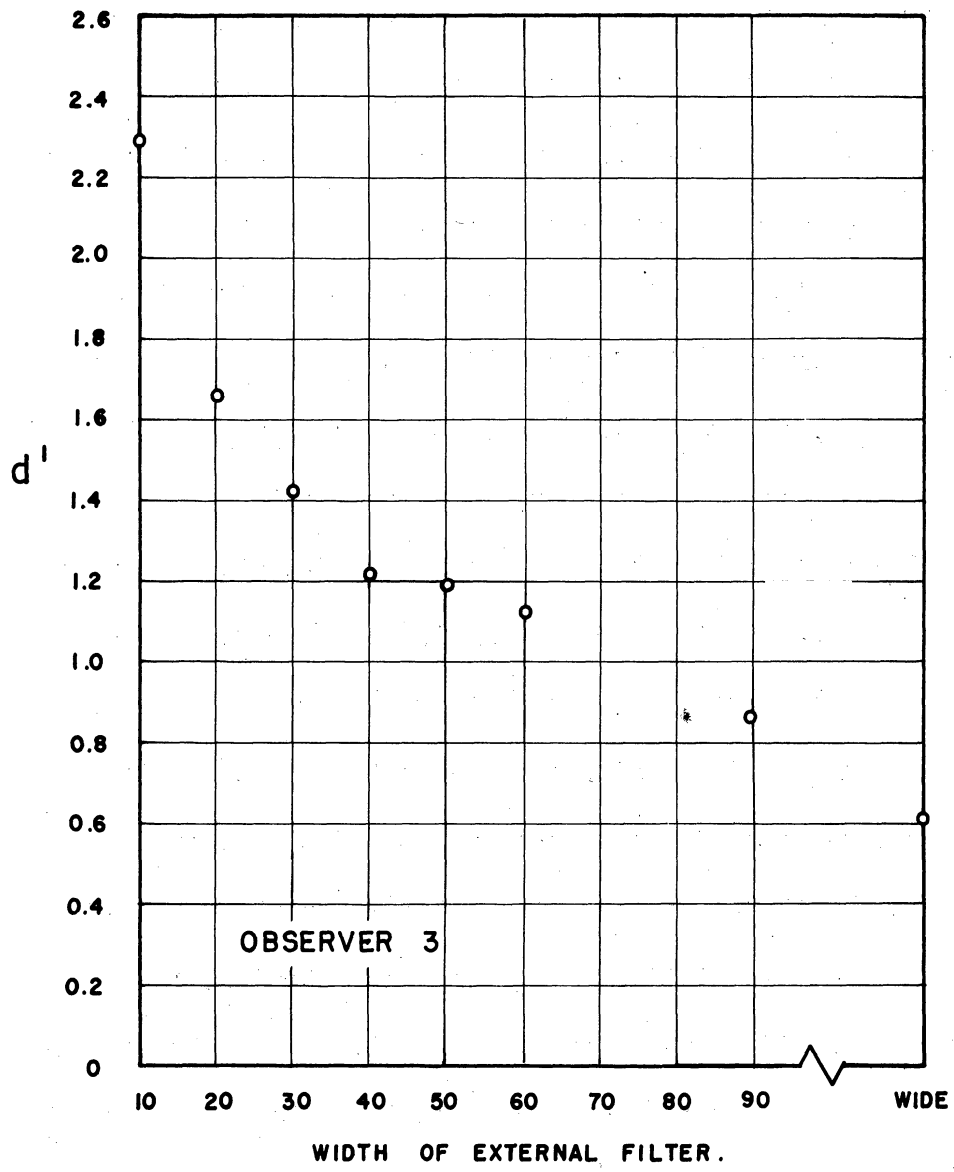

FIG. 19 d' AS A FUNCTION OF EXTERNAL BANDWIDTH FOR OBSERVER 3

Now, by using the data in Figures 14 through 19, combined with the theoretical curves in Figure 13, one may solve for the internal bandwidth of the auditory filter under each of the assumptions.

The value of d' obtained when a given external bandpass is employed may be translated into the change in noise power entering the observer's measure with the aid of Figures 14, 15, and 16. This change in noise power may be used with the theoretical curves of Figure 13 to determine the ratio of the external to internal bandwidths. Since the external bandwidth is known, the internal bandwidth may be determined. For example, it can be seen in Figure 17 that for observer 1, employing an external bandpass of 20 cps results in a d' of 1.54. From the same observer's performance in Figure 14, it can be seen that a d' = 1.54 is equivalent to a change in noise power of approximately 5.1 db. Assuming the band of attention has the characteristic of a single-tuned filter, for example, a change of a 5.1 db is seen from Figure 13 to result from a ratio of external to internal bandwidth of approximately .45. Since external bandwidth is 20 cps, the internal bandwidth under this assumption is approximately 44 cps. This same procedure is repeated for each observer, for each external bandwidth and each assumption about the characteristic of the internal band. Table III shows the results of these calculations. The mean and standard deviation of the estimates of internal bandwidth are given in the table. These two statistics are calculated from the data, not from theoretical considerations. The reader should be cautioned about the point obtained with the ten-cycle-per-second external bandwidth. As explained previously, the signal was passed through a ten cps bandpass before being added to the noise. If the center of either of these bandpasses drifted during the course of the experiment, the observer could listen to the signal at a frequency where there would be little masking noise. That this

Assumed Internal Bandpass Characteristic		Width of External Filter*							Mean	Standard Deviation
		10	20	30	40	50	60	90		
Single-Tuned	Observer 1	37	44	49	53	37	44	25	41	8.3
	Observer 2	31	35	53	58	42	50	50	46	9.2
	Observer 3	28	32	37	37	43	44	33	36	5.4
Square	Observer 1	64	87	100	114	94	113	95	95	15.7
	Observer 2	58	71	107	124	102	122	138	103	26.9
	Observer 3	55	65	82	89	104	113	106	88	20.2
Gaussian (1σ points)	Observer 1	73	91	103	117	91	109	86	96	13.8
	Observer 2	61	74	111	124	100	120	130	103	24.3
	Observer 3	58	67	82	87	102	109	97	86	17.2
Gaussian (half-power points)	Observer 1	60	77	86	98	77	92	69	80	12.2
	Observer 2	55	61	94	105	83	100	107	86	19.5
	Observer 3	50	56	69	73	86	92	78	72	14.05

* The bandwidths of the external filter listed as column headings here, and along the abscissa of the three preceding figures, are nominal. It was not practicable in the experiment to attempt to adjust the bandwidths to be exactly equal to any given figure. The actual values of external bandwidth employed in the experiment and in the calculations were 12, 20, 31, 41, 50, 60, and 90, respectively.

TABLE III. ESTIMATES OF INTERNAL BANDWIDTH ON THE BASIS OF VARIOUS EXTERNAL BANDWIDTHS result did occur is suggested by Figures 17, 18, and 19. Here one can see that the points obtained with the larger external bandpass shows consistent increase in the detection variable until the ten cps bandpass is reached. This point obtained with the small filter seems unduly high. This point is reflected in the estimates of the internal bandpass where it can be seen that the bandwidth estimate derived from experimental sessions employing the 10 cps external filter is the least in the vast majority of cases.

It will be noticed from Table III that the assumption that the band of attention has the characteristic of a single-tuned filter leads to the smallest estimates of its width. The bandwidth estimated under this assumption best agrees with the indications for its width derived from an analysis of observer efficiency reported in Section 2.8. Recall that according to the analysis in terms of efficiency, the observer's band is less than 60 cps in width. These estimates are also not very different from those reported by Fletcher and by Schafer, et. al. The standard deviations of these estimates indicate that the assumption is a reasonable one.

3.2.4 Summary. The experiment reported above is an attempt to estimate the bandwidth of the hearing mechanism with a fixed set of experimental conditions. It is assumed that the internal filter remains constant through the experiment. Further it is assumed that the change in the width of the external bandpass has the same affect as a reduction of the noise power in an experiment where the external filter is held constant and the noise power input is simply reduced. From these assumptions the internal bandwidth is estimated from the change in the detection variable as the external filter is reduced.

3.3 The Signal at Either of Two Frequencies.

The narrow-band auditory device is a mechanism capable of coding frequency information into some other type of information. The extent of this information that can be utilized by the observer is another matter. An experiment reported below was designed to determine the efficiency with which information representing different frequencies is utilized. Its result is consistent with the proposition that the observer is, at a single "instant" in time, sensitive to only a narrow range of frequencies, and that shifting the observation from one frequency to another requires a measurable amount of time.

The first part of the experiment consists in determining what percent correct an observer can attain in a four-alternative, forced-choice situation where the signal frequency is announced and held constant. In this situation the masking noise is held constant for all tests. A duration for the signal and some voltage is selected. The first part of the test consists in determining the percent correct the observer will attain at some given frequency (f_A). After this is completed, the observer is notified that the frequency of the tone will be changed to some other frequency (f_B). The observer then listens to this new frequency, and, with the same physical parameters used to test the first frequency, the second test is run to determine what percent correct the observer can attain at the second frequency (f_B).

In the last part of the experiment, the observer is told that either f_A or f_B will be presented, that is, that either one frequency or the other will be presented in one of the four time intervals constituting a forced-choice trial, and that each frequency is equally likely to occur on a given trial. On one half of the trials f_A is presented, on the other half of the tests f_B is presented, in random order. This situation may then be described as the problem of detecting one of two equally-likely frequencies. It should be noticed that the observer is not asked to identify which of the two frequencies was presented. He is asked merely to state the location in time of a signal.

Suppose that in order to listen to one frequency and then listen to another, the band of attention must sweep through the intervening frequencies. According to this hypothesis, two variables are critical in determining how the observer will behave in this experiment: the frequency separation of the tones to be detected and their duration.

Assume some maximum sweep rate, $s = \max \frac{df}{dt}$, where s is the rate of sweep, f is frequency, and t time. If the frequency separation is increased sufficiently, one would expect the probability of detection to decrease until finally the observer cannot listen for one frequency and then shift to another. When this point is reached the observer's optimum behavior is to listen for either one or the other of the two frequencies and simply ignore the fact that another frequency is being presented half the time. This point may be called the "successive observation" point. That is, the observer can observe one frequency or the other but not both during the duration of the signal.

Figure 20 shows what percent correct may be expected in the four-alternative case as a function of d'. The top curve is that obtained for the case of signal-known exactly. The definition of this curve is explained in Section 2. The lower curve assumes the observer can listen for only one of the two frequencies. For any given d', the percent correct in the successive observation case will be the conditional probability that if the frequency is sent the observer will detect it. Since, the frequency at which the observer is listening is independent of the frequency actually sent, the probability of a correct detection is the probability that the frequency was sent, (1/2), times the probability that if it was sent the observer detects it. This latter conditional probability is obtained from the signal-known-exactly case. To this probability must be added the probability that if the observer is listening to the frequency which was not sent he will choose the correct interval anyway, that is, the probability he will choose the correct interval with no information.

Combining these probabilities the expected probability correct for the successive observation point is:

$$\frac{P(c) \text{ (for signal-known-exactly)} + 1/4}{2} \qquad (16)$$

The probability of a correct response for the signal known exactly used in Equation 16 is the algebraic average of the probability correct obtained at f_A and f_B in the first part of the experiment, where only one of these is present in a set of trials.

The curve labeled "successive" should not be interpreted as a predicted curve. Rather it is a lower limit set by the experimental conditions; if the observer cannot shift from one frequency to the next but has to listen to either one frequency or the other, then this is the performance one would expect.

The curve labeled "simultaneous" is obtained in the following manner. It is supposed the observer can listen to both frequencies at the same time. He then records eight measures, two for each time interval, (one at each frequency, f_A and f_B). Among the eight measures he chooses the one most likely to be caused by the signal and chooses the time interval associated with this value. This is not the optimum decision function for this case. A better procedure is to combine the measures associated with the two frequencies for one time interval and compare the four combinations. Such a procedure would define a curve somewhere between the upper curve and the "simultaneous" curve.

The data for three observers are placed on the graphs of Figures 20, 21, 22, and 23 as follows. First the d' is determined for the signals when the

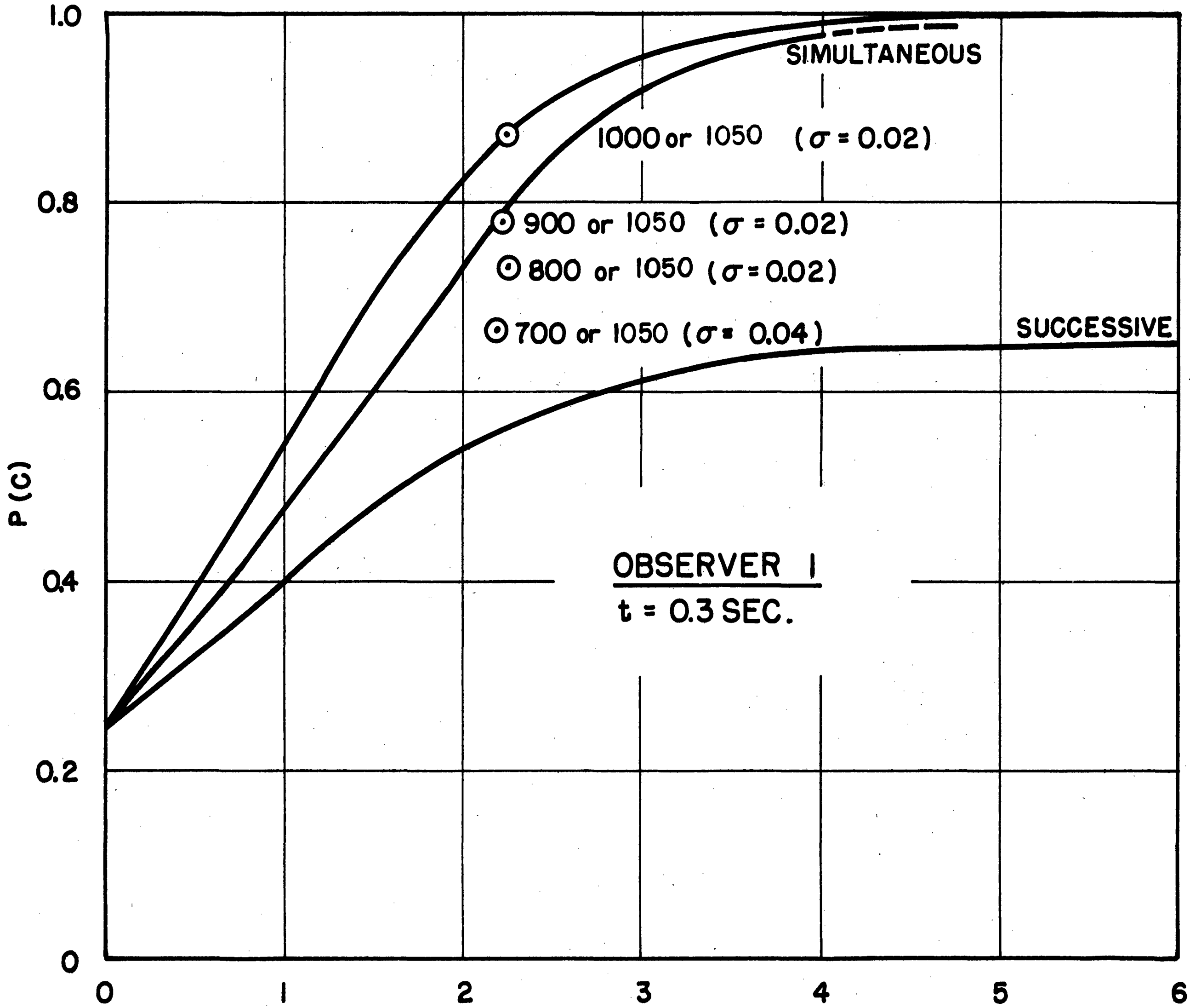

FIG. 20. RESULTS OBTAINED WITH A SIGNAL EITHER OF TWO FREQUENCIES, OBSERVER 1, t = .3 SECOND

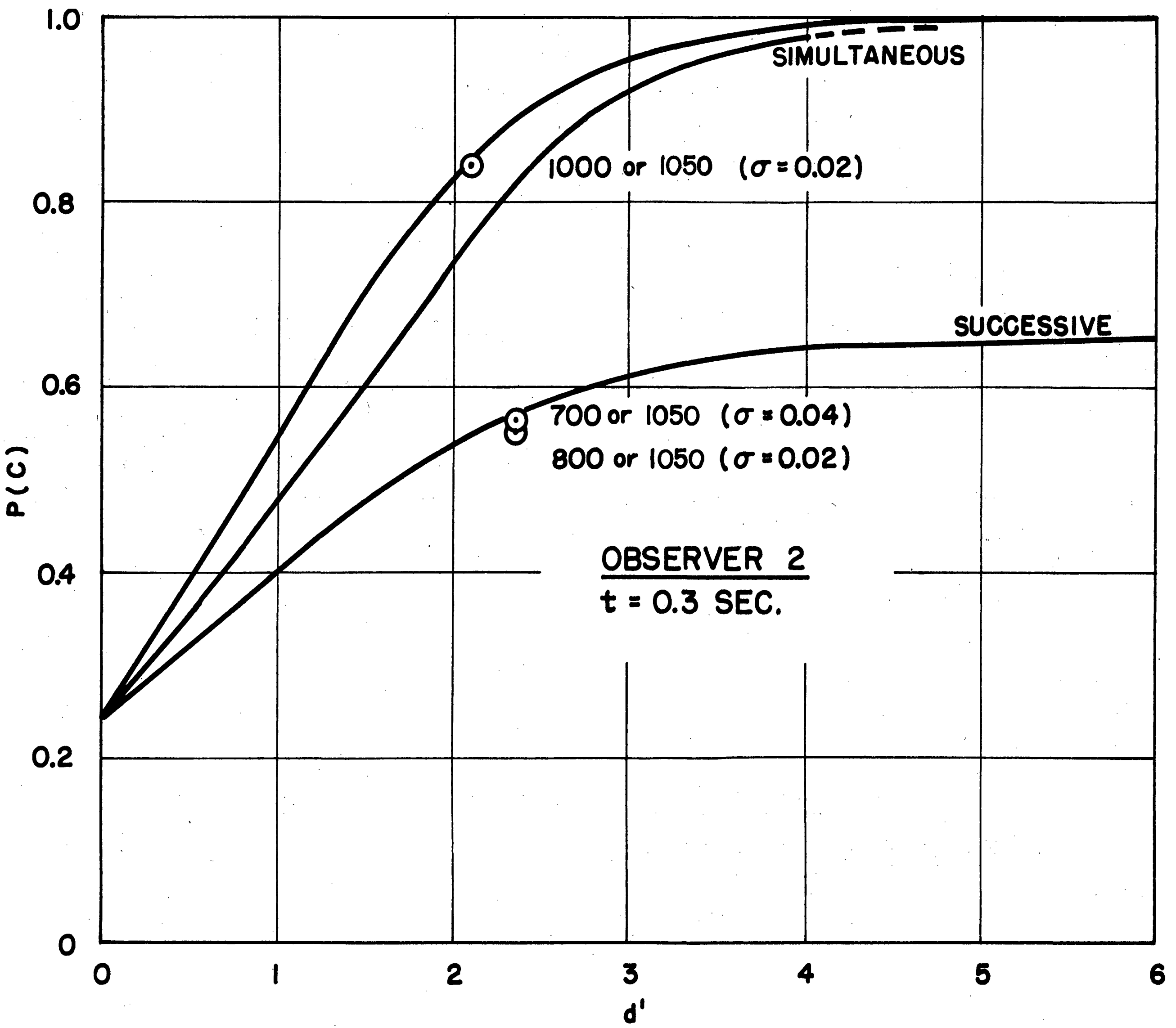

FIG. 21 RESULTS OBTAINED WITH A SIGNAL EITHER OF TWO FREQUENCIES, OBSERVER 2, t = .3 SECOND

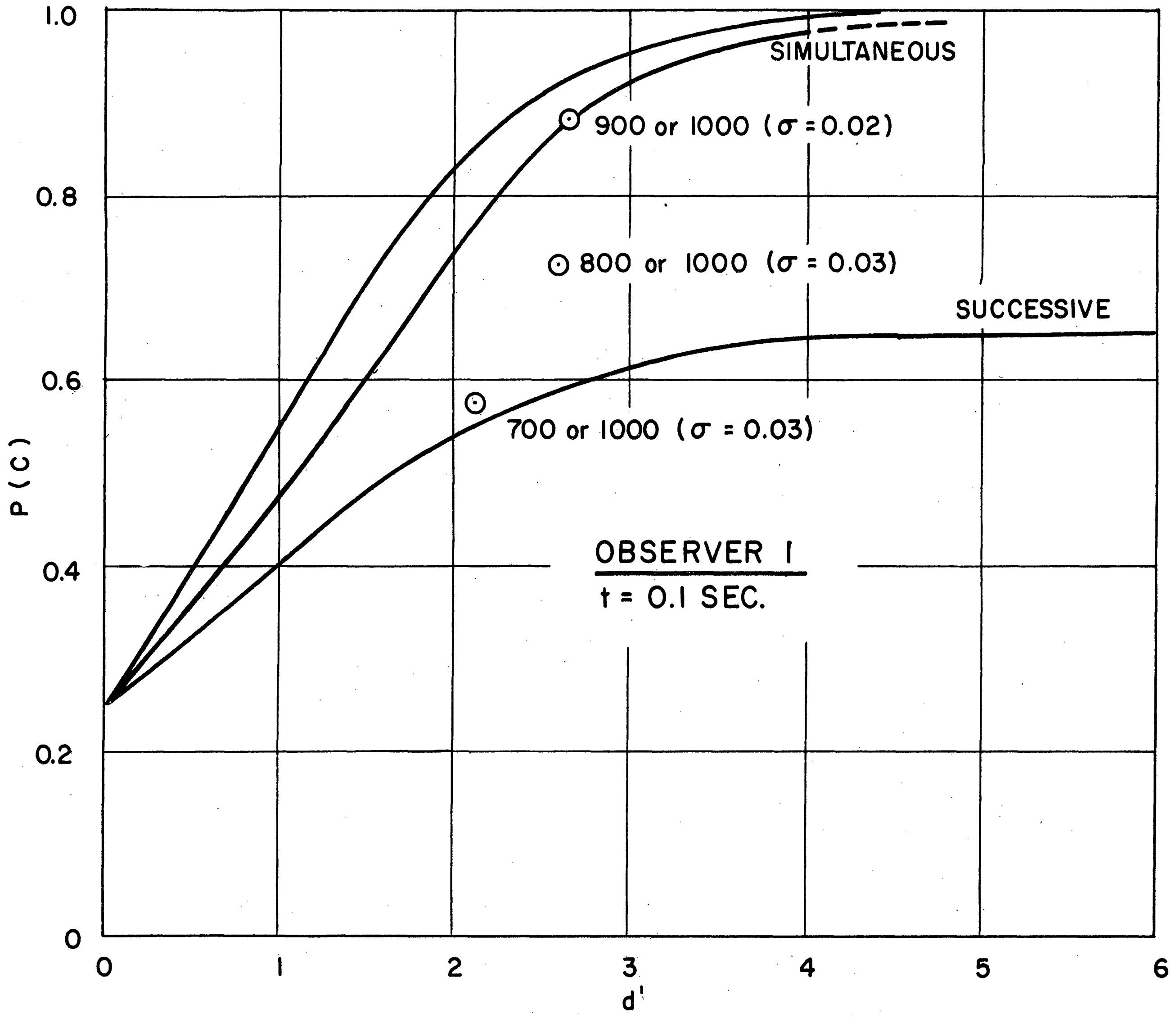

FIG. 22 RESULTS OBTAINED WITH A SIGNAL EITHER OF TWO FREQUENCIES, OBSERVER I, t = .1 SECOND

2262 A-G3-146 JEM 9-8-54

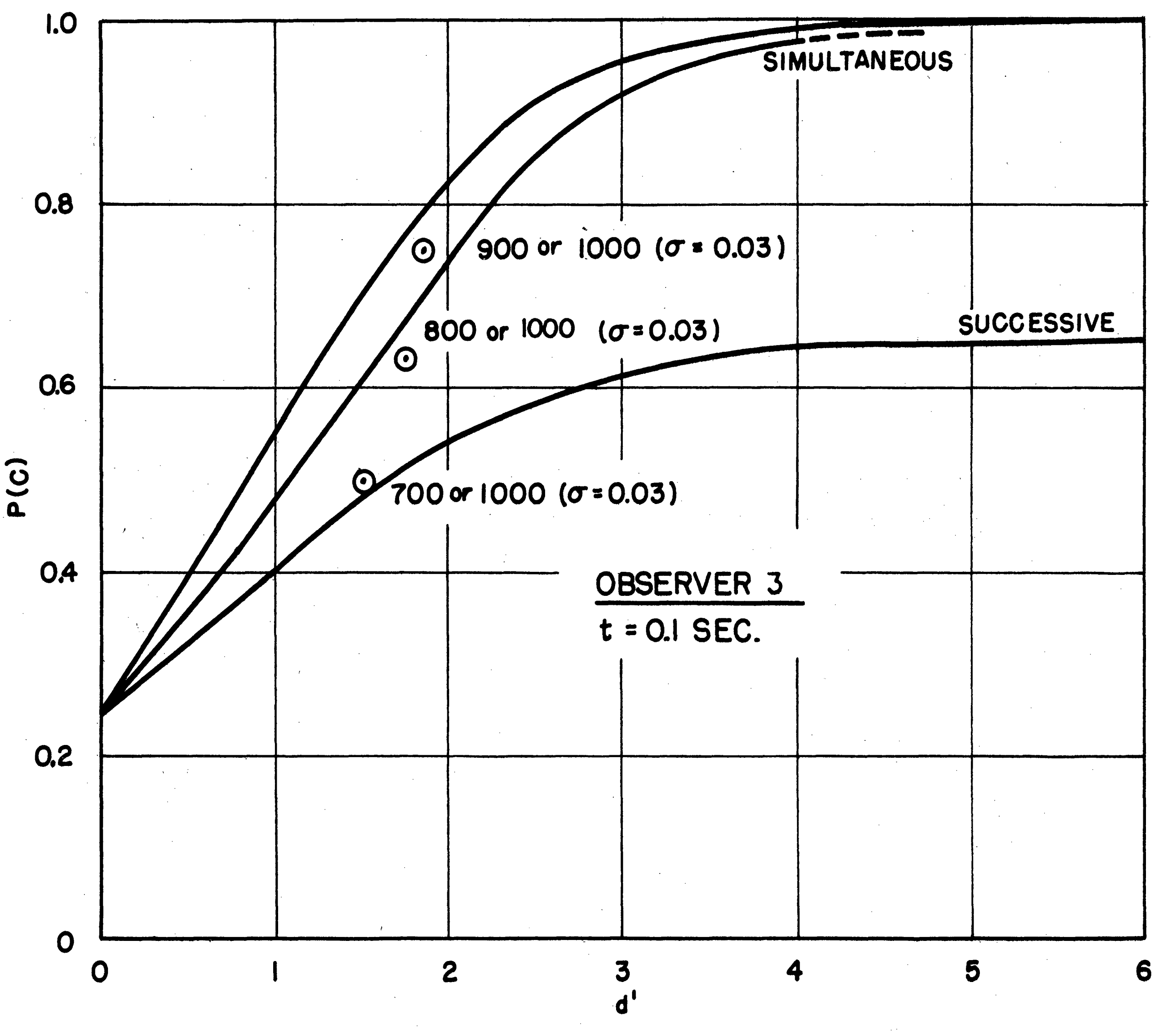

FIG. 23 RESULTS OBTAINED WITH A SIGNAL EITHER OF TWO FREQUENCIES, OBSERVER 3, t = .1 SECOND

frequency is known; approximately 700 observations define these values of d'. Then the percentage correct obtained in the experiment in which the signal is known to be one of two frequencies is entered for that d'; approximately 250 observations define these points. Thus the points on the graph represent the percentage correct obtained when the two frequencies were equally likely; each of these points is placed directly below the average percentage correct for the two frequencies when presented alone. The experimental points therefore, reflect the decrement in detection associated with the uncertainty which results when the signal may be one of two frequencies.

As the results indicate, the larger frequency separations produce a larger decrement in performance. Also there is a suggestion in the data that duration influences the decrement in detection. Since only one observer participated in the experiments conducted at both durations, this conclusion must be tentative.

These results are consistent with the following conclusions. First, the hearing mechanism requires a measurable amount of time to shift from the observation of one frequency to another. Secondly, since frequency separation affects the decrement in performance resulting when the frequency is uncertain, the mechanism shifts in some prescribed order, that is, it is apparently necessary to shift through the intervening frequencies.

4. A THEORY OF SPEECH PERCEPTION

4.1 The Problem

The results of the experiments reported above may be accounted for by ascribing to the hearing mechanism narrow-band and sweeping or shifting properties. The question arises whether these properties, inferred from detection experiments, are involved in the perception of more natural auditory stimuli, that is, in the perception of complex sounds of greater energy relative to the masking noise, or whether a different, exclusive set of properties is involved in this case. The answer given to this question is not necessarily critical with respect to the validity of the proposed theory of detection behavior. If it can be established that the narrow-band and sweeping properties are not involved in, or could not account for, perception of strong and complex signals, then it may be maintained that the problem of perceiving such signals may well be a different one and hence may involve different properties of the hearing mechanism. In the opinion of the authors, however, substantiation of the proposition that narrow-band and sweeping properties are not employed in the perception of strong or complex signals would require a thorough reconsideration of the data that lead to the imputing of these properties to the mechanism of low-energy, pure-tone detection. The interests of theory construction, it is believed, are not well served by accommodating each new datum with the postulation of new properties. Correspondingly, properties of the mechanism inferred from one class of data should be examined for relevance to related classes of data. Consideration of possible support for the proposition that the particular properties discussed are consistent with perception of strong signals is, therefore, in order.

In this section, the perception of speech is taken as an example of perception of strong, complex signals. Do the facts of speech perception force the postulation of properties exclusive of the ones discussed, or do preliminary considerations suggest that the above-named properties may be utilized in speech perception? The significance of the answer proposed to this question rests in the fact that research undertaken will be guided by it. By way of orientation. for the following discussion, the characteristics of the mechanism inferred from the detection experiments are assumed, and an attempt is made to show speech perception to be compatible with this assumption.

Characterizing the auditory mechanism in the way suggested poses a large problem for the theory of speech perception. How is it possible for a narrow-band device to intercept the frequencies serving as cues for the identification of speech sounds, given the indication, from several sources, that these frequencies vary rapidly over a range approaching 7000 cps?

4.2 The Information Contained in Frequency

In view of the assumption stated above that the major problem for the theory of speech perception is to account for the reception of a wide range of frequencies by a narrow-band device, it may be well to indicate briefly the role played by frequency in the identifiability of speech sounds. This can be done conveniently by reviewing the work on speech synthesis of Cooper, Delattre, Liberman, et. al. at Haskins Laboratories (Refs. 4, 20, 21) since further reference will be made to some of their results.

Speech synthesis is accomplished at the Haskins Laboratories by a device called the pattern playback which converts hand-painted spectrograms into sound. This apparatus makes it possible to study the effects on perception of variations in several acoustic variables. A sampling of the results indicating the importance of frequency in speech perception follows.

In one of the earlier papers of the Haskins Series (Ref. 20), a study is reported in which an attempt was made to determine the importance of the frequency position of a short burst of noise, which spanned 360 cps, in the identification of the unvoiced stop consonants. A collection of hand-painted spectrograms of burst plus vowel was prepared, in which the frequency position of the burst and the schematic vowel paired with each burst were systematically varied. It was found that the frequency position of the burst could serve as a cue for distinguishing among p, t, and k. The results indicate, however, that this particular cue is not sufficient; whereas bursts above 3000 cps were consistently judged as t, the judgment of bursts below that level as either p or k showed a dependence upon the relation of the frequency position of the burst to the vowel with which it was paired. This last mentioned result will be discussed more fully below.

Later research served to isolate a second cue to the perception of stops, namely, the curvature of the formants[1] during the vowel onset (Ref. 21). These shifts in frequency of the formants are characteristically found where speech sounds join; the formant transitions are presumed to reflect continuous movement of the articulators and the corresponding progressive changes in the relative sizes of the various resonance chambers that are involved in producing different sounds successively. The extent and direction of the transition of the second formant during vowel onset serves as a cue for distinguishing among the unvoiced stops, p, t, and k, among the corresponding voiced stops b, d, and g, and among the corresponding nasal consonants m, n, and η. Stated another way, the extent and direction of second-formant transition distinguishes p, b, m for t, d, n, from k, g, η; in fact, a single transition can serve for p, b, and m, another for t, d, and n, and a third for k, g, and η. With respect to possible cues serving to distinguish the classes of sounds from one another, it seems likely, on the basis of the Haskins experiments, that the voiced stops are distinguished from the unvoiced stops by the presence, in the former, of energy at the fundamental frequency of the voice. In addition, transitions of the first formant appear to contribute to the voicing. Also, a certain neutral and constant

1. "Formant" is the term applied to frequency regions of relatively high energy concentration. The formants appear in spectrograms as dark horizontal bars. Ordinarily, at least three formants can be identified in the vowel spectrograms; these are conventionally numbered beginning with the formant having the lowest frequency.

resonance, with frequency and intensity characteristics different from those of the vowel, will inpart the color of nasal consonants as a class, and, apparently, serves as a sufficient cue for distinguishing this class from the stop consonants.

Although neither the short burst nor the second-formant transition are entirely unambiguous cues, it appears that use of both cues would resolve many of the ambiguous cases. It should also be pointed out that the Haskins group has evidence from exploratory studies that the transition of the third formant contributes to consonant identification.

The attributes of vowel sounds important in their identification may be treated in less detail through a brief summary of the work of R. L. Miller (Ref. 23). This investigation has demonstrated by means of auditory tests with synthetic vowels that the position and relative amplitude of the first three formants and the fundamental frequency of the voice are high-ranking cues in vowel identification.

It appears, then, that several frequency cues can be isolated as fairly reliable contributors to speech sound identification. It is also evident that the frequencies involved in the multiple cues span a rather wide range. The center frequency of the third formant, for example, may be as far as 3500 cps away from the fundamental frequency of the voice. In the Haskins experiments utilizing bursts of noise, bursts above 4000 cps were found to serve as cues In addition, the results of articulation tests with low-pass filters show a contribution of frequencies in the neighborhood of 7000 cps (Ref. 10)[1].

1. French and Steinberg (Ref. 10) have shown that connected discourse is very intelligible when only the frequency components of the speech below 1900 cps reach the listener. It seems unlikely, however, that the listener habitually confines his attention to the frequencies below 1900 cps, since speech is also very intelligible when only the frequency components above 1900 cps are passed on to the listener.

4.3 The Effective Bandwidth for Speech Perception

Granting that the frequencies serving as cues in the identification of speech sounds vary rapidly over a wide range, considerations of the effective bandwidth of the auditory mechanism, with respect to a particular time base, become important. One might assume an average duration of speech sounds and calculate the range over which the mechanism could sweep in that time. However, since the experiments reported in Section 3 above were designed to demonstrate the existence of certain characteristics of the auditory mechanism, and the parameters of these characteristics have not yet been fully explored, it is not possible at the present time to report reliable estimates of the effective bandwidth attainable by a narrow-band device by virtue of a sweeping property. A rough estimate of the sweep rate in the vicinity of 1000 cps can be obtained by noting that, in the data reported in Section 3 from the study employing two equally likely frequencies, when the two frequencies are 300 cps apart, the percentage of correct detections falls on the theoretical curve predicated on "successive" observation. Acceptance of 300 cps as the maximum effective bandwidth given a 1/10 second signal, or of 3000 cycles per second per second as the maximum sweep rate, would have to be restricted to the range about 1000 cps. It seems unlikely that a sweep rate this slow would obtain throughout the audible range of frequencies; perhaps sweep rate increases logarithmically with increasing frequency.

It is also the case that estimates of sweep-rate, or of effective bandwidth derived from experiments employing low signal-to-noise ratios should probably not be extrapolated to the perception of stronger signals. It is not inconsistent with the working assumption of a narrow-band, sweeping device for strong-signal perception to suspect that certain parameters of the device will vary as a function of signal-to-noise ratio. Consider the curve labelled "successive" in Figures 20-23; it could be argued that this curve would rise again if continued further.

It may be assumed, however, for purposes of this discussion, that a mechanism utilizing narrow-band and sweeping properties in the perception of speech cannot intercept all of the frequencies contributing to the identification of speech sounds. The consideration of these properties as relevant to strong-signal perception, of course, depends upon the existence of a decrement with respect to strong signals when the range of possible signal frequencies is of this order of magnitude.

4.4 The Central Control of Sweep.

It is clear, on *a priori* grounds, that when signal frequencies of relatively short duration vary over a wide range, the probability of their interception by a narrow-band, sweeping device or the probability that the device is tuned to a given frequency at the time of its occurrence, is quite low, with a resultant loss of frequency information. This assumption, however, that a wide range of signal frequencies (wide relative to the signal duration) yields a low probability of interception, is valid only if all frequencies in the range have nearly equal probabilities of occurrence. If, at a given time, the signal frequencies differ in their probabilities and if, in addition, regulation of the position of the sweep is possible, then the probability of intercept need not be low.

It is well known that the information-carrying frequencies in speech are not, at a given point in a message, equally probable; and it may be contended, on the basis of the data reported above, that the listener is capable of regulating the sweep of the narrow-band. Thus, although some frequency information may not be picked up by such a mechanism, regulation of the sweep in accordance with the redundancy in the frequency structure of speech may result in the reception of a sufficient quantity of information to make possible the perception of speech with such a mechanism.

4.5. A Priori Probability and Speech Perception

It will be well to review briefly the extent and nature of redundancy in the frequency structure of speech in order to assess the extent to which the search of the mechanism can be guided by a priori or conditional probabilities.

The first paper to treat the statistical structure of conversation was authored by French, Carter, and Koenig (Ref. 9). The tendency for vowels and consonants to alternate, very likely the most effective form of redundancy with respect to the guidance of search, is brought out clearly by this study. Nine types of phonetic syllables occurred; no instances of two vowels in succession were found, and the syllables having two or more consecutive consonants comprised about 1/7 of the total number of syllables.[1] Since the differentiating characteristics of consonant sounds cover a wider range of frequencies than do the cues enabling vowel-sound recognition (see Potter, Kopp, and Green (Ref. 27)), there exists an excellent basis for a sweep pattern involving an alternation between a single, rapid sweep over a wide range of frequencies and an oscillating sweep over a narrower range. Since 2/3 of the syllables in the study began with consonants, there exists a basis for regulating the phase of such a pattern.

There are several other indications in the study by French, Carter, and Koenig, of redundancy in the frequency structure of speech. To give an

1. The present view of the redundancy in the frequency structure of speech will be confined to English. In terms of the present purposes, this is apparently a conservative, as well as convenient, choice; Newman (Ref. 24) for example, reports that English shows less restriction in terms of patterning than the other ten languages he studied.

example, one tabulation of their data shows that of the twenty-two vowels used in the analysis, roughly 1/2 of the occurrences are comprised of the six front vowels. That the front vowels, as a class, differ from middle and back vowels with respect to the positions of the first two formants has been demonstrated by Peterson and Barney (Ref. 25). French, Carter, and Koenig also present the conditional probability that a particular vowel will be followed by a particular consonant. Five vowels are followed by no consonant more than 50 percent of the time.

Another study of redundancy in English was made by Black (Ref. 1). With respect to the purposes of this discussion, the results are very similar to those discussed above. Black found transitional probabilities exceeding .33.

Thus, there is some basis for supposing that a narrow-band device whose sweep is susceptible to regulation by the listener, may be guided by *a priori* probabilities so that a substantial increase in the probability of intercept results. Still, the fact that this is a sweeping mechanism suggests that many of the interceptions are of sufficiently short duration to lead to considerable attenuation. If this is true, immediate decisions concerning the individual signals would lead to a high frequency of error. It may be, however, that if a portion of the speech sounds in a message are not detected, or if decisions are not made immediately concerning certain sounds, that those elements of the message are filled in on the basis of *a posteriori* probabilities, that is, on the basis of subsequent information.

4.6 A Posteriori Probability and Speech Perception

In the study made by French, Carter, and Koenig, five vowels appeared without a prior consonant more than 50 percent of the time. Black found transitional probabilities of one sound preceding another, of the same order of magnitude as the conditional probabilities of one sound following another.

It seems reasonable, then, that sounds not intercepted may be inferred solely on the basis of a priori or a posteriori probabilities. Even in this case, of course, decisions concerning individual elements of a message near the time of their transmission could lead to errors. A method of decoding that involves storage of the information received, and a single decision made at the conclusion of the message, may lead to appreciable reduction in error. This method of decoding is based on the principle discussed in Section 2 above, that the decision process is one in which information may be lost. The preservation of a posteriori probabilities, and a single decision based on the pattern of these at the end of the message, or at the end of some larger portion of the message than that represented by a single symbol, is far more efficient decoding process than a process calling for symbol-by-symbol decoding. A mathematical demonstration of the improvement which can be expected from using such a decoding process, which makes full use of the redundancy in the message structure, is reported by Tanner and Birdsall (Ref. 37).

4.7 Other Evidence of Redundancy in Speech Structure

To summarize the development to this point: it is supposed that redundancy in the frequency structure of speech plays two roles, one a forward-looking role giving information concerning where to observe next, the other involving both forward and backward conditional probabilities that are applied in the decoding process.

Up to this point, the extent of redundancy in speech has been treated in terms of statistical considerations, in terms of the patterning found in samples of the English language. In addition to the extent of redundancy in speech displayed by statistical studies, it should be pointed that speech sounds themselves often convey information about adjacent sounds. House and Fairbanks (Ref. 15) have shown, for example, that the fundamental frequency, phonetic power, and duration of vowels reflect the consonant context in which they occur. Curtis (Ref. 5) has summarized several acoustic studies reporting cues to the identification of speech sounds as residing in interactions among consecutive sounds. Thus, the listener does not have to depend entirely, for filling in missed portions of utterances, upon the a priori probability of occurrence of the various sounds and their sequential probabilities.

The primary data on the extent of redundancy in speech, of course, in terms of the present purposes, come neither from statistical nor acoustic studies of speech, but from studies devised to show directly its influence on perception. Several such studies have been performed; three of these demonstrate that information about adjacent sounds carried by the sounds themselves, independent of their sequential probabilities, is utilized by the perceiver. One of the investigations made at Haskins Laboratories, reviewed earlier in this section, showed consonant recognition to be dependent upon the formant transition of vowels following them. This suggests that consonants not received can be inferred from their influence on adjacent sounds, since, in this case, the burst of noise that is regarded as the acoustic counterpart of the consonant was not present in the message. In other studies, Harris (Ref. 14) and Carol Schatz (Ref. 30) employed spliced tape to show that vowel-consonant and consonant-vowel pairs suffer in intelligibility when each member of the pair is taken from another context.

Several other experiments have demonstrated the influence on perception of redundancy, or alternatively, of restricting the number of alternative messages. Miller, Heise, and Lichten (Ref. 22) compared performance across test materials comprised, respectively, of nonsense syllables, words in isolation, words in sentences, and digits. The size of the effect on perception of restricting the number of alternatives may be illustrated by pointing out that the same observers, under identical acoustic conditions, responded correctly to 95 percent of the digits and to 5 percent of the nonsense syllables presented. Or, again, in order for the observers to get 50 percent of the nonsense syllables correct, the signal-to-noise ratio had to be 17 db greater than the signal-to-noise ratio which resulted in a score of 50 percent for digits. Miller, Heise, and Lichten also conducted speech perception tests employing different sizes of vocabularies of monosyllabic words. At a signal-to-noise ratio of -9 db, 60 percent of the items of the 32-vocabulary were responded to correctly; when a vocabulary of approximately 1000 words was employed, a score of 60 percent required a signal-to-noise ratio of +9 db.

The very sizable effect of redundancy on the perception of speech is also shown by results published by French and Steinberg (Ref. 10). These investigators demonstrated that the intelligibility of sentences is near perfect at a signal-to-noise ratio that yields 70 percent correct responses to nonsense syllables. Apparently, speech sounds not recognized when a part of nonsense syllables are detected immediately when they are a part of meaningful sentences, or are inferred correctly on the basis of additional contextual information.

4.8 Summary

On the basis of data presented in the previous sections, it has been proposed that the hearing mechanism be characterized as a narrow-band, sweeping device. This view of the mechanism is not necessarily incompatible with the facts of speech perception. In particular, it has been suggested that the ability of the perceiver to control such parameters of the device as its sweeping property, and his ability to utilize a significantly large portion of the redundancy present in speech, serve as a framework for a theory of speech perception.

5. SUMMARY

This paper presents the results of an investigation of the auditory processes of detection and frequency analysis. The primary results may be reviewed briefly. In Section 2, data are reported that are inconsistent with the assumptions upon which the conventional threshold theory is based. The data support a different detection theory, one that incorporates a variable "cutoff" level that may be controlled by the observer in the interests of optimum behavior. The data reported in Section 3 are consistent with the existence of a certain selectivity of the auditory system with respect to frequency. In particular they are consistent with the propositions that the auditory system is sensitive to a small range of frequencies at a given time and that a change in the frequency region of sensitivity involves a process requiring time.

As stated in the introduction, the experiments reported are of a sampling nature in the sense that the signal parameters studied are restricted in range. This sort of experiment, however, is consistent with the present purpose. The interest is merely to establish the existence of certain properties of the hearing mechanism. It is unlikely that the results of a similar study employing a wider range of signal conditions would be qualitatively different than those reported above.

In an important sense, this paper is one concerned with method, with the mode of attack on psychological problems. To the extent that the viewpoint presented in this paper influences experimental design, the paper can be considered a contribution to methodology. It is the authors' opinion that the theory specifies the control of variables hitherto ignored to a large extent, that the theory emphasizes the statistical nature of the problem, and that it specifies a dependent experimental variable which has high face validity.

On the other hand, to the extent that the experimental results illuminate how the auditory mechanism functions, the paper can be viewed as one contributing to auditory theory. Further, since the data indicate that the observer is capable of exerting a significant degree of control over the parameters of the auditory mechanism, and that he uses an optimum process in selecting the parameters, the data have implications for general behavioral theory. The theory specifies ways in which sensory and central processes are related. It suggests ways in which central processes can be studied by means of psychophysical experiments.

Although the results illuminate how the auditory mechanism works, the claim is not made, of course, that it is possible to state from experiments or otherwise, what mechanism actually exists. Although the results display certain general properties of the auditory mechanism, they do not indicate the nature of the physical system that effects these properties. In discussing the data on frequency selectivity, for example, the terms "filter" and "sweeping" have been used. This terminology follows from one of several possible models for the physical system. The data could, perhaps, have been as well presented in terms of some other as-if representation. The selectivity characteristics exhibited in the data, however, are probably achieved by a mechanism that is very different in form from any of the models presently available. As a matter of fact, neither the square filter nor the Gaussian filter considered in Section 3 is physically realizable, in at least the sense that each requires an infinite time delay. Nevertheless, the possibility of convenient calculation justifies first-order comparisons of results to be expected on the two bases. The filter model is tentatively employed simply because it permits convenient assessment of the experimental results on selectivity. The same model is then used in discussing the results of the experiment requiring shifting of the range of sensitivity, the experiment employing a signal at either of two frequencies, although the data themselves do not demand the "sweeping-filter" concept. The data establish only that a price must be paid for observing signals at different frequencies. Strictly from the data reported above, it is impossible to determine whether the price is paid in terms of definition, or in terms of noise, or in terms of time as suggested by the model.

The purpose of the investigation was to derive a model useful in guiding further study of the processes of detection and frequency analysis. It is believed that this purpose has been accomplished. In any event, the results of the study have disclosed properties of the auditory system that must be taken into account in devising the procedure for many types of experiments on the auditory system.

APPENDIX

The first three assumptions listed in the text are not strictly necessary for the development of theory to handle the particular problems considered in this report. They are included because it is felt that they are assumptions defining a theory broader in scope than the particular applications described in the text. They define a class of theories, one of which is defined by the remaining postulates (4-11).

The first three postulates are included so that the reader might have a better understanding of the authors' overall theoretical position. Although the reasonableness of these postulates at this time is no more than a matter of faith, it is believed that an understanding of the faith of the authors may contribute to the understanding of their writings.

It should be recognized that the first three postulates may not be the best assumptions upon which to base the theory. For example, the following set may be better by some criteria:

A-1. A human being has a finite capacity for processing information.

A-2. A human being makes decisions only when forced to do so.

A-3. A human being optimizes.

Again, these are not intended to be complete. Many definitions are required before these postulates have any meaning. Nevertheless, it seems possible that the postulates 1-2-3 of the text could be consequences of the postulates A-1, A-2, A-3 stated above.

A set of postulates such as this furnishes a convenient basis for the design of experiments. It should be noted that all of the experiments reported in the text are such that the set of postulates describes a class of situations of which the experimental situations are members. With this basis, problems of trying to collect and analyze data involving apparently "infinite" amounts of information are not encountered.

REFERENCES

1. Black, J. W. "The Information of Sounds and Phonetic Diagrams of One- and Two-Syllable Words," JSHD, Vol. 19, pp. 397-412, 1954.

2. Blackwell, H. R., "Threshold Psychophysical Measurements, I. An Analysis and Evaluation," (in preparation).

3. Bross, I.D.J., Design for Decision, Macmillian Company, 1953.

4. Cooper, F. S., Delattre, P. C., Liberman, A.M., Borst, J. M., and Gerstman, L. J., "Some Experiments on the Perception of Synthetic Speech Sounds," J. Acoust. Soc. Am., Vol. 24, pp. 597-606, 1952.

5. Curtis, J. F., "Systematic Research in Experimental Phonetics 3. The Case for Dynamic Analysis in Acoustic Phonetics," JSHD, Vol. 19, pp. 147-157, 1954.

6. Fitzhugh, R., Personal Communication

7. Fletcher, H., "Auditory Patterns," Rev. Mod. Phys., Vol. 12, pp. 47-65, 1940.

8. Fox, W. C., "Signal Detectability: A Unified Description of Statistical Methods Employing Fixed and Sequential Observation Processes," Tech. Rpt. No. 19, Electronic Defense Group, University of Michigan, 1953.

9. French, N. R., Carter, C. W., Jr., and Koenig, W. J., "The Words and Sounds of Telephone Conversations," Bell Syst. Tech. J., Vol. IX, pp. 290-324, 1930.

10. French, N. R. and Steinberg, J. C., "Factors Governing the Intelligibility of Speech Sounds," J. Acoust. Soc. Am., Vol. 19, pp. 90-119, 1947.

11. Galambos, R. and Davis, H., "The Response of Single Auditory Nerve Fibers to Acoustic Stimulation," J. Neurophysio. 1943,6, p. 39-58.

12. Garner, W. R., "The Effect of Frequency Integration on Temporal Integration of Energy in the Ear," J. Acoust. Soc. Amer., 19, p.808-815, 1947.

13. Granit, R., Sensory Mechanisms of the Retina, Oxford University Press, 1947.

14. Harris, C. M., "A Study of the Building Blocks of Speech," J. Acoust. Soc. Am., Vol. 25, pp. 962-969, 1953.

15. House, A. S. and Fairbanks, G., "The Influence of Consonant Environment upon the Secondary Acoustical Characteristics of Vowels," J. Acoust. Soc. Am. Vol. 25, pp. 105-113, 1953.

16. Jones, L. V., and D. W. Fiske, "Models for Testing the Significance of Combined Results," Psych. Bull., Vol. 50, pp. 375-383, 1953.

17. Karoly, A. J. and Isaacson, R. L., "Scanning Mechanisms in Audition", Paper presented at Michigan Academy of Science, Ann Arbor, March, 1956.

18. Kuffler, S. W., "Electric Potential Changes at an Isolated Nerve-Muscle Junction," J. Neurophysiol., Vol. 5, p. 18, 1942.

19. Lawson, J. S. and Uhlenbeck, G. E., Threshold Signals, McGraw-Hill Book Company, p. 208, 1950.

20. Liberman, A. M., Delattre, P., and Cooper, F. S., "The Role of Selected Stimulus Variables in the Perception of the Unvoiced Stop Consonants," Amer. J. Psych., Vol. LXV, pp. 497-516, 1952.

21. Liberman, A. M., Delattre, P., Cooper, F. S., and Gerstman, L. J., "The Role of Consonant-Vowel Transitions in the Perception of the Stop and Nasal Consonants," Psych. Monographs, Vol. 68, No. 8, 1954.

22. Miller, G. A., Heise, G. A., and Lichten, W., "The Intelligibility of Speech as a Function of the Context of the Test Materials," J. Exp. Psychol, Vol. 41, 1951.

23. Miller, R. L., "Auditory Tests with Synthetic Vowels," J. Acoust. Soc. Am. 25, 1953, 114-121.

24. Newman, E. B., "The Pattern of Vowels and Consonants in Various Languages," Amer. J. Psychol., Vol. 64, pp. 369-379, 1951.

25. Peterson, G. E. and Barney, H. L., "Control Methods Used in a Study of the Vowels," J. Acoust. Soc. Am., Vol. 24, pp. 175-184, 1952.

26. Peterson, W. W. and Birdsall, T. G., "Theory of Signal Detectability," Part I and II, Tech. Rpt. No. 13, Electronic Defense Group, University of Michigan, 1953. Also available in 1954 Symposium on Information Theory, Transactions of the IRE, September, 1954.

27. Potter, R. K., Kopp, G. A. and Green, H.C., Visible Speech, Van Nostrand, New York, 1947.

28. Roberts, G. A., "An Automatic Random Programmer," Department of Electrical Engineering, Engineering Research Institute, Electronic Defense Group, University of Michigan,

29. Schafer, T. H., Gales, R. S., Shewmaker, C. A., Thompson, P. O., "Frequency Selectivity of the Ear as Determined by Masking Experiments," J. Acoust. Soc. Am., Vol. 22, pp. 410-496, 1950.

30. Schatz, Carol D., "The Role of Context in the Perception of Stops," Language, Vol. 30, No. 1, 1954.

31. Shannon, C. E. and Weaver, W., The Mathematical Theory of Communication, Univ. of Ill. Press, 1949.

32. Skoglund, C. R., "The Response to Linearly Increasing Currents in Mammalian Motor and Sensory Nerves," Acta Physiol. Scand, Vol. 4, Suppl. 12, 1942.

33. Smith, M. and Wilson, Edna, "A Model of the Auditory Threshold and Its Application to the Problem of the Multiple Observer," Psychol Monogr., Vol. 67, No. 9, 1953.

34. Swets, J. A. and Birdsall, T. G., "Detection and Recognition: Testing Multiple Hypotheses," Technical Report , Electronic Defense Group, University of Michigan (in preparation).

35. Swets, J. A., Tanner, W. P., Jr., and Birdsall, T. G., "The Evidence for a Decision-Making Theory of Visual Detection," Tech. Rpt. No. 40, Electronic Defense Group, University of Michigan, 1955.

36. Tanner, W. P., Jr., "A Theory of Recognition," Technical Report No. 50, Electronic Defense Group, University of Michigan, 1955.

37. Tanner, W. P., Jr. and Birdsall, T. G., "A Class of Binary Block Codes with Error-Reducing Properties," Technical Report No. 41, Electronic Defense Group, University of Michigan, (in preparation).

38. Tanner, W. P., Jr., Swets, J. A., "A Decision-Making Theory of Visual Detection," Psychol. Rev., Vol. 61, No. 6, 1954.

39. Van Meter, D. and Middleton, D., "Modern Statistical Approaches to Reception in Communication Theory," 1954 Symposium on Information Theory, Transactions of IRE, September, 1954.

40. Wald, A., Sequential Analysis, John Wiley and Sons, 1947.

41. Woodward, P. M. and Davies, I. L., "Information Theory and Inverse Probability in Telecommunication Proc. IEE, Vol. 99, Part III, 1952.

DISTRIBUTION LIST

1 Copy	Director, Electronic Research Laboratory Stanford University Stanford, California Attn: Dean Fred Terman
1 Copy	Commanding General Army Electronic Proving Ground Fort Huachuca, Arizona Attn: Director, Electronic Warfare Department
1 Copy	Chief, Research and Development Division Office of the Chief Signal Officer Department of the Army Washington 25, D. C. Attn: SIGEB
1 Copy	Chief, Plans and Operations Division Office of the Chief Signal Officer Washington 25, D. C. Attn: SIGEW
1 Copy	Countermeasures Laboratory Gilfillan Brothers, Inc. 1815 Venice Blvd. Los Angeles 6, California
1 Copy	Commanding Officer White Sands Signal Corps Agency White Sands Proving Ground Las Cruces, New Mexico Attn: SIGWS-CM
1 Copy	Commanding Officer Signal Corps Electronics Research Unit 9560th TSU Mountain View, California
60 Copies	Transportation Officer, SCEL Evans Signal Laboratory Building No. 42, Belmar, New Jersey FOR - SCEL Accountable Officer Inspect at Destination File No. 22824-PH-54-91(1701)

1 Copy	J. A. Boyd Engineering Research Institute University of Michigan Ann Arbor, Michigan
1 Copy	H. W. Welch, Jr. Engineering Research Institute University of Michigan Ann Arbor, Michigan
1 Copy	Document Room Willow Run Research Center University of Michigan Willow Run, Michigan
10 Copies	Electronic Defense Group Project File University of Michigan Ann Arbor, Michigan
1 Copy	Engineering Research Institute Project File University of Michigan Ann Arbor, Michigan