GREAT LAKES RESEARCH INSTITUTE

UNIVERSITY OF MICHIGAN

Contribution No. 2

A Dynamic Height Method for the Determination of Currents in Deep Lakes

John C. Ayers

Contribution No. 3

Simplified Computations for the Dynamic Height Method of Current Determination in Lakes

John C. Ayers and Roger Bachmann

ANN ARBOR, MICHIGAN

1957

Reprinted from LIMNOLOGY AND OCEANOGRAPHY
Vol. II, No. 2, April, 1957
Printed in U.S.A.

Simplified Computations for the Dynamic Height Method of Current Determination in Lakes[1]

INTRODUCTION

A dynamic height method for the determination of current directions and velocities in deep lakes has been developed by Ayers (1956). This method involves very considerable amounts of arithmetical calculation and hence contains numerous possibilities for arithmetical error. Successful reduction of the amount of computation required would achieve both economy of time and reduced chances for error. This paper presents a successful simplified computation routine for the freshwater dynamic height method.

The simplified computation routine consists of a more convenient rearrangement of the original computation method and in no way changes the theoretical considerations upon which that method was based. The reader is referred to Ayers (1956) for the full development of theory and the original method.

Formal thanks are tendered to the University of Michigan Biological Station, where much of this work was carried out.

THE SIMPLIFIED METHOD

The new method differs from the original in two ways: 1) the expansive effects of temperature and the compressive effects of pressure are independently determined and applied, and 2) new tables directly readable to 0.1°C have been prepared. Independent application of the thermal and pressure effects eliminates the laborious double interpolations commonly required in the original method.

The new method first determines the specific volume anomaly at observed temperature and zero hydrostatic pressure ($10^5\ \delta_{t,o}$), then applies a compression correction ($\Delta a_{t,p}$) at observed temperature and *in situ* pressure to obtain the anomaly at temperature and pressure. The new specific volume anomaly table (Table 1) was prepared by graphing the zero-pressure portions of Ayers' (*loc. cit.*, Table 2) original anomaly table and reading anomaly values ($10^5\ \delta_{t,o}$) at each 0.1°C. The new table of coefficients of compression (Table 2) was prepared by graphing and reading at each 0.1°C Ayers' (*loc. cit.*, Table 1) compressibility coefficients. As in the original paper, the coefficient of compressibility (in cm^3/cm^3/atmosphere) is numerically equal to compression in cm^3/atmosphere (C_t) when applied to a volume of one cubic centimeter. The pressure effects are handled, as in the old method, by the decibar system: pressure in atmospheres, p = depth in meters ÷ 10 meters/atmosphere. As before, $10^5\, C_t \times p = \Delta a_{t,p}$ and $10^5\, \delta_{t,o} - \Delta a_{t,p} = 10^5\ \delta_{t,p}$, the required anomaly at temperature and pressure.

As in the old method, the resultant anomaly ($10^5\ \delta_{t,p}$) is required for the surface temperature, for each subsurface inflection of the temperature curve, and for the temperature at the reference level.

COMPARISONS OF THE NEW AND OLD METHODS

As indicated above, the required resultant anomaly for any subsurface temperature and pressure can be obtained (in the new method) by two table readings, one division, one multiplication, and one subtraction—a total of five operations. Under the old method obtaining a single resultant anomaly at temperature and pressure involved a total of thirteen operations as follows:

---

[1] Contribution No. 3 from the Great Lakes Research Institute, University of Michigan, and Contribution No. 883 from the Woods Hole Oceanographic Institution.

TABLE 1. *Specific volume anomaly* ($10^5\,\delta_{t,o}$)

°C	0	.1	.2	.3	.4	.5	.6	.7	.8	.9
0	13.0	12.4	11.5	10.8	10.2	9.6	9.0	8.4	7.9	7.4
1	7.0	6.6	6.2	5.7	5.3	4.9	4.6	4.2	3.8	3.5
2	3.0	2.8	2.6	2.3	2.0	1.8	1.6	1.4	1.2	1.1
3	1.0	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
4	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
5	1.0	1.1	1.2	1.3	1.5	1.7	1.9	2.1	2.4	2.7
6	3.0	3.3	3.7	4.1	4.5	4.9	5.3	5.7	6.1	6.5
7	7.0	7.5	8.0	8.5	9.0	9.5	10.0	10.5	11.0	11.5
8	12.0	12.6	13.2	13.9	14.6	15.3	16.0	16.7	17.4	18.3
9	19.0	19.7	20.4	21.2	22.0	22.8	23.6	24.4	25.3	26.1
10	27.0	28.0	29.0	30.0	31.0	32.0	33.0	34.0	35.0	36.0
11	37.0	38.0	39.1	40.2	41.3	42.5	43.6	44.7	45.8	46.9
12	48.0	49.1	50.2	51.4	52.6	53.8	55.0	56.2	57.4	58.7
13	60.0	61.3	62.6	63.9	65.2	66.5	67.8	69.1	70.4	71.7
14	73.0	74.4	75.8	77.2	78.6	80.0	81.4	82.8	84.2	85.6
15	87.0	88.6	90.2	91.8	93.4	95.0	96.6	98.2	99.8	101.4
16	103.0	104.7	106.4	108.1	109.8	111.5	113.2	114.9	116.6	118.3
17	120.0	121.8	123.6	125.4	127.2	129.0	130.8	132.6	134.4	136.2
18	138.0	139.9	141.8	143.7	145.6	147.5	149.4	151.3	153.2	155.1
19	157.0	159.0	161.0	163.0	165.0	167.0	169.0	171.0	173.0	175.0
20	177.0	179.1	181.2	183.3	185.4	187.5	189.6	191.7	193.8	195.9
21	198.0	200.3	202.6	204.9	207.2	209.5	211.8	214.1	216.4	218.7
22	221.0	223.3	225.6	227.9	230.2	232.5	234.8	237.1	239.4	241.7
23	244.0	246.4	248.8	251.2	253.6	256.0	258.4	260.8	263.2	265.6
24	268.0	270.6	273.2	275.8	278.4	281.0	283.6	286.2	288.8	291.4

TABLE 2. *Coefficients of compression* ($10^5\,C_t$)

°C	0	.1	.2	.3	.4	.5	.6	.7	.8	.9
0	5.250	5.243	5.236	5.230	5.224	5.219	5.214	5.209	5.204	5.200
1	.195	.191	.187	.183	.179	.176	.173	.170	.167	.163
2	.160	.157	.154	.151	.148	.145	.142	.139	.136	.133
3	.130	.128	.125	.122	.120	.118	.115	.112	.110	.108
4	.105	.103	.100	.098	.095	.092	.089	.087	.085	.082
5	.080	.078	.076	.074	.072	.070	.068	.066	.064	.062
6	.060	.058	.057	.055	.053	.051	.049	.047	.045	.044
7	.043	.041	.039	.038	.036	.034	.033	.031	.029	.028
8	.027	.025	.024	.023	.022	.020	.019	.018	.016	.014
9	.013	.012	.011	.010	.008	.007	.006	.004	.003	.001
10	5.000	4.998	4.997	4.996	4.994	4.993	4.992	4.990	4.989	4.987
11	4.985	.984	.983	.982	.981	.980	.979	.978	.977	.976
12	.975	.974	.973	.971	.970	.969	.968	.967	.966	.965
13	.964	.962	.961	.960	.959	.958	.957	.956	.955	.954
14	.953	.953	.952	.951	.950	.949	.948	.947	.946	.945
15	.944	.943	.942	.941	.941	.940	.939	.938	.938	.937
16	.936	.936	.935	.934	.933	.932	.932	.931	.931	.930
17	.929	.929	.928	.928	.927	.926	.925	.924	.923	.923
18	.922	.921	.921	.920	.919	.919	.918	.918	.917	.916
19	.916	.915	.914	.914	.913	.913	.912	.912	.911	.911
20	.910	.910	.909	.909	.908	.908	.907	.907	.906	.906
21	.905	.905	.904	.904	.903	.903	.903	.903	.902	.902
22	.902	.902	.901	.901	.901	.901	.900	.900	.899	.899
23	.898	.898	.898	.897	.897	.896	.896	.895	.894	.894
24	.893									

a. Reading anomalies at temperatures adjacent to the observed temperature and at pressure less than observed pressure (2 table readings)

b. Interpolating to anomaly at observed temperature and less than observed pressure (1 subtraction, 1 multiplication, and 1 addition)

c. Reading anomalies at temperatures adjacent to observed temperature and at pressure greater than observed pressure (2 table readings)

d. Interpolating to anomaly at observed temperature and greater than observed pressure (3 operations as in b above)

e. Interpolating between anomalies at observed temperature (items b and d above) for anomaly at observed temperature *and* pressure (3 operations as in b above).

The new computation reduces to less than half the number of operations (and the time, labor, and chance for error) involved in obtaining each individual resultant specific volume anomaly.

Subsequent computations leading to the final dynamic height at each station are carried out as in the original paper and are no less time-consuming and laborious than before. They do benefit, however, by the lesser chance for error inherent in the new method.

As a comparison of the accuracy of the two methods, the dynamic heights of 56 stations taken in Lake Michigan on 28 June 1955 have been worked out by each method. While there were minor variations in the insignificant terminal decimal places, the final dynamic heights rounded off to the same values by either method. There is, then, no loss of accuracy resulting from the simpler new method. The total labor (with calculator) required for the two determinations of this topography was 31 man-hours by the original method and 13.5 man-hours by the new.

REFERENCES

AYERS, J. C. 1956. A dynamic height method for the determination of currents in deep lakes. Limnol. & Oceanogr., **1**: 150–161.

JOHN C. AYERS
Great Lakes Research Institute and Department of Zoology, University of Michigan; and Woods Hole Oceanographic Institution

ROGER BACHMANN
Idaho Cooperative Wildlife Research Unit, University of Idaho, Moscow, Idaho

Reprinted from LIMNOLOGY AND OCEANOGRAPHY
Vol. 1, No. 3, July, 1956
Printed in U.S.A.

A Dynamic Height Method for the Determination of Currents in Deep Lakes[1]

JOHN C. AYERS

Department of Conservation, Cornell University and Woods Hole Oceanographic Institution

ABSTRACT

An adaptation of the oceanographer's dynamic height method to freshwater conditions, the method develops directly from the Smithsonian Tables of water density as a function of temperature and from Amagat's coefficients of compressibility. Pressure effects are handled by use of the decibar system. The calculated dynamic heights are plotted and contoured. Current directions are obtained by application of the geostrophic principle; current velocities are computed from the slopes of the surface topography. A table of specific volume anomalies, necessary for the height computation, has been developed. Sources of potential error are reviewed and assessed. The method has been applied to three synoptic surveys of Lake Huron, and the results obtained are both internally consistent and in good agreement with the behaviors of other parameters. When used with proper caution the method appears to be a promising technique for the study of certain circulation phenomena of large deep lakes.

INTRODUCTION

It is well known that freshwater is slightly compressed by pressure and expanded by temperatures above and below 4° Centigrade. For some decades the oceanographer has made use of similar phenomena to calculate current direction and velocity from density data, and from the reciprocal of density—specific volume. It appears that with certain modifications the oceanographer's dynamic height method of current determination can be applied to at least the deeper lakes.

The degree to which this application may be limited by the shallowness of lakes is not yet clear. The method depends upon the selection of a subsurface reference level at which currents are virtually absent. It is imperative that the reference level be below the depth of the turbulent mixing and return flow that result from wind action. It appears from the work of Mortimer (1951, 1954) that turbulent mixing and return may, under ordinary winds, attain two or three times the mean depth of the thermocline. The depth to which summer warming penetrates may also be used as a measure of the over-all depth of wind-caused mixing; the latter was used in Lake Huron (Table 3). On the basis of our present knowledge, the method appears to be applicable in lakes where the preponderance of depths is more than three times the mean depth to the thermocline.

It is strongly urged that currents determined by this method be checked against currents obtained by other means; lack of agreement may indicate that significant bottom currents are present and that bottom stress must be considered. The dynamic height method may be unsuccessful during periods of spring and fall turnover.

THE METHOD

The dynamic (or geopotential) height method deals with the calculated lengths of water columns which are considered to be standing upon the horizontal reference level. In each component segment of the water column at each station the resultant effect, on length, of compression due to hydrostatic pressure of overlying water and expansion due to temperatures above or below 4° C is determined. Summation of the resultant effects in all the component segments gives the column's length change resulting from the expansions and/or compressions of its parts. Addition of this

---

[1] Contribution No. 2 from the Great Lakes Research Institute and Contribution No. 823 from the Woods Hole Oceanographic Institution.

length change to the theoretical length of the column under standard conditions gives the calculated height of the column top above the reference level under the prevailing conditions of pressure and temperature. The differing heights of the column tops at the various stations produce a "topography" of the water surface. From this dynamic or geopotential topography the current directions and velocities are obtained.

The freshwater method's primary length unit is the centimeter. Its use allows unusual convenience in operating with specific volume.

The effects of temperature on volume per unit mass have been obtained from the Smithsonian Tables as quoted in the *Handbook of Chemistry and Physics* (1949, Table of relative density and volume of water). The volume per unit mass is the specific volume.

$$\text{Volume/unit mass} = \text{specific volume} = a_{t,o} \text{ (at observed temperature and zero hydrostatic pressure)} \qquad (1)$$

The coefficients of compressibility used are those of Amagat (1893)[2] as given in the *Handbook of Chemistry and Physics* (*op. cit.*, Table of compressibility of liquids). Amagat's coefficients for 1 to 25 atmospheres pressure at 0, 10, and 20° C were graphed, and the graph extrapolated to 30° C; from the graph the coefficient at each degree was obtained (Table 1).

Amagat's unit "contraction in unit volume per atmosphere" (cm³/cm³/atm) when multiplied by the number of volumes involved becomes compression per atmosphere (cm³/atm); when one specific volume is used the compression is numerically the same as the coefficient and has the unit cm³/gm/atm.

TABLE 1. *Compressibility coefficients and compression in unit volume* (C_t) *at temperatures between 0 and 30 degrees Centigrade, based on Amagat (1893)*

°C	Compressibility coefficient* (cm³/cm³/atm)	Compression in 1 cm³ (C_t)* (cm³/atm)
	$\times 10^{-6}$	$\times 10^{-6}$
0	52.50	52.50
1	51.95	51.95
2	51.60	51.60
3	51.30	51.30
4	51.05	51.05
5	50.80	50.80
6	50.60	50.60
7	50.43	50.43
8	50.27	50.27
9	50.13	50.13
10	50.00	50.00
11	49.85	49.85
12	49.75	49.75
13	49.64	49.64
14	49.53	49.53
15	49.44	49.44
16	49.35	49.35
17	49.30	49.30
18	49.22	49.22
19	49.15	49.15
20	49.10	49.10
21	49.05	49.05
22	49.02	49.02
23	48.98	48.98
24	48.93	48.93
25	48.90	48.90
26	48.88	48.88
27	48.85	48.85
28	48.82	48.82
29	48.81	48.81
30	48.80	48.80

* Second decimal places only approximate.

Since one atmosphere of hydrostatic pressure = 1 bar = 10 decibars = the hydrostatic pressure of ten meters of water, and one meter of water produces one decibar of pressure, the depth in meters is therefore essentially numerically equal to the pressure in decibars. Also

$$\text{Depth in meters}/10 = \text{atmospheres pressure} = p \qquad (2)$$

Compression at observed temperature, C_t, (Table 1) multiplied by the *in situ* pressure at depth gives compression *in situ*

$$C_t \times p = \text{compression } in\ situ = \Delta a_{t,p}. \qquad (3)$$

It is assumed that a lake has constant surface area during the short period when data are being collected, i.e., the lake consists of a finite number of water columns each of constant 1 cm² cross section. Volume changes may then be considered as consisting entirely of changes in length, and each component specific volume of

---

[2] The author is aware of Beyer's (1954) conclusion that Amagat's coefficients are slightly too large. The use of Amagat's coefficients may introduce a small systematic error.

TABLE 2. *Specific volume anomaly* ($10^5\delta_{t,p}$)*
(Specific volume length anomaly when applied to columns of 1 cm^2 cross section)

Depth		Pressure, atmospheres	Temperature, °C							
feet	meters		0	1	2	3	4	5	6	7
0	0	0	13.0	7.0	3.0	1.0	0	1.0	3.0	7.0
	5	0.5	10.4	4.4	0.4	−1.6	−2.5	−1.5	0.5	4.5
	10	1.0	7.8	1.8	−2.2	−4.1	−5.1	−4.1	−2.1	2.0
	15	1.5	5.1	−0.8	−4.7	−6.7	−7.7	−6.6	−4.6	−0.6
50	15.2	1.52	5.0	−0.9	−4.8	−6.8	−7.8	−6.7	−4.7	−0.7
	25	2.5	−0.1	−6.0	−9.9	−11.8	−12.8	−11.7	−9.7	−5.6
100	30.5	3.05	−3.0	−8.8	−12.7	−14.6	−15.6	−14.5	−12.4	−8.4
	35	3.5	−5.4	−11.2	−15.1	−17.0	−17.9	−16.8	−14.7	−10.7
150	45.7	4.57	−11.0	−16.7	−20.6	−22.4	−23.3	−22.2	−20.1	−16.1
	50	5.0	−13.3	−19.0	−22.8	−24.7	−25.5	−24.4	−22.3	−18.2
	100	10.	−39.5	−45.0	−48.6	−50.3	−51.1	−49.8	−47.6	−43.4
	150	15.	−65.8	−70.9	−74.4	−76.0	−76.6	−75.2	−72.9	−68.7
	200	20.	−92.0	−96.9	−100.2	−101.6	−102.1	−100.6	−98.2	−93.9
	250	25.	−118.3	−122.9	−126.0	−127.3	−127.6	−126.0	−123.5	−119.1

Depth		Pressure, atmospheres	Temperature, °C							
feet	meters		7	8	9	10	11	12	13	14
0	0	0	7.0	12.0	19.0	27.0	37.0	48.0	60.0	73.0
	5	0.5	4.5	9.5	16.5	24.5	34.5	45.5	57.5	70.5
	10	1.0	2.0	7.0	14.0	22.0	32.0	43.0	55.0	68.0
	15	1.5	−0.6	4.5	11.5	19.5	29.5	40.5	52.6	65.6
50	15.2	1.52	−0.7	4.4	11.4	19.4	29.4	40.4	52.5	65.5
	25	2.5	−5.6	−0.6	6.5	14.5	24.5	35.6	47.6	60.6
100	30.5	3.05	−8.4	−3.3	3.7	11.8	21.8	32.8	44.9	57.9
	35	3.5	−10.7	−5.6	1.5	9.5	19.6	30.6	42.6	55.7
150	45.7	4.57	−16.1	−11.0	−3.9	4.2	14.2	25.3	37.3	50.4
	50	5.0	−18.2	−13.1	−6.1	2.0	12.1	23.1	35.2	48.2
	100	10.	−43.4	−38.3	−31.1	−23.0	−12.9	−1.8	10.4	23.5
	150	15.	−68.7	−63.4	−56.2	−48.0	−37.8	−26.6	−14.5	−1.3
	200	20.	−93.9	−88.5	−81.3	−73.0	−62.7	−51.5	−39.3	−26.1
	250	25.	−119.1	−113.7	−106.3	−98.0	−87.6	−76.4	−64.1	−50.8

Depth		Pressure, atmospheres	Temperature, °C							
feet	meters		14	15	16	17	18	19	20	21
0	0	0	73.0	87.0	103.0	120.0	138.0	157.0	177.0	198.0
	5	0.5	70.5	84.5	100.5	117.5	135.5	154.5	174.5	195.6
	10	1.0	68.0	82.1	98.1	115.1	133.1	152.1	172.1	193.1
	15	1.5	65.6	79.6	95.6	112.6	130.6	149.6	169.6	190.6
50	15.2	1.52	65.5	79.5	95.5	112.5	130.5	149.5	169.5	190.5
	25	2.5	60.6	74.6	90.7	107.7	125.7	144.7	164.7	185.7
100	30.5	3.05	57.9	71.9	88.0	105.0	123.0	142.0	162.0	183.0
	35	3.5	55.7	69.7	85.7	102.7	120.8	139.8	159.8	180.8
150	45.7	4.57	50.4	64.4	80.5	97.5	115.5	134.5	154.6	175.6
	50	5.0	48.2	62.3	78.3	95.4	113.4	132.4	152.5	173.5
	100	10.	23.5	37.6	53.7	70.7	88.8	107.9	127.9	149.0
	150	15.	−1.3	12.8	29.0	46.1	64.2	83.3	103.4	124.4
	200	20.	−26.1	−11.9	4.3	21.4	39.6	58.7	78.8	99.9
	250	25.	−50.8	−36.6	−20.4	−3.3	15.0	34.1	54.3	75.4

Depth		Pressure, atmospheres	Temperature, °C							
feet	meters		21	22	23	24	25	26	27	28
0	0	0	198.0	221.0	244.0	268.0	294.0	320.0	347.0	375.0
	5	0.5	195.6	218.6	241.6	265.6	291.6	317.6	344.6	372.6
	10	1.0	193.1	216.1	239.1	263.1	289.1	315.1	342.1	370.1
	15	1.5	190.6	213.7	236.7	260.7	286.7	312.7	339.7	367.7
50	15.2	1.52	190.5	213.6	236.6	260.6	286.6	312.6	339.6	367.6
	25	2.5	185.7	208.7	231.8	255.8	281.8	307.8	334.8	362.8
100	30.5	3.05	183.0	206.1	229.1	253.1	279.1	305.1	332.1	360.1
	35	3.5	180.8	203.8	226.9	250.9	276.9	302.9	329.9	357.9
150	45.7	4.57	175.6	198.6	221.6	245.6	271.7	297.7	324.7	352.7
	50	5.0	173.5	196.5	219.5	243.5	269.6	295.6	322.6	350.6
	100	10.	149.0	172.0	195.0	219.1	245.1	271.1	298.2	326.2
	150	15.	124.4	147.5	170.5	194.6	220.7	246.7	273.7	301.8
	200	20.	99.9	123.0	146.0	170.1	196.2	222.2	249.3	277.4
	250	25.	75.4	98.5	121.6	145.7	171.8	197.8	224.9	253.0

* Decimals are only approximate.

each column may be considered to contain unit mass in a 'cube' of 1 cm² cross section and variable length. Thus compression effects may be subtracted from the specific volume at temperature to obtain the resultant length of the specific volume 'cube' at *in situ* temperature and pressure

$$a_{t,o} - \Delta a_{t,p} = \text{specific volume length } \textit{in situ} = a_{t,p}. \qquad (4)$$

To reduce the physical size of the numbers it is convenient to work with the specific volume length anomaly which, further, is multiplied temporarily by 10^5.

$$a_{t,p} - 1.000000 = \text{specific volume length anomaly} = \delta_{t,p} \qquad (5)$$

The computations leading to the tabulated (Table 2) specific volume length anomaly of water at 15° C and 25 meters depth will serve as an example of the way the table was constructed.

$$a \text{ at } 15^\circ,\ 0 \text{ hydrostatic pressure} = a_{t,o} = 1.00087 \text{ cm}^3/\text{gm} \qquad (\text{Eq. } 1)$$

$$p = 25 \text{ meters} \div 10 \text{ meters/atm} = 2.5 \text{ atm} \qquad (\text{Eq. } 2)$$

$$\Delta a_{15,2.5} = pC_t = 2.5 \text{ atm} \times 49.44 \times 10^{-6}\text{cm}^3/\text{gm/atm} = 123.6 \times 10^{-6}\text{cm}^3/\text{gm} \qquad (\text{Eq. } 3)$$

$$a_{15,2.5} = (1.00087 \text{ cm}^3/\text{gm} \div 1 \text{ cm}^2/\text{gm}) - (123.6 \times 10^{-6}\text{cm}^3/\text{gm} \div 1 \text{ cm}^2/\text{gm}) = 1.00087 \text{ cm} - 123.6 \times 10^{-6} \text{ cm} = 1.000746 \text{ cm} \qquad (\text{Eq. } 4)$$

$$\delta_{15,2.5} = 1.000746 \text{ cm} - 1 \text{ cm} = 0.000746 \text{ cm} \qquad (\text{Eq. } 5)$$

$$10^5\delta_{15,2.5} = 74.6 \text{ cm}$$

(see $t = 15^\circ$, $p = 2.5$ atm in Table 2)

The specific volume length anomalies ($10^5\delta_{t,p}$) for each degree between 0 and 28° C and at several pressure levels are given in Table 2. This table is of minimal size so far as numbers of pressure levels are concerned. The user is advised to expand it by adding other pressure levels in the depth (pressure) range which pertains to his lake.

Computation begins with a fairly detailed reading of the temperature profile, in which the profile is broken down into its essentially linear portions and the temperature and depth (pressure) of each inflection point recorded. Enough inflection points should be read to reproduce the profile well when these points on a graph are connected by straight lines.

The required specific volume length anomaly for the surface temperature is found by interpolation between the proper values of Table 2. Similarly the anomaly for each inflection point is found, by single interpolation when the depth (pressure) falls on a tabulated pressure level, or by double interpolation when observed temperature and pressure both fall between tabulated values.

The anomaly is determined for the surface, for each inflection point, and for the reference level. The anomalies are averaged by successive pairs and each average is multiplied by the depth interval (in centimeters) covered by that pair to give the calculated length anomaly, ΔD, of that segment of the water column. Each ΔD is divided by 10^5 to return it to actual value; the ΔD's are summed cumulatively from zero at the surface down to the reference level; and the cumulated sum at the reference level is converted to meters (see Table 3). The cumulated length anomaly added to the reference level's depth under standard conditions gives the calculated length of the existing unit-cross-section water column. The final calculated length of the water column is its dynamic height or geopotential height. In Table 3 the reference level was the 60-decibar surface whose depth under standard conditions would be 60 meters.

At stations where the depth is less than that of the chosen reference level it is permissible to substitute the lower parts of a near-by station of adequate depth, as in Figure 4. When this is done the isobaric surfaces become horizontal (slopeless and currentless) between the identical parts of the two stations. This device was suggested by Nansen and introduced by Helland-Hansen (1934). It is apparently satisfactory in places where bottom currents are virtually absent, but may be seriously in error in places where currents

TABLE 3. *Dynamic height computation, Station 40, survey* Synoptic II, *27 July 1954*

Depth cm	Temperature °C	$10^5\ \delta_{t,p}$ at temp. & depth	Average	Depth interval	Product	ΔD, cm	Cumulative ΔD, cm	60-decibar ΔD	
								cm	meters
0	18.6	149.4					0		
			141.5	520	73580	0.74			
520	17.9	133.6					0.74		
			99.4	520	51688	0.52			
1040	13.8	65.2					1.26		
			49.0	480	23520	0.24			
1520	11.3	32.7					1.50		
			9.2	1530	14076	0.14			
3050	5.1	−14.3					1.64		
			−16.7	760	−12692	−0.13			
3810	4.4	−19.0					1.51		
			−21.0	760	−15960	−0.16			
4570	4.3	−23.0					1.35		
			−26.7	1430	−38181	−0.38			
6000	4.3	−30.3					0.97	0.97	0.010
			add reference level depth at standard conditions						60.000 meters
			Dynamic height in dynamic meters						60.010

along the bottom are significant. This device should be used only with great caution at stations so shallow that major parts of the water column require substitution of values.

Current directions are obtained by plotting the dynamic height for each station and drawing height contours of the surface topography at regular intervals. Current-direction arrowheads may be drawn on the contour lines by application of the geostrophic principle: the currents are situated on the slopes of the topography, flow parallel to the contours, and flow in such direction that the topographic 'high' is on the right (in the Northern Hemisphere) when the observer looks in the direction of flow.

Current velocity between each pair of stations can be obtained from: $v = 10\ i_d/0.000145 \sin \phi$ (Sverdrup *et al.* 1942: 392) where i_d is the slope of the surface between the stations and $\sin \phi$ is the natural trigonometric function of the latitude of the mid-point between the station pair. If the slope is expressed in meters of height difference per meter of horizontal distance between the stations, the velocity will be in meters per second; if the slope is in cm/m, velocity will be in centimeters per second. LaFond (1951: 98) gives a similar method of calculating current velocity from the same parameters.

Since the distance between contours is inversely proportional to the velocity, it is convenient to determine the velocities of the fastest current, the slowest current, and of a few intermediate ones, then to construct a trumpet-shaped graph (see Figs. 1, 2, and 3) from which velocity may be read by transferring the intercontour distance with dividers.

APPLICATIONS OF THE METHOD

The dynamic height method has been used to determine the surface current patterns and velocities in Lake Huron at three different times in the summer of 1954. At the end of June the major part of the lake was still in essentially its spring condition —with no thermocline yet formed. At this time the current pattern (Fig. 1) was almost identical to that deduced by Harrington (1895) from drift bottles released by lake steamers throughout the entire shipping season, approximately late March to early December. It was also in agreement with the results of Millar (1952) who confirmed Harrington's current pattern by studies of intake-water temperatures on lake steamers throughout the shipping season. For these reasons it is believed that the pattern observed in June approximates the "fundamental" winter-to-winter circulation pattern of the lake.

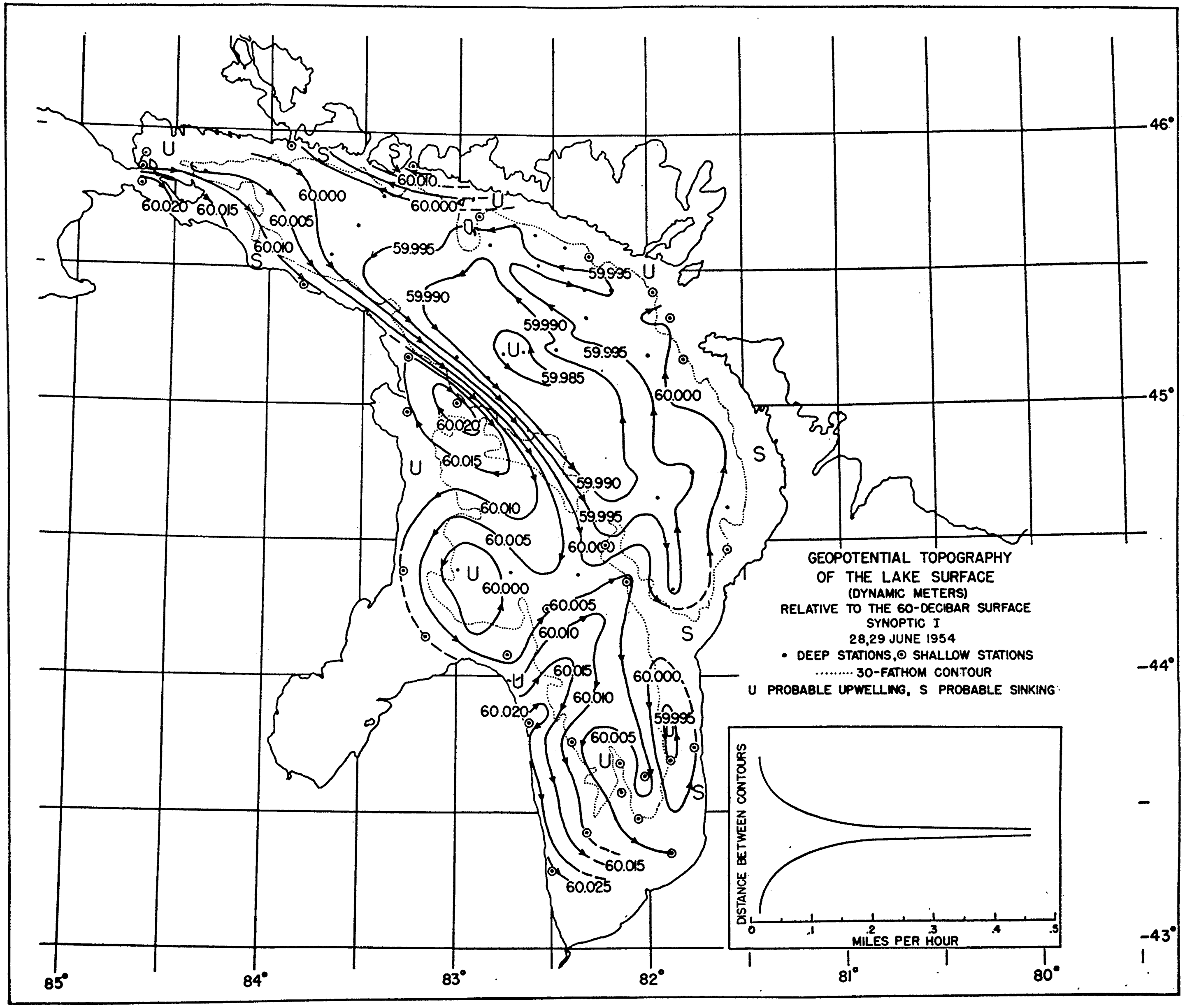

Fig. 1. The currents of Lake Huron on 28-29 June 1954, in relation to the 60-decibar surface.

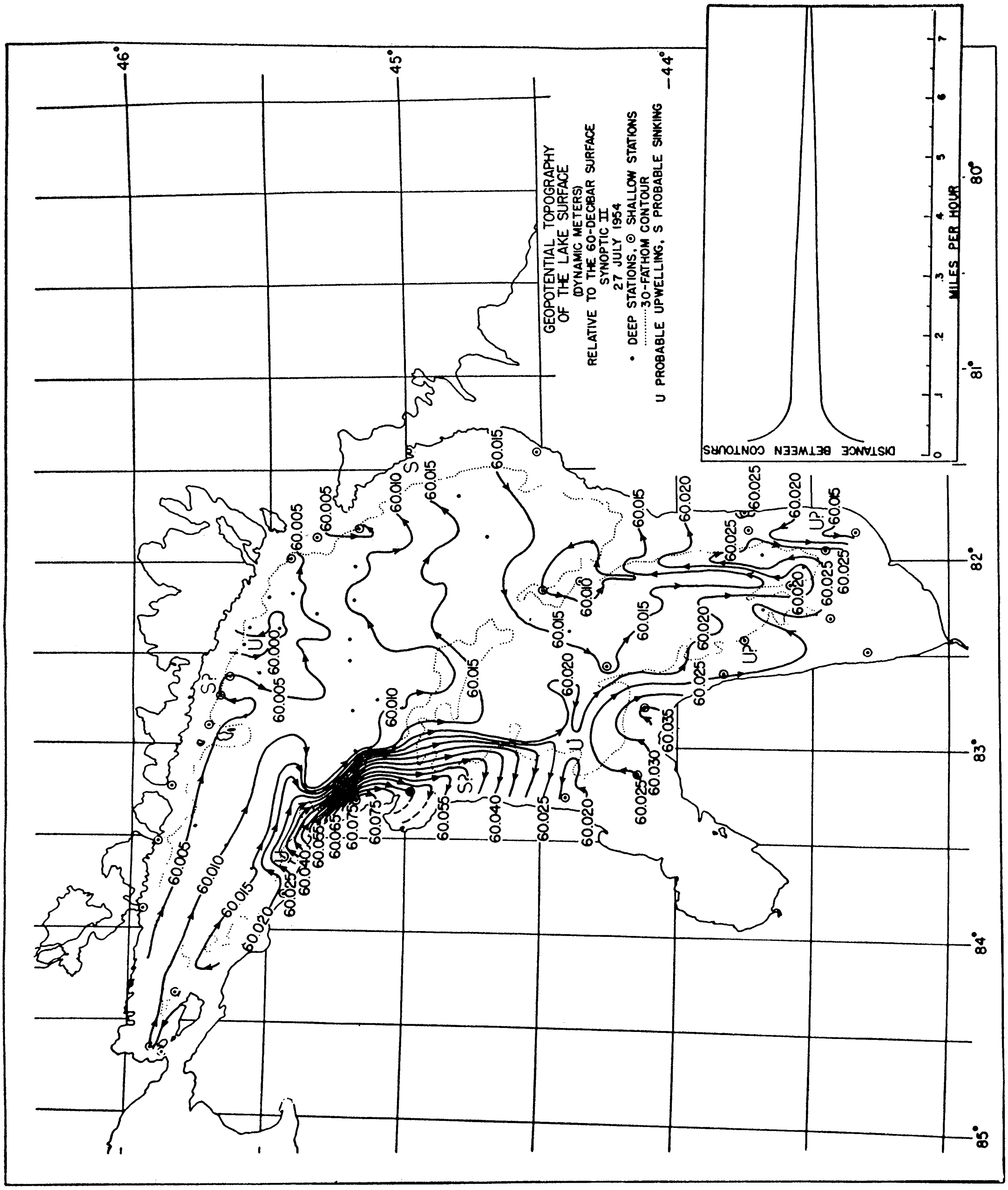

Fig. 2. The currents of Lake Huron on 27 July 1954, in relation to the 60-decibar surface.

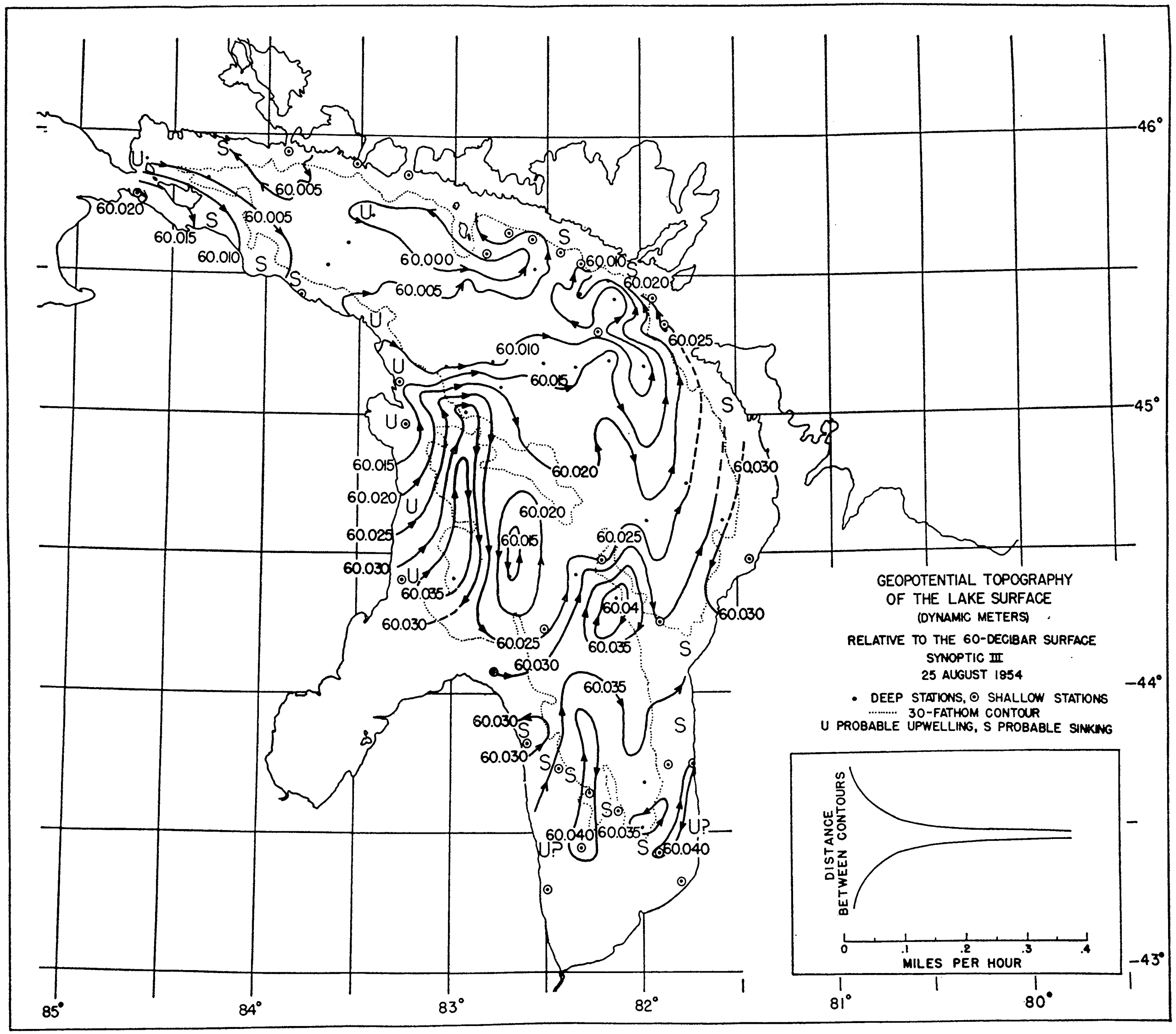

Fig. 3. The currents of Lake Huron on 25 August 1954, in relation to the 60-decibar surface. This is apparently a non-equilibrium condition.

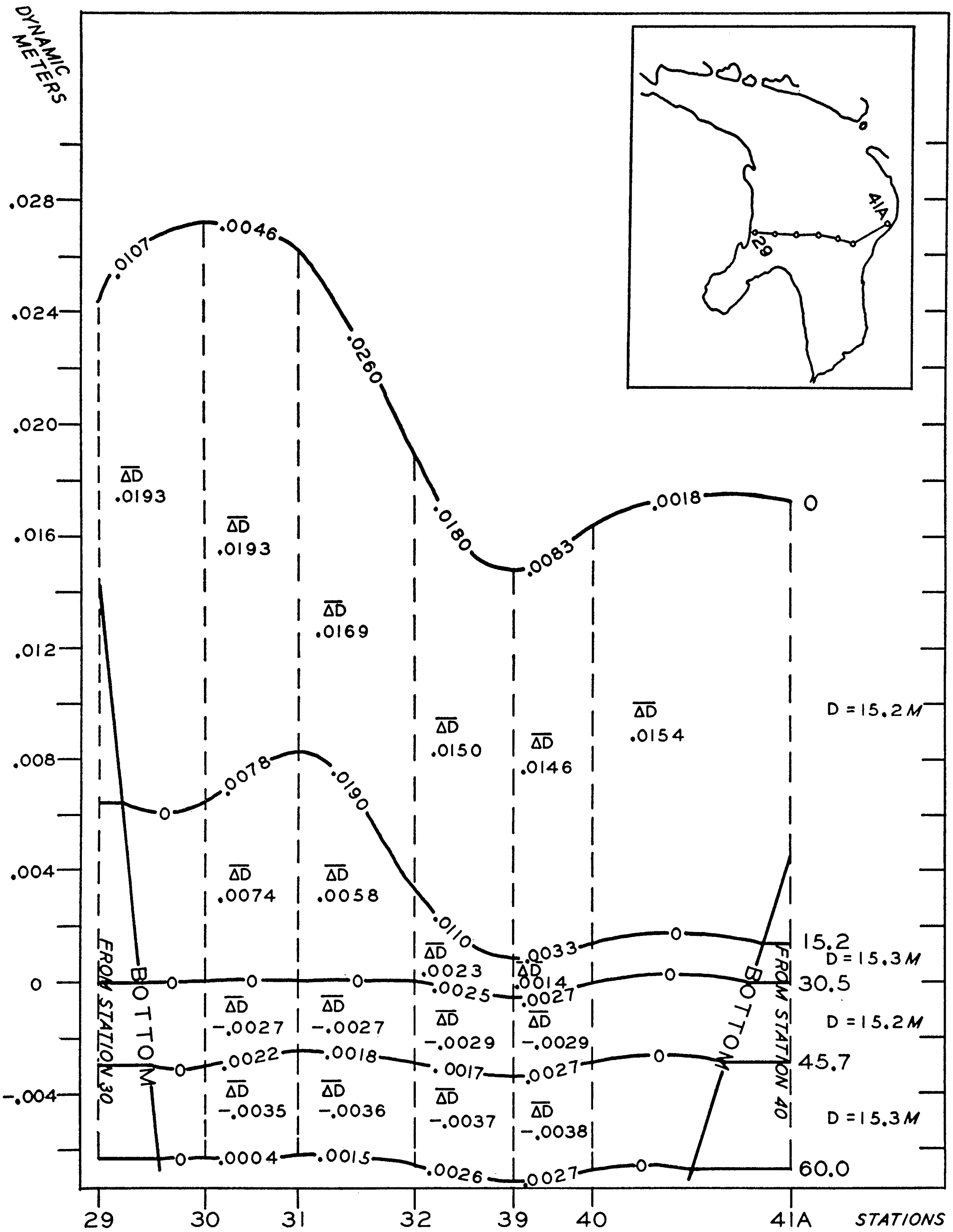

FIG. 4. The distribution of dynamic heights and current velocities in the Oscoda, Michigan-Southampton, Ontario section on 27 July 1954. Current velocities in meters/second are shown on the decibar surfaces. The $\overline{\Delta D}$ values, in dynamic meters, in the between-station intercontour blocks are averages applying to those blocks. Interception of the isobaric surfaces by the bottom is indicated at each end. D values on the right are the intercontour intervals, in meters, under standard conditions. The insert shows the location of the section.

In the interval from late June to late July a strong thermocline developed over the whole lake. The survey of 27 July took place after winds from quarters more northerly than usual, and the current pattern (Fig. 2) appeared to be a wind-induced distortion of the fundamental pattern; the warm light surface water apparently responded rather readily to the wind shift.

Prior to the survey of 25 August 1954 (Fig. 3) the winds were from more nearly normal westerly quarters, and the current pattern appeared to indicate a partial return toward the June condition. The fact that the outflows from Lakes Michigan

TABLE 4. *Computation of water transport through the Oscoda-Southampton section, 27 July 1954*
Data for second and fourth columns are from Figure 4.

Isobaric surface pair, decibars	Intersurface depth $(D + \Delta\overline{D})$, dynamic m*	Intersurface area m^2	Average velocity m/sec*	Transport m^3/sec**	Transport sum m^3/sec**
Transport between stations 29 and 30, 23979 meters apart					
0–15.2	15.2193	364943	0.0054	1970 north	
15.2–30.5	—	—	0	0	
30.5–45.7	—	—	0	0	
45.7–60.0	—	—	0	0	
					1970 north
Transport between stations 30 and 31, 21243 meters apart					
0–15.2	15.2193	323304	0.0016	517 north	
15.2–30.5	15.3074	325175	0.0039	1268 north	
30.5–45.7	15.1973	322836	0.0011	355 north	
45.7–60.0	15.2965	324944	0.0013	422 north	
					2562 north
Transport between stations 31 and 32, 26393 meters apart					
0–15.2	15.2169	401620	0.0225	9036 south	
15.2–30.5	15.3058	403966	0.0095	3838 south	
30.5–45.7	15.1973	401102	0.0009	361 south	
45.7–60.0	15.2964	403718	0.0017	686 south	
					13921 south
Transport between stations 32 and 39, 22048 meters apart					
0–15.2	15.2150	335460	0.0145	4864 south	
15.2–30.5	15.3023	337385	0.0068	2294 south	
30.5–45.7	15.1971	335066	0.0021	704 south	
45.7–60.0	15.2963	337253	0.0022	742 south	
					8604 south
Transport between stations 39 and 40, 17703 meters apart					
0–15.2	15.2146	269344	0.0058	1562 north	
15.2–30.5	15.3014	270881	0.0030	813 north	
30.5–45.7	15.1971	269034	0.0027	726 north	
45.7–60.0	15.2962	270789	0.0027	731 north	
					3832 north
Transport between stations 40 and 41A, 44096 meters apart					
0–15.2	15.2154	670938	0.0009	604 north	
15.2–30.5	—	—	0	0	
30.5–45.7	—	—	0	0	
45.7–60.0	—	—	0	0	
					604 north
				Net transport, m^3/sec	13557 south

---

* Last decimal place only approximate.

** Last two places only approximate.

and Superior, in flowing through Lake Huron, would have to cross dynamic height contours in both the upper and lower ends of the lake indicates that this survey covered a transient non-equilibrium condition in the lake.

The surface current directions of all three surveys were in excellent agreement with the movements of drift bottles released on the days of the surveys. Average current velocities obtained by the dynamic height method in June and July were not significantly different, statistically, from those of drift bottles released on those surveys. From the August survey, drift bottle returns were too few to permit velocity comparison.

As a further check on the method, a water transport estimate has been made for the 27 July section from Oscoda, Michigan, to Southampton, Ontario (Fig. 4). If the method is correct, summation of the products of average velocity and area from the component parts of the section should give a rough measure of the volume of water passing through the section. This volume was compared with the outflow of Lake Huron via the St. Clair River. The distribution of dynamic heights across the section, average dynamic height increments in between-station blocks, and velocities between station pairs are shown in Figure 4; the computations are summarized in Table 4.

Current velocity was calculated at each of the 0, 15.2, 30.5, 45.7, and 60.0 decibar surfaces between each pair of stations in the section. An average velocity was obtained for each intercontour area between each pair of stations; each intercontour average dynamic height increment, $\overline{\Delta D}$, between each pair of stations was added to the corresponding standard-condition depth interval, D, and multiplied by the distance between stations to give the area of each intercontour between-station block; the average velocity in each block was multiplied by the block area; and the resulting volume transports were summed algebraically. The resulting estimate of transport through the section was about 13,500 m^3/second or about 473,000 ft^3/second. In the last week of July 1954 the outflow of the St. Clair River was 216,000 ft^3/second (U. S. Lake Survey, Detroit, personal communication). The order-of-magnitude agreement between these figures is probably all that could be expected, for there are areas between the section ends and the shores, and below the reference level, where no transport figures could be obtained; in addition, the Helland-Hansen substitutions in the bottoms of the end stations force the supposition of no currents or transports there.

POTENTIAL ERRORS OF THE METHOD

Sverdrup *et al.* (*op. cit.*, pp. 393–4) review the sources of potential error in the dynamic height method, and every user of the method should be familiar with them. They include errors stemming from the neglect of frictional effects and errors due to the slope of the reference surface. Currents obtained from dynamic computations are relative to any currents at the reference level. In considering these relative currents as *absolute* currents an error is introduced if the reference surface slopes (has currents). Errors from these sources appear to be small in the ocean, and (in the first approximation) may be considered small in lakes that are wide in comparison to the inertia circle, whose radius is: $r = v/2\omega \sin \phi$ (where v is maximum surface velocity and $2\omega \sin \phi$ is the double angular velocity of the earth in that latitude).

The presence of internal seiches or internal waves during the period of observation may be a source of potential error, but the works of Mortimer (1951, 1954, 1955) indicate that in large deep lakes the periods of such oscillations are long compared to the ordinary work-day. In the first approximation, internal seiches of large deep lakes may be considered to cause little error in one-day synoptic surveys such as were used on Lake Huron. Their effect on the accuracy of the method in smaller shallower lakes remains to be investigated.

The most probable source of significant error in applications of the dynamic height method to lakes appears to be the direct effect of the wind. Pronounced changes in wind direction and/or velocity produce changes in the distribution of density, and significant accelerations (not taken into account by the method) may be present until equilibrium is regained. Prolonged relatively constant winds may remove surface water from the up-wind shore and pile it against the down-wind shore, imparting an 'artificial' slope to the surface. The lee-shore sinking and windward-shore upwelling produced by such winds may be expected to tilt subsurface isobaric surfaces in the opposite direction from the surface slope—further increasing the depth of warm surface water down-wind and decreasing it up-wind. Further studies on the effect of the wind, and upon the degree to which shallowness of lakes limits the use of the method, are needed.

CONCLUSIONS

Although many potential sources of error are to be considered in applications of the dynamic height method, most of them appear to lead to relatively small errors and nearly all may be minimized by careful planning and by choice of days for observation. Used with proper caution, the dynamic height method appears to be a promising technique for the investigation of certain circulation phenomena, in large lakes at least. The method appears to be more satisfactory for determinations of surface current patterns and velocities than for subsurface water transport computations. Application of the method in Lake Huron has produced logical and reasonable results which are both internally consistent and in good agreement with the behaviors of other parameters (Ayers *et al.* 1956).

REFERENCES

AMAGAT, E. H. 1893. Memoires sur l'elasticité et la dilation des fluides jusqu'aux tres hautes pressions. Ann. Chim. (Phys.), **29:** 505–574.

AYERS, J. C., D. C. CHANDLER, G. H. LAUFF, AND D. V. ANDERSON. 1956. The currents and water masses of Lake Huron. Great Lakes Research Institute, University of Michigan, Ann Arbor. (in press)

BEYER, R. T. 1954. Formulae for sound velocity in sea water. J. Mar. Res., **13:** 113–121.

HANDBOOK OF CHEMISTRY AND PHYSICS (C. D. Hodgman, Ed.) 1949. 31st Ed. Chemical Rubber Publ. Co., Cleveland, Ohio. xviii + 2737 pp.

HARRINGTON, M. W. 1895. Surface currents of the Great Lakes, as deduced from the movements of bottle papers during the seasons of 1892, 1893, and 1894. U. S. Weather Bureau, Bulletin B (rev. ed.).

HELLAND-HANSEN, BJ. 1934. The Sognefjord section. pp 257–274 *in* James Johnstone Memorial Volume. Univ. Press, Liverpool. 348 pp.

LAFOND, E. C. 1951. Processing oceanographic data. H. O. Publ. No. 614, U. S. Navy Hydrog. Off., Washington, D. C. vi + 114 pp.

MILLAR, F. G. 1952. Surface temperatures of the Great Lakes. J. Fish. Res. Bd. Canada, **9:** 329–376.

MORTIMER, C. H. 1951. Water movements in stratified lakes, deduced from observations in Windermere and model experiments. Union Géodes. et Geophys. int., Assoc. int. d'Hydrol. scientif., Assemblée gén. de Bruxelles 1951, **3:** 336–349.

———. 1954. Models of the flow-pattern in lakes. Weather, **9:** 177–184.

———. 1955. Some effects of the earth's rotation on water movements in stratified lakes. Proc. Int. Assoc. Limnol., **12:** 66–77.

SVERDRUP, H. U., M. W. JOHNSON, AND R. H. FLEMING. 1942. The oceans, their physics, chemistry, and general biology. Prentice-Hall, New York. x + 1087 pp.

11/11 6:00p