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SIGNAL DETECTION AS A FUNCTION OF FREQUENCY ENSEMBLE

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Abstract

This report presents the results of two experimental investigations of the detection of a signal in noise as a function of signal ensemble size and ensemble frequency range. Signal ensemble size is defined as the number of signals of different frequency that are equally-likely to occur and which the observer must try to detect. Ensemble frequency range is defined as the frequency separation between the highest and the lowest frequency signals included in that ensemble.

Data are presented for four observers in the first experiment and for three observers in the second experiment. The results obtained are compared with predictions based upon the null hypothesis, a narrow-band observer model, a multiple-band observer model, and the mathematically optimum detector.

The possibility is suggested that the models discussed are not necessarily mutually exclusive, but rather complementary in an over-all view of detection.

SIGNAL DETECTION AS A FUNCTION OF FREQUENCY ENSEMBLE

1. INTRODUCTION

In 1956, Tanner, Swets and Green[1] reported a decrement in signal detection as the number of different frequencies which the signal might assume increased from one to two. The decrement was not only found to be a function of the uncertainty thus created by the greater ensemble of frequencies, but also a function of the difference between f_1 and f_2 of the signal. The greater the difference between the frequencies, the greater was the decrement.

It was the purpose of the present investigations to explore more systematically detectability of auditory signals as a function of (1) ensemble size and (2) ensemble frequency range; and further, to determine the extent of agreement between the obtained experimental data and predictions made on the basis of four mathematical models.

Two major experiments were carried out and are presented separately in sections III and IV.

2. THE MATHEMATICAL MODELS

2.1 The Narrow-Band Scanning Model

This model is based on the assumption that the observer is, at a single "instant" in time, sensitive to only a narrow range of frequencies; that the hearing mechanism requires a measurable amount of time to shift from the observation of one frequency to another, and that, in so doing, it must shift through the intervening frequencies.

2.2 The Multiple Band Model

This model is based on the assumptions that the observer can attend to several frequency bands at once though these bands need not be adjacent, and that the outputs of these bands are linearly combined by the observer.

2.3 The Null Hypothesis

The Null Hypothesis predicts performance for the two and four signal ensembles to be no different from average performance on the one-signal-ensemble.

2.4 The Mathematically Optimum Detector

The mathematically optimum detector is perfect except for limitations placed upon it externally by the system. In the present experiment, accuracy of the optimum detector is limited by noise and the number of alternative signals which it must detect.

3. THE FIRST EXPERIMENT

In the first investigation, signal ensembles of one, two, and four frequencies were used. In the detection of one frequency, d' values[2] were individually obtained for eight signals: 1007, 1057, 1107, 1157, 1207, 1257, 1307, and 1357 cps. In the detection of two equally-likely frequencies the following pairs were used: 1157 and 1207; 1107 and 1257; 1057 and 1307; 1007 and 1357 cps, giving range differences of 50, 150, 250, and 350 cps respectively. In the detection of four equally-likely frequencies the following groups were used: 1107, 1157, 1207, 1257; 1057, 1107, 1257, 1307; 1007, 1057, 1307, 1357 cps; and 1007, 1157, 1207, 1357 cps, giving range differences of 150, 250, 350, and 350 cps respectively. The last two ensembles enabled the experimenter to study pattern effects under constant ensemble size and range.

All trials were two-interval forced-choice and were presented binaurally (through PDR-8 headphones) to the observers in the following time order:

(1) A warning light to indicate that the trial is about to begin, followed in 0.44 second by

(2) Test interval I. - 0.42 second in duration

(3) Test interval II. - 0.42 second in duration

The signal, which appeared in either of the test intervals, was 0.10 second in duration.

(4) Response interval - 1.06 seconds extent. A red light flashes if the observer's response is correct.

(5) 0.50 second after the response period the correct response is indicated visually.

(6) 1.36 seconds lapse between the end of one trial and the beginning of the next trial.

100 trials constituted a run. Eight runs constituted an experimental session.

Voltages of the signals, as measured at the headphones, were variously set (0.0045 v for the two highest frequencies and 0.0042 v for the others) in an attempt to keep the detection constant for all signals. All runs were done in a background of white Gaussian noise of 20 KC bandwidth at an amplitude of 0.06 volts.

Conditions were counterbalanced and so presented as to include at least one run of every ensemble size in every experimental session. Thus, in any one session those four frequencies which comprised the four-signal ensemble of the experimental session were presented also individually, making four runs of ensembles-of-one, and in pairs making two ensembles-of-two. For example, if the ensemble-of-four which was to be presented on a particular day consisted of 1007, 1157, 1207, and 1357 cps, detection indices were obtained for that ensemble; for each of these frequencies presented individually; and also for the pairs 1007 or 1357, and 1157 or 1207 cps.

Approximately sixteen runs were obtained for each condition. The data thus obtained were then compared with predictions made on the basis of the hypotheses, or models, as described in Sec. II - above.

Table I gives the obtained data for the ensemble-of-one condition both in percent correct and in d' for each of four observers.

In Tables IIA, IIB, IIC, and IID data relating to the multiple ensemble conditions are presented. Here predictions are listed in percent-correct for the first three hypotheses discussed above. The deviations of these predictions from the obtained data, and the obtained data are also listed.

3.1 Calculation of Predictions for Multiple Ensemble Performance

3.1.1 By Narrow-Band Scanning Model. The predicted percent correct is determined in the following manner: since the observer can attend to only one stimulus at a time, that stimulus in the ensemble for which detection is best (i.e., percent correct is greatest) will be the stimulus the observer will listen for. He will, therefore, have the same percent correct for that stimulus as he did in the one-of-one condition through all ensemble sizes. When the stimulus to which he is attending does not occur, the observer will not have heard a signal in either presentation interval and his response, therefore, will be based upon a guess. In a two-alternative forced-choice type of experiment such as this is, the

TABLE I

Results Obtained From Four Observers on the
Detection of Single Signals in White Gaussian Noise

Data are presented as percent correct and their respective d's.

Signal Frequency (cps)	O:SS		O:SR		O:GP		O:SD	
	P(C)	d'	P(C)	d'	P(C)	d'	P(C)	d'
1007	89.13	1.74	79.50	1.16	71.00	0.78	85.70	1.51
1057	91.28	1.92	74.17	0.91	70.11	0.75	88.28	1.68
1107	91.22	1.92	75.25	0.96	67.71	0.65	84.28	1.425
1157	88.06	1.66	74.38	0.92	66.57	0.61	82.08	1.29
1207	89.38	1.76	77.89	1.08	67.86	0.66	82.92	1.35
1257	87.61	1.63	70.92	0.78	68.71	0.68	87.00	1.59
1307	91.38	1.93	77.17	1.04	76.38	1.02	89.15	1.73
1357	91.00	1.90	80.25	1.20	72.50	0.85	89.60	1.77

probability of a guess being correct is 0.5.

3.1.2 By Multiple Band Model. More recently, Green[3] has obtained data which seem to support a multiple band observer model, rather than the single narrow-band observer described above. In the multiple band observer model, $\underline{d}'$ is a distance in sigma units on a decision axis between the means of two Gaussian distributions of equal variance, noise (N) and signal-plus-noise (S+N). The mean of N is, for simplicity, set at zero, variance at 1; the S+N distribution for a set of signals S_1 (all signals within the set being within the same critical band) has, then, mean $\underline{d}'_1$ and variance 1. Now consider two different sets of signals-in-noise, S_1 and S_2, such that $\underline{d}'_1 = \underline{d}'_2$. If these signals are presented randomly one at a time and the observer is listening for either of two, the noise distributions of the two critical bands are added (the mean remains zero, variance is multiplied by 2) and the predicted values for S_1 and S_2 in sigma units are $\frac{\underline{d}'_1}{\sqrt{2\sigma_N}}$ and $\frac{\underline{d}'_2}{\sqrt{2\sigma_N}}$, respectively. The equivalent percentages of these new $\underline{d}'$ values, weighted according to the relative frequency with which they occurred during the trial, constitute the predicted percent-correct for that condition.

The same logic is applied to 1-of-X signal sets, variance always being multiplied by the number of critical bands in operation, hence, the original $\underline{d}'$s are always divided by the square root of the number of critical bands in operation.

3.1.3 By the Null Hypothesis. Performance on the component signals is averaged to predict performance on the multi-ensemble conditions.

3.2 Comparison of the Obtained Data with Three Hypotheses.

The deviations of the predicted P(C) from the obtained P(C) by each of our three models discussed above are presented in Tables IIA through IID

TABLE IIA

A Comparison of the Obtained Data and the Probabilities of Correct Detections as Predicted by the Three Models

O:SS

Signal Ensemble (cps)	Narrow-Band Model		Multiple-Band Model		Null Hypothesis		
	Predicted P(C)	Predicted Minus Obtained	Predicted P(C)	Predicted Minus Obtained	Predicted P(C)	Predicted Minus Obtained	P(C) Obtained
1157 or 1207	60.58*	-27.82	88.72	0.32	88.7	0.30	88.4
1107 or 1257	70.45	-12.50	85.2	2.25	89.5	6.55	82.95
1057 or 1307	71.42	-11.39	86.72	3.91	91.3	8.49	82.81
1007 or 1357	71.81	- 9.44	81.9	0.65	90.0	8.75	81.25
1107, 1157, 1207 or 1357	60.17	-23.83	86.15** 82.8<>89.5	2.15	89.6	5.6	84.0
1057, 1107, 1257 or 1307	59.48	-22.12	85.96	4.36	90.3	8.7	81.6
1007, 1157, 1207 or 1357	61.23	-21.27	76.4	-6.1	89.9	7.40	82.5
1007, 1057, 1307 or 1357	61.14	-19.39	84.31	3.78	90.7	10.17	80.53

TABLE IIB

A Comparison of the Obtained Data and the Probabilities of Correct Detections as Predicted by the Three Models

O:SR

Signal Ensemble (cps)	Narrow-Band Model		Multiple-Band Model		Null Hypothesis		
	Predicted P(C)	Predicted Minus Obtained	Predicted P(C)	Predicted Minus Obtained	Predicted P(C)	Predicted Minus Obtained	P(C) Obtained
1157 or 1207	64.76*	-10.49	76.14	0.89	76.1	0.85	75.25
1107 or 1257	63.22	- 7.78	66.95	-4.05	73.1	2.10	71.00
1057 or 1307	62.96	-10.44	68.6	-4.8	75.7	2.30	73.40
1007 or 1357	66.44	-10.16	72.2	-4.4	79.9	3.30	76.6
1107, 1157, 1207 or 1257	57.26	-13.24	71.32** 68.05<>74.6	0.82	74.6	4.10	70.50
1057, 1107, 1257 or 1307	56.22	-11.08	67.78	0.48	74.4	7.10	67.30
1007, 1157, 1207 or 1357	57.98	-14.77	67.22	-5.53	78.0	5.25	72.75
1007, 1057, 1307 or 1357	58.36	-15.78	70.40	-3.74	77.8	3.66	74.14

* If one were to consider this ensemble to be within the same narrow-band, the predicted P(C) would be the same as the null.

** The midpoint of this interval, used in calculating predicted minus obtained.

TABLE IIC

A Comparison of the Obtained Data and the Probabilities of Correct Detections as Predicted by the Three Models

O:GP

Signal Ensemble (cps)	Narrow-Band Model		Multiple-Band Model		Null Hypothesis		
	Predicted P(C)	Predicted Minus Obtained	Predicted P(C)	Predicted Minus Obtained	Predicted P(C)	Predicted Minus Obtained	P(C) Obtained
1157 or 1207	60.0 *	- 6.92	67.22	0.30	67.2	0.28	66.92
1107 or 1257	60.32	- 7.09	62.95	-4.46	68.2	0.79	67.41
1057 or 1307	64.67	- 7.53	67.15	-4.85	73.2	1.20	72.00
1007 or 1357	62.90	- 9.54	65.95	-6.49	71.8	-0.64	72.44
1107, 1157, 1207 or 1257	54.56	-12.44	65.2 ** 62.7<>67.7	-1.8	67.7	0.70	67.00
1057, 1107, 1257 or 1307	55.5	-15.27	65.00	-5.77	70.8	0.03	70.77
1107, 1157, 1207 or 1357	56.7	-10.3	61.58	-5.42	69.5	2.50	67.00
1007, 1057, 1307 or 1357	57.81	-10.06	66.5	-1.37	72.5	4.63	67.87

TABLE IID

A Comparison of the Obtained Data and the Probabilities of Correct Detections as Predicted by the Three Models

O:SD

Signal Ensemble (cps)	Narrow-Band Model		Multiple-Band Model		Null Hypothesis		
	Predicted P(C)	Predicted Minus Obtained	Predicted P(C)	Predicted Minus Obtained	Predicted P(C)	Predicted Minus Obtained	P(C) Obtained
1157 or 1207	66.85 *	-15.06	82.50	0.59	82.5	0.59	81.91
1107 or 1257	67.4	-13.47	77.35	-3.54	85.7	4.83	80.87
1057 or 1307	70.79	-13.21	80.3	-3.7	88.5	4.5	84.00
1007 or 1357	69.75	-11.35	79.50	-1.60	87.7	6.60	81.10
1107, 1157, 1207 or 1257	60.17	-19.03	80.0 ** 75.9<>84.1	0.80	84.1	4.90	79.2
1057, 1107, 1257 or 1307	59.00	-21.85	78.82	-2.03	87.5	6.65	80.85
1007, 1157, 1207 or 1357	59.91	-20.69	72.65	-7.95	85.1	4.50	80.60
1007, 1057, 1307 or 1357	59.58	-19.78	79.90	0.54	88.5	9.14	79.36

* If one were to consider this ensemble to be within the same narrow-band, the predicte P(C) would be the same as the null.

** The midpoint of this interval, used in calculating predicted minus obtained.

and shown graphically in Figure 1. An examination of these data reveals a consistent underestimation of observers' performance by the narrow-band scanning model, and with one exception a consistent, though small, overestimation of observers' performance by the null hypothesis. Using non-parametric statistics, predictions by these two models are found to differ significantly from the data, predictions by the multiple band model, however, do not. The obtained data fell shorter of the null predictions in the one-of-four ensembles than in the one-of-two ensembles. This is a reflection of a greater decrement in performance with greater complexity of stimulus field.

As regards effect of pattern of performance, there is no indication in the present study that this is a relevant variable. A Chi-Square test on the distributions of P(C) for the two ensembles-of-four of equal range, showed no significant differences for all four observers.

3.3 Calculation of the Mathematically Optimum Detector

Finally, in Table III the obtained data in d' are listed along with the d' expected of the mathematically optimum detector. For a signal ensemble of one, $d'_{\text{optimum}} = \sqrt{\frac{2E}{N_o}}$, where E is signal energy and N_o is noise power per unit bandwidth. For a signal ensemble of M orthogonal signals $d'_{M \text{ optimum}} = \sqrt{\ln\left(1 - \frac{1}{M} + \frac{1}{M} e^{\frac{2E}{N_o}}\right)}$, and since $\frac{1}{M} e^{\frac{2E}{N_o}}$ is very much greater than $1 - \frac{1}{M}$, we can for simplicity eliminate $1 - \frac{1}{M}$ from the equation. Our final estimate is $d'_M \doteq \sqrt{\frac{2E}{N_o} - \ln M}$ (see footnote 4.) A comparison of the obtained with the mathematical optimum is expressed in terms of efficiency[5] or η, where $\eta = (d'\text{ obt.}/d'\text{ opt.})^2$. The efficiencies thus calculated are presented graphically for the four observers in Figs. 2A, 2B, 2C and 2D, respectively.

FIG.1

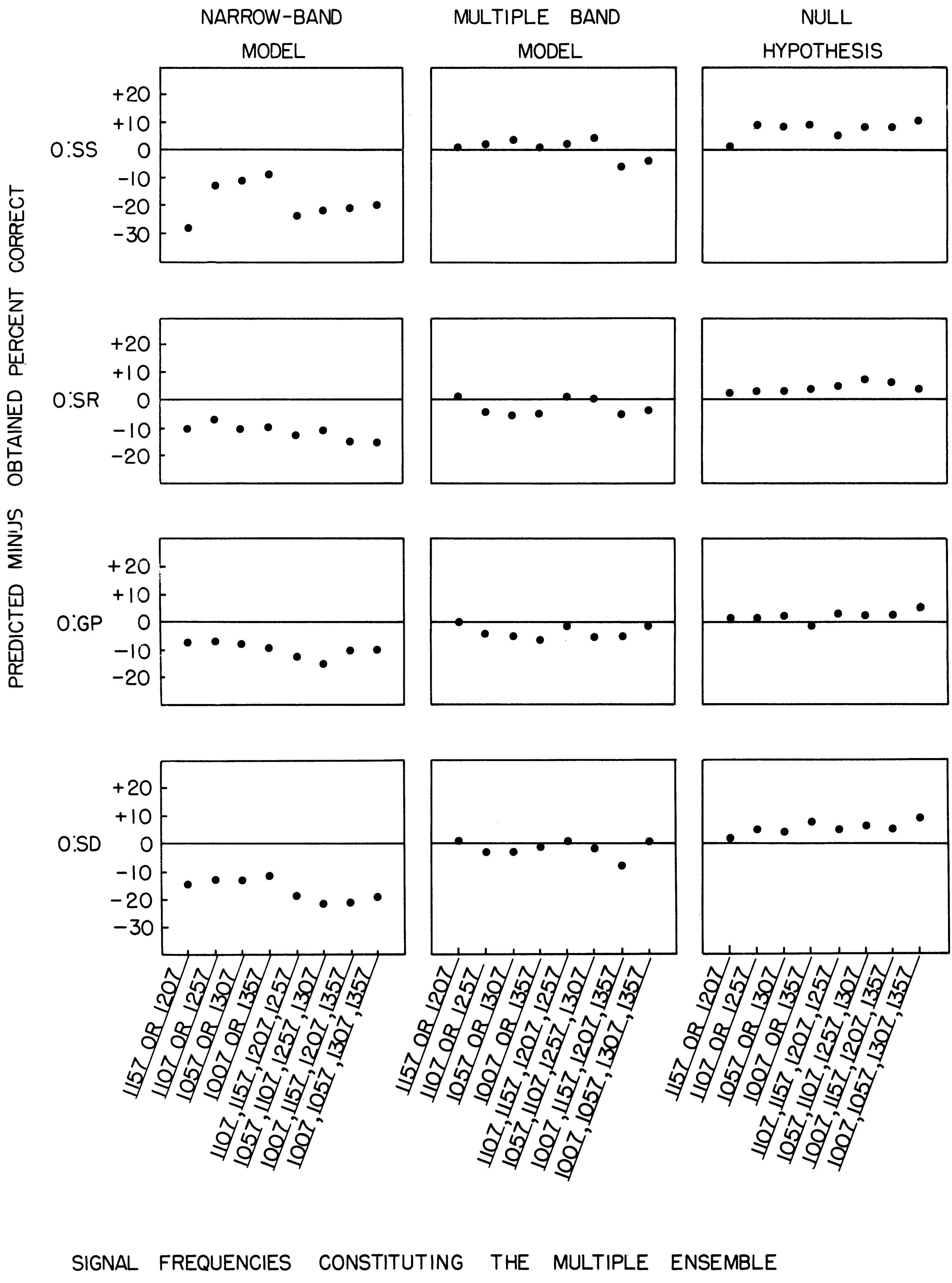
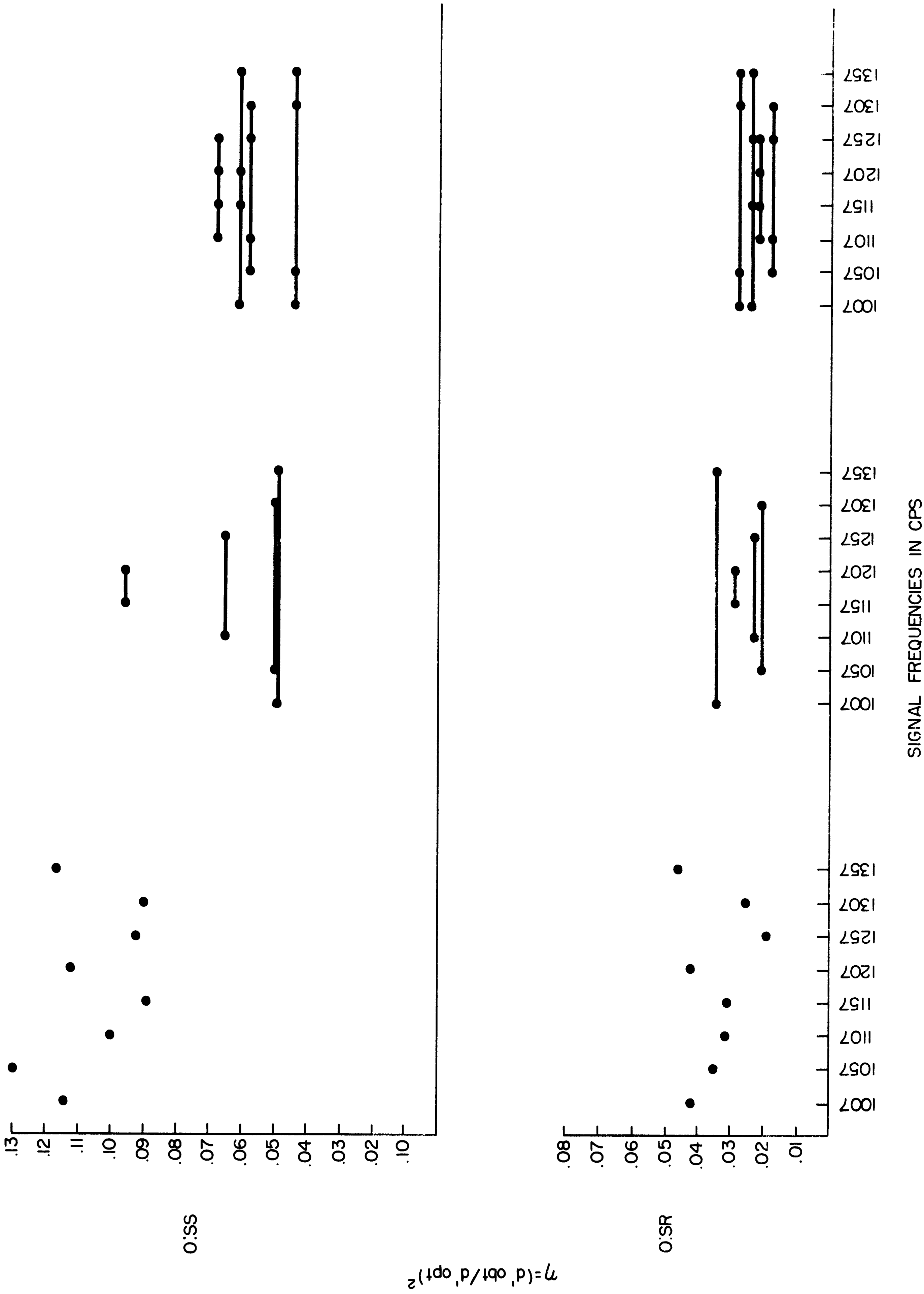

NARROW-BAND MODEL
MULTIPLE BAND MODEL
NULL HYPOTHESIS
PREDICTED MINUS OBTAINED PERCENT CORRECT
O:SS
O:SR
O:GP
O:SD
+20
+10
0
-10
-20
-30
1157 OR 1207
1107 OR 1257
1057 OR 1307
1007 OR 1357
1107, 1157, 1207, 1257
1057, 1107, 1257, 1307
1007, 1157, 1207, 1357
1007, 1057, 1307, 1357
SIGNAL FREQUENCIES CONSTITUTING THE MULTIPLE ENSEMBLE

TABLE III

A Comparison of d' for the Mathematically Optimum Detector with d' Data for Three Observers, and Efficiencies (η's) Calculated for All Experimental Conditions

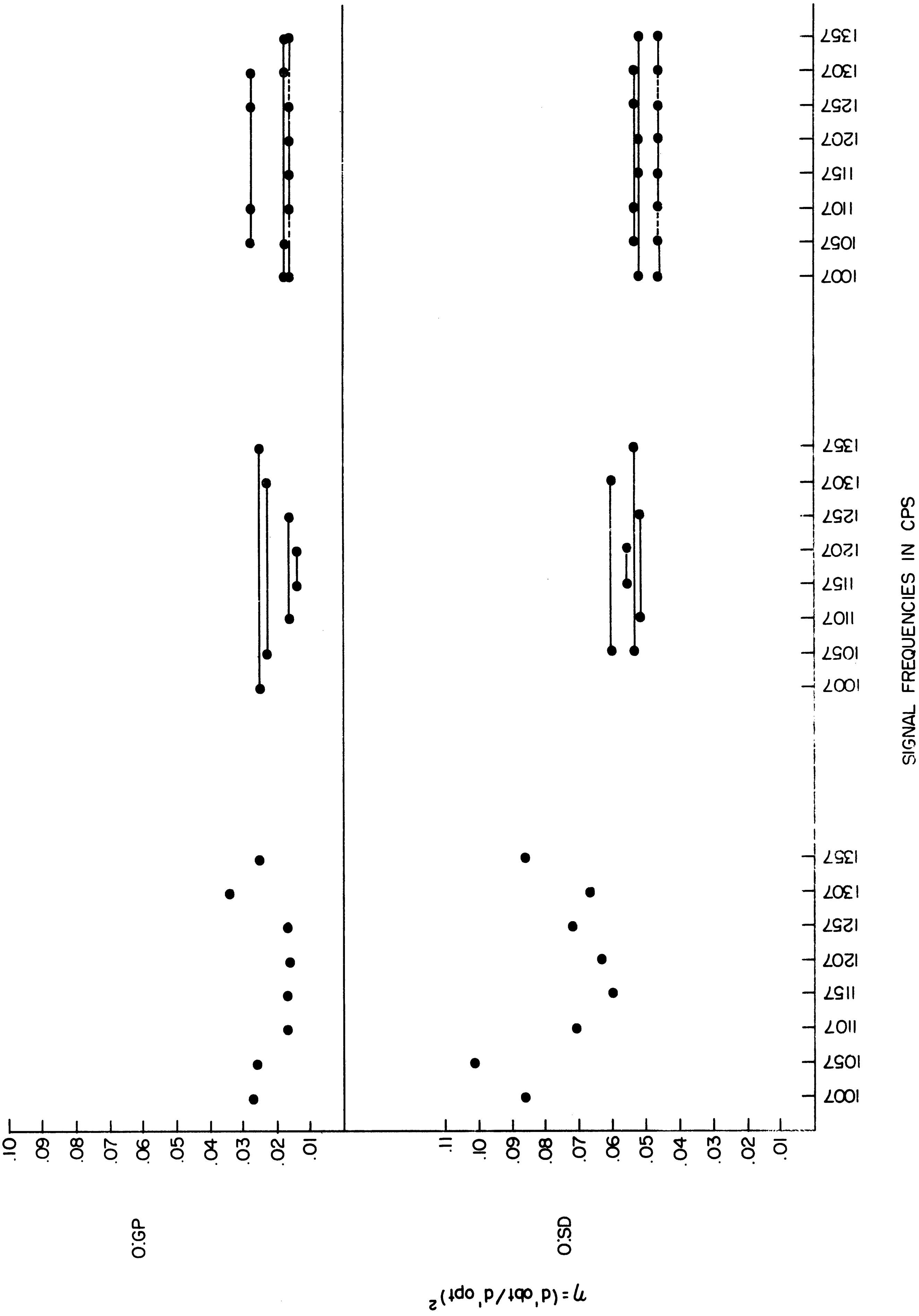
Signal Frequency (cps)	d' opt.	O:SS d' obt.	O:SS η	O:SR d' obt.	O:SR η	O:GP d' obt.	O:GP η	O:SD d' obt.	O:SD η
1007	5.66	1.92	0.114	1.16	0.042	0.94	0.027	1.66	0.086
1057	5.66	2.04	0.130	1.06	0.035	0.92	0.026	1.80	0.101
1107	5.66	1.82	0.103	1.02	0.032	0.74	0.017	1.51	0.071
1157	5.66	1.69	0.089	1.00	0.031	0.73	0.017	1.39	0.060
1207	5.66	1.90	0.112	1.16	0.042	0.72	0.016	1.42	0.063
1257	5.66	1.72	0.092	0.79	0.019	0.73	0.017	1.52	0.072
1307	6.02	1.81	0.090	0.96	0.025	1.12	0.034	1.56	0.067
1357	6.02	2.06	0.117	1.28	0.046	0.96	0.025	1.76	0.086
1157 or 1207	5.59	1.73	0.096	0.95	0.029	0.66	0.014	1.31	0.055
1107 or 1257	5.59	1.43	0.065	0.84	0.023	0.71	0.016	1.28	0.052
1057 or 1307	5.78	1.29	0.050	0.83	0.021	0.87	0.023	1.42	0.060
1107 or 1357	5.78	1.28	0.049	1.08	0.035	0.89	0.024	1.33	0.053
1107, 1157, 1207 or 1257	5.54	1.45	0.068	0.83	0.022	0.69	0.016	1.19	0.046
1057, 1107, 1257 or 1307	5.63	1.36	0.058	0.76	0.018	0.92	0.027	1.30	0.053
1007, 1157, 1207 or 1357	5.63	1.39	0.061	0.87	0.024	0.71	0.016	1.28	0.052
1007, 1057, 1307 or 1357	5.72	1.20	0.044	0.95	0.028	0.75	0.017	1.22	0.046

FIG. 2A



O:SS
O:SR
η=(d' obt/d' opt)²
SIGNAL FREQUENCIES IN CPS
1007
1057
1107
1157
1207
1257
1307
1357
.13
.12
.11
.10
.09
.08
.07
.06
.05
.04
.03
.02
.10
.08
.07
.06
.05
.04
.03
.02
.01

FIG. 2B


O:GP
O:SD
η = (d'obt/d'opt)²
SIGNAL FREQUENCIES IN CPS
1007
1057
1107
1157
1207
1257
1307
1357
.10
.09
.08
.07
.06
.05
.04
.03
.02
.01
.11

3.4 Comparison of the Obtained Data with Mathematically Optimum Detector

3.4.1 η as a Function of Ensemble Size.

If mean η's for each ensemble size are considered, it is found that efficiencies for all observers are greatest for the smallest ensemble size. Three of four observers (O:SS, O:SR, and O:SD) show a further though smaller decrease in efficiency as the ensemble size increases from two to four. One observer (O:GP) shows no difference between the multiple ensembles of different size.

If we compare efficiencies on each of the four-signal ensembles with the two-signal ensembles of equal frequency range (Table IV), we find that O:SS shows greater efficiencies, three of four times, in the larger ensembles than in the smaller ensembles of comparable range. O:SR and O:SD and, with a single exception, O:GP, show lower efficiencies in the larger ensemble conditions of comparable frequency range.

3.4.2 η as a Function of Ensemble Frequency Range.

A comparison of efficiencies within the ensemble-of-four condition, taken observer by observer, shows no systematic change as a function of frequency range. Furthermore, for O:SR and O:SD, no systematic change is noted in the ensemble-of-two. As has been noted, these results are predictable by the multiple band model.

Upon examination of the data of O:SS and O:GP, however, an interesting pattern emerges. O:SS, who was the most efficient observer through all the experiments reported herein, shows a decrease of efficiency as the two signal frequencies become more widely separated. O:GP, who was the least efficient observer, shows an increase in efficiency as a function of increasing frequency separation. If we speculate that there is a narrow-band

TABLE IV

A Comparison of Efficiencies, ($\underline{\eta}$'s), Obtained for Two Ensemble Sizes and Three Frequency Ranges

Observer	Ensemble Size	Ensemble Frequency Range		
		150 cps	250 cps	350 cps
SS	2	0.065	0.050	0.049
	4	0.068	0.058	0.061, 0.044
SR	2	0.023	0.021	0.035
	4	0.022	0.018	0.024, 0.028
GP	2	0.016	0.023	0.024
	4	0.016	0.027	0.016, 0.017
SD	2	0.052	0.060	0.053
	4	0.046	0.053	0.052, 0.046

observer, we should expect that observer to behave under these conditions of increasing frequency range as O:SS. If, on the other hand, there were a broad-band observer, we should expect him to behave as O:GP. It is interesting to note at this point, that in an experiment that may be characterized as a wide-band detection problem--one calling for the detection of noise in noise,[6] these two observers reversed themselves in their relative abilities to detect. In that experiment, O:GP's efficiencies were indeed higher than O:SS's.

These differences in the observers, while only fragmentary, indicate a possibility that there is a population of observers whose distribution ranges from narrow-band observers to broad band observers with the greatest number of observers perhaps, being those who can shift their operation from narrow- to multiple- to broad-band listening as the task requires.

4. THE SECOND EXPERIMENT

4.1 Introduction

The purpose of the second experiment was to explore more extensively the applicability the narrow-band and the multiple-band observer models. The original experimental design and equipment were retained but modifications in the experimental variables were as follows:

(1) Greater frequency separations were introduced. The eight frequencies dealt with singly were: 507, 707, 907, 1107, 1307, 1507, 1707 and 1907 cps. The pairs used to make up ensembles-of-two were: 1107 or 1307, 907 or 1507, 707 or 1707, and 507 or 1907 cps, giving range differences of 200, 600, 1000 and 1400 cps, respectively. Two different ensembles-of-four included 707, 1107, 1307, and 1707 cps, giving a range of

1000 cycles; and 507, 907, 1507, and 1907 cps, giving a range of 1400 cps.

With these ensembles the number of critical bands involved in calculating predictions by the multiple-band model is the same for every ensemble of equal size. Thus, in every ensemble-of-two, the number of critical bands is two; in every ensemble-of-four the number of critical bands is four.

(2) An ensemble of eight equally-likely occurring signals was included. This ensemble included all of the individual frequencies listed above.

(3) An increase in voltage (to 0.067 v) in the background noise was introduced with no relative increase in the signal voltages. Detection was more difficult for comparable ensembles in this experiment than in the first experiment (see section III).

(4) Only one of our former observers, O:SD, remained with us for this experiment. In addition, two new observers were employed and trained, and data for three are presented.

4.2 Results

4.2.1 Detection of a Single Signal in Noise

The results in terms of percent correct and *d'* for the detection of a single signal in noise are presented for all three observers in Table V. With the exception of O:SD in detecting the 1507 cps signal, the performance of each observer is uniform through all frequencies.

4.2.2 Detection of Multiple Ensembles

Predictions of performance on the multiple-ensemble conditions are based again upon (1) performance in detecting a single signal and (2) the mathematics described in section 3.1. The predictions and the obtained data are presented in Tables VIA, VIB, and VIC. It will be noted that the

TABLE V

Results Obtained From Three Observers on the
Detection of Single Signals in White Gaussian Noise

Data are presented as percent correct and their respective _d_'s.

	O:BH		O:CR		O:SD	
Signal Frequency (cps)	P(C)	_d_'	P(C)	_d_'	P(C)	_d_'
507	75.12	0.96	76.12	1.00	85.50	1.50
707	71.38	0.80	73.88	0.90	74.12	0.92
907	78.00	1.09	77.25	1.05	79.50	1.16
1107	72.50	0.84	77.00	1.04	77.00	1.04
1307	74.00	0.91	73.38	0.89	78.25	1.10
1507	74.37	0.92	79.50	1.16	57.00	0.25
1707	75.50	0.98	76.38	1.02	76.12	1.00
1907	74.87	0.94	75.37	0.97	80.25	1.20

TABLE VIA

A Comparison of the Obtained Data and the Probabilities of Correct Detections as Predicted by Three Models

O:BH

Signal Ensemble (cps)	Narrow Band Model		Multiple Band Model		Null Hypothesis		
	Predicted P(C)	Predicted Minus Obtained	Predicted P(C)	Predicted Minus Obtained	Predicted P(C)	Predicted Minus Obtained	P(C) Obtained
1107 or 1307	62.00	-4.62	66.97	0.35	73.25	6.63	66.62
907 or 1507	64.00	-3.00	69.25	2.25	76.18	9.18	67.00
707 or 1707	62.75	0.50	67.20	4.95	73.44	11.19	62.25
507 or 1907	62.56	-3.19	67.75	2.00	75.00	9.25	65.75
707, 1107, 1307 or 1707	56.38	-4.37	62.21	1.46	73.34	12.59	60.75
507, 907, 1507 or 1907	57.00	-3.50	63.29	2.79	75.59	15.09	60.50
507, 707, 907, 1107, 1307, 1507, 1707 or 1907	53.50	-8.06	59.08	-2.48	74.47	12.91	61.56

TABLE VIB

A Comparison of the Obtained Data and the Probabilities of Correct Detections as Predicted by Three Models

O:CR

Signal Ensemble (cps)	Narrow-Band Model		Multiple Band Model		Null Hypothesis		
	Predicted P(C)	Predicted Minus Obtained	Predicted P(C)	Predicted Minus Obtained	Predicted P(C)	Predicted Minus Obtained	P(C) Obtained
1107 or 1307	63.50	-1.38	68.50	3.62	75.19	10.31	64.88
907 or 1507	64.75	2.38	71.17	8.80	78.37	16.00	62.37
707 or 1707	63.19	-1.06	68.15	3.90	75.13	10.88	64.25
507 or 1907	63.06	2.19	68.90	8.03	75.74	14.87	60.87
707, 1107, 1307 or 1707	56.75	2.63	63.19	9.07	75.16	21.04	54.12
507, 907, 1507 or 1907	57.38	0.51	64.39	7.52	77.06	20.19	56.87
507, 707, 907, 1107, 1307, 1507, 1707 or 1907	53.69	-3.12	59.84	3.03	76.11	19.30	56.81

TABLE VIC

A Comparison of the Obtained Data and the Probabilities of Correct Detections as Predicted by Three Models

O:SD

Signal Ensemble (cps)	Narrow-Band Model		Multiple Band Model		Null Hypothesis		
	Predicted P(C)	Predicted Minus Obtained	Predicted P(C)	Predicted Minus Obtained	Predicted P(C)	Predicted Minus Obtained	P(C) Obtained
1107 or 1307	64.12	-6.00	70.40	0.28	77.62	7.50	70.12
907 or 1507	64.75	2.00	63.65	0.90	68.25	5.50	62.75
707 or 1707	63.06	0.56	68.42	5.92	75.12	12.62	62.50
507 or 1907	67.52	2.15	75.05	9.68	82.88	17.51	65.37
707, 1107, 1307 or 1707	57.06	-5.69	63.94	1.19	76.37	13.62	62.75
507, 907, 1507 or 1907	58.88	-4.62	64.06	0.56	75.56	12.06	63.50
507, 707, 907, 1107, 1307, 1507, 1707 or 1907	54.44	-6.00	62.65	2.21	75.97	15.53	60.44

difference between the predictions made by the narrow-band model and those made by the multiple-band model are smaller in this experiment than in the original experiment. (For direct comparison, see both sets of predictions for O:SD.) It is generally true, that the differences in the predictions diminish as the original percent correct, upon which the predictions are based, decreases. The deviations of the obtained data from the predictions of the three models, for all observers, are presented in Figure 3. Performance in all multiple ensembles for all observers falls short of the predictions by the null hypothesis.

Which, then of the remaining hypotheses best fits the data? Figure 4 offers a direct comparison of the narrow-band and multiple-band models for each observer. For each multiple-ensemble condition a line is drawn from the (less deviant) better prediction to the zero line. Thus, we can see that the multiple band model more closely resembles the obtained data 6 of 7 times for O:BH, only 1 of 7 times for O:CR, and 5 of 7 times for O:SD. In the first experiement, however, the multiple-band model approached the data more closely than the narrow-band model in all conditions for all observers. Whether this shift toward the narrow-band model is related to the greater frequency separations, to the smaller E/N_0 ratios, to individual differences, or to the interaction of these variables is not determinable in the present experiment. The present research indicates only that both models may be descriptive of certain detection phenomena, though neither is sufficient to handle all the data.

4.2.3 Efficiencies

Efficiencies, or η's, which represent the comparison of the obtained d's with d's expected of the mathematically optimum observer are presented for all conditions and all observers in Table VII and are shown graph-

Figure 3

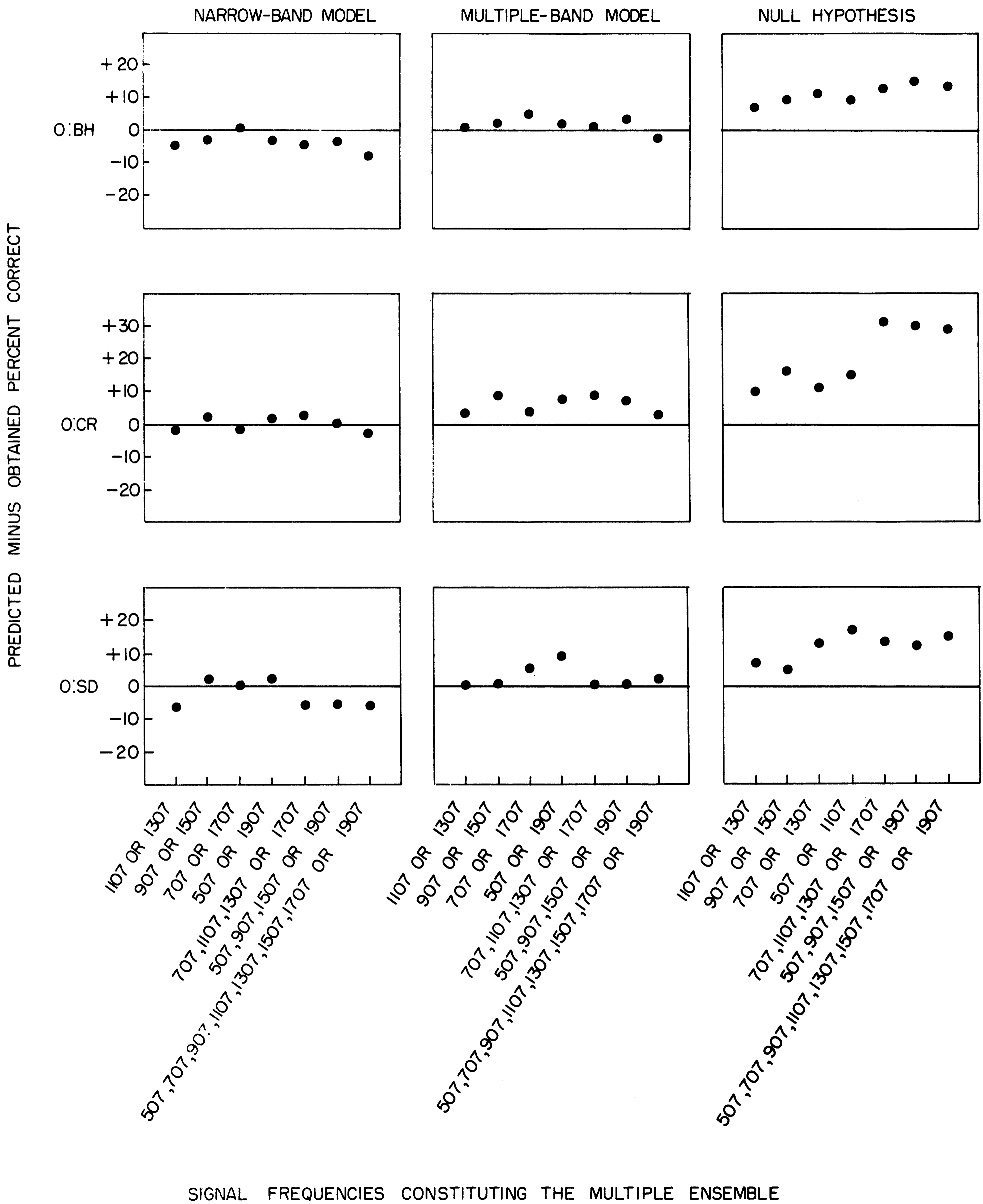
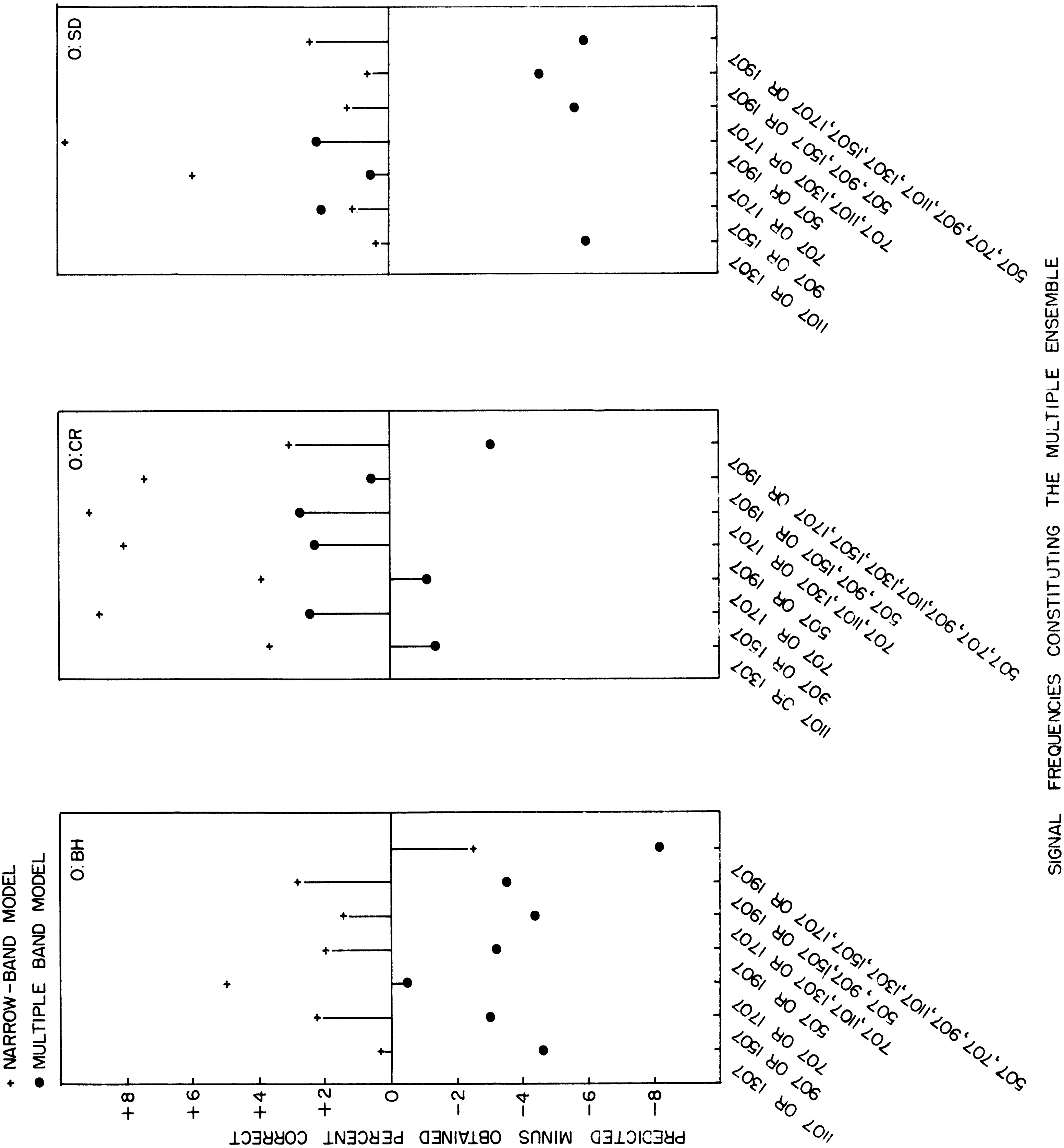

NARROW-BAND MODEL
MULTIPLE-BAND MODEL
NULL HYPOTHESIS
PREDICTED MINUS OBTAINED PERCENT CORRECT
O:BH
O:CR
O:SD
+20
+10
0
-10
-20
+30
1107 OR 1307
907 OR 1507
707 OR 1707
507 OR 1907
707, 1107, 1307 OR 1707
507, 907, 1507 OR 1907
507, 707, 907, 1107, 1307, 1507, 1707 OR 1907
1307
1107
SIGNAL FREQUENCIES CONSTITUTING THE MULTIPLE ENSEMBLE

Figure 4



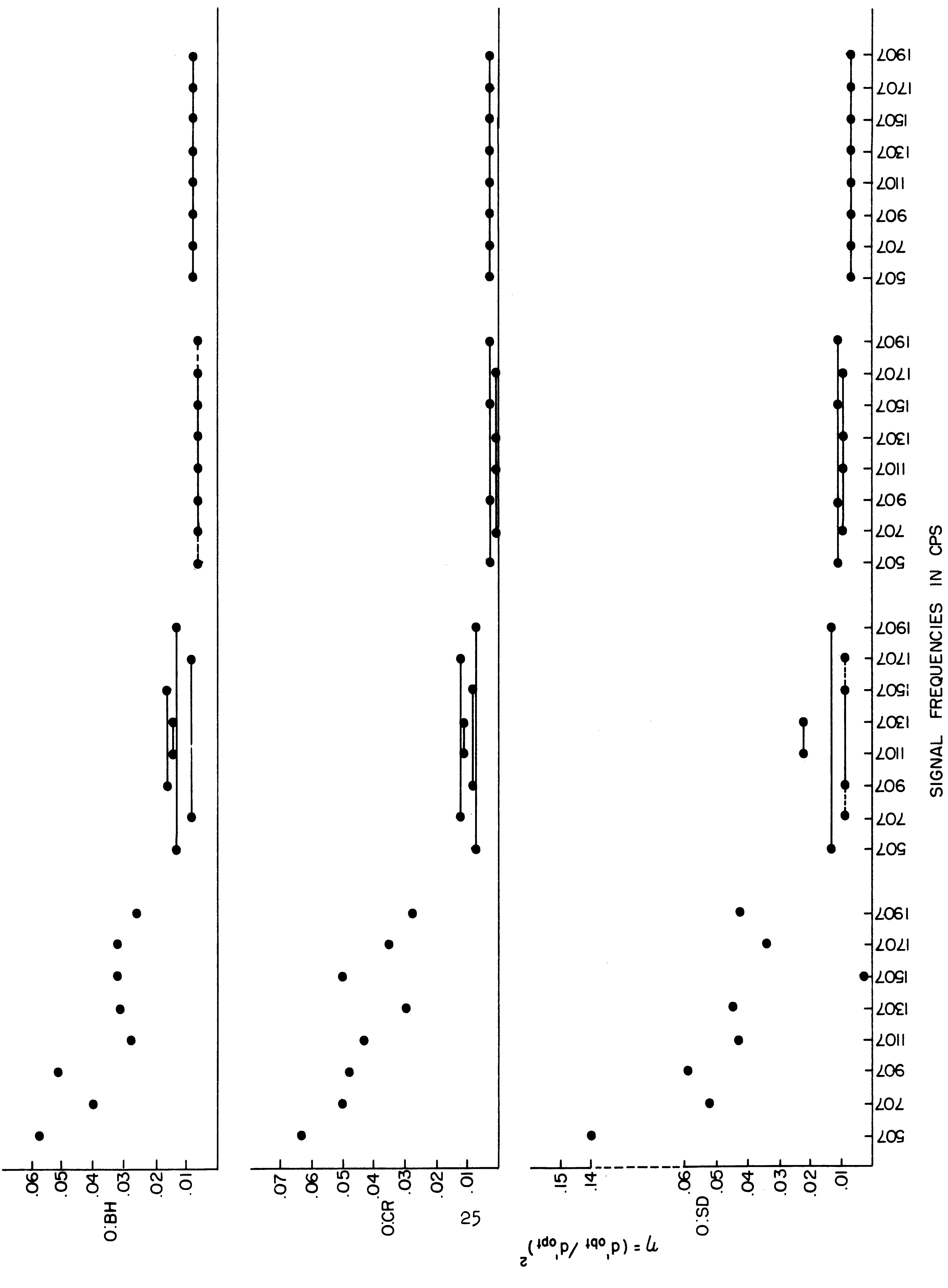
NARROW-BAND MODEL
MULTIPLE BAND MODEL
O:BH
O:CR
O:SD
PREDICTED MINUS OBTAINED PERCENT CORRECT
+8
+6
+4
+2
0
−2
−4
−6
−8
1107 OR 1307
907 OR 1507
707 OR 1707
507 OR 1907
707,1107,1307 OR 1707
507,907,1507 OR 1907
507,707,907,1107,1307,1507,1707 OR 1907
SIGNAL FREQUENCIES CONSTITUTING THE MULTIPLE ENSEMBLE

TABLE VII

A Comparison of d' for the Mathematically Optimum Detector with d' Data for Three Observers, and Efficiencies (η's) Calculated for all Experimental Conditions

Signal in cps	d' opt.	O:BH		O:CR		O:SD	
		d' obt.	η	d' obt.	η	d' obt.	η
507	4.008	0.955	0.057	1.005	0.063	1.50	0.140
707	4.008	0.795	0.039	0.898	0.050	0.915	0.052
907	4.816	1.09	0.051	1.052	0.048	1.165	0.059
1107	5.030	0.845	0.028	1.04	0.043	1.04	0.043
1307	5.199	0.91	0.031	0.886	0.029	1.102	0.045
1507	5.199	0.925	0.032	1.165	0.050	0.25	0.002
1707	5.440	0.975	0.032	1.016	0.035	1.005	0.034
1907	5.804	0.94	0.026	0.97	0.028	1.202	0.043
1107 or 1307	5.05	0.607	0.014	0.542	0.011	0.745	0.022
907 or 1507	4.95	0.62	0.016	0.443	0.008	0.46	0.009
707 or 1707	4.83	0.44	0.008	0.52	0.012	0.45	0.009
507 or 1907	4.97	0.572	0.013	0.427	0.007	0.56	0.013
707, 1107, 1307 or 1707	4.94	0.39	0.006	0.143	0.0002	0.46	0.009
507, 907, 1507 or 1907	4.87	0.38	0.006	0.246	0.002	0.49	0.10
507, 707, 907, 1107, 1307, 1507, 1707 or 1907	4.98	0.416	0.007	0.242	0.002	0.378	0.006

Figure 5



O:BH
O:CR
O:SD
η = (d'obt /d'opt)²
SIGNAL FREQUENCIES IN CPS
507
707
907
1107
1307
1507
1707
1907
.01
.02
.03
.04
.05
.06
.07
.14
.15

ically in Figure 5.

4.2.4 η and P(C) as a Function of Ensemble Size and Ensemble Frequency Range

If mean η's for each ensemble size are considered, it is found that for all observers η's (1) are greatest for the detection of the single signal; (2) decrease considerably (from 50 to 75 percent) for the ensemble-of-two; (3) decrease additionally for the ensemble-of-four. η continues to decrease for one of the observers (O:SD) as the ensemble is increased to eight. The other two observers, however, improve somewhat under this largest-ensemble condition.

The results are similar even if viewed in terms of percent correct. Although it has been shown that all performance on multiple ensemble falls short of null predictions, predictions by the null hypothesis have been based throughout upon percent correct detection of the single signals. If, however, predictions for the four-signal-ensemble are based on detection of the two-signal-ensemble; and if predictions for the eight-signal-ensemble are based upon performance in the four-signal-ensemble condition, we find the following:

(1) Percent correct detections obtained for the two-signal-ensembles are less than predicted from single signal performance.

(2) Percent correct detections obtained for the four-signal-ensembles are less than predicted from performance on two-signal-ensembles.

(3) Percent correct detections obtained for the eight-signal-ensembles are less than predicted from performance on four-signal-ensembles for only one observer, O:SD, For the other two observers, O:BH and O:CR, performance is better than predicted.

A check experiment was performed to determine whether this reversal, found both in η and in P(C) as described above, was attributable to the fact that the observers were operating at efficiencies so close to zero in the four-signal-ensemble that any decrement virtually would be unmeasurable. Wide-band noise was dropped from 0.067 v to 0.040 v and data for four and eight-signal-ensembles were obtained at the new E/N_0 levels for two observers, O:BH and O:CR. The data, based upon four runs of each of the four-signal-ensembles and eight runs of the eight-signal-ensemble are presented in Table VIII. At these new energy levels, we obtain (1) a loss of efficiency as the ensemble increases from four to eight, and (2) percent correct detections on the eight-signal-ensemble that are smaller than average performance on the four-signal-ensemble.

TABLE VIII

		O:BH			O:CR		
Signal Ensemble (cps)	$\underline{d}'$ opt.	$\underline{d}'$ obt.	η	P(C)	$\underline{d}'$ obt.	η	P(C)
707, 1107, 1307 or 1707	7.350	2.40	0.106	95.50	2.29	0.097	94.75
507, 907, 1507 or 1907	7.452	2.40	0.104	95.50	2.37	0.101	95.25
507, 707, 907, 1107, 1307, 1507, 1707 or 1907	7.354	2.13	0.084	93.37	2.09	0.081	93.00

If we compare efficiencies on each of the four-signal-ensembles with the two-signal ensembles of equal frequency range (Table IX) we find lower efficiencies for the larger ensembles of comparable range five of six times. For the frequency range of 1000 cps, however, O:SD shows no change in efficiency with increase in ensemble size. In comparing the

TABLE IX

A Comparison of Efficiencies, ($\underline{\eta}$'s), Obtained for Three Ensemble Sizes and Two Frequency Ranges

Observer	Ensemble Size	Ensemble Frequency Range	
		1000 cps	1400 cps
BH	2	0.008	0.013
	4	0.006	0.006
	8		0.007
CR	2	0.012	0.007
	4	0.0002	0.002
	8		0.002
SD	2	0.009	0.013
	4	0.009	0.010
	8		0.006

ensemble-of-eight with the ensemble-of-four of equal range, it is seen that the efficiency of O:BH increases, of O:CR remains unchanged and of O:SD decreases.

A comparison of efficiencies within ensemble size shows no systematic change as a function of frequency range for any of the three observers.

5. CONCLUSION

Within the experimental conditions described in the present paper, it has been found:

(1) that for 6 of 7 observers, performance in detection decreased as a function of increasing ensemble size, (provided that the level of performance from which decrement was measured was not so low as to make decrement measures unfeasible),

(2) that no conclusive statement can be made about performance as a function of signal frequency range of the ensemble,

(3) that the multiple band model was consistently better in predicting the results of the first experiment, but that

(4) in the second experiment

(A) the narrow-band model more often predicted ~~the obtained~~ the obtained data for at least one observer, whereas for the other two observers (6 of 7 and 5 of 7) of the multiple ensembles were more closely predicted by the multiple-band model,

(B) On the other hand, a comparison of the average deviation of the narrow-band model predictions from the obtained with the avg. deviation of the multiple-band model predictions from the obtained data shows that:

(i) the multiple-band predictions have the smaller average deviation for one observer (O:BH),

(ii) the narrow-band predictions have the smaller average deviation for one observer (O:CR),

(iii) the average deviations of the two sets of predictions from the obtained are virtually equal for one observer (O:SD).

Finally, we conclude that both the narrow-band scanning model and the multiple-band observer model may be descriptive of certain detection phenomena, though neither is sufficient to handle all the data.

REFERENCES

1. Tanner, W. P., Jr., Swets, J. A. and Green, D. M., "Some General Properties of the Hearing Mechanism," Electronic Defense Group, University of Michigan, Technical Report No. 30, March 1956.

2. $\underline{d}'$ as a detection index in a two-alternative forced-choice test is simply a normal transform of the probability of correct detection. For a more detailed treatment of $\underline{d}'$, see Tanner, W. P., Jr., and Birdsall, T. G., "Definition of $\underline{d}'$ and $\underline{\eta}$ as Psychophysical Measures," Electronic Defense Group, University of Michigan, Technical Report No. 80, February 1958.

3. Green, D. M., "Detection of Signals in Noise and the Critical Band Concept," Electronic Defense Group, University of Michigan, Technical Report No. 82, April 1958.

4. For a more detailed treatment, see Peterson, W. W. and Birdsall, T. G., "Theory of Signal Detectability," Parts I and II, Electronic Defense Group, University of Michigan, Technical Report No. 13 (1953), and Tanner, W. P., Jr. and Birdsall, T. G., op. cit. The author wishes to express her thanks to T. G. Birdsall for his guidance in the mathematical treatment of the data.

5. Tanner, W. P., Jr. and Birdsall, T. G., "Definition of $\underline{d}'$ and $\underline{\eta}$ as Psychophysical Measures," Electronic Defense Group, University of Michigan, Technical Report No. 80, February 1958.

6. Unpublished study conducted in the Electronic Defense Group laboratories, University of Michigan, by Marius Smith.

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