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Final Report

RESEARCH IN VEHICLE MOBILITY

Performance Coefficients for Free-Running Wheels in Sand

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## LIST OF SYMBOLS AND TERMS

### General

B	soil bed width, ft
D	soil bed depth, ft
R	rolling resistance of wheel, lb force
v	wheel velocity, ft/sec (axle speed parallel to undisturbed surface)
$v_h$	hypothetical wheel velocity based on rate of rotation, ft/sec
w	wheel load, lb force
S	shear stress, lb/in. <sup>2</sup>
b	wheel width, ft
d	wheel diameter, ft
g	acceleration of gravity, ft/sec <sup>2</sup>
l	length, ft
p	pressure, lb/in. <sup>2</sup>
t	time, sec
Z	sinkage, ft (axle sinkage relative to undisturbed surface)

### Bekker Soil Values

$k_\phi$	sinkage parameter, lb/in. <sup>n+2</sup>
$k_c$	sinkage parameter, lb/in. <sup>n+1</sup>
n	nondimensional sinkage parameter
$K_1$	slip parameter, in. <sup>-1</sup>
$K_2$	nondimensional slip parameter

## LIST OF SYMBOLS AND TERMS (Concluded)

### Coulomb Soil Values

- c soil cohesion, lb/in.<sup>2</sup>
- $\gamma$  soil density, lb/ft<sup>3</sup>
- $\rho$  soil density, slugs/ft<sup>3</sup>
- $\phi$  soil friction angle (nondimensional)

### Coefficients

- R/W nondimensional drag coefficient
- Z/d nondimensional sinkage coefficient
- i nondimensional slip coefficient =  $(v_h - v)/v$
- $V^2/gd$  nondimensional speed coefficient
- $W/g\rho d^3$  nondimensional load coefficient
- $\alpha$  nondimensional shape coefficient (aspect ratio) =  $b/d$
- $\mu$  nondimensional friction coefficient (soil to wheel surface)

### Special

- $\underline{\underline{D}}$  symbol used to denote dimensional equality

## ABSTRACT

This report concerns research done in a largely unexplored area of a relatively new field of engineering. Because of this, the approach and methods used should be considered to have a provisional status. However, the results are believed to be valid, and future developments in methods of analysis may increase their general applicability.

The report is loosely divided into two parts, experimental and theoretical, but owing to the nature of the subject a considerable intermixture of the two sections is necessary and desirable. The experimental sections present performance coefficients in nondimensional form for free running wheels in sand. These are the drag coefficient, the sinkage coefficient, and the slip ratio. The experimental results are compared with theoretical results where these are available. The theoretical results are interpreted in terms of the experimental.

The theoretical section is largely devoted to the dimensional analysis and to soil value systems. The theory of dimensional analysis is presented in some detail, more so than is customary in an engineering report, because apparently this is the first time it has been applied in this manner and because it is felt that the particular stress given the principle of dimensional analysis here is different and useful. Considerable emphasis is given to soil value systems because this subject presents a major obstacle to further progress in the theory of wheel mechanics, especially with respect to methods of dimensional analysis.



## INTRODUCTION

During the past two years, The University of Michigan, through its Research Institute, has done research in the field of land locomotion and particularly on the mechanics of the wheel. This work has been sponsored by the Land Locomotion Laboratory of the Ordnance Tank and Automotive Command, Detroit, Michigan. The main work has been on the study of wheel performance in off-the-road operations by the method of dimensional analysis and the use of models.

Some previous applications of the method of dimensional analysis to wheel mechanics have been made, as for example the work of Nuttall<sup>1</sup> and the work done in Europe mentioned by Bekker.<sup>2</sup> These earlier attempts have in the case of European studies been concerned with hard-surface operations only, or in Nuttall's case have considered a limited range and number of variables so that the results have questionable generality. The basic problem in a dimensional analysis of wheel performance is the correct identification of the variables which define the performance, and of these the greatest problem is with those variables which describe the soil.

The theory of wheel mechanics based on the Coulomb soil value system is not very advanced, and owing to the attendant mathematical difficulties does not offer much hope of improvement. The situation with respect to the Bekker system of soil values is better; successful analyses have been made; but the results have yet to be generally substantiated in practical full-scale experience. As a result, the method of dimensional analysis appears to offer the only immediate means for the successful correlation of wheel performance parameters.





## THEORY

### DIMENSIONAL ANALYSIS

An extended study of this subject reveals the facts that it has received much attention from the most eminent scientific philosophers and that it has aroused much controversy. A typical paper on the subject generally notes that confusion exists, offers some new ideas about or interpretations of the subject, and concludes by deploring the state of confusion. It appears, however, that the subject is basically a part of the philosophy of the physical universe and will continue to have controversial aspects as long as there are controversial aspects of the universe, which seems to be indefinitely. This, however, does not deny its usefulness, as most if not all physical theories have some limit of validity.

Despite its well-laid foundations and demonstrated usefulness, dimensional analysis is clearly not a commonly used engineering concept. Speculation about the reason for this leads one to the idea that somehow the subject has yet to be adequately presented. For example, in commenting on the nature of dimensional analysis, Weber<sup>3</sup> says: "Fundamentally dimensional analysis is identical with the analysis of physical equations, and in particular with the analysis of physical differential equations." This statement may be extremely enlightening to some people, but is not likely to move those with an engineering viewpoint. Again, Bridgman<sup>4</sup> states: "Dimensional analysis is essentially of the nature of an analysis of an analysis." This is a little better perhaps, but also seems rather unrelated to engineering matters. The problem of understanding is basically a consequence of the broad philosophical aspects of the subject, and it is not likely to be answered satisfactorily for all concerned by any single statement or expressed viewpoint. It appears, therefore, that there is justification for additional presentation of the subject.

The following comments express concepts that have been useful in the present study. These comments merely stress aspects of the subject that are apparently neglected, and offer nothing new to the theory.

In approaching a physical problem by dimensional analysis, the question naturally arises of "What is it we are trying to do?" The answer to this question is that we are trying to discover the natural or master variables that determine the performance of the system. The variables that are customary to engineering such as pressure, velocity, density, etc., are concepts of the mind and in a sense represent the operation of simple machines that are developed from fundamental ideas like time, length, and mass. A more complex structure such as a pressure vessel is not, anthropomorphically speaking, particularly aware of its internal pressure, but is only concerned with whether or not it exists as a vessel. In aerodynamics, it is found that within certain limits

the drag coefficient of a sphere is a function of the Reynolds number. In this case, the natural variables of the air and sphere system are the drag coefficient and the Reynolds number. It is unfortunate that a simple instrument to measure Reynolds number does not exist and apparently cannot be conceived. If it did exist, consideration of aerodynamic problems would be directly in terms of this variable instead of through the indirect approach of density, viscosity, velocity, and characteristic length.

Dimensional analysis is therefore a means to find these master variables of a system and to define them in terms of simple concepts such as pressure, density, and velocity. In applying dimensional analysis to a problem, the first step is to enumerate the basic variables such as pressure, density, etc., which are involved in the problem. As has been pointed out many times, these are exactly the same variables which must be used in a successful mathematical analysis of the problem. Dimensional analysis is of no assistance at this point, and success depends on adequate intuition and experience. The results will be no better than the assumption of the basic variables.

Once the assumption has been made, the rules are fairly definite and at this point dimensional analysis constitutes a method. The application of the method to the assumed basic variables yields the nondimensional master variables which are generally fewer in number than the basic variables. These master variables also have the advantage that, being nondimensional, they are subject to no restriction of mathematical manipulation imposed by dimensional homogeneity. In problems of mechanics, the number of master variables will ordinarily be the number of basic variables less three, or less the number of fundamental units in the system of measurement which for the M,L,T system equals three. This rule is not infallible, and the correct rule has been formulated by Langharr.<sup>5</sup> This rule is best introduced by an example.

Assume a system that involves as basic variables the physical quantities  $v, t, g, \rho, p, \mu, \ell$ . Using these quantities, a matrix is developed which lists the exponents of the fundamental units in terms of the basic quantities as follows.

	$v$	$t$	$g$	$\rho$	$p$	$\mu$	$\ell$
M	0	0	0	1	1	1	0
L	1	0	1	-3	-1	-1	1
T	-1	1	-2	0	-2	-1	0

Taking the columns headed  $p, \mu, \ell$ , and evaluating the third-order determinant

$$\begin{vmatrix} 1 & 1 & 0 \\ -1 & -1 & 1 \\ -2 & -1 & 0 \end{vmatrix} = 0 + 0 - 2 + 0 + 0 + 1 = -1,$$

it is found that the value of this determinant is -1 and is therefore not zero. Therefore the rank of the determinant is three. If this determinant were zero, it might be possible to find a nonzero determinant from some other combination

of three columns. If any nonzero determinant can thus be found, the rank of the matrix is three. If it cannot be found, the matrix is of rank less than three. In that case, it may be of rank two, which will be the case if it is possible to form a nonzero determinant of order two from any combination of two columns and two rows. Langharr's rule states simply that the number of master variables is equal to the number of basic variable less the rank of the matrix. In this case, the number is  $7-3 = 4$  master variables.

One of the master variables thus found will be dependent, the others independent. Any one of these may be selected as the dependent variable, but will necessarily be one that contains the dependent basic variable under consideration.

There are at least two methods of formulating the master variables from the basic variables. One method is attributed to Lord Rayleigh, the other to E. Buckingham.<sup>6</sup> Of the two, the method due to Rayleigh appears to be favored. It will be illustrated by an example. Returning to the previous set of basic variables, suppose it is desired to find the dependence of  $v$  on the other variables, or

$$v = f(t, g, \rho, p, \mu, l) \quad (1)$$

According to the theory of dimensional analysis, this functional relation may be expressed as

$$v = t^{a_1}, g^{a_2}, \rho^{a_3}, p^{a_4}, \mu^{a_5}, l^{a_6} \quad (2)$$

where the exponents are sufficiently arbitrary to allow the functions to be expressed and permit, for example, such forms of the function as

$$v = t^{a_1} \cdot g^{a_2} \cdot \rho^{a_3} \cdot p^0 \cdot \mu^0 \cdot l^0 + t^0 \cdot b^0 \cdot \rho^0 \cdot p^{b_1} \cdot \mu^{b_2} \cdot l^{b_3} \quad (3)$$

where  $t^{a_1} \cdot g^{a_2} \cdot \rho^{a_3}$  and  $p^{b_1} \cdot \mu^{b_2} \cdot l^{b_3}$  must have the dimensions of  $v$ . The objective is to express the exponents in the least arbitrary way, and this is done by equating the dimensions of exponents. Equation (2) is written in the dimensional form.

$$LT^{-1} = (T)^{a_1} (LT^{-2})^{a_2} (ML^{-3})^{a_3} (ML^{-1}T^{-2})^{a_4} (ML^{-1}T^{-1})^{a_5} (L)^{a_6} \quad .$$

Equating the exponents of  $M$  yields

$$0 = a_3 + a_4 + a_5$$

of  $L$

$$1 = a_2 - 3a_3 - a_4 - a_5 + a_6$$

of T

$$-1 = a_1 - 2a_2 - 2a_4 - a_5 .$$

There are three equations and six unknowns, so three unknowns remain arbitrary; let these be  $a_1, a_2, a_3$ . Thus

$$a_4 = 1 + a_1 - 2a_2 + a_3$$

$$a_5 = -1 - a_1 + 2a_2 - 2a_3$$

$$a_6 = 1 - a_2 + 2a_3 .$$

Entering these exponents into Eq. (2) gives

$$v = t^{a_1} g^{a_2} \rho^{a_3} p^{(1 + a_1 - 2a_2 + a_3)} \mu^{(-1 - a_1 + 2a_2 - 2a_3)} l^{(1 - a_2 + 2a_3)} ,$$

and on collecting terms of like exponents

$$\frac{v\mu}{pl} = \left(\frac{tp}{\mu}\right)^{a_1} \left(\frac{g\mu^2}{p^2l}\right)^{a_2} \left(\frac{p\rho l^2}{\mu^2}\right)^{a_3} \quad (4)$$

Since the variables are nondimensional, they may be combined to form new terms with new arbitrary exponents. Multiplying the first and fourth terms yields

$$\frac{v\mu}{pl} \cdot \frac{p\rho l^2}{\mu^2} = \frac{\rho lv}{\mu}$$

which is the Reynolds number. Discarding  $v\mu/pl$ , which can no longer be considered independent, the equation may now be written

$$\frac{\rho vl}{\mu} = \left(\frac{tp}{\mu}\right)^{b_1} \left(\frac{g\mu^2}{p^2l}\right)^{b_2} \left(\frac{p\rho l^2}{\mu^2}\right)^{b_3} . \quad (5)$$

By additional manipulations of this type, the form of the equation reduces to

$$\frac{\rho vl}{\mu} = \left(\frac{tv}{l}\right)^{c_1} \left(\frac{v^2}{gl}\right)^{c_2} \left(\frac{\rho v^2}{\mu}\right)^{c_3} , \quad (6)$$

in which the Reynolds number is an arbitrary function of the terms on the right, where the second term on the right is Froude's number, and the third term is Newton's number. This form of the master variables is not useful for solving for  $v$ , since  $v$  occurs in all terms, but Eqs. (4) and (5) would be suitable, however.

This completes the presentation of the method, or as much of it as is demanded for the purpose at hand. Before the method is applied to the subject of wheel-soil mechanics, it is necessary to investigate the basic variables

needed to describe the problem. The most difficult part is the soil value system.

## SOIL VALUE SYSTEMS

There are two systems of soil values in current use that have applicability to the soil-wheel mechanics problem. One is called the Coulomb system; the other, as far as is known, has not been named, but is here called the Bekker system. The Coulomb system describes the soil in terms of three quantities, the soil cohesion, the friction angle, and the density. An aeolotropic soil will have different values for the cohesion and friction angle in different directions. The Coulomb system has a look of fundamentality about it, because it describes the soil properties in terms of simple basic concepts. Accordingly, if these properties are valid, it should be possible to construct with them solutions to problems involving particular mechanics and geometry. On the practical application, it turns out that the procedure from the basic Coulomb quantities to the solution of a particular case involves difficult conceptual and mathematical problems, and the result is that no really satisfactory analysis of soil-wheel mechanics rests on this system.

Aside from mathematical difficulties, it appears to be an open question whether the Coulomb system is adequate for all problems of soil mechanics. The incorporation into the soil plasticity theory of ideas such as energy dissipation, velocity distribution, and strain-hardening suggests that additional soil properties are needed for a general theory. This problem does not appear to exist for sand, which is noncohesive and represents a simple, limiting type of soil. Therefore the prognosis is that the Coulomb system will be successful as a means of correlating wheel performance in sand.

The Bekker system approaches the problem of soil values from a different concept. It is the view here that the Bekker system basically constitutes an analogy of the wheel. Accordingly, an important factor in the validity of the Bekker system is whether the analogy is adequate. As it stands, this question cannot be positively answered, but experience has shown that the system is at least applicable under a limited range of conditions.

The Bekker system is apparently based on the earlier Bernstein system, which defined soil properties in terms of certain characteristic dimensions of the wheel, the load on the wheel, and the drag of the wheel. The Bernstein system yields soil values in terms of wheel performance, and this is a disadvantage from a measurement viewpoint, whatever else may be said for it. The Bekker system is independent of wheel performance measurement, and thus offers a more generalized approach.

The Bekker values are derived from two relatively simple tests. One test consists of forcing a flat plate into the ground, and recording the force-penetration relations. The plate has a characteristic dimension "b," and if

circular "b" is the diameter, and if rectangular, it is the smallest dimension of the rectangle. This is the test analogous to the wheel in its load-bearing and rolling-resistance capacity. The penetration test gives results typical of Fig. 1.

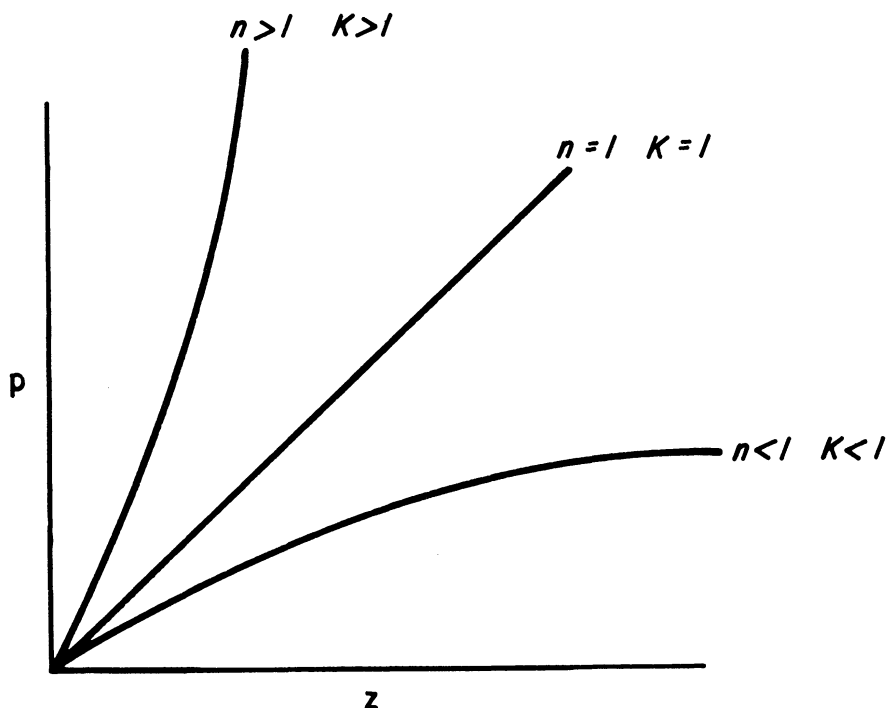


Fig. 1. Representation of soil-penetration data.

These curves showing the force-penetration relation for the test are adequately represented by an equation of the form

$$p = kz^n \quad (7)$$

where

- $k = k_\phi + k_c/b$
- $p =$  pressure under plate
- $z =$  sinkage of plate
- $b =$  characteristic dimension of plate
- $k_\phi, k_c, n =$  Bekker soil values.

As the mathematical relation consists of a single exponential term, the Bekker constants  $k_c, k_\phi, n$  may be evaluated by the use of logarithms.

The other Bekker test consists of finding the shear strength of the soil by means of a loaded circular plate pressed into the soil with the application of a torque to rotate the plate. The plate has lugs attached to improve the grip on the soil. The results for a single soil are represented by the curves of Fig. 2.

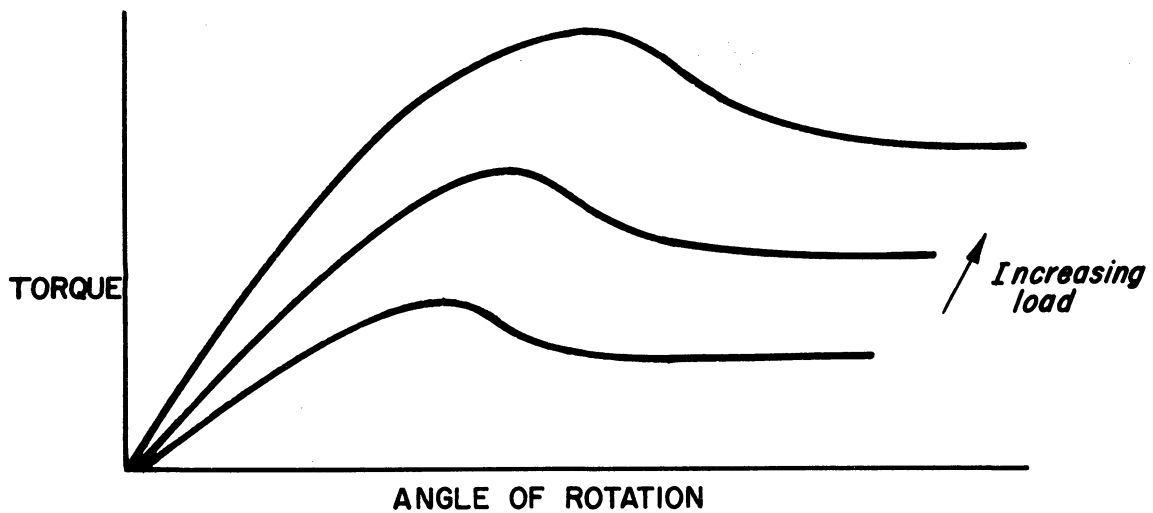


Fig. 2. Representation of soil-shear data.

These curves may be adequately represented for the present by a mathematical expression of the type

$$S = c + p \tan \phi \left\{ \frac{e^{(-K_2 + \sqrt{K_2^2 - 1}) K_1 d} - e^{(-K_2 - \sqrt{K_2^2 - 1}) k_1 d}}{[e^{(-K_2 + \sqrt{K_2^2 - 1}) K_1 d} - e^{(-K_2 - \sqrt{K_2^2 - 1}) k_1 d}]_m} \right\}, \quad (8)$$

where

- S = shear stress under plate
- c = cohesion of soil
- p = pressure under plate
- $\phi$  = soil-friction angle
- $K_1 K_2$  = Bekker soil values
- m = maximum value of term in brackets
- d = strain in soil under plate.

It is noted that this test makes use of the Coulomb values  $c$  and  $\phi$ , and thus the Bekker system incorporates these values. This test is analogous to the wheel in its traction capacity. Since this work is not concerned with wheel traction, no further comment will be made on this part of the Bekker system.

Returning to the first set of Bekker constants,  $k_c$  and  $k_\phi$  and  $n$ , it seems certain that these quantities are in some way related to the Coulomb soil values, the system boundaries, and perhaps to presently undefined soil properties. It would be most desirable if a workable relation between these systems could be established, as the soil-measurement procedures would be simplified. The relation would also be useful in the dimensional analysis of the soil-wheel mechanics, as will be seen presently.

While there is no analytical relation between Bekker's constants  $k_\phi$ ,  $k_c$ , and  $n$  and the Coulomb values, dimensional analysis is able to suggest some plausible forms for the relations. A dimensional analysis of the Bekker constants shows them to have the dimensions

$$k_\phi \stackrel{D}{=} \frac{F}{L^{n+2}}$$

$$k_c \stackrel{D}{=} \frac{F}{L^{n+1}}$$

$$n \stackrel{D}{=} 0 .$$

The Coulomb constants have the dimensions

$$c \stackrel{D}{=} F/L^2$$

$$\phi \stackrel{D}{=} 0$$

$$\gamma \stackrel{D}{=} F/L^3$$

Beginning with the Bekker constant  $n$ , it is noted from experience that the value of  $n$  is strongly influenced by the nonhomogeneity of the soil, or more specifically by the depth of the soil over a hard bottom. Thus,  $n$  may be considered to be a function of the soil depth in a homogeneous soil, but since depth is dimensional, the depth must be canceled dimensionally by another characteristic length which can logically be the pressure plate's characteristic dimension. Thus the group  $b/B$  can be formed as a master variable. It is further noted that soils without cohesion exhibit nonzero values of  $n$  so that " $n$ " does not appear to be a function of  $c$ . The friction angle of the soil presumably affects  $n$ , although no direct experiments to establish this fact are known.

Nothing is known about the influence of  $\gamma$  on " $n$ ," but if this influence exists,  $\gamma$  must enter the relation along with  $c$  and a characteristic length in order to be dimensionally correct. But  $c$  has already been excluded so it is probable that

$$n = f(b/B, \phi) . \quad (9)$$

Experiments also show that soils without cohesion have the Bekker constant  $k_c = 0$ . This means that the soil cohesion certainly enters the relation for  $k_c$ . If this holds, then  $\gamma$  and a characteristic length must also enter the relation to satisfy dimensional requirements. It is presumed that  $\phi$  may also influence  $k_c$  but no experimental data on this point are known, so that probably

$$k_c = f(c, \gamma, B, b, \phi) . \quad (10)$$



In the same way as before, it is noted that for cohesionless soils,  $k_\phi$  has a nonzero value, so that  $c$  does not enter the relation for  $k_\phi$ . In this case  $\gamma$  is necessary along with a characteristic length, so that

$$k_\phi = f(\gamma, b, B, \phi) \quad (11)$$

Tests in deep sand with a friction angle of  $33^\circ$  have shown values of  $n \cong 1.0$  so for this case the Bekker constants have the dimensions

$$k_\phi = F/L^3$$

$$k_c = F/L^2$$

$$n = 0$$

This suggests that under these conditions

$$k_\phi = f(\gamma, \phi) \quad (12)$$

$$k_c = f(c, \phi) \quad (13)$$

$$n = 1.0 = f(\phi) \quad (14)$$

This is about all that dimensional considerations can yield on the problem at the present. The possibility remains as suggested before, that the Coulomb system is inadequate for a full description of the soil within the concepts of vehicle mobility. If this is true, then it is to be expected that the Bekker values will depend on additional soil properties beyond the ones considered here.

#### APPLICATION OF DIMENSIONAL ANALYSIS TO SOIL-WHEEL MECHANICS

In setting up the experimental test conditions, certain variables are selected as contributing to the soil-wheel mechanics. As pointed out in the section on dimensional analysis, the selection of the appropriate variables depends largely on experience and intuition. This further presumes that there are conceptual variables adequate to describe the system under consideration. This point was mentioned in the section on soil values and it is with the soil values that the greatest difficulty rests. For the present, however, it is assumed that either the Coulomb or the Bekker soil value system is adequate to represent the properties of sand, and this, taken with the conditions listed in the section on test procedure, leads to the following basic test variables in the Coulomb system.

##### Dependent Variables

- Z - sinkage
- R - rolling resistance
- i - slip

## Independent Variables

d	- wheel diameter	
D	- soil depth	
B	- soil bin width	
$\alpha$	- aspect ratio (wheel width/diameter)	
$\mu$	- coefficient of friction - wheel to soil	
g	- acceleration of gravity	
v	- wheel speed (axle)	
W	- wheel load	
$\rho$	- soil density	} Coulomb soil properties
$\phi$	- soil friction angle	
c	- soil cohesion	

Since the tests are in sand, the cohesion  $c$  is taken equal to zero. In the Bekker system the same quantities apply except that  $k\phi$ ,  $k_c$ ,  $n$  are substituted for the Coulomb soil properties, with  $k_c = 0$ .

A dimensional analysis of these variables for the Coulomb system leads to the following master variables and relations.

$$\begin{aligned}
 R/W &= f [(V^2/gd) (W/gpd^3) (\alpha) (\phi) (\mu) (d/D)] \\
 Z/d &= g [(V^2/gd) (W/gpd^3) (\alpha) (\phi) (\mu) (d/D)] \\
 i &= h [(V^2/gd) (W/gpd^3) (\alpha) (\phi) (\mu) (d/D)]
 \end{aligned} \tag{15}$$

In the Bekker system the variables and relations are

$$\begin{aligned}
 R/W &= f \left[ \left( \frac{d^{n+2} k\phi}{W} \right), \left( \frac{k_c}{d k\phi} \right), \alpha, \mu, n \right] \\
 Z/d &= g \left[ \left( \frac{d^{n+2} k\phi}{W} \right), \left( \frac{k_c}{d k\phi} \right), \alpha, \mu, n \right] \\
 i &= h \left[ \left( \frac{d^{n+2} k\phi}{W} \right), \left( \frac{k_c}{d k\phi} \right), \alpha, \mu, n \right]
 \end{aligned} \tag{16}$$

It was stated before under procedures that certain limitations made it impractical to vary the soil properties, and of course gravity, so that the effective independent basic variables are  $W$ ,  $v$ ,  $d$ ,  $\alpha$ . This means that the functional relations to be actively sought are

$$R/W = f [(V^2/gd) (W/gpd^3) (\alpha)] \quad \text{Coulomb System} \tag{17}$$

$$R/W = f \left[ \left( \frac{d^{n+2} k\phi}{W} \right), (\alpha), (n) \right] \quad \text{Bekker System} \tag{18}$$

In the Coulomb system, it is assumed that B and D are large enough so that the effect on the groups these terms enter is asymptotic and therefore practically do not enter as variables.

For the Coulomb system, the model rules require that

$$\begin{aligned} V^2/gd &= k_1 & \text{or} & & V &= k_1 \sqrt{d} \\ W/g\rho d^3 &= k_2 & \text{or} & & W &= k_2 d^3 \end{aligned}$$

When the model rules are applied to the Bekker system, it is seen that no result can be obtained without specifying  $k\phi$  and  $n$ . This is the immediate reason for avoiding the Bekker system in the present study and is the reason why the necessity for developing the relation between the Bekker system and the Coulomb system was stressed in the last section.

If, as seems possible, the value of  $n$  may be taken to be unity for the present work, then the Bekker system reduces to

$$R/W = f \left[ \left( \frac{d^{n+2} k\phi}{W} \right), (\alpha) \right] = g \left[ \left( \frac{g\rho d^3}{W} \right), \alpha \right] \quad (19)$$

According to Eq. (19), the Bekker system reduces to the Coulomb system for sand, when  $n = 1$ . The only difference is that the speed does not enter the Bekker relation. This is because the gravitational constant is not essential to the Bekker system of constants, or more particularly because the soil density is not one of the Bekker constants. It therefore appears immaterial when testing in sand which system is used, except that the Coulomb system allows the effects of speed to be investigated, and is therefore the system used here.



## EXPERIMENTAL RESEARCH

### TECHNIQUES

One of the unexpected things about wheel model tests is the magnitude of experimental difficulties encountered. Part of this problem is simply the range and number of variables which must necessarily be examined. Instrumentation itself is not difficult. The force, displacements, etc., to be measured are relatively large, and no serious problem of transients or frequency response are encountered. The greatest single difficulty, and one which has not been altogether successfully countered, is the preparation of the soil bed to a uniform state. This more than anything else probably accounts for the large scatter in test results normally encountered in work of this type. The properties of sand and other soils are quite sensitive to their prior history of treatment. Various mechanical agitators and packers have been tried as a means to bring the sand to a uniform state, but the best method yet found is the use of an air lift under the sand. Unfortunately, this method could not be applied to the tests here because the blowing of the sand is detrimental to the delicate mechanical parts of the wheel measuring apparatus. The method finally used involves passing a coarse rake through the bed to its depth a definite number of times, followed by a single pass with a leveling drag. Tests were generally repeated to take advantage of the mean, although in some sensitive areas the tests were tripled.

The next most difficult problem encountered is the balance system which allows the wheel to reach its natural sinkage under an applied load. This is a direct consequence of the large range of loads tested, varying from 1.5 to 100 lb. If the response of the balance system is to be  $\approx 5\%$  on load, the sensitivity must be .075 lb or about one ounce. This in a system with a dead weight of perhaps 25 lb calls for very good mechanical performance along with necessary ruggedness to handle the large loads. It is assumed that the motion of the carriage imparts a sensitivity that is based on dynamic friction, rather than static friction, and that the results are somewhat better than those based on static measurements of the balance sensitivity.

### APPARATUS

The test facility consists of an elongated metal bin filled with sand on which the wheel is dragged. The soil bin is 40 ft long, 2.0 ft wide and 1.0 ft deep. It was filled with sand to a depth of 7.50 in. for making the wheel tests. Provisions are made for measuring wheel rotation, drag, sinkage, and speed. The carrier which holds the wheel is arranged so that various fixed loads may be applied to the wheel.

The wheel carriage rolls on a monorail track mounted above the test bin,

and is moved by a cable-windlass device which is in turn driven by a variable-speed d-c motor. The motor is part of a Ward-Leonard type system operating from an an amplidyne generator. The speed of the motor is controlled and regulated by electrical feedback and amplification from the motor speed indicator to the generator field. The speed range available is 0-6 ft/sec.

The test wheels are made from aluminum tubing with side plates. They are of rectangular cross section with smooth surfaces. A general view of the test facility is shown in Fig. 3. A more detailed view is given in Fig. 4.

Wheel Rotation.—Wheel rotation is measured by means of an electrical potentiometer that has its rotating shaft attached to the wheel axle. A d-c voltage is applied to the potentiometer and the output is carried through cables to the recorder. The angular resolution of the system is about 1/2 degree of rotation. The potentiometer is shown at the extreme left of the test wheel axle in Fig. 5.

Wheel Sinkage.—The sinkage measurement is incorporated with the part of the carriage that permits vertical motion of the wheel and axle. The dead weight of the wheel holder is counterbalanced by a flexible steel cable passing over a pulley to a pan on which weights are applied. The rotation of the pulley due to paying in and out of cable is measured with a potentiometer and serves as an indication of sinkage. The resolution of the sinkage measurements is about 1/32 in. This apparatus is visible in the upper left of Fig. 6.

Wheel Drag.—The wheel drag is determined by measuring the horizontal reaction of that part of the carriage system which supports the wheel and the sinkage apparatus. These particular parts are supported by a parallelogram frame with vertical sides which ultimately attach to the main carriage frame. The vertical parts of the parallelogram are not capable of horizontal reaction since they have pin joints, and therefore the total horizontal reaction of the wheel drag is applied to a single point and is measured there by means of a strain gauge. This system is capable of resolution of about 1% of the peak drag for which the recorder is set, the resolution being mainly limited by the recorder. The parallelogram support, strain gauge and oscillation damper are shown in Fig. 6.

Wheel Speed.—The speed of the wheel axle is determined by measuring the rate of rotation of the windlass motor driving the carriage. An electrical tachometer is attached to this motor, the output calibrated directly in ft/sec of wheel velocity. The tachometer output is also used for speed regulation purposes in the feedback system. These elements are shown in Fig. 3.

Apparatus Notes.—The parallelogram wheel support provides a convenient and satisfactory means of separating the horizontal wheel reactions from the vertical forces, but it will do this only if the sinkage motion of the wheel is in a vertical line. Any other motion will yield cross components between

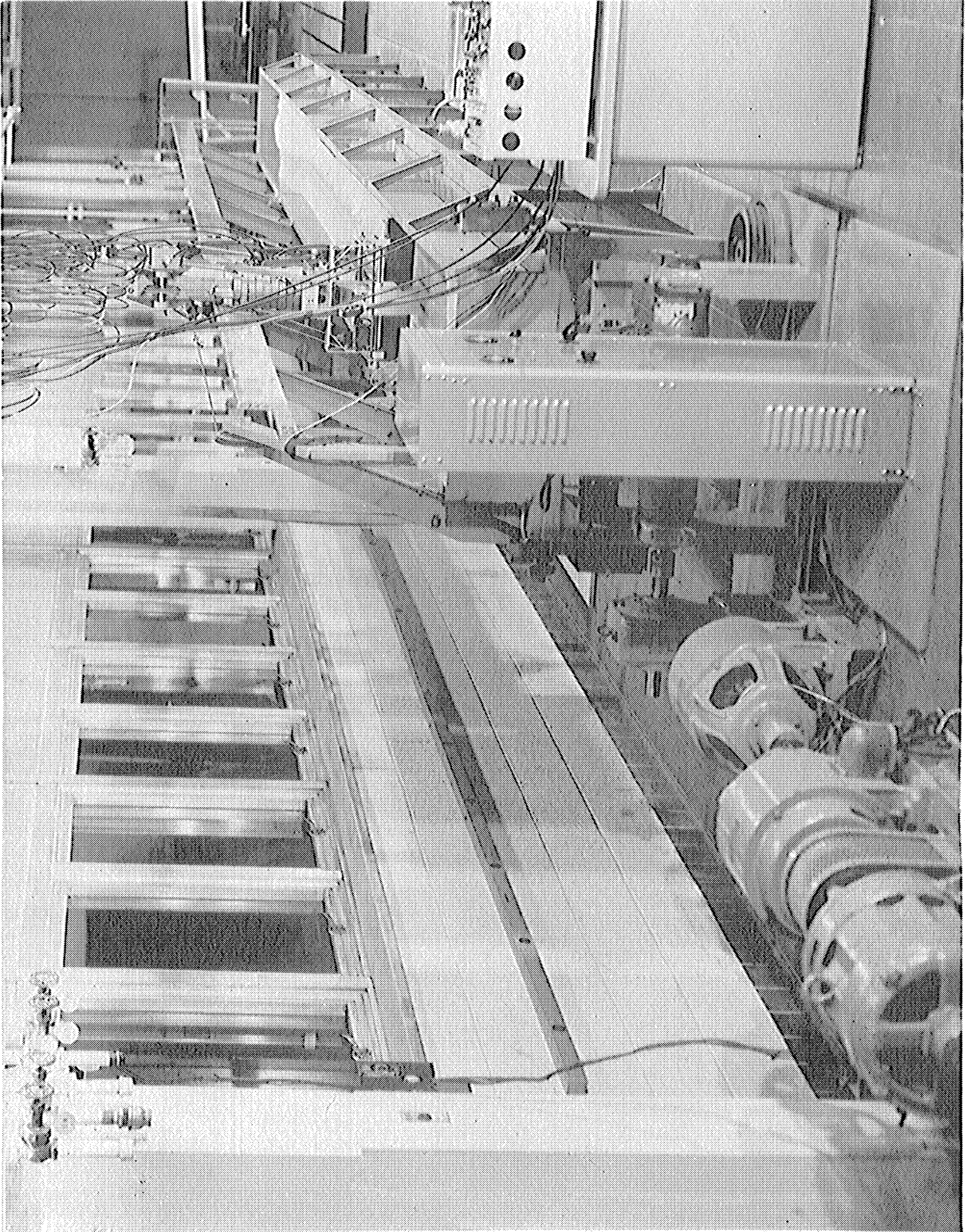


Fig. 3. General view of test facility.



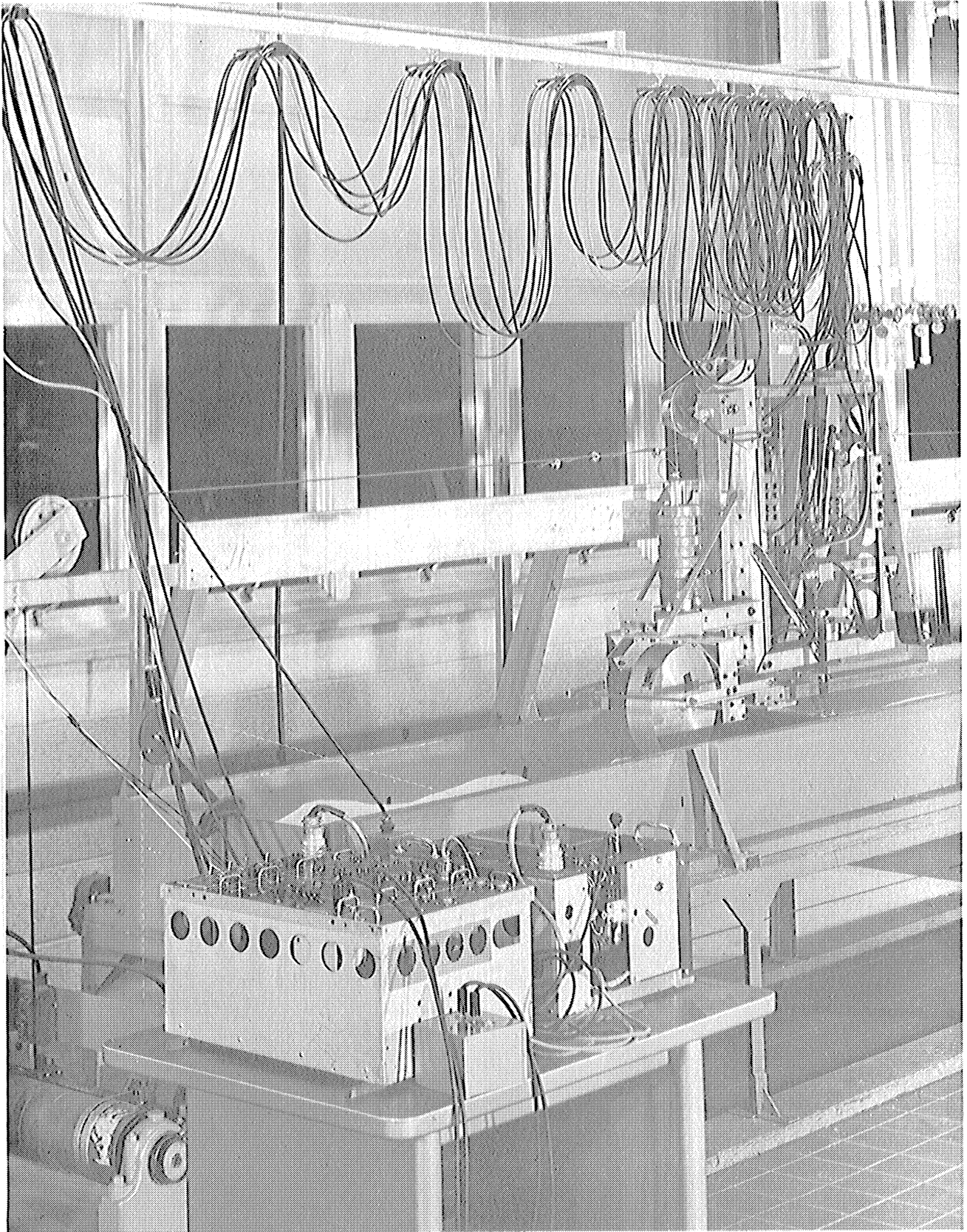


Fig. 4. Detailed view of test facility.



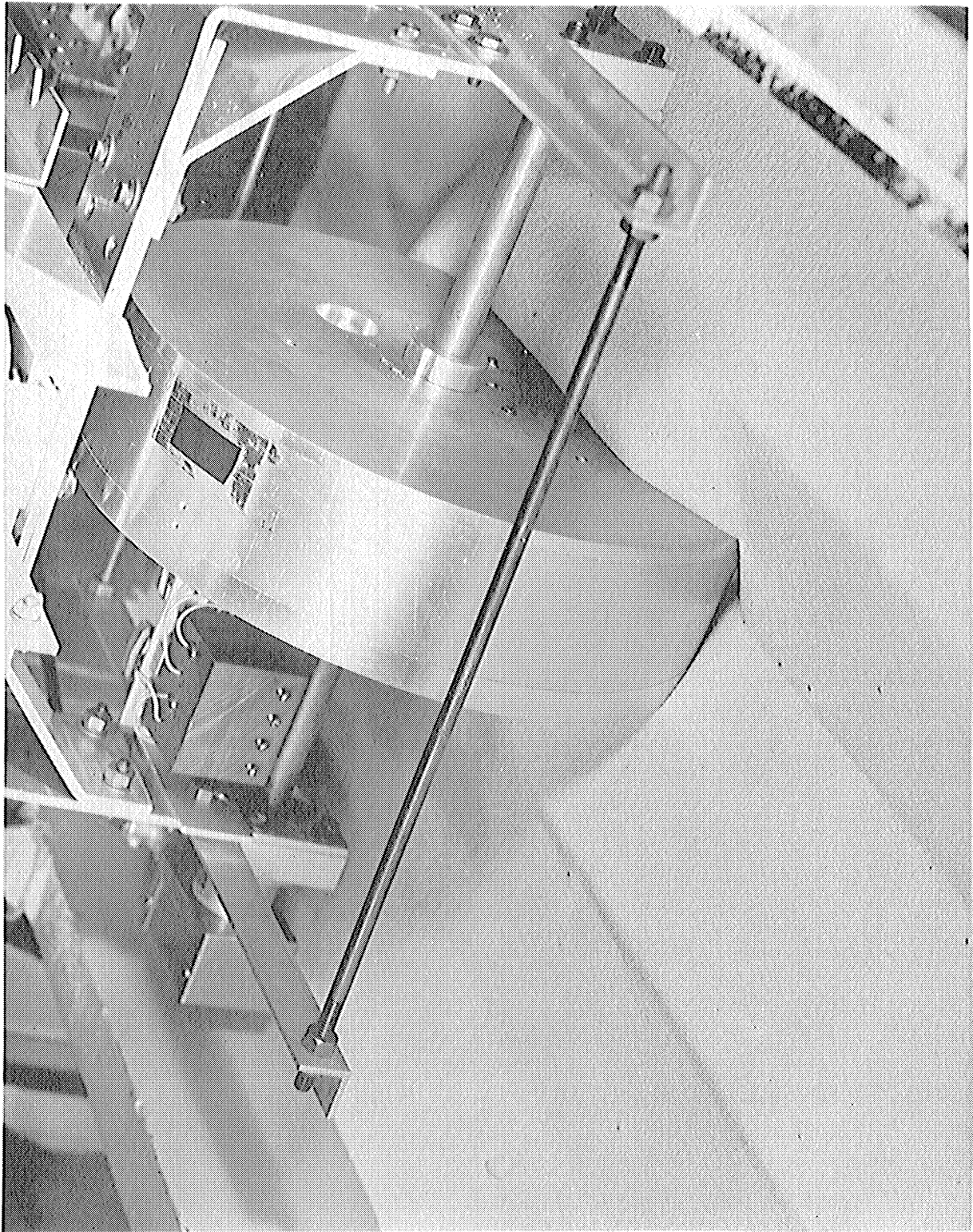


Fig. 5. Detailed view of wheel under test.

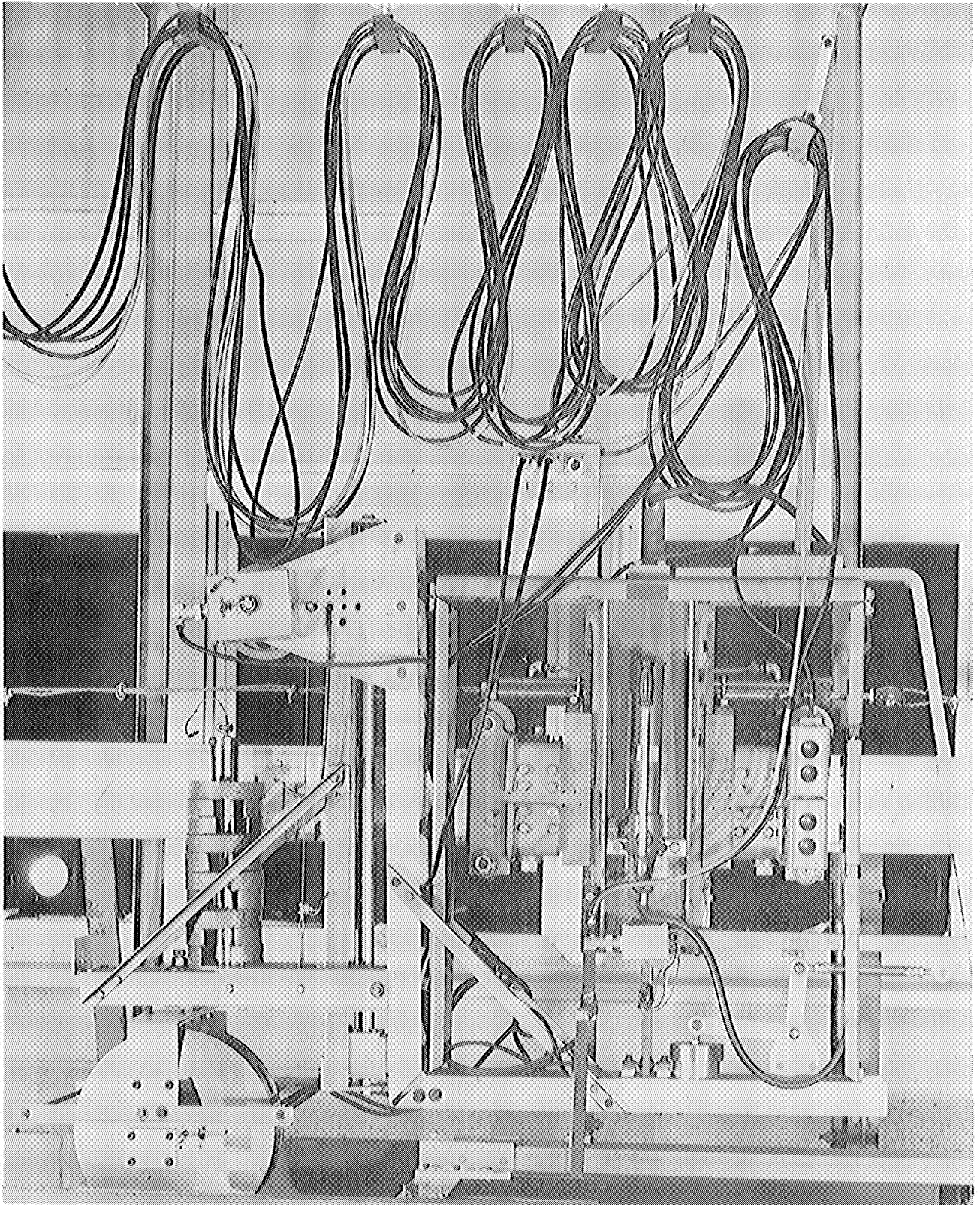


Fig. 6. Detailed view of test carriage.

the load and the drag. The vertical motion of the wheel was obtained in these tests by means of linear ball bearings rolling on polished and hardened steel ways. This system is compact and rugged but still has inconveniently large friction considering the range of forces to be measured. It is proposed that a better system can be developed using pin-link components which have inherently very low friction. Various straight-line motions using pin-link components have been developed. Most of these give approximate straight-line motion, but some come very close. The Watt linkage appears to be well suited to this application.

## PROCEDURE

On setting up the experimental program, it was apparent that a complete study of all the possible variables that might influence wheel performance was impractical. For this reason, the study has been limited to those variables which judgment and past experience indicate to be most important. The nondimensional variables selected for study are the dependent variables  $R/W$  (drag coefficient),  $Z/d$  (sinkage coefficient), and  $i$  (slip coefficient), and the independent variables  $V^2/gd$  (speed coefficient),  $W/gd^3$  (load coefficient), and  $\alpha$  (aspect ratio).

The influence of the independent variable  $\phi$  (friction angle),  $\mu$  (friction coefficient),  $d/D$  (depth ratio), and  $b/B$  (width ratio) was not investigated for several reasons, the main reason being that the inclusion in the study of even one of these variables led to an excessively ambitious program at such an early stage of investigation. With respect to  $\phi$ ,  $d/D$ ,  $b/B$ , it was felt that these quantities do not change greatly under practical wheel operating conditions in sand, and could therefore be left out of the study without serious consequences. The omission of  $\mu$  is more serious, however, as there is good reason to believe that it has a significant influence on wheel performance. A choice had to be made between  $\mu$  and  $V^2/gd$ , and the latter was chosen because of the potential significance of this variable. That is, it was desired to observe at what speed dynamic effects might become significant, since at very low speeds, no detectable influence was observed.

The experimental work has been performed on smooth, rigid, rectangular cross-section wheels made of aluminum, ranging in diameter from 0.552 to 1.040 ft and having aspect ratios (width/diameter) of from 0.27 to 0.84. Loads of from 1.48 to 100 lb have been applied and speeds of from 0.73 to 5.0 ft/sec. The wheels were unpowered and free running.

To hold the nondimensional independent variables constant for the different wheel diameters, the following test conditions were prescribed.



TABLE OF TEST CONDITIONS

Wheel Diam, ft	Speeds, ft/sec				
1.040	1.00	2.00	3.00	4.00	5.00
0.885	0.925	1.85	2.78	3.70	4.63
0.714	0.825	1.65	2.48	3.30	4.12
0.552	0.73	1.46	2.19	2.92	3.65
	Loads, lb				
1.040	10.00	20.00	40.00	60.00	100.00
0.885	6.20	12.35	24.70	37.0	62.0
0.714	3.22	6.44	12.90	19.40	32.2
0.552	1.48	2.96	5.93	8.90	14.80
	Aspect Ratios				
1.040	0.27	0.52	0.84		
0.885	0.27	0.52	0.84		
0.714	0.27	0.52	0.84		
0.552					

The test conditions yield the following values for the independent nondimensional variables.

$$W/gpd^3 - 0.890, 0.535, 0.356, 0.267, 0.178, 0.089$$

$$v^2/gd - 0.745, 0.475, 0.277, 0.119, 0.030$$

$$\alpha - 0.84, 0.52, 0.27$$

The test were conducted in a bed of dry beach sand 0.58 ft deep, 2.0 ft wide, and 40 ft long. The sand has a weight density of 100 lb/ft<sup>3</sup>, friction angle 34°.

The procedure in making the tests was to hold the aspect ratio and speed coefficient constant, and to vary the load coefficient. Then a different speed coefficient would be selected and the tests were repeated until all combinations of the variables were exhausted. By this means it was possible to evaluate the influence of each independent variable on the various dependent variables under study.

## ANALYSIS OF DATA

### APPROACH

The graphical form of the data suggests the type of mathematical equation that will be required for its expression. This is the so-called empirical equation. Actually, if an empirical equation has general validity, it necessarily expresses the results that would be obtained from a correct theoretical analysis of the problem, if such an analysis could be made. Empirical equations are generally understood to have only a limited range of applicability, but this is a practical and not a necessary conclusion. The generality of any empirical relation must depend on the range of its applicability and this range should be clearly stated along with its presentation.

The theory of dimensional analysis shows that any equation representing a physical phenomenon may be dimensionally homogeneous, and if so must be expressible as a sum of products of powers of the participating variables. Thus if

$$a = f(b, c) ,$$

equations of the type

$$a = b^\alpha c^\beta$$

$$a = b^\alpha c^\beta + b^\gamma c^\delta , \text{ etc. } ,$$

are permissible, but the variable and the exponent must be such that

$$a \stackrel{D}{=} b^\alpha c^\beta \stackrel{D}{=} b^\gamma c^\delta ;$$

where D indicates dimensional equality. This places a considerable restriction on the possible relationships that may be employed.

On the other hand, if a, b, c are nondimensional variables, there is no restriction on the exponents so that equations of the form

$$a = b^0 c^\beta + b^\gamma c^0 = c^\beta + b^\gamma$$

are permissible, or more generally

$$a = f_1(b, c) + f_2(b, c) + f_3(b, c) \dots ,$$

where  $f_1, f_2, f_3$  are not restricted but may be in the form of products of powers.

It is now assumed that the variables of a system have been put into proper nondimensional form, and that what follows applies to nondimensional variables.

Suppose a system involves three variables a,b,c and that experiments are made to determine the functional relationship that exists between these variables. This relation may be such that the data can be represented by curves of the type shown in Fig. 7.

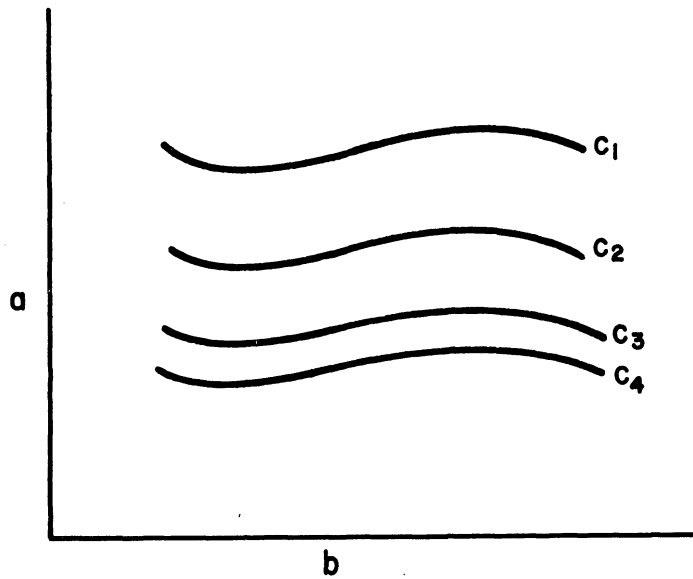


Fig. 7. Representation of linear relationship between variables.

If this is the case, the functional relationship between a and b is independent of c and the relationship between the variables may be expressed as

$$a = f(b) + g(c)$$

On the other hand, the data may be represented by curves of the type in Fig. 8, in which case the functional relationship between a and b is not independent of

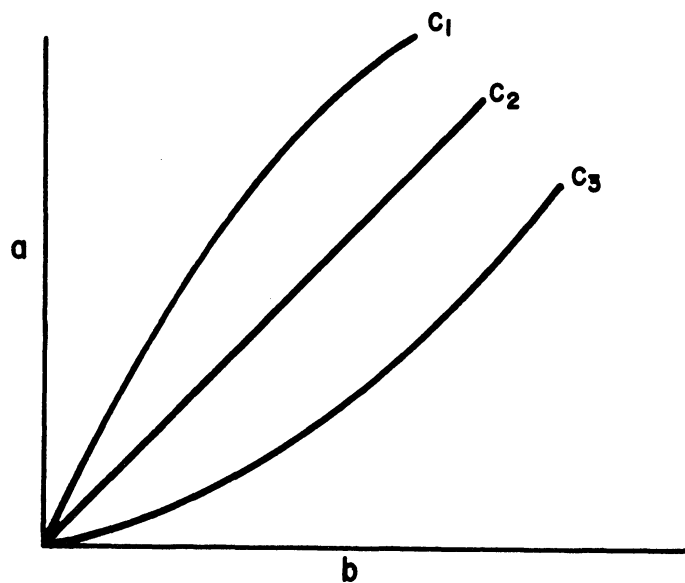


Fig. 8. Representation of nonlinear relationship between variables.

c, or as it is generally stated, a nonlinear relationship exists. In this case, the relationship between the variables is expressed as

$$a = f(b,c) .$$

In view of this, consideration can be given to developing the empirical equations which will represent the data here obtained.

#### ROLLING RESISTANCE

The data from the load-drag tests are presented in Figs. 9a, b, c, d, and e. An inspection of the graphs reveals that the functional relation between drag coefficient  $R/W$  and load coefficient  $W/gpd^3$  does not change significantly with aspect ratio, and only slightly with speed coefficient. This latter conclusion is more evident when the drag coefficient is plotted against the speed coefficient as on Fig. 10.

It is reasonable, therefore, to assume that the relationship between drag coefficient ( $R/W$ ) and load coefficient ( $W/gpd^3$ ) is practically independent of the aspect ratio  $\alpha$  and the speed coefficient  $V^2/gd$ . Therefore a mathematical expression of the form

$$R/W = f\left(\frac{W}{gpd^3}\right) + g(\alpha) + p\left(\frac{V^2}{gd}\right)$$

is to be expected. To a good approximation, the speed coefficient may be neglected, and if this is done, the relation is between  $R/W$ ,  $W/gpd^3$ , and  $\alpha$  (Fig. 11). The relation between  $R/W$  and  $W/gpd^3$  may be represented with fair accuracy by a straight-line function for  $W/gpd^3 < 1.0$ . The relation between  $R/W$  and  $\alpha$  may also be represented by a straight-line function for  $\alpha < 1.0$ . Utilizing these approximations, the equation for  $R/W$  is

$$R/W = 0.40 (W/gpd^3 + 1) - 0.28 \alpha \quad (20)$$

and is valid for  $W/gpd^3, V^2/gd, \alpha < 1$ . This equation predicts drag coefficients that are somewhat high for values of the load coefficient in the range 0.1 - 0.5 and somewhat low values in the range 0.5 - 0.9. It does not predict a zero value for the drag coefficient within the range of its validity and this is a significant point that will be taken up later.

The data including speed effects may be represented graphically and are presented in this form in Fig. 12. In this figure the principal influence on  $R/W$  is due to the load coefficient, and the influence of the aspect ratio and the speed coefficient are shown as variations. To find the drag coefficient, it is evaluated for the particular load coefficient, and the variations due to the speed coefficient and aspect ratio are applied.

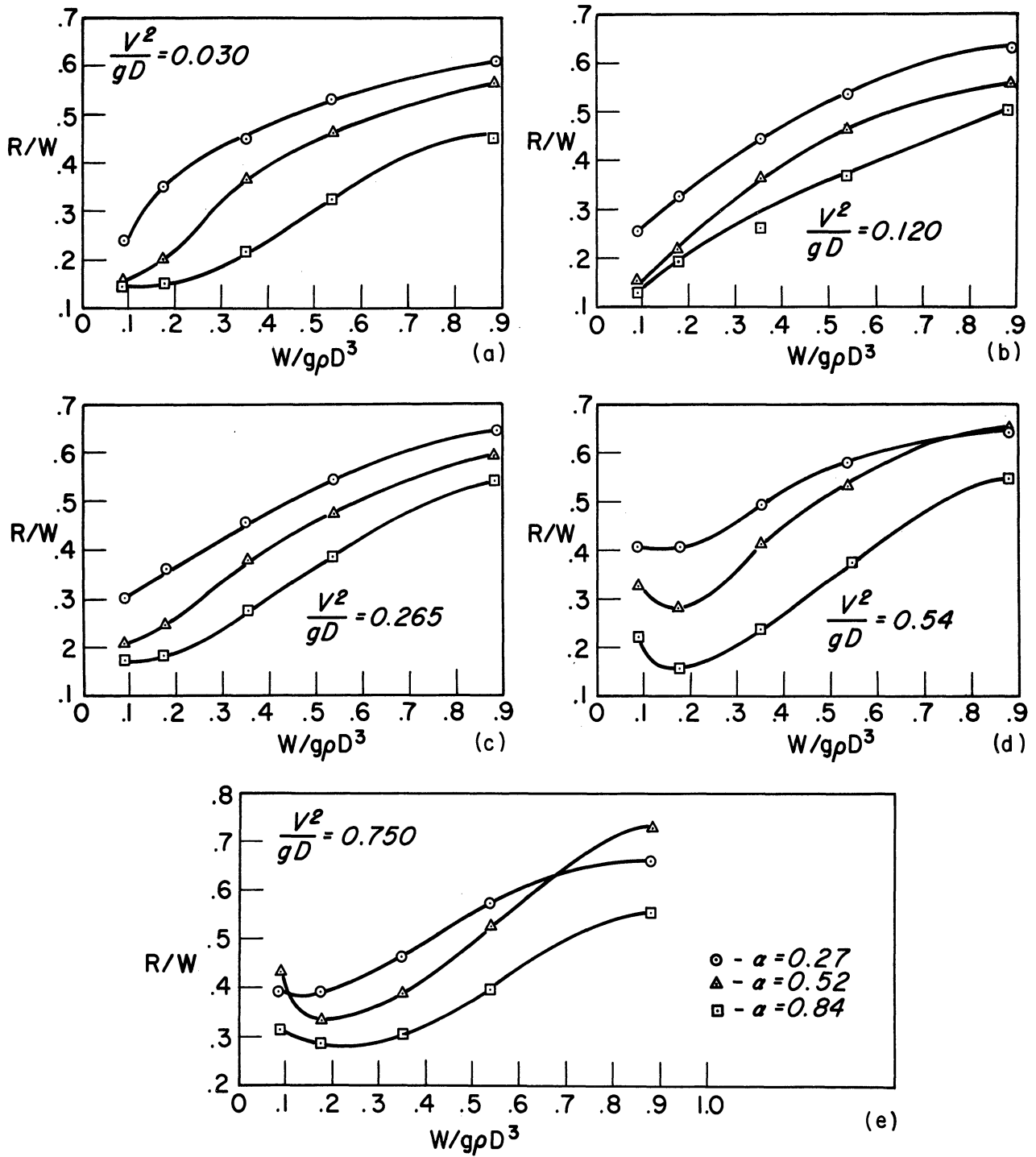


Fig. 9. Experimental data relating the drag coefficient to the independent system variables.



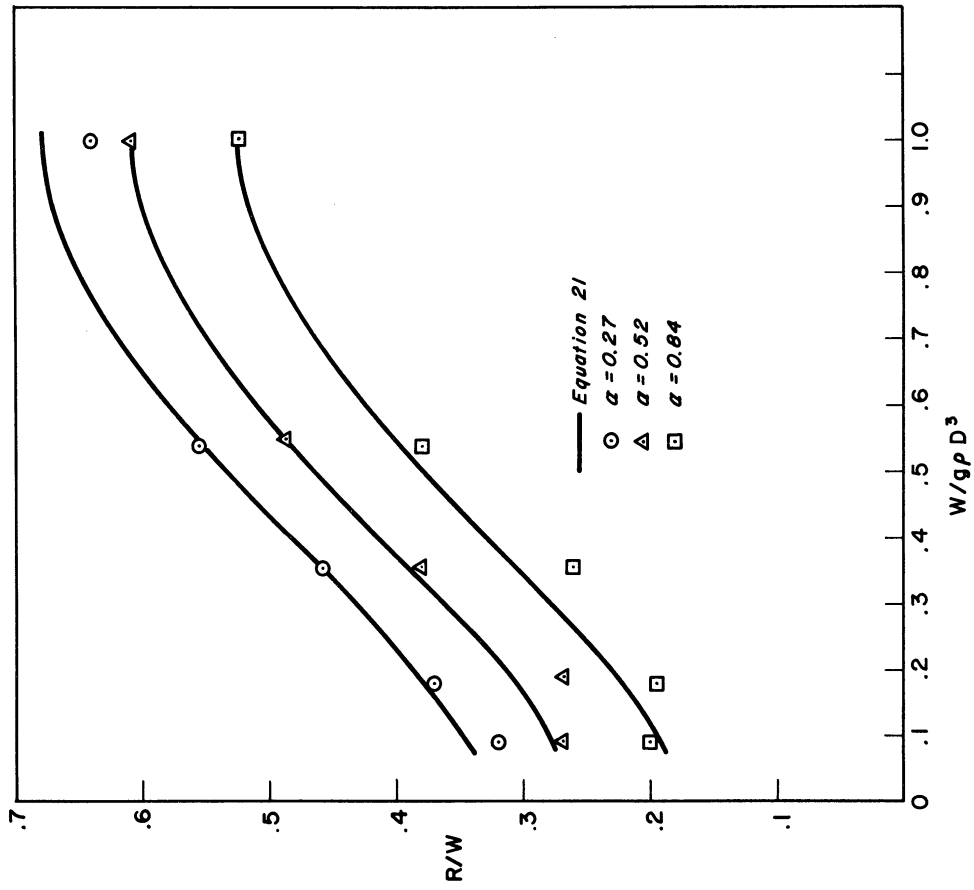


Fig. 11. Experimental data relating the drag coefficient to the load coefficient and the aspect ratio, at the average value of the speed coefficient. Curves of equations.

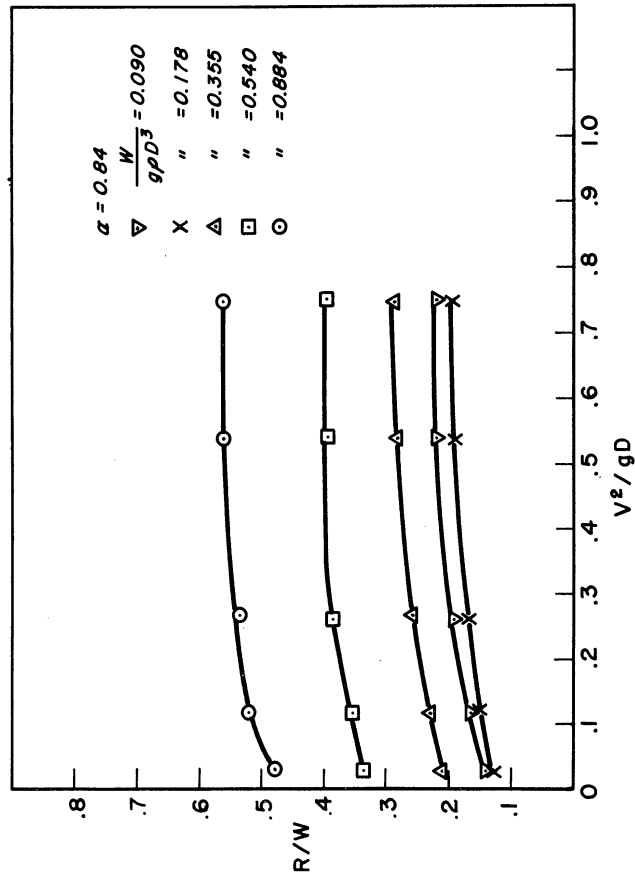


Fig. 10. Experimental data relating the drag coefficient to the speed coefficient.

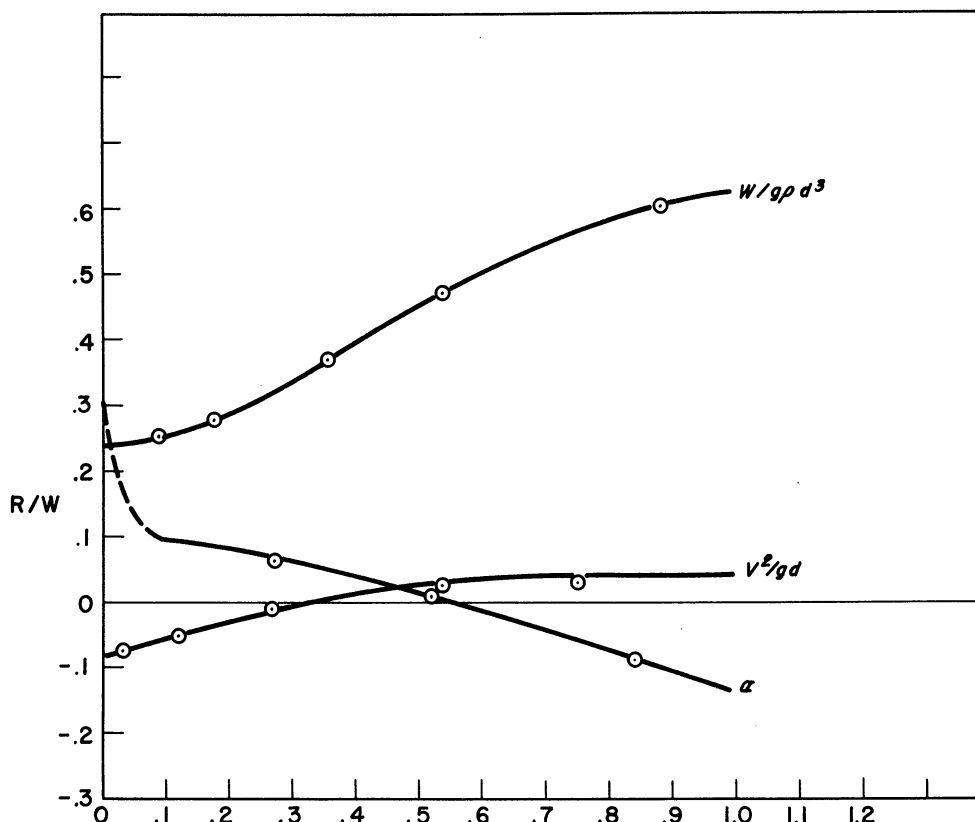


Fig. 12. Linearized experimental curves relating the drag coefficient to the independent system variables.

A considerably better representation of the data of Fig. 11 is given by the following equation which involves a power series of the load coefficient

$$R/W = 0.40 + 0.35 [(W/gpd^3)^{1.5} - (W/gpd^3)^3] - 0.27 \alpha \quad (21)$$

This equation is also valid in the range

$$W/gpd^3, \quad v^2/gd, \quad \alpha < 1.0 \quad .$$

Previous work has consisted mainly of the theoretical studies of Bernstein<sup>2</sup> and the experimental work by Nuttall.<sup>1</sup> Bernstein's considerations depend on a different system of soil value than the one used here and involves a constant designated as "n" and another constant designated as k to represent soil properties. If the value of n is taken to be unity, as appears to apply to the present studies, then the Bernstein formula for drag becomes

$$R/W = 0.86 \left( \frac{W}{bk} \right)^{0.33} (d)^{-0.67} = 0.86 \left( \frac{W}{bkd} \right)^{0.33} (d)^{-0.33} \quad (22)$$

This may be put in the form

$$R/W = .86 \left( \frac{W}{\alpha kd^2} \right)^{0.33} (d)^{-0.33} \quad , \quad (23)$$

and if  $\alpha$  is allowed to be the variable,

$$R/W = K (1/\alpha)^{0.33} \quad (24)$$

According to this, the drag coefficient decreases with increasing  $\alpha$  in accordance with the results previously indicated.

A more detailed consideration of this Bernstein formula reveals, however, that this is not the best way to demonstrate the effects of  $\alpha$ , since changing  $\alpha$  in this way also changes the unit load under the wheel. A better way is to consider wheels which have a constant load "c" per unit width. On this assumption, the Bernstein formula becomes

$$R/W = 0.86 \left( \frac{cb}{bkd} \right)^{0.33} (d)^{-0.33} = 0.86 \left( \frac{c}{kd} \right)^{0.33} (d)^{-0.33}, \quad (25)$$

and it is apparent that the drag coefficient of the wheel does not depend on the aspect ratio, since b drops out. This implies that the end effects have no influence on the drag coefficient, and is hardly a reasonable conclusion for wheels loaded in this manner. Nuttall has subjected a modified form of the Bernstein equation to a dimensional analysis and, by making various assumptions and approximations, has defined the results in terms of the Coulomb system of soil values, which is the system used for our own studies in sand.

Nuttall's analysis of the Bernstein equation leads to the following equation for the drag coefficient:

$$R/W = 0.5 \left( \frac{W}{g \theta b^{.5} d^{2.5}} \right)^{0.4}, \quad (26)$$

and applies to a sand having a weight density of 90 lb/cu ft and a friction angle of  $34^\circ$ . These are substantially the properties of the sand used for the tests here. Nuttall's equation predicts results that are substantially in agreement with the test results made here, as for example at the conditions:

$$D = 1.05 \text{ ft}$$

$$\gamma = 100 \text{ lb/ft}^3$$

$$\alpha = 0.27$$

$$W = 60 \text{ lb}$$

Nuttall's equation predicts  $R/W = 0.50$ ; the data given here predict  $R/W = 0.54$  and Eq. (1) predicts  $R/W = 0.54$ . Nuttall further states that the nondimensional coefficient

$$\frac{W}{gpb^{.5} d^{2.5}}$$

is the proper coefficient for expanding model tests or for generalizing mobility data in friction soils. This does not seem to be a defensible conclusion as can be seen by applying the Nuttall equation to a constant unit width loaded wheel. This puts the Nuttall equation into the form

$$R/W = 0.5 \left( \frac{cb}{gpb^{.5} d^{2.5}} \right)^{.4} = 0.5 \left( \frac{c}{gpd^2} \right)^{.4} \alpha^{.2} \quad (27)$$

According to this viewpoint,  $R/W \rightarrow 0$  as  $\alpha \rightarrow 0$ , and  $R/W \rightarrow \infty$  as  $\alpha \rightarrow \infty$ . These results are not similar to those predicted by the Bernstein formula when subjected to the same test, and are even less in accordance with experience, for if an infinite aspect ratio is considered, it is perfectly feasible to simulate this condition in the laboratory by a two-dimensional test, where the wheel is exactly as wide as the test bed. There is absolutely nothing in our experience to indicate that such a test would result in an infinite drag coefficient. As a matter of fact, such a test would provide a convenient end point for testing the effect of variations of  $\alpha$ . On the other hand, for zero aspect ratio, the Nuttall formula predicts zero drag coefficient, and if our experience means anything, this ought to be the condition where the drag coefficient becomes large. The Nuttall equation is valid within the range of test conditions examined, but the claim for general validity is not warranted. Even an examination of Nuttall's data does not lead to a strong conviction that  $R/W$  approaches zero under any condition.

The basic reason that both the Bernstein and Nuttall equations fail at the limits of  $\alpha$  is that they both involve  $\alpha$  implicitly in a nonlinear relation with the load coefficient. Nuttall's equation can be put in the form

$$R/W = 0.5 \left( \frac{W}{gpd^3 \alpha^{.5}} \right)^{0.4} = 0.5 \left( \frac{W}{gpd^3} \right)^{.4} \left( \frac{1}{\alpha} \right)^{.2}, \quad (28)$$

and Bernstein's with  $n = 1$  into the form

$$R/W = 0.86 \left[ \left( \frac{W}{gpd^3} \right) \cdot \left( \frac{gp}{k} \right) \cdot \left( \frac{1}{\alpha} \right) \right]^{.33} \quad (29)$$

Equations of this form must have limits of zero or infinity at the limits of  $\alpha$ . Yet the data indicate that the load coefficient and the aspect ratio are largely independent of each other in their contribution to drag coefficient. On a more basic level of reasoning, these relations probably fail because of the basic assumption of Bernstein on which both relations ultimately rest. Bernstein's basic assumption implicitly involves the idea that for a wheel to have rolling resistance, it must compress the soil because it is carrying a load. However, there may be other sources of drag that do not contribute to the load-carrying

ability of the wheel. Friction at the sides of the wheel would act this way. Thus it is more generally true that for a wheel to have drag, it only must have a trail behind it. This trail does not have to be a visible track or furrow, but only a change of state in the soil, a kind of entropy trail.

It seems, therefore, that the relatively simple formulæ of Bernstein and Nuttall, while valid in restricted ranges of application, cannot be generally valid and are in fact likely to be quite misleading when applied to wheels of practical geometry. Of the two equations, Bernstein's has the more rational form.

What remains for the time being is the reliance on graphical data such as Fig. 12 and on the empirical formulas describing them. This has been the accustomed approach of aeronautical and marine engineers to many of their model problems.

If the most violent liberties are taken with the data, then the curves of Fig. 11 may be represented by curves as in Fig. 13.

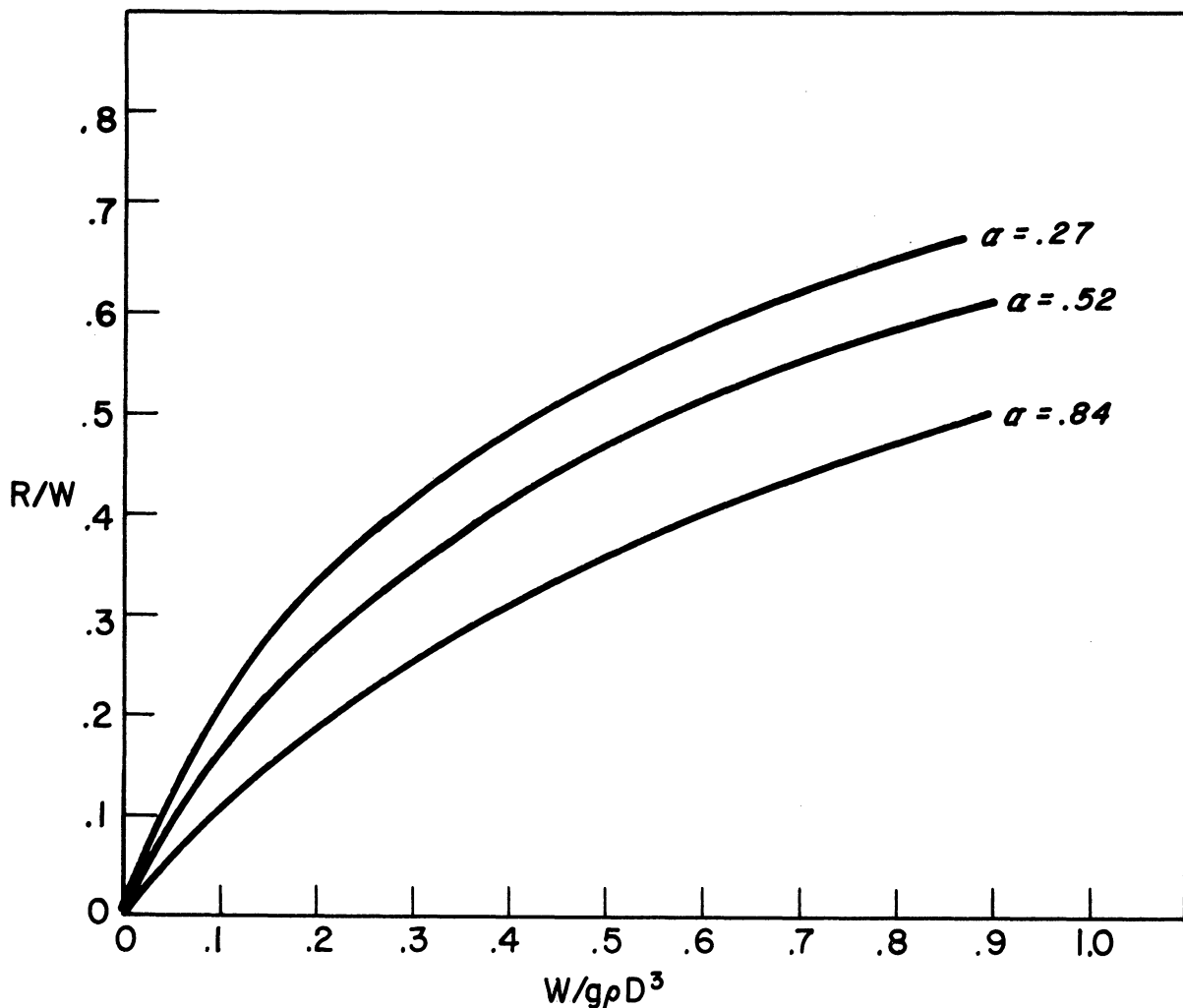


Fig. 13. Experimental drag coefficient curves faired according to Nuttall.

An equation that closely fits these curves is given by

$$R/W = 0.53 \left( \frac{W}{gpd^3} \right)^{.445} \left( \frac{1}{\alpha} \right)^{.40} \quad (30)$$

This equation is presented to show how the data may be worked into this form and is compared to Nuttall's equation.

$$R/W = 0.50 \left( \frac{W}{gpd^3} \right)^{.40} \left( \frac{1}{\alpha} \right)^{.20} \quad (31)$$

#### WHEEL SINKAGE

The sinkage of the wheel is a more primitive concept than the rolling resistance from an analytical viewpoint, but the mathematical difficulties attendant to the analysis are about the same for either case. Experimentally, the measurement of sinkage is simpler than the measurement of drag. There are fewer extraneous factors to influence the sinkage, so it can be expected that the sinkage data will show less scatter than the drag data. This is evident from Fig. 14, which gives the relations between the nondimensional sinkage coefficient  $Z/d$  and the load coefficient  $W/gpd^3$  and the aspect ratio  $\alpha$ . No systematic variation in the sinkage coefficient was found with respect to the speed coefficient at any speed tested. The maximum speeds for the model tests correspond to a full-scale (3-ft-diameter) wheel operating at 8.70 ft/sec, or about 6 miles per hour.

The form of the curves in Fig. 14 show that  $Z/d$  is a nonlinear function of the load coefficient, and the aspect ratio. An acceptable equation to represent these curves is

$$Z/d = 0.071 \alpha^{-1.5} \left( \frac{W}{gpd^3} \right)^{1.345}, \quad (32)$$

valid for  $[0 < (W/gpd^3) < 1.0]$  and  $(0.2 < \alpha < 1.0)$ .

Equation (32) gives values for  $Z/d$  that are too large for small values of  $\alpha$ , and a better representation is provided by

$$Z/d = (0.115 \alpha^{-1} - 0.044 \alpha^{0.55}) (W/gpd^3)^{1.35}, \quad (33)$$

which is valid within the same limits. Equation (32) may be rounded off somewhat without doing too much violence to the data and becomes

$$Z/d = 0.08 \alpha^{-1.35} (W/gpd^3)^{1.35} = 0.08 (W/g\alpha bd^2)^{1.35} \quad (34)$$

This form is interesting because of its comparison with analytical results.

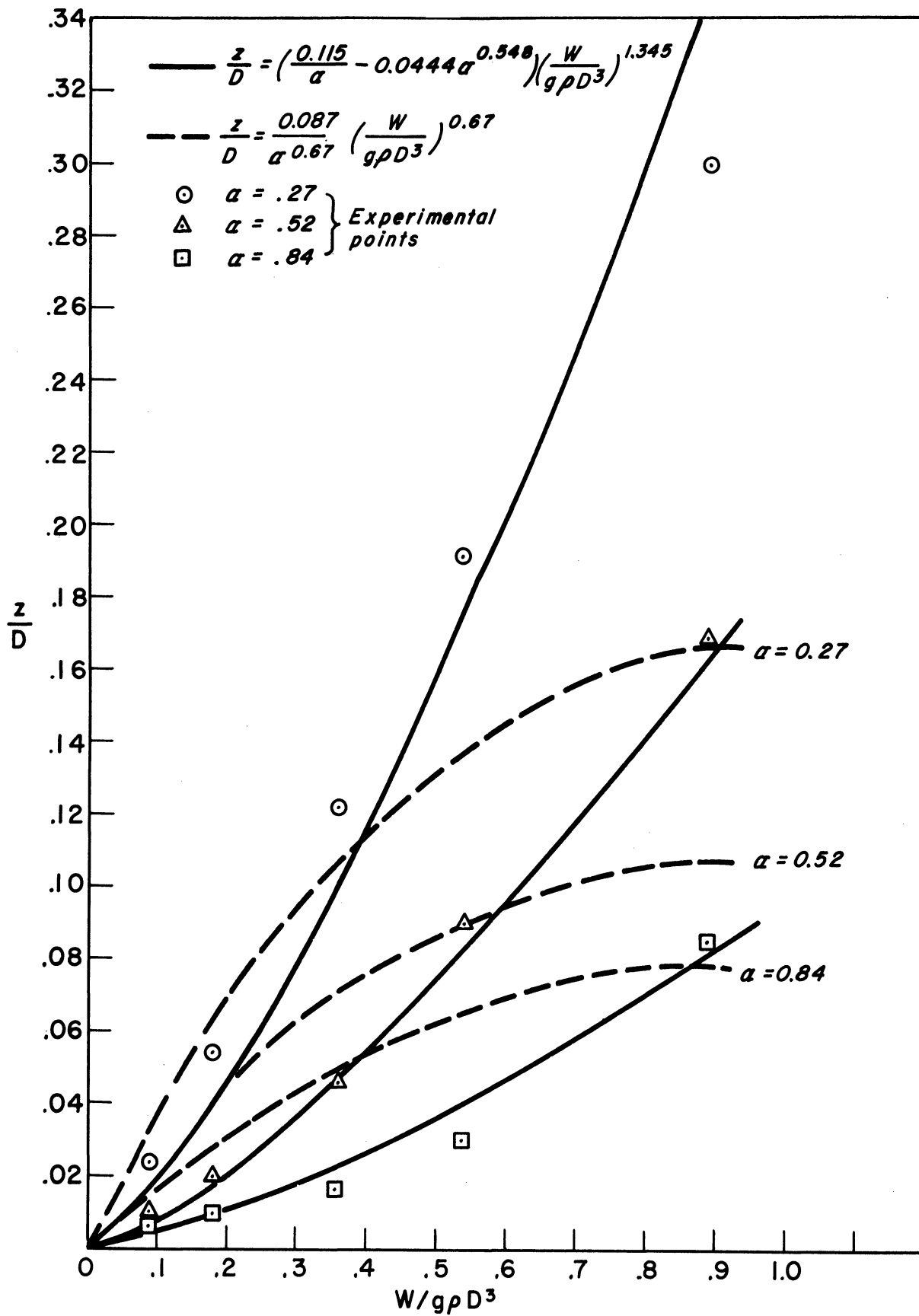


Fig. 14. Experimental values of sinkage coefficient and curves of equations.

Bekker<sup>7</sup> gives the analytical equation for sinkage

$$Z = \left[ \frac{3W}{(3-n)bk_{\phi} d^{0.5}} \right]^{2/(2n+1)} \quad (35)$$

This for  $n = 1$ , which applies to the tests made here, reduces to

$$Z = \left[ \frac{3W}{(2bk_{\phi} d^{0.5})} \right]^{0.67}$$

Putting this equation into nondimensional forms gives

$$Z/d = 1.31 \left[ \frac{W}{\alpha k_{\phi} d^3} \right]^{0.67}$$

Since the tests were made under conditions that  $n \cong 1.0$  and since the dimensions of  $k_{\phi}$  are  $F/L^{n+2}$ , it is seen that  $k_{\phi}$  has the dimensions of  $\gamma$  or  $g_{\phi}$ , namely  $F/L^3$ , so that  $k_{\phi} = c\gamma = cg_{\phi}$ . Utilizing this information and noting that for the tests  $k_{\phi} = 3.3 \text{ lb/in.}^3 = 5700 \text{ lb/ft}^3$  is representative, Bekker's equation becomes

$$Z/d = 0.087 \alpha^{-0.67} (W/g_{\phi} d^3)^{0.67} = 0.087 (W/g_{\phi} d^2)^{0.67} \quad (36)$$

With Bekker's equation written so that the load coefficient is given explicitly [as in Eq. (36)], it is seen that the exponent of this coefficient is less than unity. This means that the relative effect of this group will decrease as the load coefficient increases. It must be remembered of course that this applies only to the case where  $n \cong 1.0$ . It also must be remembered that the assumption on which Bekker's equations are based apply only to small sinkages, but even at small sinkages the form of the Bekker equation is not in compliance with the experimental results of Fig. 12. This suggests that additional refinements are necessary in the analytical approach, assuming, of course, that the experimental data are valid.

For the Bekker equation to yield the same exponential for load coefficient, it is necessary to adopt a value for  $n$  of 0.25. Under these assumptions it is necessary to make  $k_{\phi}$  a function of some characteristic length such as the diameter in addition to the soil density. Some data<sup>7</sup> relating to this point have recently become available, and show that for a constant depth of sand  $k_{\phi}$  does vary with wheel diameter. By assuming  $n = .25$ , and using this information, the Bekker equation becomes

$$Z/d = (1.10)^{1.33} \left( \frac{W}{\alpha k_{\phi} d^{2.25}} \right)^{1.33}, \quad (37)$$

and since



$$k_{\phi} \stackrel{D}{=} F/L^{2.25} \stackrel{D}{=} F/L^3 (L)^{0.75} = c \gamma d^{0.75} ,$$

$$c = \frac{k_{\phi}}{\gamma d^{0.75}} \quad 270 k_{\phi} \left( \frac{1b}{in.^{2.25}} \right) = k_{\phi} \left( \frac{1b}{ft^{2.25}} \right) \left( \begin{array}{c} \text{conversion} \\ \text{factor} \end{array} \right) .$$

The data mentioned above give for example

$$k_{\phi} = 1.80 \text{ lb/in.}^{2.25}$$

$$\gamma = 100 \text{ lb/ft}^3$$

$$d = 1.0 \text{ ft} ,$$

from which

$$c = \frac{1.80 \times 270}{100 \times 1.0} = 4.88 .$$

Substituting the term  $k_{\phi} = c\gamma d^{0.75}$  into the Bekker equation (37) gives

$$Z/d = 0.134 \alpha^{-1.33} (W/g\phi d^3)^{1.33} = 0.134 (W/g\phi d^2)^{1.33} \quad (38)$$

This form of the Bekker equations gives the same trend as the data in Fig. 10. The predicted sinkage is about 50% greater. To place Bekker's equation in this form, it is necessary to assume a value of  $n$  that seems unreasonably low, and also a law of variation of  $k_{\phi}$  that may not be generally valid, and in particular may not be valid under the radically changed value of  $n$ , must be depended upon.

The Bekker equation predicts excessively large sinkage, in the small sinkage range, so perhaps the theory should be re-examined to see if full account is being taken of the load-bearing capacity of the soil. When examined as an analogy of the pressure plate, it is seen that the characteristic dimension of the wheel changes from the fore and aft length for the first point of contact (very light loads) until at moderate load the characteristic length becomes the width of the wheel, for normally proportioned wheels. It is not believed that this point has been utilized in theoretical wheel-sinkage analysis.

#### WHEEL SLIP

Slip is defined by the following mathematical expression

$$i = \frac{V_h - V}{V} ,$$

where

$v$  = speed of axle parallel to ground

$v_h$  = hypothetical speed of wheel based on turning rate if no slip occurred.

Defined this way, slip is a nondimensional coefficient, and has negative value for unpowered wheels.

As with the drag and the sinkage, slip exhibited no detectable systematic variation with speed in the ranges tested, so the speed-coefficient data are used in establishing a mean.

The slip-coefficient data as a function of aspect ratio and the load coefficient are given in Fig. 15. It seems logical that the slip should become

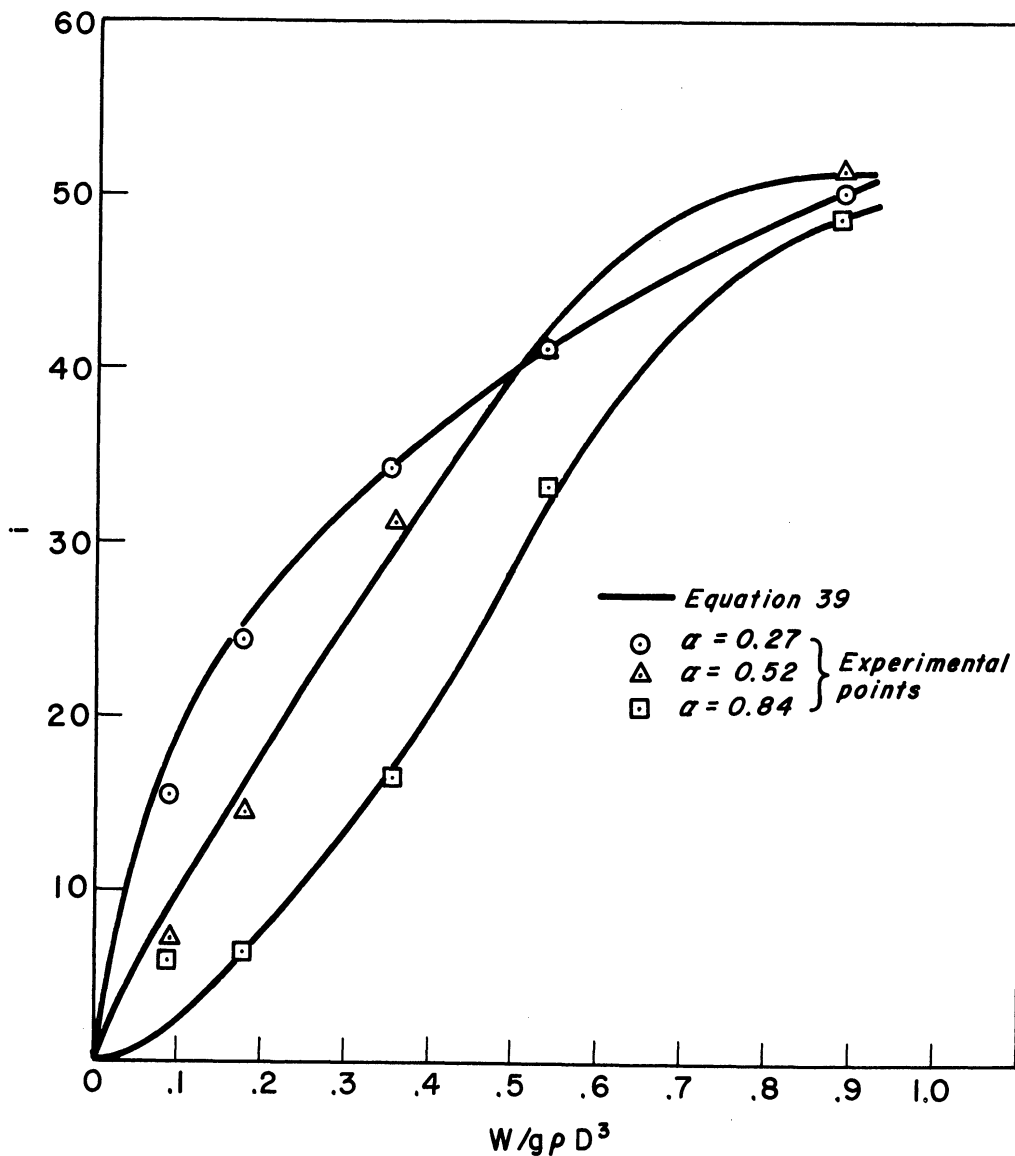


Fig. 15. Experimental values of slip coefficient and curves of equations.

zero at zero load coefficient for a wheel with a frictionless axle, and the following equation for slip coefficient was developed on this assumption.

$$i = k_1 \left( \frac{W}{g\rho d^3} \right)^{n_1} - k_2 \left( \frac{W}{g\rho d^3} \right)^{n_2}, \quad (39)$$

where

$$k_1 = 93 - \frac{(0.68 - \alpha)^2}{0.52 \times 10^{-2}}$$

$$k_2 = 45 - \frac{(0.71 - \alpha)^2}{0.53 \times 10^{-2}}$$

$$n_1 = 1.85 \alpha$$

$$n_2 = 3n_1 = 5.55 \alpha$$

As far as is known, there is no theoretical analysis of the slip of a free running wheel, so no evaluation of the theory can be made.

The unexpected thing about the slip data is the way the slip values coalesce at large loads for the different aspect ratios. It hardly seems likely that the same soil deformations exist at this point, but whatever does happen appears to give equivalent results.

The slip is the most easily measured of the variables studied and the probability is large that the data of Fig. 15 are a true measure of wheel slip. The largest error is in the load assumed, which may be inaccurate because of friction in the test apparatus.

Since both slip and sinkage are related to the same load, some idea of the error introduced by factors other than load may be obtained by plotting the slip against the sinkage (Fig. 16). It is seen from this that there is a high degree of predictability in the data when the load coefficient is removed.

While no known analysis of passive slip exists, studies now being made at the University may provide a theoretical correlation of the slip data. It is strongly suspected that there may be important relations between the slip and the drag.

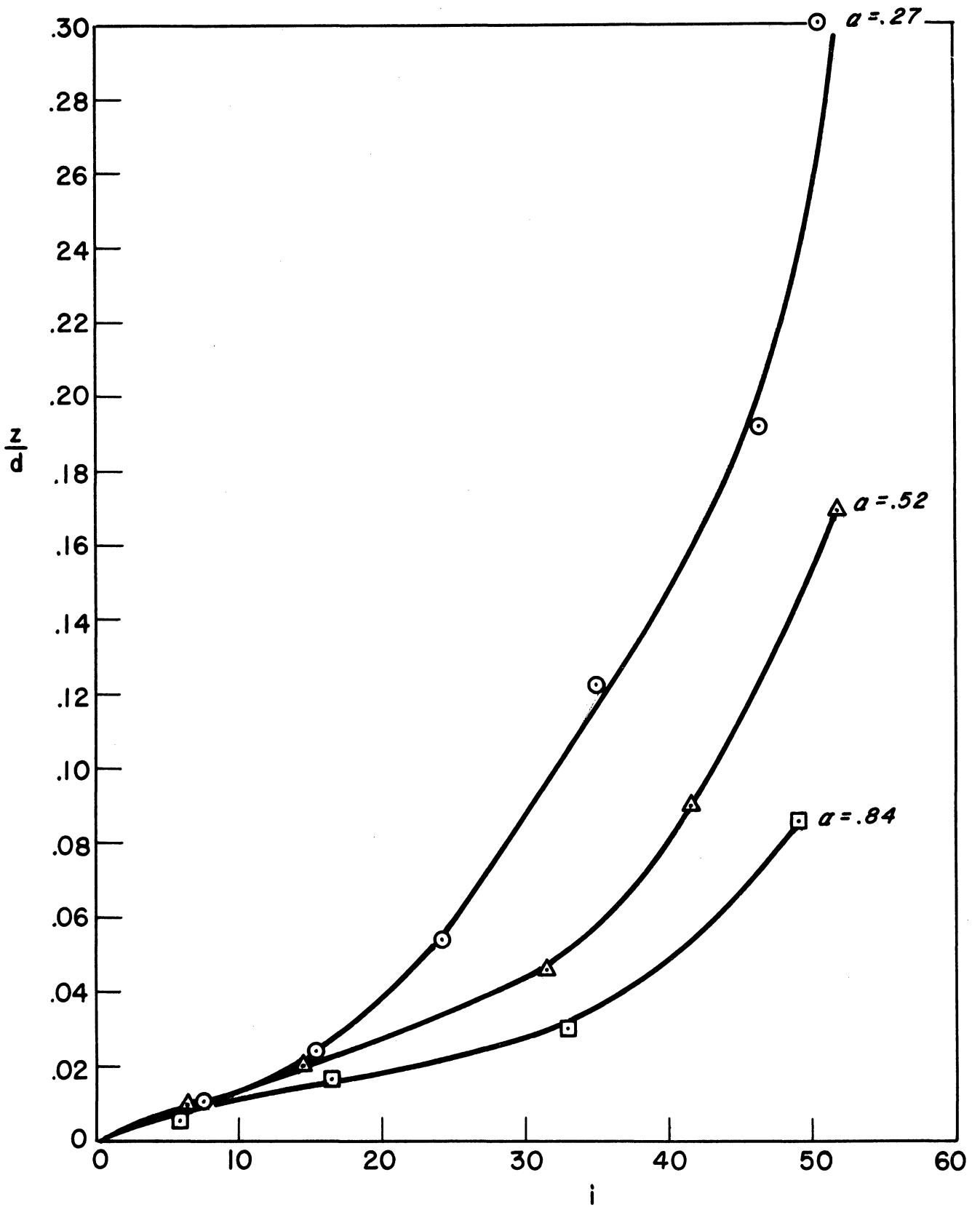


Fig. 16. Relation between slip and sinkage.

## SUMMARY AND EVALUATION OF WORK

As far as is known, the present study encompasses the broadest range of variables that have been considered in any related series of wheel tests. In evaluating experimental studies, a factor to be constantly considered is the experimental error. A critical analysis of the range of error has not yet been made for these experiments, but a cursory evaluation indicates that the largest individual error made would barely invalidate the results.

Three wheel-performance coefficients have been included in this investigation and of these the drag coefficient presents the most unexpected results. These are the experimental findings that the drag coefficient does not simultaneously become zero with the load coefficient. Also surprising is the discovery that the load coefficient and the aspect ratio are nearly independent of each other in their effect on the drag coefficient. The speed coefficient has little effect on the drag coefficient in the speed ranges tested. The findings concerning load coefficient and aspect ratio should stimulate a re-examination of the theoretical approach to the rolling resistance. The analyses made of existing theoretical results indicate that these results have certain irrational aspects, which signal the danger of using them beyond their experimental verification.

The mechanism which could account for the drag coefficient remaining greater than zero at zero load coefficient is not at all clear, but the finding does not appear irrational when the matter is viewed as a limit problem. The practical aspects of this finding would particularly apply to lightly loaded and narrow wheels.

The evaluation of the sinkage coefficient involves less experimental error than that of the drag coefficient as was expected, but the trend of the results is quite different from the results of the theoretical analysis. The sinkage for light loads is less than that theoretically predicted, and for heavy loads the sinkage is greater. Here again, the results seem to indicate a need for a re-examination of the theoretical approach.

The measurement of the slip coefficient probably involves the least experimental error, and it appears from the data that the slip coefficient becomes zero along with the load coefficient. This seems reasonable, since if it were possible to have a wheel without axle friction, cohesion between the wheel and the soil would insure zero slip even with zero load. As far as is known, there is no theoretical analysis for slip with which the results may be compared. The slip coefficient curves also show that, under heavy load, a wide wheel slips about as much as a narrow one even though the sinkage is much less. This may be due to the approach to two-dimensional operations of the wheel, and may have significant theoretical implications. The shape of the slip coefficient curves (Fig. 15) suggest that slip is a

complex function of the variables involved. Considering the over-all results, it appears that the variables selected in the dimensional analyses for predicting wheel performance are adequate for operations in sand. Some doubt existed at first whether the test bed depth and width were large enough to represent infinite boundaries effectively, but the experimental results show no indication of this limitation.

#### RECOMMENDATION FOR FUTURE STUDIES

The logical step after studying free running wheels is to undertake the study of powered wheels. This is an essential and necessary part of the minimum knowledge of wheel performance, but it is felt that preceding or concurrent with powered wheel tests, additional useful work can be done with free running wheels in friction soil. In particular, the coefficient of friction between the soil and the wheel should be investigated. It is believed that this factor may have significant effects on rolling resistance. On powered wheels it is probably predominant in determining traction. In addition, it is felt that the effects of aspect ratio should be determined out to infinite values which may be simulated by a two-dimensional test. Some tests on very narrow wheels should also be performed to establish the effect of the other limits of this variable. These tests can be quite easily run at this time and would complete the information about this variable which also plays an important part in theoretical analyses. Also of importance is the effect of tread. Some consideration given to this subject leads to the idea that tread design may be reducible to a friction factor. If this can be done, it would increase the importance of friction studies further.

The Bekker soil value system should be brought into the scope of dimensional analysis. This is the system that currently offers the best basis for practical analytical work. Some encouraging progress has already been made in this direction, but a definite program of study and experiment is needed so that test programs can be analyzed on a rational basis.

## CONCLUSIONS

At the beginning of this work, it was assumed that some generalizations of wheel-performance data could be found through the applications of dimensional principles to model studies. This assumption now appears justified, as the results obtained for "similitude" conditions are regular and predictable.

This conclusion is less certain for full scale, since the tests did not extend to these sizes and comparable data are almost nonexistent in the literature. It is believed that there is a scale effect involved for larger wheels which would modify the results here obtained. Some preliminary tests have been made which indicate that this scale effect is related to the ratio of the wheel diameter to the depth of the equivalent homogeneous soil. It further appears that the scale effect is essentially zero as long as the above-mentioned ratio is less than one. Inasmuch as most full-scale operations probably do not meet this requirement, it can be expected that the scale effect will emerge and that the drag, sinkage, and slip of full-scale wheels for usual operations will be somewhat less than is predicted here.

The Coulomb soil values appear adequate for sand but should be replaced by the Bekker values for cohesive soils. The two systems are apparently equivalent for deep beds of sand.





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