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HYBRID COMPUTER STUDY OF AN IMPACTING TWO DEGREE OF FREEDOM SYSTEM

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ABSTRACT

The following report is concerned with a hybrid analog computer study of an impacting two degree of freedom system. Such systems play an important role in many types of industrial machines, e.g., impact dampers, shakers, rock drills, etc. To optimize a given behavior of these machines, it is imperative that analytical models and tools be available.

The analysis of such systems is complicated by the non-linearity introduced in the impact process. Where analysis of such systems does exist it generally assumes certain steady-state behavior and then indicates the parameter ranges for which the assumed form of solution both exists and is stable. It will be the purpose of the report to provide a stability check on the existing solutions.

It is found that although the response predicted by the existing solutions is quite good, the stability behavior is considerably more sensitive than experimental evidence indicates. Thus, an alternate model is suggested and studied in the report which has a relatively small effect on response but much improves the stability of the system. A limited parameter study is done to indicate the two distinct types of periodic, stable solutions encountered, i.e., periodicity with and without a beat, and to show the approximate trends of the stability regions.

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LIST OF SYMBOLS

a	constant
b	constant
c	damping coefficient
e	coefficient of restitution
F_o	dimensionless mean force (P_o/P_1)
F_1	dimensionless hold down force (H/P_1)
g	acceleration of gravity
H	hold down force
k	spring constant between mass M_1 and M_2
k_1	spring constant above mass M_1
K	ratio of spring constant (k_1/k)
M_1	mass of case
M_2	mass of piston
p	natural frequency of floating system $[\sqrt{k(M_1+M_2)/(M_1/M_2)}]$
$P(t)$	actuator force
P_o	mean actuator force
P_1	amplitude of oscillating force
s	displacement from equilibrium position of spring k_1
S	dimensionless displacement from equilibrium position of spring k_1 $[s/(P_1/k)]$
t	time
v	velocity
W	dimensionless weight of the system $[(M_1+M_2/P_1)g]$
x_1	displacement of mass M_1

x_2 displacement of mass M_2

y_1 coordinate transformation for displacement of M_1

y_2 coordinate transformation for displacement of M_2

α phase angle between forcing function and impact

β critical damping in the one spring or floating system $(2k/p)$

γ dimensionless damping coefficient (c/β)

δ equilibrium distance of mass M_2 from base in force limiter case

ξ_1 dimensionless position of mass M_1 $[x_1/(P_1/k)]$

ξ_2 dimensionless position of mass M_2 $[x_2/(P_1/k)]$

τ dimensionless time (pt)

φ dimensionless phase angle $(p\alpha)$

ω forcing frequency

Ω dimensionless forcing frequency (ω/p)

I. Introduction

An interesting class of dynamic systems exists which involves impact between masses within the system or impact with an external system. Some examples of such systems are the following. In the case of an impact damper [1] a single mass or a number of masses is constrained to move linearly colliding with the other masses or the system wall. The energy loss in the collision is utilized as a damping mechanism. This technique has been suggested as a possible wing flutter damper [2]. Impact vibrators have been used [3] extensively to insure the flow of material in troughs, bins, etc. In this case a mass is made to impact against the bin wall and the resultant vibration prevents any flow stoppage, etc. A third class of machines utilizing the phenomena might be called impact tools. In this case the impact is used as a means of driving, removing, penetrating another system. Pile drivers, scale removers, rock drills are all examples of this class.

The problem which, in fact, provided the orginal motivation for the study was a "jack hammer" or "paving breaker". The model for this system is shown in fig. 1. The mass M_1 corresponds to the mass of the case, the mass M_2 the piston mass, k is the piston return spring constant, H is the operator "hold down" force, and P(t) is the actuating force. The piston in this case impacts against a bit or moil shown schematically as simply a base. The energy loss in the impact is accounted for through an effective coefficient of restitution e. Although this was the specific motivation for the problem the general mathematical model representing fig. 1 might well apply to other systems. Hence, a general study of this system is of interest.

The mathematical study of such systems is quite difficult. The impact process introduces a non-linearity in the system or put another way, the system is piecewise linear between impacts. Of major interest here are the steady-state solutions. Two problems exist; first finding the steady-state solution and then showing that it is, in fact, a stable solution. It will be the main purpose of the present paper to provide an analog computer program for the general dynamic system. Analytic solutions including stability analysis exist for special subcases. It will also be the purpose of the paper to check these solutions. In addition, the effects of damping, variation of hold down force, initial condition perturbation, and variation of spring constants, will be investigated for both the general and special systems. It should be noted, however, that it is not the purpose of this paper to provide a systematic parameter variation but rather to demonstrate the various types of stable, periodic solutions possible plus other interesting features for various choices of the parameters.

In the next section of the report the available analytic steady-state solutions including stability analysis are outlined. In addition, improvements in the model are discussed and the corresponding governing equations of motion derived. In section III the analog computer program is presented and discussed. Section IV contains the numerical results of the study while the discussion of results, and some tentative conclusions are outlined in section V.

II. Governing Equations and Solutions

A. Existing Solutions

The idealized two degree of freedom system for which analytic solutions are available [4] is shown in fig. (1-A). The differential equations of motion, valid between impacts of M_2, for this system are:

$$M_1 \frac{d^2x_1}{dt^2} + k(x_1-x_2) = P_o + P_1 \cos \omega(t+\alpha) - H$$

$$M_2 \frac{d^2x_2}{dt^2} + k(x_2-x_1) = -(P_o + P_1 \cos \omega(t+\alpha))-(M_1 + M_2)g \qquad \text{(II-1)}$$

It is assumed for convenience here that the force H is constant and the driving force P(t) is sinusoidal. It should be noted that gravity effects play an important role here, i.e., note the second of equations (II-1). Physically the system here is "floating", or is unattached to the base. M_2 strikes the base, rebounds, and at some later time, due to gravity, the system falls and M_2 again strikes the base. The height of rebound, time between impacts, etc., all depend on the system parameters.

The essential purpose of the present study is to provide analog simulation of the above and several closely related systems. In particular this will include checking the stability of predicted steady state solutions of the above system [5], investigating the effects of damping on the system behavior, and looking for other types of steady state behavior than that predicted by the analysis.

The solution presented in [4] and the subsequent stability analysis presented in [5] are for the undamped, "floating" system shown in fig. (1-A). Equations (II-1) are the governing equations of motion. Since steady-state behavior is desired the analytical approach in [4] assumes that the response of the masses M_1 and M_2 have the same period as that of the driving force $P(t)$ (this is based on experimental observation). This assumption is shown schematically in fig. 2. It can be seen from the figure that mass M_2 rebounds from the surface with a velocity ev, where e is the coefficient of restitution and v is the velocity of M_2 immediately before impact.

Several interesting features emerge from the solution of reference [4]. First, it is shown that not all values of the coefficient of restitution permit the assumed solution. It is shown that as e decreases the one bounce per forcing period solution breaks down. The possible one bounce solution region is shown in fig. 3. It should be noted that fig. 3 is a dimensionless plot. The dimensionless variables are defined later. Second, two distinct single bounce solutions, which satisfy the equations of motion between impacts and all conditions at the beginning and end of the impact cycle, are possible. It is shown in [5] that one of these branches always leads to an unstable solution and further that the stability region for

for the other branch is very limited, see fig. 3 again. It is also shown in [5] that this branch is stable only for very small perturbations of the motion. This stability will be checked using the analog solution.

Since experimental results indicate a much more insensitive stability behavior, it was felt that the choice of model shown in fig.(1-A) was, in fact, poor. Several modifications to fig. (1-A) which hopefully increase stability but do not radically change the response suggest themselves. In fig. (1-B) a displacement limited model is shown. In this case, large deviations from the mean steady state position are prevented by providing an upper bound on the displacement of M_2. It should be noted that this does not necessarily insure a stable solution, however. A second possibility is shown in fig. (1-C) essentially a force limited model. Large changes from the mean steady-state position here are resisted by large forces due to the added spring k_1. It is felt that the force limited model is somewhat closer to reality in the present case. For example, in the case of a "jack hammer" an operator would apply force much in the manner of this spring, k_1, i.e., if the system tended to go unstable with increasing amplitude, the operator would apply increasing force to bring it back. The equations of motion for this model will be derived and non-dimensionalized.

B. Force Limited Model

Derivation of the equations of motion for the force limited model presents one additional problem, namely, the choice of coordinate system. We can either stay with the present coordinate system, i.e., x_2 is measured positive upward from the base and $x_1 = 0$ is the equilibrium position of M_1 for $P(t) = 0$ and $k_1 = 0$, or we can consider a second, somewhat more commonly used coordinate system. Here the entire system hangs suspended under its own weight from the upper support. y_1 and y_2 are measured positive upward from the positions of static equilibrium (for $P(t) = H(t) = 0$) of M_1 and M_2 respectively. This eliminates the gravity term. Also, in general, there will be some "gap" distance, δ, between the equilibrium position of M_2 and the base. The problem could be formulated in either coordinate system, the systems being related by a linear transformation of the type,

$$\begin{aligned} y_1 &= x_1 + a \\ y_2 &= x_2 + b \end{aligned} \qquad \text{(II-2)}$$

It was decided that the original coordinate system is the more realistic one for this particular problem since the unstretched spring position k_1 (this spring is modeling the operator) is also a function of system paramters.

The equations of motion for this system are:

$$M_1 \frac{d^2x_1}{dt^2} + c\left(\frac{dx_1}{dt} - \frac{dx_2}{dt}\right) + k(x_1-x_2) + k_1x_1 =$$

$$P_o + P_1 \cos \omega(t+\alpha) + k_1s - H$$

$$M_2 \frac{d^2x_2}{dt^2} - c\left(\frac{dx_1}{dt} - \frac{dx_2}{dt}\right) - k(x_1-x_2) = -P_o - P_1 \cos \omega(t+\alpha)$$

$$- (M_1+M_2)g \tag{II-3}$$

where k_1s is the spring force in k_1 when $x_1 = 0$. Thus, s fixes the unstretched position of the spring k_1. Note that for no damping ($c = 0$) and no upper spring ($k_1 = 0$) the equations reduce to equations (II-1).

The following dimensionless variables are chosen:

$$\begin{aligned} \tau &= pt & F_o &= P_o/P_1 \\ \xi_1 &= x_1/(P_1/k) & F_1 &= H/P_1 \\ \xi_2 &= x_2/(P_1/k) & W &= (M_1+M_2)g/P_1 \\ \Omega &= \omega/p & K &= k_1/k \\ \varphi &= p\alpha & \gamma &= c/\beta \\ \mu &= M_1/M_2 & S &= s/(P_1/k) \end{aligned} \tag{II-4}$$

where $p^2 = k[M_1+M_2/M_1M_2]$. It should be noted that this is not the natural frequency for the two spring system of equations (II-3). However, for $k_1 << k$, the shift in natural frequency is small. Moreover, using the old natural frequency is considerably more convenient numerically. It should also be noted that there are two frequencies in the latter case, one of which is very close to zero.

γ in equations (II-4) is the dimensionless damping constant where $\beta = 2k/p$ and represents critical damping in the one spring system. Substituting these dimensionless variables into equations (II-3), the following dimensionless differential equations of motion are obtained.

$$\frac{d^2\xi_1}{d\tau^2} + 2\frac{\gamma}{(1+\mu)}\left(\frac{d\xi_1}{d\tau} - \frac{d\xi_2}{d\tau}\right) + \frac{1}{1+\mu}(\xi_1-\xi_2) + \frac{K}{1+\mu}\xi_1 =$$

$$\frac{1}{1+\mu}\{F_o + \cos\Omega(\tau+\varphi) + S - F_1\} \qquad \text{(II-5)}$$

$$\frac{d^2\xi_2}{d\tau^2} + \frac{2\mu\gamma}{1+\mu}\left(\frac{d\xi_2}{d\tau} - \frac{d\xi_1}{d\tau}\right) + \frac{\mu}{1+\mu}(\xi_2-\xi_1) = -\frac{\mu}{1+\mu}\{F_o + \cos\Omega(\tau+\varphi) + W\}$$

In the following section these equations are programmed for the Applied Dynamics (2-64PB) hybrid analog computer.

III. Analog Computer Program

The analog computer lends itself to rapid solutions of differential equations. Furthermore two advantages which make this computer useful for the impact problem are 1) any variable in the system can be continuously monitored and 2) various parameters can be changed in the system and their effects immediately obtained. This is especially useful for studying system responses and in particular, checking stability. However, there are disadvantages too. The error introduced by the computer is directly proportional to the number of components used in the program, i.e., the more amplifiers and potentiometers the greater the error. Furthermore, the computer has only two place accuracy. Therefore, if the solution was sensitive to small disturbances, it would be entirely possible that the above disadvantages could introduce inaccuracies great enough to influence the solution.

The analog program representing the equations (II-5) is of the standard type for two second-order coupled differential equations and is shown in fig. 4. Because of the characteristics of impact, there is need for external control over the integrating amplifiers. A pure analog computer has no such capability which necessitated the use of the hybrid computer.

The computer used for this impact study was Applied Dynamics' Model (2-64PB), hybrid analog computer. This machine, with logic control, has the additional capability of temporarily storing computer variables and/or the automatic switching and sequencing of computer components. The usefulness of the logic control is especially apparent in simulation of non-linear problems, i.e., the impact phenomenon.

In this problem, impact occurs when $\xi_2 = 0$ and is characterized by a velocity before impact of $\dot{\xi}_2$ and a velocity after impact of $-e\dot{\xi}_2$ where e is the coefficient of restitution. The computer routine takes the following form. Between impacts, the tracking amplifier continuously tracks the velocity $\dot{\xi}_2$. When impact is sensed by the comparators ($\xi_2 \leq 0$), a logic signal is triggered placing the main system and tracking amplifier into a hold mode, i.e., storing $\dot{\xi}_2$ just before impact. After an increment of time, a pulser of the logic system places the velocity integrator ($\dot{\xi}_2$) into a mode to receive the new initial conditions. Then the output of the storage amplifier (tracking amplifier in hold) is first passed through an inverting amplifier and then through a potentiometer set at the coefficient of restitution. The output of the potentiometer is placed on the $\dot{\xi}_2$ velocity integrator as the new initial condition. Again after a short time delay the main program is again placed into the operate mode until the comparator signals the next impact.

The entire logic sequence, fig. 5, takes about three seconds. The advantages of the hybrid computer can readily be seen when compared with the pure analog computer. Without the use of a hybrid, the impact phenomena would have to be simulated by the operator who would place the computer in hold, determine the velocity $\dot{\xi}_2$ before impact, multiply by coefficient of restitution, re-initialize the $\dot{\xi}_2$ integrator, and finally turning the computer back on to await the next impact. To say the least, this would be a very time consuming and inaccurate method.

IV. Numerical Results

It will be the purpose of this section to present analog computer solutions to the governing differential equations of motion of both the force limited and floating systems, i.e., equations (II-5). The equations of motion of the floating system are obtained by setting K = S = 0 in equations II-5. A steady-state analytic solution [4] and stability analysis [5] exists for equations (II-5) for $K = S = \gamma = 0$. These solutions will be checked and the floating system studied for $\gamma \neq 0$. In addition, the full set of equations (force limited model, see fig. (1-C) will be studied to see what changes exist in both stability and response. Since no analytic solution exists for this latter case, the purpose of this study is mainly to indicate the various possible types of steady-state behavior plus the effects of damping, spring constants, "hold down" force, coefficient of restitution and initial conditions on the response and stability of the system.

A. Steady-State Response of the Floating System

The steady-state solution of [4] assumes that the system response follows the forcing frequency; i.e., is periodic with period equal to the forcing period. Since the analytic solution is obtained in one period only, it further assumes that system response is identical for all periods. This will henceforth be referred to as a non-beat periodic solution. Under this assumption two such steady-state solutions are found, both of which satisfy the equations of motion and the "boundary" conditions or conditions at the beginning and end of the period. The stability analysis of [5] shows that only one of these two steady-state solutions is asymtotically stable. The results of this stability analysis are summarized in fig. 3. The choice of parameters in fig. 3 is:

$$\begin{aligned} F_o &= 1.160 \\ F_1 &= 0.453 \\ W &= 0.362 \\ \mu &= 10.83 \end{aligned} \tag{IV-1}$$

To show how instability developes, consider the following values of e and Ω;

$$\begin{aligned} e &= .6 \\ \Omega &= 1.44 \end{aligned} \tag{IV-2}$$

These values correspond to actual operating conditions of a particular machine, see [4]. The two predicted steady-state branches of [4] yield the following conditions at the beginning of the period for the masses M_1 and M_2.

$$\begin{array}{ll} \underline{\text{Branch I}} & \underline{\text{Branch II}} \\ \xi_1(0) = 1.71 & \xi_1(0) = 3.46 \\ \dfrac{d\xi_1(0)}{d\tau} = 0.0375 & \dfrac{d\xi_1(0)}{d\tau} = 0.0375 \end{array} \tag{IV-3}$$

$$\xi_2(0) = 0 \qquad\qquad \xi_2(0) = 0$$

$$\frac{d\xi_2(0)}{d\tau} = 1.22 \qquad\qquad \frac{d\xi_2(0)}{d\tau} = 1.22 \tag{IV-3}$$

If these two solutions were stable, then using these conditions as initial conditions, steady-state motion would be present from time $\tau = 0$. The motion for this set of parameters is shown in figs. 6 and 7. It is interesting to note that the solution has the assumed periodicity in both branches for approximately five cycles and then begins to deviate markedly. As predicted in fig. 3 the motion is indeed unstable for this set of parameters.

To check the stability analysis of [5] consider the following parameters:

$$e = 0.7$$

$$\Omega = 3.0 \tag{IV-4}$$

It can be verified from fig. 3 that this is a stable point. The analysis of [5] predicts stability for the second branch only and only for very small perturbations. The steady-state initial conditions for the second branch are;

$$\xi_1(0) = 1.11$$

$$\frac{d\xi_1(0)}{d\tau} = .0127$$

$$\xi_2(0) = 0 \tag{IV-5}$$

$$\frac{d\xi_2(0)}{d\tau} = .644$$

Substituting these into the analog program the response shown in fig. 8 results. The results here are somewhat inconclusive in that results show some deviation after a large number of cycles. The analog computer itself introduces a perturbation in the system, however, which may be sufficient to introduce the deviation shown. In any event, fig. 8 indicates that the solution stability is indeed sensitive to small perturbations as indicated in [5].

The inclusion of damping in the system provides some particularly interesting results. Considering the parameters of equations (IV-1, IV-4, IV-5) plus a damping coefficient of $\gamma = 0.2$ we find a new type of periodic (stable) solution entering, see fig. 9.* Here we have a beat phenomena present which repeats every ten cycles. This clearly violates the assumed form of the steady-state solution of [4]. If the damping is now increased to $\gamma = 0.3$, the non-beat periodic solution again exists, (see fig. 10). Of particular interest here is the behavior of $\xi_1(\tau)$. The motion of $\xi_1(\tau)$ dies out almost completely. Thus, we have, in effect, a single degree of freedom system which is quite similar to the "vibration absorber" [8]; i.e., a two mass system is "tuned" until the motion of the upper mass goes to zero.

*This will be referred to, henceforth, as a beat periodic solution.

If the damping factor is further increased, e.g. to $\gamma = 0.4\text{-}0.5$, the solution periodicity breaks down completely within several cycles. It should be noted that over the periodic portion, the response of ξ_2 is not strongly dependent on the damping factor. Also, it should be noted that the behavior of the upper (larger) mass M_1 is the best indicator of stability in the system; i.e. trends in $\xi_1(\tau)$ provide the best insight as regards stability.

It can also be shown that the inclusion of damping increases the stability region. If we consider

$$\begin{aligned} e &= 0.8 \\ \Omega &= 3.0, \end{aligned} \qquad \text{(IV-6)}$$

a point clearly outside the stable region of fig. 3, we find approximately the same behavior as for e = 0.7. In fig. 11, $\xi_1(\tau)$, $\xi_2(\tau)$ are plotted for $\gamma = 0.30$ and 0.35.

The initial conditions for the parameters of equations (IV-6) were the same as those for equations (IV-4), i.e., equations (IV-5), which, in effect, provides a perturbation of the initial conditions. This can be seen in the beginning of motion on fig. 11. With damping the stability seems less sensitive to initial condition perturbation.

B. Steady-State Response of the "Force Limited" System

As was previously mentioned in section II, the reason for studying the "force limited" model is to provide a better approximation to the physical system. In this model a second spring is added (see fig.1-C) to provide a mechanism for correcting any large deviations of the motion which might arise, and, hopefully thereby increase stability. The governing equations of motion for this case are equations (II-5). To simplify matters S will be set equal to zero. It should be noted that from the point of view of the computer, however, S and F_1 (hold down force) are identical.

It was shown above that $e = 0.6$, $\Omega = 1.44$ leads to unstable solutions (both branches). In this study both e and Ω will be fixed at these values and the effects of K, γ, F_1 will be considered.

The periodic behavior of the solution seems to be markedly improved by the addition of the spring k_1. Fig. 12 shows the system behavior for $\gamma = 0$, $K = .06$ and $F_1 = .045$. Although the oscillations are large there is a definite "beat" periodicity. This compares with the system response of fig. 7, and in this case for less hold down force, i.e., F_1 and K are small. It will be shown later that increasing hold down force tends to increase stability. If the damping factor is increased very slightly, $\gamma = 0.01$, and the spring constant ratio is increased to $K = .154$ we have the periodic behavior shown in fig. 13. We still have the beat phenomena present, but much less violently. If we increase the spring constant still further $K = .21$ and increase damping to $\gamma = 0.1$, we tend to the non-beat periodicity of [4], (see fig. 14). Figs. 14-A and 14-B also show that the solution converges to the steady state value for a fairly large range of initial conditions. In fig. 14-A we have $\xi_1(0) = 1.7$ and in fig. 14-B we have $\xi_1(0) = 5.0$. The other initial conditions remain the same. Both converge to the same steady-state solution indicating that stability of the "force limited system" is less sensitive to initial disturbance than the floating system . These two sets of initial conditions correspond roughly to the two solution branches of [4], indicating only one stable solution in the real system.

Comparing the response shown in fig. 14 with the steady-state solution predicted in [4] it is interesting to note a close correlation. [4] predicts the maximum of $\xi_2(\tau)$ to be approximately 4.4 while fig. 14 indicates a maximum displacement of $\xi_2(\tau)$ of 4.0. Although, the parameters in the two cases are slightly different they are close enough to warrant comparison. It also should be noted that the period in the force limited system is equal to the forcing period. This would seem to indicate that the steady state solution of [4] is correct wherever periodicity of the type assumed is found.

It should be clear from the above discussion that stable periodic solutions exist for the force limited system. It is now of interest to consider the effects of K, γ and F_1 on the system response. Fig. 15 considers a variation of K, all other parameters fixed. Only $\xi_2(\tau)$ is shown here. For K = .05 results are somewhat inconclusive. A steady-state stability was not reached, but this could, however, possibly be due to initial condition sensitivity. For K = .10, stability was finally achieved; however, it contains a strong beat. As K increases through K = .15, .20 we tend toward the non-beat periodic behavior of [4]. As K is increased beyond this the results again tend to be inconclusive. No solution of the non-beat type seems to exist. Thus K seems to have a stabilizing influence on the system but only for limited ranges.

To consider the effects of damping we again show only $\xi_2(\tau)$, see fig. 16. In general, damping shows the same effects on the system as K. For small values of damping, results are inconclusive (strong beat effects). As γ is increased, γ = .125, .15, the beat phenomena tends to die out and at γ = .175 the non-beat periodic behavior is again evident. For larger values of damping, results again tend to be inconclusive. No solution of the non-beat type seems possible.

Fig. 17 shows the effects of the hold down force F_1. Again, the variation of F_1 causes approximately the same effects as K and γ. For low values of F_1, $F_1 = .09$, results are inconclusive. The beat phenomena begins to show up at $F_1 = .23$. At $F_1 = .32$ the non-beat periodic behavior is found. This holds through $F_1 = .41$. For larger values of F_1 results again tend to be inconclusive. It is of interest to note that the non-beat periodic solution holds over a fairly wide range of hold down forces. We also note that as the hold down force increases, the amplitude of $\xi_2(\tau)$ decreases as expected.

V. Results and Conclusions

The purpose of this report has been to utilize the analog computer in the solution of dynamic impact problems. Of particular interest here has been the generation of periodic, stable, steady-state solutions. Since very little analytical work is available to direct the computer studies, the main objective has been to simply identify the various types of response and to provide some indication of what effects damping, spring constants, hold down force, and initial condition perturbations have on the response and stability of the system.

A class of hammer impact machines has been idealized as a floating two mass system. Stability analysis of this system indicates that it has very poor stability characteristics. This does not seem to be the case in real machines of this type. It has, thus, been proposed in this report that another system, namely a force limited model, would provide a more realistic model. The response and stability of this system seem to indicate that this is so, i.e., stability characteristics are better, while the response is largely unchanged.

Due to the limited and somewhat random character of the present study, trends in the system response is perhaps a better term than conclusions. The observed trends are listed below.

1. Stability in the undamped floating system, of the non-beat type is very limited and sensitive to perturbations of the system.

2. Damping in the floating system has the following effects. For small damping a beat type of periodic solution exists (for the parameters studied). As damping is increased the periodic solution tends to the non-beat type. For large damping no periodic solutions were observed. Inclusion of damping in general seems to enlarge the stability region.

3. For the non-beat type of periodic solution, the motion of the large mass, ξ_1 , goes to zero in a manner similar to the "vibration absorber".

4. The inclusion of damping in the floating system tends to reduce the sensitivity of stability.

5. The stability characteristics of the force limited system seems to be much improved. Moreover a comparison of response with the floating system seems to indicate that the added spring tends to effect only the stability and not the response, indicating that the solutions of [4].

are acceptable wherever stability of the type assumed exists.

6. The effects of F_1, γ, K on the force limited model are all approximately the same. Stability increases as each parameter increases, going first through a beat type stability, and finally to a non-beat type. As the parameters are increased beyond some critical point, the periodic behavior seems to break down.

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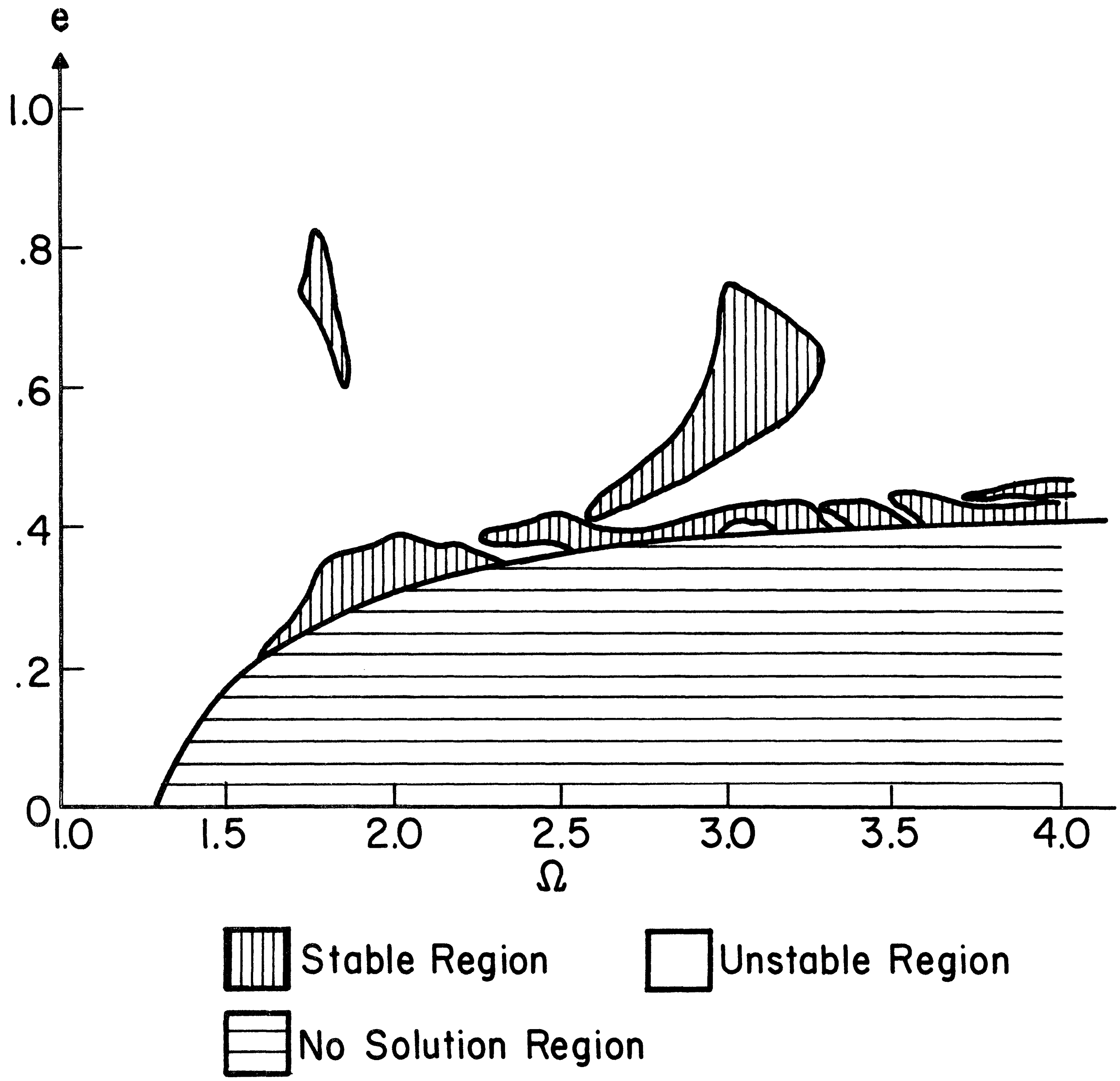

Fig. 3 Regions of stability for second-branch steady-state solution (after Fu & Paul).

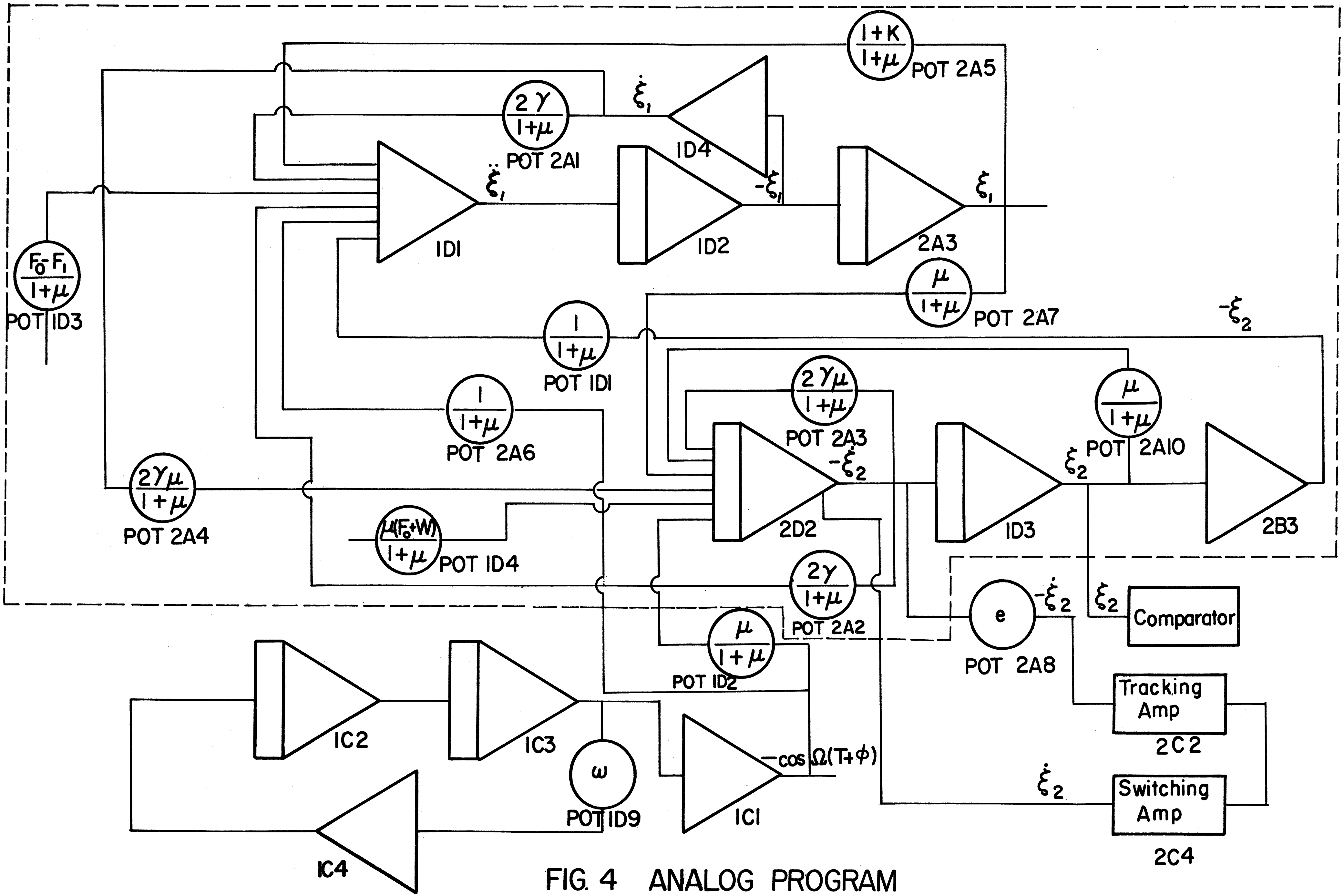

FIG. 4 ANALOG PROGRAM

DOTTED LINE ENCLOSES MAIN PROGRAM