THE USE OF COMPUTERS IN MECHANICAL ENGINEERING EDUCATION

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ABSTRACT

The Mechanical Engineering Department at The University of Michigan has taken an active part in computing work at the University with a total of nine faculty members serving as participants in the Project on the Use of Computers in Engineering Education during the past two-year period. In addition, there have been twenty-six Mechanical Engineering professors from other universities who participated in Project activities for periods varying from one week to a full semester.

All undergraduate Mechanical Engineering students at the University are required to take an introductory sophomore level digital computer course taught by Computing Center and Mathematics Department personnel. Digital and analog computer work has been assigned in 17 departmental courses during the past year, giving students an opportunity to gain practice in the application of computers in the solution of their engineering problems. The Mechanical Engineering curriculum, the areas where computer activity has been strongest, and a sampling of opinions of faculty members as to the effectiveness of computer usage in engineering instruction are described.

The Department is responsible for the operation of an analog computer laboratory and students are required to become familiar with these computers as well as with the digital machine. A selected set of eleven example problems prepared by faculty Project participants and other faculty members in the Department is also included. These may be considered as a supplement to the 64 example engineering problems, including several related to Mechanical Engineering subject areas, which have been published previously by the Project.

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USE OF COMPUTERS IN MECHANICAL ENGINEERING EDUCATION

I. INTRODUCTION

The Department of Mechanical Engineering at the University of Michigan has participated extensively in the study of the educational utility of computers. Many of the faculty have learned to use the computer in the classroom and the number of new uses increases in breadth and depth continually. Both analog and digital computer techniques have been employed successfully. It is the purpose of this report to show how these techniques have been introduced and to present the general philosophy surrounding the application of computers in the departmental courses.

In order to orient the computer and its educational value to the degree programs of the Department of Mechanical Engineering, the structure of the undergraduate and graduate degree programs will be considered first.

II. THE UNDERGRADUATE PROGRAM

The program in Mechanical Engineering leading to the degree of Bachelor of Science in Engineering may be thought of as two sections. The first is technical and devoted to the preparation of engineers for professional development through industrial service or graduate study. The second is devoted to the humanities and social sciences and is designed to prepare the student for an effective life in society. The technical section emphasizes the basic sciences and engineering sciences. Courses in these areas include associated laboratories that are the direct responsibility of the teachers in the corresponding lecture courses. To this foundation is added a group of courses in analysis and design to foster the association of theory, experiment and practice that is vital to good engineering practice.

Flexibility to meet the varied needs and aims of the students is provided by substitutions, combined programs with other departments, and electives in the area of the student's interest. Variation in the abilities of the students is accounted for in a level of attainment requirement for graduation, which permits gifted students to acquire the degree in less than the usual number of credit hours by taking special sequences of courses that cover the same material in fewer class hours. Hence the program may vary from one student to another, but they are all variations on a theme, which is the recommended program.

The current recommended program in Mechanical Engineering is listed below, and followed by a flow chart of the program. All students are required to complete this program or its equivalent.

The program as laid out in the flow diagram (Fig. 1B) indicates the prerequisite and corequisite courses and the order in which the courses should be taken to complete the program in eight semesters and a summer session. The rows on the chart give the semester schedule and credit hour loads. The columns show the various area sequences involved. For example, the first column shows the sequence in mathematics running from the first semester through the fifth.

TABLE IB

Undergraduate Program in Mechanical Engineering, 1962-63

Course Number*	Subject	Hours
Elective	Humanities, Arts & Social Sciences	11
English I	English Composition and Speech	6
English II	English Composition, Speech or Literature	2
English III	Literature	2
Econ. 401	Modern Economic Society	3
Math. 233, 234, 371, 372	Analytic Geometry and Calculus	16
Math. 373	Elementary Computer Techniques	1
Math. 404	Differential Equations	3
Physics 145	Mechanics, Sound, and Heat	5
Physics 146	Electricity and Light	5
Chem. 103 or 104	General or Inorganic Chemistry	4
Chem. 105 or 106	General and Inorganic Chemistry	4
Chem/Met. Eng. 250	Principles of Engineering Materials	3
Chem/Met. Eng. 270	Metals and Alloys	3
Eng. Graphics 101	Engineering Drawing	3
Eng. Graphics 104	Introduction to Descriptive Geometry	2
Eng. Mech. 208	Statics	3
Eng. Mech. 210	Mechanics of Materials	4
Eng. Mech. 212	Laboratory in Strength of Materials	1
Eng. Mech. 324	Fluid Mechanics	3
Eng. Mech. 343	Dynamics	3
Mech. Eng. 251	Manufacturing Processes I	3
Mech. Eng. 324	Fundamentals of Fluids Machinery	4
Mech. Eng. 335	Thermodynamics I	3
Mech. Eng. 336	Thermodynamics II	4
Mech. Eng. 340	Dynamics of Machinery	4
Mech. Eng. 362	Machine Design I	3
Mech. Eng. 371	Heat Transfer I	4
Mech. Eng. 381	Manufacturing Processes II	3
Mech. Eng. 460	Machine Design II	3
Elec. Eng. 315	D. C. and A. C. Apparatus and Circuits	4
Elec. Eng. 442	Motor Control and Electronics	4
Technical Electives		12
	Total	138

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*Descriptions of these courses may be found in the University of Michigan College of Engineering Announcement of 1962-63.

Undergraduate Mechanical Engineering Program 1962-63

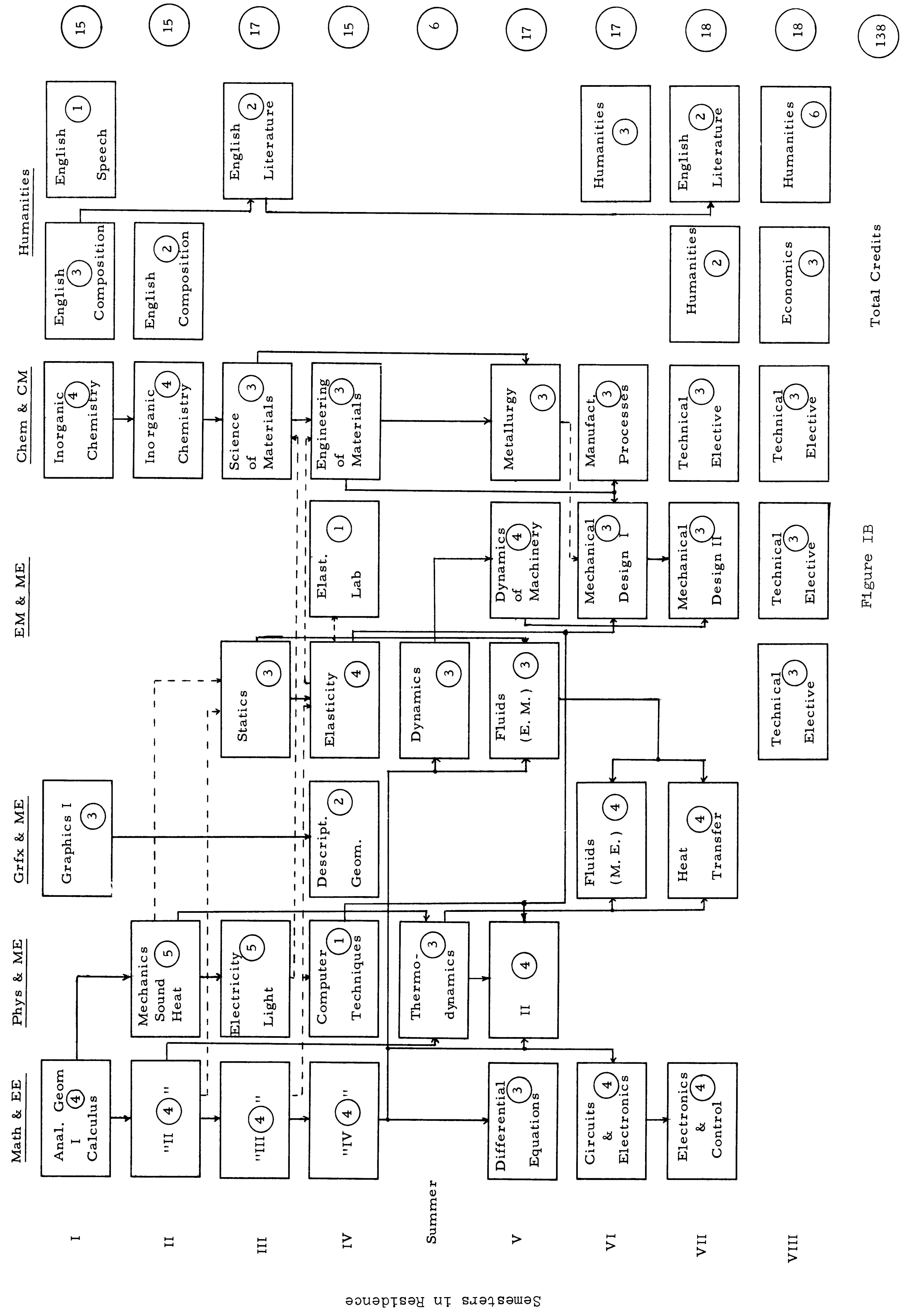

Figure IB

In the area bounded by rows five through seven and columns two through four the flow of thermodynamics, fluid mechanics and heat transfer becomes evident. The sequence in engineering mechanics occurs in column four, starting with Statics, and flows downward and to the right through Dynamics to Dynamics of Machinery which initiates a mechanical design sequence. The design sequence is also fed by the sequence in Chemistry, Materials and Processing that occurs in the sixth column. The diagram indicates clearly the interrelations of the various areas of the program and should be helpful in relating the use of computers to the fundamental purposes of such a program.

Training in the use of computers is introduced to the program in two ways. First, a specific course entitled Elementary Computer Techniques (Math 373) occurs in the second column in the fourth semester, as shown in the flow chart. In that position this course may be used as a prerequisite for any or all junior technical subjects. It is at present a specific prerequisite for the second course in thermodynamics (M.E. 336) and the course in mechanical design (M.E. 362) and is used in that manner by others, fluid mechanics (M.E. 324), and heat transfer (M.E. 371) in particular. Secondly, training in the techniques of analog computers is an integral part of the course content of Dynamics of Machinery (M.E. 340). Experience shows that it is usually better to separate the introduction to the digital computer because of the dominance of the language aspects of the problem in the early stages. Analog computers may be introduced either way depending upon the complexity of the problems to be handled. Coupled with an introduction to mechanical vibrations, it is most effective to integrate the computer training. In Dynamics of Machinery (M.E. 340) the analog computer is used in both classroom and laboratory experiences.

It should be observed that the early occurence of the Computer Techniques course makes possible a large number of classroom applications throughout the upper class years. Only the most general of these applications will be reported here. However, the successes in these areas are continually germinating new applications in other course areas.

III THE GRADUATE PROGRAM

The graduate program of the Mechanical Engineering Department is structured in six major areas. These are:

1) Fluid Flow
2) Heat Transfer
3) Thermodynamics
4) Stress Analysis
5) Dynamics and Vibration
6) Materials and Manufacturing

The requirements for the master's degree include at least one course in four of the engineering science areas cited above, and also advanced calculus. No thesis is required but the majority of the students gain some experience in research either through work on one of the research projects or through study or research in selected Mechanical Engineering Topics (ME 600), a three credit individual-research-type course.

The requirements for the doctorate involve thirty credits of course work beyond the Master of Science degree, two languages, the successful passing of written and oral qualifying examinations, and the doctoral thesis. Courses at the 600, 700, and 800 level are primarily for doctoral students.

Table IIB lists the graduate Mechanical Engineering courses available to show the breadth and depth of the offerings. The Table is divided somewhat arbitrarily into Basic and Professional courses. The intent is to indicate those courses whose content is taken primarily from the fundamental aspects of the science and those extending the student into the frontiers of current practice. It should be apparent that such a separation is most difficult because of the ever changing state of technology. However, the reader may grasp some feeling of current emphasis from this tabulation.

Table IIB

Graduate Courses in Mechanical Engineering

Basic Courses

ME 417	Plastic Forming of Metals I
ME 421	Dynamics and Thermodynamics of Compressible Flow
ME 432	Combustion
ME 461	Automatic Control
ME 517	Plastic Forming of Metals II
ME 524	Fluid Mechanics II
ME 531	Statistical Thermodynamics
ME 535	Advanced Thermodynamics
ME 540	Mechanical Vibrations
ME 541	Synthesis of Mechanisms
ME 550	Physical Behavior of Materials
ME 556	Stress Considerations in Design
ME 567	Reliability Consideration in Design
ME 571	Heat Transfer II (Conduction)
ME 583	Machinability
ME 625	Introduction to Viscous Flow Theory
ME 672	Heat Transfer III (Convection)
ME 673	Heat Transfer IV (Radiation)
ME 772	Heat Transfer V
ME 835	Seminar in Thermodynamics

Professional Courses

ME 422	Design Theory of Fluid Machinery
ME 437	Applied Energy Conversion
ME 463	Wear Considerations in Design
ME 465	Mechanical Analysis Laboratory
ME 467	Lubrication and Bearing Analysis
ME 480	Design of Manufacturing Equipment
ME 482	Manufacturing Engineering
ME 486	Manufacturing Considerations in Design
ME 491	Heating and Air Conditioning
ME 492	Design of Heating and Air Conditioning Systems
ME 493	Gas Turbine Engines
ME 494	Design of Gas Turbine Engines
ME 495	Analysis and Design of Rocket Engines
ME 496	Internal Combustion Engines
ME 497	Automotive Laboratory
ME 498	Automotive Chassis
ME 539	Cryogenics and Refrigeration
ME 594	Advanced Theory of Internal Combustion Engines

IV COMPUTERS IN MECHANICAL ENGINEERING

The philosophy of the use of computers in Mechanical Engineering is related to the program structure just presented. The fundamental conviction is that Mechanical Engineering students should be introduced to the use of computers, both analog and digital, as an integral part of their educational experience. Further, the computer experience must be a graduated experience, repeated in many meaningful situations in the various subject areas of the student's degree program.

Within the course work several valuable attributes of the introduction and application of computers are evident. First, the computer challenges the student with a new degree of discipline in his work. The solution of problems on the computer forces the student to think of the logical structure of the problem. The various alternatives presented by the typical engineering problem are better treated by the creation of procedures of generality for the computer than by the usual set of restricted cases considered without the availability of the computer. Second, the student may extend his problem solving experience by considering more complex and realistic problems. The student has the satisfaction of obtaining the results of quite involved procedures more frequently. Less often must he be content with a superficial treatment of analytical procedures. Third, the student is extended in his mathematical experience, through the association with numerical analysis. Even "trial and error" methods are made more rigorous in the consideration of Newton-Raphson methods, half-interval methods and other methods applicable to the solution of the non-linear expressions common to engineering practice. Vector and matrix analysis may be applied to appropriate problems in realistic situations with little more effort than was previously expended in the consideration of situations constructed so as to be simple enough to be solved by hand. In addition, it should be observed that graduate students, at the doctoral level, may now undertake many problems that were unreasonable or even impossible heretofore.

Finally, it has been observed that the solution of problems on the computer frequently produces a greater interaction between the student and the faculty. Possibly, this has been due to the fact that both the faculty and the students have been sharing the learning of the computer techniques. However, it is more reasonable to suppose that the ability to engage problems of more realistic proportions than the typical academic excercise is stimulating to both faculty and students, thus creating an atmosphere of mutual interest.

V APPLICATION OF COMPUTERS IN SPECIFIC SUBJECT AREAS

Because of the dynamic growth of computer applications it is certain that any presentation of the applications is, at once, obsolete. Therefore, a set of representative subject areas have been selected to illustrate the nature of activities at one point in time. The subject areas chosen include Thermodynamics, Heat Transfer, Fluid Flow, Kinematics and Dynamics. Thus it is possible, in most instances, to display problem materials that are of varying complexity and thus suitable for courses ranging from the undergraduate through the graduate level. The comments quoted concerning the problem materials and the applications of computers have been extracted from written commentaries submitted by the faculty concerned with the subject areas. It is hoped that their candid frankness will help the reader to orient the experiences of one faculty body to the context in which he may find himself.

a) Thermodynamics

In this area the student has a large number of opportunities to meet situations that are suitable for computer solution. While a few problems have been assigned in Thermodynamics I (ME 335, 3 credits), it is more likely that the first computer problems in Thermodynamics will be encountered in Thermodynamics II (ME 336, 4 credits). This is because of the nature of the material presented in the first course and because of the extreme diversity of students present in the course. Many students from departments other than Mechanical Engineering are found in Thermodynamics I.

The second course is taught by a number of faculty and thus provides an opportunity to observe the reaction of several staff members to the presentation of a common problem. The problem presented in all these classes was the computation of the compressibility factors of gases from the Beattie-Bridgeman equation. (See example problem No. 15 of the First Annual Report of the Ford Foundation Computer Project).

The comments of the staff reflect a general feeling that the problem was of value to the student.

"The problem was of considerable benefit in understanding the use of equations of state and their limitations."

"I think the problem was helpful and should be continued."

"This computer problem served as a good demonstration as to the use of digital computers as an engineering tool".

"The assignments have been useful as far as the objectives go".

The comments also indicate the value of previous course work in computers (e.g. Elementary Computer Techniques) prior to courses given in the professional departments. At the time this problem was given relatively few Mechanical Engineering students had had the opportunity to take such a course. (This course, Math 373, is now a prerequisite for ME 336 and ME 362.)

"However, it has been very time consuming for both students and myself when the students have not had any previous experience...".

"More sophisticated problems can be covered when the students have had a formal and more thorough training program such as is now required in the curriculum".

One staff member mentioned the "drag-out" of the problem "when service at the Computing Center is poor". This comment is mentioned because it emphasizes the incompatibility of effective education and long queues. The Computing Center currently processes more than 6,000 problem runs per month and is anticipating delivery of equipment of much higher capacity. The comment may thus be interpreted as a warning against the diversification of resources into small low-capacity machines and a strong argument for support of the very largest capacity facility that can be afforded.

The use of the digital computer as a demonstrating device was made in Advanced Thermodynamic (ME 535, 3 credits, graduate level). Here the problem of determining the effects of pressure and percent theoretical air on the adiabatic flame temperature of methane,including the effects of dissociation of the products of combustion,was presented. The students developed the solution but did not program or run the problem. Instead the program and results were given to the class. The objective was to evaluate the combustion process with quantitative evidence to support the discussion. (It should be observed that the lack of previous computer experience is even more noticeable among the upper level students. In a few semesters it may become commonplace to expect and receive student programs for more complicated problems than this one.)

The Seminar in Thermodynamics (ME 835, 3 credits, upper graduate level) considered the problem of the analysis of rocket engine performance as a function of propellant mixture ratio and chamber pressure. This was programmed individually by the students and "was beneficial to the students' understanding of chemical equilibrium and to the approach to the solution of advanced problems in Thermodynamics".

The problem statements and solutions for a set of problems of graduated complexity in the area of Thermodynamics are given in the problem section of this report.

b) Heat Transfer and Fluid Flow

These areas are representative of many that are strongly founded in engineering principles and build from these foundations into engineering practice. Accordingly, the computer finds considerable application and utility in developing these materials. Each area will be treated separately but the level of the material covered is comparable, the first encounter coming in the junior year.

Heat Transfer I (ME 371, 4 credits, junior level) is concerned with the study of the mechanisms of heat transfer processes. For the past two semesters, a problem in two-dimensional heat conduction using finite difference techniques has been programmed for the digital computer. The students themselves have prepared this program. Their instructor comments....

"In assigning problems for programming by the students the primary objective was to give the students an experience in setting up an algorithm for the solution of a problem. In so doing they must think in terms of unambiguous logical operations, an experience which has value in all phases of engineering. A secondary objective was to instill in the students confidence in solving specific heat transfer problems with a digital computer."

A comparison of analytic, graphical, analog computer and numerical methods with actual experimental evidence is readily possible. In this course the students "make physical measurements of a transient heat conduction phenomenon and compare these with the results of an analytic solution, a numerical solution using hand computations, a graphical solution, the analog computer solution, and a numerical digital computer solution. The advantages and limitations of each of these is discussed."

An apparent benefit of such an approach is the opportunity to teach the limitations, as well as the advantages. This aspect is sometimes ignored in an attempt to "prove a point".

"This program demonstrates interaction of errors due to discretization, or grid-size spacing, and due to round-off, inherent in the iteration techniques. The objective was to demonstrate potential pitfalls in the use of numerical solutions, and also to demonstrate their value with complex geometries and boundary conditions."

A representative set of problems to be treated are the following:

I. A steady-state two dimensional conduction problem with heat transfer coefficients and fluid temperatures specified. (Example 3-4 in "Principles of Heat Transfer" by Frank Kreith.)

II. Transient one dimensional heat conduction in an infinite slab with a step change in one surface temperature. (Both analog and digital computer solutions obtained for this problem.)

III. Transient one dimensional heat conduction in an infinite slab with a step change in one fluid temperature.

The comments concerning the difficulties associated with the computer solutions are also interesting.

"In first presenting a digital computer problem some apathy on the part of students was detected. It was even found necessary to provide a certain amount of incentive. However, once having gained some experience and success in programming the attitude in most cases was changed to one of enthusiasm."

"It has been my personal experience that it is unwise to give a student problems in learning the language and learning the problem at the same time. For this reason I think computers can be very nicely used in the classroom because most of the students are familiar with the problems being discussed, and hence are not only oriented but motivated to their solution on a high speed computer."

It might have been observed that the foregoing remarks apply equally to faculty as well as students. In fact one rather expects that the use of the computer in the classroom must "prove itself" with many faculty. These men are highly qualified in their respective areas and quite naturally may hesitate to venture into an area in which they are much less certain of themselves initially. A staff member comments....

"In order to introduce the staff to the use of digital computers, a program (such as the Project on the Use of Computers in Engineering Education) is invaluable in the early stages".

The computer has also found much application in the graduate level Heat Transfer courses. A partial list of problems programmed in these courses is given to indicate a little of the scope of the work done.

I. Evaluation of the double integral to obtain the geometric view factor for radiation between two surfaces of arbitrary shape and orientation. (Radiative Heat Transfer, ME 673, 3 credits.)

II. Solution of the radiation between a multiplicity of gray surfaces. (ME 673)

III. Linear problems of diffusion with simultaneous convective flow (Convective Heat Transfer, ME 672, 3 credits).

IV. Convective heat flow in a duct with simultaneous axial conduction. (ME 672)

The application of computers in fluid flow is first encountered in Fundamentals of Fluid Machinery (ME 324, 4 credits). This course was using the IBM 650 computer in classroom activities as early as 1957-58. It was recognized that the availability of a compiler was essential to effective classroom use. The IT (Internal Translator) and later the GAT (General Algebraic Translator) compilers were employed to allow the solution of the non-linear energy dissipation equations in turbulent flow by the students.

In this connection an approach was developed that seems effective when it is necessary, as it was at that time, to teach programming to the students as well as the problem. "First, select a problem that may be solved a small piece at a time, each piece leading into a larger problem. In this case, the first problem was to solve the Colebrook equation for turbulent flow at Reynolds numbers greater than 4000 and with general relative roughness. This function has the form:

$$\frac{1}{\sqrt{f}} = 1.14 - 2.*\log_{10} \left[\quad e/D + \frac{9.35}{N_R \sqrt{f}} \right]$$

This is solved as a rather simple but meaningful program using the Newton-Raphson method, and allows the student to put most of his attention on the computer solution after a brief introduction to the fluid mechanics. It is sometimes desirable to assign one solution to be done by hand so that the method is made completely clear (and so that the student can verify his computer solution). Next, this program is converted into a subroutine and the problem is generalized to allow any Reynolds number and a general set of conduit and fluid parameters to solve for pressure drop given the flow, or flow given the pressure drop. This causes the student to consider the action to be taken in the "transition zone", a topic frequently avoided otherwise. Finally, this program is converted into a subroutine and applied to the solution of the flow in a network of inter-connected conduits."

In this way, the student may begin with a fairly simple exercise, gain familiarity and confidence but retain the early work as a valuable part of later, more complicated and meaningful programs.

Other problems that have been treated successfully and effectively by junior level students are:

I. Supersonic flow along a wall using the method of characteristics.

II. Representation of characteristics of two phase flow with variable temperature, pressure and fluid parameters.

III. Surge system oscillations.

IV. Dynamic behavior of positive displacement fluid machinery with associated external systems.

Representative problems in Heat Transfer and Fluid Flow are included in the problem section of this report.

c) Kinematics, Dynamics and Machine Design

Considerable activity has occurred using analog computers in the courses in Machine Design, specifically in Dynamics of Machinery, (ME 340, 4 credits).

"One objective in the use of the analog in ME 340 is simply to introduce the student to the use of the analog computer. A second is to demonstrate the validity of derived relationships of parameters for the systems involved. Important is the ability to observe visually the frequency response effect, and the response of these simple systems to various input functions."

An important use of the analog computer occurs in the laboratory. Here the student simulates the mechanism used in a torsional vibration experiment. The physical model and the simulated model are compared under the excitation of various frequencies.

"The analog computer is a stimulant to the student learning the dynamics of mechanisms. Students better understand dynamics and are better equipped to solve problems in that area as a result of their classroom and laboratory experiences."

Another faculty member states..."Many vibrations problems are quite abstract and in solving them the student is often so far from physical reality that it is merely a mathematical exercise. Next to an actual laboratory demonstration the EDA (Electronic Differential Analyzer) is the best tool to establish a connection between the physics and the mathematics of a problem".

The course in Mechanical Vibrations (ME 540, 3 credits, graduate level) further extends the use of analog computers. In this course, the student is brought into contact with vibrating systems with several degrees of freedom. In addition certain non-linear problems may also be treated very effectively. The Mechanical Engineering Department analog computer has the equipment to allow function generation and square law multiplication. The multipliers may be extended in usefulness by using the built-in Zener diodes to obtain the appropriate sign on the results of the multiplication.

In general, the results with the analog equipment have been regarded as successful. The feeling is strong that additional work will be incorporated into the courses using these machines.

The Machine Design sequence also uses the digital computer in many varied applications. The first point at which contact is made with the digital computer is in Design of Machine Elements (ME 362, 3 credits, junior level). Typical problems have included a Bevel Gear speed reducer force analysis and variation in Hertz contact stresses in cam systems. The kinematic analysis has also been treated successfully....

"To me, however, the problem of kinematic analysis, as distinguished from synthesis, has been reduced almost at one stroke to pure routine. In other words, the act of programming the computer has produced a set of rules which can be followed by any technician (if computer time is too expensive) to analyze any mechanism problem". (It should be noted that "too expensive" was clarified later in a set of solutions using the IBM 704 in which the cost per solution was found to be of the order of 5 cents per point.)

Other applications in Experimental Research in Mechanical Engineering (ME 408, 3 credits, senior level) included: Deflections in shafts with complex loading and discontinuous cross sections; Determination of transverse shaft vibration frequencies for multiple degrees of freedom by the Stodola iterative method.

An evaluation of the use of computers in engineering education may be found in the following remarks submitted by one of the faculty:

"1. It contains an automatic feedback control on the learning experience of the student. Problem solution by the student with subsequent correction by the instructor has long been a basis of our approach, but the insistence that the student correct his errors and resubmit until correct has not been part of it. The use of the computer furnishes the discipline that insists upon this complete approach to the learning process.

2. The element of reward for success is an inherent part of dealing with computers. This is also a vital part of the learning process. It is very interesting to observe the stimulation of interest, and the enthusiasm which is generated by an experience with the computer.
3. It induces conscious, careful, deliberate organization of analytical approach, and thereby brings about a deeper understanding of the problem at hand. (It seems to me that this is probably the most important virtue of the computer at this point in the program (ME 362). The introduction of the problem to go on the computer should be related to a method of approach to complex analytical problems. Up to this point students have relied a great deal upon intuitive, mathematical approaches, and have tended to deal with solutions which could be seen as a whole. At this point they should be dealing with several variables simultaneously, and should learn to derive what they want from the several equations involved.)
4. A rational approach to problems coupled with the use of applied mathematics is encouraged.
5. The temptation to generalize, and the ease with which it can be done on computers is helpful in studying alternative solutions to design problems.
6. The complexity and length of problems that can be handled is increased. Like all tools, the computer has its faults as well as virtues. It does absorb time at this stage, cutting down the number of problems that can be handled in a semester. This will be compensated for by practice, of course, and by the fact that fewer more complex problems may serve just as well educationally, any way. Sometimes students become more fascinated by the tool than by the problem. The search for punctuation marks is sometimes as tedious and more frustrating than the job of hand computation."

Others are more "restrained" but show definite tendencies toward being "sold" within a reasonable period. The following two paragraphs are taken, intact, from a report submitted by one of the faculty to illustrate this point:

"Of course it is a natural consequence of the Computer Project, but I think the digital computer's role in engineering education has been somewhat overdone at Michigan. I have noted particularly that industrialists take a long hard look at a problem before considering a computer solution. Not often are they looking for the exact answers the computer will provide. Many times they will be perfectly satisfied if they can find something about the general trend. Hard justification is necessary before they will consider using a computer. I think this same justification should be employed in education.

On the other hand problems exist today, and even more are foreseen for the future, in which computer solutions will be the only available means. In general these seem to be in the design area where optimization theory, reliability, and statistical information, are important factors. These will be dealt with , for the most part, at the Senior and the Senior-graduate level."

Problems in the machine design area, including both analog and digital applications, are included in the problem section of this report.

VI. AVAILABLE COMPUTING FACILITIES

The digital computing facilities available to faculty and students in the Mechanical Engineering Department consist of the University Computing Center's IBM 709 with an associated IBM 1401 for off-line card, tape, and paper handling. The 709 will be replaced by an IBM 7090 during the coming summer.

Considerable analog computing power is also available to the Department. This includes a Department-operated analog laboratory with five 8-16 amplifier Applied Dynamics (AD1) analog computers, each equipped with Sanborn hot-wire recorders. Additional equipment includes x-y plotters and square law multipliers. Diode function generators will be installed in the near future.

In addition, a large Applied Dynamics computer (AD64) containing 32 amplifiers, built-in multipliers and diode function generators, is available for student use and laboratory demonstrations. The machine is equipped with hardware for repetitive operation and an oscilloscope for display of problem solutions. On occasion, students also take more comprehensive analog computer courses taught in the Instrumentation Department which has several large analog computers available for student use.

VII. CONCLUSIONS

The consensus, taken from the faculty reports submitted and condensed here, is that computers serve a vital function in the education of Mechanical Engineering students. In the main, the applications have grown conservatively and cautiously from small beginnings to the rather substantial uses quoted.

It is useful, especially for those considering similar computer activities in engineering education, to review briefly some of the salient aspects of the use of computers as gleaned from the reports of the faculty of the Mechanical Engineering Department .

First of all, the learning process is considerably assisted by the proper use of computers in the class work. This benefit seems to be due to three major factors that are particularly active in the use of computers:

1. The student must organize the solution of problems more explicitly and carefully when using the computer than when performing the solution by hand. This is because the student is always "present" when solving problems by hand. When using the computer, only the student's instructions for solving the problem can be "present" at solution time.
2. Rewards for a successful effort are very positive and immediate. This greatly enhances the learning process, stimulates interest and generates enthusiasm.
3. Discipline to submit and resubmit problems until completely correct is inherent in the use of computers by the student. It is interesting to note that the student will readily accept the flagging of errors and termination of a computer run by the computer while he would probably complain if his instructor were to make the same correction in his work.

Secondly, the courses themselves are improved by the use of computers. Some of these benefits are:

1. The extension of the student's mathematical experience through the association with numerical methods.
2. The treatment of more realistic and meaningful problems in the various course areas, with the added benefit of studying the results of these solutions under many varying conditions. This means that the courses are also extended into areas not previously touched.
3. The stimulation of greater student - faculty interaction through the treatment of interesting problems and solution techniques.

Of course, there have been some dissatisfactions and criticisms. These are the natural byproducts of any creative process. It is significant that the reactions have almost always been stated positively with the obvious intent to correct and improve the use of computers in education. A part of the difficulties arose from the fact that both the faculty and the students were learning together. This is a little frustrating to both. As time progresses, more and more students will have had the basic computer course (now a required course for graduation in the undergraduate program). This will mean that less time will be spent in the classroom on programming details with more value extracted in the engineering training of the student.

The evolution of engineering education has progressed into another stage. It is safe to say that the Mechanical Engineering curriculum will be molded for a long time to come by the forces generated during this effort.

The processes of natural selection administered by an alert and conscientious faculty will insure the dynamic growth of the effective use of computers in the classroom. What has been observed and reported here is a glimpse of this process during its early, vigorous, formative period.

VIII. EXAMPLE PROBLEMS

Several problems from the various Mechanical Engineering subject areas, many prepared by faculty participants in the Ford Project, are listed below. Complete solutions follow for most of the problems. For a few, only the problem statement is shown. All the digital computer programs are written in the MAD language which is fully described in A Computer Primer for the MAD Language by E. I. Organick.

These problems may be considered as a supplement to Problems 1 through 45 published in the First Annual Report of the Project, Problems 46 through 56 published in the Second Annual Report, and Problems 57 through 64 in a recent Project publication, Use of Computers in Industrial Engineering Education. It should be remembered that many of these problems represent the instructor's first effort at a computer solution and may not necessarily illustrate sophisticated programming techniques.

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Example Problem No. 65

ISENTROPIC PROCESS FOR IDEAL GAS WITH VARIABLE SPECIFIC HEAT

by

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Course: Thermodynamics I   Credit hours: 3   Level: Junior

Statement of Problem

The entropy change for an ideal gas can be computed with an equation as follows:

$$S_2 - S_1 = \int_1^2 C_P \frac{dT}{T} - R \ln\left[\frac{P_2}{P_1}\right] \qquad (1)$$

Solutions for this equation are simple when the specific heat is constant, but when C_P is a function of temperature the solution is more tedious.

In the case of the isentropic process, the equation can be used to compute one value of a variable if the other three values are known. For example, knowing the initial temperature T_1 and the pressures P_1 and P_2, the final temperature T_2 can be computed. When specific heats are constant the familiar equation

$$\frac{T_2}{T_1} = \left[\frac{P_2}{P_1}\right]^{\frac{k-1}{k}} \qquad (2)$$

can be applied. Tables of thermodynamic properties are available for certain substances (namely air) which account for variations in specific heat, but a trial and error solution of equation (1) must be used for other substances.

Write and test a MAD program which will accept as data an initial temperature T_1 (degrees R), an initial pressure, psia , and a final pressure, psia , for which the temperature, T_2, will be computed if C_P is a function of temperature as follows:

$$C_P = 11.515 - \frac{172}{\sqrt{T}} + \frac{1530}{T} \qquad (3)$$

Note: The above equation is for oxygen and gives the specific heat BTU/(lb mol °R) if T is in degrees R.

A solution to an accuracy of ± 2 degrees will be satisfactory.

Solution

Since this problem can be solved easily by a simple trial and error technique the problem statement to the student can be very brief. In this problem the reason for using the computer can be justified by two arguments

Isentropic Process for Ideal Gas With Variable Specific Heat

(1) Several solutions will be needed.

(2) Although the solution is simple the arithmetic is such that the possibility of error is quite high.

In solving the problem, S_2-S_1 in equation (1) is set equal to zero because the process to be analyzed is isentropic, and then the equation for C_P (equation (3)) is introduced:

$$0 = (11.515 - \frac{172}{T^{1/2}} + \frac{1530}{T}) \frac{dT}{T} - R \ln \left[{P_2}/{P_1} \right]$$

$$0 = 11.515 \ln \left[\frac{T_2}{T_1} \right] + 344 \left[\frac{1}{T_2^{1/2}} - \frac{1}{T_1^{1/2}} \right] - 1530 \left[\frac{1}{T_2} - \frac{1}{T_1} \right] -R \ln \left[{P_2}/{P_1} \right]$$

For facility in program writing, an internal function

$$F_o \; (T_2, T_1) = 11.515 \ln \left[\frac{T_2}{T_1} \right] + 344 \left[\frac{1}{T_2^{\frac{1}{2}}} - \frac{1}{T_1^{\frac{1}{2}}} \right] - 1530 \left[\frac{1}{T_2} - \frac{1}{T_1} \right] \qquad (5)$$

was defined so that the following equation could be written

$$\text{DIFF} = F_o \; (T_2, T_1) - \ln \left[{P_2}/{P_1} \right] \qquad (6)$$

Thus if the value of DIFF is exactly 0, the exact solution is obtained. Since the tolerance of ± 2 degrees permits some simplification, the solution follows a simple pattern.

(1) The initial guess for T_2 is made by assuming the constant specific heat case and applying equation (2). The value thus obtained is substituted into equation (6) and the value for DIFF could theoretically be positive or negative. Note: This may not be so but a good computer program should handle either eventuality even though it is improbable that DIFF will be negative. In either case the value of T_2 is changed in steps of 10 degrees until a sign change occurs, at which time the value of T_2 is altered in steps of 1 in the opposite direction until the sign changes again. When the second sign change occurs the current value of T_2 is called the answer.

<u>List of Symbols</u>

T1 = T_1, Initial temperature, °R (Data).

T2 = T_2 = Final temperature, °R.

P1 = P_1 = Initial pressure, psia (data) .

P2 = P_2 = Final pressure, psia (data) .

TISEN = Temperature calculated with equation (2), °R .

DIFF = Difference calculated from equation (6).

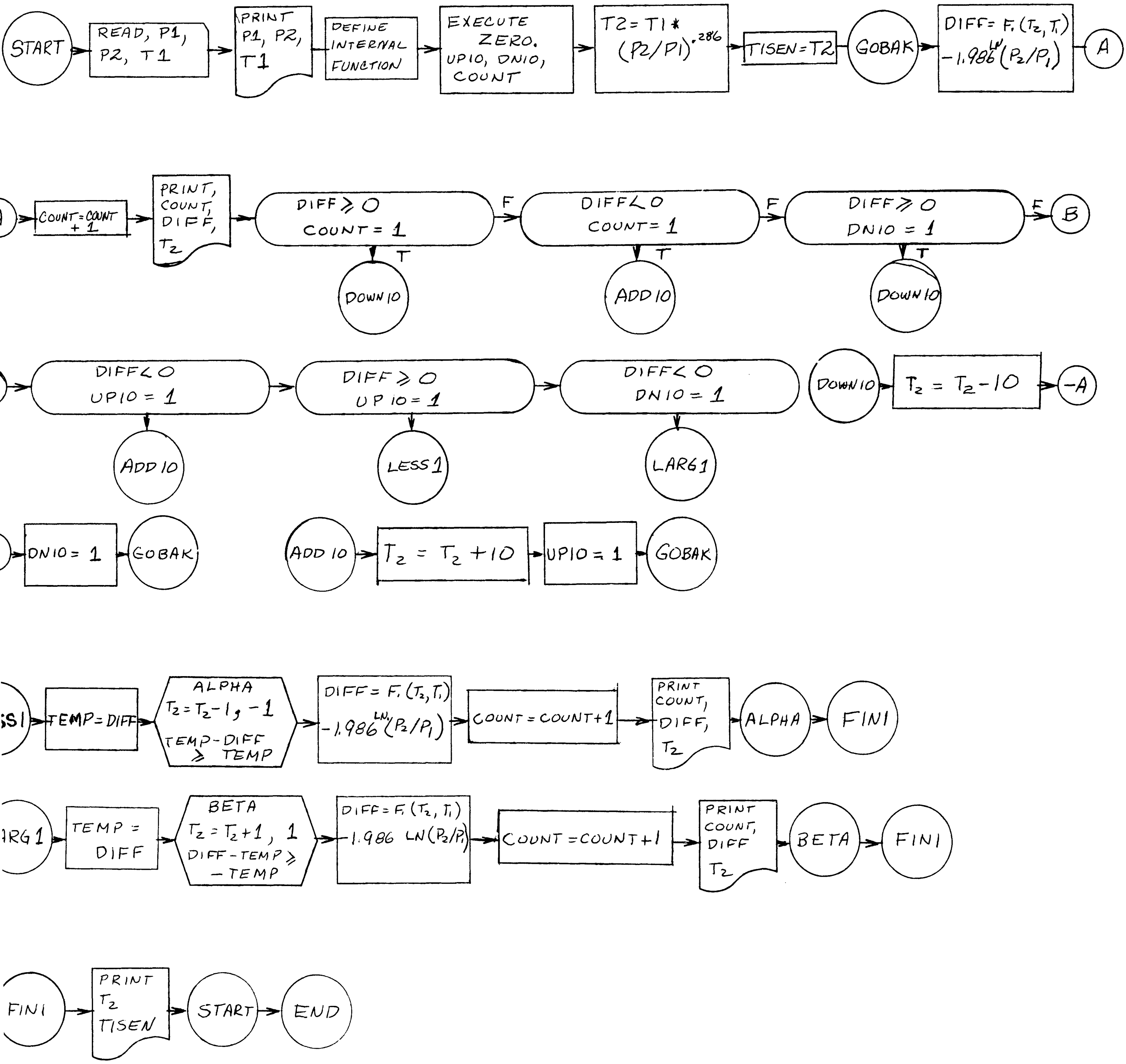
COUNT = A counter to count iterations.

DNIO,UPIO = A switch to indicate path taken.

TEMP = A temporary storage location.

Example Problem No. 65

Flow Diagram



Isentropic Process for Ideal Gas With Variable Specific Heat

MAD Program and Data

```
 ROBERT SLONNEGER               D044N                    1   005   050
$COMPILE MAD,EXECUTE.PRINT OBJECT.DUMP
          R       THIS PROGRAM WILL CALCULATE THE TEMPERATURE AFTER
          R       AN ISENTROPIC CHANGE OF STATE WHEN THE SPECIFIC
          R       HEATS ARE A FUNCTION OF TEMPERATURE)
           INTEGER          UP10, DN10, COUNT
START      READ FORMAT INPUT, P1, P2, T1
           PRINT FORMAT HEAD, P1, P2, T1
           INTERNAL FUNCTION F.(S2,S1) = 11.515*ELOG.(S2/S1)+344.*
          1((1./SQRT.(S2))-(1./SQRT.(S1)))-1530.*((1./S2)-(1./S1))
           EXECUTE ZERO.(          UP10, DN10, COUNT)
           T2 = T1*((P2/P1) .P. .286)
           TISEN = T2
GOBAK      DIFF = F.(T2,T1) - 1.986*ELOG.(P2/P1)
           COUNT = COUNT + 1
           PRINT FORMAT CHECK, COUNT, DIFF, T2
           WHENEVER DIFF .GE. 0.0 .AND. COUNT .E. 1,TRANSFER TO DOWN10
           WHENEVER DIFF.L.0. .AND.COUNT.E.1,TRANSFER TO ADD10
           WHENEVER DIFF.GE.0.0.AND.DN10.E.1,TRANSFER TO DOWN10
           WHENEVER DIFF.L.0. .AND.UP1 .E.1,TRANSFER TO ADD10
           WHENEVER DIFF .GE. .0.AND.UP10.E.1,TRANSFER TO LESS1
           WHENEVER DIFF.L.0. .AND.DN1 .E.1,TRANSFER TO LARG1
DOWN10     T2 = T2-10.0
           DN1  = 1
           TRANSFER TO GOBAK
ADD10      T2 = T2+10.0
           UP1  = 1
           TRANSFER TO GOBAK
LESS1      TEMP = DIFF
           THROUGH ALPHA, FOR T2=T2-1, 1,TEMP-DIFF.GE.TEMP
           DIFF = F.(T2,T1) - 1.986*ELOG.(P2/P1)
           COUNT = COUNT+1
           PRINT FORMAT CHECK, COUNT,DIFF,T2
ALPHA      CONTINUE
           TRANSFER TO FINI
LARG1      TEMP = DIFF
           THROUGH BETA,FOR T2=T2+1.,1.,DIFF-TEMP .GE.-TEMP
           DIFF = F.(T2,T1) - 1.986*ELOG.(P2/P1)
           COUNT = COUNT +1
           PRINT FORMAT CHECK, COUNT,DIFF,T2
BETA       CONTINUE
FINI       PRINT FORMAT ANS,T2,TISEN
           TRANSFER TO START
           VECTOR VALUES INPUT =$F10.3,F10.3,F10.0*$
           VECTOR VALUES HEAD=$1H1,74HTEMPERATURE AFTER ISENTROPIC CHANG
          1E OF STATE WITH VARIABLE SPECIFIC HEAT  //19H INITIAL PRESSUR
          2E = F15.5, 5HPSIA./17H FINAL PRESSURE = F15.5,5HPSIA./
          322H INITIAL TEMPERATURE = F5.0, 15HDEGREES RANKINE*$
           VECTOR VALUES CHECK=$8H COUNT - I5, S2, 12HDIFFERENCE = F10.5
          1,S2, 19HFINAL TEMPERATURE = F5.0*$
           VECTOR VALUES ANS = $20H0FINAL TEMPERATURE =F5.0 ,
          115HDEGREES RANKINE/44H0IF SPECIFIC HEAT CONSTANT THE TEMPERAT
          2URE = F5.0,15HDEGREES RANKINE*$
           END OF PROGRAM
$DATA
 14.7      100.      520.
 10.       500.      600.
```

Computer Output

```
TEMPERATURE AFTER ISENTROPIC CHANGE OF STATE WITH VARIABLE SPECIFIC HEAT

INITIAL PRESSURE =        14.70000PSIA.
FINAL PRESSURE =         100.00000PSIA.
INITIAL TEMPERATURE = 520.DEGREES RANKINE
COUNT =      1  DIFFERENCE =   0.13093  FINAL TEMPERATURE = 900.
COUNT =      2  DIFFERENCE =   0.04739  FINAL TEMPERATURE = 890.
COUNT =      3  DIFFERENCE =  -0.03694  FINAL TEMPERATURE = 880.
COUNT =      4  DIFFERENCE =  -0.02847  FINAL TEMPERATURE = 881.
COUNT =      5  DIFFERENCE =  -0.02001  FINAL TEMPERATURE = 882.
COUNT =      6  DIFFERENCE =  -0.01156  FINAL TEMPERATURE = 883.
COUNT =      7  DIFFERENCE =  -0.00311  FINAL TEMPERATURE = 884.
COUNT =      8  DIFFERENCE =   0.00532  FINAL TEMPERATURE = 885.

FINAL TEMPERATURE = 886.DEGREES RANKINE

IF SPECIFIC HEAT CONSTANT THE TEMPERATURE = 900.DEGREES RANKINE

TEMPERATURE AFTER ISENTROPIC CHANGE OF STATE WITH VARIABLE SPECIFIC HEAT

INITIAL PRESSURE =        10.00000PSIA.
FINAL PRESSURE =         500.00000PSIA.
INITIAL TEMPERATURE = 600.DEGREES RANKINE
COUNT =      1  DIFFERENCE =   0.81400  FINAL TEMPERATURE =1837.
COUNT =      2  DIFFERENCE =   0.76852  FINAL TEMPERATURE =1827.
COUNT =      3  DIFFERENCE =   0.72282  FINAL TEMPERATURE =1817.
COUNT =      4  DIFFERENCE =   0.67690  FINAL TEMPERATURE =1807.
COUNT =      5  DIFFERENCE =   0.63077  FINAL TEMPERATURE =1797.
COUNT =      6  DIFFERENCE =   0.58442  FINAL TEMPERATURE =1787.
COUNT =      7  DIFFERENCE =   0.53784  FINAL TEMPERATURE =1777.
COUNT =      8  DIFFERENCE =   0.49104  FINAL TEMPERATURE =1767.
COUNT =      9  DIFFERENCE =   0.44401  FINAL TEMPERATURE =1757.
COUNT =     10  DIFFERENCE =   0.39675  FINAL TEMPERATURE =1747.
COUNT =     11  DIFFERENCE =   0.34926  FINAL TEMPERATURE =1737.
COUNT =     12  DIFFERENCE =   0.30153  FINAL TEMPERATURE =1727.
COUNT =     13  DIFFERENCE =   0.25356  FINAL TEMPERATURE =1717.
COUNT =     14  DIFFERENCE =   0.20536  FINAL TEMPERATURE =1707.
COUNT =     15  DIFFERENCE =   0.15691  FINAL TEMPERATURE =1697.
COUNT =     16  DIFFERENCE =   0.10822  FINAL TEMPERATURE =1687.
COUNT =     17  DIFFERENCE =   0.05928  FINAL TEMPERATURE =1677.
COUNT =     18  DIFFERENCE =   0.01009  FINAL TEMPERATURE =1667.
COUNT =     19  DIFFERENCE =  -0.03936  FINAL TEMPERATURE =1657.
COUNT =     20  DIFFERENCE =  -0.03440  FINAL TEMPERATURE =1658.
COUNT =     21  DIFFERENCE =  -0.02945  FINAL TEMPERATURE =1659.
COUNT =     22  DIFFERENCE =  -0.02450  FINAL TEMPERATURE =1660.
COUNT =     23  DIFFERENCE =  -0.01955  FINAL TEMPERATURE =1661.
COUNT =     24  DIFFERENCE =  -0.01460  FINAL TEMPERATURE =1662.
COUNT =     25  DIFFERENCE =  -0.00966  FINAL TEMPERATURE =1663.
COUNT =     26  DIFFERENCE =  -0.00472  FINAL TEMPERATURE =1664.
COUNT =     27  DIFFERENCE =   0.00022  FINAL TEMPERATURE =1665.

FINAL TEMPERATURE =1666.DEGREES RANKINE

IF SPECIFIC HEAT CONSTANT THE TEMPERATURE =1837.DEGREES RANKINE

* * ALL DATA PROCESSED.
```

Isentropic Process for Ideal Gas With Variable Specific Heat

Critique

The selection of a trial and error technique was deliberate to emphasize to the student the approach that can be used to such a problem. Since this problem is intended for first semester junior students it was felt that more elegant numerical techniques should not be considered.

It is an example of an early computer problem which is easy to solve, but the student should realize that the computer can solve routine, tedious problems as well as the very elegant problems. It seems to this writer that this type of problem, solvable by conventional means but tedious and filled with chances for error, are excellent problems for the junior level engineering student. Moreover it is possible that a more sophisticated problem might discourage a beginning student.

The selection of 10 degree steps for successive approximations was quite arbitrary. In the original instructor's solution some conditional print statements were included and for fairly low pressures (P2) the 10 degree guess was excellent. For higher pressures machine time could be reduced by specifying larger steps based on the magnitude of the term DIFF.

Example Problem No. 66

COMPRESSIBILITY FACTORS USING THE BEATTIE-BRIDGEMAN EQUATION

by

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Course: Thermodynamics II Credit hours: 3 Level: Junior

Statement of Problem

Write a program for the I.B.M. 709 Computer to calculate compressibility factor as a function of reduced temperature and pressure for any pure substance following the Beattie-Bridgeman equation of state. For any T_r, consider values of P_r from 0.2 to an arbitrary $P_{r_{max}}$ in steps of 0.2.

Plot the resulting reduced isotherms, compare with the generalized charts, and discuss the correlation.

Test the program using data for nitrogen with values of T_r = 1.0, 1.1, 1.3, 1.6, and 1.8. Let $P_{r_{max}}$ be 3.0 for all values of T_r.

Solution

The Beattie-Bridgeman Equation of State is

$$P = \frac{RT}{v} + \frac{\beta}{v^2} + \frac{\gamma}{v^3} + \frac{\delta}{v^4}$$

where

$$\beta = B_oRT - A_o - cR/T^2$$

$$\gamma = -B_obRT + A_oa - B_ocR/T^2$$

$$\delta = B_obc\ R/T^2$$

Newton's method of solving iteratively for v is as follows.

$$f(v) = Pv^4 - RTv^3 - \beta v^2 - \gamma v - \delta = 0$$

$$f'(v) = 4Pv^3 - 3RTv^2 - 2\beta v - \gamma$$

$$v_{i+1} = v_i - \frac{f(v_i)}{f'(v_i)} = v_i(1 - E_i)$$

where

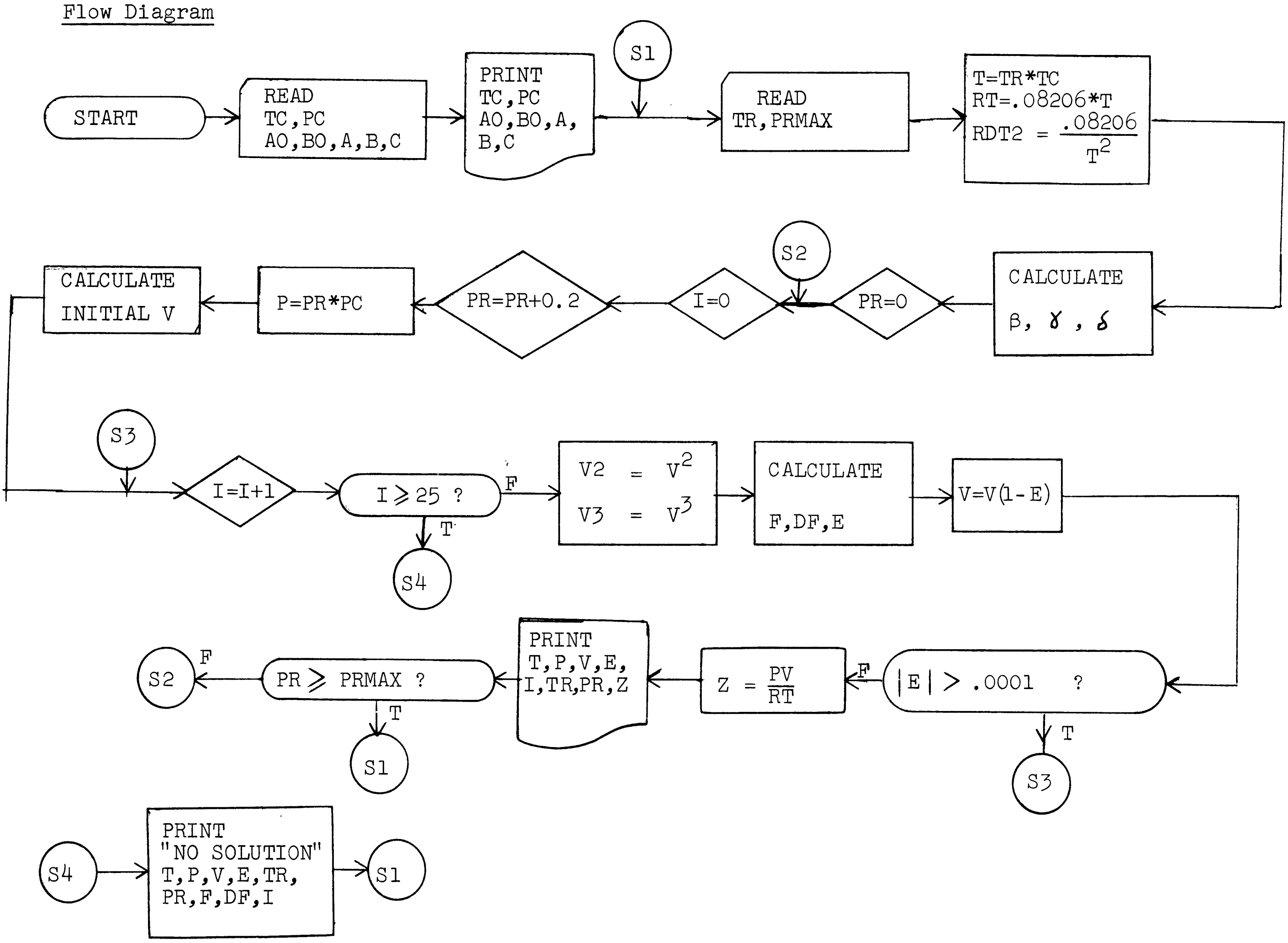
$$E_i = \frac{f(v_i)/f'(v_i)}{v_i}$$

For the first trial,

$$v = \frac{RT}{P} + \frac{\beta}{Pv} + \frac{\gamma}{Pv^2} + \cancelto{\sim 0}{\frac{\delta}{Pv^3}} \approx \frac{RT}{P} + \frac{\beta}{RT} + \frac{\gamma P}{(RT)^2}$$

Compressibility Factors Using the Beattie-Bridgeman Equation

Flow Diagram



MAD Program and Data

```
$COMPILE MAD, EXECUTE, DUMP
           READ DATA
           PRINT RESULTS TC,PC,AO,BO,A,B,C
S1         READ DATA
           T=TR*TC
           RT=.08206*T
           RDT2=.08206/(T*T)
           BETA=BO*RT-AO-C*RDT2
           GAMMA=AO*A-BO*B*RT-BO*C*RDT2
           DELTA=BO*B*C*RDT2
           PR=0.
S2         I=0
           INTEGER I
           PR=PR+0.2
           P=PR*PC
           V=RT/P+BETA/RT+GAMMA*P/(RT*RT)
S3         I=I+1
           WHENEVER I.GE. 25, TRANSFER TO S4
           V2=V*V
           V3=V*V2
           F=P*V*V3-RT*V3-BETA*V2-GAMMA*V-DELTA
           DF=4.*P*V3-3.*RT*V2-2.*BETA*V-GAMMA
           E=F/(V*DF)
           V=V*(1.-E)
           WHENEVER.ABS.E.G.0.0001, TRANSFER TO S3
```

MAD Program and Data (continued)

```
           Z=P*V/RT
           PRINT RESULTS T,P,V,E,I,TR,PR,Z
           WHENEVER PR.GE.PRMAX, TRANSFER TO S1
           TRANSFER TO S2
S4         PRINT COMMENT $0 NO SOLUTION $
           PRINT RESULTS T,P,V,E,TR,PR,F,DF,I
           TRANSFER TO S1
           END OF PROGRAM
```

```
$ DATA

TC=126.1,PC=33.5,AO=1.3445,BO=0.05046,A=0.02617,B=-0.00691,C=0.42E+05*
TR=1.0,PRMAX=3.0*
TR=1.1,PRMAX=3.0*
TR=1.3,PRMAX=3.0*
TR=1.6,PRMAX=3.0*
TR=1.8,PRMAX=3.0*
```

Computer Results for this program are not shown.

Example Problem No. 67

THE EFFECT OF PRESSURE AND PROPELLANT RATIO ON HYDROGEN-OXYGEN ROCKET PERFORMANCE

by

R. E. Sonntag

Course: Seminar in Thermodynamics Credit Hours: 3 Level: Graduate

Statement of the Problem

Consider the following rocket motor. Liquid hydrogen at -423°F and liquid oxygen at -290°F are fed to the combustion chamber in a molar ratio of m:1/2, where m may range from 1.0 to 2.0. The combustion chamber pressure may range from 200 - 300 psia.

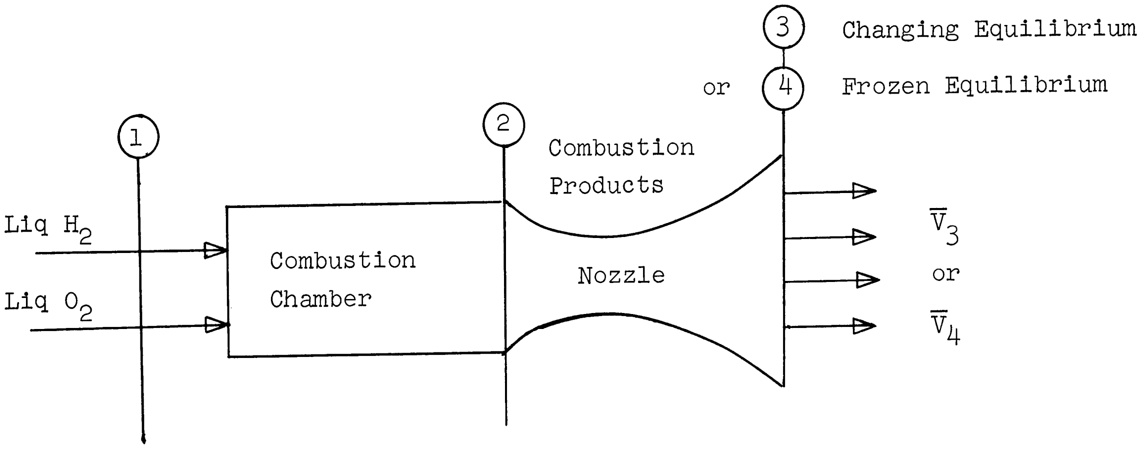
a) Assuming no heat transfer from the chamber, determine the flame temperature and composition, considering the products to consist of H_2O, H_2, O_2, H, O, OH.

b) These products are then expanded through a reversible adiabatic nozzle to 14.7 psia. Consider the following two possible situations.

   1) frozen equilibrium; i.e., there is no change in composition through the nozzle, although specific heat varies with temperature.

   2) changing equilibrium; i.e., at each point in the nozzle there is a condition of equilibrium among the six constituents.

For each of these cases, determine the nozzle exit temperature and velocity, and in 2), the composition at the nozzle exit.



Data

Liquid O_2 at -290°F $\qquad \bar{h}^o_{liq\ O_2} = -5400 \frac{BTU}{Lb\text{-}Mole}$

Liquid H_2 at -423°F $\qquad \bar{h}^o_{liq\ H_2} = -3380 \frac{BTU}{Lb\text{-}Mole}$

Both relative to 0 for gaseous O_2 and H_2 at 537 R.

$C_p = A + BT + CT^2 + DT^3 \ \frac{\text{BTU}}{\text{Mole-R}}$, $T = {}^oR$

	A	$B \cdot 10^3$	$C \cdot 10^6$	$D \cdot 10^9$
H_2O:	6.970	1.925	-.149	---
H_2:	6.424	.576	-.0241	---
O_2:	6.732	.835	-.0554	---
OH:	7.1663	-.3365	.310	-.0358
O:	5.3621	-.360	.1046	-.00942
H:	4.968	---	---	---

	$\bar{h}^o_{537}$ $\frac{\text{BTU}}{\text{Mole}}$	$\bar{g}^o_{537}$ $\frac{\text{BTU}}{\text{Mole}}$	$\bar{s}^o_{537}$ $\frac{\text{BTU}}{\text{Mole-R}}$
H_2O:	-104,071	-98,344	45.106
H_2:	0	0	31.191
O_2:	0	0	49.003
OH:	18,100	16,080	43.888
O:	106,500	98,900	38.469
H:	94,000	87,500	27.393

Solution

Consider the combustion

$$mH_2 + \frac{1}{2} O_2 \longrightarrow H_2O + (m - 1) H_2, \ m \geqslant 1 \tag{1}$$

subject to the dissociation reactions

1. $H_2O \rightleftharpoons H_2 + \frac{1}{2} O_2$ (2)
2. $H_2O \rightleftharpoons \frac{1}{2} H_2 + OH$ (3)
3. $\frac{1}{2} H_2 \rightleftharpoons H$ (4)
4. $\frac{1}{2} O_2 \rightleftharpoons O$ (5)

The four simultaneous equilibrium equations are

$$K_1 = \frac{x_{H_2} \, x_{O_2}^{1/2}}{x_{H_2O}} \, P^{1/2} \tag{6}$$

$$K_2 = \frac{x_{H_2}^{1/2} x_{OH}}{x_{H_2O}} \, P^{1/2} \tag{7}$$

$$K_3 = \frac{x_H}{x_{H_2}^{1/2}} \; P^{1/2} \tag{8}$$

$$K_4 = \frac{x_O}{x_{O_2}^{1/2}} \; P^{1/2} \tag{9}$$

The mass balance ratio of H to O is

$$\frac{\text{GM-ATOMS H}}{\text{GM-ATOMS O}} = \frac{2m}{1} = \frac{2x_{H_2O} + 2x_{H_2} + x_{OH} + x_H}{x_{H_2O} + 2x_{O_2} + x_{OH} + x_O} \tag{10}$$

and the sum of the mole fractions must equal unity, or

$$x_{H_2O} + x_{H_2} + x_{O_2} + x_H + x_O + x_{OH} = 1 \tag{11}$$

Substituting equations (6), (7), (8), (9) into (10) and (11),

$$(A4)\; x_{H_2}\, x_{O_2}^{1/2} + 4m\, x_{O_2} + (A5)\; x_{O_2}^{1/2} + (A6)\; x_{H_2}^{1/2}\, x_{O_2}^{1/2} - 2\, x_{H_2} - (A7)\; x_{H_2}^{1/2} = 0 \tag{12}$$

and

$$(A1)\; x_{H_2}\, x_{O_2}^{1/2} + x_{H_2} + x_{O_2} + (A2)\; x_{H_2}^{1/2}\, x_{O_2}^{1/2} + (A7)\; x_{H_2}^{1/2} + (A3)\; x_{O_2}^{1/2} - 1 = 0 \tag{13}$$

where (A1),...,(A7) are constants for a given temperature, pressure, and m, given by

$$\left.\begin{aligned} A1 &= P^{1/2}/K_1 \\ A2 &= K_2/K_1 \\ A3 &= K_4/P^{1/2} \\ A4 &= 2(m-1)(A1) \\ A5 &= 2m(A3) \\ A6 &= (2m-1)(A2) \\ A7 &= K_3/P^{1/2} \end{aligned}\right\} \tag{14}$$

The equilibrium constants are functions of temperature only, and can be evaluated from the relation

$$\ell n\, K_T = \ell n\, K_{T_o} + \frac{\Delta H^o_{T_o}}{R}\left[\frac{1}{T_o} - \frac{1}{T}\right] + \int_{T_o}^{T}\left[\int_{T_o}^{T} \sum_{\substack{\text{Prod-}\\ \text{React.}}} (\nu C_p)\, dT\right]\frac{dT}{RT^2} \tag{15}$$

The resulting four equations are those listed in the program for K1, K2, K3, and K4 in terms of temperature T.

The procedure for solution for a given pressure P and ratio m is to assume a temperature, evaluate the four equilibrium constants, and then the coefficients A1 through A7 according to (14). The set of equations (12), (13) must then be solved for x_{H_2} and x_{O_2}.

The Newton-Raphson iterative method was used for this solution, in terms of the variables

$$\left.\begin{aligned} Y &= x_{H_2}^{1/2} \\ Z &= x_{O_2}^{1/2} \end{aligned}\right\} \quad (16)$$

Once these values have been found, all the mole fractions can be calculated from equations (6) through (9).

To check the assumed temperature, it will be necessary to make use of the First Law of Thermodynamics at state 2 and the Second Law for either state 3 or 4.

Thus, for Part a) of the problem, assuming negligible velocities,

$$H_2 - H_1 = 0 \quad (17)$$

where

$$H_1 = mh_{liq\ H_2} + \frac{1}{2} h_{liq\ O_2} \quad (18)$$

and

$$H_2 = n_{H_2O}\, h_{H_2O} + n_{H_2}\, h_{H_2} + n_{O_2}\, h_{O_2} + n_{OH}\, h_{OH} + n_O\, h_O + n_H\, h_H \quad (19)$$

The problem introduced here is that it is necessary to know the number of moles of each component, rather than the mole fractions, which have been determined.

Consider that "a" moles of H_2O dissociate according to reaction 1 (Eq. 2), thereby forming a moles of H_2 and a/2 moles of O_2. Similarly, let b moles of H_2O, c/2 moles of H_2, and d/2 moles of O_2 dissociate according to the other three reactions. The composition at equilibrium, then is

$$\left.\begin{aligned} n_{H_2O} &= 1 - a - b \\ n_{H_2} &= m - 1 + a + b/2 - c/2 \\ n_{O_2} &= a/2 - d/2 \\ n_{OH} &= b \\ n_H &= c \\ n_O &= d \end{aligned}\right\} \quad (20)$$

Therefore

$$n_T = \sum_i n_i = m + \frac{1}{2}(a + b + c + d) \quad (21)$$

and

$$x_i = \frac{n_i}{n_T} \quad (22)$$

Since the mole fractions have already been determined at the assumed temperature, a, b, c, and d can be evaluated by reduction of the six equations (22), resulting in

$$a = \frac{X_2 \left[1 - m + X_3\right] + X_4 \left[2m - 1 - X_1\right]}{X_2 + X_3 \left[1 + X_2\right] + \frac{1}{2}\left[1 - X_1\right] - X_4 \left[1 + X_1\right]} \quad (23)$$

$$b = \frac{X_4 - \left[X_4 + \frac{1}{2} \right] a}{X_4 - X_2/2} \tag{24}$$

$$c = bX_1 \tag{25}$$

$$d = bX_2 \tag{26}$$

where

$$\left. \begin{aligned} X_1 &= \frac{x_H}{x_{OH}} \\ X_2 &= \frac{x_O}{x_{OH}} \\ X_3 &= \frac{x_{H_2}}{x_{H_2O}} \\ X_4 &= \frac{x_{O_2}}{x_{H_2O}} \end{aligned} \right\} \tag{27}$$

These values are then substituted into (20) to obtain the number of moles of each substance.

Assuming ideal gases, the partial enthalpy of each component is equal to its enthalpy at one atmosphere pressure, so that for each component

$$h_{i_T} = h_{i_{T_o}} + \int_{T_o}^{T} C_{P_i} \, dT \quad . \tag{28}$$

The resulting six equations, to be found in the program, are given according to the nomenclature HH2O for h_{H_2O}, etc.

If, at the assumed temperature, the First Law (17) is not satisfied, then the temperature must be incremented by some ΔT, the direction depending on the sign of (17). The entire procedure is then repeated for the new temperature. Whenever the sign of (17) changes from one trial to another, the ΔT is cut in half and the temperature incremented in the opposite direction the procedure being repeated until (17) is as close to zero as desired.

For part b), the frozen equilibrium calculation is performed first, because the composition is the same as that just determined at state 2. The half-interval method of assuming a temperature and incrementing is again used, the condition to be satisfied being the Second Law,

$$S_4 - S_2 = 0 \tag{29}$$

where

$$S = n_{H_2O} \, \overline{S}_{H_2O} + n_{H_2} \, \overline{S}_{H_2} + n_{O_2} \, \overline{S}_{O_2} + n_{OH} \, \overline{S}_{OH} + n_H \overline{S}_H + n_O \overline{S}_O \tag{30}$$

and

$$\overline{S}_i = S^o_{i_T} - R \, \ln(x_i P) \tag{31}$$

$$S^o_{i_T} = S^o_{i_{T_o}} + \int_{T_o}^{T} C_{P_i} \frac{dT}{T} \tag{32}$$

S_2 is determined from the temperature, pressure and composition at state 2. Values for T_4 are assumed until Eq. (29) is as close to zero as desired. Upon solution, the velocity can be calculated from the First Law,

$$H_1 = H_4 + (2m + 16) \frac{\overline{V}_4^2}{2g_c} \qquad (33)$$

by using the set of enthalpy equations (28) at the temperature T_4.

In evaluating the changing equilibrium part, the requirement to be met is that

$$S_3 - S_2 = 0 \qquad (34)$$

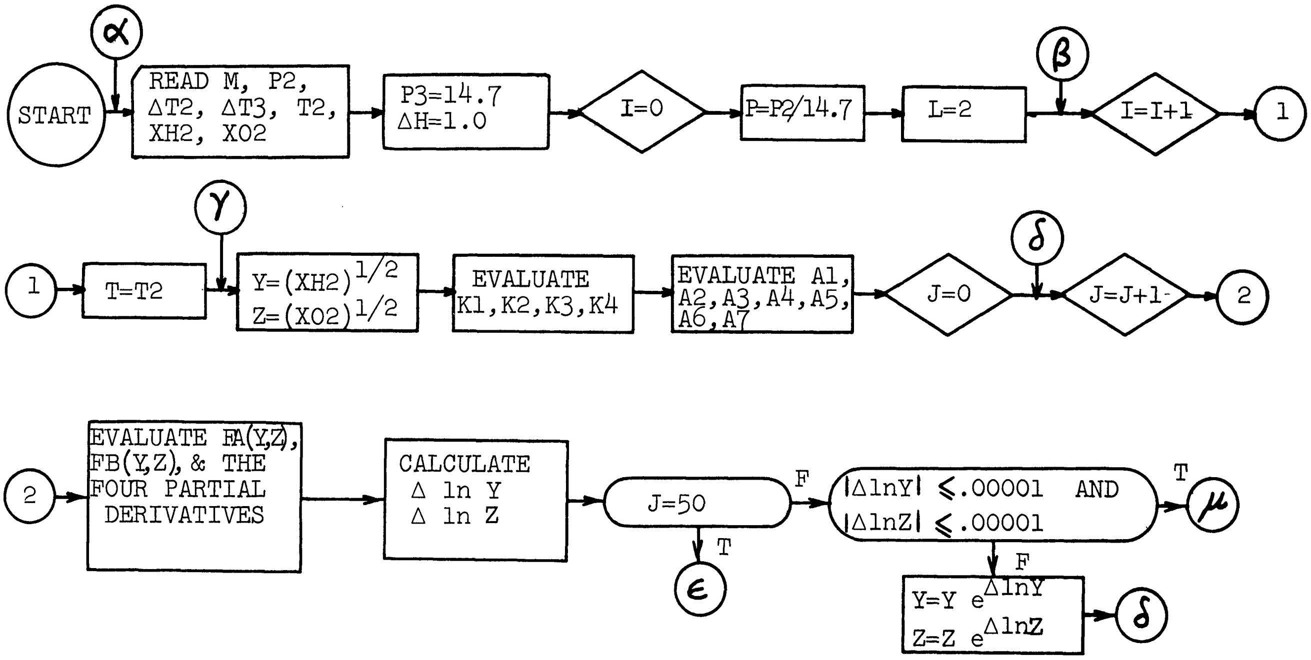
where equations (30), (31), (32) give the entropies. The difference between this part and the preceding is that the composition no longer is the same as at state 2, but instead must satisfy the equilibrium equations (6) through (9). Thus, the procedure is similar to that of the first part of the problem except that Eq. (34) must be satisfied. Upon determination of the temperature T_3 and composition satisfying these equations, the velocity is found from the First Law,

$$H_1 = H_3 + (2m + 16) \frac{\overline{V}_3^2}{2g_c} \qquad (35)$$

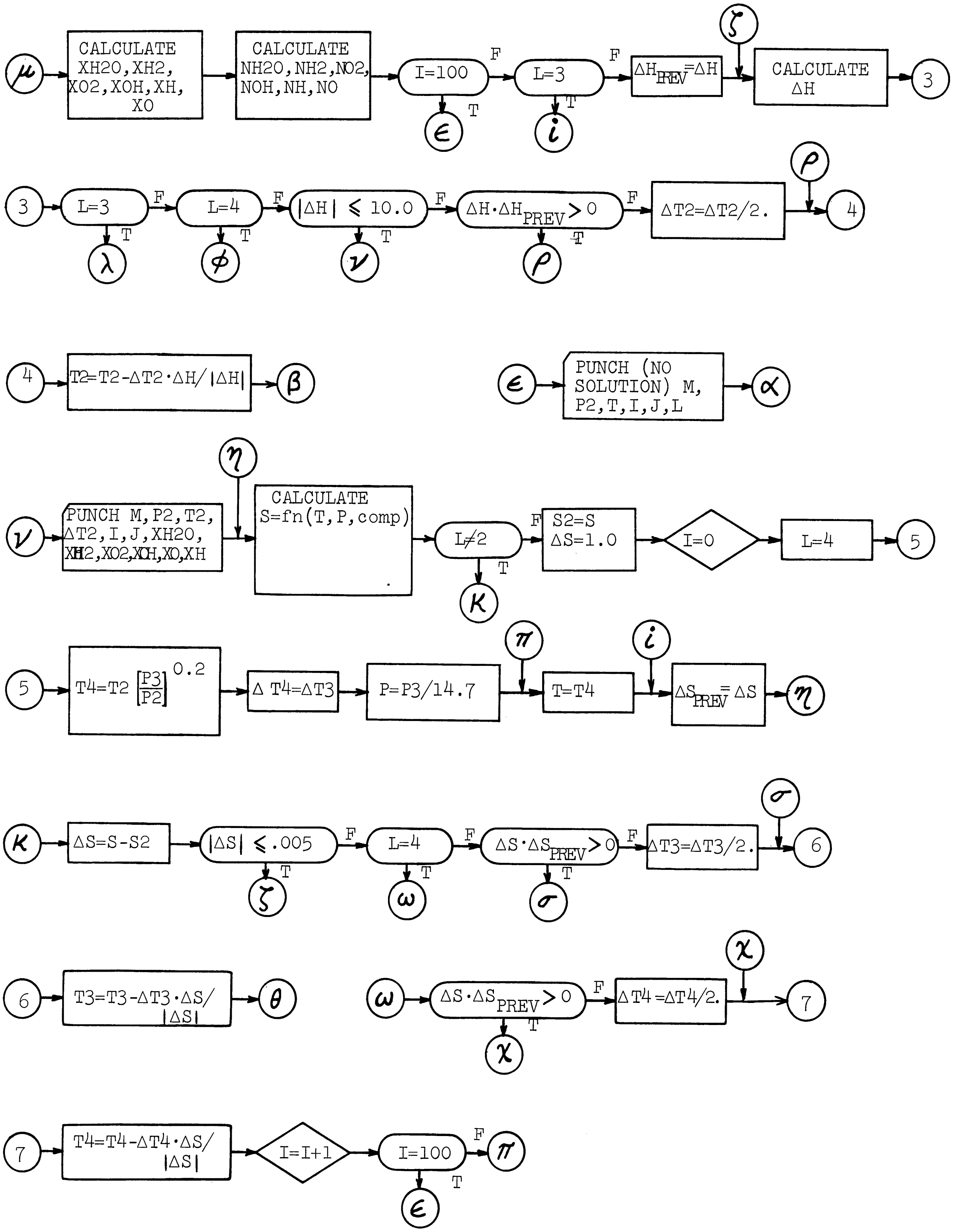
where the enthalpy equations are now used at T_3.

It should be pointed out that the six equations corresponding to (22) could be substituted into the equilibrium equations (6) through (9), resulting in a set of four nonlinear equations in the four unknowns a, b, c, d, which could be solved simultaneously. The problem was not solved by this procedure due to the additional computational time required for such a solution.

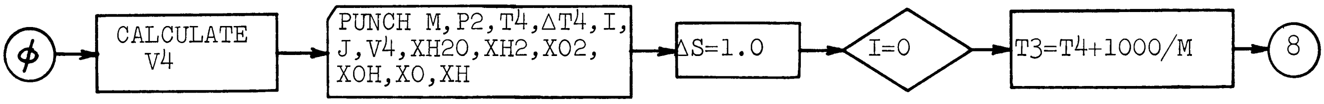
Flow Diagram

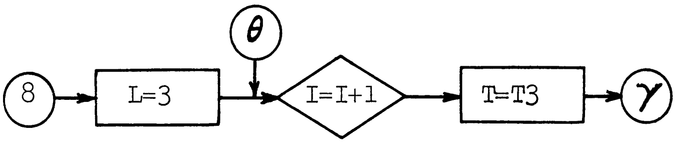


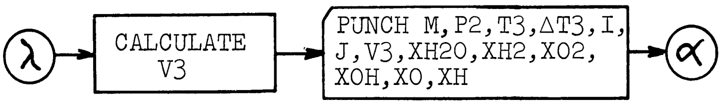
Flow Diagram, Continued



Flow Diagram, Continued







NOTE: FA(Y,Z), FB(Y,Z) APPEARING AFTER DELTA ARE THE SET OF EQUATIONS (12), (13) IN TERMS OF Y, Z, TO BE SOLVED BY THE NEWTON-RAPHSON ITERATIVE METHOD.

MAD Program and Data

```
   R. E. SONNTAG                        X18-N                        003    000    050
 *COMPILE MAD, EXECUTE,PUNCH OBJECT, DUMP


 ALPHA       READ FORMAT DATA,M,P2,DELT2,DELT3,T2,XH2,XO2                                      *001
             VECTOR VALUES DATA=$7F10.5*$                                                      *002
             P3=14.7                                                                           *003
             DELH=1.0                                                                          *004
             I=0                                                                               *005
             P=P2/14.7                                                                         *006
             L=2                                                                               *007
 BETA        I=I+1                                                                             *008
             T=T2                                                                              *009
 GAMMA       Y=SQRT.(XH2)                                                                      *010
             Z=SQRT.(XO2)                                                                      *011
             K1=EXP.(-4.750 + 1.42*ELOG.(T) - 0.000234*T + 0.00816*10..P.-                     *012
            16*T*T - 51702./T)                                                                 *012
             K2=EXP.(-4.694 + 1.716*ELOG.(T) - 0.000497*T + 0.0375*10..P.                      *013
            1-6*T*T - 0.0015*10..P.-9*T*T*T - 60810./T)                                        *013
             K3=EXP.(-0.256 + 0.884*ELOG.(T) - 0.0000725*T + 0.001005*10..                     *014
            1P.-6*T*T - 46896./T)                                                              *014
             K4=EXP.(-.023 + 1.005*ELOG.(T) -0.000196*T + 0.0111*10..P.-6*                     *015
            1T*T - 0.000395*10..P.-9*T*T*T - 53132./T)                                         *015
             A1=SQRT.(P)/K1                                                                    *016
             A2=K2/K1                                                                          *017
             A3=K4/SQRT.(P)                                                                    *018
             A4=2.*(M - 1)*A1                                                                  *019
             A5=2.*M*A3                                                                        *020
             A6=(2.*M - 1)*A2                                                                  *021
             A7=K3/SQRT.(P)                                                                    *022
             J=0                                                                               *023
 DELTA       J=J+1                                                                             *024
             FA=A4*Y*Y*Z    + 4.*M*Z*Z + A5*Z + A6*Y*Z - 2.*Y*Y - A7*Y                         *025
             FB=A1*Y*Y*Z    + Y*Y + Z*Z + A2*Y*Z + A7*Y + A3*Z - 1.                            *026
             DFADY=2.*A4*Y*Y*Z    + A6*Y*Z -4.*Y*Y - A7*Y                                      *027
             DFADZ=   A4*Y*Y*Z    + 8.*M*Z*Z + A5*Z + A6*Y*Z                                   *028
             DFBDY=2.*A1*Y*Y*Z    + 2.*Y*Y + A2*Y*Z + A7*Y                                     *029
             DFBDZ=   A1*Y*Y*Z    + 2.*Z*Z + A2*Y*Z + A3*Z                                     *030
             DENOM=DFADY*DFBDZ - DFADZ*DFBDY                                                   *031
             DELY=(FB*DFADZ - FA*DFBDZ)/DENOM                                                  *032
             DELZ=(FA*DFBDY - FB*DFADY)/DENOM                                                  *033
             WHENEVER J .E. 50, TRANSFER TO EPSIL                                              *034
             WHENEVER .ABS.(DELY) .LE. 0.00001 .AND. .ABS.(DELZ) .LE.                          *035
            1 0.00001, TRANSFER TO MU                                                          *035
             Y=Y*EXP.(DELY)                                                                    *036
             Z=Z*EXP.(DELZ)                                                                    *037
             TRANSFER TO DELTA                                                                 *038
 MU          XH2=Y*Y                                                                           *039
             XO2=Z*Z                                                                           *040
             XH2O=XH2*Z*A1                                                                     *041
             XOH=Y*Z*A2                                                                        *042
             XH=Y*A7                                                                           *043
             XO=Z*A3                                                                           *044
             X1=XH/XOH                                                                         *045
             X2=XO/XOH                                                                         *046
             X3=XH2/XH2O                                                                       *047
             X4=XO2/XH2O                                                                       *048
             A=(X2*(1. - M + X3) + X4*(2.*M - 1. - X1))/(X2 + X3*(1. + X2)                     *049
            1 + 0.5*(1. - X1) - X4*(1. + X1))                                                  *049
             B=(X4 - A*(X4 + 0.5))/(X4 - 0.5*X2)                                               *050
             C=B*X1                                                                            *051
             D=B*X2                                                                            *052
             NH2O=1. - A - B                                                                   *053
             NH2=M - 1. + A + 0.5*B - 0.5*C                                                    *054
             NO2=0.5*(A - D)                                                                   *055
             NOH=B                                                                             *056
             NH=C                                                                              *057
             NO=D                                                                              *058
             WHENEVER I .E. 100, TRANSFER TO EPSIL                                             *059
             WHENEVER L .E. 3, TRANSFER TO IOTA                                                *060
             DELHPR=DELH                                                                       *061
```

MAD Program and Data, Continued

```
 ZETA       TSQ=0.001*T*T                                                   *062
            TCUBE=0.001*T*TSQ                                               *063
            HH20=-108081. + 6.970*T + 0.9625*TSQ - 0.0497*TCUBE             *064
            HH2=-3532. + 6.424*T + 0.288*TSQ - 0.00803*TCUBE                *065
            HO2=-3737. + 6.732*T + 0.4175*TSQ - 0.01847*TCUBE               *066
            HOH=14283. + 7.1663*T- 0.16825*TSQ + 0.1033*TCUBE - 0.0000089   *067
           15*T*TCUBE                                                       *067
            HO=103668. + 5.3621*T - 0.18*TSQ + 0.0349*TCUBE - 0.00000235*   *068
           1T*TCUBE                                                         *068
            HH=91330. + 4.968*T                                             *069
            H=NH20*HH20 + NH2*HH2 + NO2*HO2 + NOH*HOH + NO*HO + NH*HH       *070
            DELH=H + 3380.*M + 2700.                                        *071
            WHENEVER L .E. 3, TRANSFER TO LAMBDA                            *072
            WHENEVER L .E. 4, TRANSFER TO PHI                               *073
            WHENEVER .ABS. DELH .LE. 10., TRANSFER TO NU                    *074
            WHENEVER (DELH*DELHPR) .G. 0, TRANSFER TO RHO                   *075
            DELT2=0.5*DELT2                                                 *076
 RHO        T2=T2 - DELT2*DELH/.ABS.DELH                                    *077
            TRANSFER TO BETA                                                *078
 EPSIL      PUNCH FORMAT NONE,M,P2,T,I,J,L                                  *079
            VECTOR VALUES NONE = $4H M =F5.2, 7H   P2 =F6.1, 6H   T =F7.    *080
           11, 6H   I =I4,6H   J =I4,6H   L =I4*$                           *080
            TRANSFER TO ALPHA                                               *081
 NU         PUNCH FORMAT FIRST,M,P2,T2,DELT2,I,J,XH20,XH2,XO2,XOH,XO,XH     *082
            VECTOR VALUES FIRST =$4H M =F5.2,7H   P2 =F6.1,7H   T2 =F7.     *083
           11, 10H   DELT2 =F6.2,6H   I =I3, 6H   J =I3/11H     XH20 =      *083
           2F10.7,9H    XH2 =F10.7, 9H    XO2 =F10.7/11H     XOH =F10.7     *083
           3,9H    XO =F10.7,9H    XH =F10.7*$                              *083
  ETA       LNT = ELOG.(T)                                                  *084
            TSQ = T*T*10.0.P.-6                                             *085
            SH20 = 0.2935 + 6.97*LNT + 0.001925*T -0.0745*TSQ               *086
            SH2 = -9.415 + 6.424*LNT +0.000576*T -0.01205*TSQ               *087
            SO2 = 6.312 + 6.732*LNT + 0.000835*T -0.0277*TSQ                *088
            SOH =-0.974+7.1663*LNT-0.0003365*T+0.155*TSQ-0.00001193*T       *089
           1    *TSQ                                                        *089
            SO =4.935+5.3621*LNT-0.00036*T+0.0523*TSQ-0.00000314*T*TSQ      *090
            SH = -3.8 + 4.968*LNT                                           *091
            S=NH20*SH20+NH2*SH2+NO2*SO2+NOH*SOH+NO*SO+NH*SH-1.986*(NH20*    *092
           1 ELOG.(XH20)+NH2*ELOG.(XH2)+NO2*ELOG.(XO2)+NOH*ELOG.(XOH)+NO*   *092
           1 ELOG.(XO)+NH*ELOG.(XH)+ELOG.(P.P.(NH20+NH2+NO2+NOH+NO+NH)))    *092
            WHENEVER L .NE.2, TRANSFER TO KAPPA                             *093
            S2 = S                                                          *094
            DELS = 1.0                                                      *095
            I = 0                                                           *096
            L=4                                                             *097
            T4=T2*(P3/P2).P.0.2                                             *098
            DELT4=DELT3                                                     *099
            P = P3/14.7                                                     *100
 PI         T=T4                                                            *101
  IOTA      DELSPR = DELS                                                   *102
            TRANSFER TO ETA                                                 *103
  KAPPA     DELS = S - S2                                                   *104
            WHENEVER .ABS. DELS .LE. 0.005, TRANSFER TO ZETA                *105
            WHENEVER L .E. 4, TRANSFER TO OMEGA                             *106
            WHENEVER (DELS*DELSPR).G. 0, TRANSFER TO SIGMA                  *107
            DELT3=0.5*DELT3                                                 *108
  SIGMA     T3 = T3 - DELT3*DELS/.ABS.DELS                                  *109
            TRANSFER TO THETA                                               *110
 OMEGA      WHENEVER (DELS*DELSPR) .G. 0, TRANSFER TO CHI                   *111
            DELT4=0.5*DELT4                                                 *112
 CHI        T4=T4 - DELT4*DELS/.ABS. DELS                                   *113
            I=I+1                                                           *114
            WHENEVER I .E. 100, TRANSFER TO EPSIL                           *115
            TRANSFER TO PI                                                  *116
 PHI        V4=SQRT.(-25031.*DELH/(M+8.0))                                  *117
            PUNCH FORMAT CONST ,M,P2,T4,DELT4,I,J,V4,XH20,XH2,XO2,XOH,      *118
           1 XO,XH                                                          *118
            VECTOR VALUES CONST  =$2HM=F5.2,5H  P2=F6.1,5H  T4=F7.1,        *119
           18H  DELT4=F6.2,4H  I=I3,4H  J=I3,5H  V4=F8.1/11H     XH20 =     *119
           2F10.7,9H    XH2 =F10.7,9H    XO2 =F10.7/11H     XOH =F10.7,     *119
           39H    XO =F10.7,9H    XH =F10.7*$                               *119
            DELS=1.0                                                        *120
            I=0                                                             *121
            T3 = T4 + 1000./M                                               *122
            L = 3                                                           *123
 THETA      I = I + 1                                                       *124
            T = T3                                                          *125
            TRANSFER TO GAMMA                                               *126
 LAMBDA     V3=SQRT.(-25031.*DELH/(M+8.0))                                  *127
            PUNCH FORMAT RESULT,M,P2,T3,DELT3,I,J,V3,XH20,XH2,XO2,XOH,      *128
           1 XO,XH                                                          *128
            VECTOR VALUES RESULT =$2HM=F5.2,5H  P2=F6.1,5H  T3=F7.1,        *129
           18H  DELT3=F6.2,4H  I=I3,4H  J=I3,5H  V3=F8.1/11H     XH20 =     *129
           2F10.7,9H    XH2 =F10.7,9H    XO2 =F10.7/11H     XOH =F10.7,     *129
           39H    XO =F10.7,9H    XH =F10.7*$                               *129
            TRANSFER TO ALPHA                                               *130
            INTEGER I,J,L                                                   *131
            END OF PROGRAM                                                  *132
```

Computer Output

```
  M = 1.00   P2 = 200.0   T2 = 6086.5   DELT2 =  0.50   I = 19   J =  2
      XH2O = 0.6703460    XH2 = 0.1243038    XO2 = 0.0397145
       XOH = 0.0936007     XO = 0.0227281     XH = 0.0493072
 M= 1.00  P2= 200.0  T4= 3827.0  DELT4=  8.00  I=  9  J=  2  V4=  9011.5
      XH2O = 0.6703460    XH2 = 0.1243038    XO2 = 0.0397145
       XOH = 0.0936007     XO = 0.0227281     XH = 0.0493072
 M= 1.00  P2= 200.0  T3= 4954.0  DELT3=  1.00  I= 11  J=  2  V3=  9418.6
      XH2O = 0.8103833    XH2 = 0.0808551    XO2 = 0.0296413
       XOH = 0.0475383     XO = 0.0090634     XH = 0.0225200
  M = 1.20   P2 = 200.0   T2 = 6042.2   DELT2 =  0.25   I = 22   J =  2
      XH2O = 0.6532267    XH2 = 0.1955084    XO2 = 0.0133436
       XOH = 0.0673720     XO = 0.0123252     XH = 0.0582241
 M= 1.20  P2= 200.0  T4= 3780.7  DELT4=  4.00  I= 10  J=  2  V4=  9479.1
      XH2O = 0.6532267    XH2 = 0.1955084    XO2 = 0.0133436
       XOH = 0.0673720     XO = 0.0123252     XH = 0.0582241
 M= 1.20  P2= 200.0  T3= 4830.1  DELT3=  8.00  I= 10  J=  3  V3=  9891.9
      XH2O = 0.7765779    XH2 = C.1697389    XO2 = 0.0034956
       XOH = 0.0226282     XO = 0.0023376     XH = 0.0252219
  M = 1.40   P2 = 200.0   T2 = 5899.0   DELT2 =  1.00   I = 16   J =  2
      XH2O = 0.6179716    XH2 = 0.2747403    XO2 = 0.0038769
       XOH = 0.0416415     XO = 0.0053182     XH = 0.0564521
 M= 1.40  P2= 200.0  T4= 3663.7  DELT4=  4.00  I=  8  J=  2  V4=  9831.4
      XH2O = 0.6179716    XH2 = 0.2747403    XO2 = 0.0038769
       XOH = 0.0416415     XO = 0.0053182     XH = 0.0564521
 M= 1.40  P2= 200.0  T3= 4486.0  DELT3=  4.00  I=  7  J=  3  V3= 10210.0
      XH2O = 0.7002676    XH2 = 0.2788168    XO2 = 0.0001843
       XOH = 0.0058107     XO = 0.0002233     XH = 0.0146972
  M = 1.60   P2 = 200.0   T2 = 5694.0   DELT2 =  2.00   I = 12   J =  2
      XH2C = 0.5739799    XH2 = 0.3527253    XO2 = 0.0010338
       XOH = 0.0231559     XO = 0.0019589     XH = 0.0471491
 M= 1.60  P2= 200.0  T4= 3502.1  DELT4=  4.00  I=  7  J=  2  V4= 10088.3
      XH2O = 0.5739799    XH2 = 0.3527253    XO2 = 0.0010338
       XOH = 0.0231559     XO = 0.0019589     XH = 0.0471491
 M= 1.60  P2= 200.0  T3= 4083.1  DELT3=  4.00  I= 11  J=  3  V3= 10408.3
      XH2O = 0.6217212    XH2 = 0.3714413    XO2 = 0.0000074
       XOH = 0.0011077     XO = 0.0000132     XH = 0.0057092
  M = 1.80   P2 = 200.0   T2 = 5457.7   DELT2 =  0.25   I = 18   J =  2
      XH2O = 0.5287101    XH2 = 0.4230364    XO2 = 0.0002635
       XOH = 0.0120117     XO = 0.0006494     XH = 0.0353290
 M= 1.80  P2= 200.0  T4= 3320.0  DELT4=  2.00  I= 10  J=  2  V4= 10273.8
      XH2O = 0.5287101    XH2 = 0.4230364    XO2 = 0.0002635
       XOH = 0.0120117     XO = 0.0006494     XH = 0.0353290
 M= 1.80  P2= 200.0  T3= 3705.5  DELT3=  2.00  I= 14  J=  3  V3= 10524.3
      XH2O = 0.5548105    XH2 = 0.4431828    XO2 = 0.0000003
       XOH = 0.0001862     XO = 0.0000006     XH = 0.0018196
  M = 2.00   P2 = 200.0   T2 = 5211.0   DELT2 =  1.00   I = 15   J =  2
      XH2O = 0.4859722    XH2 = 0.4833094    XO2 = 0.0000656
       XOH = 0.0059508     XO = 0.0002004     XH = 0.0245026
 M= 2.00  P2= 200.0  T4= 3133.6  DELT4=  2.00  I=  6  J=  2  V4= 10398.9
      XH2O = 0.4859722    XH2 = 0.4833094    XO2 = 0.0000656
       XOH = 0.0059508     XO = 0.0002004     XH = 0.0245026
 M= 2.00  P2= 200.0  T3= 3376.6  DELT3=  1.00  I= 14  J=  3  V3= 10587.4
      XH2O = 0.4998301    XH2 = 0.4996101    XO2 = 0.0000000
       XOH = 0.0000299     XO = 0.0000000     XH = 0.0005299
  M = 1.00   P2 = 225.0   T2 = 6116.2   DELT2 =  0.25   I = 21   J =  2
      XH2O = 0.6730543    XH2 = C.1235838    XO2 = 0.0393344
       XOH = 0.0934934     XO = 0.0222903     XH = 0.0482439
 M= 1.00  P2= 225.0  T4= 3764.2  DELT4=  4.00  I= 10  J=  2  V4=  9190.4
      XH2O = 0.6730543    XH2 = 0.1235838    XO2 = 0.0393344
       XOH = 0.0934934     XO = 0.0222903     XH = 0.0482439
 M= 1.00  P2= 225.0  T3= 4925.2  DELT3=  1.00  I= 12  J=  2  V3=  9610.9
      XH2O = 0.8188794    XH2 = 0.0778967    XO2 = 0.0286360
       XOH = 0.0454017     XO = 0.0083451     XH = 0.0208426
  M = 1.20   P2 = 225.0   T2 = 6071.0   DELT2 =  1.00   I = 17   J =  2
      XH2O = 0.6555651    XH2 = 0.1952693    XO2 = 0.0130606
       XOH = 0.0670443     XO = 0.0120062     XH = 0.0570548
 M= 1.20  P2= 225.0  T4= 3719.9  DELT4=  2.00  I= 10  J=  2  V4=  9658.2
      XH2O = 0.6555651    XH2 = 0.1952693    XO2 = 0.0130606
       XOH = 0.0670443     XO = 0.0120062     XH = 0.0570548
 M= 1.20  P2= 225.0  T3= 4789.3  DELT3=  4.00  I=  9  J=  3  V3= 10092.2
      XH2O = 0.7828021    XH2 = 0.1687971    XO2 = 0.0029589
       XOH = 0.0204497     XO = 0.0019510     XH = 0.0230412
  M = 1.40   P2 = 225.0   T2 = 5923.7   DELT2 =  0.25   I = 19   J =  2
      XH2O = 0.6197767    XH2 = 0.2750221    XO2 = 0.0037410
       XOH = 0.0411674     XO = 0.0051223     XH = 0.0551705
```

C= 70%

Computer Output, Continued

```
 M= 1.40  P2= 225.0  T4= 3600.6  DELT4=  8.00  I=  6  J=  2  V4= 10016.2
      XH2O = 0.6197767     XH2 = 0.2750221     XO2 = 0.0037410
       XOH = 0.0411674      XO = 0.0051223      XH = 0.0551705
 M= 1.40  P2= 225.0  T3= 4422.9  DELT3=  4.00  I=  7  J=  3  V3= 10408.4
      XH2O = 0.7027159     XH2 = 0.2796673     XO2 = 0.0001302
       XOH = 0.0047577      XO = 0.0001574      XH = 0.0125715
  M = 1.60    P2 = 225.0    T2 = 5713.5    DELT2 =  0.50    I = 18    J =  2
      XH2O = 0.5752619     XH2 = 0.3533399     XO2 = 0.0009828
       XOH = 0.0227106      XO = 0.0018614      XH = 0.0458436
 M= 1.60  P2= 225.0  T4= 3436.8  DELT4=  2.00  I=  9  J=  2  V4= 10276.2
      XH2O = 0.5752619     XH2 = 0.3533399     XO2 = 0.0009828
       XOH = 0.0227106      XO = 0.0018614      XH = 0.0458436
 M= 1.60  P2= 225.0  T3= 4007.8  DELT3=  2.00  I= 10  J=  3  V3= 10600.5
      XH2O = 0.6224668     XH2 = 0.3721416     XO2 = 0.0000044
       XOH = 0.0008276      XO = 0.0000080      XH = 0.0045516
  M = 1.80    P2 = 225.0    T2 = 5473.0    DELT2 =  1.00    I = 13    J =  2
      XH2O = 0.5295160     XH2 = 0.4237242     XO2 = 0.0002478
       XOH = 0.0117066      XO = 0.0006106      XH = 0.0341956
 M= 1.80  P2= 225.0  T4= 3253.4  DELT4=  2.00  I= 10  J=  2  V4= 10459.6
      XH2O = 0.5295160     XH2 = 0.4237242     XO2 = 0.0002478
       XOH = 0.0117066      XO = 0.0006106      XH = 0.0341956
 M= 1.80  P2= 225.0  T3= 3625.0  DELT3=  8.00  I=  9  J=  3  V3= 10715.9
      XH2O = 0.5550157     XH2 = 0.4435018     XO2 = 0.0000001
       XOH = 0.0001274      XO = 0.0000003      XH = 0.0013547
  M = 2.00    P2 = 225.0    T2 = 5222.0    DELT2 =  2.00    I = 11    J =  2
      XH2O = 0.4864771     XH2 = 0.4839343     XO2 = 0.0000609
       XOH = 0.0057587      XO = 0.0001862      XH = 0.0235864
 M= 2.00  P2= 225.0  T4= 3066.0  DELT4=  8.00  I=  4  J=  2  V4= 10584.6
      XH2O = 0.4864771     XH2 = 0.4839343     XO2 = 0.0000609
       XOH = 0.0057587      XO = 0.0001862      XH = 0.0235864
 M= 2.00  P2= 225.0  T3= 3298.0  DELT3=  4.00  I= 12  J=  3  V3= 10773.7
      XH2O = 0.4998824     XH2 = 0.4997238     XO2 = 0.0000000
       XOH = 0.0000192      XO = 0.0000000      XH = 0.0003747
  M = 1.00    P2 = 250.0    T2 = 6143.0    DELT2 =  1.00    I = 20    J =  2
      XH2O = 0.6754867     XH2 = 0.1229302     XO2 = 0.0389939
       XOH = 0.0933852      XO = 0.0219012      XH = 0.0473030
 M= 1.00  P2= 250.0  T4= 3711.4  DELT4=  2.00  I= 10  J=  2  V4=  9340.5
      XH2O = 0.6754867     XH2 = 0.1229302     XO2 = 0.0389939
       XOH = 0.0933852      XO = 0.0219012      XH = 0.0473030
 M= 1.00  P2= 250.0  T3= 4898.4  DELT3=  1.00  I= 11  J=  2  V3=  9780.2
      XH2O = 0.8265142     XH2 = 0.0751978     XO2 = 0.0277152
       XOH = 0.0434714      XO = 0.0077222      XH = 0.0193809
  M = 1.20    P2 = 250.0    T2 = 6096.5    DELT2 =  0.50    I = 23    J =  2
      XH2O = 0.6577448     XH2 = 0.1950338     XO2 = 0.0128015
       XOH = 0.0667130      XO = 0.0117151      XH = 0.0559919
 M= 1.20  P2= 250.0  T4= 3665.0  DELT4=  2.00  I= 11  J=  2  V4=  9817.7
      XH2O = 0.6577448     XH2 = 0.1950338     XO2 = 0.0128015
       XOH = 0.0667130      XO = 0.0117151      XH = 0.0559919
 M= 1.20  P2= 250.0  T3= 4751.4  DELT3=  1.00  I= 14  J=  2  V3= 10264.7
      XH2O = 0.7880678     XH2 = 0.1680439     XO2 = 0.0025201
       XOH = 0.0185633      XO = 0.0016422      XH = 0.0211647
  M = 1.40    P2 = 250.0    T2 = 5946.0    DELT2 =  2.00    I = 15    J =  2
      XH2O = 0.6213724     XH2 = 0.2752699     XO2 = 0.0036226
       XOH = 0.0407417      XO = 0.0049522      XH = 0.0540435
 M= 1.40  P2= 250.0  T4= 3545.7  DELT4=  4.00  I=  7  J=  2  V4= 10174.4
      XH2O = 0.6213724     XH2 = 0.2752699     XO2 = 0.0036226
       XOH = 0.0407417      XO = 0.0049522      XH = 0.0540435
 M= 1.40  P2= 250.0  T3= 4363.9  DELT3=  8.00  I=  6  J=  3  V3= 10581.0
      XH2O = 0.7046486     XH2 = 0.2804004     XO2 = 0.0000931
       XOH = 0.0039253      XO = 0.0001125      XH = 0.0108201
  M = 1.60    P2 = 250.0    T2 = 5731.0    DELT2 =  1.00    I = 13    J =  2
      XH2O = 0.5763867     XH2 = 0.3538786     XO2 = 0.0009391
       XOH = 0.0223164      XO = 0.0017780      XH = 0.0447019
 M= 1.60  P2= 250.0  T4= 3379.7  DELT4= 64.00  I=  2  J=  2  V4= 10437.7
      XH2O = 0.5763867     XH2 = 0.3538786     XO2 = 0.0009391
       XOH = 0.0223164      XO = 0.0017780      XH = 0.0447019
 M= 1.60  P2= 250.0  T3= 3936.7  DELT3=  4.00  I=  8  J=  3  V3= 10773.0
      XH2O = 0.6230291     XH2 = 0.3726952     XO2 = 0.0000027
       XOH = 0.0006221      XO = 0.0000049      XH = 0.0036460
  M = 1.80    P2 = 250.0    T2 = 5486.5    DELT2 =  0.50    I = 16    J =  2
      XH2O = 0.5302249     XH2 = 0.4243281     XO2 = 0.0002343
       XOH = 0.0114350      XO = 0.0005775      XH = 0.0332003
 M= 1.80  P2= 250.0  T4= 3194.9  DELT4=  2.00  I= 10  J=  2  V4= 10620.9
      XH2O = 0.5302249     XH2 = 0.4243281     XO2 = 0.0002343
       XOH = 0.0114350      XO = 0.0005775      XH = 0.0332003
 M= 1.80  P2= 250.0  T3= 3554.5  DELT3=  4.00  I=  9  J=  3  V3= 10877.8
      XH2O = 0.5551524     XH2 = 0.4437221     XO2 = 0.0000001
       XOH = 0.0000901      XO = 0.0000002      XH = 0.0010352
```

Computer Output, Continued

```
  M = 2.00   P2 = 250.0   T2 = 5232.0   DELT2 =  4.00   I = 12   J =  3
      XH2O = 0.4869036    XH2 = 0.4844709    XO2 = 0.0000570
       XOH = 0.0055936     XO = 0.0001745     XH = 0.0228005
 M= 2.00  P2= 250.0  T4= 3006.5  DELT4=  2.00  I=  8  J=  3  V4= 10745.5
      XH2O = 0.4869036    XH2 = 0.4844709    XO2 = 0.0000570
       XOH = 0.0055936     XO = 0.0001745     XH = 0.0228005
 M= 2.00  P2= 250.0  T3= 3228.5  DELT3=  2.00  I= 11  J=  3  V3= 10934.0
      XH2O = 0.4999161    XH2 = 0.4997991    XO2 = 0.0000000
       XOH = 0.0000127     XO = 0.0000000     XH = 0.0002721
  M = 1.00   P2 = 275.0   T2 = 6167.5   DELT2 =  0.50   I = 22   J =  2
      XH2O = 0.6776363    XH2 = 0.1223475    XO2 = 0.0386900
       XOH = 0.0932926     XO = 0.0215587     XH = 0.0464750
 M= 1.00  P2= 275.0  T4= 3665.3  DELT4=  8.00  I=  7  J=  2  V4=  9470.0
      XH2O = 0.6776363    XH2 = 0.1223475    XO2 = 0.0386900
       XOH = 0.0932926     XO = 0.0215587     XH = 0.0464750
 M= 1.00  P2= 275.0  T3= 4873.3  DELT3= 16.00  I=  8  J=  3  V3=  9932.5
      XH2O = 0.8334940    XH2 = 0.0726958    XO2 = 0.0268582
       XOH = 0.0416983     XO = 0.0071714     XH = 0.0180824
  M = 1.20   P2 = 275.0   T2 = 6119.5   DELT2 =  0.50   I = 19   J =  2
      XH2O = 0.6597564    XH2 = 0.1948089    XO2 = 0.0125652
       XOH = 0.0663916     XO = 0.0114506     XH = 0.0550273
 M= 1.20  P2= 275.0  T4= 3616.5  DELT4=  2.00  I= 12  J=  2  V4=  9957.9
      XH2O = 0.6597564    XH2 = 0.1948089    XO2 = 0.0125652
       XOH = 0.0663916     XO = 0.0114506     XH = 0.0550273
 M= 1.20  P2= 275.0  T3= 4713.9  DELT3=  8.00  I= 10  J=  3  V3= 10423.7
      XH2O = 0.7928145    XH2 = 0.1674065    XO2 = 0.0021382
       XOH = 0.0168233     XO = 0.0013790     XH = 0.0194385
  M = 1.40   P2 = 275.0   T2 = 5965.5   DELT2 =  0.50   I = 21   J =  2
      XH2O = 0.6229082    XH2 = 0.2754911    XO2 = 0.0035114
       XOH = 0.0403108     XO = 0.0047923     XH = 0.0529862
 M= 1.40  P2= 275.0  T4= 3494.8  DELT4=  2.00  I=  9  J=  2  V4= 10319.4
      XH2O = 0.6229082    XH2 = 0.2754911    XO2 = 0.0035114
       XOH = 0.0403108     XO = 0.0047923     XH = 0.0529862
 M= 1.40  P2= 275.0  T3= 4309.1  DELT3=  4.00  I=  8  J=  3  V3= 10732.2
      XH2O = 0.7061817    XH2 = 0.2810280    XO2 = 0.0000677
       XOH = 0.0032658     XO = 0.0000816     XH = 0.0093752
  M = 1.60   P2 = 275.0   T2 = 5747.0   DELT2 =  1.00   I = 14   J =  2
      XH2O = 0.5773730    XH2 = 0.3543539    XO2 = 0.0009014
       XOH = 0.0219682     XO = 0.0017062     XH = 0.0436980
 M= 1.60  P2= 275.0  T4= 3327.2  DELT4= 64.00  I=  2  J=  2  V4= 10581.6
      XH2O = 0.5773730    XH2 = 0.3543539    XO2 = 0.0009014
       XOH = 0.0219682     XO = 0.0017062     XH = 0.0436980
 M= 1.60  P2= 275.0  T3= 3874.2  DELT3=  2.00  I= 10  J=  3  V3= 10918.4
      XH2O = 0.6234293    XH2 = 0.3731059    XO2 = 0.0000017
       XOH = 0.0004799     XO = 0.0000031     XH = 0.0029800
  M = 1.80   P2 = 275.0   T2 = 5498.5   DELT2 =  0.50   I = 16   J =  2
      XH2C = 0.5308623    XH2 = 0.4248680    XO2 = 0.0002225
       XOH = 0.0111884     XO = 0.0005486     XH = 0.0323104
 M= 1.80  P2= 275.0  T4= 3142.8  DELT4=  2.00  I= 10  J=  2  V4= 10761.6
      XH2O = 0.5308623    XH2 = 0.4248680    XO2 = 0.0002225
       XOH = 0.0111884     XO = 0.0005486     XH = 0.0323104
 M= 1.80  P2= 275.0  T3= 3490.4  DELT3= 16.00  I=  6  J=  3  V3= 11020.9
      XH2O = 0.5552511    XH2 = 0.4438827    XO2 = 0.0000000
       XOH = 0.0000650     XO = 0.0000001     XH = 0.0008030
  M = 2.00   P2 = 275.0   T2 = 5240.5   DELT2 =  0.50   I = 16   J =  2
      XH2O = 0.4872964    XH2 = 0.4849561    XO2 = 0.0000536
       XOH = 0.0054411     XO = 0.0001641     XH = 0.0220888
 M= 2.00  P2= 275.0  T4= 2953.2  DELT4=  4.00  I=  6  J=  2  V4= 10886.6
      XH2O = 0.4872964    XH2 = 0.4849561    XO2 = 0.0000536
       XOH = 0.0054411     XO = 0.0001641     XH = 0.0220888
 M= 2.00  P2= 275.0  T3= 3167.2  DELT3=  2.00  I= 12  J=  3  V3= 11072.5
      XH2O = 0.4999384    XH2 = 0.4998501    XO2 = 0.0000000
       XOH = 0.0000087     XO = 0.0000000     XH = 0.0002028
  M = 1.00   P2 = 300.0   T2 = 6189.5   DELT2 =  0.50   I = 23   J =  2
      XH2O = 0.6797450    XH2 = 0.1217697    XO2 = 0.0384016
       XOH = 0.0931626     XO = 0.0212306     XH = 0.0456907
 M= 1.00  P2= 300.0  T4= 3622.1  DELT4=  4.00  I=  8  J=  2  V4=  9590.9
      XH2O = 0.6797450    XH2 = 0.1217697    XO2 = 0.0384016
       XOH = 0.0931626     XO = 0.0212306     XH = 0.0456907
 M= 1.00  P2= 300.0  T3= 4850.1  DELT3=  4.00  I= 10  J=  3  V3= 10067.0
      XH2O = 0.8397476    XH2 = 0.0704265    XO2 = 0.0260783
       XOH = 0.0401037     XO = 0.0066934     XH = 0.0169506
  M = 1.20   P2 = 300.0   T2 = 6140.7   DELT2 =  0.25   I = 23   J =  2
      XH2O = 0.6615463    XH2 = 0.1946086    XO2 = 0.0123545
       XOH = 0.0661047     XO = 0.0112165     XH = 0.0541693
 M= 1.20  P2= 300.0  T4= 3573.4  DELT4=  2.00  I= 13  J=  2  V4= 10080.7
      XH2O = 0.6615463    XH2 = 0.1946086    XO2 = 0.0123545
       XOH = 0.0661047     XO = 0.0112165     XH = 0.0541693
```

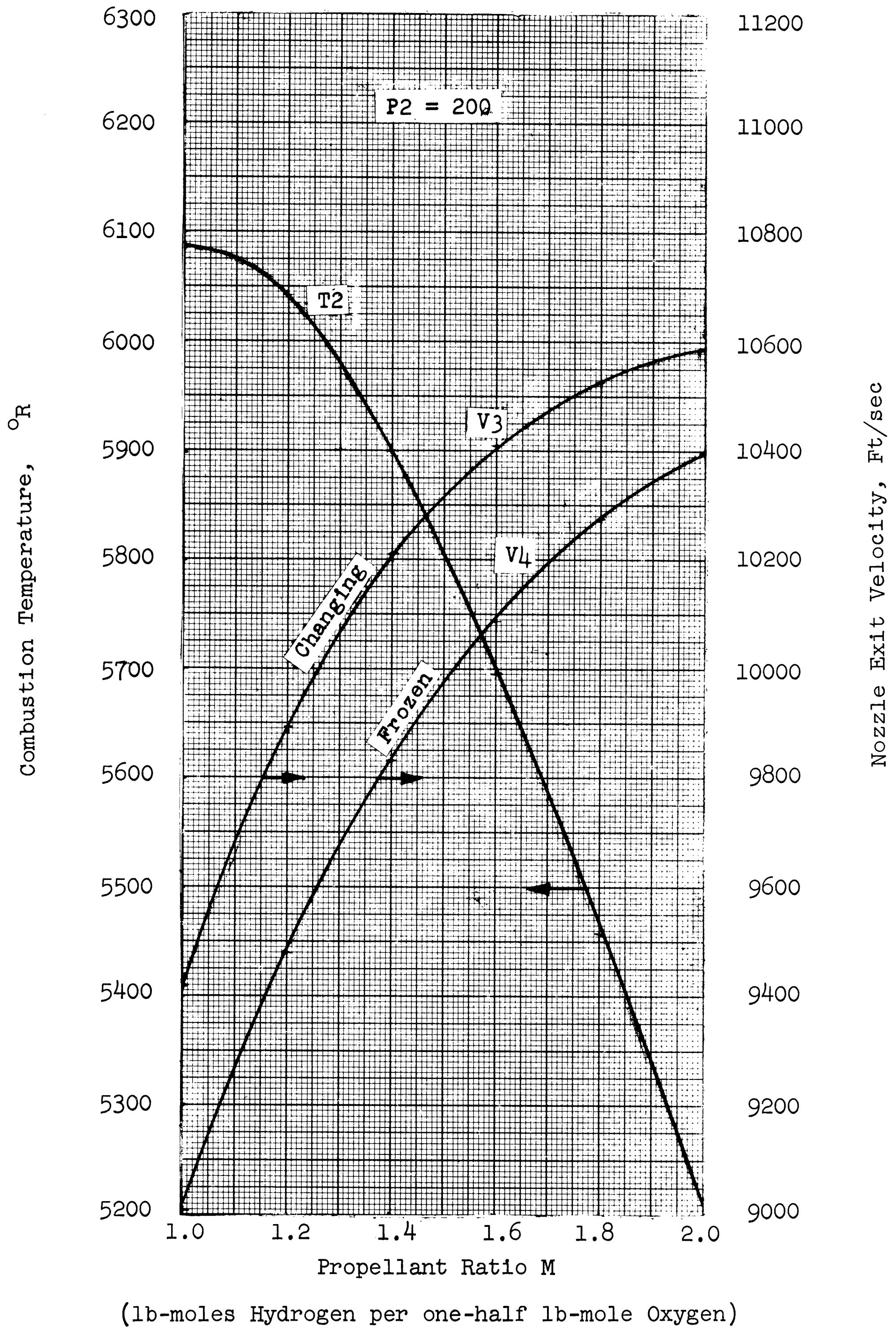
Computer Output, Continued

```
 ~~M= 1.20  P2= 300.0  T3= 4679.7  DELT3=  1.00  I= 16  J=  2  V3= 10559.6~~
         XH2O = 0.7967626     XH2 = 0.1669135     XO2 = 0.0018328
          ~~XOH = 0.0153473      XO = 0.0011721      XH = 0.0179739~~
   M = 1.40   P2 = 300.0    T2 = 5983.5    DELT2 =  0.50   I = 18   J =  2
         ~~XH2O = 0.6242743     XH2 = 0.2756902     XO2 = 0.0034135~~
          XOH = 0.0399251      XO = 0.0046520      XH = 0.0520451
 ~~M= 1.40  P2= 300.0  T4= 3449.4  DELT4= 16.00  I=  5  J=  2  V4= 10446.7~~
         XH2O = 0.6242743     XH2 = 0.2756902     XO2 = 0.0034135
          ~~XOH = 0.0399251      XO = 0.0046520      XH = 0.0520451~~
 M= 1.40  ┘2= 300.0  T3= 4257.7  DELT3=  2.00  I= 11  J=  3  V3= 10866.7
         ~~XH2O = 0.7074178     XH2 = 0.2815687     XO2 = 0.0000497~~
          XOH = 0.0027359      XO = 0.0000599      XH = 0.0081678
   ~~M = 1.60   P2 = 300.0    T2 = 5761.5    DELT2 =  0.50   I = 18   J =  2~~
         XH2O = 0.5782698     XH2 = 0.3547839     XO2 = 0.0008678
          ~~XOH = 0.0216473      XO = 0.0016422      XH = 0.0427891~~
 M= 1.60  P2= 300.0  T4= 3281.9  DELT4=  2.00  I= 10  J=  2  V4= 10705.3
         ~~XH2O = 0.5782698     XH2 = 0.3547839     XO2 = 0.0008678~~
          768 = ).0216473      XO = 0.0016422      OH 3 0.04278 1
 ~~M= 1.60  P2= 300.0  T3= 3816.9  DELT3=  2.00  I= 10  J=  3  V3= 11047.1~~
         XH2O = 0.6237312     XH2 = 0.3734272     XO2 = 0.0000011
          ~~XOH = 0.0003755      XO = 0.0000020      XH = 0.0024629~~
   M = 1.80   P2 = 300.0    T2 = 5508.0    DELT2 =  2.00   I =  9   J =  2
           ~~26 = 0.5315038     XH2 = 0.4253911     XO2 3 0.0002111~~
                        3      XO = 0.0005206      XH 3 0.0314422
     ~~.80  P2= 300.0  T4= 3095.2  DELT4=  2.00  I= 10  J=  2  V4= 10886.1~~
         XH2O = 0.5315038     XH2 = 0.4253911     XO2 = 0.0002111
          ~~XOH = 0.0109343      XO = C.0005206      XH 3 0.0314422~~
 M= 1.80  P2= 300.0  T3= 3432.8  DELT3=  2.00  I= 11  J=  3  V3= 11146.7
         ~~XH2O = 0.5553180     XH2 = 0.4439999     XO2 = 0.0000000~~
          XOH = 0.0000479      XO = 0.0000001      XH = 0.0006340
   ~~M = 2.00   P2 = 300.0    T2 = 5248.0    DELT2 =  2.00   I = 13   J =  2~~
         XH2O = 0.4876552     XH2 = 0.4853934     XO2 = 0.0000507
          ~~XOH = 0.0053020      XO = 0.0001550      XH = 0.0214474~~
 M= 2.00  P2= 300.0  T4= 2906.0  DELT4=  1.00  I=  8  J=  2  V4= 11009.1
         ~~XH2O = 0.4876552     XH2 = 0.4853934     XO2 = 0.0000507~~
          XOH = 0.0053020      XO = 0.0001550      XH = 0.0214474
 ~~M= 2.00  P2= 300.0  T3= 3112.0  DELT3=  2.00  I= 13  J=  3  V3= 11195.0~~
         XH2O = 0.4999538     XH2 = 0.4998859     XO2 = 0.0000000
          ~~XOH = 0.0000061      XO = 0.0000000      XH = 0.0001541~~
```

Discussion of Results

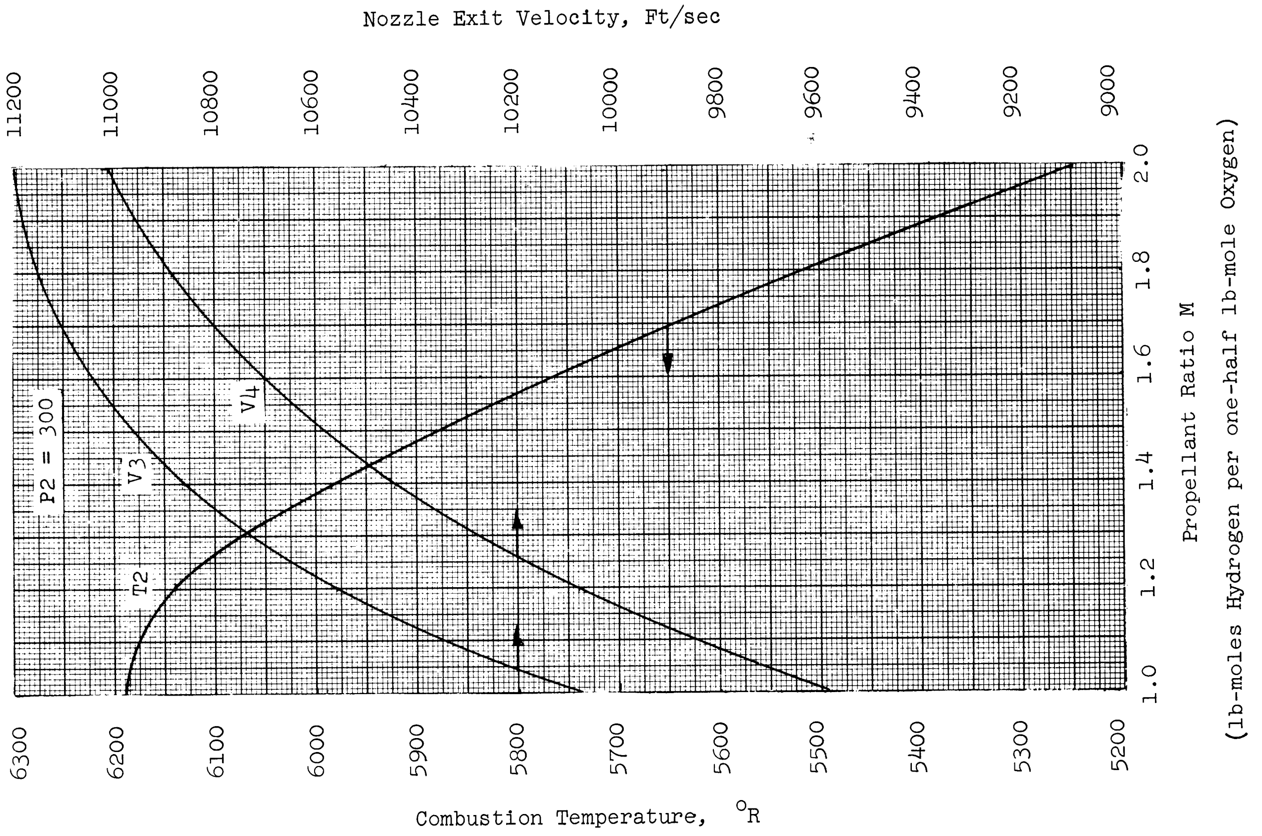
The program output gives the values of chamber pressure in psia, combustion temperature in ^{o}R, and the various mole fractions at state 2. The second set lists the temperature and velocity, ft/sec, at the nozzle exit state 4, resulting from a frozen equilibrium calculation. The composition here is necessarily the same as at state 2. Finally, the temperature, velocity and composition at the nozzle exit state 3, resulting from a changing equilibrium calculation, are given. Accuracy of the calculated temperatures in the half-interval method is indicated in the output by the smallest ΔT values, although the true accuracy of solution is somewhat less, due to inaccuracies in the specific heat equations.

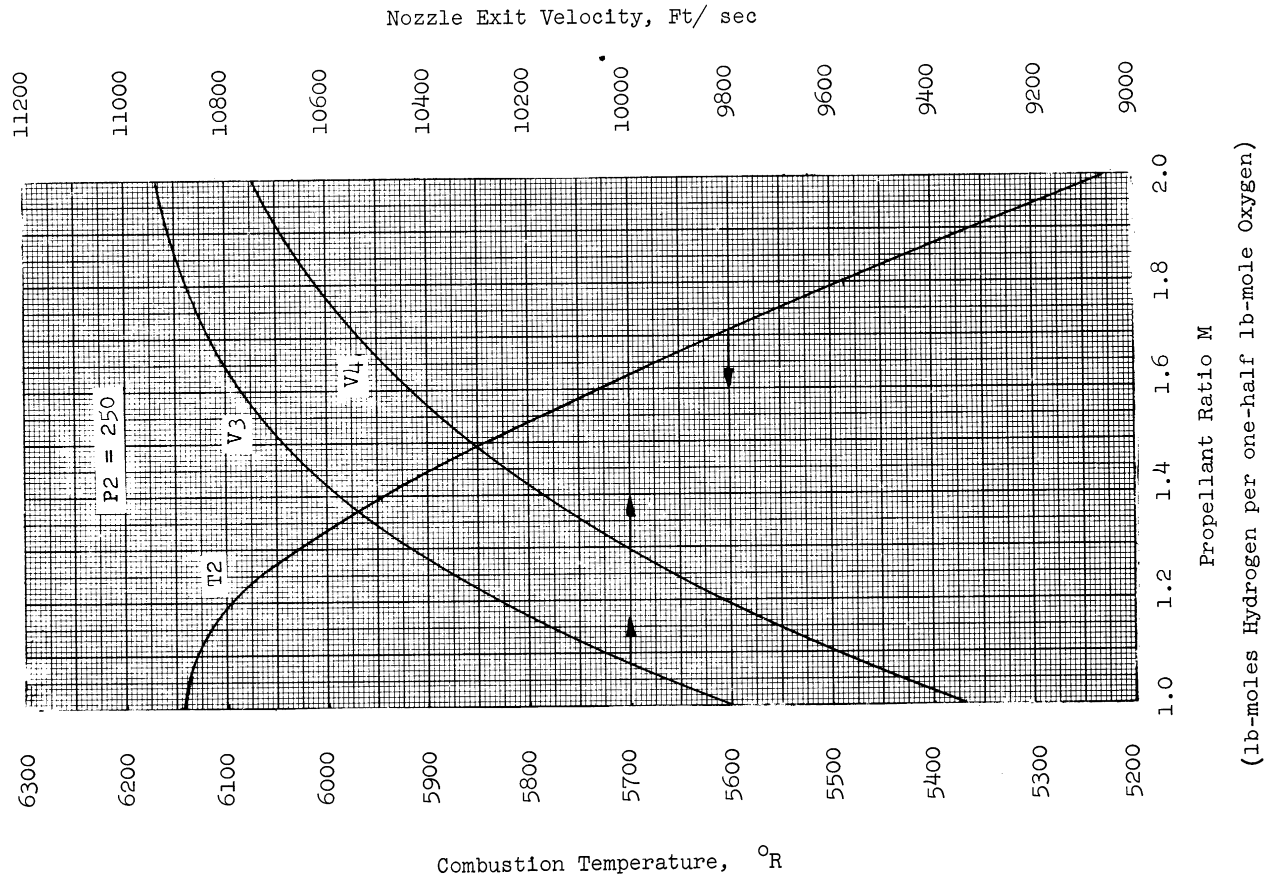
The combustion temperature and both exit velocities have been plotted versus propellant ratio at three pressures. Of particular interest is the increase in exit velocity with increasing m, even though the combustion temperature decreases, this being due to the decreased molecular weight of the products from the excess hydrogen. Eventually the velocity would reach a maximum and then decrease for higher values of m, but this range was not studied since the nozzle exit temperatures become so low as to make the dissociation reactions negligible and consequently introduce convergence problems in the solution. It should also be mentioned that the actual nozzle process, although not isentropic, should lie somewhere between the two extreme conditions considered here.



(lb-moles Hydrogen per one-half lb-mole Oxygen)

Example Problem No. 67





Critique

The problem solved is a realistic one, although simplified as the interest was in the thermodynamics of the problem rather than in rocket design. The project was experimental, that of assigning a very difficult problem to a class with no programming experience. The class consisted of only five students, thereby permitting more individual attention than would normally be possible, and the students had previously studied the basic subject matter, enabling them to begin the solution fairly early in the semester.

The class showed unusual interest in the problem and progressed extremely well. The students attended the evening lectures, which were very beneficial to them. Problems common to all the students were discussed frequently in class, and each benefited from the others' mistakes. The MAD language proved to be ideal, due to its simplicity and freedom from unnatural statements and conventions.

By the semester's end, all five students had working programs, two obtaining complete results, the others having only partial success because of convergence problems. All were hindered by the relatively long processing time. Complete success could have been achieved if the processing time had been less, or if the instructor had pushed the class harder early in the semester.

The students' knowledge of the computer, programming, use of trial and error methods, and interest in progression to other problems exceeded the instructor's expectations, although there were more difficulties in convergence than anticipated. The solution of a problem of this complexity also aided materially in their overall understanding of the fundamental principles involved.

Example Problem No. 68

DETERMINATION OF THE COMPOSITION OF THE PRODUCTS OF COMBUSTION

by

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School of Mechanical Engineering
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Course: Thermodynamics or Combustion Credit Hours: 3 Level: Senior

Statement of the Problem

Calculate the composition of the products of combustion of a stoichiometric ethylene and oxygen flame at 2700°C. The following products are believed to exist: CO_2, CO, H_2O, O_2, H_2, OH, O and H. It is further assumed that the total pressure is one atmosphere and that chemical equilibrium exists in the burnt gases. All gases are considered to be ideal gases.

Solution

The total pressure P is given by,

$$P = p_{CO_2} + p_{CO} + p_{H_2O} + p_{O_2} + p_{H_2} + p_{OH} + p_O + p_H \tag{1}$$

In general we know,

$$N_C = p_{CO_2} + p_{CO} \tag{2}$$

$$N_O = 2p_{CO_2} + 2p_{CO} + p_{H_2O} + 2p_{O_2} + p_{OH} + p_O \tag{3}$$

$$N_H = 2p_{H_2O} + 2p_{H_2} + p_{OH} + p_H \tag{4}$$

The five equilibrium constants are given by,

$$K_1 = \frac{p_{CO}\sqrt{p_{O_2}}}{p_{CO_2}} = 0.297 \tag{5}$$

$$K_2 = \frac{p_{H_2}\sqrt{p_{O_2}}}{p_{H_2O}} = 0.0435 \tag{6}$$

$$K_3 = \frac{p_{OH}\sqrt{p_{H_2}}}{p_{H_2O}} = 0.0465 \tag{7}$$

$$K_4 = \frac{p_H}{\sqrt{p_{H_2}}} = 0.154 \tag{8}$$

$$K_5 = \frac{p_O}{\sqrt{p_{O_2}}} = 0.110 \tag{9}$$

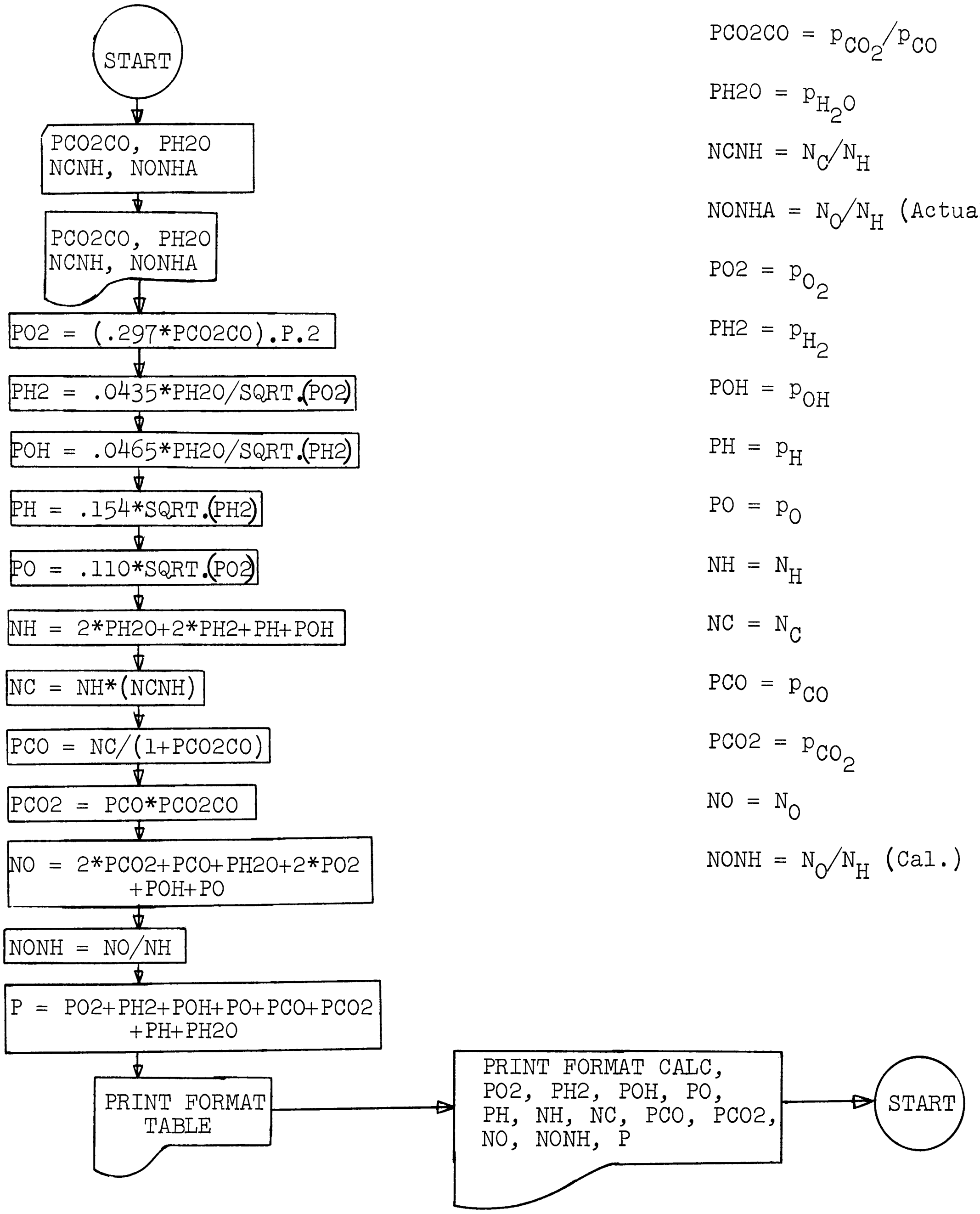
From the above equations the eight unknown partial pressures of Equation 1 are calculated.

We also know that,

$$N_C/N_H = 0.5 \text{ and } N_O/N_H = 1.5$$

The method of Damkohler and Edse (1)* is used to solve the system of equations. The values of equilibrium constants are obtained from Gaydon and Wolfhard (2).

Flow Diagram



Nomenclature

PCO2CO = p_{CO_2}/p_{CO}

PH2O = p_{H_2O}

NCNH = N_C/N_H

NONHA = N_O/N_H (Actual)

PO2 = p_{O_2}

PH2 = p_{H_2}

POH = p_{OH}

PH = p_H

PO = p_O

NH = N_H

NC = N_C

PCO = p_{CO}

PCO2 = p_{CO_2}

NO = N_O

NONH = N_O/N_H (Cal.)

---

* The numbers in parentheses refer to the references at the end.

Example Problem No. 68

MAD Program and Data

```
   R M SHASTRI                          D054N               001    006    025
   R M SHASTRI                          D054N               001    006    025
 * COMPILE MAD, EXECUTE, PRINT OBJECT, DUMP
              RDETERMINATION OF THE COMPOSITIONOF THE PRODUCTS OF COMBUSTION
              ROF STOCHIOMETRIC ETHYLENE AND OXYGEN MIXTURE, WHEN CHEMICAL
              REQUILIBRIUM EXISTS AT HIGH TEMPERATURES
 START         READ FORMAT DATA, PCO2CO,PH2O,NCNH,NONHA
               PUNCH FORMAT DDATA,PCO2CO,PH2O,NCNH,NONHA
               PRINT FORMAT DDATA, PCO2CO,PH2O,NCNH,NONHA
               PO2=(.297*PCO2CO).P.2
               PH2= .0435*PH2O/SQRT.(PO2)
               POH= .0465*PH2O/SQRT.(PH2)
               PH= .154*SQRT.(PH2)
               PO= .110*SQRT.(PO2)
               NH=2*PH2O+2*PH2+PH+POH
               NC=NH*NCNH
               PCO=NC/(1+PCO2CO)
               PCO2=PCO*(PCO2CO)
               NO=2*PCO2+PCO+PH2O+2*PO2+POH+PO
               NONH=NO/NH
               P=PO2+PH2+POH+PO+PCO+PCO2+PH+PH2O
               PUNCH FORMAT TABLE
               PRINT FORMAT TABLE
               PUNCH FORMAT CALC,PO2,PH2,POH,PO,PH,NH,NC,PCO,PCO2,NO,NONH,P
               PRINT FORMAT CALC,PO2,PH2,POH,PO,PH,NH,NC,PCO,PCO2,NO,NONH,P
               TRANSFER TO START
               VECTORVALUESDATA=$4F10.3*$
               VECTORVALUESDDATA=$1H1 S5,6HPCO2CO S6,4HPH2O S5,4HNCNH S4,5HN
              1ONHA/1H S5,F7.4,S3,F7.4,S2,F7.4,S2,F7.4*$
               VECTORVALUESTABLE=$1H S1,3HPO2 S4,3HPH2 S3,3HPOH S3,2HPO S4,2
              1HPH S4,2HNH S4,2HNC S4,3HPCO S2,4HPCO2 S3,2HNO S3,4HNONH S3,1
              1HP*$
               VECTORVALUESCALC=$1H 12F6.4*$
               END OF PROGRAM
 * DATA
        1.5       0.35      0.5       1.5
        0.75      0.35      0.5       1.5
        0.75      0.25      0.5       1.5
        1.03      0.307     0.5       1.5
```

Computer Output (Rearranged)

Results from the first three data points:

```
     PCO2CO     PH2O    NCNH    NONHA

     1.5000    0.3500  0.5000  1.5000

     0.7500    0.3500  0.5000  1.5000

     0.7500    0.2500  0.5000  1.5000

  PO2    PH2    POH    PO     PH     NH     NC     PCO    PCO2   NO     NONH   P

0.19850.03420.08800.04900.02850.88490.44240.17700.26551.59191.79901.19 6

0.04960.06840.06230.02450.04030.93920.46960.26830.20131.20691.28501.0646

0.04960.04880.05260.02450.03400.68430.34210.19550.14660.91511.33730.8017
```

Results from the fourth data point:

```
    PCO2CO     PH2O    NCNH    NONHA

     1.0300    0.3070  0.5000  1.5000

  PO2    PH2    POH    PO     PH     NH     NC     PCO    PCO2   NO     NONH   P

0.09360.04370.06830.03370.03220.80180.40090.19750.20341.20051.49720.9793
```

Discussion of Results

Initial trial assumptions are made for p_{H_2O} and p_{CO_2}/p_{CO}. Now all the remaining partial pressures are calculated. A check is made to see whether the total pressure and the ratio N_O/N_H will have the correct values. If they do not check, new assumptions are made and the procedure is repeated. After 3 trial calculations a plot is made of p_{CO_2}/p_{CO} vs. p_{H_2O} as shown in Figure 1. Using this figure, better approximations are made.

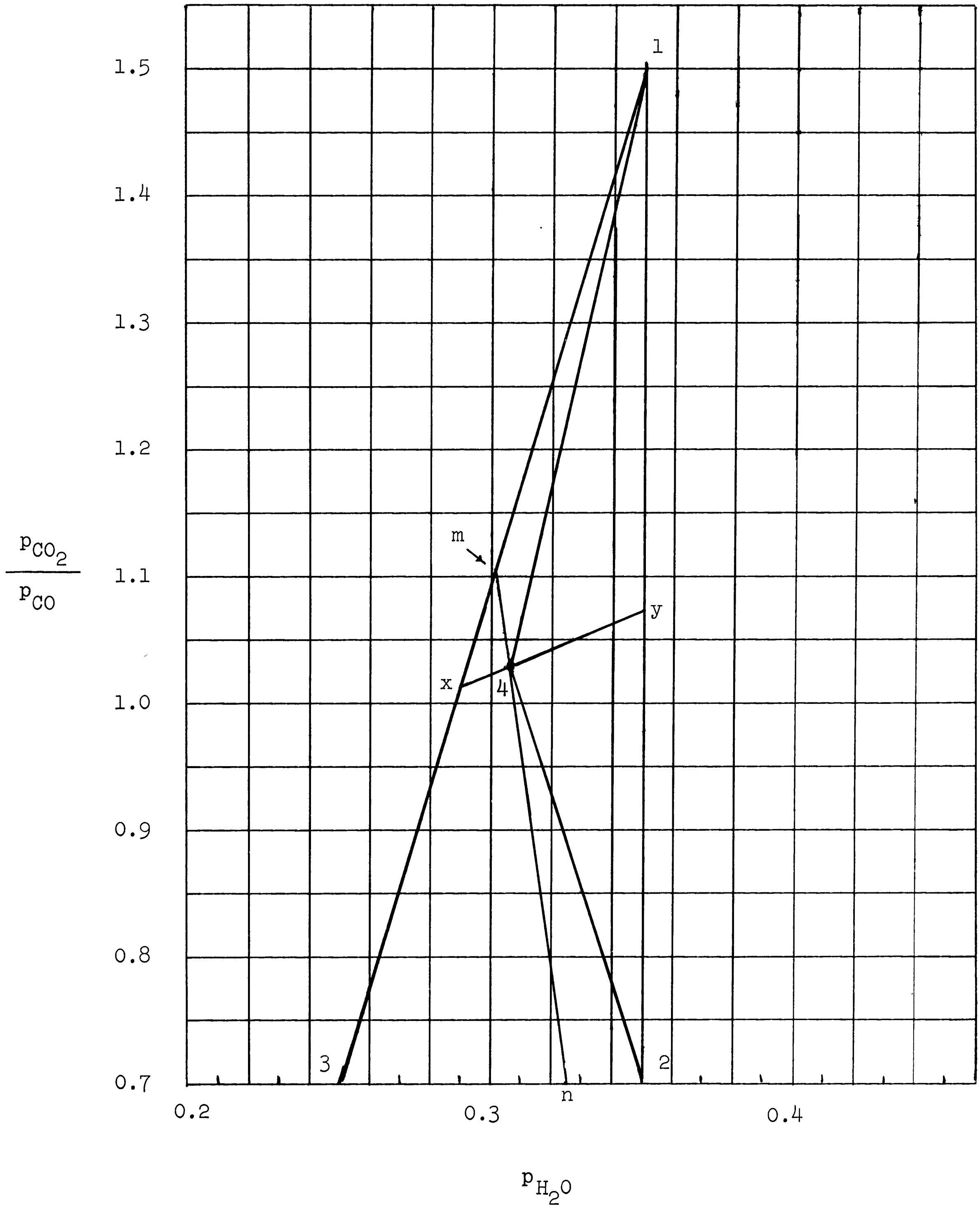


Figure 1. Plot of p_{CO_2}/p_{CO} vs. p_{H_2O}

Calculations showed a total pressure of more than one atmosphere at Point 1 and less than 1 atmosphere at Point 3. A linear variation of pressure is assumed. The Point 'm' is determined where the total pressure is approximately one atmosphere. Similarly, a point 'n' is established between Points 2 and 3. The whole procedure is repeated for $N_O/N_H = 1.5$ and points 'x' and 'y' are established. The values of p_{CO_2}/p_{CO} and p_{H_2O} are given by the point 4; the point of intersection of the lines m-n and a-b will be the values for the 4th assumption. The procedure is now repeated using Points 1, 2, 4 to get Point 5. The procedure is repeated until the calculations give the required values for P and N_O/N_H. This, then, will also give the values of the correct composition.

Critique

This problem has been taken from Gaydon and Wolfhard (2). The method could be applied when there are any number of products of combustion. The equilibrium constants for the products at the desired temperature must be known. Here the total pressure has been assumed a constant before and after the reaction. With slight changes the above method could also be used for constant volume processes.

References

1. Damkohler, G. and Edse, R., Z. Elektrochem 49, 178 (1943).
2. Gaydon, A. G. and Wolfhard, H. G. "Flames," Chapman and Hall Ltd., London (1953).

Example Problem No. 69

ADIABATIC FLAME TEMPERATURE FOR COMBUSTION OF AIR AND METHANE

by
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The University of Michigan

Course: Advanced Thermodynamics Credit hours: 3 Level: Graduate

Statement of Problem

It is desired to determine the effects of pressure and percent theoretical air on the adiabatic flame temperature and product composition for the combustion of methane with air over the following range:

1 to 50 atmospheres pressure

100 to 150 percent theoretical air

Assume all gases behave ideally, and that the reactants enter at 77°F.

Let $100 \times a$ = percent theoretical air

where $a \geqslant 1.0$

Then, for the combustion at 77°F,

$$CH_4 + (2a)\ O_2 + (7.52a)\ N_2 \rightarrow CO_2 + 2H_2O + 2\ (a-1)\ O_2 + (7.52a)\ N_2$$

As the temperature of the products is increased to the flame temperature, consider that only two dissociation reactions occur:

$$A: \quad CO_2 = CO + \frac{1}{2}\ O_2 \qquad (1)$$

$$B: \quad H_2O = H_2 + \frac{1}{2}\ O_2 \qquad (2)$$

In the solution, use the following specific heat equation and data:

$$\overline{C}_{p_o} = \alpha + \beta \cdot 10^{-3}T + \gamma \cdot 10^{-6}T^2 \quad \frac{\text{Btu}}{\text{Lb-Mole-}^{\circ}\text{R}}$$

where $T = {}^{\circ}R$

	α	β	γ
CO_2	6.214	5.776	-1.094
CO	6.420	0.925	-0.060
O_2	6.148	1.722	-0.285
H_2O	7.256	1.277	0.087
H_2	6.947	-0.111	0.148
N_2	6.524	0.694	-0.001

The resulting specific heat equations will not prove to be particularly accurate at temperatures above 3500°R, but are sufficient for this illustration.

At 77°F,

	$\bar{h}^o$	$\bar{g}^o$ (Btu/lb-Mole)
CH_4	-32,200	
CO_2	-169,293	-169,667
H_2O	-104,071	-98,344
CO	-47,548	-59,054

Solution

Let y and z be defined as the moles involved in reactions A and B respectively, so that the change in each component is

$$\text{A:} \quad CO_2 = CO + \frac{1}{2} O_2 \qquad -y \quad +y \quad +\frac{1}{2} y \tag{1}$$

$$\text{B:} \quad H_2O = H_2 + \frac{1}{2} O_2 \qquad -z \quad +z \quad +\frac{1}{2} z \tag{2}$$

Since the initial moles of each component are given in the product side of the combustion equation, the number of moles at equilibrium (flame temperature) is

$$\begin{aligned}
n_{CO_2} &= 1 - y \\
n_{H_2O} &= 2 - z \\
n_{O_2} &= 2(a-1) + \frac{1}{2} y + \frac{1}{2} z \\
n_{CO} &= y \\
n_{H_2} &= z \\
n_{N_2} &= 7.52 a \\
\hline
n_{total} &= 1 + 9.52a + \frac{1}{2} y + \frac{1}{2} z
\end{aligned}$$

Note that although the nitrogen is inert insofar as the reactions are concerned, it must be considered in the total number of moles present at equilibrium.

The two equilibrium equations are now written as

$$K_A = K_A(T) = \frac{x_{CO}\, x_{O_2}^{\frac{1}{2}}}{x_{CO_2}} P^{\frac{1}{2}} = \left(\frac{y}{1-y}\right) \left(\frac{2a-2 + \frac{1}{2}y + \frac{1}{2} z}{1+9.52a + \frac{1}{2} y + \frac{1}{2} z}\right)^{\frac{1}{2}} P^{\frac{1}{2}} \tag{3}$$

Adiabatic Flame Temperature for Combustion of Air and Methane

$$K_B = K_B(T) = \frac{x_{H_2} \, x_{O_2}^{\frac{1}{2}}}{x_{CO_2}} \quad P^{\frac{1}{2}} = \left(\frac{z}{2-z}\right) \left(\frac{2a-2 + \frac{1}{2}y + \frac{1}{2}z}{1+9.52a + \frac{1}{2}y + \frac{1}{2}z}\right)^{\frac{1}{2}} P^{\frac{1}{2}} \qquad (4)$$

It will, therefore, be necessary to express the equilibrium constants as functions of temperatur

It is known that

$$d(\ln K)_p = \frac{\Delta H_T^o}{RT^2} \, dT_p \qquad (5)$$

However, the standard-state enthalpy change ΔH^o is a function of temperature, and can be written

$$\Delta H_T^o = \Delta H_{T_o}^o + \sum_{\text{prod-react}} \gamma(\bar{h}_T^o - \bar{h}_{T_o}^o)$$

$$= \Delta H_{T_o}^o + \int_{T_o}^{T} \sum_{P-R} (\nu \bar{C}_{p_o}) \, dT \qquad (6)$$

Substituting equation (6) into (5), and integrating between any two temperatures T_1 and T_2,

$$\ln\left[\frac{K_{T_2}}{K_{T_1}}\right] = \frac{\Delta H_{T_o}^o}{R} \left[\frac{1}{T_1} - \frac{1}{T_2}\right] + \frac{1}{R} \int_{T_1}^{T_2} \left[\int_{T_o}^{T} \sum_{P-R} (\nu \bar{C}_{p_o}) \, dT\right] \frac{dT}{T^2} \qquad (7)$$

Reaction A

At 537°R, using the enthalpy and free energy of formation data,

$$\Delta H_{A_1}^o = 121{,}745 \frac{\text{Btu}}{\text{Mole } CO_2} ; \quad \Delta G_{A_1}^o = 110{,}613 \frac{\text{Btu}}{\text{Mole}}$$

Therefore,

$$\ln K_{A_1} = -\frac{\Delta G_{A_1}^o}{RT} = -103.6$$

Also,

$$\sum_{P-R} (\nu \bar{C}_{p_o}) = 3.28 - .00399 \, T + .892 \cdot 10^{-6} T^2$$

Substituting these values into equation (7) and integrating,

$$\ln K_A = 1.65 \ln T - .001 \, T + .075 \cdot 10^{-6} T^2 - \frac{60759}{T} - 0.41 \qquad (8)$$

Reaction B

At 537°R,

$$\Delta H_{B_1}^o = 104{,}071 \frac{\text{Btu}}{\text{Mole } H_2O} ; \quad \Delta G_{B_1}^o = 98{,}344 \frac{\text{Btu}}{\text{Mole}}$$

Therefore,

$$\ln K_{B_1} = -\frac{\Delta G_{B_1}^o}{RT} = -92.2$$

Also

$$\sum_{P-R} (\nu \overline{c}_{p_o}) = 2.765 - .000527\ T - .082 \cdot 10^{-6} T^2$$

As before,

$$\ln K_B = 1.393 \ln T - .000133 \cdot T - .007 \cdot 10^{-6}\ T^2 - \frac{51673}{T} - 4.61 \qquad (9)$$

The third equation to be set up is the First Law, which requires that

$$H_R = H_p \qquad (10)$$

where

$$H_R = \overline{h}^o_{CH_4} = -\ 32{,}200 \text{ Btu} \qquad (11)$$

$$H_p = n_{CO_2}\ \overline{h}_{CO_2} + n_{H_2O}\ \overline{h}_{H_2O} + n_{CO}\ \overline{h}^o_{CO}$$

$$+ \int_{537R}^{T} \Big[\ n_{CO_2}\ c_{p_{CO_2}} + n_{H_2O}\ c_{p_{H_2O}} + n_{O_2}\ c_{p_{O_2}} + n_{CO}\ c_{p_{CO}}$$

$$+ n_{H_2}\ c_{p_{H_2}} + n_{N_2}\ c_{p_{N_2}}\ \Big]\ dT \qquad (12)$$

Substituting numerical values into equation (12),

$$H_p = -\ 382{,}646 - 34{,}163a + 120{,}511y + 102{,}665\ z + T(8.43 + 61.356\ a$$

$$+ 3.28\ y + 2.765\ z) + \left[\frac{T}{100}\right]^2 (24.44 + 43.31\ a - 19.96\ y - 2{,}635\ z)$$

$$- \left[\frac{T}{100}\right]^3 (0.117 + 0.1923\ a - 0.297\ y + 0.028\ z) \qquad (13)$$

Therefore, Equation (10) becomes

$$T(8.43 + 61.356\ a + 3.28\ y + 2.765\ z) + \left[\frac{T}{100}\right]^2 (24.44 + 43.31\ a - 19.96\ y$$

$$- 2.635\ z) - \left[\frac{T}{100}\right]^3 (0.117 + 0.1923\ a - 0.297\ y + 0.028\ z\ ) + 120{,}511\ y$$

$$+ 102{,}665\ z - 34{,}163\ a - 350{,}446 = 0 \qquad (14)$$

Thus, for given values of a and P, there are three equations (3, 4, 14) in three unknowns (T, y, z), since K_A, K_B are given in terms of T by equations (8), (9).

For an assumed T, y and z are determined from (3) and (4), this necessitating the simultaneous solution of two non-linear equations. This could be accomplished by elimination of one of the variables in terms of the other, but a more general method will be used here as an illustration, applying as well in the case of more complex reactions or systems of reactions.

Specifically, initial values of y and z are assumed, and then incremented by linearized correction equations obtained from the partial derivatives of the equations involving y and z (3 and 4). The process is repeated until the correction is sufficiently small. Once y and z have been so determined for the assumed T, all three are substituted into equation (14) to check the assumption of T. The value of T is then incremented and the procedure repeated until the error in equation (14) is less than some predetermined amount.

For any given T, there are two equations

$$F_A\ (y,z) = 0, \qquad F_B\ (y,z) = 0$$

Assume initial values for y, z. Evaluating F_A (y,z), F_B (y,z) for this y, z, it is found that, in general, both are unequal to zero. Therefore y and z must be corrected by some amounts Δy and Δz such that the required conditions are met. That is,

$$F_A + \Delta F_A = 0$$

$$F_B + \Delta F_B = 0$$

where

$$\Delta F_A = \left[\frac{\partial F_A}{\partial \ell n\ y}\right] \Delta \ell n\ y + \left[\frac{\partial F_A}{\partial \ell n\ z}\right] \Delta \ell n\ z$$

$$\Delta F_B = \left[\frac{\partial F_B}{\partial \ell n\ y}\right] \Delta \ell n\ y + \left[\frac{\partial F_B}{\partial \ell n\ z}\right] \Delta \ell n\ z$$

Note that a logarithmic form has been used here for the incrementation of y and z, this being done to prevent y or z from becoming negative during the course of the solution. Otherwise it would be very possible to obtain a physically non-real solution of the equations, with one or more of the equilibrium mole fractions having negative values. The solution desired is, of course, always the one with all positive masses.

Solving the last four equations,

$$\Delta \ell n\ y = \frac{\begin{vmatrix} -F_A & \dfrac{\partial F_A}{\partial \ell n\ z} \\ -F_B & \dfrac{\partial F_B}{\partial \ell n\ z} \end{vmatrix}}{\begin{vmatrix} \dfrac{\partial F_A}{\partial \ell n\ y} & \dfrac{\partial F_A}{\partial \ell n\ z} \\ \dfrac{\partial F_B}{\partial \ell n\ y} & \dfrac{\partial F_B}{\partial \ell n\ z} \end{vmatrix}} \qquad (15)$$

$$\Delta \ln z = \frac{\begin{vmatrix} \frac{\partial F_A}{\partial \ln y} & -F_A \\ \frac{\partial F_B}{\partial \ln y} & -F_B \end{vmatrix}}{\begin{vmatrix} \frac{\partial F_A}{\partial \ln y} & \frac{\partial F_A}{\partial \ln z} \\ \frac{\partial F_B}{\partial \ln y} & \frac{\partial F_B}{\partial \ln z} \end{vmatrix}} \qquad (16)$$

Therefore, the new values of y and z to be used in the succeeding trial are

$$\ln y = \ln y + \Delta \ln y, \quad \ln z = \ln z + \Delta \ln z \qquad (17)$$

or,

$$y = y \cdot (e)^{\ln y}, \quad z = z \cdot (e)^{\ln z}$$

and the iteration process is repeated until $\Delta \ln y$ and $\Delta \ln z$ are both sufficiently small.

For the problem at hand, the quantities to be used in evaluating equations (15) and (16) are found by rewriting equations (3) and (4) in the form

$$F_A = \ln \left[\left[\frac{y}{1-y}\right]^2 \cdot \frac{(D2)}{(D1)} \cdot P \right] - 2 \cdot \ln K_A \qquad (18)$$

$$F_B = \ln \left[\left[\frac{z}{2-z}\right]^2 \cdot \frac{(D2)}{(D1)} \cdot P \right] - 2 \cdot \ln K_B \qquad (19)$$

where, for convenience,

$$(D1) = n_{total} = 1 + 9.52 \cdot a + \frac{1}{2} y + \frac{1}{2} z \qquad (20)$$

$$(D2) = n_{O_2} = 2a - 2 + \frac{1}{2} y + \frac{1}{2} z \qquad (21)$$

$\ln K_A$ and $\ln K_B$ are given by equations (8) and (9) respectively.

Then, since

$$\frac{\partial F_A}{\partial \ln y} = \left[\frac{\partial F_A}{\partial y}\right]\left[\frac{dy}{d \ln y}\right] = \left[\frac{\partial F_A}{\partial y}\right] \cdot y \quad , \quad \text{etc.}$$

$$\frac{\partial F_A}{\partial \ln y} = \frac{2}{1-y} + (D3) \cdot y \qquad (22)$$

$$\frac{\partial F_A}{\partial \ln z} = (D3) \cdot z \qquad (23)$$

$$\frac{\partial F_B}{\partial \ln y} = (D3) \cdot y \qquad (24)$$

$$\frac{\partial F_B}{\partial \ln z} = \frac{4}{2-z} + (D3) \cdot z \qquad (25)$$

where

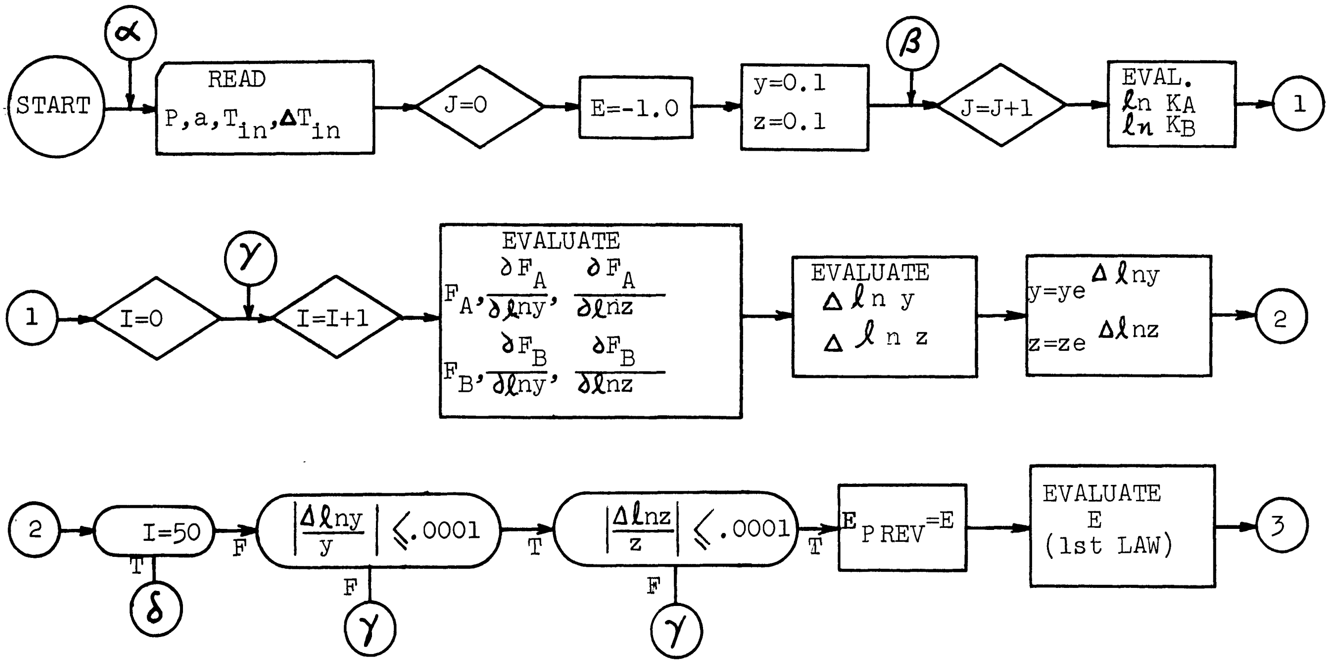
$$(D3) = \frac{1.5 + 3.76 \cdot a}{(D1) \cdot (D2)} \qquad (26)$$

Thus, for an initial assumption of y, z, equations (18), (19), (22), ... , (25) are evaluated and substituted into equations (15) and (16), thereby giving the corrections to be applied to y, z (according to equation (17)) for the next trial.

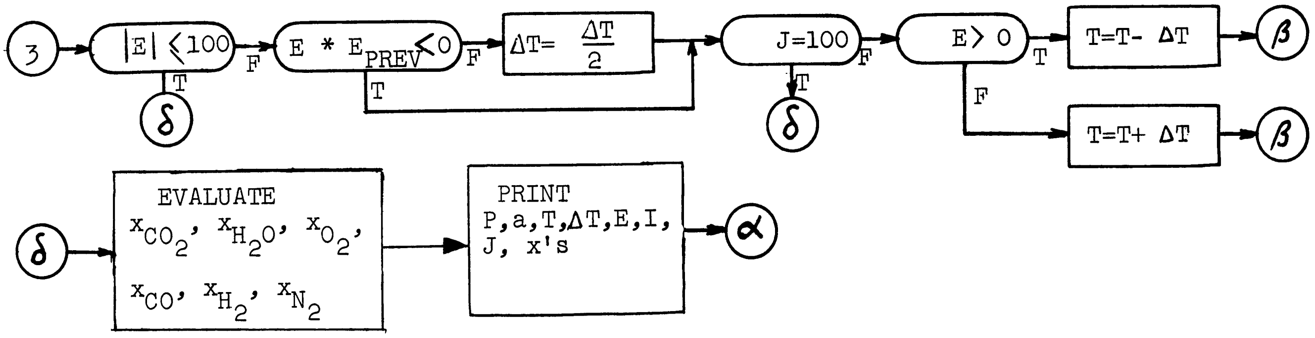
It should be stated here that convergence to a solution using this method is certainly a real problem, since the successive incrementations may very well grow and cause divergence from any solution if the initial assumptions of y and z are not in the general region of the true values. There are special techniques for handling this particular problem, namely, means for predicting whether or not the assumed values are inside the area of convergence. If not, then different assumptions are generated.

Finally, it should be obvious that all three equations (3,4, 14) could be solved simultaneously using the same method that is used here to solve (3) and (4) for any assumed T. This would not follow as closely the paths taken to solve such a system by hand, but is perhaps somewhat more rigorous mathematically. Such a method naturally will pose a more severe test of the convergence, requiring that the initial assumption of T also be a reasonably good one (which is not the case in our method of solution). The three equations lead to a set of three linear correction equations, the set then being solved for the corrections ($\Delta y, \Delta z, \Delta T$, or logarithmic counterparts) by matrix inversion. These same remarks apply also to the general case of n simultaneous equilibrium reactions, resulting in a set of n+1 linear correction equations (including the First Law and T), which can be solved by the same method.

Flow Diagram



Flow Diagram (continued)



MAD Program

```
$ COMPILE MAD, EXECUTE

ALPHA         READ FORMAT DATA,P,A,T, DELTAT
              VECTOR VALUES DATA=$4F10.2*$
              J=0
              E=-1.0
              Y=0.1
              Z=0.1
BETA          J=J+1
              LNKA=1.65*ELOG.(T)-.001*T+.075*(T/1000.).P.2-60759./T-.41
              LNKB=1.393*ELOG.(T)-.000133*T-.007*(T/1000.).P.2-51673./T-4.61
              I=0
              INTEGER I,J
GAMMA         I=I+1
              D1=1.0+9.59*A+0.5*(Y+Z)
              D2=2.0*(A-1.0)+0.5*(Y+Z)
              D3=(1.5+3.76*A)/(D1*D2)
              FA=ELOG.(Y*Y*D2*P/(D1*(1.0-Y).P.2))-2.0*LNKA
              FB=ELOG.(Z*Z*D2*P/(D1*(2.0-Z).P.2))-2.0*LNKB
              DFADZ=D3*Z
              DFBDY=D3*Y
              DFADY=2.0/(1.0-Y)+DFBDY
              DFBDZ=4.0/(2.0-Z)+DFADZ
              DENOM=DFADY*DFBDZ-DFADZ*DFBDY
              DY=(FB*DFADZ-FA*DFBDZ)/DENOM
              DZ=(FA*DFBDY-FB*DFADY)/DENOM
              Y=Y*EXP.(DY)
              Z=Z*EXP.(DZ)
              WHENEVER I.E.50, TRANSFER TO DELTA
              WHENEVER.ABS.(DY/Y).G.0.0001, TRANSFER TO GAMMA
              WHENEVER.ABS.(DZ/Z).G.0.0001, TRANSFER TO GAMMA
              EPREV=E
              E=(8.43+61.356*A+3.28*Y+2.765*Z)*T+(24.44+43.31*A-19.96*Y-2.635*Z)*(T/100.
             1 ).P.2-(.117+.1923*A-.297*Y+.028*Z)*(T/100.).P.3+120511.*Y+102665*Z-
             2 34163.*A-350446.
              WHENEVER.ABS.(E).LE.100., TRANSFER TO DELTA
              WHENEVER (E*EPREV).L.0,DELTAT=0.5*DELTAT
              WHENEVER J.E.100, TRANSFER TO DELTA
              WHENEVER E.G.0
              T=T-DELTAT
              OTHERWISE
              T=T+DELTAT
              END OF CONDITIONAL
```

MAD Program (continued)

```
          TRANSFER TO BETA
DELTA     XCO2=(1.0-Y)/D1
          XH20=(2.0-Z)/D1
          XO2=D2/D1
          XCO=Y/D1
          XH2=Z/D1
          XN2=7.52*A/D1
          PRINT FORMAT RESULT ,P,A,T,DELTAT,E,I,J,XCO2,XH20,XO2,XCO,XH2,XN2
          VECTOR VALUES RESULT=$1H0,5F10.2,2I4,6F10.7*$
          TRANSFER TO ALPHA
          END OF PROGRAM
```

Computer Output

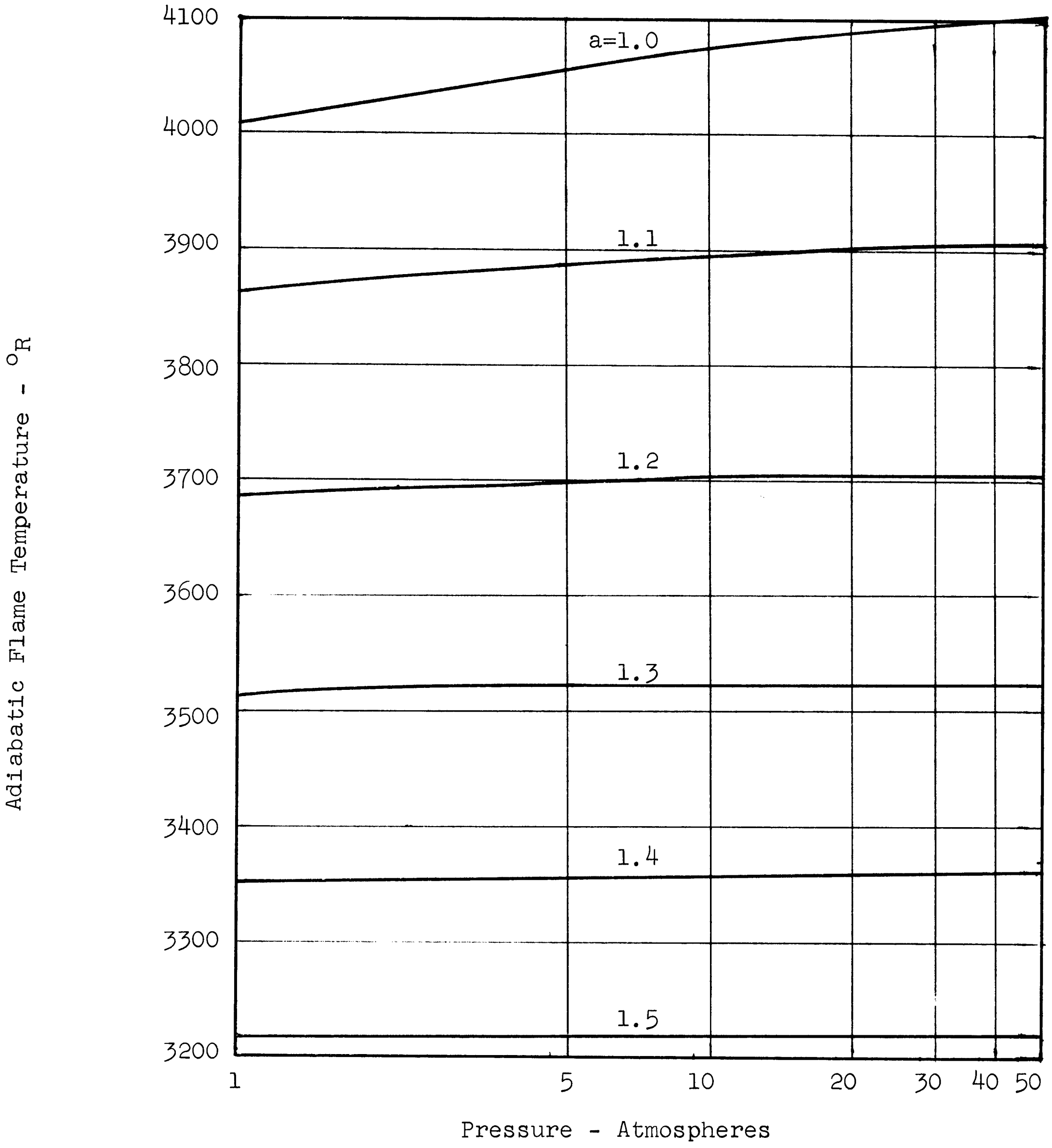
The following table has been obtained from the computer output.

P_{ATM}	a	T	x_{CO_2}	x_{H_2O}	x_{O_2}	x_{CO}	x_{H_2}	x_{N_2}
1	1.0	4008	.084702	.185716	.006456	.009741	.003171	.710214
5	1.0	4056	.088028	.187242	.004333	.006617	.002048	.711732
10	1.0	4072	.089196	.187751	.003599	.005519	.001679	.712256
20	1.0	4086	.090203	.188178	.002971	.004572	.001371	.712705
30	1.0	4094	.090714	.188393	.002654	.004090	.001217	.712932
40	1.0	4098	.091062	.188536	.002439	.003763	.001115	.713085
50	1.0	4102	.091304	.188636	.002289	.003535	.001043	.713193
1	1.1	3866	.083812	.172937	.019501	.003174	.001033	.719543
5	1.1	3890	.085438	.173630	.018493	.001637	.000520	.720282
10	1.1	3897	.085894	.173819	.018213	.001206	.000380	.720488
20	1.1	3902	.086240	.173961	.018001	.000879	.000275	.720644
30	1.1	3904	.086401	.174027	.017902	.000727	.000227	.720716
40	1.1	3906	.086496	.174066	.017844	.000636	.000199	.720759
50	1.1	3906	.086566	.174095	.017801	.000570	.000178	.720790
1	1.2	3690	.079338	.160483	.032907	.001092	.000378	.725802
5	1.2	3698	.079948	.160748	.032529	.000513	.000176	.726086
10	1.2	3701	.080101	.160813	.032435	.000369	.000126	.726156
20	1.2	3702	.080212	.160861	.032366	.000263	.000090	.726208
30	1.2	3704	.080261	.160881	.032336	.000217	.000074	.726231
40	1.2	3704	.080292	.160894	.032317	.000188	.000064	.726245
50	1.2	3704	.080312	.160904	.032305	.000168	.000057	.726254
1	1.3	3516	.074354	.149337	.045110	.000387	.000145	.730667
5	1.3	3520	.074574	.149437	.044973	.000177	.000066	.730773
10	1.3	3521	.074628	.149462	.044939	.000126	.000047	.730798
20	1.3	3522	.074667	.149479	.044915	.000090	.000033	.730816
30	1.3	3522	.074684	.149487	.044905	.000073	.000027	.730824
40	1.3	3522	.074694	.149491	.044898	.000064	.000024	.730829
50	1.3	3522	.074701	.149494	.044894	.000057	.000021	.730833
1	1.4	3357	.069643	.139514	.055930	.000144	.000058	.734711
5	1.4	3358	.069725	.139554	.055878	.000065	.000026	.734752
10	1.4	3359	.069745	.139564	.055865	.000046	.000019	.734761
20	1.4	3359	.069758	.139570	.055857	.000033	.000014	.734768
30	1.4	3359	.069765	.139573	.055853	.000027	.000011	.734771
40	1.4	3359	.069768	.139575	.055850	.000024	.000010	.734773
50	1.4	3359	.069771	.139576	.055849	.000021	.000009	.734774
1	1.5	3212	.065387	.130861	.065482	.000055	.000024	.738191
5	1.5	3213	.065419	.130877	.065462	.000025	.000011	.738206
10	1.5	3213	.065427	.130881	.065457	.000017	.000008	.738210
20	1.5	3213	.065432	.130883	.065454	.000012	.000006	.738213
30	1.5	3213	.065434	.130885	.065452	.000010	.000005	.738214
40	1.5	3213	.065436	.130885	.065451	.000009	.000004	.738215
50	1.5	3213	.065437	.130886	.065450	.000008	.000003	.738216

Discussion of Results

The reason for limiting the range of theoretical air to 150 per cent in this example is that for greater amounts the flame temperature is decreased sufficiently that there is no appreciable dissociation. In this case, the solution is reduced to a relatively simple calculation.

The results of this program are shown in graphical form in the figure below. As a point of interest, the time required to completely solve the equations for one set of P and a (i.e., determine the flame temperature) was less than one second. This would have been reduced somewhat if a good initial assumption for T had been made in each case. With the values actually used, the temperature had to be incremented an average of twelve times per solution to reduce the error in equation (14) to the equivalent of less than one degree Rankine.



Example Problem No. 70

TRANSIENT TEMPERATURE CALCULATION FOR A JACKETED MIXING KETTLE

by

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Course: Heat Transfer Credit Hours: 3 Level: Graduate

Statement of the Problem

In developing a certain chemical mixing process, the temperature of the mixture is to be maintained at fixed values which may be varied over a moderate range near 150 degrees F. A scheme for doing this is to locate a thermocouple in the mixture at some representative spot and use its output voltage as the input to an automatic controller, as indicated in Figure 1. The controller would regulate the heat to or from the mixture by controlling the flow of hot and cold water to the jacket spaces in the steel vessel. Either the hot or cold water valve will be open depending on whether a temperature increase or decrease is desired. Zero dead time is to be assumed.

In such a system, how may the temperature in the mixture be expected to vary with time after the controller is suddenly reset to call for a new temperature?

The nomenclature, physical constants and initial values are shown in Table 1. The jacket is assumed to be perfectly insulated.

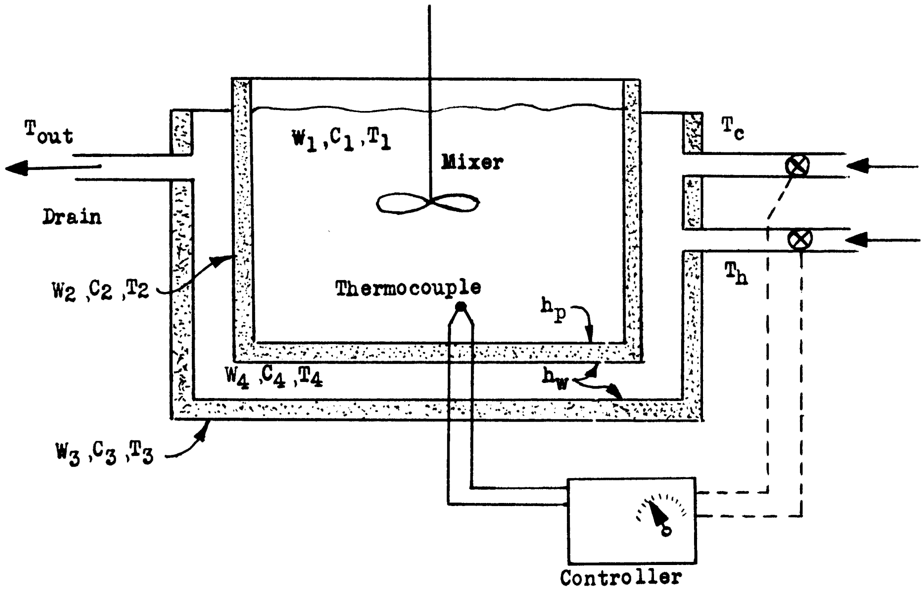

Fig. 1. Mixing Kettle with Insulated Jacket

Nomenclature

Subscripts

1 and p	mixture
2	vessel
3	jacket
4 and w	circulating water
in	inlet conditions
out	outlet conditions
c	cold water
h	hot water

Constants and Variables	1	2	3	4
W, weight, lbs	7700	3500	1500	84000 (lbs/hr)
C, specific heat, BTU/lb.deg F	0.95	0.1	0.1	1.0
A, area, ft^2	400	400	400	

HP = 75 BTU/hr-ft^2 deg F, convective heat transfer coef. for mixture

HW = 500 BTU/hr-ft^2 deg F, convective heat transfer coef. for water

TH = 200 deg F, hot water temperature

TC = 50 deg F, cold water temperature

T_1 = 100 deg F, initial value, product temperature

T_2 = 100 deg F, initial value, vessel temperature

T_3 = 100 deg F, initial value, jacket temperature

TCONTL = 150 deg F, controller temperature setting

H = 0.001 hours, time increment for Runge-Kutta Integration Procedure

TIMAX = 0.7 hours, maximum time

Solution

A. Derivation of Equations

Apply the principle of conservation of energy to the mixture, vessel, water and the jacket.

Assumptions

(a) Walls sufficiently thin so that the temperatures are constant within.

(b) Sufficient mixing to cause the mixture temperature to be uniform.

(c) Jacket water temperature is the average of the inlet and outlet temperatures.

(d) Mixing energy negligibly small.

Transient Temperature Calculation for a Jacketed Mixing Kettle

Apply the principle of conservation of energy. Increase in internal energy of mixture = heat flow from kettle to mixture.

$$C_1W_1 \frac{dT_1}{dt} = h_pA_2 \ (T_2-T_1) \tag{1}$$

Increase in internal energy of kettle = heat flow from water to kettle
- heat flow from kettle to product.

$$C_2W_2 \frac{dT_2}{dt} = h_wA_2 \ (T_w-T_2) - h_pA_2 \ (T_2-T_1) \tag{2}$$

Increase in internal energy of jacket = heat flow from water to jacket.

$$C_3W_3 \frac{dT_3}{dt} = h_wA_3 \ (T_w-T_3) \tag{3}$$

Heat loss by water = heat flow to kettle and jacket.

$$C_4W_4 \ (T_{in}-T_{out}) = h_w \ A_2 \ (T_w-T_2) + h_wA_3 \ (T_w-T_3) \tag{4}$$

Average temperature exists within water duct.

$$T_w = (T_{in} + T_{out})/2 \tag{5}$$

The five unknown temperatures are T_1, T_2, T_3, T_w and T_{out}. T_{out} can be eliminated by combining equations (4) and (5), and constants introduced to give

$$T_w = \left[T_{in} + K_5T_2 + K_6T_3\right] /K_7 \tag{6}$$

$$F_1 = K_1(T_2-T_1) \tag{7}$$

$$F_2 = K_2(T_w-T_2) - K_3(T_2-T_1) \tag{8}$$

$$F_3 = K_4(T_w-T_3) \tag{9}$$

where

$$K_1 = \frac{h_pA_2}{C_1W_1}, \quad K_2 = \frac{h_wA_2}{C_2W_2}, \quad K_3 = \frac{h_pA_2}{C_2W_2}, \quad K_4 = \frac{h_wA_3}{C_3W_3}$$

$$K_5 = \frac{h_wA_2}{2W_4C_4}, \quad K_6 = \frac{h_wA_3}{2W_4C_4}, \quad K_7 = 1 + K_5 + K_6$$

B. Computer Program

The input data is as follows: W_1, W_2, W_3, W_4, C_1, C_2, C_3, C_4, A_1, A_2, A_3, h_p, h_w, initial time (0), time increment H, TIMAX, T_1, T_2, T_3, T_h, T_c, $T_{control}$. The numerical values are shown both in Table 1, and in the tabulated results.

The constants K_i are first calculated and the plotting subroutine given the information pertaining to the form of the plotting grid (PLOT1.)[1] and the maximum and minimum values of the ordinate and abscissa (PLOT2.). The Runge-Kutta subroutine is then initialized with the

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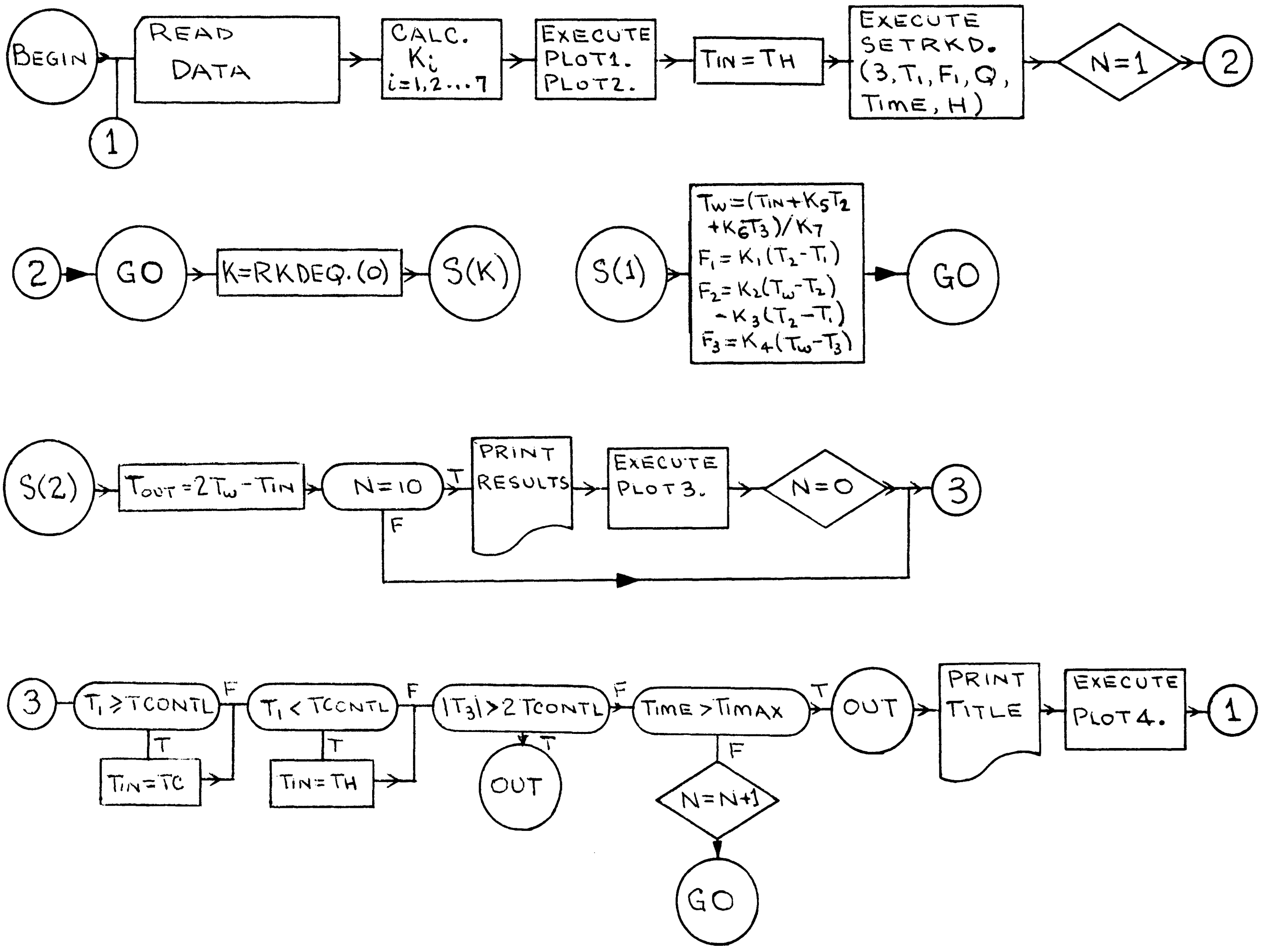
[1] Plot Subroutine by Brice Carnahan and Larry Evans.

number of equations, names of first unknown, first derivative, a dummy variable, initial time and the time increment (SETRKD.)^2. By trial, it was found that an increment of H = 0.01 was unstable and H = 0.001 was used. However, a print-out at intervals of time equal to 0.01 is satisfactory, so that a counter N is used and the Runge-Kutta subroutine executed (RKDEQ.). On each pass, the mixture temperature T_1 is checked against the control temperature setting and switched if necessary. Because of the possibility of instability, the absolute value of the jacket temperature T_3 is checked against the control temperature. It was found by trial that T_3 was the most sensitive to instability. If instability is sensed, the solution of the equations is stopped and the final stages of the plotting routine entered (PLOT4.).

On each tenth pass all temperatures and the time are printed out, and the same information is given to the plotting routine (PLOT3.) for future plotting print-out.

The final instructions of the program are to print remarks on the plot and to return to the read-in instruction. If more sets of data are available, the program will continue; if not, the program ends.

Flow Diagram



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2) Runge-Kutta Subroutine by Bruce Arden.

MAD Program and Data

```
 TED KIRKPATRICK               D002N  8              002    005    150       T1NK0000
$COMPILE MAD, EXECUTE
           R
           R***TRANSIENT TEMPERATURE IN A MIXING VESSEL
           R*****E.T.KIRKPATRICK
           RUSE SUBROUTINES RKDEQ AND PLOT
           R
            DIMENSION T(3),F(3),Q(3),DUMMY(732)
            INTEGER NPLOTS ,K
            INTEGER N
           R
            READ FORMAT DATA1,W1,W2,W3,W4,C1,C2,C3,C4,A1,A2,A3,HP,HW,TIME
           1,H,TIMAX,T(1),T(2),T(3),TH,TC,TCONTL
            VECTOR VALUES DATA1=$4F10.3/4F10.3/3F10.3/2F10.3/3F10.3/6F10.
           13*$
            PRINT FORMAT HEAD1,$    W1          W2          W3          W4    $
            VECTOR VALUES HEAD1=$1H2,7C6*$
            PRINT FORMAT ECHO1,W1,W2,W3,W4
            VECTOR VALUES ECHO1=$1H ,4F10.2*$
            PRINT FORMAT HEAD2,$    C1          C2          C3          C4    $
            VECTOR VALUES HEAD2=$1H0,7C6*$
            PRINT FORMAT ECHO2,C1,C2,C3,C4
            VECTOR VALUES ECHO2=$1H ,4F10.2*$
            PRINT FORMAT HEAD3,$    A1          A2          A3    $
            VECTOR VALUES HEAD3=$1H0,5C6*S
            PRINT FORMAT ECHO3,A1,A2,A3
            VECTOR VALUES ECHO3=$1H ,3F10.2*$
            PRINT FORMAT HEAD4,$    HP          HW         TIME        INCR=H
           1  TIMAX      $
            VECTOR VALUES HEAD4=$1H0,9C6*$
            PRINT FORMAT ECHO4,HP,HW,TIME,H,TIMAX
            VECTOR VALUES ECHO4=$1H ,5F10.2*$
            PRINT FORMAT HEAD5,$    T1          T2          T3          TH
           1   TC      TCONTL  $
            VECTOR VALUES HEAD5=$1H0,10C6*$
            PRINT FORMAT ECHO5,T(1),T(2),T(3),TH,TC,TCONTL
            VECTOR VALUES ECHO5=$1H ,6F10.2*$
            K1=(HP*A2)/(C1*W1)
            K2=(HW*A2)/(C2*W2)
            K3=(HP*A2)/(C2*W2)
            K4=(HW*A3)/(C3*W3)
            K5=(HW*A2)/(2.*W4*C4)
            K6=(HW*A3)/(2.*W4*C4)
            K7=1 K5+K6
            PRINT FORMAT KAYS, $CONSTANTS K1 TO K7$ ,K1,K2,K3,K4,K5,K6,
           1K7
            VECTOR VALUES KAYS =$1H0, 3C6/1H0,7F10.4*$
            VECTOR VALUES MARGIN=$                         TEMPERATURES$
            VECTOR VALUES N=1, ,1,0,1
START       EXECUTE PLOT1.(N,6,10,7,10)
            EXECUTE PLOT2.(DUMMY,0.7,0.,200.,50.)
            PRINT FORMAT HEAD
            VECTOR VALUES HEAD=$1H1,S14,58HTABULATED SOLUTION OF SIMULTAN
           1EOUS DIFFERENTIAL EQUATIONS /1H0,S14,47HGIVING TRANSIENT TEMP
           2ERATURES IN A MIXING TANK /1H0,S6,4HTIME,S5,2HT1,S8,2HT2,S8,2
           3HT3,S8,2HTW,S7,3HTIN,S6,4HTOUT*$
           R
            TIN=TH
            H=H/10.
            EXECUTE SETRKD.(3,T(1),F(1),Q(1),TIME,H)
GO1         N=1
GO          K=RKDEQ.(0)
            TRANSFER TO S(K)
S(1)        TW=(TIN+(K5*T(2))+(K6*T(3)))/K7
            F(1) =K1*(T(2)-T(1))
            F(2) =K2*(TW-T(2)) K3*(T(2)  T(1))
            F(3) =K4*(TW-T(3))
            TRANSFER TO GO
```

MAD Program and Data, Continued

```
S(2)      TWATR =TW
          TOUT=2*TWATR-TIN
          WHENEVER N.GE.10, TRANSFER TO CONT
          TRANSFER TO CONT2
CONT      PRINT FORMAT RESULI,TIME,T(1),T(2),T(3),TWATR,TIN,TOUT
          VECTOR VALUES RESULT = $1H , 1F10.3, 6F10.2*$
          EXECUTE PLOT3.($E$,TIME,TOUT,1)
          EXECUTE PLOT3.($*$,TIME,T(1),1)
          EXECUTE PLOT3.($V$,TIME,T(2),1)
          EXECUTE PLOT3.($J$,TIME,T(3),1)
          EXECUTE PLOT3.($W$,TIME,TWATR,1)
          N=
CONT2     WHENEVER T(1).GE.TCONTL,TIN=TC
          WHENEVER T(1).L.TCONTL,TIN=TH
          WHENEVER TIME.G.TIMAX,TRANSFER TO OUT
          N= N 1
          TRANSFER TO GO
OUT       PRINT FORMAT TITLE
          VECTOR VALUES TITLE=$1H1,S4 ,40HTRANSIENT TEMPERATURES IN A M
         1IXING TANK *$
          EXECUTE PLOT4.(32,MARGIN)
         R
          PRINT FORMAT BOTTOM
          VECTOR VALUES BOTTOM=$1H0,S40,30HTHE INDEPENDENT VARIABLE TIM
         1E //1H ,S27,55HPLOTTING CHARACTERS. T1(*),T2(V),T3(J),TWATR(W
         2),TOUT(E)*$
          END OF PROGRAM
$ DATA
7700.    350 .    1500.    84000.
.95      .1       .1       1.
400.     400.     400.
75.      500.
0.0      0.01      .7
100.0    100.     100.0    200.0    50.0     150.
```

Computer Output

```
 W1         W2         W3         W4
7700.00    3500.00    1500.00   84000.00

 C1         C2         C3         C4
  0.95       0.10       0.10       1.00

 A1         A2         A3
400.00     400.00     400.00

 HP         HW        TIME       INCR=H     TIMAX
  75.00     500.00      0.00       0.01       0.70

 T1         T2         T3         TH         TC        TCONTL
100.00     100.00     100.00     200.00      50.00     150.00
```

Computer Output (Tabular)

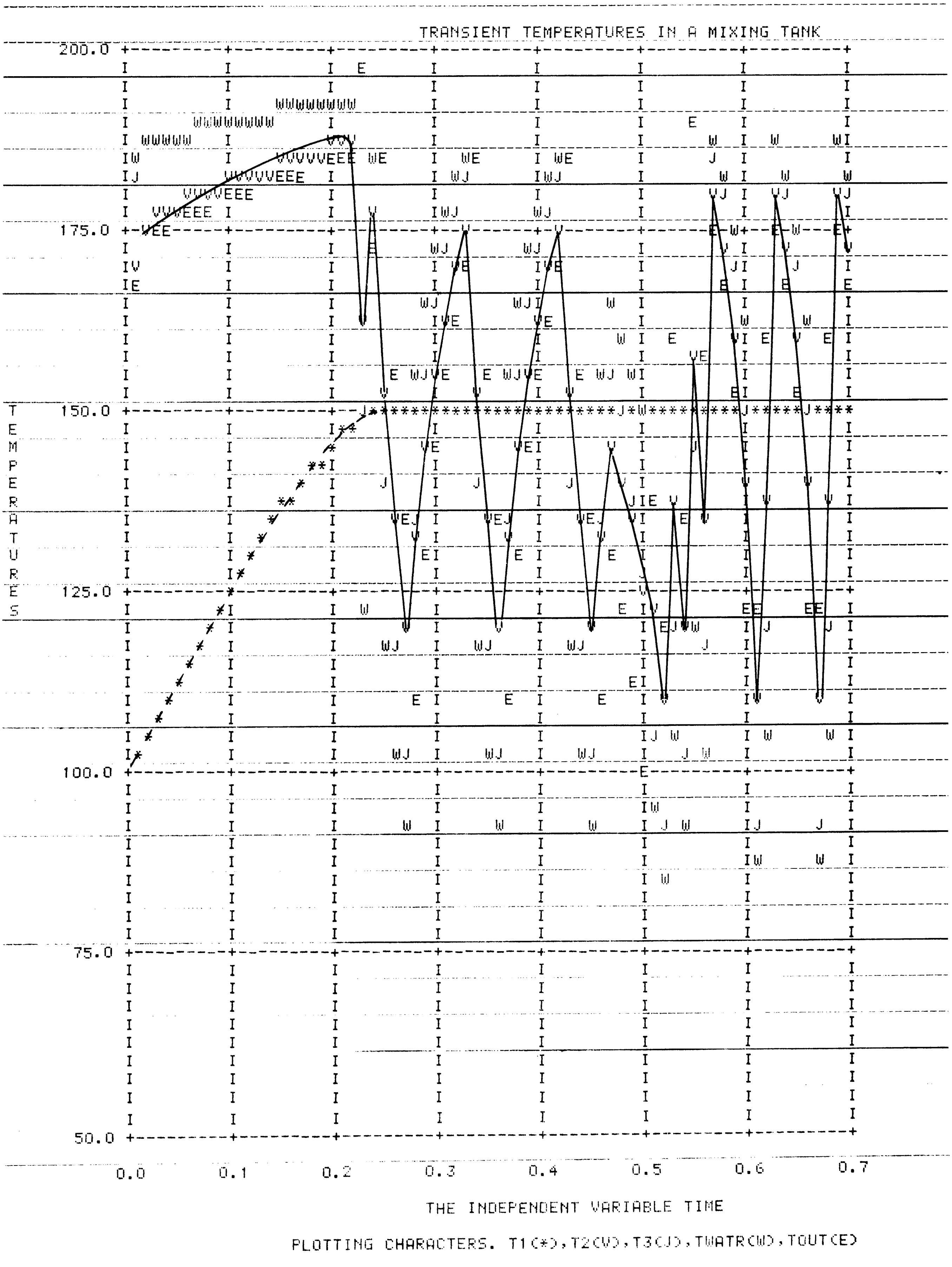
```
           TABULATED SOLUTION OF SIMULTANEOUS DIFFERENTIAL EQUATIONS
           GIVING TRANSIENT TEMPERATURES IN A MIXING TANK

  TIME       T1         T2         T3         TW         TIN        TOUT

 0.010     101.99     171.10     183.03     183.86     200.00     167.71
 0.020     104.90     176.01     186.85     186.92     200.00     173.84
 0.030     107.78     176.95     187.42     187.46     200.00     174.91
 0.040     110.58     177.66     187.82     187.84     200.00     175.69
 0.050     113.29     178.34     188.18     188.21     200.00     176.42
 0.060     115.91     178.99     188.54     188.57     200.00     177.14
 0.070     118.46     179.63     188.89     188.92     200.00     177.83
 0.080     120.93     180.25     189.23     189.25     200.00     178.50
 0.090     123.33     180.85     189.55     189.58     200.00     179.15
 0.100     125.65     181.43     189.87     189.89     200.00     179.79
 0.110     127.90     181.99     190.18     190.20     200.00     180.40
 0.120     130.09     182.53     190.47     190.50     200.00     180.99
 0.130     132.21     183.06     190.76     190.78     200.00     181.57
 0.140     134.26     183.58     191.04     191.06     200.00     182.13
 0.150     136.25     184.07     191.31     191.33     200.00     182.67
 0.160     138.18     184.56     191.58     191.60     200.00     183.19
 0.170     140.06     185.03     191.83     191.85     200.00     183.70
 0.180     141.87     185.48     192.08     192.10     200.00     184.20
 0.190     143.63     185.92     192.32     192.34     200.00     184.68
 0.200     145.34     186.35     192.55     192.57     200.00     185.14
 0.210     147.00     186.76     192.78     192.79     200.00     185.59
 0.220     148.60     187.16     193.00     193.01     200.00     186.03
 0.230     150.11     162.93     149.36     123.66      50.00     197.31
 0.240     150.01     176.57     184.18     186.20     200.00     172.41
 0.250     150.10     153.07     141.25     117.35      50.00     184.70
 0.260     150.07     134.62     117.50     103.11      50.00     156.21
 0.270     149.98     120.01     102.37      92.88      50.00     135.76
 0.280     149.89     133.39     135.85     154.94     200.00     109.87
 0.290     149.85     145.72     154.19     165.16     200.00     130.32
 0.300     149.85     155.80     165.46     172.46     200.00     144.91
 0.310     149.89     163.65     173.00     177.78     200.00     155.57
 0.320     149.96     169.64     178.30     181.72     200.00     163.44
 0.330     150.05     174.16     182.15     184.65     200.00     169.30
 0.340     150.11     152.94     141.14     117.27      50.00     184.53
 0.350     150.08     134.62     117.49     103.10      50.00     156.21
 0.360     149.99     120.02     102.37      92.88      50.00     135.76
 0.370     149.90     133.39     135.85     154.94     200.00     109.88
 0.380     149.86     145.72     154.19     165.16     200.00     130.33
 0.390     149.86     155.80     165.46     172.46     200.00     144.92
 0.400     149.90     163.65     173.00     177.78     200.00     155.57
 0.410     149.97     169.64     178.30     181.72     200.00     163.44
 0.420     150.06     174.16     182.15     184.65     200.00     169.30
 0.430     150.12     152.94     141.14     117.27      50.00     184.53
 0.440     150.09     134.62     117.49     103.10      50.00     156.21
 0.450     150.00     120.02     102.37      92.88      50.00     135.76
 0.460     149.90     133.39     135.85     154.94     200.00     109.88
 0.470     149.86     145.73     154.19     165.16     200.00     130.33
 0.480     149.65     139.28     148.77     161.01     200.00     122.02
 0.490     149.89     134.88     137.07     155.89     200.00     111.78
 0.500     149.72     124.93     128.50     149.42     200.00      98.84
 0.510     149.98     122.02     104.02      94.16      50.00     138.32
 0.520     149.87     108.88      92.09      85.44      50.00     120.88
 0.530     150.08     137.30     119.70     104.82      50.00     159.64
 0.540     150.02     120.17     102.49      92.98      50.00     135.95
 0.550     150.10     156.52     144.11     119.56      50.00     189.13
 0.560     150.12     134.82     117.65     103.23      50.00     156.46
 0.570     150.03     178.95     186.15     187.73     200.00     175.46
 0.580     149.82     172.36     180.69     183.50     200.00     167.01
 0.590     149.68     160.50     170.40     175.77     200.00     151.54
 0.600     149.67     140.15     149.48     161.56     200.00     123.13
 0.610     149.88     110.31      93.27      86.36      50.00     122.71
 0.620     150.11     137.38     119.77     104.87      50.00     159.75
 0.630     150.05     179.09     186.27     187.82     200.00     175.64
 0.640     149.84     172.37     180.70     183.51     200.00     167.03
 0.650     149.70     160.50     170.41     175.77     200.00     151.54
 0.660     149.70     140.16     149.48     161.57     200.00     123.13
 0.670     149.90     110.32      93.28      86.36      50.00     122.72
 0.680     150.13     137.39     119.77     104.88      50.00     159.75
 0.690     150.07     179.10     186.27     187.82     200.00     175.65
 0.700     149.86     172.38     180.70     183.52     200.00     167.03
```

Example Problem No. 70

Computer Output (Plotted)



Discussion of Results

The tabulated results show the temperatures of the mixture, vessel, jacket, tank water, and inlet and outlet conditions for time increments of 0.01 hours. The plotted results show the same information in graphical form, with the characters denoting the temperatures as follows:

Character	Temperature
*	Mixture
V	Vessel
J	Jacket
W	Water
E	Exit Water

A curve has been drawn through points V to show the cyclical variations of temperature within the vessel walls.

Critique

Probably the most instructive features of this problem are the difficulties incurred by the instability of the Runge-Kutta solutions when too large a step interval is chosen. In order to get a solution to this problem a very small interval must be taken, and this means that many iterations must be used to yield just one solution. It is true that the temperature curves could be made smoother by a different choice of the water flow rates, and the heat transfer convective coefficients. However, the values chosen for the sample problem are probably close to the actual values. The temperature changes would initially be quite rapid.

It should be pointed out that an analog computer could be used to "solve" this problem with no stability problems of the type incurred with the digital computer. However, it would be difficult to study both the small temperature differences at the beginning and end of the curve, together with the intermediate results unless a fairly complicated switching circuit was used to change the scale factor at the appropriate points in the solution.

Example Problem No. 71

SURGE SYSTEM OSCILLATIONS

by

Harry P. Hale
Department of Mechanical Engineering
Wayne State University

Course: Hydraulic Machinery   Credit hours: 3   Level: Senior

Statement of Problem

In hydroelectric plants quite a lot of study has been given to unsteady flow in the long hydraulic tunnels from the headwater to the hydraulic turbine. The calculations are tedious since the equations involved are nonlinear differential equations. On the next page is shown a schematic of the system involved and the various dimensions. The control valve is ordinarily connected to a turbine through a speed control governor. The overall problem involving tunnels, surge tanks, turbine, and governor is quite complicated. In this problem you will not be concerned with the turbine or governor.

The control valve is so adjusted that the water velocity in the tunnel is 6.85 feet/second.

(a) Set up the differential equations for unsteady flow. Assume the mass of water in the surge tank is negligible, the tunnel walls are inelastic, and the fluid is incompressible. (The 6.85 ft/sec is the maximum velocity in the tunnel.)

(b) Find the period of mass oscillation neglecting friction if the control valve is suddenly closed.

(c) Set up the differential equation considering friction if the control valve is suddenly closed.

(d) Solve the differential equation of part (c) using the Runge-Kutta routine. To evaluate the friction factor, f, use the Moody Chart and assume the tunnel is concrete with the roughness factor, ϵ , equal to 0.005 feet. As a first approximation assume f is constant and evaluate it at a tunnel velocity of 6.85 ft/sec. Assume the water temperature is 70° F. Carry your solution out to about 5 cycles.

References:

Engineering Fluid Mechanics by Jaeger

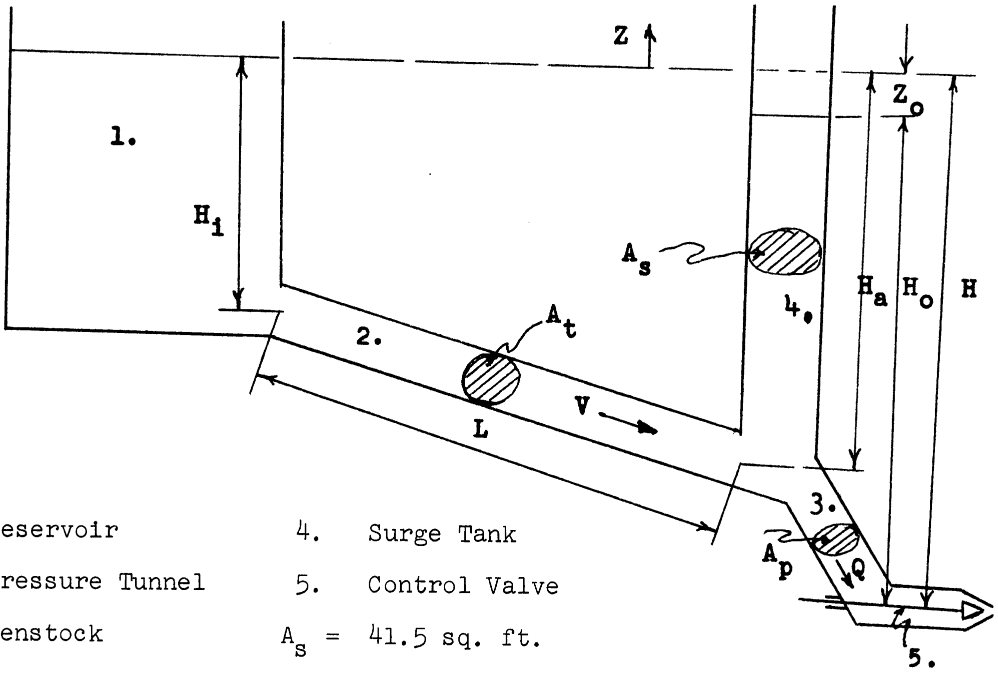
Hydraulic Transients by Rich

Solution

Refer to the figures shown below.

$$dW\sin\alpha \quad = \quad \gamma A_t dL\sin\alpha \quad = \quad \gamma A_t dh$$

$$\gamma A_t dh \; - \; A_t dp \; - \; \gamma A_t dh_f \quad = \quad \frac{\gamma}{g} A_t dL \frac{dV}{dt}$$



1. Reservoir
2. Pressure Tunnel
3. Penstock

L = 8500 feet

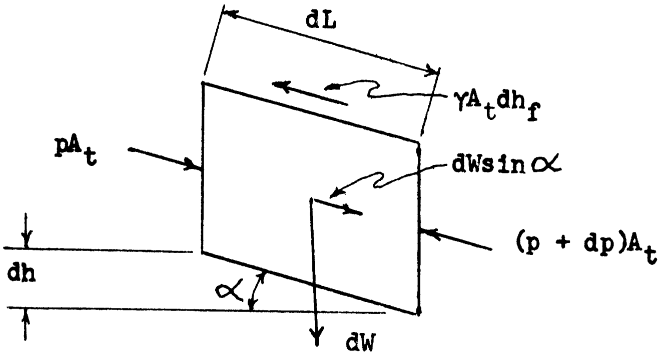
A_t = 31.5 sq. ft.

4. Surge Tank
5. Control Valve

A_s = 41.5 sq. ft.

V_o = 6.85 ft/sec.

Schematic of Hydraulic System



Free Body Diagram

Integrating dL between 0 and L, dh between $H_1 - V^2/2g$ and H_a, dp/γ between H_1 and $H_a + Z$, and dh_f between 0 and L, the following equation is obtained:

I. $$\frac{L}{g}\frac{dV}{dt} + Z + \frac{V^2}{2g} + h_f = 0$$

From the continuity equation the following is obtained:

II. $$VA_t = A_s\frac{dZ}{dt} + Q$$

where, γ = specific weight, lb/ft^3

V = tunnel velocity, ft/sec

h_f = head loss due to friction, feet.

Q = flow in penstock, cubic feet/second.

(b) At $t = 0-$, $Q = Q_o$; At $t = 0+$, $Q = 0$ and $A_tV = A_s dZ/dt$. Also if the $V^2/2g$ term is neglected, $(L/g)(dV/dt) + Z = 0$. Combining these two equations,

$$\frac{d^2Z}{dt^2} + \frac{gA_t}{LA_s} = 0$$

From this the natural frequency, $\omega = \sqrt{gA_t/LA_s}$ radians/sec. The period $T = 2\pi/\omega$ = 117 seconds.

(c) Neglect the $V^2/2g$ term as it will be small compared with the friction loss. Using the Darcey-Weisback formula, $h_f = f\ (L/D)(V^2/2g)$, the following formula is obtained:

$$d^2Z/dt^2 + 0.00287Z + 0.00208\ (dZ/dt)^2 = 0$$

(The friction factor, f, turns out to be about 0.02)

(d) The above second order differential equation will have to be broken down into two first order equations as follows:

Let $Z(1) = Z$; let $Z(2) = dZ/dt = F(1)$

The two first order equations are:

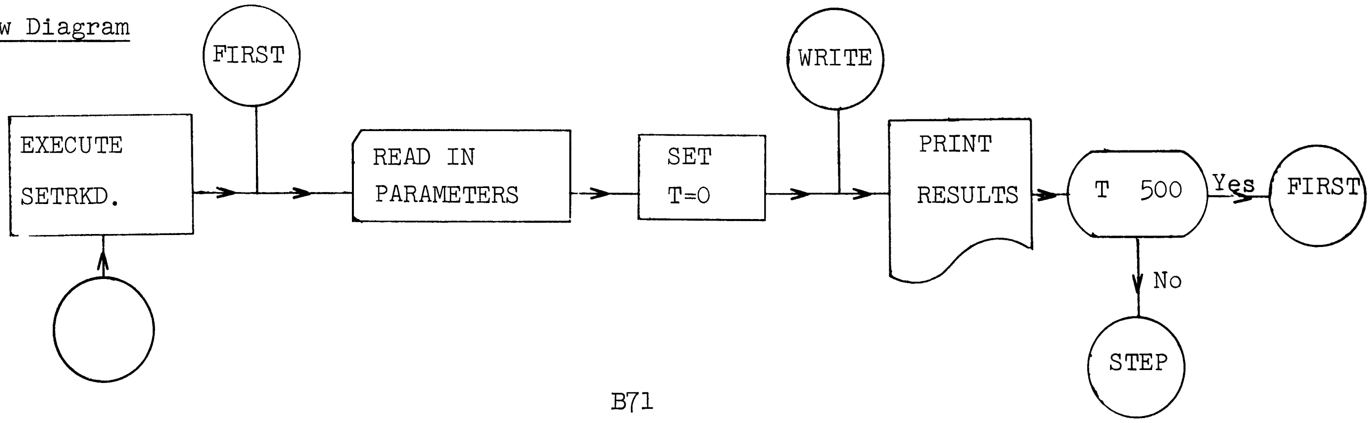
$F(1) = Z(2)$

$F(2) = -A\cdot Z(1) - B\cdot Z(2)\cdot Z(2)$

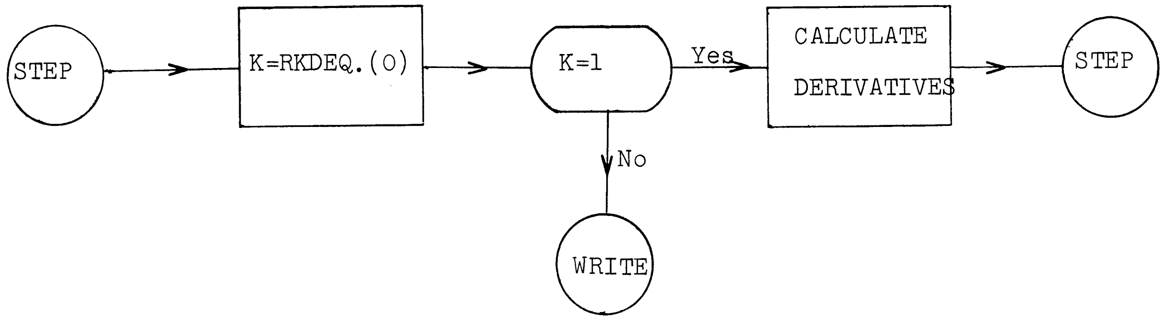
At $t = 0$, dZ/dt or $F(1) = 0$

At $t = 0$, Z or $Z(1) = Z_o = h_f = -19.6$ feet.

Flow Diagram



Flow Diagram (continued)



MAD Program

```
          RSURGE SYSTEM OSCILLATIONS
           DIMENSION F(2),Q(2),Z(2)
           INTEGER K
           EXECUTE SETRKD.(2,Z(1),F(1),Q,T,H)
FIRST      READ FORMAT INPUT,A,B,Z(1),Z(2),H
           PRINT FORMAT HEAD
           T=0.
WRITE      PRINT FORMAT RESULT,T,Z(1),Z(2)
           WHENEVER T.G.500., TRANSFER TO FIRST
STEP       K=RKDEQ.(0)
           WHENEVER K.E.1
               F(1)=Z(2)
               F(2)=-A*Z(1)-B*.ABS.Z(2)*Z(2)
               TRANSFER TO STEP
           END OF CONDITIONAL
           TRANSFER TO WRITE
           VECTOR VALUES HEAD=$1H1,S13,47HTABULATED SOLUTION OF THE DIFF
          1ERENTIAL EQUATION/1H0,S16,1HT,S18,1HZ,S18,1HV*$
           VECTOR VALUES INPUT=$5F10.5*$
           VECTOR VALUES RESULT=$1H,6F19.4*$
           END OF PROGRAM
```

Example Problem No. 71

Computer Output

```
TABULATED SOLUTION OF THE DIFFERENTIAL EQUATION

      T                  Z                  W
   0.0000            -19.6000            0.0000
   4.0000            -19.1518            0.2231
   8.0000            -17.8295            0.4353
  12.0000            -15.6981            0.6261
  16.0000            -12.8617            0.7864
  20.0000             -9.4574            0.9090
  24.0000             -5.6477            0.9884
  28.0000             -1.6118            1.0217
  32.0000              2.4635            1.0082
  36.0000              6.3926            0.9491
  40.0000              9.9996            0.8478
  44.0000             13.1255            0.7095
  48.0000             15.6343            0.5404
  52.0000             17.4181            0.3483
  56.0000             18.4010            0.1414
  60.0000             18.5412           -0.0715
  64.0000             17.8337           -0.2808
  68.0000             16.3132           -0.4763
  72.0000             14.0542           -0.6486
  76.0000             11.1663           -0.7896
  80.0000              7.7882           -0.8928
  84.0000              4.0801           -0.9540
  88.0000              0.2153           -0.9710
  92.0000             -3.6284           -0.9436
  96.0000             -7.2767           -0.8738
 100.0000            -10.5670           -0.7653
 104.0000            -13.3546           -0.6234
 108.0000            -15.5184           -0.4546
 112.0000            -16.9653           -0.2663
 116.0000            -17.6337           -0.0667
 120.0000            -17.4951            0.1357
 124.0000            -16.5570            0.3314
 128.0000            -14.8654            0.5110
 132.0000            -12.5025            0.6657
 136.0000             -9.5824            0.7885
 140.0000             -6.2445            0.8739
 144.0000             -2.6461            0.9183
 148.0000              1.0455            0.9205
 152.0000              4.6615            0.8807
 156.0000              8.0387            0.8016
 160.0000             11.0270            0.6871
 164.0000             13.4953            0.5426
 168.0000             15.3364            0.3747
 172.0000             16.4712            0.1907
 176.0000             16.8511           -0.0014
 180.0000             16.4603           -0.1932
 184.0000             15.3179           -0.3756
 188.0000             13.4796           -0.5398
 192.0000             11.0345           -0.6779
 196.0000              8.0998           -0.7836
 200.0000              4.8148           -0.8524
 204.0000              1.3335           -0.8815
 208.0000             -2.1832           -0.8702
 212.0000             -5.5751           -0.8195
 216.0000             -8.6899           -0.7322
 220.0000            -11.3895           -0.6127
 224.0000            -13.5556           -0.4665
```

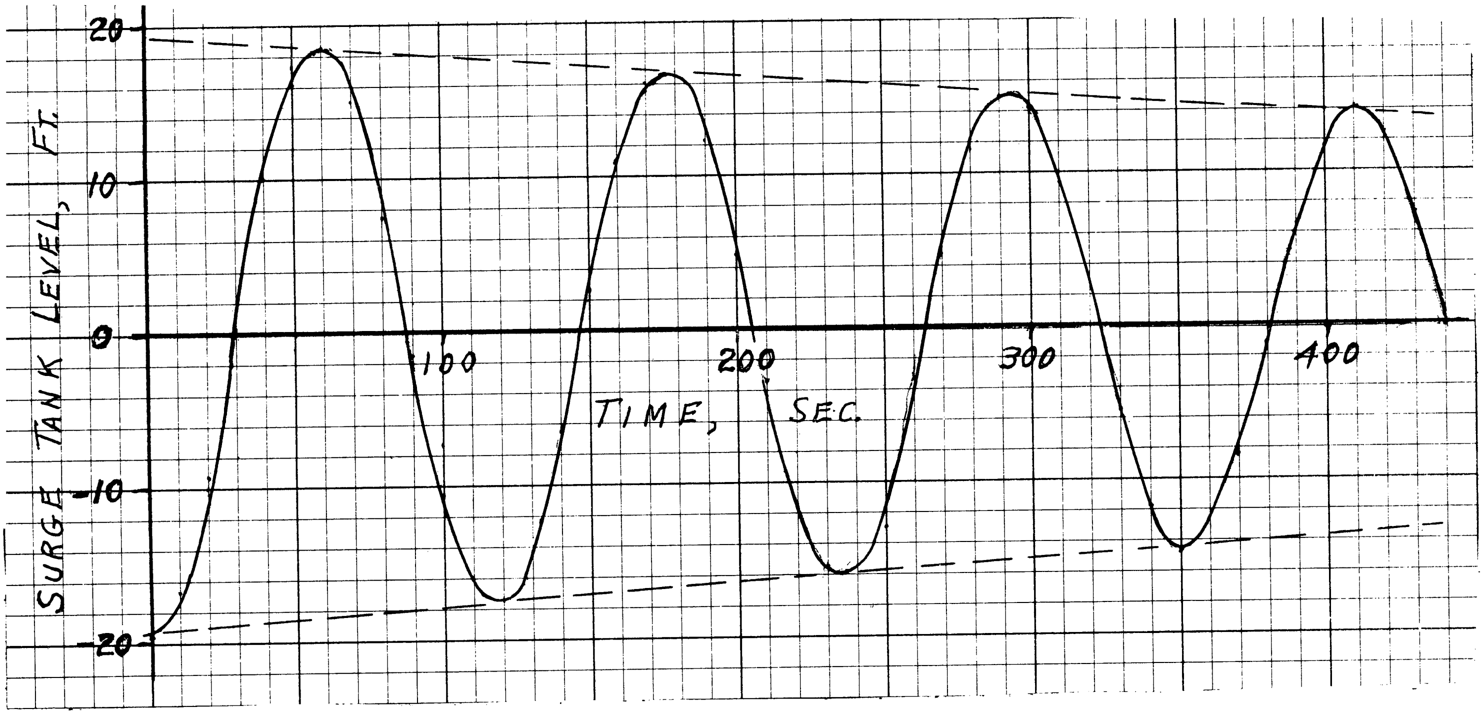
Computer Output (continued)

```
  228.0000          -15.0944          -0.3001
  232.0000          -15.9396          -0.1209
  236.0000          -16.0550           0.0634
  240.0000          -15.4362           0.2447
  244.0000          -14.1136           0.4140
  248.0000          -12.1511           0.5633
  252.0000           -9.6433           0.6856
  256.0000           -6.7099           0.7753
  260.0000           -3.4894           0.8287
  264.0000           -0.1319           0.8437
  268.0000            3.2082           0.8201
  272.0000            6.3795           0.7596
  276.0000            9.2399           0.6653
  280.0000           11.6630           0.5418
  284.0000           13.5428           0.3947
  288.0000           14.7979           0.2306
  292.0000           15.3742           0.0565
  296.0000           15.2469          -0.1199
  300.0000           14.4229          -0.2905
  304.0000           12.9420          -0.4470
  308.0000           10.8759          -0.5820
  312.0000            8.3234          -0.6892
  316.0000            5.4059          -0.7639
  320.0000            2.2603          -0.8029
  324.0000           -0.9674          -0.8049
  328.0000           -4.1296          -0.7703
  332.0000           -7.0834          -0.7011
  336.0000           -9.6969          -0.6009
  340.0000          -11.8549          -0.4742
  344.0000          -13.4632          -0.3270
  348.0000          -14.4519          -0.1656
  352.0000          -14.7785           0.0029
  356.0000          -14.4291           0.1711
  360.0000          -13.4207           0.3310
  364.0000          -11.8022           0.4750
  368.0000           -9.6514           0.5962
  372.0000           -7.0708           0.6890
  376.0000           -4.1825           0.7495
  380.0000           -1.1213           0.7752
  384.0000            1.9714           0.7653
  388.0000            4.9547           0.7207
  392.0000            7.6942           0.6439
  396.0000           10.0680           0.5386
  400.0000           11.9717           0.4097
  404.0000           13.3221           0.2630
  408.0000           14.0608           0.1050
  412.0000           14.1557          -0.0576
  416.0000           13.6034          -0.2174
  420.0000           12.4304          -0.3667
  424.0000           10.6935          -0.4983
  428.0000            8.4756          -0.6062
  432.0000            5.8821          -0.6854
  436.0000            3.0351          -0.7326
  440.0000            0.0669          -0.7458
  444.0000           -2.8861          -0.7250
  448.0000           -5.6897          -0.6715
  452.0000           -8.2182          -0.5881
  456.0000          -10.3594          -0.4786
  460.0000          -12.0190          -0.3482
  464.0000          -13.1248          -0.2027
  468.0000          -13.6287          -0.0484
  472.0000          -13.5091           0.1080
  476.0000          -12.7720           0.2591
  480.0000          -11.4529           0.3978
  484.0000           -9.6153           0.5174
  488.0000           -7.3468           0.6124
  492.0000           -4.7545           0.6786
  496.0000           -1.9600           0.7133
  500.0000            0.9075           0.7150
  504.0000            3.7167           0.6843
```

Discussion of Results and Critique

The results indicate that the surge tank oscillations die out at a slow rate, as might be expected. Although these results are not startling, the main purpose of such a problem is to show the student how a rather tedious problem can be solved rather easily with the aid of a computer. Also, more realistic conditions could be used such as setting up an equation for the friction factor, f, in terms of Reynolds Number instead of using a constant value for f. Also a more complicated system could be analyzed by including the turbine and governor and checking stability of the overall system. The data for this problem was taken from an actual installation as noted in the reference, Engineering Fluid Mechanics by Jaeger.

It would probably be better to analyze the differential equation with an analog computer first so that the student could quickly see how changes in parameters affected the results. Then, for more precise answers, the digital computer should be used.



Surge System Oscillations

Example Problem No. 72

ANALOG ANALYSIS OF SINUSOIDALLY EXCITED SPRING-MASS-DASHPOT SYSTEM

by

C. W. Messersmith

School of Mechanical Engineering

Purdue University

Course: Mechanical Engineering Laboratory Credit Hours: 2 Level: Junior

Statement of the Problem

A spring-mass-dashpot system has linear characteristics. If x is the displacement of the mass (M) from its rest position, if -kx is the spring force tending to restore the mass to its rest position, and if the damping force is - $c\frac{dx}{dt}$; write the differential equation for the system assuming the mass to be acted upon by a force of the form F sin ωt. For a particular system the mass is 3.22 lb_m, the spring constant (k) is 400 lb_f per ft, the dashpot factor (c) is 2 lb_f sec per ft and the exciting force has a maximum amplitude of 20 lb_f. Use the analog computer to determine the amplitude of vibration of the mass for frequencies of the forcing function varying from about one-half to about twice the natural frequency of the system and compare your results with analytically determined values.

Solution

$$0.1 \frac{d^2x}{dt^2} + 2 \frac{dx}{dt} + 400x = 20 \sin \omega t$$

let $s = \frac{d}{dt}$

then $4000x + 20\,sx + s^2x - 200 \sin \omega t = 0$

the natural frequency = $\sqrt{4000} \approx 63$ radians per sec

If $x_{max} = 0.16$, then for optimum amplifier operation,

display 4000(0.16) = 640, say 600x;

$\frac{600x}{63}$, say 10 sx; and $\frac{600x}{(63)^2}$, say $\frac{s^2x}{6}$

The equation for implementing the computation then becomes

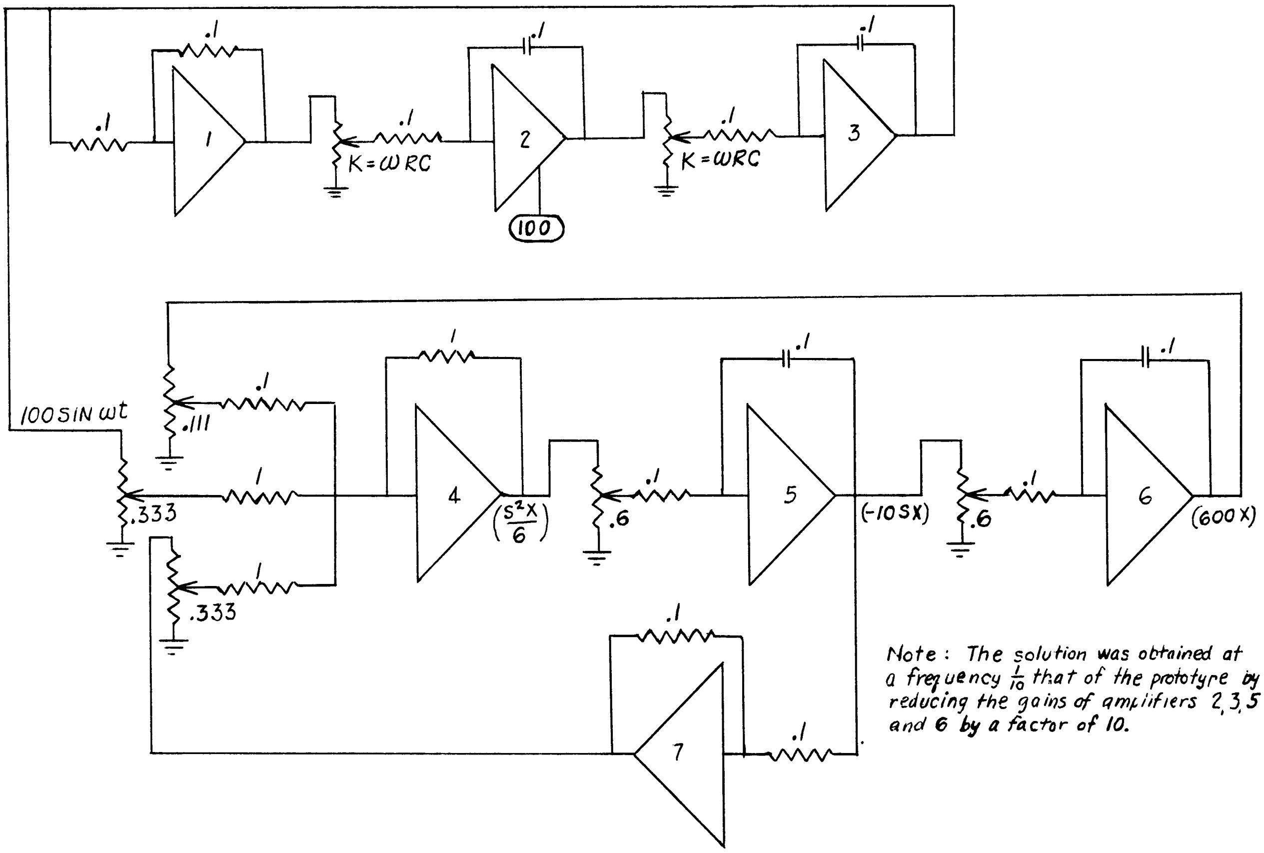
$$\frac{4000}{600}(600x) + \frac{20}{10}(10\,sx) + 6\left(\frac{s^2x}{6}\right) - 200 \sin \omega t = 0$$

or

$$1.111(600x) + 0.333\,(10\,sx) + \left(\frac{s^2x}{6}\right) - 33.3 \sin \omega t = 0$$

Computer Circuit

The computer circuit to solve the equation including generation of the forcing function is shown below. The computer was operated at frequencies one-tenth that of the problem as indicated by the note on the sketch.



Note: The solution was obtained at a frequency $\frac{1}{10}$ that of the prototype by reducing the gains of amplifiers 2, 3, 5 and 6 by a factor of 10.

Computer Circuit

Computer Output

Fig. 1 shows a curve for the analytical steady-state solution and various points obtained from the computer solution. Figs. 2a, 2b, and 2c show records of the computer solution made at three different frequencies of the forcing function.

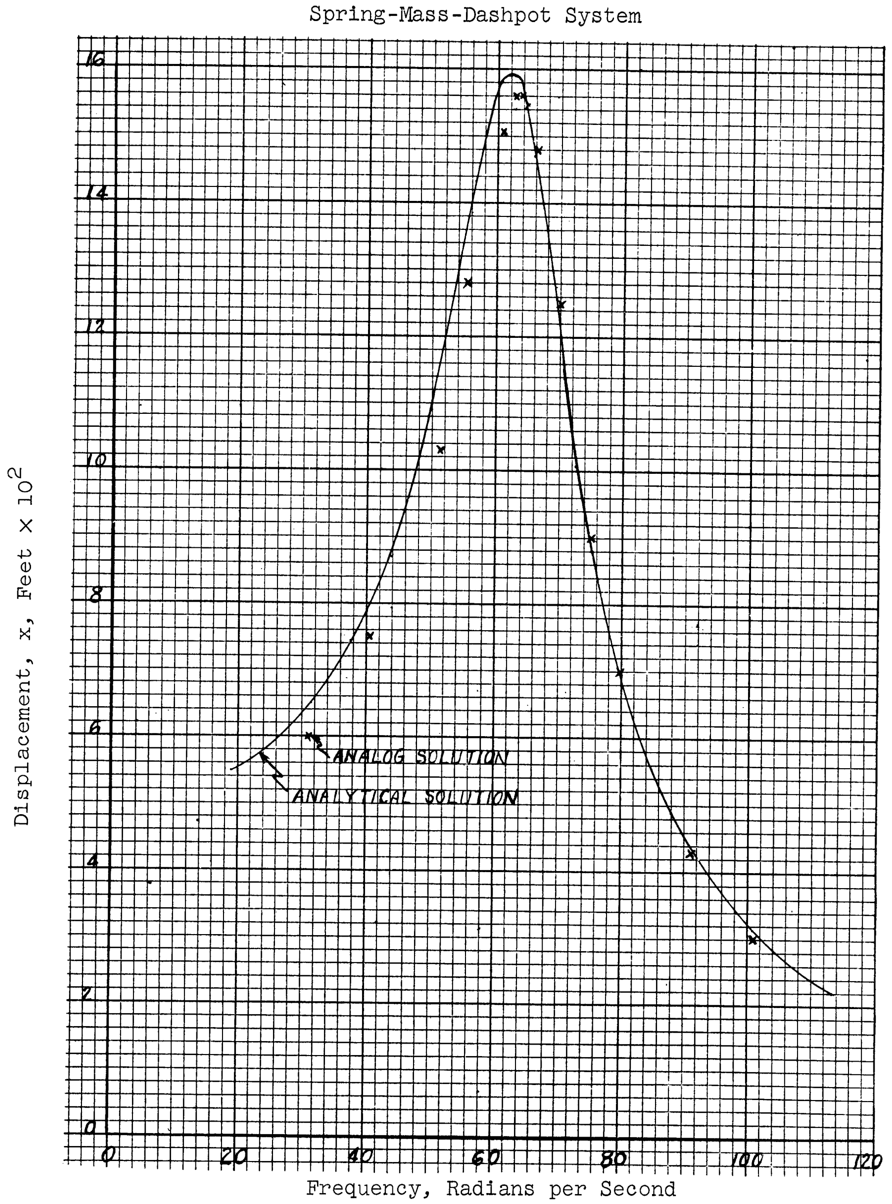


Fig. 1. Solution of the Equation $0.1 \frac{d^2x}{dt^2} + 2 \frac{dx}{dt} + 400x = 20 \sin \omega t$

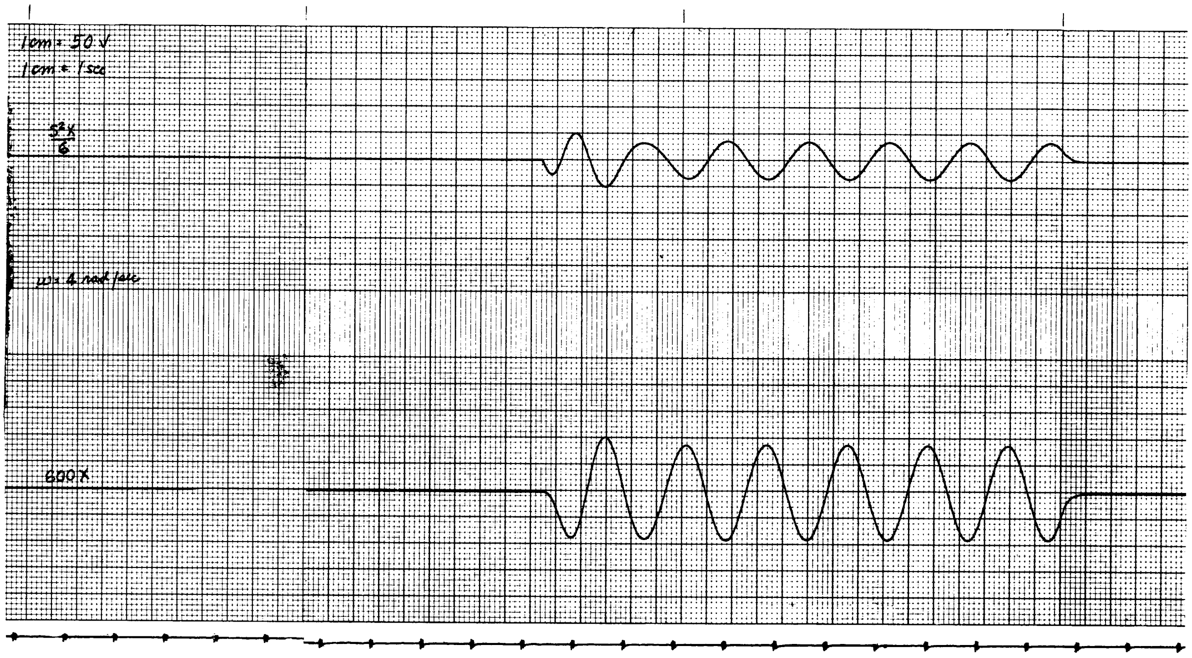


Fig. 2a
Typical Graphical Results

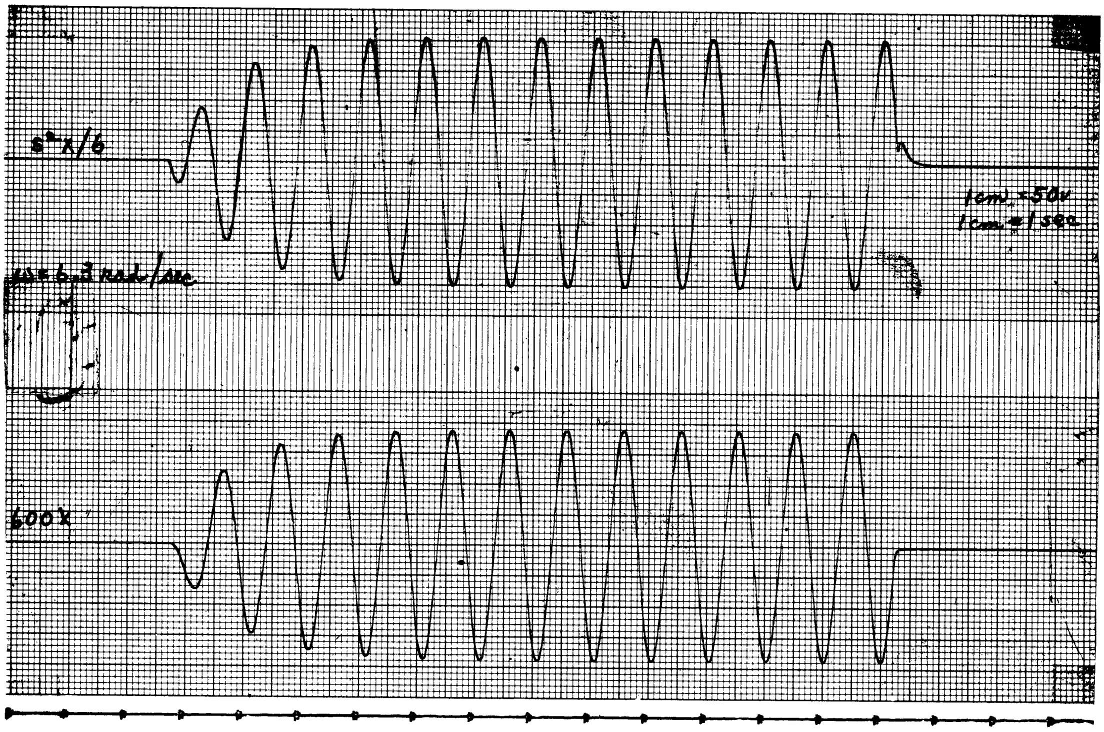


Fig. 2b

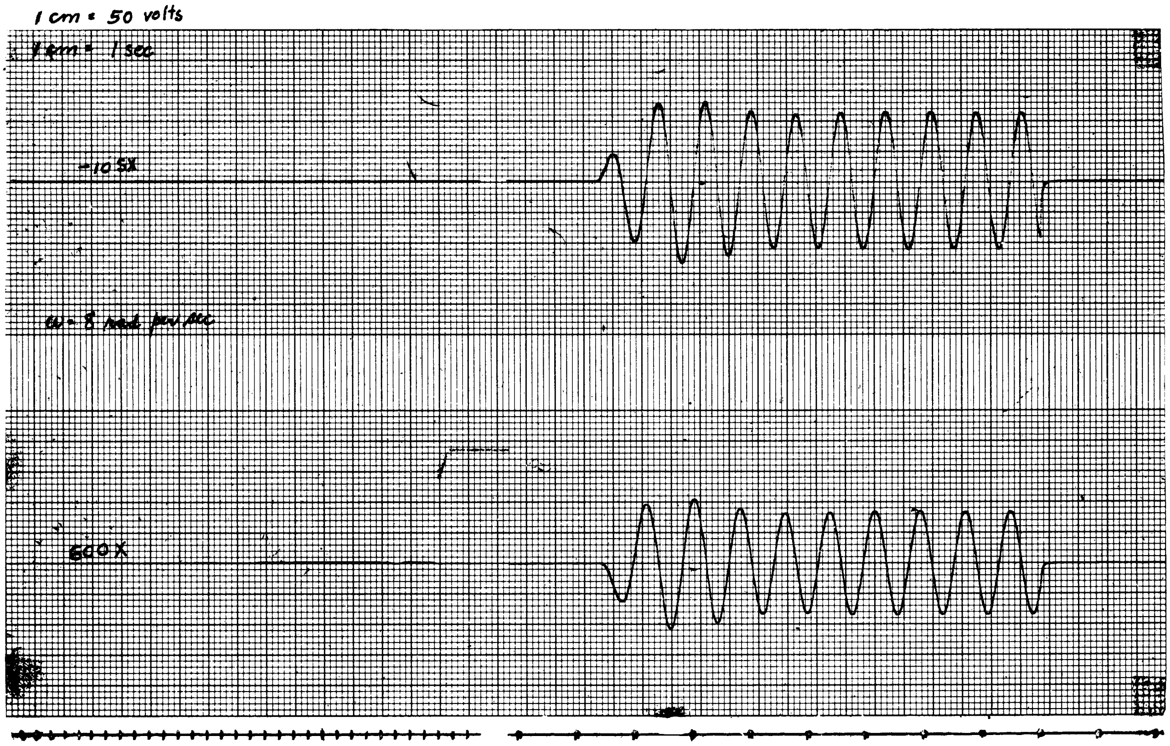


Fig. 2c

Critique

This problem, with certain modifications caused by available equipment, has been used in a laboratory course in which the students had their first experience with analog computers. The fact that they were able to determine the solution analytically as a check on the computer tended to build confidence in the computer. The students were very much interested and readily realized the ease with which results could be obtained for a variety of parameter values.

Example Problem No. 73

BEVEL GEAR SPEED REDUCER FORCE ANALYSIS

by

J. Raymond Pearson

Department of Mechanical Engineering

The University of Michigan

Course: Machine Design I Credit hours: 3 Level: Senior

Statement of Problem

A firm wishes to produce speed reducers of this type for various ratings, and plans to replace the tapered roller bearings with ball bearings. The ball bearings are to be mounted, as shown in Figure 12 of Machine Analysis and Design Problems by Alvord, Pearson, and Hall, so that the thrust reactions occur on one bearing at a time.

Input data and symbols to be used:

Input torque and external forces on the shafts

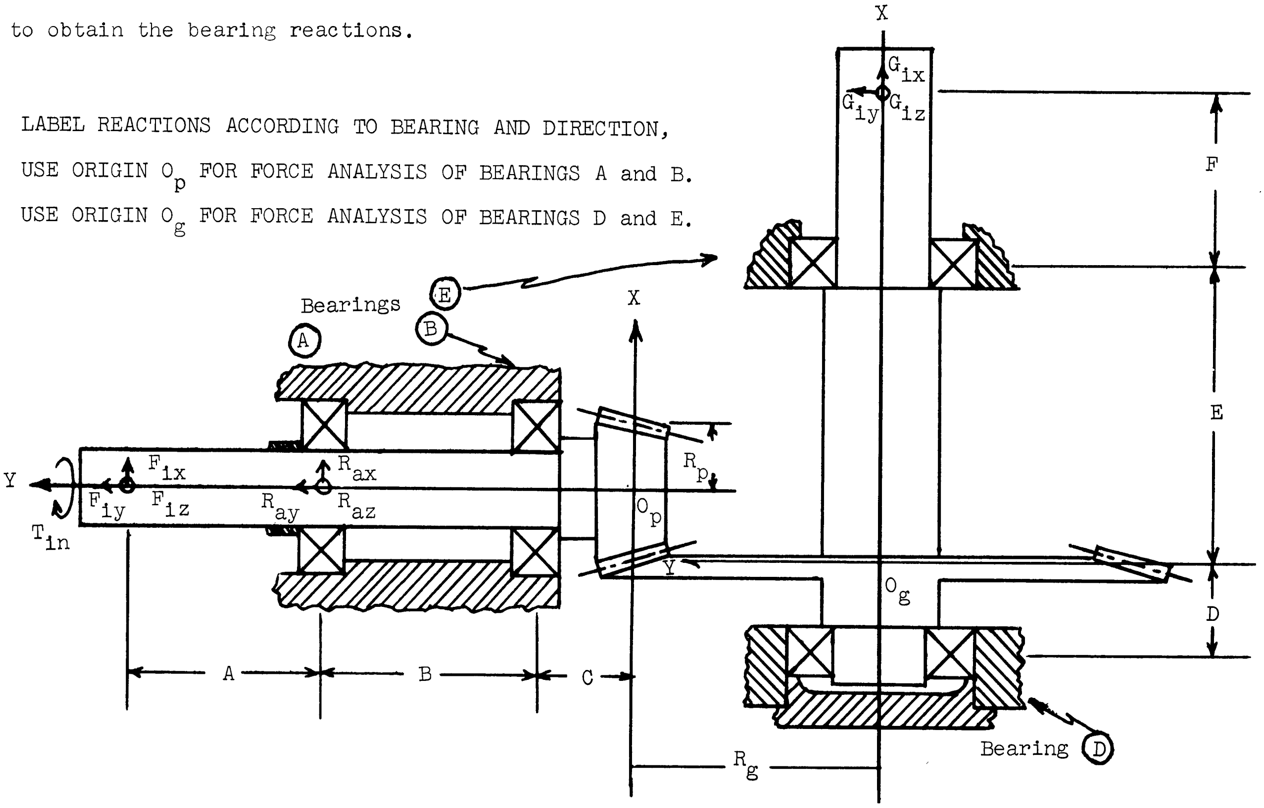
Tin = 500 in-lb	Fix = 100lb	Gix = 200lb
	Fiy = 150lb	Giy = 300lb
	Fiz = 200lb	Giz = 400lb

Physical dimensions:

Pressure angle (ϕ) = 20°, velocity ratio = 4:1, R_p = ___, Rg = ___, A = ___, B = ___, C = ___, D = ___, E = ___, F = ___.

Measure Figure 12 to obtain dimensions where they are not given. Label the reactions on the bearings as shown.

Prepare a general force analysis to determine all bearing reactions. Write a MAD program for solution on the digital computer. Run the program, using the given input data, to obtain the bearing reactions.


LABEL REACTIONS ACCORDING TO BEARING AND DIRECTION,

USE ORIGIN O_p FOR FORCE ANALYSIS OF BEARINGS A and B.

USE ORIGIN O_g FOR FORCE ANALYSIS OF BEARINGS D and E.

Example Problem No. 74

CAM DESIGN PROPOSAL ANALYSIS

by

J. Raymond Pearson
Department of Mechanical Engineering
The University of Michigan

Course: Machine Design II   Credit hours: 3   Level: Senior

Statement of the Problem

Attached is the drawing of a proposed design for a cam drive with a straight line follower. The cam is an eccentric circular disk.

Space limitations require that the sum of the cam and roller radii be held at 2.500 inches and that the roller width be 0.5xradius of the roller. The output load and the friction load are negligible. The spring load S equals - 100 - kX.

The following is a suggested list of symbols and significant constants:

R_2 = Eccentric radius   in.
R_3 = Follower roller radius   in.
D = R_3/R_2 ratio   -
M_s = Follower mass = 0.01 + roller & pin mass $\# s^2/in.$
B = Roller width   in.
A = Follower acceleration in/s^2
ω = Cam speed   rad/s
X = Disk eccentricity   in.
θ = Cam angle   deg
I = Inertia force   #
S = Spring force   #
K = Spring stiffness rate   #/in
β = Angle of line of centers   deg
F = Contact force   #
σ = Contact stress $\# /in^2$
P = Roller pin bearing projected pressure $\# /in^2$
Y = Follower displacement   in.
M_{d1} = Modulus of Elasticity of roller material in/s^2
M_{d2} = Modulus of Elasticity of cam material   "
C = Sum of cam & roller radii   "

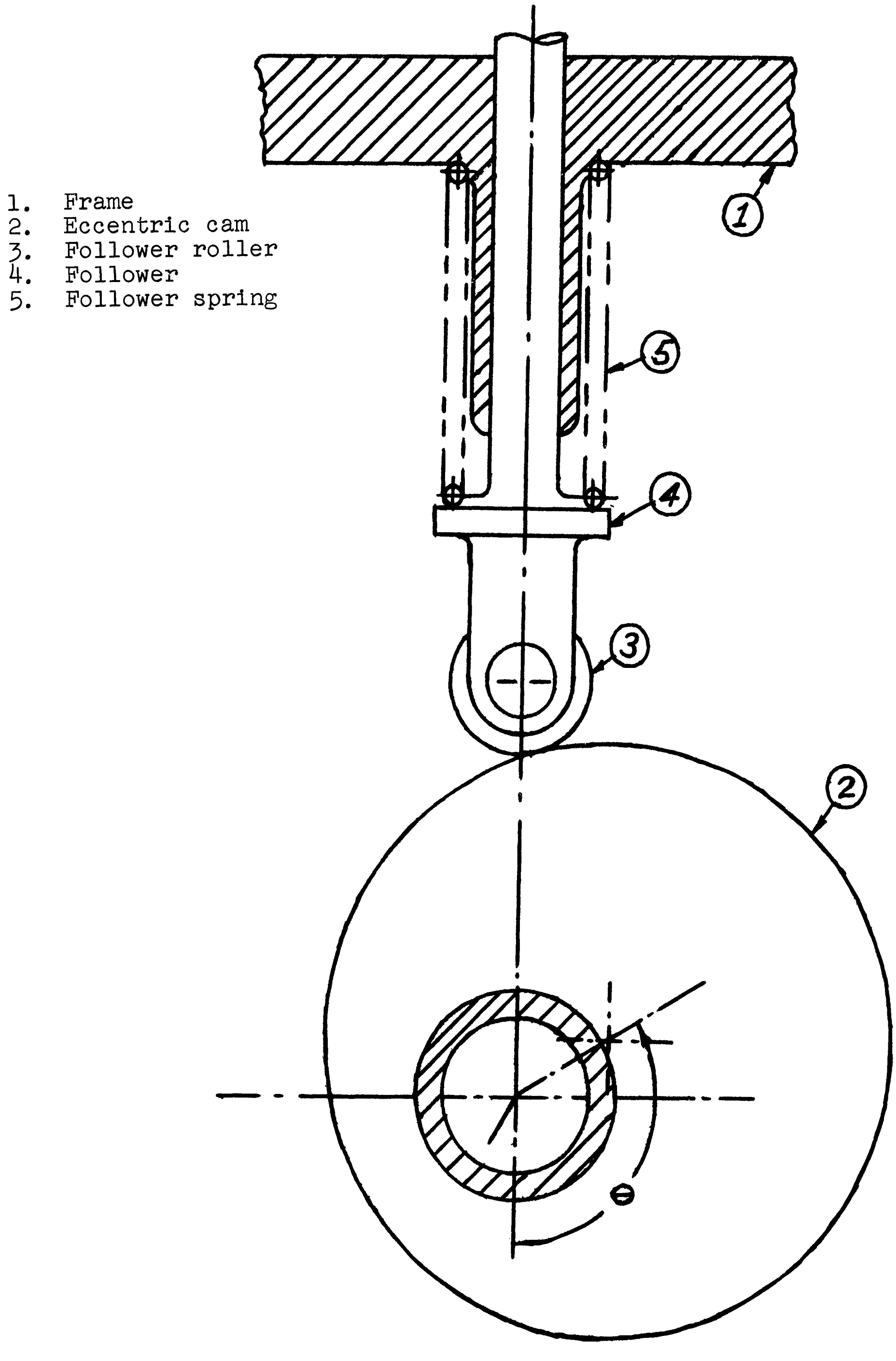
Cam Design Proposal Analysis

It seems likely that the critical factors of this design will be either the contact stresses between the roller and cam surfaces or the follower roller bearing pressure. The type of bearing proposed here is an ordinary sleeve bearing. The effects of change in radius ratio and speed on these two factors is desired.

Write out an analysis to determine the variation of stress and pressure with cam angle for the two parameters, radius ratio and speed.

Prepare and run a computer program to determine the necessary values and prepare a set of curves to demonstrate these effects.

Use values of 0.125, 0.250, 0.375, 0.500 for the radius ratio and 75, 100, 125, and 150 radians/sec. for the speed parameter.



1. Frame
2. Eccentric cam
3. Follower roller
4. Follower
5. Follower spring

Example Problem No. 75

USE OF THE ELECTRONIC DIFFERENTIAL ANALYZER TO STUDY THE DYNAMICS OF MACHINERY*

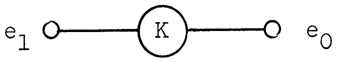
Department of Mechanical Engineering
The University of Michigan

Course: Dynamics of Machinery Credit Hours: 4 Level: Junior

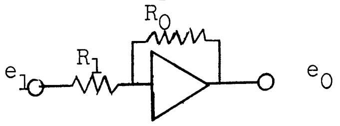
A series of exercises has been prepared to introduce the student to the use of the electronic differential analyzer as a means of studying the dynamics of machinery. Typical examples will be presented of some of the material which has been prepared. The examples consist of: a typical set of homework problems for which a written solution is required, eight computer problems to be solved on the electronic differential analyzer, an experiment combined with a computer problem, and a typical, one-hour examination.

Typical Set of Homework Problems

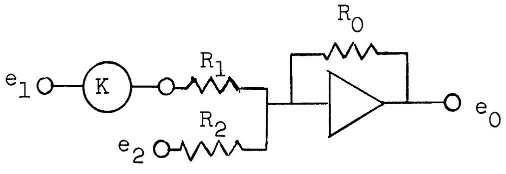
Fill in Answers

1.


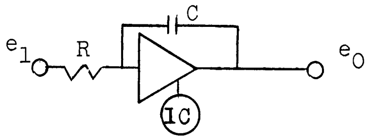
e_1 =

2.


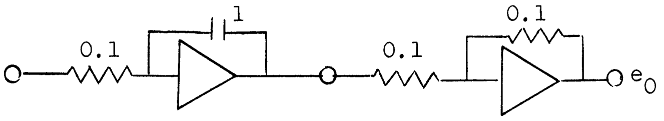
e_1 =

3.


e_1 =
e_2 =

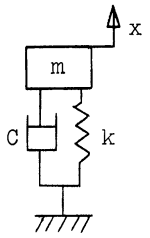
4.


e_0 =

5. $e_1 = \alpha t$


e_0 =

6. Draw the analog circuit to simulate the following system:



$x_0 = 0.10$ in
$m = 1.0$ lb_f-sec^2/in
$c = 1.0$ lb_f-sec/in
$k = 100$ lb_m/in

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* These exercises were prepared jointly by several members of the Mechanical Engineering faculty.

Typical Set of Problems to be Solved on the Electronic Differential Analyzer

Analog Computer Problem Number 1:

Solve the equation $m\ddot{x} + c\dot{x} + kx = 0$ where $m = 1$ lb-sec^2/in., $c = 1.5$ lb-sec/in., and $k = 100$ lb/in.

1. For starting conditions use $x = 1\text{-}1/2$ in, $\dot{x} = 0$. Obtain a record of x and $\dot{x}$ on the oscillograph. Be sure that they are both positive in the same direction. How long does it require for the amplitude to decay to 1/4 in? What voltage did you use for x? For $\dot{x}$? On the tape, a centimeter represents what displacement? What velocity?
2. Repeat item 1 for $\dot{x} = 15$ in/sec, $x = 0$. What differences are noted?
3. Disconnect the velocity loop and start the motion with $x = 1\text{-}1/2$ in, $\dot{x} = 0$. What is the measured frequency? The calculated frequency?
4. Change the feedback capacitors to 1 mfd each and repeat item 3. What happens?

Analog Computer Problem Number 2:

A mechanical vibrating system has $W = 10$ lb, $c = 0.305$ lb-sec/in, and $k = 120$ lb/in. It is to be excited by a step forcing function of 8 lb.

1. Draw the circuit diagram and determine suitable scales for the displacement, time, and force. This should be done before coming to the computer lab.
2. Wire up the computer and operate it so that you can check the values of your scale factors, and also to get data for item 4. The transient force should be wired up through the function switch so that you can turn the force on and off while the computer is in "operate." Take recordings of the force on one channel and the displacement on the other. Record the actual scale on the tape.
3. Disconnect the damping loop and reproduce both parts of Fig. 18-7.*
4. Measure one of the records taken in item 2 and determine the log decrement, and, from this, calculate the damping ratio. How does this damping ratio compare with the damping ratio determined from the original differential equation?

Analog Computer Problem Number 3:

Use the analog computer to calculate a set of points for one of the curves of Fig. 18-24.* Since a function generator will not usually be available, you should generate the forcing function on the computer itself. Draw the circuit diagram in such a manner that changing the frequency of the forcing function will have no effect on the amplitude of the forcing voltage. Note also that there is an easier way to do this than the method illustrated in your EDA** notes.

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* Shigley, Joseph E., Dynamic Analysis of Machines, McGraw-Hill Book Co., New York (1961). or Shigley, Joseph E., Theory of Machines, McGraw-Hill Book Co., New York (1961).

** A set of notes entitled, "Introduction to the Electronic Differential Analyzer," prepared by the faculty of the Department of Mechanical Engineering at The University of Michigan. These notes are distributed as introductory material to beginning students using the analog computer.

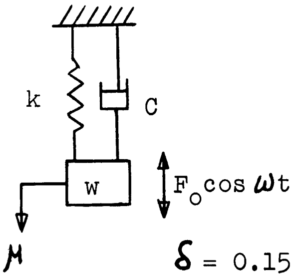
Analog Computer Problem Number 4:

A mechanical vibrating system has k = 100 lb/in, c = 0.485 lb-sec/in, and W = 10 lb. It is to be excited by a transient force of 10 lbs.

1. Draw the circuit diagram and determine suitable scales for the displacement, time, and force. The circuit should be arranged so that the force can be switched on and off.
2. Wire up the computer and operate it so that you can check the values of your scale factors. Take several recordings of amplitude and force and calculate the percentage error between the machine solution and a phase-plane solution.
3. Operate the system by turning the force to "on". After 2-1/2 cycles of vibration, turn the force off. What is the amplitude of the first peak after turning off the force?
4. Some of the problems that occur in applying constant forces to elastic systems are that the system "overshoots" its eventual equilibrium position and vibrates too much. This can be solved by introducing more damping. If too much damping is introduced, however, then the system requires excessive time to reach its position of equilibrium. A happy medium would be obtained if only a slight overshoot were obtained provided the system settled down quickly. This is really a problem in optimization. So optimize your system by finding a value for c such that the system quickly comes to the equilibrium position after the force is applied without overshooting too much.
5. Connect x to the y axis of an oscilloscope or x-y plotter, and $\dot{x}/\omega_n$ to the x axis and photograph or draw the phase-plane diagrams for parts 2 and 3.

Analog Computer Problem Number 5:

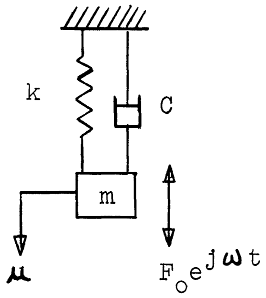
Using the computer, calculate the amplitude ratio $\frac{u_o}{F_o/k}$ corresponding to various ω/ω_n ratios from zero to 4. Plot the results with $\frac{u_o}{F_o/k}$ as the ordinate, and ω/ω_n as the abscissa. This is called a frequency-response chart. Get enough points to construct a smooth curve. Do not use a sine generator. Instead, generate the forcing function on the computer itself. Include specimens of the computer tape containing the forcing function as one graph and the response as the other graph. Be sure that both charts have positive values in the same direction.



On the same chart draw a frequency-response diagram obtained by calculation. What is the largest error found?

Analog Computer Problem Number 6:

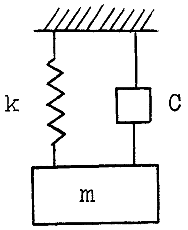
For the system shown in the figure r = 0.15, using the analog computer, calculate the amplitude ratio $\frac{u_o}{F_o/k}$ corresponding to various ω/ω_n ratios from zero to 4. Plot the results with $\frac{u_o}{F_o/k}$ as the ordinate and ω/ω_n as the abscissa. This is called a frequency response chart. Get enough points to construct a smooth curve. Do not use a sine-wave generator. Instead, generate the forcing function on the computer itself. Include specimens of the computer tape containing the forcing function as one graph and the response as the other graph. Be sure that both charts have positive values in the same direction.



Analog Computer Problem Number 7:

Prepare the analog solution for the system shown in the schematic diagram, using the following procedure:

1. System equation
2. Check solution
3. Variables range
4. Parameter range
5. Time scale adjusted equations
6. Amplitude scale adjusted equation
7. Circuit diagram
8. Potentiometer settings
9. Tabulation of runs
10. Curve plots and conclusions



$m = 10$ #s^2/in
$c = 1.0$ #s/in
$k = 2.0$ #/in
$x(0) = 1.0$ in
$\dot{x}(0) = 0$

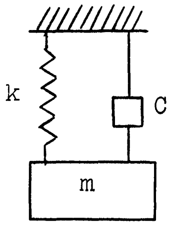
Determine experimentally:

a. the period
b. the amplitude after 2 cycles of vibration
c. the value of the critical damping constant

Analog Computer Problem Number 8:

Prepare the analog solution for the system shown in the schematic diagram, using the following procedure:

1. System equation
2. Check solution
3. Variables range
4. Parameter range
5. Time scale adjusted equation
6. Amplitude scale adjusted equation
7. Circuit diagram
8. Potentiometer settings
9. Tabulation of runs
10. Curve plots and conclusions



$m = 1.60$ #s^2/in
$c = 24.0$ #s/in
$k = 250$ #/in
$F_o = 75$ #

Instructions:

a. Determine the response ratio, x_o/x_{st} for $\omega/\omega_n = 0.5, 0.8, 1.0, 1.41, 2.0$
b. Plot response ratio vs. frequency.
c. Plot the computed curve and determine the discrepancy.

Experiment Combined with Computer Problem

Purpose: To experimentally analyze a torsional vibration model, and to simulate the model on the analog computer.

Apparatus:

1. Torsional vibration model with crank and lever forcing mechanism.
2. Necessary instrumentation.
3. Analog Computer.

Experimental Procedure:

1. Connect motor to variable speed control, range 0-1725 rpm, and run at low speed for a short period of time to see how it operates. DO NOT EXCEED 600 RPM OF CRANKSHAFT AT ANY TIME.
2. Calibrate both potentiometers separately. If this is accurately done, they should read the same when re-installed and the model turned over by hand.
3. Display on the scope and record with the camera the displacement of each end, superimposed, at 550 rpm of the crankshaft. Do not over speed!
4. Measure the spring rate of the model, but do not stress the torsion bar beyond 20,000 psi shear.
5. Display and record the damped free vibration of the flywheel.
6. Using a 3-string torsional pendulum, determine the moment of inertia of a similar flywheel, and calculate the moment of inertia of the flywheel shaft, spacer, and coupling.

Analytical Procedure:

1. Calculate the damping coefficient from the data of 5 above.
2. Write the differential equation of motion for the model, and using the measured data from 4, 5, and 6, above, simulate the model on the analog computer, reproducing as nearly as possible the displacement diagrams obtained from the model at 550 rpm.

Report Should Contain:

1. A copy of these instructions.
2. Simple but neat sketch of the test set-up.
3. Photographs or copies, neatly labeled.
4. Curve showing spring rate.
5. Observed data.
6. Calculations, computer program diagram, and recorder tape from Analytical Procedure.
7. List of important equipment used.

Typical One-Hour Examination

I. A forced-damped system of one degree of freedom consists of a weight of 38.6 pounds supported by a spring of rate 10 pounds per inch and a dashpot with a viscous damping constant of 1.0 pound-seconds/inch. The system is excited by an external harmonic force with an amplitude of 1.50 pounds and a frequency of 105.05 cycles per minute.

Determine:

a. Displacement amplitude ratio x_o/x_{st}.

b. Displacement amplitude x_o, inches.

c. Damping factor, r, dimensionless.

d. Spring force amplitude, pounds.

e. Damping force amplitude, pounds.

f. Inertia force amplitude, pounds.

g. Phase angle, ϕ, degrees.

h. Draw the force amplitude polygon to a scale of 1 inch equals 0.5 pounds. Identify the vectors.

II. a. Write the differential equation of motion for the system of problem I.

b. Draw a circuit diagram to represent the mechanical system of problem I on the electronic differential analyzer. Give all necessary capacitor, resistor, and potentiometer values.