Inverse Scattering Investigation

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by

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FOREWORD

This report was prepared by the Radiation Laboratory of the Department of Electrical Engineering of The University of Michigan. The work was performed under Contract No. F 19628-61-C-0190, "Inverse Scattering Investigation" and covers the period 3 January to 3 April 1967. Dr. Vaughan H. Weston is the Principal Investigator and the contract is under the direction of Professor Ralph E. Hiatt, Head of the Radiation Laboratory. The contract is administered under the direction of the Electronic Systems Division, Air Force Systems Command, United States Air Force, Laurence G. Hanscom Field, Bedford, Massachusetts 01730, by Lt. H. R. Betz, ESSXS. This Quarterly Report was submitted by the author on 28 April 1967.

This technical report has been reviewed and is approved.

Prior to release of this report to CFSTI (formerly OTS) it must be reviewed by the ESD Office of Public Information, ESTI, Laurence G. Hanscom Field, Bedford, Massachusetts 01730

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ABSTRACT

The problem in question consists of determining means of solving the inverse scattering problem where the transmitted field is given and the received fields are measured, and this data is used to discover the nature of the target.

The problem of what information can be determined about the body if the scattering matrix (phase and amplitude) is known only over an angular sector and measured in the far field, is studied. Some of the results are presented in the second section. In particular, the necessity of using short pulse measurements to separate out the signals from the various scattering centers is pointed out. The study of the short pulse bistatic measurements is under investigation. For smooth convex surface, particular positions of the scattering body can be obtained.

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I

INTRODUCTION

The problem in question consists of determining means of solving the inverse scattering problem where the transmitted field is given and the received fields are measured, and this data is used to discover the nature of the target. In connection with this, a review of some of the results given in (Weston et al, 1966) is outlined below.

In Chapter III, (Weston et al 1966), the concept of equivalent sources was introduced wherein the scattered field may be thought of as arising from a set of equivalent sources located on or within the body. This concept was introduced since the radii of the minimum convex surface which enclosed the equivalent sources is related to the convergence of any expansion technique utilized to derive the near scattered field of the target from the observed far field. Thus in some cases the expansions may be convergent part way inside the body. (In particular for smooth convex bodies).

One particular expansion technique that was investigated, was based upon the representation of the field in the form

$$\underline{E} = \frac{e^{ikr}}{r} \sum_{n=0}^{\infty} \frac{\underline{E}_n(\theta, \phi)}{r^n}$$

where (r, θ, ϕ) are the coordinates of a spherical polar coordinate system. The leading term given by $n = 0$ is the observed far field, from which the remaining terms can be derived through a set of recurrence relations. This expansion is convergent outside the minimum sphere enclosing the equivalent sources.

In addition the approach to the inverse scattering problem based upon the representation of the scattered field in terms of plane waves, was investigated. An explicit expression for the scattered field, valid in a half-space which depends upon the coordinate axis, was given in terms of an integral operating on the far scattered field. By rotation of axes the same expression can be used to find the near field everywhere outside the minimum convex shape enclosing the equivalent sources.

It was shown that the plane wave representation converges part way inside smooth convex portions of the body, thus extablishing the concept that the minimum convex shape enclosing the equivalent sources often may be inside the actual scattering body.

In connection with representations which are valid in the interior of the minimum convex shape enclosing the equivalent sources, a practical representation was given in the second half of Chapter IX, (Weston et al, 1966). This representation, based on an expansion in terms of spherical harmonics and Bessel functions, can be used to find the scattered field in cavity regions that penetrate the body of the minimum convex shape.

As an extension of the results derived in Weston et al, 1966), the problem of what information can be determined about the body if the scattering matrix (phase and amplitude) is known only over an angular sector of the body and measured in the far field, was studied during this quarter. Some of the results are presented in the second section. In particular, the necessity of using short pulse measurements to separate out the signals from the various scattering centers is pointed out. The study of the short pulse bistatic measurements is under investigation.

In Section 3, the use of combining information of bistatic measurements obtained for various transmitter positions is explored, as means of obtaining additional bistatic measurements for a fixed transmitter position.

II

FAR FIELD INFORMATION LIMITED TO A SOLID ANGLE

In most practical situations the scattering matrix (phase and amplitude) is measured in the far field only over a region of limited bistatic angles. In particular, the quantity $\underline{E}_o(\theta, \phi)$, related to the far zone electric intensity by the relation

$$\underline{E}(r, \theta, \phi) \sim \frac{e^{ikr}}{r} \underline{E}_o(\theta, \phi) \quad , \tag{2.1}$$

is measured over a finite range of values of θ and ϕ. In this case it is important to know what information can be determined about the body, when the scattering matrix is measured only over an angular sector. In order to study this problem the plane wave representation will be used as an analytical tool. It should be recalled from (2.1) that if, $\underline{E}_o(\theta, \phi)$ is known over the complete

$$\underline{E}(\underline{x}) = \frac{ik}{2\pi} \int_0^{2\pi} \int_0^{\frac{\pi}{2} - i\infty} e^{i\underline{k} \cdot \underline{x}} \underline{E}_o(\alpha, \beta) \sin \alpha \, d\alpha \, d\beta, \tag{2.2}$$

and

$$\underline{k} = k(\sin \alpha \cos \beta, \ \sin \alpha \sin \beta, \ \cos \alpha).$$

This particular expression holds everywhere in the half-space $z > z^*$ above equivalent sources of the scattered field.

In order to study the problem where $\underline{E}_o(\theta, \phi)$ is not known on the complete unit sphere, it will be assumed at present that the far field data is given over the solid angle

$$0 \leq \theta \leq \theta^o, \quad \text{and } 0 \leq \phi \leq 2\pi.$$

With this in mind the following defination will be presented.

Cone C: The cone which is tangent to the minimum convex shape enclosing the body or the equivalent source, and which approaches the ring $\theta = \theta_o$ asymptotically in the far field (i.e., $r \to \infty$).

As a first step the following lemma will be proved.

Lemma: If $\underline{x}$ is contained inside the upper part of the cone C, then $\underline{E}(\underline{x})$ is given in terms of the far field data by the relation

$$\underline{E}(\underline{x}) = \frac{ik}{2\pi} \int_0^{2\pi} \int_0^{\theta_o - i\infty} e^{i\underline{k} \cdot \underline{x}} \underline{E}_o(\alpha, \beta) \sin \alpha \, d\alpha \, d\beta \ . \tag{2.3}$$

Proof:

The results can be obtained through analysis similar to that of Section 5.1 in Weston et al, (1966). However, it is easier to see that expression (2.3) is derivable from (2.2) by a suitable change in the contour involving the complex variable α. From the relation

$$\underline{E}_o(\theta, \phi) = \left(\frac{i}{4\pi \epsilon_o \omega} \right) \underline{k} \wedge \underline{k} \wedge \int_V \underline{J}(\underline{x}') \, e^{-i\underline{k} \cdot \underline{x}'} \, d\underline{x}' \tag{2.4}$$

where V is the volume containing the equivalent sources, it is seen that in order to change the upper limit of integration for the variable α from $\pi - i\infty$ to $\theta_o - i\infty$, the real part of the exponent

$$i \underline{k} \cdot (\underline{x} - \underline{x}')$$

must be negative for $\alpha \to \theta - i\infty$, with θ and β restricted to the ranges $\theta_o \leq \theta \leq \pi$, $0 \leq \beta \leq 2\pi$. This implies that, for $t \to \infty$ where $\alpha = \theta - it$,

$\sinh t\, f(\underline{x}, \underline{x}') > 0$, reducing to the condition

$$f(\underline{x}, \underline{x}') > 0 \ , \tag{2.5}$$

where

$$f(\underline{x}, \underline{x}') = -\cos\theta \left[(x-x')\cos\beta + (y-y')\sin\beta\right] + (z-z')\sin\theta . \tag{2.6}$$

To interpret inequality (2.5) from the geometrical point of view, let B be the minimum convex surface enclosing the equivalent sources. Let $\pi(\theta, \beta)$ be the plane tangent to B, with normal $\underline{n}(\theta, \beta)$ given by

$$\underline{n}(\theta, \beta) = (-\cos\theta \cos\beta, \ -\cos\theta \sin\beta, \ \sin\theta).$$

If (x_o', y_o', z_o') is the point of tangency, then the equation for $\pi(\theta, \beta)$ is given by

$$-(x-x_o')\cos\theta\cos\beta - (y-y_o')\cos\theta\sin\beta + (z-z_o')\sin\theta = 0.$$

It then follows that for $\underline{x}$ "above" the plane (in the half-space which does not contain B),

$$f(\underline{x}, \underline{x}_o') > 0,$$

and hence for $\underline{x}'$ any point inside B, it must follow that

$$f(\underline{x}, \underline{x}') > 0.$$

Inequality (2.5) must hold for all values of β and θ such that $0 \leq \beta \leq 2\pi$, and $\theta_o \leq \theta \leq \pi$. Thus holding β fixed, and varying θ between θ_o and π, $\underline{x}$ must be "above" all the planes $\pi(\theta, \beta)$, i.e. in the intersection of the half-spaces. Finally by varying β it is seen that $\underline{x}$ must lie in the common intersection of the half-spaces. Since the unit vector $\underline{k}_o$ = (sin θ cos β, sin θ cos β, sin θ) lies in the plane π(θ, β), it follows that the intersection is the cone C which is tangent to B, and has generators approaching the ring $\theta = \theta_o$ in the far field.

The above lemma can be qualitatively used to determine the amount of information about the scatterer which is obtainable from measurements of the scattering matrix limited to an angular sector. For a fixed direction of incidence with respect to the scatterer, assume that the far field (phase, amplitude and polarization) is measured at a set of points (θ_n, ϕ_n) in an angular sector about the direction θ = 0. In particular assume that enough measurements are made so that $\underline{E}_o(\theta, \phi)$ can be represented by a sufficiently smooth function over a solid angle, given by $0 \leq \theta \leq \theta_o$, $0 \leq \phi \leq 2\pi$. The actual number of points needed will depend upon the number of oscillations of the field quantity $\underline{E}_o(\theta, \phi)$ in the above solid angle, and this in turn depends upon the size of the body. As a rough estimate, if D is the maximum diameter of the body, then the minimum period of the important oscillations of the quantity $\underline{E}_o(\theta, \phi)$ (i.e. the fine structure of the field), will be $2\lambda/D$. Since $\underline{E}_o(\theta, \phi)$ can be approximated sufficiently in the above solid angle, its analytic continuation in the complex θ plane can be found from a Taylor series approach. However, because information of $\underline{E}_o(\theta, \phi)$ is limited to the finite line segment $0 \leq \theta \leq \theta_o$, the analytic continuation can only be found on the line segment $0 \leq \theta \leq \theta_o + \epsilon - i\,\delta$, where ε and δ are positive numbers. The maximum value of $(\epsilon^2 + \delta^2)^{1/2}$ that can be taken will depend upon the number of terms that can be computed in the Taylor series expansion about the point $\theta = \theta_o$, and the degree that such a finite expansion approximates the analytic continuation of $\underline{E}_o(\theta, \phi)$. A precise answer to the latter statement requires a significant amount of hard analysis.

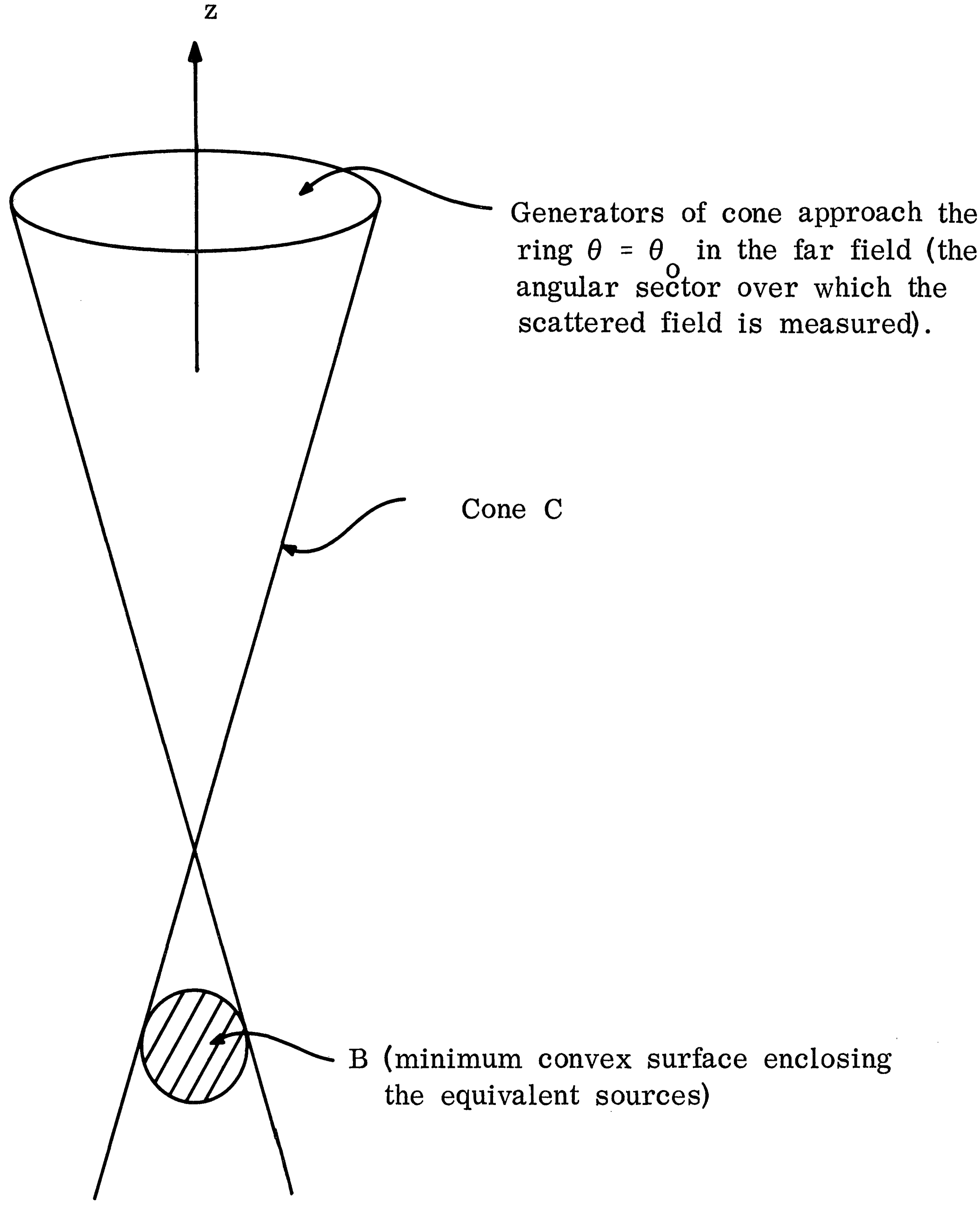

FIG. 2-1: GEOMETRY FOR CONE C RELATED TO THE FINITE REGION WHERE THE SCATTERED FIELD IS MEASURED.

From the above comments, it can be seen that the field quantity

$$\underline{E}^*(\underline{x}) = \frac{ik}{2\pi} \int_0^{2\pi} \int_0^{\theta_o + \epsilon - i\delta} e^{i\underline{k} \cdot \underline{x}} \underline{E}_o(\alpha, \beta) \sin\alpha \, d\alpha \, d\beta$$

can be obtained. Thus at a point $\underline{x}$ in the near field, the error in the electric density due to far field measurements limited to a solid angle, is given by

$$\left|\underline{E} - \underline{E}^*\right| \sim \frac{k}{2\pi} \left| \int_0^{2\pi} \int_{\theta_o + \epsilon - i\delta}^{\theta_o + \epsilon - i\infty} \underline{E}_o(\alpha, \beta) \sin\alpha \, d\alpha \, d\beta \right| .$$

From Eqs. (2.4) and (2.6) it can be seen that the dependence of $\underline{E} - \underline{E}$ on the position $\underline{x}$, is the order

$$\exp\left[-k \sin\delta \, f(\underline{x}, \underline{x}'_o)\right]$$

where $\underline{x}'_o$ is the value of $\underline{x}'$ for which $f(\underline{x}, \underline{x}')$ is a minimum, $\underline{x}'$ being a point of tangency of a line passing through the point $\underline{x}$, with the minimum convex surface enclosing the equivalent sources. Thus it is seen that if $\underline{x}$ lies in the upper part of the cone C given by the lemma, the error is exponentially small.

Since the cone C is tangent to the minimum convex surface enclosing the equivalent sources, and since such a surface lies inside a smooth convex scattering obstacle, the upper part of the cone will intersect the surface of a smooth scattering obstacle. Thus far field bistatic measurements taken over a solid angle can be used to determine a portion of a smooth convex scattering obstacle. Such a result is seen from physical considerations in the high frequency case, where the geometric optics reflected field is the dominate contribution (except for forward scattering). In this case, the process of tracing the rays back and combining the resultant scattered field with the incident field, will yield a portion of the scattering surface.

For complex type targets the upper part of the cone C will not cut the surface of the body. In this case, the far scattered field first has to be decomposed into components arising from the various scattering centers, and thus components treated separately. Techniques for separation of the various components should be sought. One possible natural approach is to use short pulse measurements to separate out the various components by way of range resolution. Alternatively, optical processing could be used, such as the holograph.

III

MEANS OF OBTAINING ADDITIONAL INFORMATION

For the position of the transmitter fixed with respect to the scattering object, the number of receivers or receiver positions that are allowable from the practical standpoint, may be too limited to insure sufficient determination of the far scattered field, either over the complete unit sphere, or over a solid angle. Thus additional information is required to determine the scattered field. One possible approach to the problem is to combine sets of measurements, each set comprising a particular transmitter position and several receiver positions, and from these, extrapolate by numerical means the additional bistatic measurements needed to sufficiently determine the scattered field for a fixed transmitter position. The fundamental relationships needed to combine the information in order to obtain the desired results are based upon the reciprocal nature of the scattered field as a function of transmitter and receiver positions and polarization, and for non-absorbing bodies, the unitary properties of the scattering matrix. A brief outline of these relationships is given below. Let $\underline{\theta}_T = (\theta_T, \phi_T)$ be the angular coordinates of the transmitter (far field), $\underline{\theta}_R = (\theta_R, \phi_R)$ be the angular coordinates of the receiver (far field). The far scattered field is given in the general form

$$\underline{E}^s = \frac{e^{ikR}}{R}\left\{\hat{\underline{\theta}}\left[E_{11}(\underline{\theta}_R,\underline{\theta}_T) + E_{12}(\underline{\theta}_R,\underline{\theta}_T)\right] + \underline{\phi}\left[E_{21}(\underline{\theta}_R,\underline{\theta}_T) + E_{22}(\underline{\theta}_R,\underline{\theta}_T)\right]\right\} \qquad (3.1)$$

where the subscripts are defined as follows. The first subscript of the general term E_{ij} referes to receiver polarization, whereas the second subscript refers to the transmitter polarization, such that a particular subscript is unity when the polarization is in the $\underline{\theta}$ direction and two when the polarization is in the ϕ

direction. In particular the term E_{11} referes to the $\underline{\theta}_R$ component of the scattered field with the transmitter polarized in the $\underline{\theta}_T$ direction.

Because of reciprocity it follows that

$$\left.\begin{aligned} E_{11}(\underline{\theta}_R, \underline{\theta}_T) &= E_{11}(\underline{\theta}_T, \underline{\theta}_R) \\ E_{22}(\underline{\theta}_R, \underline{\theta}_T) &= E_{22}(\underline{\theta}_T, \underline{\theta}_R) \\ E_{12}(\underline{\theta}_R, \underline{\theta}_T) &= E_{21}(\underline{\theta}_T, \underline{\theta}_R) \ . \end{aligned}\right\} \qquad (3.2)$$

Additional relationships between the scattered fields produced by incident waves arising from different directions (i.e., different positions of the transmitter) can be obtained for non-absorbing type scattering obstacles. These obstacles may be perfectly conducting, lossless coated conducting, or just lossless dielectric bodies. An outline of the derivation of the relationships are given below.

Let $(\underline{E}_1, \underline{H}_1)$ and $(\underline{E}_2, \underline{H}_2)$ be two different Maxwellian fields associated with the presence of the lossless scattering obstacles. Harmonic tine dependence exp(-iωt) will be assumed. It then follows from Maxwell's equations that

$$\underline{\nabla} \cdot (\underline{E}_1 \wedge \underline{H}_2^*) = i\omega\mu\, \underline{H}_2^* \cdot \underline{H}_1 - i\omega\epsilon\, \underline{E}_1 \cdot \underline{E}_2^*$$

$$\underline{\nabla} \cdot (\underline{E}_2 \wedge \underline{H}_1^*) = i\omega\mu\, \underline{H}_1^* \cdot \underline{H}_2 - i\omega\epsilon\, \underline{E}_2 \cdot \underline{E}_1^*$$

where the * represents the complex conjugate. It can be seen that

$$\text{Real}\, \underline{\nabla} \cdot \left[\underline{E}_1 \wedge \underline{H}_2^* + \underline{E}_2 \wedge \underline{H}_1^*\right] = i\omega \left\{\mu \left[\underline{H}_2^* \cdot \underline{H}_1 + \underline{H}_1^* \cdot \underline{H}_2\right] - \right.$$

$$\left. - \epsilon \left[\underline{E}_2^* \cdot \underline{E}_1 + \underline{E}_1 \cdot \underline{E}_2^*\right]\right\} = 0,$$

and similarly that

$$\mathrm{Im}\ \nabla \cdot \left[\underline{E}_1 \wedge \underline{H}_2^* - \underline{E}_2 \wedge \underline{H}_1^* \right] = 0.$$

From the divergence theorem, it can be shown that

$$\mathrm{Real} \int_s \underline{n} \cdot \left[\underline{E}_1 \wedge \underline{H}_2^* + \underline{E}_2 \wedge \underline{H}_1^* \right] ds = 0 \qquad (3.3)$$

$$\mathrm{Im} \int_s \underline{n} \left[\underline{E}_1 \wedge \underline{H}_2^* - \underline{E}_2 \wedge \underline{H}_1^* \right] ds = 0 \qquad (3.4)$$

where s is a sphere of infinite radius. Each of the different fields will be comprised of two parts, an incident plane wave, and a scattered wave given as follows for the far field

$$\underline{E}_1 = \frac{e^{ikR}}{R} \underline{\mathcal{E}}_1 + \underline{a}_1 e^{\underline{ik} \cdot \underline{R}} , \quad R \rightarrow \infty$$

where

$$\underline{\mathcal{E}}_1 = \underline{\hat{\theta}}\, f_1 (\underline{\theta}, \underline{\theta}_1) + \underline{\hat{\phi}}\, g_1 (\underline{\theta}, \underline{\theta}_1) ,$$

and with

$\underline{\theta}$ = angular coordinates of scattered field

$\underline{\theta}_1$ = angular coordinates of transmitter

$\underline{\tilde{\theta}}$ = angular coordinates of the forward scattered direction.

The vector $\underline{k}_1$ representing the direction of propagation of the incident wave is given by

$$\underline{k}_1 = k \left[\sin \tilde{\theta}_1 \cos \tilde{\phi}_1, \sin \tilde{\theta}_1 \sin \tilde{\phi}_1, \cos \tilde{\theta}_1 \right] .$$

The magnetic intensity in the far field ($R \rightarrow \infty$) is given

$$Z_o \underline{H}_1 = \frac{e^{ikR}}{R} \underline{\mathcal{H}}_1 + \hat{\underline{k}}_1 \wedge \underline{a}_1 \, e^{i\underline{k}_1 \cdot \underline{R}} ,$$

where

$$\underline{}_1 = \left\{ - \hat{\underline{\theta}} g_1 (\underline{\theta}, \underline{\theta}_1) + \hat{\underline{\phi}} f_1 (\underline{\theta}, \theta_1) \right\},$$

$$Z_o = \sqrt{\mu_o / \epsilon_o} \quad .$$

A similar relationship will hold for the other field ($\underline{E}_2$, $\underline{H}_2$). It can be shown that on the sphere S with $\underline{n} = \hat{\underline{R}}$, the follwoing holds

$$Z_o \underline{n} \cdot \underline{E}_1 \wedge \underline{H}_2^* = \frac{1}{R^2} \left\{ f_1 f_2^* + g_1 g_2^* \right\} +$$

$$+ \frac{1}{R} \left\{ \underline{a}_1 \cdot \underline{\mathcal{E}}_2^* \exp i \left[\underline{k}_1 \cdot \underline{R} - kR \right] \right.$$

$$\left. + (\underline{k}_2 \wedge \underline{a}_2^*) \cdot \underline{\mathcal{H}}_1 \exp i \left[- \underline{k}_2 \cdot \underline{R} + kR \right] \right\}$$

$$+ \hat{\underline{R}} \cdot \underline{a}_1 \wedge (\hat{\underline{k}}_2 \wedge \underline{a}_2^*) \exp i \left[\underline{k}_1 \cdot \underline{R} - \underline{k}_2 \cdot \underline{R} \right] \qquad (3.5)$$

Since Eqs. (3.3) and (3.4) can be represented in the form

$$\int_S \underline{n} \cdot \left[\underline{E}_1 \wedge \underline{H}_2^* + \underline{E}_2^* \wedge \underline{H}_1 \right] ds = 0$$

it follows from relation (3.5) that

$$2 \int \left[f_1 f_2^* + g_1 g_2^* \right] d\Omega = - \lim_{R \rightarrow \infty} (I_{12} + I_{21}^*)$$

where

$$I_{21} = \int R \left[\underline{a}_1 \cdot \underline{\mathcal{E}}_2^* + (\underline{k}_1 \wedge \underline{a}_1) \cdot \underline{\mathcal{H}}_2^* \right] \exp i \left[\underline{k}_1 \cdot \underline{R} - kR \right] d\Omega .$$

The integrals are over the unit sphere with $d\Omega = \sin\theta \; d\theta \; d\phi$. It can be shown that

$$\lim_{R \to \infty} I_{12} = -2 i \lambda \underline{a}_1 \cdot \underline{\mathcal{E}}_2^* (\theta = \tilde{\theta}_1)$$

yielding

$$\int \left[f_1 f_2^* + g_1 g_2^* \right] d\Omega = i\lambda \left[\underline{a}_1 \cdot \underline{\mathcal{E}}_2^* (\theta = \tilde{\theta}_1) - \underline{a}_2^* \cdot \underline{\mathcal{E}}_1 (\theta = \tilde{\theta}_2) \right] . \qquad (3.6)$$

For non-absorbing bodies, the above equation gives a relationship between the scattered fields produced by incident radiation from two different directions with arbitrary polarization. For complete generality it is best to express the above relationship in matrix form. Using the notation given previously,

$$f_1 (\underline{\theta}, \underline{\theta}_1) = \mathcal{E}_{11} (\underline{\theta}, \underline{\theta}_1) + \mathcal{E}_{12} (\underline{\theta}, \underline{\theta}_1)$$

$$g_1 (\underline{\theta}, \underline{\theta}_1) = \mathcal{E}_{21} (\underline{\theta}, \underline{\theta}_1) + \mathcal{E}_{22} (\underline{\theta}, \underline{\theta}_1)$$

such that when the positions of the transmitter is given by $\underline{\theta}_1$, the scattered field is given by

$$\underline{\mathcal{E}}_1 = \hat{\underline{\theta}} \; \mathcal{E}_{11} + \hat{\phi} \; \mathcal{E}_{12} ,$$

and

$$\underline{\mathcal{E}}_1 = \hat{\underline{\theta}}\, \mathcal{E}_{21} + \hat{\underline{\phi}}\, \mathcal{E}_{22}$$

for the incident field polarized in the $\hat{\underline{\theta}}_1$ and $\hat{\underline{\phi}}_1$ directions respectively. Define the matrix $H(\underline{\theta}, \underline{\theta}_1)$ as follows

$$H(\underline{\theta}, \underline{\theta}_1) = \begin{bmatrix} \mathcal{E}_{11}(\underline{\theta}, \underline{\theta}_1) & \mathcal{E}_{12}(\underline{\theta}, \underline{\theta}_1) \\ \mathcal{E}_{21}(\underline{\theta}, \underline{\theta}_1 & \mathcal{E}_{22}(\underline{\theta}, \underline{\theta}_1) \end{bmatrix} \qquad (3.7)$$

with its transpose denoted by a superscript T. It follows from reciprocity that

$$H(\underline{\theta}, \underline{\theta}_1) = H^T(\underline{\theta}_1, \underline{\theta}) \quad .$$

Equation (3.6) can be written in the general matrix form

$$\int H(\underline{\theta}, \underline{\theta}_1) H^*(\underline{\theta}_2, \theta)\, d\Omega = i\lambda \left[H^*(\underline{\theta}_2, \tilde{\underline{\theta}}_1) - H(\tilde{\underline{\theta}}_2, \underline{\theta}_1) \right] \qquad (3.8)$$

where the integration is over the individual elements of the matrix. The above relation is equivalent to the unitary property of the scattering matrix.

The unitary and symmetrical properties of the scattering matrix, are given in Waterman (1966) where in particular, attention is given to the expansion of the scattering matrix in terms of spherical vector eigenfunctions. In order to obtain a complete description of the far scattered field for a fixed transmitter position, the spherical harmonic representation is the best to use. However, in order to use relations (3.2) and (3.8) in conjunction with the sets of measurements corresponding to different transmitter position, the representation must be an expansion in terms of spherical harmonics of the angular coordinate of both the transmitter and receiver. For large objects the number of terms needed in the series is too large to handle, and such a representation is best for scattering objects whose dimensions are less than a wavelength. Thus except for Rayleight scattering, it is best to complete the scattered field only over a solid angle or sector instead of the complete unit sphere. An outline of a technique for performing this task is discussed below.

Let N sets of measurements be obtained, where each set of measurements consists of measurement of the scattered field. At M(N) different receiver positions, with a fixed position and polarization of the transmitter. From this data, the scattered field is required for the solid angle $-\beta \leq \phi_R \leq \beta$, $\pi/2 - \alpha \leq \theta_R \leq \pi/2 + \alpha$, with the angular coordinates of the transmitter given by $\underline{\theta}_T = (\pi/2,\ 0)$. The scattered field may be represented in this domain in the form of a fourier series e.g.,

$$E_{11} = \sum a_{nm}^{n'm'} \exp\left\{ i\pi \left[\frac{n\theta_R}{\alpha} + \frac{n'\theta_T}{\alpha} + \frac{m\phi_R}{\beta} + \frac{m'\phi_T}{\beta} \right] \right\}$$

where the summantion is over the indices. Due to reciprocity, the particular matrix given by $\left\{ a_{nm}^{n'm'} \right\}$ must be symmetric. The munber of terms required in the series depends upon the number of oscillations of E_{11} in the solid angle and this in turn depends upon the size of the object. From the sets of measurements E_{11} is given for is given for a finite set of values of $\underline{\theta}_R$ and $\underline{\theta}_T$, thus producing a set of linear algebraic equations. The coefficients $a_{nm}^{n'm'}$ are then found from inversion of the matrix. From the numerical standpoint there is a limit to the size of matrix which can be handled. This will place restrictions upon the size of the solid angle over which the scattered field is to be determined for a fixed direction of incidence.

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13. ABSTRACT

The problem in question consists of determining means of solving the inverse scattering problem where the transmitted field is given and the received fields are measured, and this data is used to discover the nature of the target.

The problem of what information can be determined about the body if the scattering matrix (phase and amplitude) is known only over an angular sector and measured in the far field, is studied. Some of the results are presented in the second section. In particular, the necessity of using short pulse measurements to separate out the signals from the various scattering centers is pointed out. The study of the short pulse bistatic measurements is under investigation. For smooth convex surface, particular positions of the scattering body can be obtained.

UNCLASSIFIED

Security Classification

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Inverse Scattering Electromagnetic Theory Radar Identification of Objects Near Field Representation in Terms of Far Field Data						

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