THE UNIVERSITY OF MICHIGAN

INDUSTRY PROGRAM OF THE COLLEGE OF ENGINEERING

DESIGN STUDY OF A LIQUID AND A SOLID ROCKET PROPELLANT SYSTEM

William Whicher
Theodore Petersen

January, 1960

IP-411

PREFACE

There are few places in the literature wherein the rocket propulsion system and vehicle performance have been treated together. In addition, a design study of two propulsion systems - one based on a liquid propellant and the other upon a solid propellant is quite interesting for comparison purposes. The two design studies presented here were carried out by the authors as part of the requirements for the Rocket Propulsion course in the Department of Aeronautical and Astronautical Engineering. In the interest of analytical facility, simplified assumptions were made for both preliminary designs. These assumptions, however, would not seriously affect the results.

This report was reproduced in the interest of supplying the Industry Program members with a concise design study of two systems based on the same design objectives and employing the same techniques of construction. It also clears up some of the arguments concerning the relative advantages and disadvantages of solid and liquid rocket motor systems.

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LIQUID ROCKET DESIGN

William Whicher

NOMENCLATURE

sp gr - specific gravity

psi - pounds per square inch

Ae - exit area of rocket nozzle

q - dynamic pressure

P_c - rocket chamber pressure

F - thrust (also T is used for thrust)

λ - rocket nozzle correction factor to account for divergence of flow at rocket exit

V_e - gas velocity at exit of rocket nozzle

$\dot{m}$ - mass flow

T_c - exit temperature of rocket gas

P_e - exit pressure of rocket gas

A_T - rocket nozzle throat area

P_2/P_1 - pressure ratio across oblique shock in exit of rocket nozzle

Θ - cone angle of rocket nozzle at exit

M_1 - Mach number before oblique shock in exit of rocket nozzle

Γ - $\gamma^{\frac{1}{2}} \left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma + 1}{2(\gamma - 1)}}$

t - time

T - thrust

D - drag on rocket

M - mass of rocket at time, t (also M - Mach number)

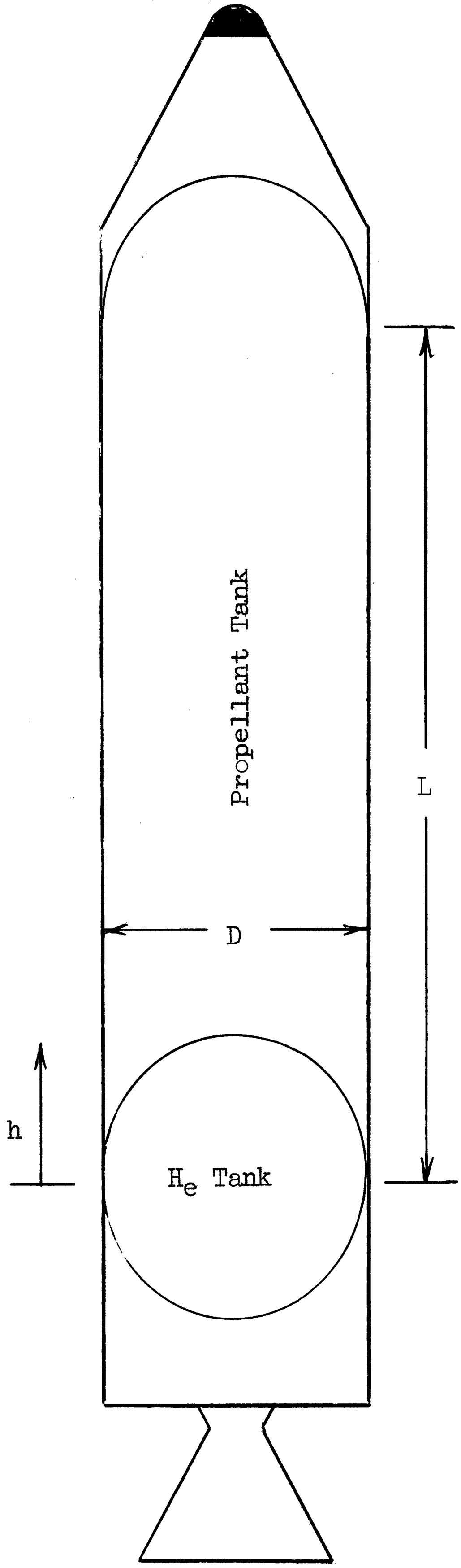
h - altitude of the rocket

g - gravitational constant

$\mathring{w}$ - weight flow

V	-	velocity of the rocket
L	-	length
C_o	-	drag coefficient
R_e	-	Reynold's number
S	-	area

Problem Number 3



Given the following information, design a rocket.

Mass ratio = 20/1 (includes payload)

Payload wt. = 10% dry wt. of bird.

Fineness ratio = $\frac{L}{D} = \frac{15}{1}$

Make bird out of Fiberglas.

Sp.gr. Fiberglas = 1.80

Working stress = σ = 80000 psi

Use a monopropellant sp.gr. = 1.00

Motor $\frac{400 \text{ lbs. thrust}}{\text{lb. engine wt.}}$

Use a gas pressurization system.
Ae in a spherical container.
P_{He} = 3500 psia initially.

Use a heater charge in Helium tank so that Helium always expands isothermally.

25% pressure drop across plumbing.

25% pressure drop across injector face.

Lift off wt. = 25,000 lbs.

Also make plots of:

a) head suppression vs. time.

b) chamber pressure vs. time.

c) drag vs. time.

d) acceleration vs. time.

e) altitude vs. time.

f) Mach number vs. time (only until q drops off appreciably)

g) 9 vs. time ($q = \frac{1}{2} \rho v^2$)

Additional information

Use an engine with same geometry as in Problem 2* except chop off nozzle so that flow never separates.

When designing tankage only consider hoop tension.

Assume that a regulator valve is in the Helium line that controls the pressure on top of the propellant at a constant value until the helium runs out.

Assume a head suppression valve is installed in the propellant lines so as to regulate the pressure in the chamber at a constant 150 psia until the helium runs out.

Size rocket

Mass ratio = 20/1

Wt. fuel at lift off = (0.95)(25000)

= 23750 lbs.

$$\text{Tank volume} = \frac{\text{Wt. fuel}}{\text{density fuel}}$$

$$= \frac{23750}{62.4}$$

$$= 381 \text{ ft}^3$$

Calculate tank dimensions

$$V = \frac{L\pi D^2}{4} \qquad \frac{L}{D} = 15 \qquad L = 15D$$

$$V = \frac{15D^3\pi}{4}$$

$$D^3 = \frac{(381)(4)}{15\pi} = 32.4$$

$$D = 3.18 \text{ ft.}$$

$$L = 47.7 \text{ ft.}$$

Use L/D = 3 for nose cone

$$L_{Total} = 47.7 + (3)(3.18)$$

$$= 57.24 \text{ ft.}$$

* Refers to design of a rocket motor chamber and nozzle previously carried out

Calculate Engine Characteristics

Characteristics for P_c = const. = 150 psia

Calculate F_∞

$$F_\infty = \lambda \overset{\circ}{m} V_e + P_e A_e$$

from Problem 2

$$F_\infty = 577\ M_e T_e^{1/2} + P_e A_e$$

for $\frac{A_e}{A_T} = 3.011$; $\frac{P_e}{P_c} = 0.06526$; $\frac{T_e}{T_c} = 0.6345$

$M_e = 2.40$ (from gas tables)

so

$$F_\infty = (577)(2.40)(3172)^{1/2} + (144)(9.789)(2.68)(3.011)$$

$$= 78000 + 11350$$

$$= 89350 \text{ lb.}$$

The thrust at any altitude is given by

$$F = F_\infty - A_e P_a$$

Verify that nozzle does not separate

$$\frac{P_2}{P_1} = 1 + \frac{\gamma M_1^2 \theta}{(M_1{}^2-1)^{1/2}} = 1 + \frac{(1.2)(2.4)^2(0.1745)}{[(2.4)^2-1]^{1/2}}$$

$$= 1 + \frac{1.205}{(4.76)^{1/2}} = 1 + 0.552$$

$$P_2 = (150)(0.06526)(1.552) = 15.19 \text{ psia.}$$

This is above atmospheric pressure of 14.7 psia so separation does not occur.

Calculate thrust after 45.0 seconds

P_c from graph

$$F = 600\ P_c$$

TABLE I

Altitude ft	Atmospheric Pressure lb/ft^2	A_eP_a lb	F lb
0	2116	17060	72290
4000	1828	14730	74620
8000	1572	12680	76670
12000	1346	10830	78520
16000	1147	9240	80110
20000	972.5	7840	81510
24000	820.2	6610	82740
28000	687.8	5550	83800
32000	573.3	4670	84680
36000	474.7	3825	85525
40000	391.7	3156	86194
44000	323.2	2604	86746
48000	266.7	2148	87202
55000	190.5	1535	87815
60000	149.8	1207	88143
65000	117.8	950	88400
70000	93.51	753	88597
75000	73.70	594	88756
80000	58.01	467	88883

$t = 50.0$

$F = (600)(130.6)$

$= 78360$ lbs.

$t = 55.0$ sec.

$F = (600)(118.4)$

$= 71040$ lbs.

$t = 60.0$

$F = (600)(105.0)$

$= 63000$ lbs.

$t = 65.0$

$F = (600)(89.0)$

$= 53400$ lbs.

$t = 67.26 = t_{BO}$

$F = (600)(83.3)$

$= 49980$ lbs.

Determine chamber pressure as a function of time

A head suppression valve is installed in the propellant lines to keep the pressure in the chamber constant at 150 psia. However due to the limitations on the amount of helium that can be carried the chamber pressure decreases during the last part of burning. At some time the head suppression term goes to zero and the chamber pressure is determined solely by the helium pressure, the hydraulic head, and the line plus injector impedance. After the critical time (the time at which the chamber pressure starts to fall) the chamber pressure can be determined by an equilibrium equation relating the line impedance, the helium pressure and the hydraulic head.

$$P_{He} + H - Z\,P_c = 0$$

The chamber pressure was constant up to 45 seconds. Therefore it is convenient to take our time origin at 45 seconds. Doing this we can rewrite the equation

$$\frac{P_i V_i}{V_{45} + \int_0^t \overset{\circ}{V} dt} + \frac{(h_{45} - \int_0^t \overset{\circ}{h} dt)\rho_P F}{W_{45} - \int_0^t \overset{\circ}{\omega} dt} = Z\, P_c(t)$$

Where

P_i = initial helium pressure

V_i = initial helium volume

V_{45} = helium volume at 45 seconds

$\overset{\circ}{V}$ = time rate of change of helium volume

h_{45} = level of the propellant at 45 seconds

$\overset{\circ}{h}$ = time rate of change of propellant level

ρ_P = density of propellant

W_{45} = weight of rocket at 45 seconds

$\overset{\circ}{\omega}$ = weight flow

F = thrust

$P_c(t)$ = chamber pressure

Z = ratio of tank pressure to chamber pressure, previously calculated as 16/9 .

The chamber pressure and the chamber temperature are related through the equilibrium constant K_p. However since we do not know the chemical composition of the monopropellant we are forced to assume that the reaction products and therefore the chamber temperature are constant. As a consequence of this assumption we note that the mass flow is sensitive only to changes in chamber pressure.

$$\overset{\circ}{m} = \frac{\Gamma P_c A_t}{a_c}$$

$\therefore$

$$m = \frac{\overset{\circ}{m}_i}{P_{c_i}} P_c$$

By using this relation we can solve for $\overset{\circ}{V}$, $\overset{\circ}{h}$, and $\overset{\circ}{\omega}$ as functions of P_c alone. At 45 seconds we are at about 100,000 ft. Therefore we can ignore drag, and also assume that we are exhausting into a vacuum. The last assumption allows us to solve for F as a function of P_c alone.

$$F = \lambda \overset{\circ}{m} V_e + P_e A_e$$

$$= \lambda \frac{\overset{\circ}{m}_{45}}{P_{c_{45}}} P_c V_e + \frac{P_e}{P_c} P_c A_e$$

$$= [\lambda \frac{\overset{\circ}{m}_{45}}{P_{c_{45}}} V_e + \frac{P_e}{P_c} A_e] P_c$$

$$\lambda = 0.9924$$

$$\frac{\overset{\circ}{m}_{45}}{P_{c_{45}}} = \frac{11.9}{150} = 0.07933$$

$$V_e = M_e \sqrt{\gamma R \frac{T_e}{T_c} T_c} = 6600 \text{ ft/sec}$$

$\therefore$

$$F = 599.9 P_c$$

$$\overset{\circ}{\omega} = g \overset{\circ}{m} = (32.2)(0.07933) P_c = 2.554 P_c$$

$$\overset{\circ}{V} = \frac{\overset{\circ}{\omega}}{\rho_P} = \frac{2.554 P_c}{62.4} = 0.04094 P_c$$

$$\overset{\circ}{h} = \frac{\overset{\circ}{V}}{\text{inside tank area}} = \frac{0.04094 P_c}{7.94} = 0.005155 P_c$$

Substituting these values into the original equation we obtain

$$\frac{(3500)(16.83)}{293.4 + 0.04094 \int_0^t P_c(t)dt} + \frac{(13.15 - 0.005155 \int_0^t P_c(t)dt)(62.4)(599.9 P_c(t))}{(7765 - 2.554 \int_0^t P_c(t)dt)(144)}$$

$$= P_c(t)$$

The initial values at t=0 (45 seconds) have been included in the above equation. This equation gives P_c as a function of time.

$$P_c = P_c(t)$$

We note that this equation is a non-linear integral equation. An analytical solution is not possible. However if we make certain assumptions we can obtain an approximate solution to this equation.

First from the first mean value theorem of integral calculus we note

$$\int_0^t P_c(t)dt = P_c(\xi)t$$

where $P_c(\xi)$ is some average value in the interval. Now unless we know ξ we cannot locate $P(\xi)$ in the interval. However for calculation purposes we will assume that ξ lies in the middle of the interval. This is true if the interval 0 to t is small. For our purposes we will use 2.5 second intervals.

The equation to be solved is now

$$\frac{P_i V_i}{V_o + 0.04094\ P_c(\xi)t} + \frac{[h_o - (0.005155)\ P_c(\xi)t]\ \ 259.96\ P_c(\xi)]}{W_o - 2.554\ P_c(\xi)t} = \frac{16}{9} P_c(\xi)$$

where V_o, h_o, and W_o are the values of the helium volume, the propellant level and the rocket weight at the end of the previous interval. The rest of the terms are as noted above.

The above equation is cubic in $P_c(\xi)$ and can be solved most easily by iteration. The final calculation for each interval is presented below.

$$\frac{58905}{293.4 + 0.10235\ P_c} + \frac{(13.15 - 0.1289\ P_c)(260)(P_c)}{7765 - 6.385\ P_c} = \frac{16}{9} P_c$$

45 0 $\leq t \leq$ 47.5 try P_c = 142

$$\frac{58905}{293.4 + 14.53} + \frac{[(13.15)-(1.830)](260)(142)}{7765 - 906} = \frac{16}{9} P_c$$

$$\frac{58905}{307.93} + \frac{(11.32)(260)(142)}{6859} = \frac{16}{9} P_c$$

$$P_c = \frac{9}{16} (191.2 + 60.9) = \frac{9}{16} (252.1) = 141.9$$ Use P_c = 142.0

47.5 $\leq$ t $\leq$ 50.0 try P_c = 134.5

$$\frac{58905}{307.93 + 13.78} + \frac{(11.32 - 1.732)(260)(134.5)}{6859 - 858} = \frac{16}{9} P_c$$

$$\frac{16}{9} P_c = \frac{58905}{321.71} + \frac{(9.588)(260)(134.5)}{6001}$$

$$P_c = \frac{9}{16} (183.0 + 55.9) = \frac{9}{16} (238.9) = 134.5$$ Use P_c = 134.5

50.0 $\leq t \leq$ 52.5 try P_c = 127.5

$$\frac{58905}{321.71 + 13.07} + \frac{(9.588 - 1.642)(260)(127.5)}{6001 - 814} = \frac{16}{9} P_c$$

$$\frac{16}{9} P_c = \frac{58905}{334.78} + \frac{(7.946)(260)(127.5)}{5187}$$

$$P_c = \frac{9}{16} (175.9 + 50.7) = \frac{9}{16} (226.6)$$

$$= 127.5$$ Use P_c = 127.5

$52.5 \leq t \leq 55.0$

try $P_c = 121.0$

$$\frac{58905}{334.78 + 12.39} + \frac{(7.946 - 1.560)(260)(121)}{5187 - 773} = \frac{16}{9} P_c$$

$$\frac{58905}{347.17} + \frac{(6.386)(260)(121)}{44\ 14} = \frac{16}{9} P_c$$

$$P_c = \frac{9}{16} (169.8 + 45.4) = \frac{9}{16} (215.2)$$

$= 121.0$ <u>Use $P_c = 121.0$ Psia</u>

$55.0 \leq t \leq 57.5$

$$\frac{16}{9} P_c = \frac{58905}{347.17 + 11.75} + \frac{(6.386 - 1.479)(260)(114.8)}{4414 - 732}$$

$$= \frac{58905}{358.92} + \frac{(4.907)(260)(114.8)}{3682}$$

$$P_c = \frac{9}{16} (164.0 + 39.8) = \frac{9}{16} (203.8)$$

$P_c = 114.8$ <u>Use $P_c = 114.8$</u>

$57.5 \leq t \leq 60.0$

try $P_c = 108.0$

$$\frac{16}{9} P_c = \frac{58905}{358.92 + 11.06} + \frac{(4.907 - 1.391)(260)(108)}{3682 - 689}$$

$$= \frac{58905}{369.98} + \frac{(3.516)(260)(108)}{2993}$$

$$P_c = \frac{9}{16} (159.1 + 33.0) = \frac{9}{16} (1924)$$

$= 108.0$ <u>Use $P_c = 108.0$</u>

$60.0 \leq t \leq 62.5$

try $P_c = 101.0$

$$\frac{16}{9} P_c = \frac{58905}{369.99 + 10.35} + \frac{(3.516 - 1.291)(260)(101)}{2993 - 645}$$

$$= \frac{58905}{380.33} + \frac{(2.225)(260)(101)}{2348}$$

$$P_c = \frac{9}{16}(154.8 + 24.9) = \frac{9}{16}(179.7)$$

$= 101.0$ <u>Use $P_c = 101.0$</u>

$62.5 \leq t \leq 65.0$

try 93.0 psi

$$\frac{16}{9} P_c = \frac{58905}{380.33 + 9.53} + \frac{(2.225 - 1.198)(260)(93)}{2348 - 593}$$

$$= \frac{58905}{389.86} + \frac{(1.027)(260)(93)}{1755}$$

$$P_c = \frac{9}{16}(151.0 + 14.1) = \frac{9}{16}(165.1)$$

$= 93.0$ psia <u>Use 93</u>

To determine burn out time we note that the helium volume at burn out is given by

V_{He} = Vol propellant tank + initial vol He.

$= 397.83\ ft^3$

From this we can determine the chamber pressure at burn out.

$$P_{c_{BO}} = \frac{(3500)(16.83)}{397.83}$$

$= 83.287$ x 83.3 psia

Burnout will occur before 67.5 seconds. We can find the burnout time if the average chamber pressure during the last period is known. Estimate this from the average pressure for the last two times

$$P_{c_{65}} = 93.0 - \frac{(101 - 93)}{2}$$
$$= 89.0$$

The average pressure over the interval $65 \rightarrow t_{BO}$ is

$$P_{c_{av}} = \frac{83.3 + 89.0}{2}$$

$$= 86.15 \text{ psia}$$

Now since the helium volume must be 397.83 ft^3 at burnout

$$V_{He_{65}} + (0.04094)\ P_c(\xi)\ t_b' = 397.83$$

$$389.86 + (0.04094)(86.15)\ t_b' = 397.83$$

$$t_b' = \frac{7.97}{(86.15)(0.04094)}$$

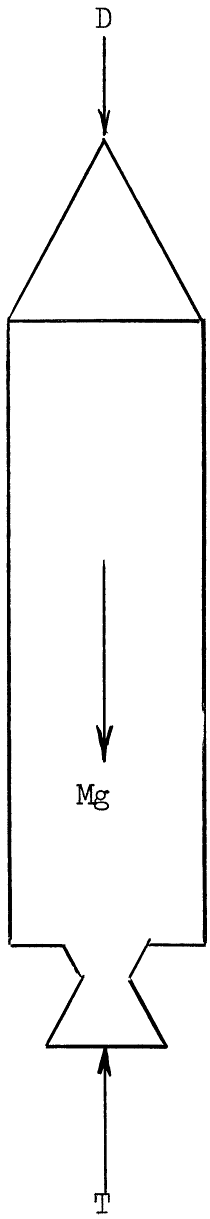
$$= 2.26$$

$$t_{BO} = 65.0 + 2.26$$

$$= \underline{67.26 \text{ seconds after lift off}}.$$

TABLE II

Time Sec	Chamber Pressure psia	Weight lb	$\dot{w}$ lb/sec (over interval)
45.0	150.0	7765	
46.25	142.0		362.2
47.50		6859	
48.75	134.5		343.2
50.00		6001	
51.25	127.5		325.6
52.50		5187	
53.75	121.0		309.4
55.00		4414	
56.25	114.8		293.0
57.50		3682	
58.75	108.0		275.6
60.00		2993	
61.25	101.0		258.0
62.50		2348	
63.75	93.0		237.2
65.00	89.0	1755	223.8
67.26	83.3	1250	



Performance Calculations

Using Newtons second law

$$M \frac{dV}{dt} = T - D - Mg$$

Now if we consider thrust and drag constant over the time interval and express M as M(t) we obtain.

$$\frac{dV}{dt} = \frac{T - D}{M_o - \dot{m}t} - g$$

$$= \frac{g(T - D)}{W_o - \dot{\omega}t} - g$$

$$\int_{V_o}^{V} dV = \int_{o}^{t} \frac{g(T-D)dt}{W_o - \dot{\omega}t} - \int_{o}^{t} gdt$$

$$V = V_o + \frac{T - D}{\dot{m}} \ell n \frac{W_o}{W_o - \dot{\omega}t} - gt$$

$$V = \frac{dh}{dt}$$

$$\int_{h_o}^{h} dh = \int_{o}^{t} V_o dt + \frac{T-D}{\dot{m}} \int_{o}^{t} \ell n \frac{W_o \; dt}{W_o - \dot{\omega}t} - \int_{o}^{t} gtdt$$

after consulting an integral table we find

$$h = h_o + V_{ot} - \frac{gt^2}{2} + \frac{T-D}{\dot{m}}[t - \frac{W_o - \dot{\omega}t}{\dot{\omega}} \ell n \frac{W_o}{W_o - \dot{\omega}t}]$$

Note that I have integrated from 0 to t. This is equivalent to considering only the time intervals with initial conditions applying from the end of the previous time interval.

$0 \leq t \leq 5 \qquad F - D = 72290$ lb.

$t = 5$

$$V = \frac{72290}{11.9} (\ell n\ 25000 - \ell n\ 23085) - 161$$

$$= 6070\ [10.12663 - 10.04693] - 161$$

$$= 6070\ (0.07970) - 161$$

$$= 484 - 161$$

$$= \underline{323 \text{ ft/sec.}}$$

$$h = 6070\ [5 - \frac{(23085)(0.07970)}{383}] - (25)(16.1)$$

Since the [ ] involve the difference of numbers very close together, use log tables instead of a slide rule so as to improve accuracy.

$$\begin{aligned} \log 23085 &= 4.36335 \\ \log 0.07970 &= \underline{8.90146 - 10} \\ &\ 13.26479 - 10 \\ \log 383 &= \underline{2.58320} \\ &\ 10.68159 - 10 = \log 4.8049 \end{aligned}$$

$$h = 6070(5 - 4.8049) - 403$$

$$= 1183 - 403$$

$$= \underline{780 \text{ ft}}$$

$$Re = \frac{\rho V L}{\mu}$$

$$L = 47.7 + (3.18)(3) = 47.7 + 9.54$$

$$= 57.24$$

$$Re = \frac{(0.002310)(323)(57.24)}{3.699 \times 10^{-7}}$$ (ρ and μ from NACA TN 3182)

$$= 105.5 \times 10^{6}$$

$$M = \frac{323}{1114} = \underline{0.29}$$ (a at various altitudes from NACA TN 3182)

From graph

$$C_D = 0.18$$

$$D = C_D q\ S \qquad \text{(S based on frontal area)}$$

$$q = \frac{\rho}{2} V^2 = \frac{(2.310 \times 10^{-3})(323)^2}{2}$$

$$= \underline{120.3}\ \text{lb/ft}^2$$

Since the estimation of C_D from the graph is not precise use S based on the inside area of tank.

$$S = (1.59)^2\ \pi = 7.94$$

$$D = (0.18)(120.3)(7.94)$$

$$= \underline{172.1\ \text{lbs.}}$$

$$5 \leq t \leq 10$$

Thrust at 780 ft = 72700 lb

$$T - D = 72700 - 172 = 72528\ \text{lb}$$

t = 10

$$V = 323 + \frac{72528}{11.9}\ [\ell n\ 23085 - \ell n\ 21170] - 161$$

$$V = 323 + 6100\ (10.04693 - 9.96034) - 161$$

$$= 162 + 6100\ (0.08659)$$

$$= 162 + 528$$

$$= \underline{690\ \text{ft/sec}}$$

$$h = 780 + (323)(5) - 403 + 6100\ [5 - \frac{(21170)}{383}\ (0.08659)]$$

4.32572			
8.93747 - 10			
13.26319 - 10			
2.58320			5.0000
10.67999 - 10		→	4.7862

$$h = 780 + 1212 + 6100\ (0.2138)$$

$$= 1992 + 1302$$

$$= \underline{32\ 94\ \text{ft}}$$

$$Re = \frac{(0.002158)(690)(57.24)}{3.669 \times 10^{-7}}$$

$$= 232 \times 10^{6}$$

$$M = \frac{6.90}{1107} = \underline{0.123}$$

$$C_D = 0.17$$

$$q = \frac{(0.002158)(690)^2}{2} = \underline{514\ \text{lb/ft}^2}$$

$$D = (0.17)(514)(7.94)$$

$$= \underline{695\ \text{lbs}}$$

$$10 < t \leq 15$$

$$T - D = 74200 - 695 = 73505\ \text{lbs}$$

$t = 15$

$$V = 690 - 161 + \frac{73505}{11.9}\ [\ell n\ 21170 - \ell n\ 19255]$$

$$= 529 + 6170\ [9.96034 - 9.86552]$$

$$= 529 + 6170\ [0.09482]$$

$$= 529 + 584$$

$$= \underline{1113\ \text{ft/sec}}$$

$$h = 3294 + (690)(5) - 403 + 6170\ [5 - \frac{(19255)}{383}(0.09482)]$$

$$\begin{array}{lll} & 4.28454 & \\ & 8.97690 - 10 & \\ \hline & 13.26144 - 10 & \\ & 2.58320 & 5.0000 \\ \hline & 10.67824 - 10 & \rightarrow 4.7671 \end{array}$$

$$h = 6341 + (6170)(0.2329)$$

$$h = 6341 + 1437$$

$$= \underline{7778\ \text{ft}}$$

$$Re = \frac{(0.001881)(1113)(57.24)}{3.561 \times 10^{-7}}$$

$$= 336 \times 10^{6}$$

$$M = \frac{1113}{1100} = \underline{1.011}$$

$$C_D = 0.295$$

$$q = \frac{(0.001881)(1113)^2}{2} = \underline{1164 \text{ lbs/ft}^2}$$

$$D = (0.295)(1164)(7.94)$$

$$= \underline{2726 \text{ lbs}}$$

$15 < t \leq 20$

$T - D = 76600 - 2726 = 73874$ lb

$t = 20$

$$V = 1113 + \frac{73874}{11.9}[\ln 19255 - \ln 17340] - 161$$

$$= 952 + 6205\,[9.86552 - 9.76077]$$

$$= 952 + (6205)(0.10475)$$

$$= 952 + 651$$

$$= \underline{1603 \text{ ft/sec}}$$

$$h = 7778 + 5565 - 403 + 6205[5 - \frac{(17340)(0.10475)}{383}]$$

$$\begin{array}{l} \ \ 4.23905 \\ \ \ \underline{9.02015 - 10} \\ 13.25920 \\ \ \ \underline{2.58320} \\ 10.67600 - 10 \quad \rightarrow 4.7424 \end{array}$$

$$h = 12940 + (6205)(0.2576)$$

$$= 12940 + 1597$$

$$= \underline{14537 \text{ ft}}$$

$$Re = \frac{(1.520 \times 10^{-3})(1603)(57.24)}{3.441 \times 10^{-7}}$$

$$= 405 \times 10^{6}$$

$$M = \frac{1603}{1060} = \underline{1.512}$$

$$C_D = 0.275$$

$$q = \frac{(1.520 \times 10^{-3})(1603)^2}{2} = \underline{1953 \text{ lb/ft}^2}$$

$$D = (0.275)(1953)(7.94)$$

$$= \underline{4260 \text{ lb}}$$

$20 \leq t \leq 25$

$T - D = 79500 - 4260 = 75240$ lb

$t = 25$

$$V = 1603 - 161 + \frac{75240}{11.9} [\ln 17340 - \ln 15425]$$

$$V = 1442 + 6320 \; (9.76077 - 9.64374)$$

$$= 1442 + (6320)(0.11703) \qquad = 1442 + 740$$

$$= \underline{2182 \text{ ft/sec}}$$

$$h = 14537 + (1603)(5) - 403 + 6320 \left[5 - \frac{(15425)(0.11703)}{383}\right]$$

$$\begin{array}{r} 4.18823 \\ 9.06830 - 10 \\ \hline 13.25653 - 10 \\ 2.58320 \\ \hline 10.67333 - 10 \end{array} \rightarrow 4.7133$$

$$h = 22149 + (6320)(0.2867)$$

$$= 22149 + 1812$$

$$= \underline{23961 \text{ ft}}$$

$$Re = \frac{(1.103 \times 10^{-3})(2182)(57.24)}{3.234 \times 10^{-7}}$$

$$= 426 \times 10^6$$

$$M = \frac{2182}{1021} = \underline{2.12}$$

$C_D = 0.245$

$q = \frac{(1.103 \times 10^{-3})(2182)^2}{2} = \underline{2620 \text{ lb/ft}^2}$

$D = (0.245)(2620)(7.94)$

$= \underline{5100 \text{ lb}}$

$25 < t \leq 30$

$T - D = 82700 - 5100 = 77600$

$t = 30$

$V = 2182 - 161 + \frac{77600}{11.9} [\ell n\ 15425 - \ell n\ 13510]$

$= 2021 + 6520\ (9.64374 - 9.51118)$

$= 2021 + (6520)(0.13256)$

$= 2021 + 864$

$= \underline{2885 \text{ ft/sec}}$

$h = 23961 + (2182)(5) - 403 + 6520 \left[5 - \frac{(13510)(0.13256)}{383} \right]$

$$\begin{array}{l} 4.13066 \\ \underline{9.12241 - 10} \\ 13.25307 - 10 \\ \underline{2.58320} \\ 10.66987 - 10 \quad \rightarrow 4.6760 \end{array}$$

$h = 34468 + (6520)(0.324)$

$= 34468 + 2110$

$= \underline{36578 \text{ ft}}$

$Re = \frac{(6.92 \times 10^{-4})(2885)(57.24)}{2.96 \times 10^{-7}}$

$= 386 \times 10^6$

$M = \frac{2885}{968} = \underline{2.98}$

$C_D = 0.215$

$q = \frac{(6.92 \times 10^{-4})(2885)^2}{2} = \underline{2880 \text{ lb/ft}^2}$

$$D = (0.215)(2880)(7.94)$$

$$= \underline{4910 \text{ lb}}$$

$30 < t \leq 35$

$$T - D = 85600 - 4910 = 80690$$

$t = 35$

$$V = 2885 - 161 + \frac{80690}{11.9} [\ln 13510 - \ln 11595]$$

$$= 2724 + 6790 \; (9.51118 - 9.35832)$$

$$= 2724 + (6790)(0.15286)$$

$$= 2724 + 1038$$

$$= \underline{3762 \text{ ft/sec}}$$

$$h = 36578 + (2885)(5) - 403 + 6790 \left[5 - \frac{(11595)(0.15286)}{383}\right]$$

$$\begin{array}{r} 4.06426 \\ 9.18429 - 10 \\ \hline 13.24855 - 10 \\ 2.58320 \\ \hline 10.66535 - 10 \end{array} \rightarrow 4.6276$$

$$h = 50600 + (6790)(0.3724)$$

$$= 50600 + 2524$$

$$= \underline{53124 \text{ ft}}$$

$$Re = \frac{(3.118 \times 10^{-4})(3762)(5724)}{2.96 \times 10^{-7}}$$

$$= 226.5 \times 10^6$$

$$M = \frac{3762}{968} = \underline{3.88}$$

$$C_D = 0.19$$

$$q = \frac{(3.118 \times 10^{-4})(3762)^2}{2} = \underline{2205 \text{ lb/ft}^2}$$

$$D = (0.19)(2205)(7.94)$$

$$= \underline{3320 \text{ lb}}$$

$35 < t \leq 40$

$$T - D = 87700 - 3320 = 84380$$

$t = 40$

$$V = 3762 - 161 + \frac{84380}{11.9} [\ell n\ 11595 - \ell n\ 9680]$$

$$= 3601 + 7100\ [9.35833 - 9.17782]$$

$$= 3601 + (7100)(0.18051)$$

$$= 3601 + 1281$$

$$= \underline{4882 \text{ ft/sec}}$$

$$h = 53124 + (3762)(5) - 403 + 7100\ [5 - \frac{(9680)(0.18051)}{383}]$$

$$\begin{array}{l} \ \ 3.98588 \\ \ \ \underline{9.25650 - 10} \\ 13.24238 - 10 \\ \ \ \underline{2.58320} \\ 10.65918 - 10 \quad \rightarrow 4.5622 \end{array}$$

$$h = 71531 + (7100)(0.4378)$$

$$= 71531 + 3104$$

$$= \underline{74635 \text{ ft}}$$

$$Re = \frac{(1.115 \times 10^{-4})(4882)(57.24)}{2.96 \times 10^{-7}}$$

$$= 105 \times 10^{6}$$

$$M = \frac{4882}{971} = \underline{503}$$

$$C_D = 0.17$$

$$q = \frac{(1.115 \times 10^{-4})(4882)^2}{2} = \underline{1328 \text{ lb/ft}^2}$$

$$D = (0.17)(1328)(7.94)$$

$$= \underline{1790 \text{ lb}}$$

$40 < t \leq 45$

$T - D = 88700 - 1790 = 86910$ lb

t = 45

$$V = 4882 - 161 + \frac{86910}{11.9} [\ell n\ 9680 - \ell n\ 7765]$$

$$= 4721 + 7300\ (9.17781 - 8.95738)$$

$$= 4721 + 1610$$

$$= \underline{6331\ ft/\ sec}$$

$$h = 74635 + (4882)(5) - 403 + 7300\ [5 - \frac{(7765)(0.22043)}{383}]$$

$$\begin{array}{l} 3.890141 \\ \underline{9.343271 - 10} \\ 13.233412 - 10 \\ \underline{2.583199} \\ 10.650213 - 10 \quad \rightarrow 4.4690 \end{array}$$

$$h = 98642 + (7300)(0.5310)$$

$$= 98642 + 3880$$

$$= \underline{102,522\ ft}$$

$$Re = \frac{(2.96 \times 10^{-5})(6331)(57.24)}{2.96 \times 10^{-7}}$$

$$= 36.2 \times 10^6$$

$$M = \frac{6331}{971} = \underline{6.52}$$

$$C_D = 0.18$$

$$q = \frac{(2.96 \times 10^{-5})(6331)^2}{2} = \underline{595\ lb/ft^2}$$

$$D = (0.18)(595)(7.94)$$

$$= \underline{848\ lb}$$

An expression for altitude as a function of time can be obtained analytically if an approximate expression is used for chamber pressure as a function of time. However, the altitude integral becomes rather messy, so it was decided to continue the numerical integration using 2.5 second intervals and the average mass flow over each interval.

$t = 47.5$

$$V = 6331 - 80 + 7565 \; \ell n \frac{7765}{6859}$$

$$= 6251 + 933$$

$$= \underline{7184 \text{ ft/sec}}$$

$$h = 102522 - 100 + (6331)(2.5) + 7565 \; [2.5 - \frac{6859}{362.2} \; \ell n \frac{7765}{6859}]$$

$$= 118249 + 7565 \; [2.5000 - 2.348]$$

$$= 118249 + (7565)(0.152)$$

$$= 118249 + 1150$$

$$= \underline{119399 \text{ ft}}$$

$t = 50.0$

$$V = 7184 - 80 + 7565 \; \ell n \frac{6859}{6001}$$

$$= 7104 + 1013$$

$$= \underline{8117 \text{ ft/sec}}$$

$$h = 119399 + (7184)(2.5) - 100 + 7565 \; [2.5 - \frac{6001}{343.2} \; \ell n \frac{6859}{6001}]$$

$$= 137259 + (7565)(2.500 - 2.342)$$

$$= 137259 + (7565)(0.158) \qquad = 137259 + 1194$$

$$= \underline{138453 \text{ ft}}$$

$t = 52.5$

$$V = 8117 - 80 + 7565 \; \ell n \frac{6001}{5187}$$

$$= 8037 + 1104$$

$$= \underline{9141 \text{ ft/sec}}$$

$$h = 138453 - 100 + (8117)(2.5) + 7565 \; [2.5 - \frac{5187}{325.6} \; \ell n \frac{6001}{5187}]$$

$$= 158645 + (7565)(2.500 - 2.324)$$

$$= 158645 + (7565)(0.176)$$

$$= 158645 + 1332$$

$$= \underline{159977 \text{ ft}}$$

$t = 55.0$

$$V = 9141 - 80 + 7565 \ \ell n \frac{5187}{4414}$$

$$= 9061 + 1221$$

$$= \underline{10282 \text{ ft/sec}}$$

$$h = 159977 - 100 + (9141)(2.5) + 7565 \ [2.5 - \frac{4414}{309.4} \ \ell n \frac{5187}{4414}]$$

$$= 182729 + (7565)(2.500 - 2.304)$$

$$= 182729 + (7565)(0.196)$$

$$= 182729 + 1483$$

$$= \underline{184212 \text{ ft}}$$

$t = 57.5$

$$V = 10282 - 80 + 7565 \ \ell n \frac{4414}{3682}$$

$$= 10202 + 1375$$

$$= \underline{11577 \text{ ft/sec}}$$

$$h = 184212 + (2.5)(10282) - 100 + 7565 \ [2.5 - \frac{3682}{293} \ \ell n \frac{4414}{3682}]$$

$$= 209817 + 7565 \ (2.500 - 2.282)$$

$$= 209817 + (7565)(0.218)$$

$$= 209817 + 1650$$

$$= \underline{211467 \text{ ft}}$$

$t = 60.0$

$$V = 11577 + 7565 \ \ell n \frac{3682}{2993} - 80$$

$$= 11497 + 1570$$

$$= \underline{13067 \text{ ft/sec}}$$

$$h = 211467 - 100 + (11577)(2.5) + 7565 \ [2.500 - \frac{2993}{275.6} \ \ell n \frac{3682}{2993}]$$

$$= 240309 + (7565)(2.500 - 2250)$$

$$= 240309 + (7656)(0.250)$$

$$= 240309 + 1890$$

$$= \underline{242199 \text{ ft}}$$

t = 62.5

$$V = 13067 - 80 + [\ln \frac{2993}{2348}]\ 7565$$

$$= 12987 + (7565)(2432)$$

$$= \underline{14829 \text{ ft/sec}}$$

$$h = 242199 - 100 + (13067)(2.5) + 7565\ [2.5 - \frac{2348}{258} \ln \frac{2993}{2348}]$$

$$= 274766 + 7565\ (2.500 - 2.216)$$

$$= 274766 + (7565)(0.284)$$

$$= 274766 + 2150$$

$$= \underline{276916 \text{ ft}}$$

t = 65.0

$$V = 14829 - 80 + 7565 \ln \frac{2348}{1755}$$

$$= 14749 + 2204$$

$$= \underline{16953 \text{ ft/sec}}$$

$$h = 276916 - 100 + (2.5)(14829) + 7565\ [2.5 - \frac{1755}{237.2} \ln \frac{2348}{1755}]$$

$$= 313888 + 7565\ (2.500 - 2.158)$$

$$= 313888 + (7565)(0.342)$$

$$= 313888 + 2590$$

$$= \underline{316478 \text{ ft}}$$

$t = 67.26 = t_{BO}$

$$V = 16953 - (32.2)(2.26) + 7565 \ln \frac{1755}{1250}$$

$$= 16953 - 73 + 2565$$

$$= \underline{19445 \text{ ft/sec}}$$

$$h = 316478 - (161)(2.26)^2 + (16953)(2.26) + 7565\ [2.26 - \frac{1250}{223.8} \ln \frac{1755}{1250}]$$

$$= 354710 + 7565\ [2.26 - 1.895]$$

$$= 354710 + (7565)(0.365)$$

$$= 354710 + 2690$$

$$= \underline{357400 \text{ ft}}$$

If missle flies in a constant gravitational field then

$$\frac{Md^2h}{dt^2} = - Mg$$

or

$$\frac{d^2h}{dt^2} = - g$$

$$h = \frac{-gt^2}{2} + C_1 t + C_2$$

at $t = 0$ $\quad \mathring{h} = 19445 \quad h = 357400$

$$h = \frac{-gt^2}{2} + 19445t + 357400$$

$$\mathring{h} = - gt + 19445 = 0$$

$$t_{summit} = \frac{19445}{32.2} = 604.5 \text{ sec}$$

$$h_{summit} = - (16.1)(604.5)^2 + (19445)(604.5) + 357400$$

$$= - 5875000 + 11740000 + 357400$$

$$= 5865000 + 357400$$

$$= 6222400 \text{ ft}$$

$$= \underline{1180 \text{ miles}}$$

Calculate hydraulic head

$$H = \text{hydraulic head} = (L - \mathring{h}t)\rho_p(a + 1)$$

$$\mathring{h} = \frac{\mathring{V}}{\pi R^2} = \frac{6.15}{\pi(1.59)^2}$$

$$= 0.775 \text{ ft/sec}$$

$0 \leq t < 5$

$$H = \frac{(47.7)(62.4)(2.89)}{144}$$

$$= \underline{59.7 \text{ psi}}$$

$5 \leq t < 10$

$$H = \frac{(47.7-3.875)(62.4)(3.14)}{144}$$

$$= \frac{(43.825)(62.4)(3.14)}{144}$$

$$= \underline{60.4 \text{ psi}}$$

$10 \leq t < 15$

$$H = \frac{(43.825-3.875)(62.4)(3.47)}{144}$$

$$= \frac{(39.950)(62.4)(3.47)}{144}$$

$$= \underline{60.0 \text{ psi}}$$

$15 \leq t < 20$

$$H = \frac{(39.950-3.875)(62.4)(3.83)}{144}$$

$$= \frac{(36.075)(62.4)(3.83)}{144}$$

$$= \underline{59.8 \text{ psi}}$$

$20 \leq t < 25$

$$H = \frac{(36.075-3.875)(62.4)(4.34)}{144}$$

$$= \frac{(32.2)(62.4)(4.34)}{144}$$

$$= \underline{60.5 \text{ psi}}$$

$25 \leq t < 30$

$$H = \frac{(32.300-3.875)(62.4)(5.03)}{144}$$

$$= \frac{(28.325)(62.4)(5.03)}{144}$$

$$= \underline{61.6 \text{ psi}}$$

$30 \leq t < 35$

$$H = \frac{(28.325-3.875)(62.4)(5.98)}{144}$$

$$= \frac{(24.45)(62.4)(5.98)}{144}$$

$$= \underline{63.3 \text{ psi}}$$

$35 \leq t < 40$

$$H = \frac{(24.450-3.875)(62.4)(7.29)}{144}$$

$$= \frac{(20.575)(62.4)(7.29)}{144}$$

$$= \underline{65.0 \text{ psi}}$$

$40 \leq t < 45$

$$H = \frac{(20.575-3.875)(62.4)(8.99)}{144}$$

$$= \frac{(16.7)(62.4)(8.99)}{144}$$

$$= \underline{65.1 \text{ psi}}$$

45 = t

$$H = \frac{(13.15)(62.4)(11.48)}{144}$$

$$= \underline{65.4 \text{ psi}}$$

For t > 45 sec the hydraulic head was a necessary part of calculating the chamber pressure curve. The values listed below are from those calculations

time, sec	Hydraulic head, psi
46.25	60.9
48.75	55.9
51.25	50.7
53.75	45.4
56.25	39.8
58.75	33.0
61.25	24.9
53.75	14.1
67.26	0

Calculate accelerations

$$F\text{-}D\text{-}W = \frac{W}{g} a$$

$$a = [\frac{F-D}{W} - 1]g$$

t = 0 to 5

$$a = (\frac{72290}{25000} - 1)g$$

$$= (2.89 - 1)g$$

$$= \underline{1.89 \text{ g}}$$

$t = 5$ to 10

$$a = \left(\frac{72528}{23085} - 1\right)g$$
$$= (3.14 - 1)g$$
$$= \underline{2.14\ g}$$

$t = 10$ to 15

$$a = \left(\frac{73505}{21170} - 1\right)g$$
$$= (3.47 - 1)g$$
$$= \underline{2.47\ g}$$

$t = 15$ to 20

$$a = \left(\frac{73874}{19255} - 1\right)g$$
$$= (3.83 - 1)g$$
$$= \underline{2.83\ g}$$

$t = 20$ to 25

$$a = \left(\frac{75240}{17340} - 1\right)g$$
$$= (4.34 - 1)g$$
$$= \underline{3.34\ g}$$

$t = 25$ to 30

$$a = \left(\frac{77600}{15425} - 1\right)g$$
$$= (5.03 - 1)g$$
$$= \underline{4.03\ g}$$

t = 30 to 35

$$a = \left(\frac{80690}{13510} - 1\right)g$$

$$= (5.98 - 1)g$$

$$= \underline{4.98\ g}$$

t = 35 to 40

$$a = \left(\frac{84380}{11595} - 1\right)g$$

$$= (7.22 - 1)g$$

$$= \underline{6.22\ g}$$

t = 40 to 45

$$a = \left(\frac{86910}{9680} - 1\right)g$$

$$= (8.99 - 1)g$$

$$= \underline{7.99\ g}$$

t = 45

$$a = \left(\frac{89100}{7765} - 1\right)g$$

$$= (11.48 - 1)g$$

$$= \underline{10.48\ g}$$

t = 50

$$a = \left(\frac{78360}{6001} - 1\right)g$$

$$= (13.05 - 1)g$$

$$= \underline{12.05\ g}$$

t = 55

$$a = (\frac{71040}{4414} - 1)g$$

$$= (16.1 - 1)g$$

$$= \underline{15.1\ g}$$

t = 60

$$a = (\frac{63000}{2993} - 1)g$$

$$= (21.05 - 1)g$$

$$= \underline{20.05\ g}$$

t = 65.0

$$a = (\frac{53400}{1755} - 1)g$$

$$= (30.4 - 1)g$$

$$= \underline{29.4\ g}$$

t = 67.26

$$a = (\frac{49980}{1250} - 1)g$$

$$= (40.0 - 1)g$$

$$= \underline{39.0\ g}$$

Calculate Helium Data

Let the inside diameter of the helium tank equal the inside diameter of the propellant tank.

$$\text{Volume} = \frac{4}{3}\pi r^3 = \frac{(4\pi)(1.59)^3}{3} = 16.83\ ft^3$$

Use ideal gas law for helium calculations

$PV = mRT$ (use standard temperature)

$$m = \frac{(3500)(144)(16.83)}{\frac{(1545)(518)}{4}}$$

$$= \underline{42.4 \text{ lbs He}}$$

Find the required He pressure at the top of the propellant

25% drop across the injector face

$$X - 0.25X = 150$$

$$X = \frac{4}{3} 150 = 200 \text{ psia}$$

25% drop across the plumbing

$$X - 0.25X = 200$$

$$X = \frac{4}{3} 200 = 266.6 \text{ psia}$$

$$\text{hydraulic head just before lift off} = \frac{(47.7)(62.4)}{144}$$

$$= \underline{20.6 \text{ psi}}$$

throttle the helium so that its pressure at the top of the propellant is 246 psia. Find the time at which the pressure of the helium throughout the system is 246 psia.

$$\mathring{V} = \frac{ft^3}{sec} = \frac{\mathring{m}g}{\rho_P}$$

$$= \frac{(11.9)(32.2)}{62.4}$$

$$= 6.15 \text{ ft}^3/\text{sec}$$

$$P_1V_1 = mRT = P_2V_2$$

$$V_2 = \frac{(3500)(16.83)}{246} = 239.6 \text{ ft}^3$$

Vol of He in tank = 239.6 - 16.8 (assume volume of helium pipes negligible)

= 222.8

$$t_1 = \frac{222.8}{6.15} = \underline{36.3 \text{ sec}}$$

Calculate BTU's necessary to make process isothermal

Starting with the first law

$$\delta q = dU + PdV$$

$$= mC_V dT + PdV \qquad \text{since we assume helium is an ideal gas}$$

Then dT = 0 so

$$\delta q = PdV$$

But

$$P = \frac{RT}{V}$$

so

$${}_1Q_2 = \int_{V_1}^{V_2} RT \frac{dV}{V}$$

$$= \frac{(1544)(518)}{(4)(778)} \ln \frac{397.83}{16.83}$$

$$= \underline{814.5 \text{ BTU/lb He}}$$

$$= \underline{34{,}450 \text{ BTU Total}}$$

Calculate head suppression

$$\Delta P = \text{head suppression} = P_{He} + H - \frac{16}{9} P_c$$

$0 \leq t < 5$

$$\Delta P = 246 + 59.7 - 266.7$$

$$= \underline{39 \text{ psi}}$$

$5 \leq t < 10$

$$\Delta P = 246 + 60.4 - 266.7$$

$$= \underline{39.1 \text{ psi}}$$

$10 \leq t < 15$

$$\Delta P = 246 + 60.0 - 266.7$$

$$= \underline{39.3 \text{ psi}}$$

$15 \leq t < 20$

$$\Delta P = 246 + 59.8 - 266.7$$
$$= \underline{39.1 \text{ psi}}$$

$20 \leq t < 25$

$$\Delta P = 246 + 60.5 - 266.7$$
$$= \underline{39.8 \text{ psi}}$$

$25 \leq t < 30$

$$\Delta P = 246 + 61.6 - 266.7$$
$$= \underline{40.9 \text{ psi}}$$

$30 \leq t < 35$

$$\Delta P = 246 + 63.3 - 266.7$$
$$= \underline{42.6 \text{ psi}}$$

$t = 35 - 40$

$$\Delta P = 246 + 65.0 - 266.7$$
$$= \underline{44.3 \text{ psi}}$$

At $t = 36.3$ the pressure throughout the helium system is 246 psia. For each time interval ($t > 36.3$) the helium pressure must be calculated by

$$P_1V_1 = P_2V_2$$
$$P_{He} = \frac{P_i\ V_i}{V_i + \mathring{V} t}$$

Where

P_i = initial He pressure

V_i = initial He volume

$\mathring{V}$ = time rate of change of He volume

t = time

t = 40

$$P_{He} = \frac{(3500)(16.83)}{16.83 + (6.15)(40)}$$

$$= \frac{(3500)(16.83)}{262.8}$$

$$= \underline{233 \text{ psia}}$$

$$\Delta P = 233 + 65.1 - 266.7$$

$$= \underline{31.4 \text{ psi}}$$

Find time that chamber pressure begins to decrease. The condition for this is:

$$P_{He} + P_H - \frac{16}{9} P_c = \Delta P = 0$$

$$\frac{(3500)(16.83)}{16.83 + 6.15t} + 65.1 = 266.7$$

$$58900 = (16.83 + 6.15t)(201.6)$$

$$= 3390 + 1240t$$

$$t = \frac{55510}{1240}$$

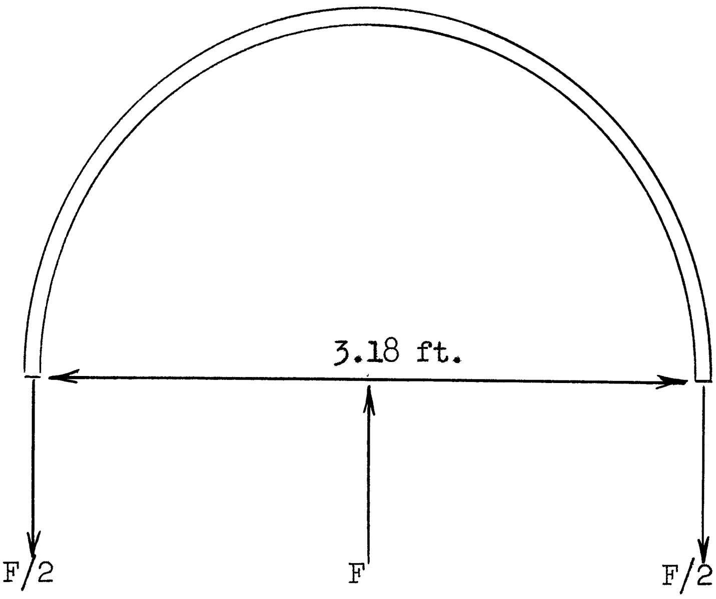
$$= \underline{44.8 \text{ seconds}}$$

Since this is very close to 45 seconds consider for calculations that the chamber pressure starts to decrease at 45 seconds.

Calculate amount of Fiberglas needed

From the head suppression graph it is seen that the maximum head suppression is 44.6 psi. Therefore, the maximum pressure at the bottom of the tank is 266.6 + 44.6 psia

Maximum pressure = 311.2 psia



$$F = (311.2)(144)(3.18)(1)$$

$$= 142{,}300 \text{ lb/ft}$$

$$\text{Working Stress} = 80{,}000 \text{ lb/in}^2$$

Neglect atmospheric pressure in calculating wall thickness. This gives an additional safety factor.

$$t = \frac{142{,}300}{(2)(80{,}000)(12)}$$

$$= \underline{.0741 \text{ inch}}$$

$$V_T = \pi r^2 h + \frac{2}{3}\pi r^3$$

$$\Delta V = 2\pi r h \Delta r + 2\pi r^2 \Delta r$$

or

$$\text{Vol Fiberglas} = (\pi D h + 2\pi r^2)\Delta r$$

$$= \pi(Dh + 2r^2)t$$

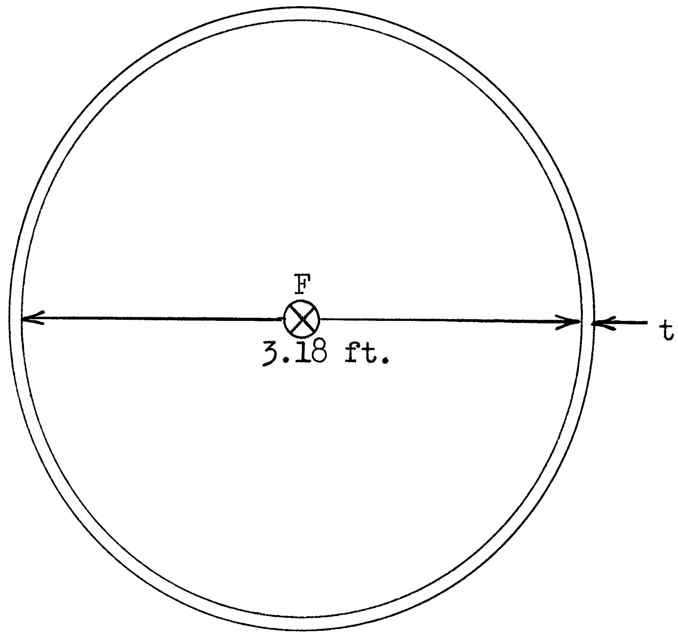
$$= \pi[(3.18)(47.7) + (2)(1.59)^2]\ \frac{0.0741}{12}$$

$$= \frac{(151.7 + 5.05)(0.0741)\pi}{12}$$

$$= \frac{(156.75)(0.0741)\pi}{12}$$

$$= \underline{3.04\ ft^3}$$

Helium Tank



$$F = (3500)(144)(1.59)^2\pi$$

$$= 4,000,000\ lb$$

$$A = \pi r^2$$

$$dA = 2\pi r dr$$

$$\Delta A = \pi D \Delta r = \pi D t$$

$$t = \frac{(4,000,000)(12)}{(144)(80,000)(3.18)\pi}$$

$$= (348)(12)$$

$$= \underline{0.417\ inches}$$

$$\Delta V = 4\pi r^2 t$$

$$V = \frac{(4\pi)(1.59)^2(0.417)}{12}$$

$$= \underline{1.106 \text{ ft}^3}$$

Total volume of fiberglas = 3.04 + 1.106

= 4.146

Wt. fiberglas = (4.146)(1.8)(62.4)

= <u>465.0 lbs.</u>

Wt. engine $= \frac{89,350}{400}$

= 223.37 lb

Total Wt. used

Wt. Helium = 42.4

Wt. fiberglas = 465.0

Wt. engine = 223.4

Total Wt. used = 730.8 lb.

Wt. left over for payload, plumbing, telemetry, etc.

= 519.2 lb

Payload from problem statement = 125.0 lb.

Wt. left over for plumbing, telemetry, etc.

= <u>394.2 lb</u>

The following performance curves and data sheet present a summary of the calculations for the liquid rocket motor system.

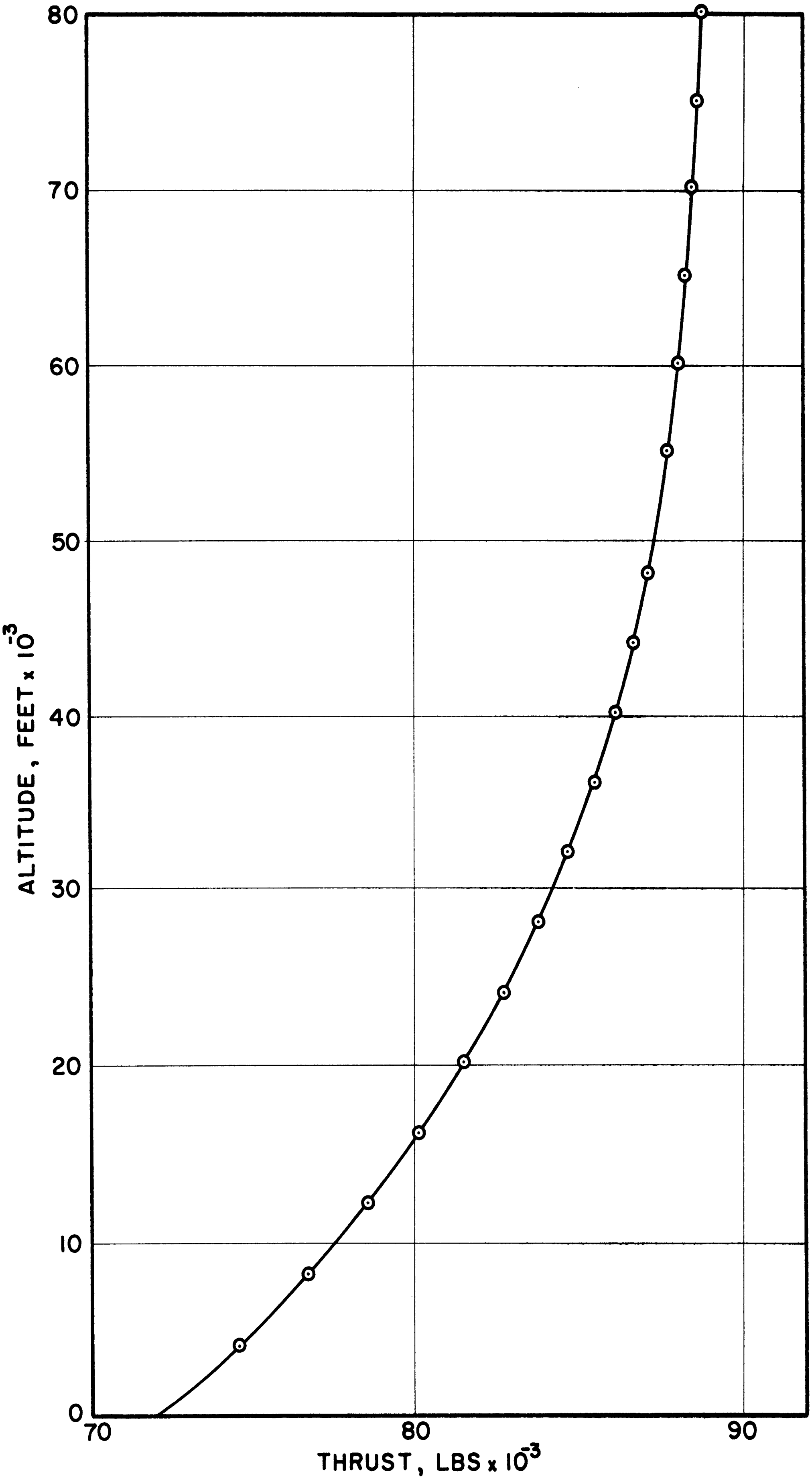


Figure 1. Rocket Motor Thrust as a Function of Altitude.

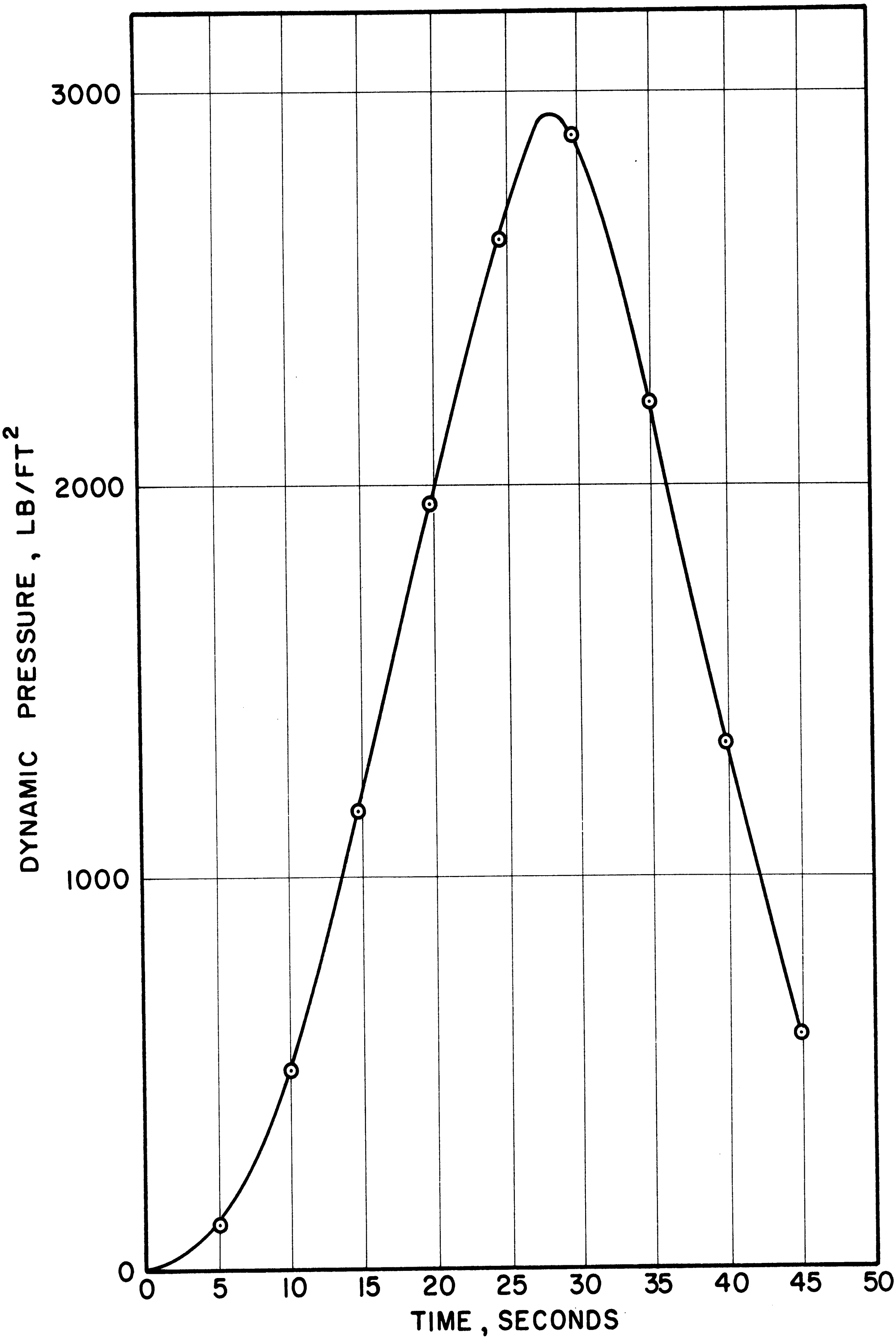


Figure 2. Dynamic Pressure Versus Altitude.

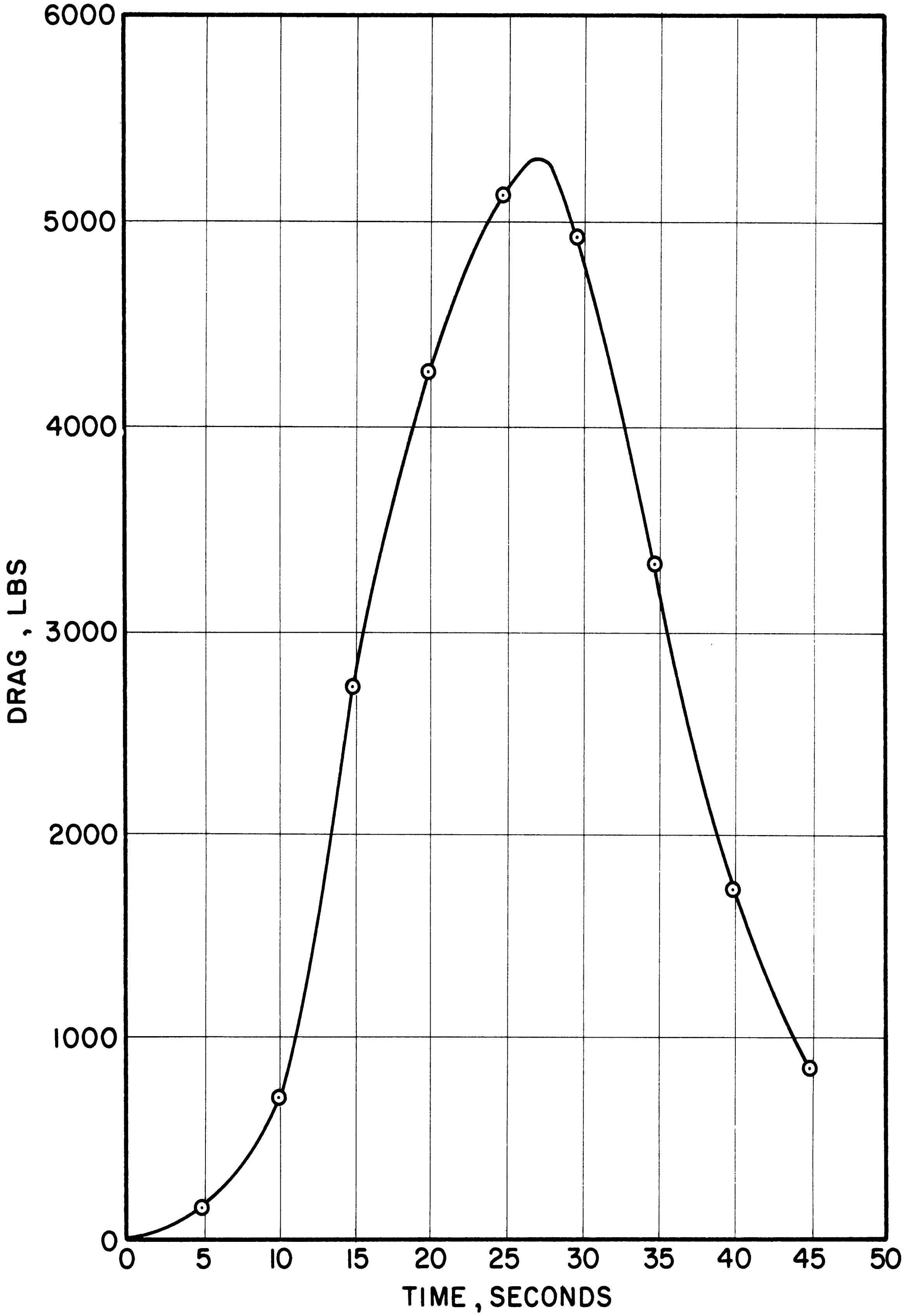


Figure 3. Drag Versus Time

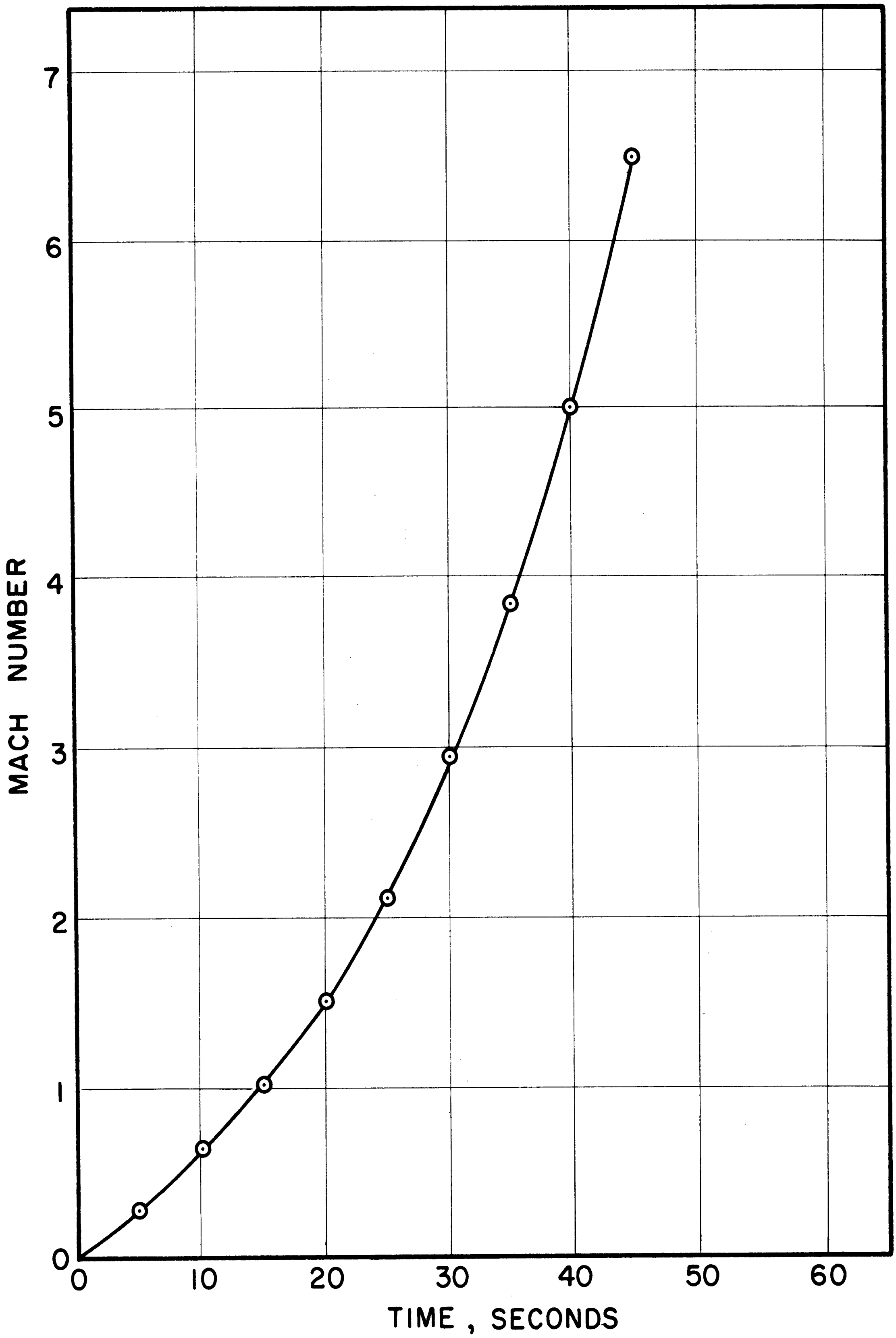


Figure 4. Mach Number as a Function of Time.

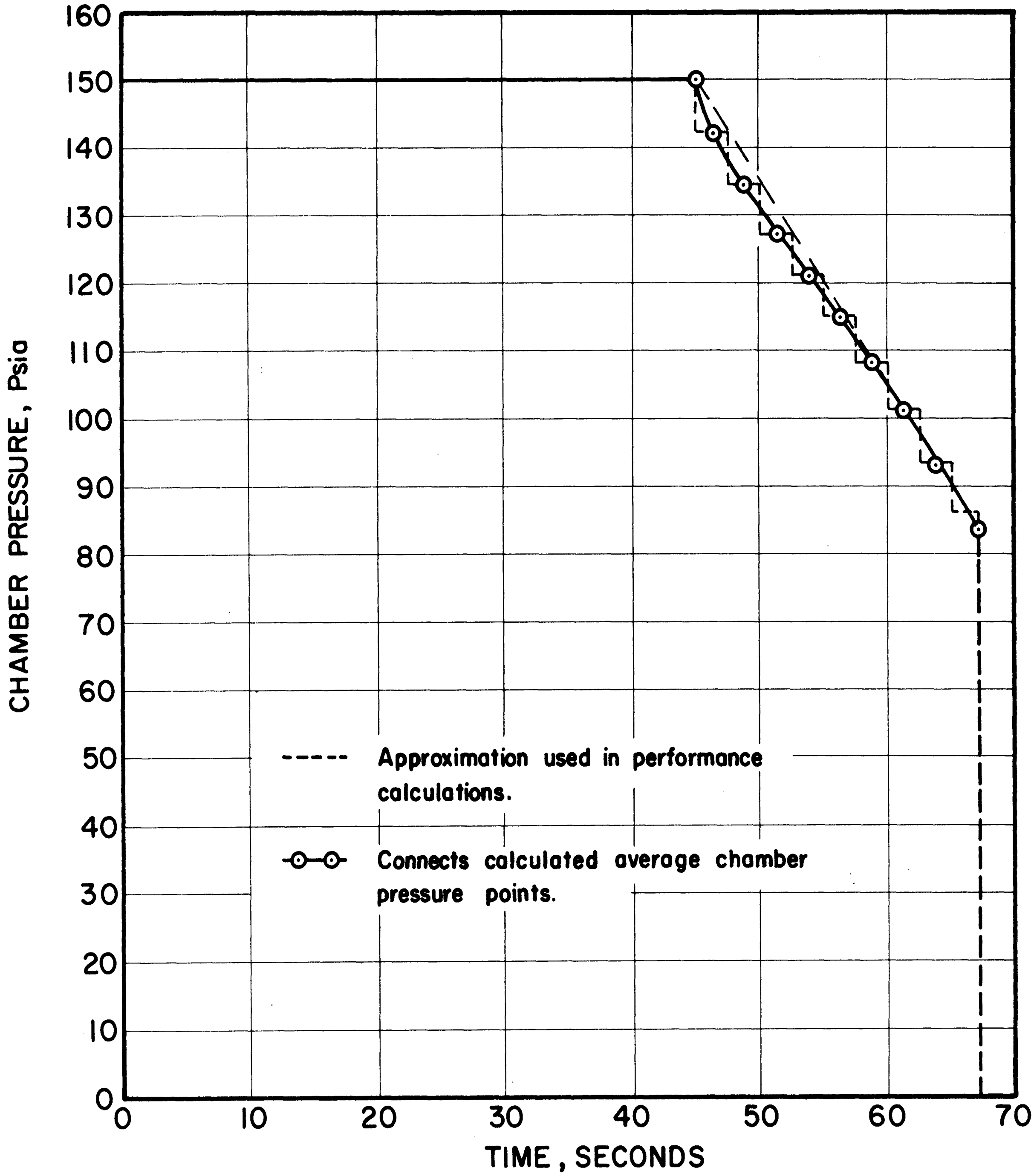


Figure 5. Rocket Chamber Pressure as a Function of Time.

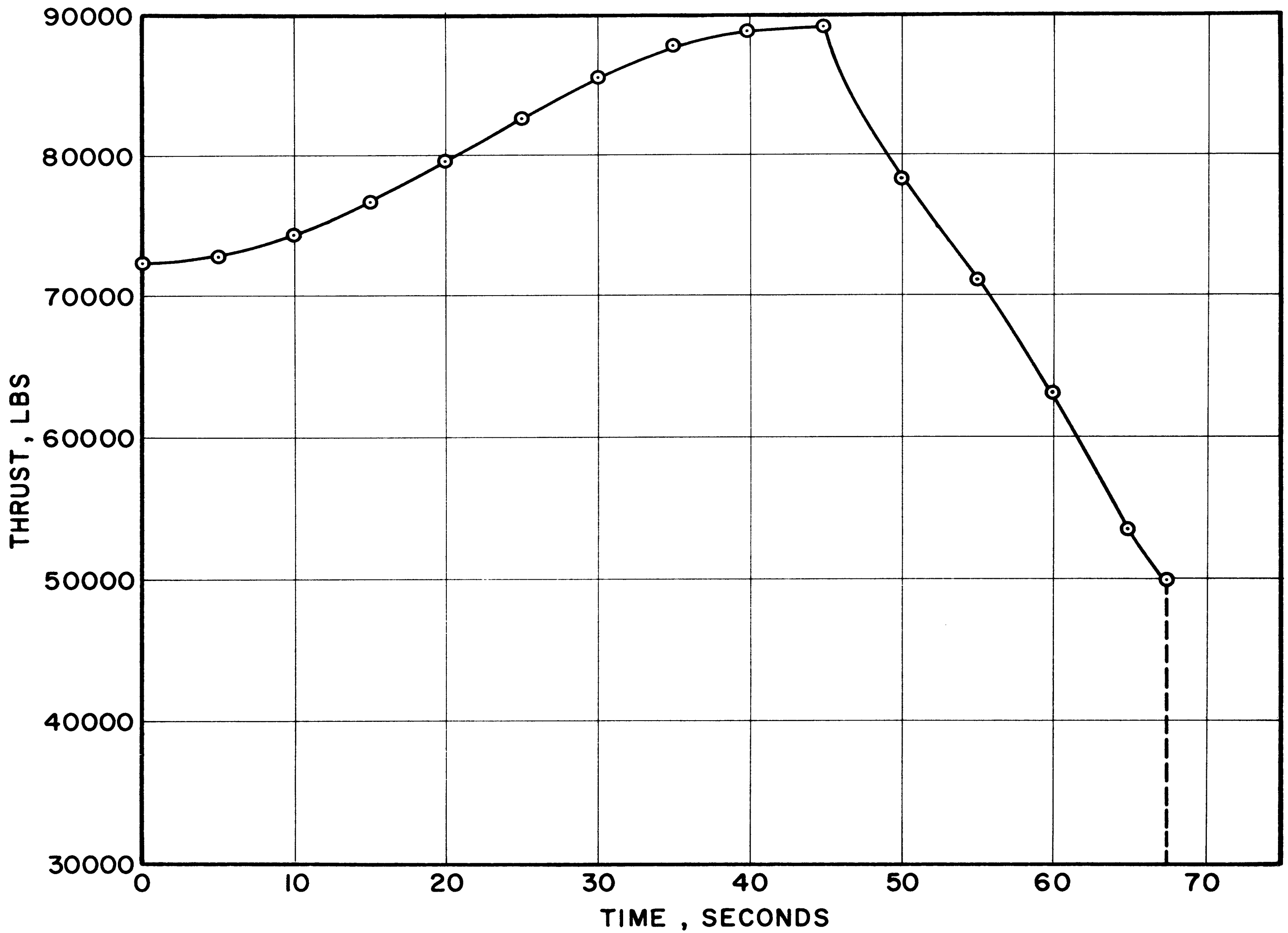


Figure 6. Rocket Motor Thrust Versus Time Until Burnout.

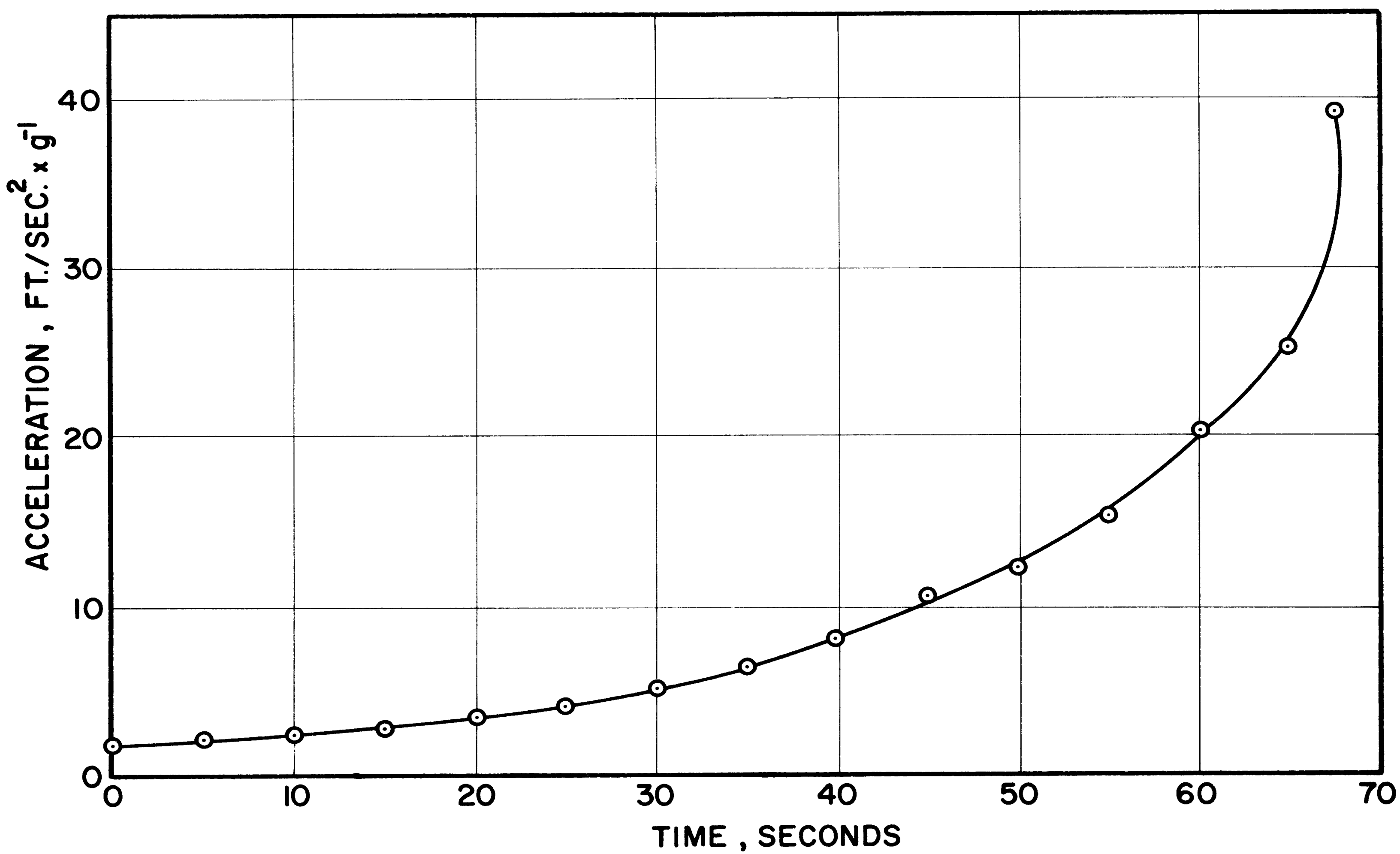


Figure 7. Vehicle Acceleration Versus Time.

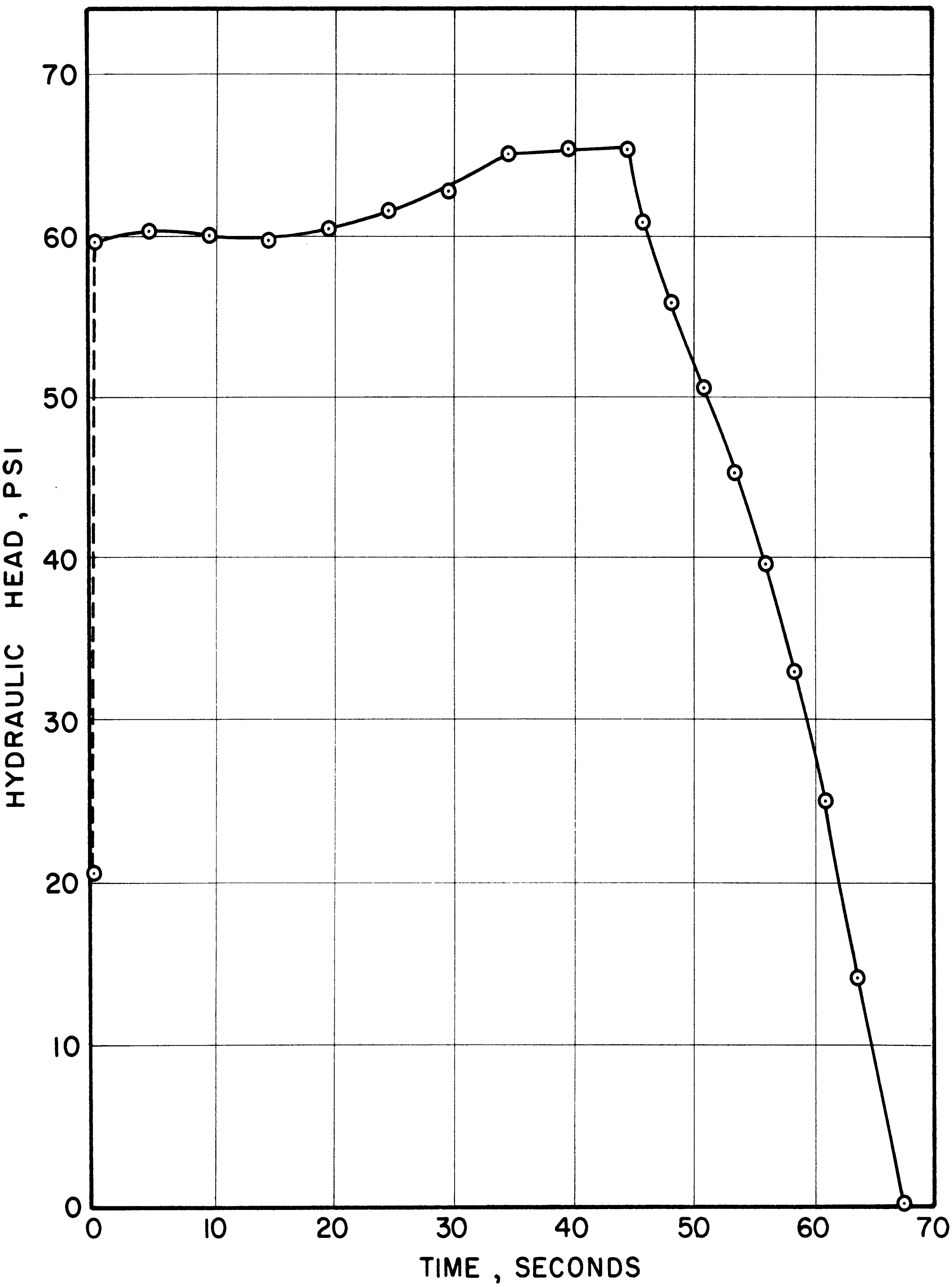


Figure 8. Hydraulic Head Versus Time.

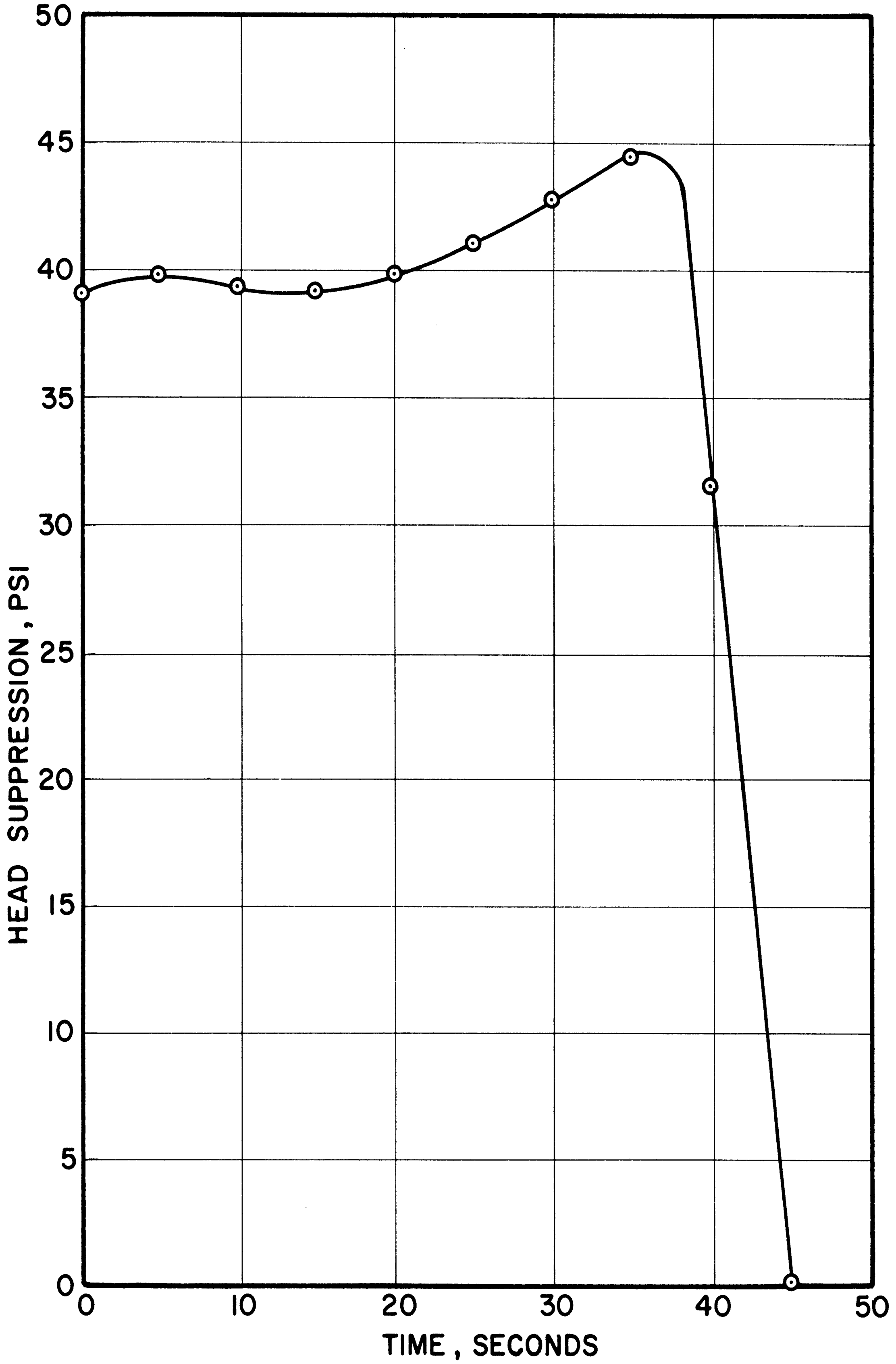


Figure 9. Pressure Drop Across the Head Suppression Valve as a Function of Time.

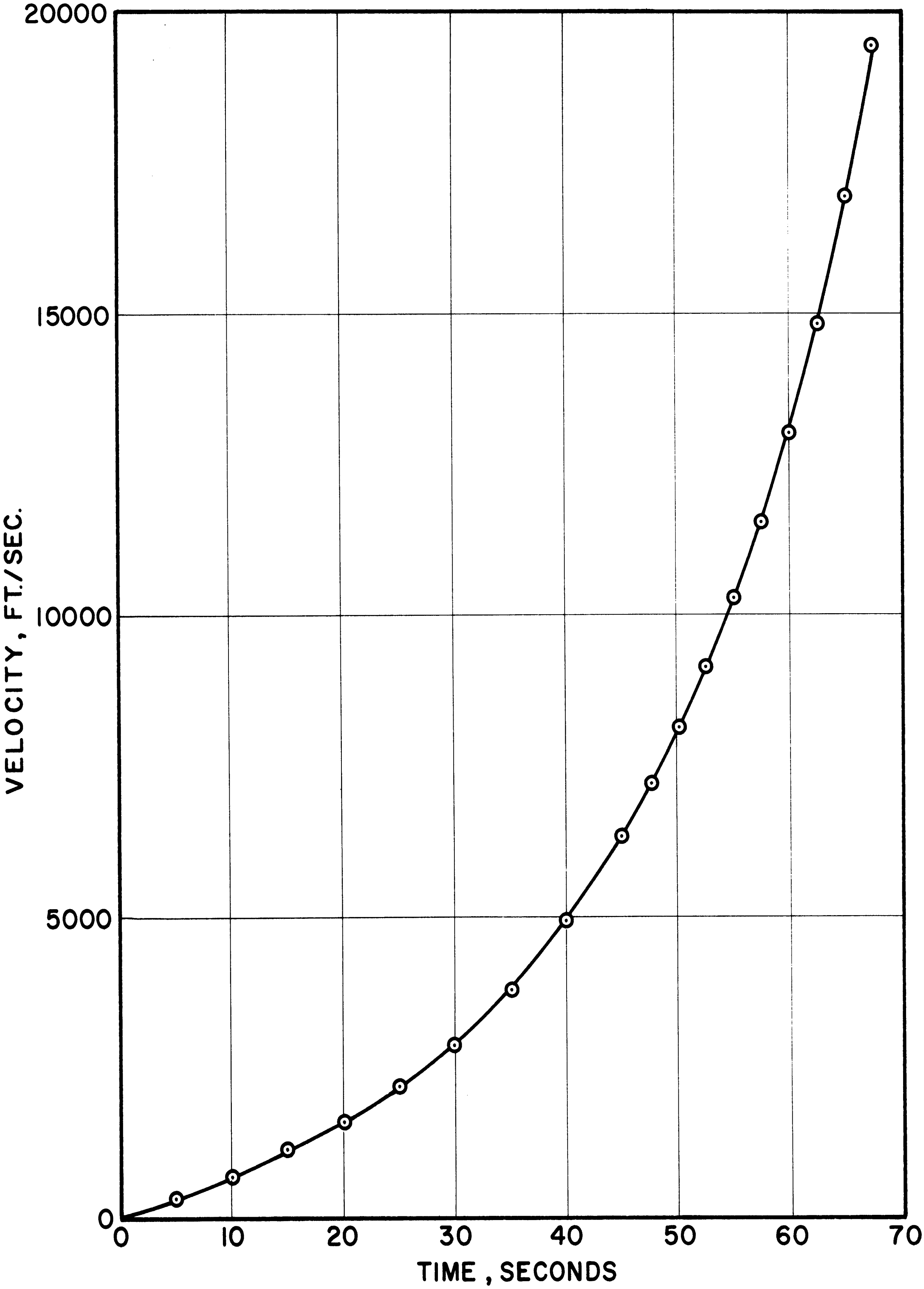


Figure 10. Vehicle Velocity Versus Time During Powered Phase.

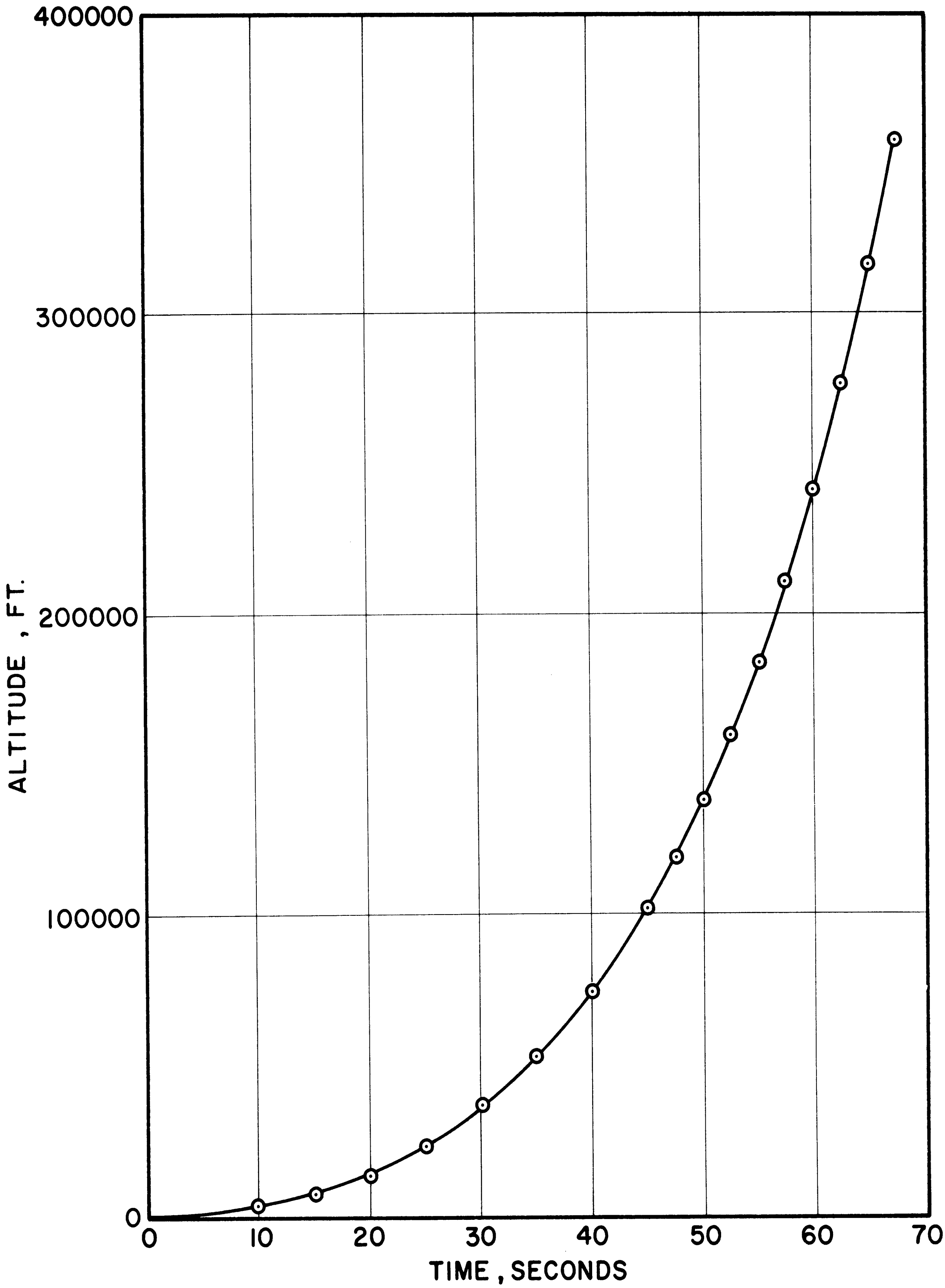


Figure 11. Altitude Versus Time During Powered Phase.

DATA SHEET

Time Sec	Thrust lb	Dynamic Pressure lb/ft^2	Drag lb	Acceleration ft/sec^2	Head Suppression Psi	Chamber Pressure Psi	Hydraulic Head Psi	Velocity ft/sec	Altitude ft	Mach Number
0-	72290	0	0		0	150	20.6			
0+	72290	0	0	1.89g	39.0	150	59.7	0	0	0
5	72700	120.3	172.1	2.14g	39.7	150	60.4	323	780	0.29
10	74200	514.0	695	2.47g	39.3	150	60.0	690	3294	0.623
15	76600	1164	2726	2.83g	39.1	150	59.8	1113	7778	1.011
20	79500	1953	4260	3.34g	39.8	150	60.5	1603	14537	1.512
25	82700	2620	5100	4.03g	40.9	150	61.6	2182	23961	2.12
30	85600	2880	4910	4.98g	42.6	150	63.3	2885	36578	2.98
35	87700	2205	3320	6.22g	44.3	150	65.0	3762	53124	3.88
40	88700	1328	1790	7.99g	31.4	150	65.1	4882	74635	5.03
45	89100	595	848	10.48g	0	150	65.4	6331	102522	6.52
46.25						142.0	60.9			
47.50								7184	119399	
48.75						134.5	55.9			
50.00	78360			12.05g				8117	138453	
51.25						127.5	50.7			
52.50								9141	159977	
53.75						121.0	45.4			
55.00	71040			15.10g				10282	184212	
56.25						114.8	39.8			
57.50								11577	211467	
58.75						108.0	33.0			
60.00	63000			20.05g				13067	242199	
61.25						101.0	24.9			
62.50								14829	276916	
63.75						93.0	14.1			
65.00	53400			29.40g		89.0		16953	316478	
67.26	49980			39.00g		83.3	0	19445	357400	

Summit Altitude is 1800 Miles.

SOLID ROCKET DESIGN

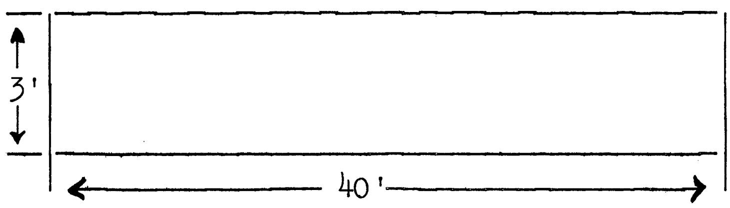
Theodore Petersen

NOMENCLATURE

$\mathring{m}$	-	mass flow
V_{EX}	-	velocity at exit of rocket nozzle
M	-	Mach number
T	-	temperature
c	-	denotes chamber conditions
P	-	pressure
a	-	speed of sound
γ	-	ratio of specific heats
A	-	area
t	-	denotes throat conditions
Ex	-	denotes the conditions at the exit of the rocket nozzle
F	-	thrust
$\int$	-	density
n	-	denotes exponent in burning rate low
r	-	burning rate
q	-	dynamic head
Γ	-	$\gamma^{\frac{1}{2}} \left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma + 1}{2(\gamma - 1)}}$
R	-	gas constant

DESIGN SPECIFICATIONS

Assume propellant chamber with internal dimensions as shown:



Propellant: NH_4NO_3/C_2H_4O/CATALYST

Properties:

Adiabatic Flame Temperature	=	2700°F.
Average Molecular Weight	=	22 lb./mol.
γ	=	1.26
Typical Sea Level I_{SP}	=	195 Sec.
Characteristic Velocity	=	4000 ft./sec.
r @ P_c = 1000 PSI 70 °F	=	0.1 in./sec.
r Exponent - n	=	0.4
Specific Weight	=	0.056 lb./in.^{3}
Lower Combustion Limit	<	100 PSI
Pressure Limit	>	3000 PSI

Specifications:

General:

Fill up to Base of Nozzle
Consider Star Grain
Burning time 20 → 60 seconds
Payload = 125 lb.
"Fiberglas" Casing - 80,000 PSI Design Stress
Protective Heat Material - .050 in. of Gunk at same Sp. Gr. as "Fiberglas."
Motor Weight:

$$\frac{70 \text{ lb. Thrust}}{\text{Lb. motor wt.}}$$

Leave out Volume Increase with Respect to Time but Comment on its Effect.
Estimate Velocity at Back End of Grain.
Size Nozzle so $P_{EX} = 1/2\ P_{ATM}$ at Sea Level so Don't Need to Worry About Separation.
Maximum Pressure Fluctuation = 50%
Junk Weight = 150 lb.
Lift Off at 4g.

Weight Estimate:

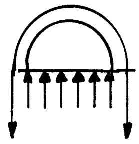
Assume a loading fraction $\epsilon = \frac{\text{Total Cross-Sect. Area of Propellant}}{\text{Cross-Sectional Area of Motor}}$

$= .80$ (Preliminary Assumption)

Volume of propellant charge: $(.80) \cdot \frac{\pi D^2}{4} (40) = 226 \text{ Ft.}^3$

Weight of charge: $(226)(.056)(1728) = 21{,}850$ lb.

Weight of fiberglas casing: 36 in. x 1000 PSI = 36,000 lb.



working stress = 80,000 PSI.

So the thickness is:

$$\frac{36{,}000}{80{,}000 \times 2} = .225 \text{ in. thick}$$

The volume of fiberglas for the walls of the chamber is then given by

$$(\pi)(3)(40)(.225)(1/12) = 7.07 \text{ ft.}^3$$

Estimate the same thickness of fiberglas for the top of the chamber and roughly assume 1 ft. of lap joint:

$$\pi \frac{3^2}{4} + \pi(3)\,1 = 3\pi(1 + \frac{3}{4})$$

Volume = $3\pi(1.75)(.225)(1/12) = .309 \text{ ft.}^3$
Total volume = 7.379 ft.^3
Weight = 7.379 x 62.4 x 1.8 = 828 lb.

Weight of insulating material:

Volume of insulating material:

$$(\pi)(3)(40)(.050)(1/12) + (\pi)(9/4)(.050)(1/12)$$
$$= 1.6 \text{ ft.}^3$$

Weight = (1.6) x (62.4) x (1.8) = 179.8 lb.
Payload weight = 125 lb.
Junk weight = 150 lb.
Total weight - Motor weight = 23,133 lb.
Assume motor weight = 1400 lb.

Total Weight = 24,533 lb.

For 4g lift-off thrust - thrust = 98,132 lb. - which corresponds to an engine weight of 1400 lb.

NOZZLE SIZING

It was specified to size the nozzle so that $P_{EX} = 1/2\ P_{ATM}$. This specification, along with a knowledge of the chamber properties and the desired thrust at sea level, enables the nozzle to be sized using isentropic relations and the basic thrust equation; the relations to be used are:

$$(1)\quad F = \overset{o}{m}\,V_{EX} + (P_{EX} - P_{ATM})\Delta_{EX} \quad \text{(Assuming Ideal Nozzle)}$$

$$(2)\quad \overset{o}{m} = \frac{\gamma MAP}{a}$$

$$(3)\quad \frac{Tc}{T} = (1 + \frac{\gamma-1}{2} M^2)$$

$$(4)\quad \frac{Pc}{P} = (1 + \frac{\gamma-1}{2} M^2)^{\gamma/\gamma-1}$$

$$(5)\quad \frac{A}{A_t} = \frac{1}{M}\left[\left(\frac{2}{\gamma-1}\right)\left(1 + \frac{\gamma-1}{2} M^2\right)\right]^{\frac{\gamma+1}{2(\gamma-1)}}$$

$$(6)\quad V_{EX} = M_{EX}\, a_{EX} = M_{EX}\sqrt{\gamma aRT_{EX}}$$

Now, substituting (2) into (1):

$$(7)\quad F = \frac{\gamma M_t A_t P_t}{a_t} \cdot V_{EX} + (P_{EX} - P_{ATM})A_{EX}$$

and substituting in (6) and cancelling constant terms:

$$(8)\quad F = \gamma M_t A_t P_t M_{EX}(T_{EX}/T_t)^{1/2} + (P_{EX} - P_{ATM})\, A_{EX}$$

and, from the isentropic relationships:

Hence,

$$(9)\quad \left(\frac{T_{EX}}{Tt}\right)^{1/2} = \left[\left(\frac{\gamma+1}{2}\right)^{\gamma/\gamma-1} \frac{P_{EX}}{P_c}\right]^{\gamma-1/z\gamma'}$$

Also,

$$(10)\quad P_t = \left(\frac{2}{\gamma+1}\right)^{\gamma/\gamma-1} P_c$$

and,

$$\frac{P_c}{P_{EX}} = \left(1 + \frac{\gamma-1}{2} M_{EX}^2\right)^{\gamma/\gamma-1}$$

$$M_{EX}^2 = \frac{2}{\gamma-1}\left[\left(\frac{P_c}{P_{EX}}\right)^{\gamma-1/\gamma} -1\right]$$

$$(11)\quad M_{EX} = \left\{\frac{2}{\gamma-1}\left[\left(\frac{P_c}{P_{EX}}\right)^{\gamma-1/\gamma} -1\right]\right\}^{1/2}$$

Now, divide (8) by A_t:

$$(12)\quad \frac{F}{A_t} = \gamma P_t M_{EX} (T_{EX}/T_t)^{1/2} + (P_{EX} - P_{ATM})A_{EX}/A_t$$

and the terms on the right of (12) can be evaluated with the known conditions:

$$P_c = 1000 \text{ PSI} \qquad T_c = 3160°R$$

$$P_{EX} = 7.35 \text{ PSI} \qquad \gamma = 1.26$$

From (10):

$$P_t = (0.5532)(1000) = 553.2 \text{ PSI}$$

From (11):

$$M_{EX} = \{7.69\ [(136.1)^{.2064} - 1]\}^{1/2}$$

$$= \{7.69\ [2.76-1]\}^{1/2} = 3.678$$

From (9):

$$\left(\frac{T_{EX}}{T_t}\right)^{1/2} = \left[(1.13)^{4.85} \cdot (.00735)\right]^{.1031} = .64$$

From (5):

$$\frac{A_{EX}}{A_t} = \frac{1}{3.678}\left[(.885)(2.758)\right]^{4.345}$$

$$= 13.05$$

Then, from (12)

$$\frac{98,132}{A_t} = (1.26)(553.2)(3.678)(.64) - (7.35)(13.05)$$

$$98,132 = (1638 + 96)\ A_t$$

$$A_t = 63.6 \text{ Sq. In.}$$

and now the mass flow can be computed:

$$\overset{o}{m} = \frac{\gamma M A P}{a_t} = \frac{(1.26)(63.6)(553.2)}{[(1.26)(32.2)(\frac{1544}{22})(\frac{3160}{1.13})]^{1/2}}$$

$$= 15.7 \text{ slugs/sec.}$$

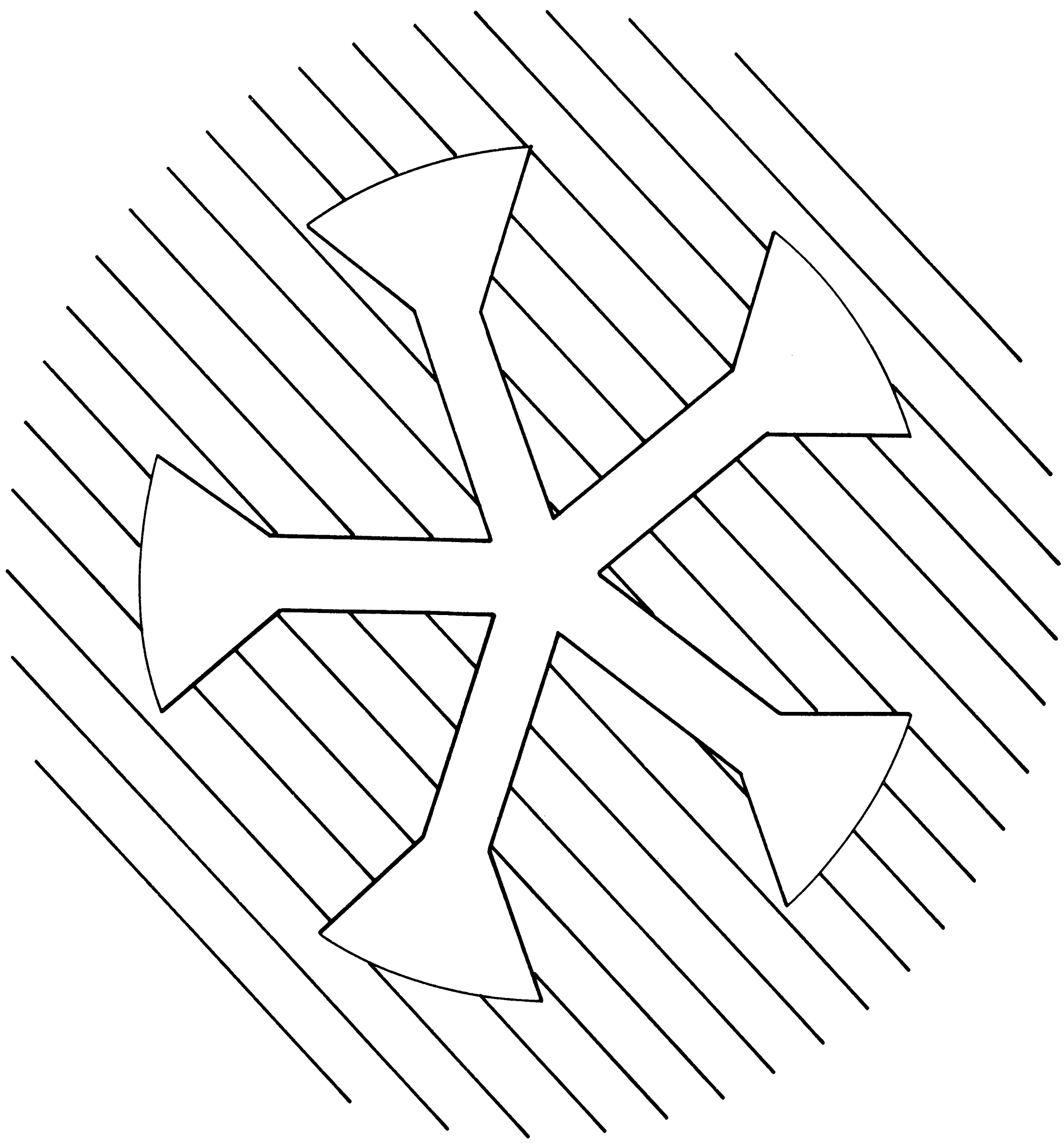


Figure 1. Wagon Wheel Grain Configuration.

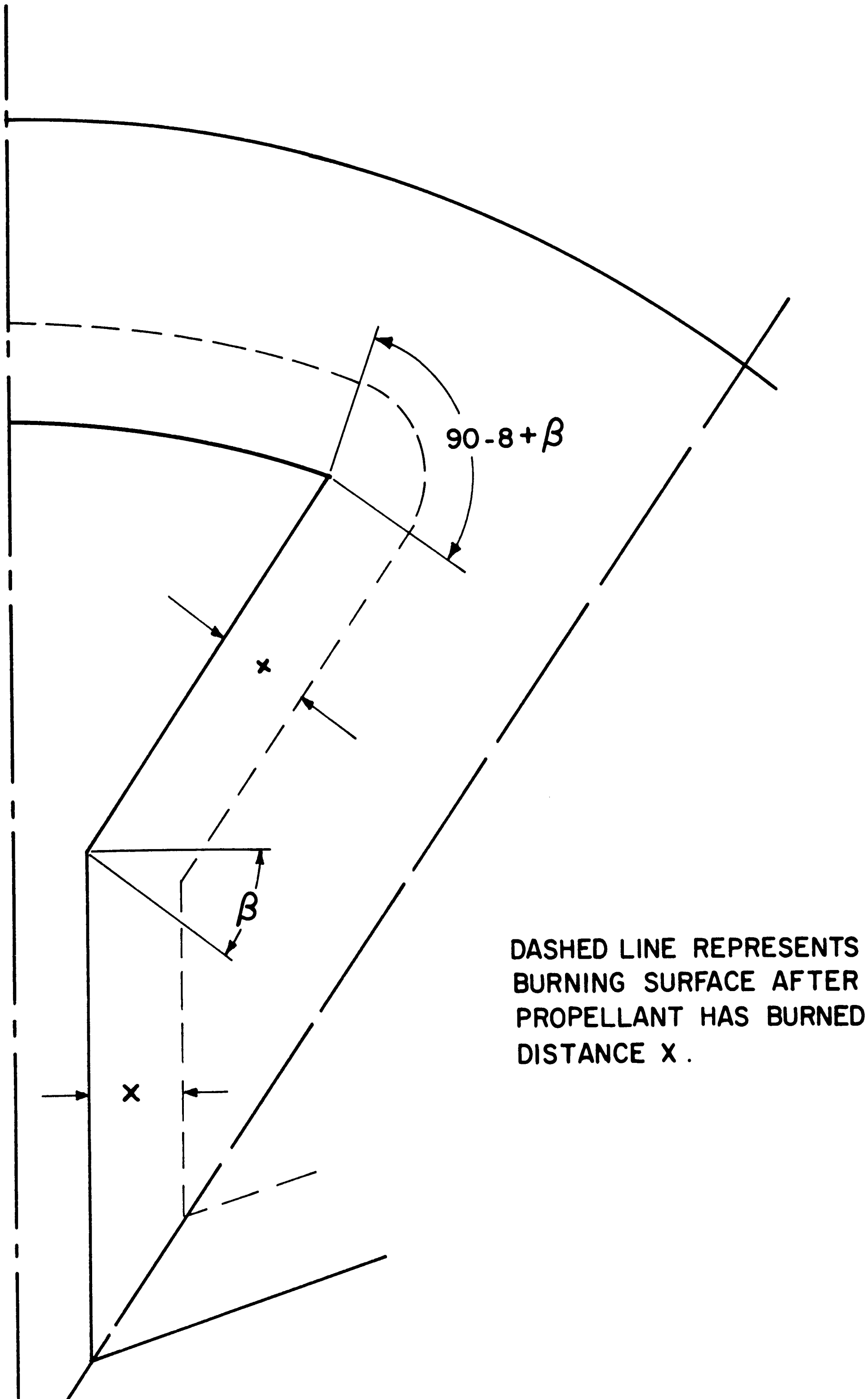

90-8+β
X
β
X
DASHED LINE REPRESENTS BURNING SURFACE AFTER PROPELLANT HAS BURNED DISTANCE X.

Figure 2.

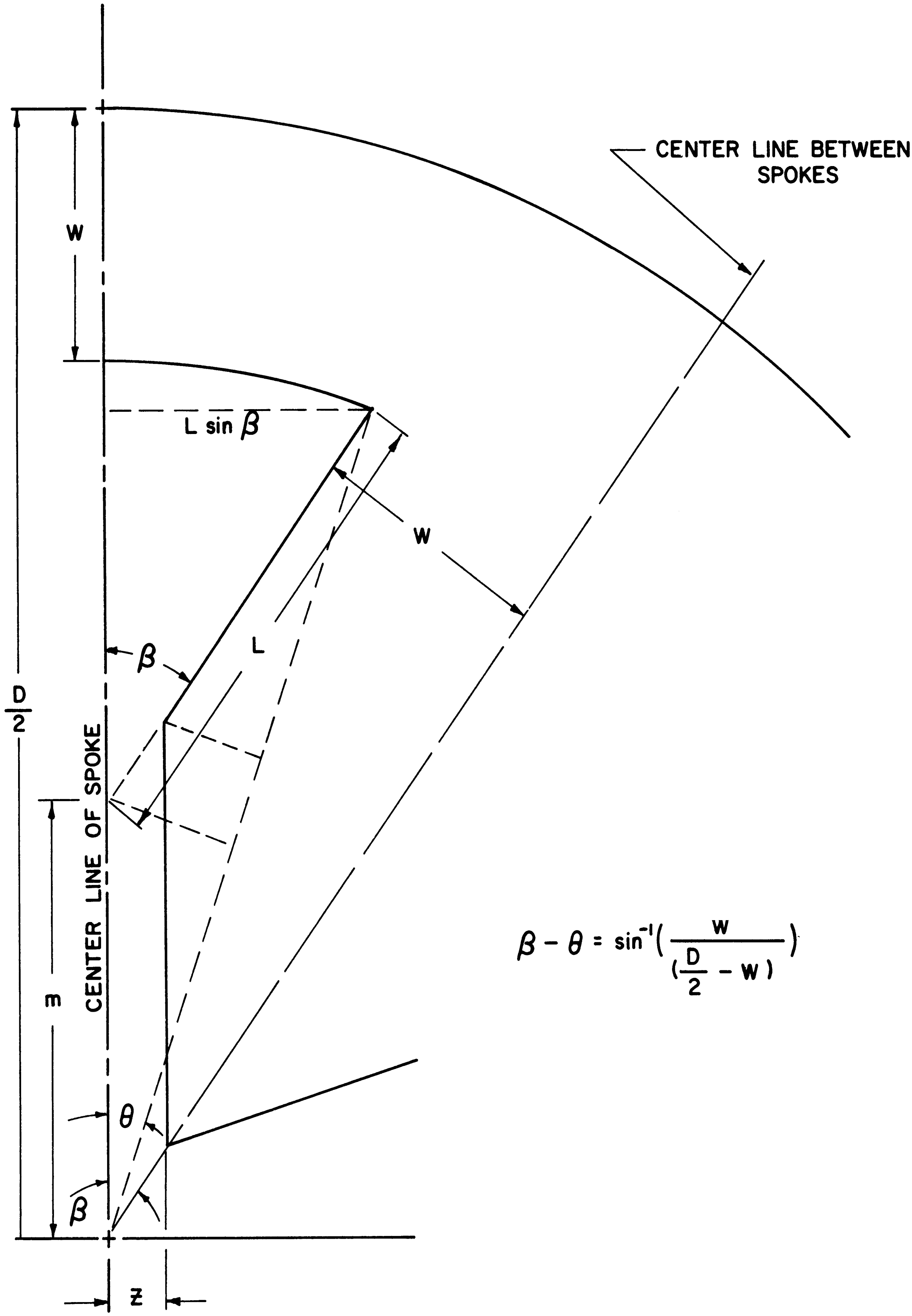

CENTER LINE BETWEEN SPOKES
W
L sin β
W
L
β
D/2
CENTER LINE OF SPOKE
m
θ
β
Z
β - θ = sin⁻¹(W / (D/2 - W))

Figure 3.

GRAIN SIZING

Now, in general, the basic differential equation governing continuity for solid propellant rockets is:

$$A_c r \rho_P = \frac{d}{dt}(\rho_c V_c) + \frac{\gamma M_t A_t P_t}{Q_t}$$

$A_c r \rho_P$ — Propellant Mass Burned

$\frac{d}{dt}(\rho_c V_c)$ — Storage Term

$\frac{\gamma M_t A_t P_t}{Q_t}$ — Mass Flow through Nozzle

and assuming that the storage term is negligible, this equation becomes:

$$A_c r \rho_P = \frac{\gamma M_t A_t P_t}{a_t}$$

and from the propellant properties:

$$\rho_P = (.056)(1728) = 96.8 \text{ lb./ft.}^3$$

$$r = a P_c^n = a(1000)^{.4} = a(15.9) = .1 \text{ in./sec.} = .00833 \text{ ft./sec.}$$

$$a = .1/15.9 = .00629$$

so,

$$A_c = \frac{(15.7)(32.2)}{(.00833)(96.8)} = 626 \text{ ft.}^2$$

and, as the chamber is 40 ft. long the burning perimeter must be 15.66 ft.

DERIVATION OF EXPRESSIONS RELATING PACKING FRACTION AND BURNING PERIMETER TO CONFIGURATION DIMENSIONS

Burning Perimeter:

The burning perimeter may be found by considering the half spoke shown in the following sketch and multiplying the result by 10.

The perimeter is made up of three portions: The arc subtended by the angle θ, the line of the spoke inclined at the angle β to the center-line and the vertical line of length m. The burning perimeter and loading fraction will be analytically determined.

The burning perimeter is initially given by the following relationship:

$$B.P._i = 10(S_1 + S_2 + S_3)$$

where:

$$S_1 = (\frac{D}{2} - W) \cdot \theta$$

$$S_2 = L - \frac{Z}{\sin\beta} = \frac{(\frac{D}{2} - W - \frac{W}{\sin\beta} \cdot \cos\theta)}{\cos(\beta - \theta)} - \frac{Z}{\sin\beta}$$

$$S_3 = m = \frac{W}{\sin\beta}$$

Now, the cross-sectional area of the propellant can be found. This will be done by subtracting the pie-shaped segments from the total area - then adding back in the triangles of base m and altitude L sinβ - and finally subtracting out the parallelograms of base m and altitude Z. Hence the cross-sectional area of the propellant is given by:

$$A_i = \frac{\pi D^2}{4} - 5(\frac{D}{2} - W)^2 \cdot \theta + 5\frac{W}{\sin\beta}\frac{(\frac{D}{2} - W - \frac{W}{\sin\beta} \cdot \cos\theta)}{\cos(\beta - \theta)} - 10\,mZ$$

Now, after the propellant has burned a distance x the burning perimeter is:

$$B.P._x = 10\,(S_1 + S_2 + S_3 + S_4)$$

where:

$$S_1 = (\frac{D}{2} - [W-x])\,\theta$$

$$S_2 = x\,(\frac{\pi}{2} - \theta + \beta)$$

$$S_3 = \frac{(\frac{D}{2} - W - \frac{W}{\sin\beta} \cdot \cos\theta)}{\cos(\beta - \theta)} - \frac{Z}{\sin\beta} - x\tan\beta/2$$

$$S_4 = m - \frac{x}{\sin\beta}$$

and the cross-sectional area of the propellant after it has burned a distance x can be found as:

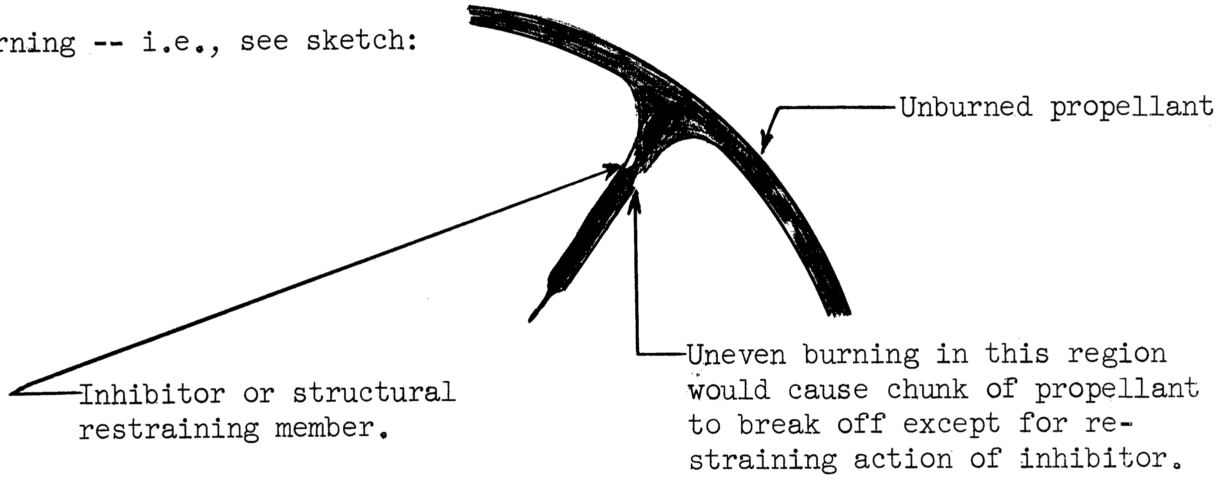
$$A_x = A_i - 5\left\{\left(\frac{D}{2} - W + x\right)^2 - \left(\frac{D}{2} - W\right)^2\right\}\theta - 5x^2\left\{\frac{\pi}{2} - \theta + \beta\right\}$$
$$- 10\,mx - \frac{10x}{\tan\beta}\cdot\frac{x}{2} - 10\left\{\frac{\left(\frac{D}{2} - W - \frac{W}{\sin\beta}\cdot\cos\theta\right)}{\cos(\beta - \theta)} - \frac{Z}{\sin\beta} - \frac{x}{\tan\beta}\right.$$
$$\left. - x\tan\frac{\beta}{2}\right\}$$

DISCUSSION OF CHOICE OF GRAIN CONFIGURATION

A sketch of the grain configuration chosen is shown on Page 60. Preliminary calculations were carried out for several star configurations but these all showed that it would be extremely difficult to meet the 4g lift-off specification for any decent loading factor without a highly regressive burning configuration resulting. It was therefore decided not to consider a star (internal) grain configuration any further. A cylindrical grain was briefly considered as it has the advantage of a uniform burning surface but it was immediately obvious that it would not be possible to meet the 4g lift-off requirement with any sort of loading fraction that was acceptable. Also, as it was specified in class not to use this configuration, its consideration was dropped.

It was then decided to try the wagon wheel configuration, and a five spoke wagon wheel, as shown in the sketch on Page 60, was decided upon as it has only slightly progressive burning as can be seen by the pressure trace on Page 61. An analytical procedure was then determined to size the configuration for a 4g lift-off and an analytic method of determining the burning surface as a function of x, the distance burned, was derived.

The actual design would differ slightly from the sketch on Page 60 in that the acute angles of the open or port area should be replaced by small fillets to prevent cracking during storing and handling. Also, although theoretically none are needed, there should be some sort of an inhibitor or structural member (i.e., a fine screen) placed along the center line between the spokes to guard against chunks of the propellant being carried downstream to the nozzle as a result of uneven burning -- i.e., see sketch:



The analytic solution of the optimum grain configuration follows:

SELECTION OF CORRECT GRAIN CONFIGURATION

It is first necessary to know the relationship between burning perimeter and loading fraction ϵ for a 4g lift-off. This relationship may be plotted by proceding in a manner similar to that used in the preliminary calculations from the calculations on Page 62.

$$A_t = \frac{F}{1542} \text{ in.}^2$$

and from the calculations on Page 63:

$$A_c = \frac{\overset{\circ}{m}}{r\rho_p} = .0064\ F \text{ ft.}^2$$ when F is in units of pounds.

and to determine the trust necessary for a given loading factor it can be seen from the calculations on Pages 57and58that:

$$\text{Total weight} = \text{T.W.} = 1283 + (27350)\epsilon + \text{motor weight}$$
$$\text{Motor weight} = F/70; \; F = 4 \times \text{T.W.}$$

and the values of thrust and loading factor that satisfy these relationships are given below along with the corresponding values of A_c and burning perimeter:

for ϵ = .60:	Total weight	= 18753 lb.
	Thrust	= 75,012 lb.
	A_c	= 480 ft^2
	B. P.	= 12 ft.
for ϵ = .70	Total weight	= 21648 lb.
	Thrust	= 86,600 lb.
	A_c	= 554 $ft.^2$
	B. P.	= 13.85 ft.
for ϵ = .75	Total weight	= 23103 lb.
	thrust	= 92,412 lb.
	A_c	= 591 $ft.^2$
	B. P.	= 14.79 ft.
for ϵ = .80	Total weight	= 24533 lb.
	Thrust	= 98,132 lb.
	A_c	= 627 $ft.^2$
	B. P.	= 15.67 ft.
for ϵ = .85	Total weight	= 25983 lb.
	Thrust	= 103932 lb.
	A_c	= 665 $ft.^2$
	B. P.	= 16.63 ft.

GRAIN CHARACTERISTICS

Consider a grain with a four inch web and five spokes (the five spoke configuration was chosen as it gives the most neutral progressivity ratio). Then:

$\underline{W = 4"}$; $\frac{D}{2} = 18$; $\beta = 36° = .628$ rad.; $\sin\beta = .588$; $\theta = 36° - \sin^{-1}\frac{4}{14}$

$$= 36.0° - 16.6° = 19.4° = .3385 \text{ rad.}$$

$$\cos\theta = .943; \; \cos(\beta-\theta) = \cos 16.6° = .959$$

then, $S_1 = (14)(.3385) = 4.74$ in.

$$S_2 = \frac{(14 - 6.42)}{.959} - 1.7z = 7.91 - 1.7z$$

$S_3 = 6.80$ in.

Therefore,

$$B.P._i = \frac{10}{12}(19.45 - 1.7z) \text{ ft.}$$

and the cross-sectional area is:

$$A_i = 1018 - 5 \times 196 \times .3385 + 5 \times 6.8 \times 7.91 \times .588 - 10 \times 6.8\, z$$

$$= 1018 - 332 + 158 - 68z = 844 - 68z$$

so the loading factor is given by:

$$\epsilon_i = \frac{844-68z}{1018}$$

Then, for $z = 0$, $B.P._i = 16.20$ ft., $\epsilon_i = .829$

for $z = 1$, $B.P._i = 14.8$ ft., $\epsilon_i = .761$

for $z = 2$, $B.P._i = 13.37$ ft., $\epsilon_i = .695$

<u>W = 3.9</u>", $\frac{D}{2} = 18$; $\beta = 36°$; $\theta = 36° - \sin^{-1}\frac{3.9}{14.1} = 36° - 16.05° = 19.95°$

$= .3475$ rad.

$\cos\theta = .940$; $\cos(\beta-\theta) = .961$

then, $S_1 = (14.1)(.3475) = 4.90$ in.

$$S_2 = \frac{(14.1 - 6.24)}{.961} - 1.7z = 8.18 - 1.7z$$

$S_3 = 6.64$ in.

Therefore,

$$B.P._i = \frac{10}{12}(19.72 - 1.7z)$$

and the cross-sectional area is:

$$A_i = 1018 - 5 \times 198.8 \times .3475 + 5 \times 6.64 \times 8.18 \times .588 - 10 \times 6.64z$$

$$= 1018 - 346 + 159.5 - 66.4z = 832 - 66.4z$$

so, the loading factor is given by:

$$\epsilon_i = \frac{832 - 66.4z}{1018}$$

Then, for $z = 0$ $\quad B.P._i = 16.42$ ft., $\quad \epsilon_i = .817$

for $z = 1$ $\quad B.P._i = 15$ ft., $\quad \epsilon_i = .752$

for $z = 2$ $\quad B.P._i = 13.6$ ft., $\quad \epsilon_i = .687$

<u>W = 3.95"</u>, $\frac{D}{2} = 18$, $\beta = 36°$, $\theta = 36° - \sin^{-1} \frac{3.95}{14.05} = 36° - 16.31° = 19.69°_5$

$\cos\theta = .942$, $\cos(\beta-\theta) = .960$ $\qquad \theta = .3435$ rad.

then, $S_1 = (14.05)(.3435) = 4.83$ in.

$$S_2 = \frac{(14.05 - 6.32)}{.960} - 1.7z = 8.06 - 1.7z$$

$S_3 = 6.71$ in.

Therefore,

$$B.P._i = \frac{10}{12}(19.6 - 1.7z)$$

and the cross-sectional area is:

$$A_i = 1018 - 5 \times 197.3 \times .3435 + 5 \times 6.71 \times 8.06 \times .588 - 10 \times 6.71z$$

$$= 1018 - 339 + 159 - 67.1z = 838 - 67.1z$$

so the loading factor is given by:

$$\epsilon_i = \frac{838 - 67.1z}{1018}$$

Then, for $z = 0$ $\quad B.P._i = 16.32$ ft., $\quad \epsilon_i = .823$

for $z = 1$ $\quad B.P._i = 14.91$ ft., $\quad \epsilon_i = .757$

for $z = 2$ $\quad B.P._i = 13.5$ ft., $\quad \epsilon_i = .691$

DISCUSSION OF INITIAL CONFIGURATION DESIGN PLOT

The plot on the preceding page was used to analytically size the wagon wheel configuration for a 4g lift-off thrust. It should be noted that the configuration with W = 3.95" and z = 1" is a design point and this was the

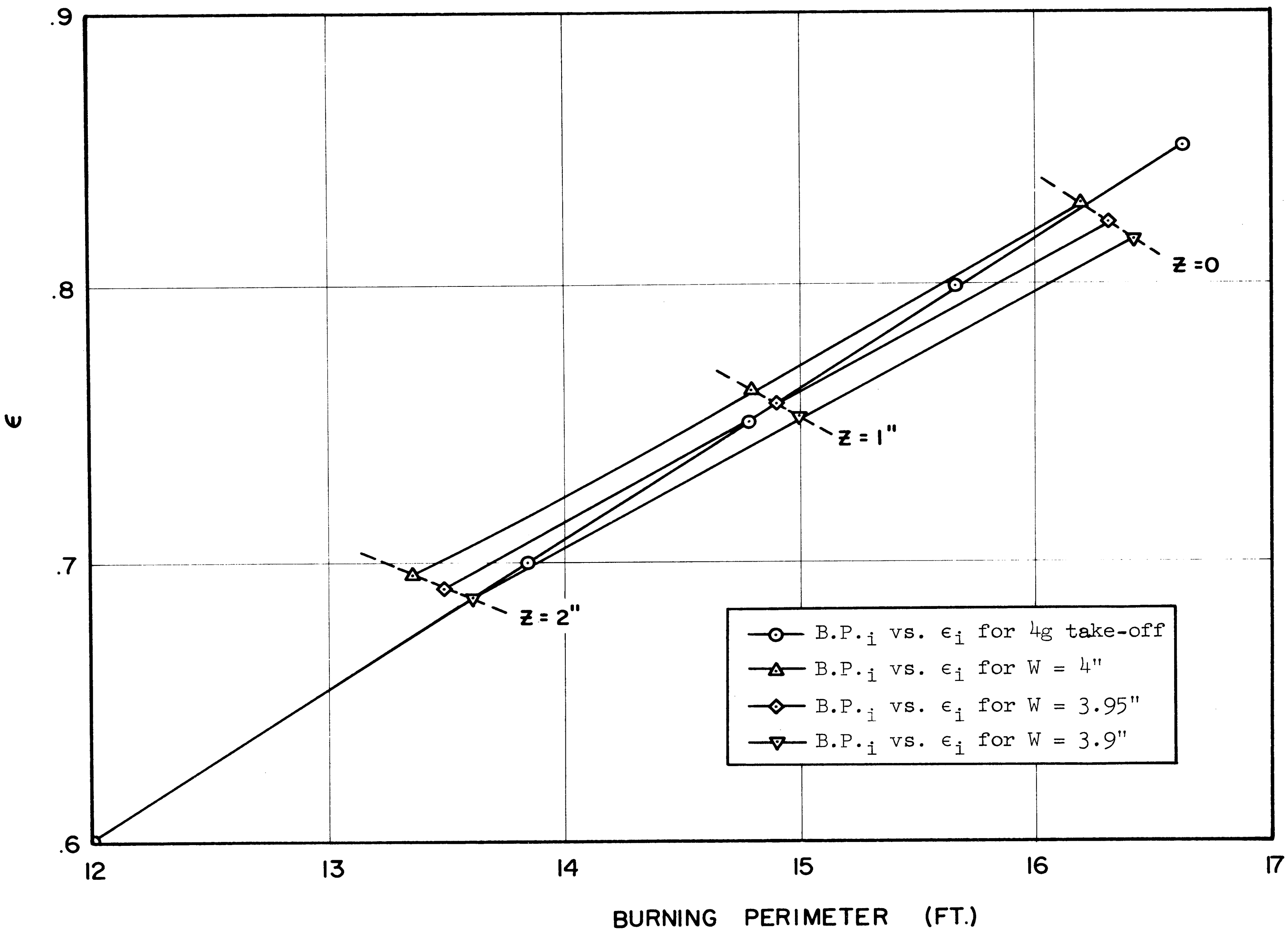

.9
.8
.7
.6
ε
12
13
14
15
16
17
z = 0
z = 1"
z = 2"
B.P.i vs. εi for 4g take-off
B.P.i vs. εi for W = 4"
B.P.i vs. εi for W = 3.95"
B.P.i vs. εi for W = 3.9"
BURNING PERIMETER (FT.)

Figure 4.

design point chosen for the configuration as it was felt that z = 1" was a reasonable value to keep the internal velocity at a reasonable magnitude and still maintain a fairly high loading factor.

This plot could be extended to cover various g take-off conditions and more values of W so as to make a complete design chart for a given size propellant chamber and the five spoke wagon wheel configuration.

It should also be noted from the plot that the value of W = 3.95" is a very good choice for a 4g lift-off as it comes very close to giving this for a range of values of ϵ and z.

DETERMINATION OF BURNING PERIMETER AS FUNCTION OF TIME

Using the five spoke wagon wheel with a web of 3.95" and z = 1" the burning perimeter as a function of x is:

$$B.P._x = \frac{10}{12}\{(14.05 + x)(.3435) + x(\frac{\pi}{2} - .3435 + .628) + (6.36 - x[.3249]) + (6.71) - 1.7x\}$$

$$= \frac{10}{12}\{4.825 + .3435x + 1.8553x + 6.36 - .3249x + 6.71 - 1.7x\}$$

$$= \frac{10}{12}\{17.9 + .1739x\} \text{ ft.}$$

Now, for ϵ = .757 the total propellant weight is 20680 lb. and the mass flow through the nozzle is given as:

$$\frac{\gamma M_t A_t P_t}{a_t} = \frac{\gamma A_t P_t}{a_t} = \frac{\gamma(\frac{2}{\gamma+1})^{\gamma/\gamma-1} A_t P_t}{(\frac{2}{\gamma+1})^{1/2} a_c} = \frac{\gamma(\frac{2}{\gamma+1})^{\gamma+1/2(\gamma-1)} A_t P_t}{a_c}$$

the rate of generation of mass is:

$$A_c r \rho_P = B.P. \ (40) \ aP_c^n \rho_P$$

and these must be equal so

$$(B.P.)(40)(.00629)P_c^{.4} (96.8)(1/12) = \frac{.7408 \ A_t \ P_c \ (144)}{\sqrt{(1.26)(32.2)(\frac{1544}{22})(3160)}}$$

at $t = 0$, B.P. = 14.91 ft., P_c = 1000 PSI, so,

$$(14.91)(40)(.00629)(96.8)(1/12) = \frac{(.7408)(144)A_t(1000)^{.6}(32.2)}{3000}$$

$$A_t = .4195 \text{ ft}^2$$

and then from the expressions on Page :

$$F = (1532)(.4195)(144) = 93{,}200 \text{ lb.}$$

so the motor weight will be 1332 lb and the total weight will be 23,295 lb. so the lift-off will be at 4g's as expected.

Meanwhile, back at the burning perimeter and hence chamber pressure as a function of time -- the governing equation is:

$$(B.P.)(40)aP_c^k \rho_P = \frac{\Gamma A_t P_c}{a_c}$$

or, rearranging:

$$P_c^{1-n} = \frac{a_c \cdot 40 \cdot a \rho_P (B.P.)}{\Gamma A_t}$$

Now, as the geometry is fixed and the combustion is assumed to take place at constant temperature the only variables in the above equation are P_c and (B.P.). Taking the logarithms of both sides:

$$1\text{-}n \text{ Log } P_c = \text{Log } \left\{\frac{a_c \cdot 40 \cdot a \cdot \rho_P}{\Gamma A_t}\right\} + \text{Log (B.P.)}$$

where the first term on the right hand side is a constant. This constant may be evaluated by substituting in the values for (B.P.) and P_c at time $t = 0$. Hence

$$(1-n) \text{ Log } P_c = C + \text{Log (B.P.)}$$

$$.6 \text{ Log } (1000) = C + \text{Log } (179)$$

Therefore $C = 1.8 - 2.25285 + 10 - 10$

$= - .45285$

so, $.6 \text{ Log } P_c = -.45285 + \text{Log (B.P.)}$

$\text{Log } P_c = -.75475 + 1.66667 \text{ Log (B.P.)}$

Now the procedure for determining the chamber pressure as a function of time will be as follows. At time $t = 0$ P_c is known and a burning rate can be determined from this pressure. This burning rate will be assumed constant for a five second time interval -- thereby giving the distance, x, that the propellant has burned over the interval. Then a burning perimeter corresponding to that x can be determined and from the above equation a new value of pressure.

$t = 0 \rightarrow 5$ seconds: $x = 0 \rightarrow .5''$

$\text{B.P.} = 10[17.9 + .0869] = 179.87 \text{ in.}$

and

$\text{Log } P_c = -.75475 + (1.66667) \text{ Log } (179.87)$

$= -.75475 + (1.66667)(2.25496)$

$= -.75475 + 3.75826 = 3.00351$

Therefore: $P_{c_5} = 1008$ PSI

and small pressure change will be assumed to have a negligible effect on r

$t = 5 \rightarrow 10$ seconds: $x = .5'' \rightarrow 1''$

$\text{B.P.} = 10[17.9 + .1739] = 180.739 \text{ in.}$

and

$$\text{Log } P_c = -.75475 + (1.66667) \text{ Log } (180.739)$$

$$= -.75475 + (1.66667)(2.25706)$$

$$= -.75475 + 3.76176 = 3.00701$$

Therefore: $P_{c_{10}} = 1016.3$ PSI

and this pressure will again cause negligible change in burning rate.

$t = 10 \rightarrow 15$ seconds: $x = 1'' \rightarrow 1.5''$

$$\text{B.P.} = 10[17.9 + .2608] = 181.608 \text{ in.}$$

and

$$\text{Log } P_c = -.75475 + (1.66667) \text{ Log } (181.608)$$

$$= -.75475 + (1.66667)(2.25914)$$

$$= -.75475 + 3.76523 = 3.01048$$

Therefore: $P_{c_{15}} = 1024.4$ PSI

and this pressure corresponds to an r of .1007 in./sec. so $r = .101$ in./sec. will be used.

$t = 15 \rightarrow 20$ seconds: $x = 1.5'' \rightarrow 2.005''$

$$\text{B.P.} = 10[17.9 + .3487] = 182.487 \text{ in.}$$

and

$$\text{Log } P_c = -.75475 + (1.66667) \text{ Log } (182.487)$$

$$= -.75475 + (1.66667)(2.26123)$$

$$= -.75475 + 3.76872 = 3.01397$$

Therefore: $P_{c_{20}} = 1032.7$ PSI $\rightarrow$ $r = .101$ in./sec.

$t = 20 \rightarrow 25$ seconds: $x = 2.005'' \rightarrow 2.510''$

$$B.P. = 10[17.9 + .4365] = 183.365 \text{ in.}$$

and

$$\text{Log } P_c = -.75475 + (1.66667) \text{ Log } (183.365)$$

$$= -.75475 + (1.66667)(2.26332)$$

$$= -.75475 + 3.77220 = 3.01745$$

Therefore: $P_{c_{25}} = 1041.0$ PSI $\rightarrow$ $r = .1013$ in./sec. = .101 in./sec.

$t = 25 \rightarrow 30$ seconds: $x = 2.510'' \rightarrow 3.015''$

$$B.P. = 10[17.9 + .5243] = 184.243 \text{ in.}$$

and

$$\text{Log } P_c = -.75475 + (1.66667) \text{ Log } (184.243)$$

$$= -.75475 + (1.66667)(2.26539)$$

$$= -.75475 + 3.77565 = 3.02090$$

Therefore: $P_{c_{30}} = 1049.3$ PSI $\rightarrow$ $r = .102$ in./sec.

$t = 30 \rightarrow 35$ seconds: $x = 3.015'' \rightarrow 3.525''$

$$B.P. = 10[17.9 + .6130] = 185.866 \text{ in.}$$

and

$$\text{Log } P_c = -.75475 + (1.66667) \text{ Log } (185.130)$$

$$= -.75475 + (1.66667)(2.26748)$$

$$= -.75475 + 3.77913 = 3.02438$$

Therefore: $P_{c_{35}} = 1057.8$ PSI $\rightarrow$ $r = .1017$ $\rightarrow$ $r = .102$ in./sec.

$t = 35 \rightarrow 39.150$ seconds: $x = 3.525'' \rightarrow 3.948''$

$$B.P. = 10[17.9 + .6866] = 185.866 \text{ in.}$$

and

$$\text{Log } P_c = -.75475 + (1.66667) \text{ Log } (185.866)$$

$$= -.75475 + (1.66667)(2.26921)$$

$$= -.75475 + 3.78202 = 3.02727$$

Therefore: $P_{c_{39.15}} = 1064.8$ PSI $\rightarrow$ $r = .102$ in./sec.

$t = 39.150 \rightarrow 39.167$: $x = 3.948'' \rightarrow 3.95''$

$$B.P. = 10[1.8553x] = (18.553)(3.95) = 73.284 \text{ in.}$$

and

$$\text{Log } P_c = -.75475 + (1.66667) \text{ Log } (73.284)$$

$$= -.75475 + (1.66667)(1.86501)$$

$$= -.75475 + 3.10835 = 2.35360$$

Therefore: $P_{c_{39.167}} = 225.74$ PSI

and the r may be computed as:

$$\text{Log } (P_c)^{.4} = .94144 \quad \rightarrow \quad (P_c)^{.4} = 8.7386$$

$$r = (.00629)(8.7386) = .054 \text{ in./sec.}$$

$t = 39.167 \rightarrow 42.5$: $x = 3.95'' \rightarrow 4.13''$

and from the plot on the following page:

$$B.P. = 10[(4.13)(1.25)] = 51.625''$$

and

$$\text{Log } P_c = -.75475 + (1.66667) \text{ Log } (51.625)$$

$$= -.75475 + (1.66667)(1.71286)$$

$$= -.75475 + 2.85477 = 2.10002$$

Therefore: $P_{c_{42.5}} = 125.9$ PSI

and the r may be computed as:

$$\text{Log } P_c^{.4} = .84001 \quad \Rightarrow \quad P_c^{.4} = 6.9185$$

$$r = (.00629)(6.9185) = .044 \text{ in. /sec.}$$

$t = 42.5 \rightarrow 44.5$: $x = 4.13" \rightarrow 4.22"$

$$\text{B.P.} = 10\ [(4.22)(1.098)] = 46.3"$$

and

$$\text{Log } P_c = -.75475 + (1.66667) \text{ Log } (46.3)$$

$$= -.75475 + (1.66667)(1.66558)$$

$$= -.75475 + 2.77596 = 2.02121$$

Therefore: $P_{c_{44.5}} = 105.1$ PSI

and the r may be computed as:

$$\text{Log } P_c^{.4} = .80848 \qquad P_c^{.4} = 6.434$$

$$r = (.00629)(6.434) = .040$$

$t = 44.5 \rightarrow 45$: $x = 4.22" \rightarrow 4.24"$

$$\text{B.P.} = 10\ [(4.24)(1.053)] = 44.65$$

and

$$\text{Log } P_c = -.75475 + (1.66667) \text{ Log } (44.65)$$

$$= -.75475 + (1.66667)(1.64982) = 1.99495$$

Therefore: $P_c = 98.85$ PSI

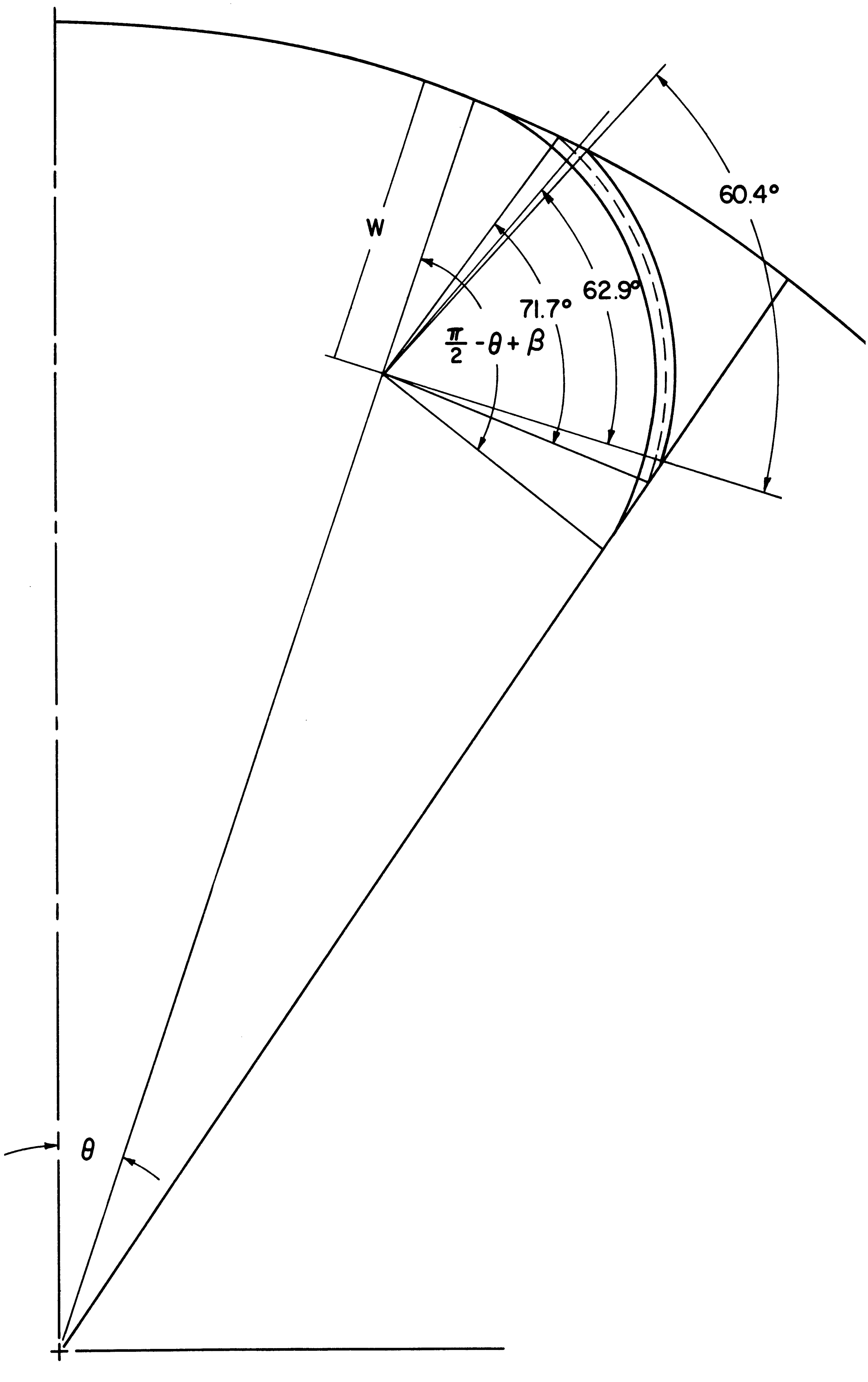


Figure 5. Burning Grain Geometry.

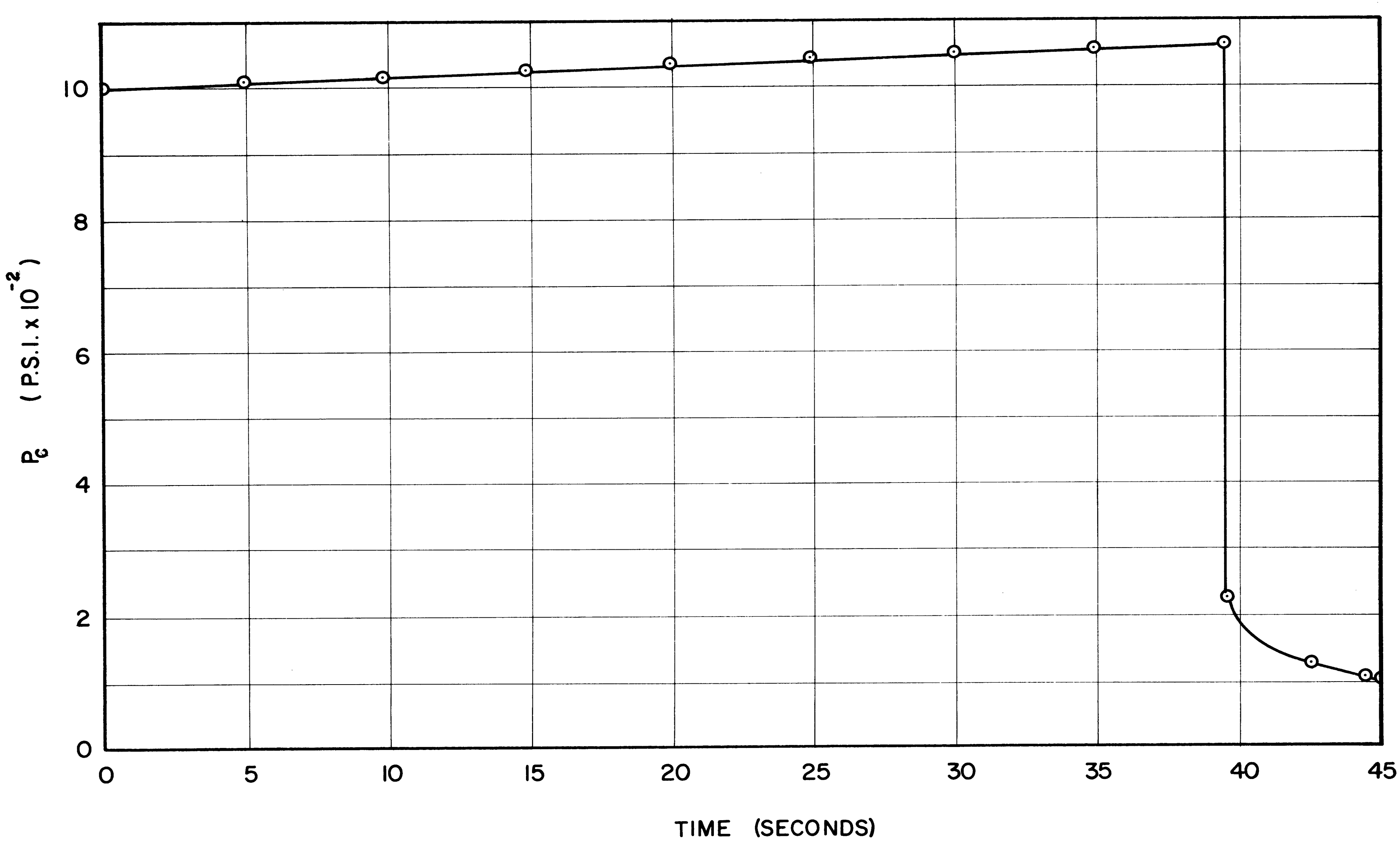


Figure 6. Chamber Pressure with Time.

DETERMINATION OF PROPELLANT CROSS-SECTIONAL AREA AND WEIGHT AS A FUNCTION OF TIME

The cross-sectional area is given by the expression on Page

as:

$$A_x = A_i - 5\left\{\left(\frac{D}{2} - W + x\right)^2 - \left(\frac{D}{2} - W\right)^2\right\}\theta - 5x^2\left\{\frac{\pi}{2} - \theta + \beta\right\} - 10\frac{Wx}{\sin\beta}$$

$$- \frac{5x^2}{\tan\beta} - \left\{\frac{\left(\frac{D}{2} - W - \frac{W}{\sin\beta}\cdot\cos\theta\right)}{\cos(\beta-\theta)} - \frac{Z}{\sin\beta} - \frac{x}{\tan\beta} - x\tan\frac{\beta}{2}\right\}x$$

and for the particular configuration chosen this reduces to:

$$A_x = 770.63 - 5\{(14.05 + x)^2 - (14.05)^2\}(.3435) - 5\{1.8553\}x^2 - 10(6.71)x$$

$$- 10(.6882)x^2 - 10\{8.06 - 1.7 - 1.3764x - .3249x\}x$$

$$= 770.63 - 1.7175\{28.1x + x^2\} - 9.2765x^2 - 67.1x - 6.882x^2$$

$$- (80.6 - 17)x + 17.013x^2$$

$$= 770.63 - 178.962x - .862x^2$$

and evaluating at the following times

at $t = 0$ $\quad A_x = 770.63 \text{ in}^2$ $\quad \epsilon = .757$

at $t = 5$ $\quad A_x = 770.63 - 89.481 - .216 = 680.93 \text{ in}^2$ $\quad \epsilon = .669$

at $t = 10$ $\quad A_x = 770.63 - 178.962 - .862 = 590.81 \text{ in}^2$ $\quad \epsilon = .580$

at $t = 15$ $\quad A_x = 770.63 - 268.443 - 1.94 = 500.25 \text{ in}^2$ $\quad \epsilon = .492$

at $t = 20$ $\quad A_x = 770.63 - 358.7 - 3.46 = 408.47 \text{ in}^2$ $\quad \epsilon = .401$

at $t = 25$ $\quad A_x = 770.63 - 449.5 - 5.43 = 315.70 \text{ in}^2$ $\quad \epsilon = .310$

at $t = 30$ $\quad A_x = 770.63 - 539.5 - 7.83 = 223.3 \text{ in}^2$ $\quad \epsilon = .219$

at $t = 35$ $\quad A_x = 770.63 - 630.5 - 10.70 = 129.43 \text{ in}^2$ $\quad \epsilon = .127$

$$\begin{aligned}
&\text{at } t = 39.15 \quad \left\{ \begin{array}{l} A_x = 770.63 - 707.0 - 13.43 = 50.20 \text{ in}^2 \qquad \epsilon = .0493 \\ A_x = 770.63 - 707.0 - 13.43 = 50.20 \text{ in}^2 \qquad \epsilon = .0493 \end{array} \right. \\
&\text{at } t = 39.167 \\
&\text{at } t = 44.8 \quad A_x = 50.20 \left[\left\{ \frac{(4.23)^2}{2} + \frac{(3.95)^2}{2} \right\} \xi - \{4.23 - 3.95\}\{.8\} \right] 10
\end{aligned}$$

where $\xi = 60.4" = 1.053$ radians

$$A_x = 50.20 - [(.5265)(2.29) + .224]\ 10 = 35.91 \text{ in}^2; \ \epsilon = .0353$$

and similarly the weight as a function of time is given as:

$$W = 23.295 + (\epsilon - .757)(27350)$$

so, at $t = 0$ seconds	W = 23,295 lb.
$t = 5$ seconds	W = 20,888 lb.
$t = 10$ seconds	W = 18,455 lb.
$t = 15$ seconds	W = 16,045 lb.
$t = 20$ seconds	W = 13,555 lb.
$t = 25$ seconds	W = 11,075 lb.
$t = 30$ seconds	W = 8,585 lb.
$t = 35$ seconds	W = 6,055 lb.
$t = 39.15$ seconds	W = 3,935 lb.
$t_{b.o.} = 44.8$ seconds	W - 3,545 lb.

THRUST FOR VARYING P_c AND P_A

We know from Equation (12) on Page 58 that:

$$\frac{F}{A_t} = \gamma\, P_t M_{EX} (T_{EX}/T_t)^{1/2} + (P_{EX} - P_A) \frac{A_{EX}}{A_t}$$

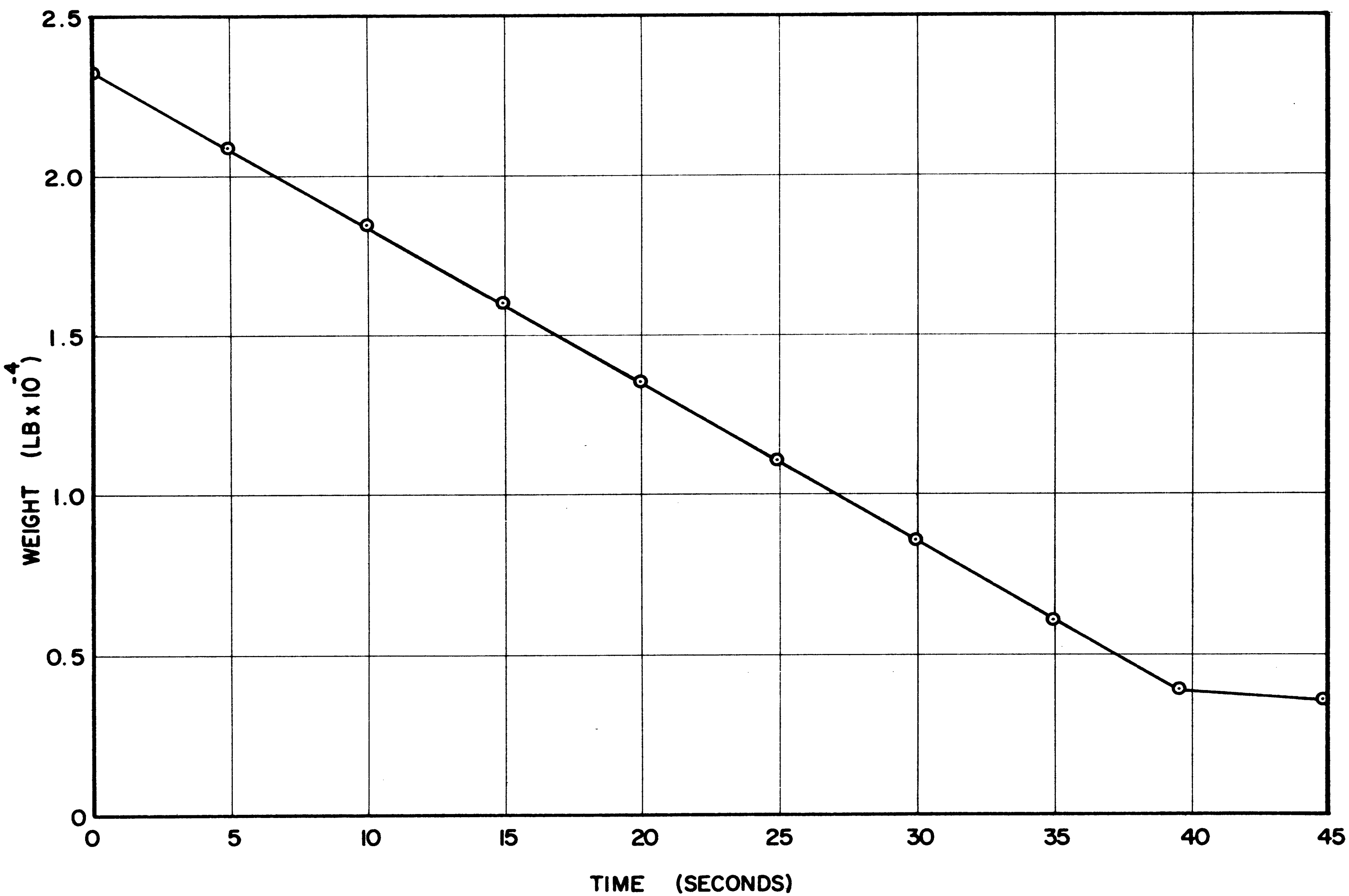


Figure 7. Weight Versus Time.

now, multiply by A_t and divide by P_c

$$\frac{F}{P_c} = \gamma \frac{P_t}{P_c} M_{EX} (T_{EX}/T_t)^{1/2} A_t + (\frac{P_{EX}}{P_c} - \frac{P_A}{P_c}) A_{EX}$$

and now the first two terms on the right hand side are known constants from the propellant specifications and nozzle geometry and may be evaluated:

$$\frac{F}{P_c} = (1.26)(.5532)(3.678)(.64)(60.4) + (.00735)(13.05)(60.4) - \frac{P_A}{P_o}(13.05)(60.4)$$

so,

$$F = (99.0 + 5.8)\, P_c - 788.5\, P_A$$

$$= 104.8\, P_c - 788.5\, P_A$$

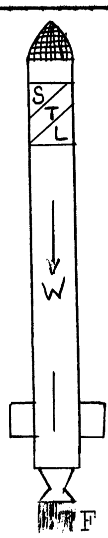
for a check, consider lift-off:

$$F = (104.8)(1000) - (788.5)(14.7)$$

$$= 104{,}800 - 11{,}600 = 93{,}200 \text{ lb.}$$

and this checks with the previous calculation of this value found in the section dealing with the variation of burning perimeter as a function of time.

PERFORMANCE EQUATIONS



Consider a rocket vehicle in a vertical trajectory. Assuming drag negligible as specified for performance calculations, and computing the velocity and altitude relations in time increments, it was shown in the previous design study that:

$$V_f = V_o + \frac{gT\Delta t}{W_o - W_f} \ln \frac{W_o}{W_f} - g\Delta t$$

and

$$h_f = h_o + V_o\Delta t + \frac{gT\Delta t}{W_o - W_f} \left\{ \ln \left(\frac{W_f}{W_o}\right)^{\frac{W_f\ \Delta t}{W_o - W_f}} + \Delta t \right\} - \frac{g}{2}\Delta t^2$$

Where: f represents values at the end of the interval

o represents values at the beginning of the interval

Δt represents the time interval

It should be noted that the average mass flow over the interval is being used. Although this is more accurate than taking the mass flow at the beginning of the interval the change is very slight due to the almost constant P_c and the average is being used primarily for ease of computation. The thrust is evaluated at the beginning of the interval. Because of the nearly constant P_c 5 second intervals will be used as it is doubtful that any significant amount of accuracy would be added by considering smaller increments:

From $t = 0 \rightarrow 5$ seconds

$$V_f = V_o + \frac{gF\Delta t}{W_o - W_f} \left\{ \ln W_o - \ln W_f \right\} - g\Delta t$$

$$\underline{V_5} = 0 + 6230 \left\{ \ln 23{,}295 - \ln 20{,}888 \right\} - 161$$

$$= 6230 \left\{ 10.05599 - 9.94693 \right\} - 161 = 6230 \left\{ .10906 \right\} - 161$$

$$= 680 - 161 = \underline{519 \text{ ft./sec.}}$$

and

$$h_f = h_o + V_o\Delta t + \frac{gF\Delta t}{W_o - W_f}\left\{\frac{W_f\Delta t}{W_o - W_f}\{\ln(W_f) - \ln(W_o)\} + \Delta t\right\} - \frac{g}{2}\Delta t^2$$

$$\underline{h_s} = 0 + 0 + 6230\{43.3\{-.10906\} + 5\} - 402.5$$

$$= 6230\{-4.73 + 5\} - 402.5 = 1682 - 402 = \underline{1280 \text{ ft.}}$$

and at 1280 ft. $P_A = 14.03$ PSI, $P_c = 1008$ PSI

$$\underline{F} = 104.8\,(1008) - 788.5\,(14.03)$$

$$= 105{,}700 - 11070 = \underline{94{,}630} \text{ lb.}$$

From $t = 5 \rightarrow 10$ seconds

$$V_{10} = 519 + \frac{161 \cdot 94{,}630}{(20{,}888 - 18{,}455)}\{\ln 20{,}888 - \ln 18{,}455\} - 161$$

$$= 519 + 6260\{9.94693 - 9.82309\} - 161$$

$$= 519 + 775 - 161 = 1133 \ \underline{\text{ft./sec.}}$$

$$h_{10} = 1280 + (519)(5) + 6260\{37.95\{-.12384\} + 5\} -402.5$$

$$= 1280 + 2595 + 6260\{-4.695 + 5\} -402.5 = 1280 + 2595 + 1910 -$$

$$= \underline{5383 \text{ ft.}}$$

and at 5383 ft. $P_A = 11.99$ PSI, $P_c = 1016.3$

$$\underline{F} = 104.8\,(1016.3) - 788.5\,(11.99)$$

$$= 106{,}300 - 94.45 = \underline{96{,}855 \text{ lb.}}$$

From $t = 10 \rightarrow 15$ seconds

$$V_{15} = 1133 + \frac{161 \cdot 96{,}855}{(18{,}455 - 16{,}045)}\{\ln 18{,}455 - \ln 16{,}045\} - 161$$

$$= 1133 + 6470\{9.82309 - 9.68315\} - 161$$

$$= 1133 + 905 - 161 = \underline{1877 \text{ ft./sec.}}$$

$\underline{h_{15}} = 5383 + 1133(5) + 6470 \{33.35 \{-.13994\} + 5\} -402.5$

$= 5383 + 5665 + 6470 \{-4.67+5\}-402.5 = 5383 + 5665 + 2135 - 402$

$= \underline{12781 \text{ ft.}}$

and at 12,781 ft. $P_A = 9.060$ PSI, $P_c = 1024.4$ PSI

$\underline{F} = 104.8 \, (1024.4) - 788.5 \, (9.060)$

$= 107350 - 7145 = \underline{100,205 \text{ lb.}}$

<u>From t = 15 → 20 seconds</u>

$$\underline{V_{20}} = 1877 + \frac{161 \cdot 100,205}{(16,045 - 13,555)} \{\ln 16045 - \ln 13555\} - 161$$

$= 1877 + 6480 \{9.68315 - 9.51451\} - 161$

$= 1877 + 1093 - 161 = \underline{2809 \text{ ft./sec.}}$

$\underline{h_{20}} = 12781 + 1877(5) + 6480 \{27.2 \{-.16864\} + 5\} - 402.5$

$= 12781 + 9385 + \{-4.58 + 5\} \, 6480 - 402.0 \, F + 2781 + 9385$

$+ 2720 - 402$

$= \underline{24,484 \text{ ft.}}$

and at 24,484 ft. $P_A = 5.574$ PSI, $P_c = 1032.7$ PSI.

$F = 104.8 \, (1032.7) - 788.5 \, (5.574)$

$= 108,250 - 4395 = \underline{103,855 \text{ lb.}}$

<u>From t = 20 → 25 seconds</u>

$$\underline{V_{25}} = 2809 + \frac{161 \cdot 103,855}{(13555 - 11075)} \{\ln 13555 - \ln 11075\} - 161$$

$= 2809 + 6740 \{9.51451 - 9.31245\} - 161$

$= 2809 + 1362 - 161 = \underline{4010 \text{ ft./sec.}}$

$$\underline{h_{25}} = 24,484 + (2809)5 + 6740 \{22.34 \{-.20206\} + 5\} - 402$$

$$= 24,484 + 14045 + 6740 \{-4.515 + 5\} - 402$$

$$= 24,484 + 14045 + 3267 - 402$$

$$= \underline{41,394 \text{ ft.}}$$

and at 41,394 ft. $P_A = 2.545$ PSI. $P_c = 1041$ PSI

$$\underline{F} = 104.8 (1041) - 788.5 (2.545)$$

$$= 109,050 - 2006 = \underline{107,044 \text{ lb.}}$$

<u>From t = 25 → 30 seconds</u>

$$\underline{V_{30}} = 4010 + \frac{161 \cdot 107,044}{(11075 - 8585)} \{\ln 11075 - \ln 8585\} - 161$$

$$= 4010 + 6950 \{9.31245 - 9.05777\} - 161$$

$$= 4010 + 1770 - 161 = \underline{5619 \text{ ft./sec.}}$$

$$\underline{h_{30}} = 41,394 + (4010)5 + 6950 \{17.22 \{-.25468\} + 5\} - 402$$

$$= 41,394 + 20050 + \{-4.390 + 5\}^{(6950)} - 402 = 41,394 + 20,050 + 4240 - 402$$

$$= \underline{65,282 \text{ ft.}}$$

and at 65,282 ft. $P_A = .808$ PSI, $P_c = 1049.3$ PSI.

$$\underline{F} = 104.8 (1049.3) - 788.5 (.808)$$

$$= 109,900 - 637 = \underline{109,263 \text{ lb.}}$$

<u>From t = 30 → 35 seconds</u>

$$\underline{V_{35}} = 5619 + \frac{161 \cdot 109,263}{(8585 - 6055)} \{\ln 8585 - \ln 6055\} - 161$$

$$= 5619 + 6960 \{9.05777 - 8.70864\} - 161$$

$$= 5619 + 6960 \{.34913\} - 161 = \underline{7888 \text{ ft./sec.}}$$

$\underline{h_{35}} = 65{,}282 + (5619)5 + 6960 \{11.97\ (-.34913) + 5\} - 402$

$= 65{,}282 + 28095 + 6960\ (.825) - 402 = \underline{98{,}715 \text{ ft.}}$

and at 98,715 ft. $P_A = .1641$ PSI $P_c = 1057.8$ PSI.

$\underline{F} = 104.8\ (1057.8) - 788.5\ (.1641)$

$= 110{,}900 - 129 = \underline{110{,}771 \text{ lb.}}$

From t = 35 → 39.167 seconds

$$\underline{V_{39.167}} = 7888 + \frac{(4.167)(32.2)(110{,}771)}{(6055 - 3905)} \{\ln 6055 - \ln 3905\} - 135$$

$= 7888 + 6910 \{8.70864 - 8.27001\} - 135$

$= 7888 + 3030 - 135 = \underline{10783 \text{ ft./sec.}}$

$\underline{h_{39.167}} = 98{,}715 + 7888\ (4.167) + 6910 \{7.57\ (-.43863) + 5\} - 280$

$= 98{,}715 + 32820 + 11620 - 280 = \underline{142{,}875 \text{ ft.}}$

and at 142,875 ft. $P_A = .026$ PSI, $P_c = 225.74$ PSI

$\underline{F} = 104.8\ (225.74) - 788.5\ (.026)$

$= 23{,}650 - 20 = \underline{23{,}630 \text{ lb.}}$

From t = 39,167 seconds → 42.5 seconds

$$\underline{V_{42.5}} = 10783 + \frac{(3.333)(32.2)(23{,}630)}{(3905 - 3620)} \{\ln 3905 - \ln 3620\} - 107$$

$= 10783 + 8900 \{8.27001 - 8.19423\} - 107$

$= 10783 + 674 - 107 = \underline{11{,}350 \text{ ft./sec.}}$

$\underline{h_{42.5}} = 142{,}875 + (10783)(3.333) + 8900 \{42.3\ (-.07578) + 5\} - 111$

$= 142{,}875 + 35900 + 15930 - 111 = \underline{194{,}594 \text{ ft.}}$

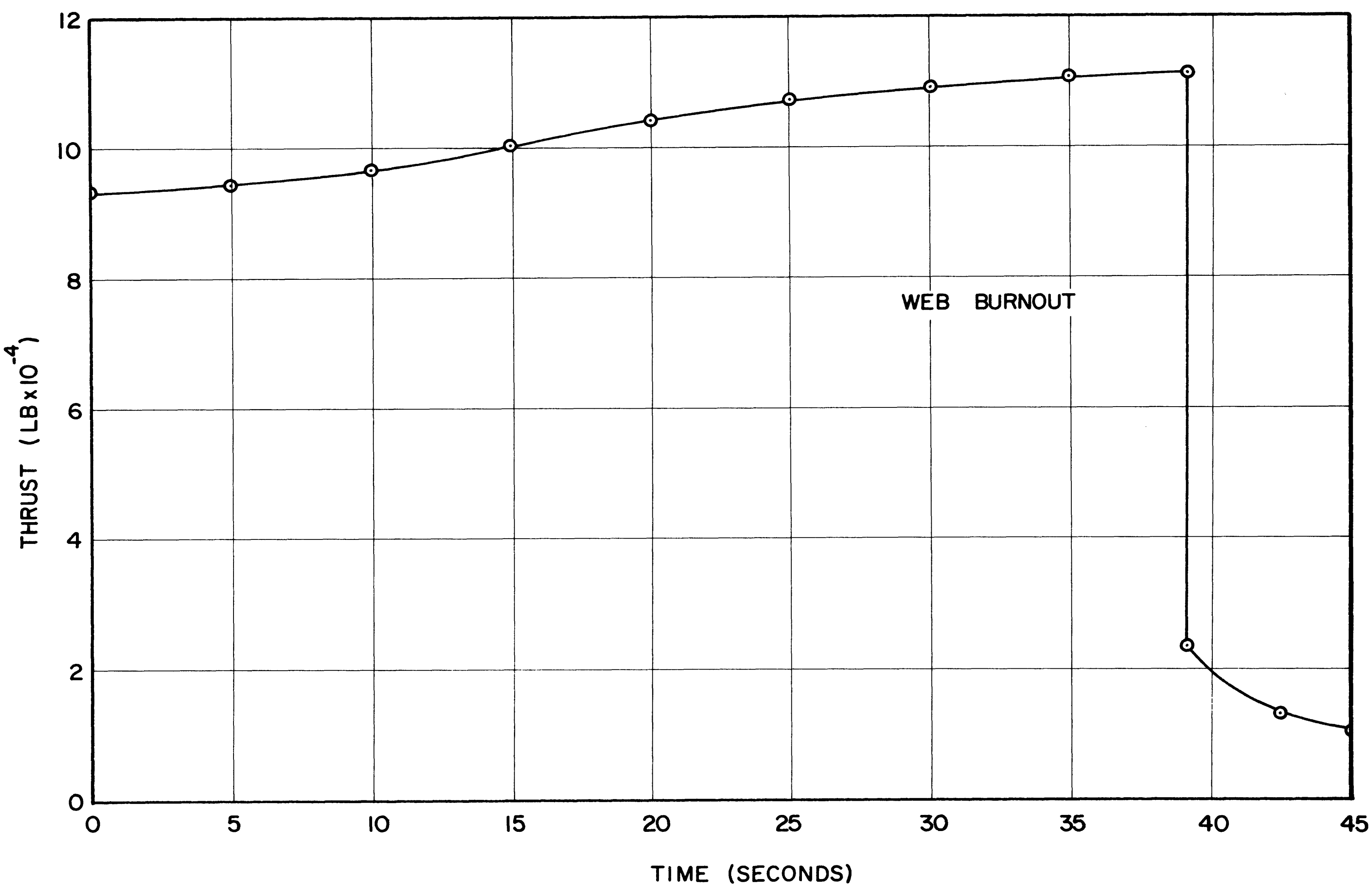


Figure 8. Thrust Versus Time.

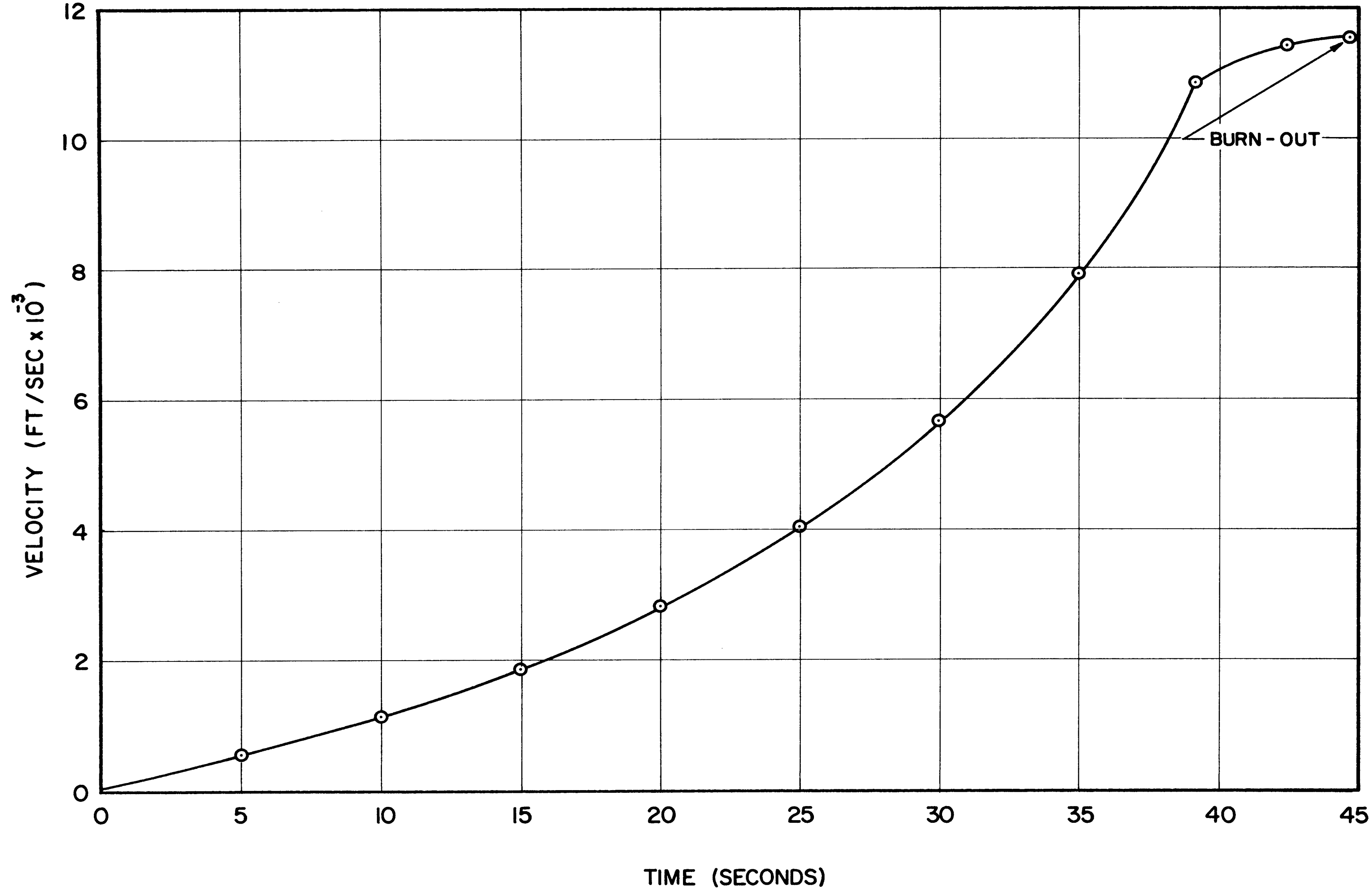


Figure 9. Rocket Velocity as a Function of Time.

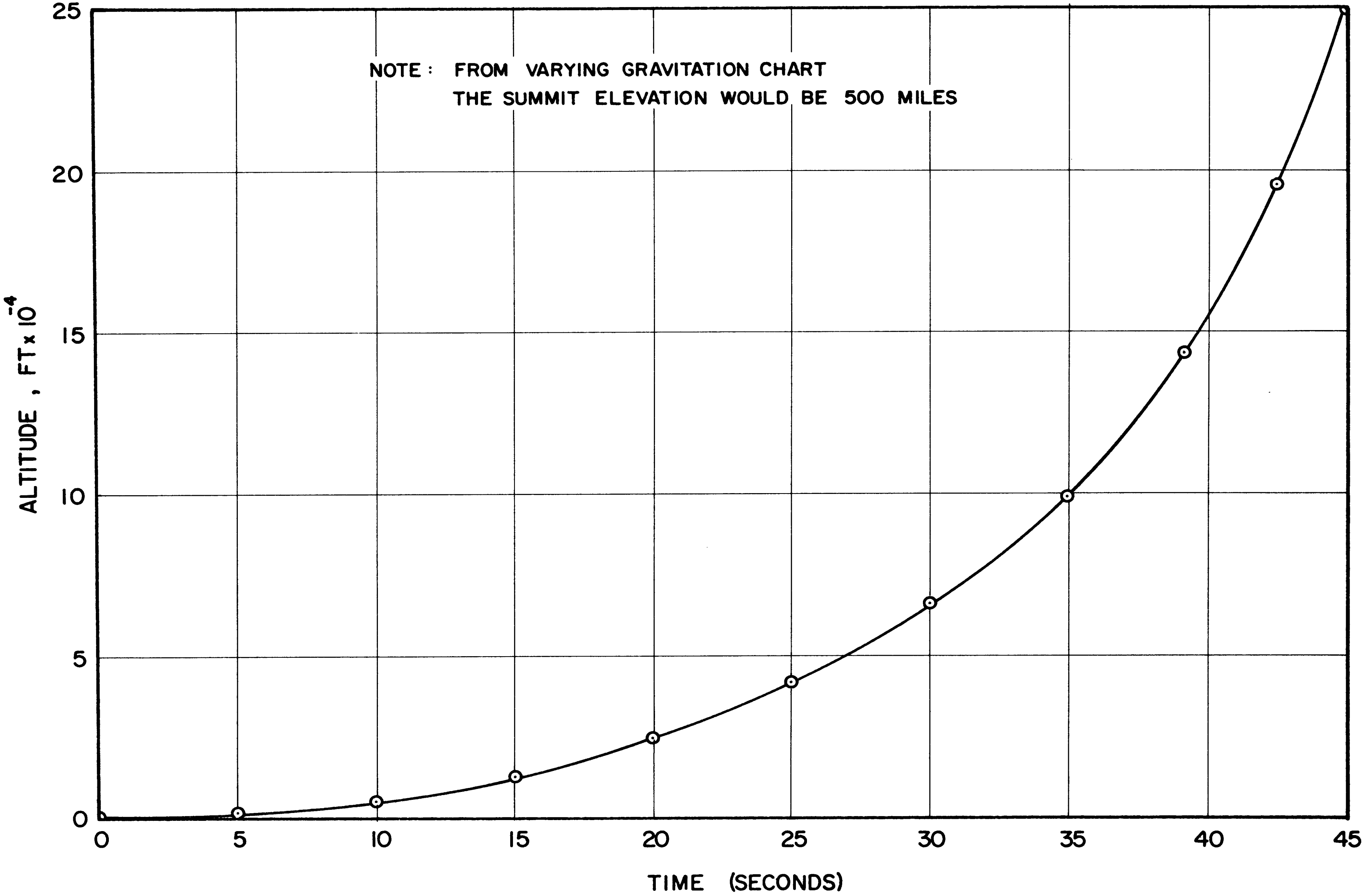


Figure 10. Rocket Altitude Versus Time During Powered Phase.

and at 194,594 P_A is negligible and P_c = 125.9

$$F = 104.8\ (125.9) = \underline{13{,}200\ \text{lb.}}$$

From t = 42.5 → 44.8 seconds (Burn-out)

$$V_{44.8} = 11{,}350 + \frac{(2.3)(32.2)(13{,}200)}{(3620 - 3545)}\ \{\ln 3620 - \ln 3545\} - 74$$

$$= 11{,}350 + 9870\ \{8.19423 - 8.17329\} - 74$$

$$= 11{,}350 + 198 - 74 = \underline{11{,}474\ \text{ft./sec.}}$$

$$h_{44.8} = 194{,}594 + (11{,}350)(2.3) + 9870\ \{108.8\ \{-.02094\} + 5\} - 85$$

$$= 194{,}594 + 26100 + 26800 - 85 = \underline{247{,}409\ \text{ft.}}$$

UTATION OF MACH NUMBER AND DYNAMIC HEAD

At t = 0 seconds

$$\underline{M = 0} \qquad \underline{q = 0}$$

At t = 5 seconds

$$c = 1112\ \text{ft./sec.} \qquad M = \frac{519}{1112} = \underline{.466}$$

$$\rho = (.9632)(.002378) = .00229\ \frac{\text{lb.sec.}^2}{\text{ft.}^4}$$

$$q = 1/2(.00229)(519)^2 = \underline{309\ \text{lb./ft.}^2}$$

At t = 10 seconds

$$c = 1096\ \text{ft./sec.} \qquad M = \frac{1133}{1096} = \underline{1.035}$$

$$\rho = (.8518)(.002378) = .002024\ \frac{\text{lb.sec.}^2}{\text{ft.}^4}$$

$$q = 1/2\ (.002024)(1133)^2 = \underline{1300\ \text{lb./ft.}^2}$$

At t = 15 seconds

$$c = 1067\ \text{ft./sec.} \qquad M = \frac{1877}{1067} = \underline{1.76}$$

$$\rho = (.6761)(.002378) = .001608\ \frac{\text{lb./sec.}^2}{\text{ft.}^4}$$

$$q = 1/2\ (.001608)(1877)^2 = \underline{2833\ \text{lb./ft.}^2}$$

At t = 20 seconds

$c = 1018$ ft./sec. $\qquad M = \frac{2809}{1018} = 2.76$

$\rho = (.4564)(.002378) = .001085 \ \frac{\text{lb.sec.}^2}{\text{ft.}^4}$

$q = 1/2\ (.001085)(2809)^2 = 4277\ \text{lb./ft.}^2$

At t = 25 seconds

$c = 968.5$ ft./sec. $\qquad M = \frac{4010}{968.5} = 4.15$

$\rho = (.2302)(.002378) = .000547 \ \frac{\text{lb.sec.}^2}{\text{ft.}^4}$

$q = 1/2\ (.000547)(4010)^2 = 4395\ \text{lb./ft.}^2$

At t = 30 seconds

$c = 968.5$ ft./sec. $\qquad M = \frac{5619}{968.5} = 5.8$

$\rho = (.07305)(.002378) = .0001738 \ \frac{\text{lb.sec.}^2}{\text{ft.}^4}$

$q = 1/2\ (.0001738)(5619)^2 = 2745\ \text{lb./ft.}^2$

At t = 35 seconds

$c = 1002$ ft./sec. $\qquad M = \frac{7888}{1002} = 7.87$

$\rho = (.01407)(.002378) = (.00003343) \ \frac{\text{lb.sec.}^2}{\text{ft.}^4}$

$q = 1/2\ (.00003343)(7888)^2 = 1040\ \text{lb./ft.}^2$

At t = 39.167 seconds

$c = 1086$ ft./sec. $\qquad M = \frac{10783}{1086} = 9.93$

$\rho = (.001967)(.002378) = .000004675 \ \frac{\text{lb.sec.}^2}{\text{ft.}^4}$

$q = 1/2\ (.000004675)(10783)^2 = 272\ \text{lb./ft.}^2$

At t = 44.8 seconds

Too high an altitude for values to be significant.

NOTE: The values of density and speed of sound used above were interpolated from the Pratt and Whitney handbook.

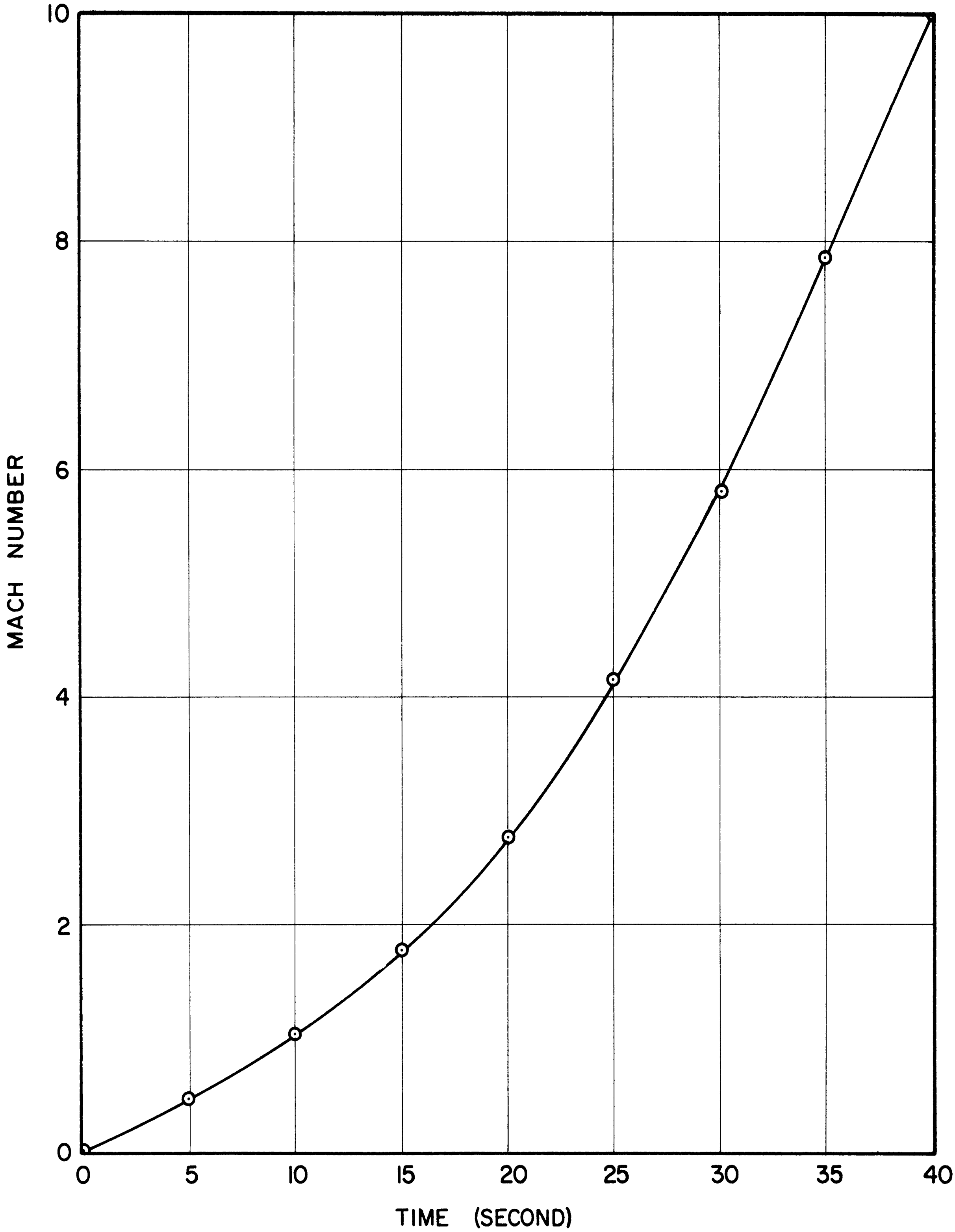


Figure 11. Mach Number Versus Time.

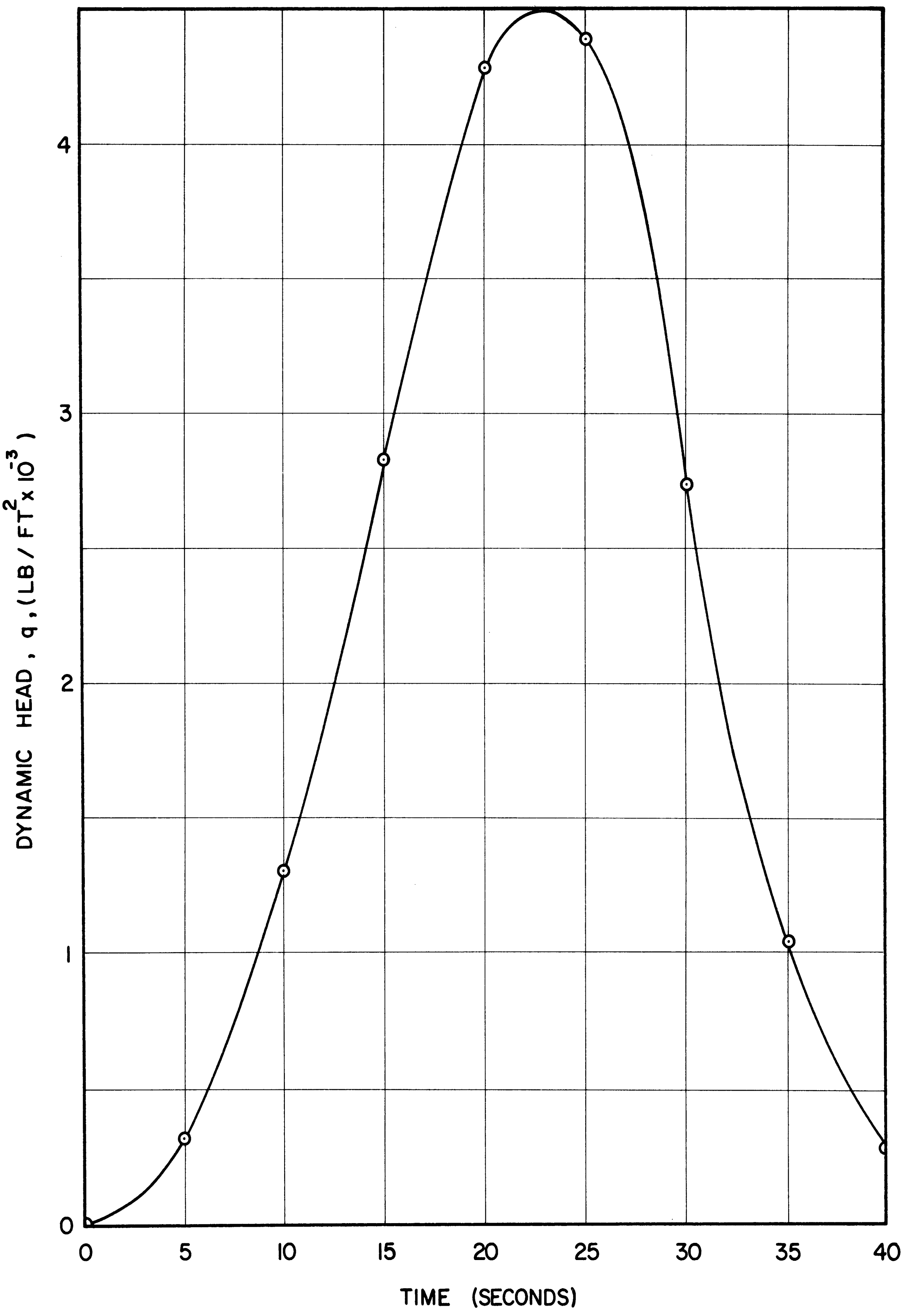


Figure 12. Dynamic Head Versus Time.

DRAG CALCULATION

As was specified, drag effects were ignored in the performance evaluation. Using the results of the performance evaluation the drag will now be calculated and the amount of additional fuel necessary to counteract the negative impulse due to the drag determined. Assuming an $\ell/D = 3$ for the nose cone:

$$L = 40 + 3(3) = 49'$$

Hence a characteristic dimension of 60' will be assumed in computing the Reynolds Number -- this should be conservative. The drag is given by the relationship:

$$D = q\, C_D\, S$$

where S will be conservatively assumed to be 7.1 square feet.

At $t = 0$ seconds

$$\underline{D = 0}$$

At $t = 5$ seconds

$$R_e = \frac{\rho V L}{\mu}$$

so,

$$R_e = \frac{(.00229)(519)(60)}{3.693 \times 10^{-7}} = 193.2 \times 10^6$$

and from C_D plot handed out in class it can be seen that for a nose cone with $\ell/D = 3$:

$$C_D = .175$$

and then the drag may be evaluated:

$$\underline{D} = (.175)(7.10)(309) = \underline{383 \text{ lb}}.$$

At t = 10 seconds

$$R_e = \frac{(.002024)(1183)(60)}{3.610 \times 10^{-7}} = 381 \times 10^6$$

$$C_D = .295$$

$$D = (.295)(7.10)(1300) = \underline{2720 \text{ lb.}}$$

At t = 15 seconds

$$R_e = \frac{(.001608)(1877)(60)}{3.453 \times 10^{-7}} = 525 \times 10^6$$

$$C_D = .255$$

$$D = (.255)(7.10)(2833) = \underline{5130 \text{ lb.}}$$

At t = 20 seconds

$$R_e = \frac{(.001085)(2809)(60)}{3.218 \times 10^{-7}} = 567 \times 10^6$$

$$C_D = .215$$

$$D = (.215)(7.10)(4277) = \underline{6530 \text{ lb.}}$$

At t = 25 seconds

$$R_e = \frac{(.000547)(4010)(60)}{2.961 \times 10^{-7}} = 445 \times 10^6$$

$$C_D = .18$$

$$D = (.18)(7.10)(4395) = \underline{5610 \text{ lb.}}$$

At t = 30 seconds

$$R_e = \frac{(.0001738)(5619)(60)}{2.961 \times 10^{-7}} = 198 \times 10^6$$

$$C_D = .155$$

$$D = (.155)(7.10)(2745) = \underline{3025 \text{ lb.}}$$

At t = 35 seconds

$$R_e = \frac{(.00003343)(7888)(60)}{2.961 \times 10^{-7}} = 53 \times 10^6$$

$$C_D = .16$$

$$D = (.16)(7.10)(1040) = \underline{1180 \text{ lb.}}$$

At t = 39.167 seconds

$$R_e = \frac{(.000004675)(10783)(60)}{4.032 \times 10^{-7}} = 7.48 \times 10^6$$

$$C_D = .19$$

$$\underline{D} = (.19)(7.10)(272) = \underline{366 \text{ lb.}}$$

The total impulse (drag) is equivalent to 1.34 seconds of sea level thrust or an additional (1.34)(481.4) = 645 lb. of propellant. This impulse was obtained by integrating the area under the following curve.

ACCELERATION COMPUTATION

The accelerations may be computed by the second law credited to some obscure physicist. This was used in the derivation of the performance equations and is in general stated as:

$$T - D - W = \frac{W}{g} a$$

But as we are neglecting drag:

$$T - W = \frac{W}{g} a$$

$$a = [\frac{T}{W} - 1] g$$

and computing the accelerations:

At t = 0 seconds

$$\underline{a} = [\frac{93,200}{23,295} - 1] g$$

$$= [4.00 - 1] g = \underline{3.0 g}$$

At t = 5 seconds

$$\underline{a} = [\frac{94,630}{20,888} - 1] g$$

$$= [4.53 - 1] g = \underline{3.53 g}$$

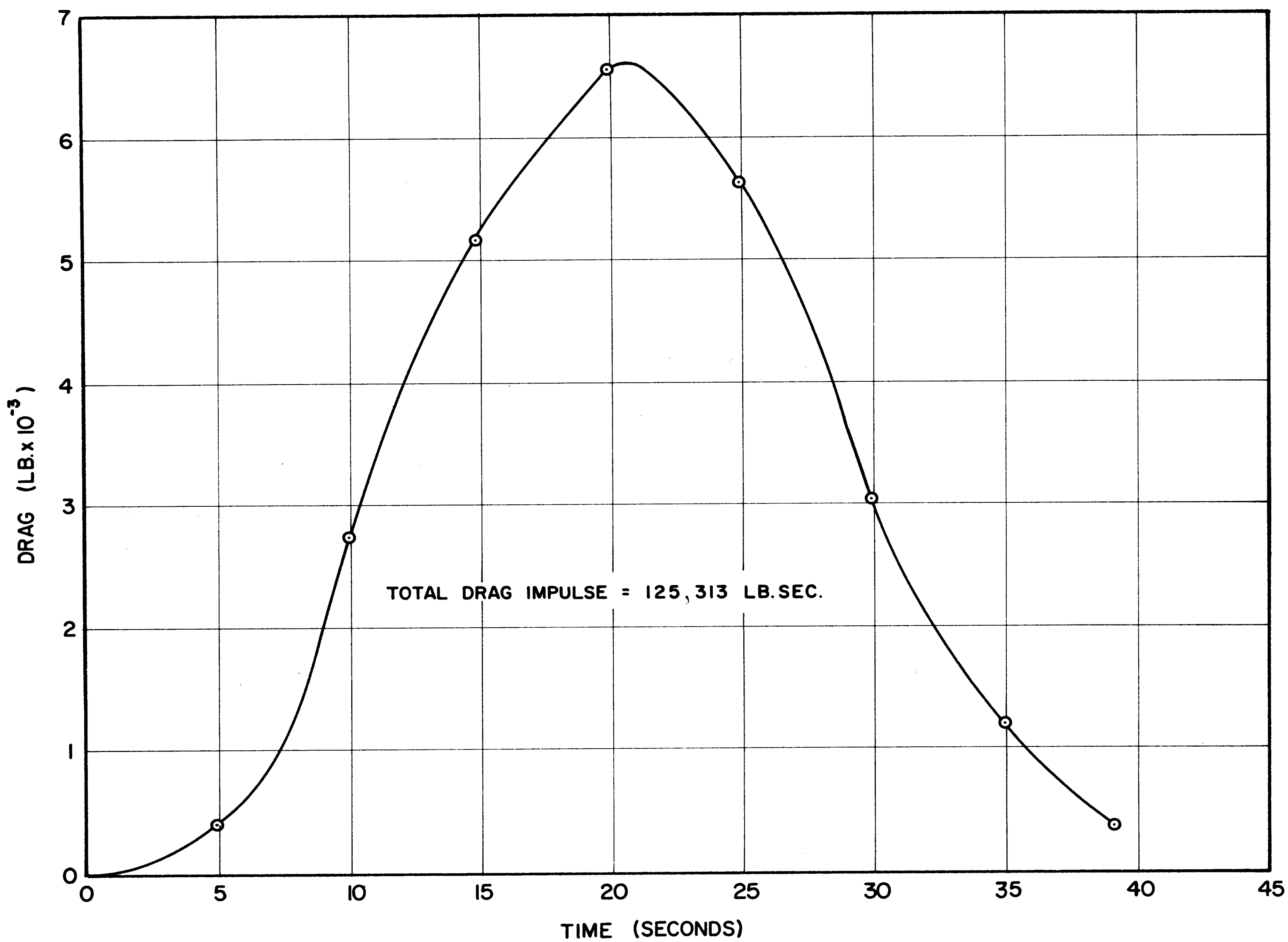


Figure 13. Drag Versus Time.

At t = 10 seconds

$$\underline{a} = \left[\frac{96,855}{18,455} - 1\right] g$$

$$= [5.25 - 1]\ g = \underline{4.25\ g}$$

At t = 15 seconds

$$\underline{a} = \left[\frac{100,205}{16,045} - 1\right] g$$

$$= [6.25 - 1]\ g = \underline{5.25\ g}$$

At t = 20 seconds

$$\underline{a} = \left[\frac{103,855}{13,555} - 1\right] g$$

$$= [7.66 - 1]\ g = \underline{6.66\ g}$$

At t = 25 seconds

$$\underline{a} = \left[\frac{107,044}{11,075} - 1\right] g$$

$$= [9.67 - 1]\ g = \underline{8.67\ g}$$

At t = 30 seconds

$$\underline{a} = \left[\frac{109,263}{8585} - 1\right] g$$

$$= [12.73 - 1]\ g = \underline{11.73\ g}$$

At t = 35 seconds

$$\underline{a} = \left[\frac{110,771}{6055} - 1\right] g$$

$$= [18.3 - 1]\ g = \underline{17.3\ g}$$

At t = 39.15 seconds

$$\underline{a} = \left[\frac{111,400}{3935} - 1\right] g$$

$$= [28.3 - 1]\ g = \underline{27.3\ g}$$

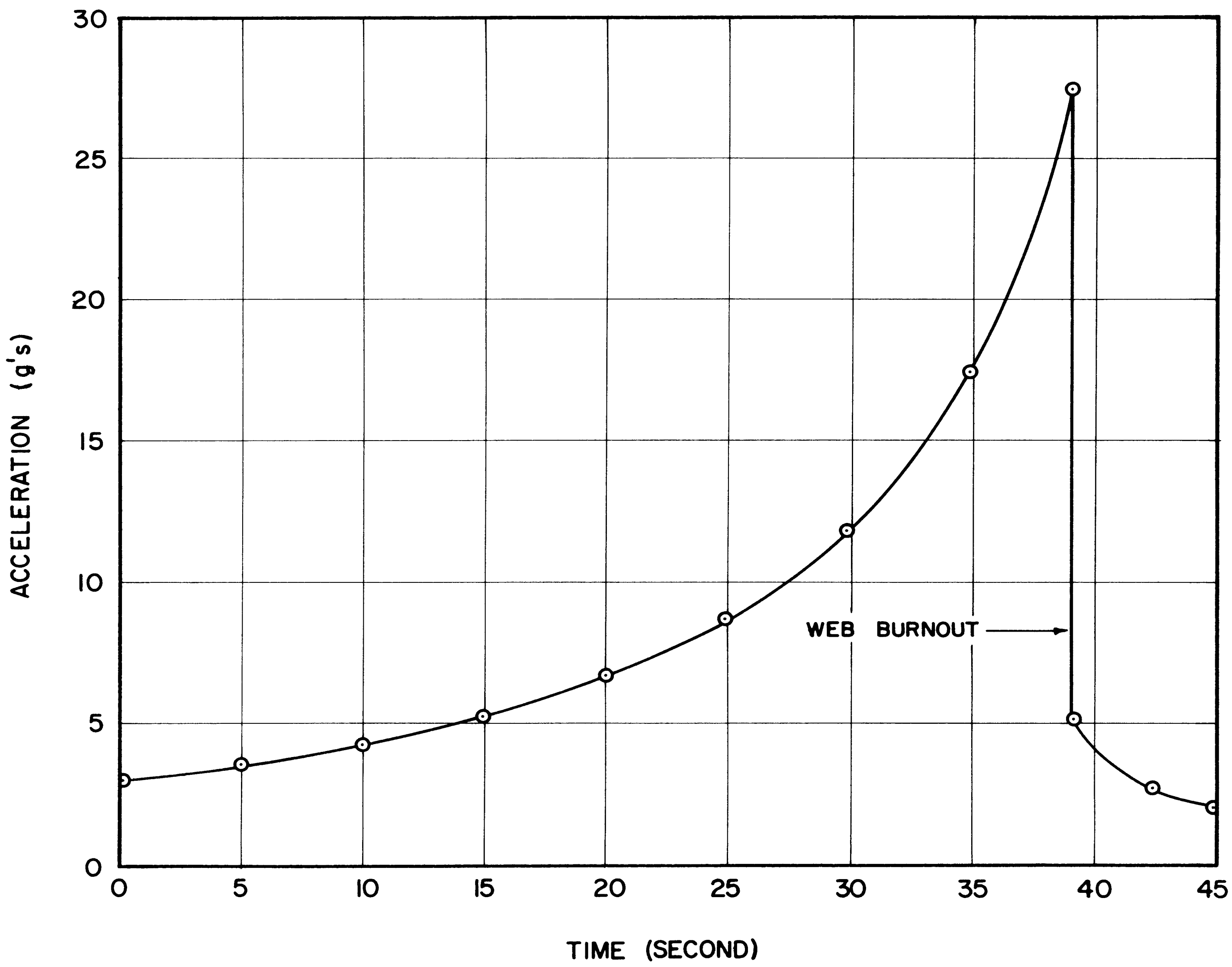


Figure 14. Rocket Acceleration as a Function of Time.

At t = 39.167 seconds

$$\underline{a} = [\frac{23,630}{3930} - 1]\ g$$

$$= [6.02 - 1]\ g = \underline{5.02\ g}$$

At t = 42.5 seconds

$$\underline{a} = [\frac{13,200}{3620} - 1]\ g$$

$$= [3.65 - 1]\ g = \underline{2.65\ g}$$

At t = 44.8 seconds

$$\underline{a} = [\frac{10,480}{3545} - 1]\ g$$

$$= [2.96 - 1]\ g = \underline{1.96\ g}$$

DISCUSSION OF STORAGE TERM

As stated on Page the basic differential equation governing continuity in solid propellant rockets is:

$$A_c r \rho_p = \frac{d}{dt}(\rho_c V_c) + \frac{\gamma M_t A_t P_t}{a_t}$$

and the mass flow out the nozzle can also be written as:

$$\frac{\gamma M_t A_t P_t}{a_t} = \Gamma' \frac{P_c A_t}{a_c}$$

where:

$$\Gamma' = \Gamma\sqrt{\gamma} = \gamma\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}}$$

and introducing the equation of state:

$$P_c = \rho_c R T_c$$

and defining the abbreviation:

$$P_p = \rho_p R T_c$$

The equation becomes:

$$A_c r \frac{P_P}{RT_c} = \frac{d}{dt}\left(\frac{P_c V_c}{RT_c}\right) + \Gamma' \frac{P_c A_t}{a_c}$$

assuming T_c is constant:

$$A_c r P_P = \frac{d}{dt}(P_c V_c) + \Gamma' \frac{P_c A_t RT_c}{a_c}$$

$$= \frac{d}{dt}(P_c V_c) + \Gamma\, P_c A_t \sqrt{RT_c} \quad *$$

now consider the storage term:

$$V_c = V_{c_o} + \int_o^t r\, A_c dt$$

$$\frac{dV_c}{dt} = r\, A_c$$

and

$$\frac{d}{dt}(P_c V_c) = P_c \frac{dV_c}{dt} + V_c \frac{dP_c}{dt}$$

$$= P_c r\, A_c + V_c \frac{dP_c}{dt}$$

so the equation can be written:

$$V_c \frac{dP_c}{dt} = A_c r\,(P_P - P_c) - \Gamma P_c A_t \sqrt{RT_c}$$

$$= \frac{A_c}{A_t}\left\{r(P_P - P_c)A_t - \Gamma P_c \frac{A_t^2}{A_c}\sqrt{RT_c}\right\}$$

$$= A_c\left\{r(P_P - P_c) - \Gamma P_c \frac{A_t}{A_c}\sqrt{RT_c}\right\}$$

Now, introduce the expression for the burning rate:

$$r = a\, P_c^{\,n}$$

Then, the equation becomes:

$$V_c \frac{dP_c}{dt} = A_c \left\{a P_c^{\,n}(P_P - P_c) - \Gamma P \frac{A_t}{A_c}\sqrt{RT_c}\right\}$$

Now, P_c can be neglected in comparison to P_P in the term $(P_P - P_c)$ as P_P is of the order of a hundred times as large (i.e., for asphalt potassium perchlorate propellants it is 125,000 PSI and for ballistite it is 230,000 PSI) - then:

$$V_c \frac{dP_c}{dt} = A_c \left\{ a P_P P_c^n - \Gamma P_c \frac{A_t}{A_c} \sqrt{RT_c} \right\}$$

and it can be seen that the above equation is not easily solvable for P_c. Consider now the case of equilibrium neutral burning so $dP_c/dt = 0$, then:

$$a A_c P_P P_c^n = \Gamma P_c \frac{A_t}{A_c} \sqrt{RT_c} \cdot A_c$$

$$P_c^{1-n} = \frac{a A_c P_P}{\Gamma \frac{A_t}{A_c} \cdot A_c \sqrt{RT_c}}$$

$$= \frac{a A_c P_P}{\Gamma A_t \sqrt{RT_c}}$$

now consider the equation when the storage term is neglected:

$$A_c r P_P = \Gamma P_c A_t \sqrt{RT_c}$$ (from Equat. * on previous page)

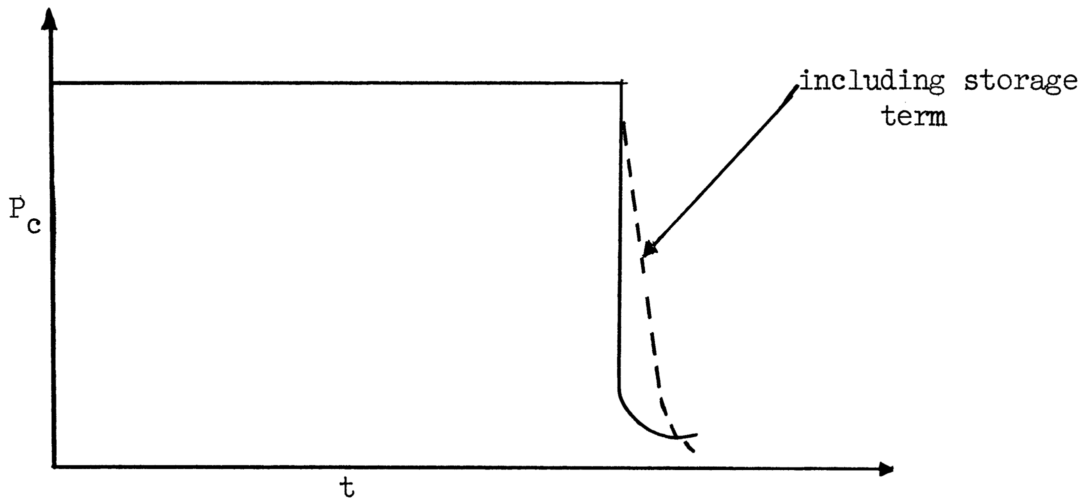
or

$$A_c a P_c^n = \Gamma P_c A_t \sqrt{RT_c}$$

so,

$$P_c^{1-n} = \frac{a A_c P_P}{\Gamma A_t \sqrt{RT_c}}$$

so it can be seen that neglecting the storage term will have no effect on the pressure solution as long as there is neutral burning ($dP_c/dt = 0$) -- hence it would have negligible effect on the pressure trace of this design up to the point of web burnout. At web burnout its effect would be as shown in the sketch below:



That is, it would not allow an instantaneous change in pressure - consequently the rocket would maintain a higher thrust level but burn out sooner.

DETERMINATION OF VELOCITY AT END OF GRAIN

The maximum velocity at the back of the grain will occur at the instant of starting as the mass flow is relatively constant, actually increasing slightly for the first 39.167 seconds of burning -- but, the port area is increasing more rapidly so the critical time will be starting.

The mass flow is given by:

$$\overset{o}{m} = \rho_c A_P V = 481.4 \text{ lb./sec.}$$

where: $A_P = 1018 - 770.63 = 247.37 \text{ in.}^2 = 1.718 \text{ ft.}^2$

and from the equation of state:

$$\rho_c = \frac{P_c}{RT} = \frac{(1000)(144) \text{ lb.°R lb.}}{(\frac{1544}{22})(3160) \text{ ft.lb.°R ft.}^2}$$

$$= .649 \text{ lb./ft.}^3$$

Hence,

$$V = \frac{481.4 \text{ lb. ft.}^3}{(649)(1.718)\text{ sec.lb.ft.}^2} = 432 \text{ ft./sec.}$$

and this velocity will vary linearly to its value of 0 at the front of grain.

COMPARISON WITH LIQUID PROPELLANT ROCKET DESIGN **

Comparison	Liquid	Solid
Payload Weight - lbs.	125 *	125
Lift-Off Weight - lbs.	25.000	23,295
Mass Ratio	20/1	6.58/1
Lift-Off g Loading	2.95	4.0
Maximum Acceleration	38.65 g	27.3 g
Burn-Out Velocity - ft./sec.	16,151	11,474
Burn-Out Altitude - ft.	326,298	247,409
Summit Altitude - ft.	5.73×10^6	2.64×10^6

* Although 125 lb. was the specified payload weight the design analysis showed that this figure could be easily doubled for the performance shown above as there was a weight margin of 404.5 lb.

From the above comparison it is obvious that the liquid propellant design is the better of the two. The two things that hurt the solid design most are high motor weight and unburned propellant. The high motor weight is due to two factors: 1) the high thrust level necessary to attain a 4g lift-off, and 2) the relatively high ratio of motor weight

** The comparison presented above between the Liquid Motor System and Solid Motor System uses a liquid motor design comparable to the one presented here but different in detail. The comments are pertinent.

to thrust produced, which is necessary as there is no way to cool the motor. Although the unburned propellant only represented 3.53% of the volume of the propellant chamber it contributed 965 lb. of dead weight which severely hampers the performance. It would be an interesting study to consider other grain configurations to minimize this factor.

SUMMARY

The design of a solid propellant rocket to meet the specifications listed on Page was carried out. Preliminary calculations were based on a loading (or packing) fraction of .80. After consideration of several grain configurations the five spoke wagon wheel was chosen as it provided a nearly neutral burning configuration that would meet the lift-off requirements for a fairly high packing fraction. An analytic procedure was developed to determine the optimum sizing of the wagon wheel to meet the thrust requirements. An analytic procedure was also determined to compute the burning perimeter as it was felt that this would be more accurate than graphical methods. From this procedure the chamber pressure can be determined as a function of time and the performance then analyzed.

The performance analysis was carried out ignoring drag effects as was specified. The total negative impulse due to drag was computed (with aid of the "rambler" C_D plots) and the amount of propellant necessary to counteract this impulse at sea level thrust conditions specified.

The results of the performance calculations were plotted and are included in the previous pages.

Although this is at best a rough analysis as it does not consider such things as heat transfer or propellant stability it is hoped that some of the methods of analysis presented herein would be useful in the more thorough consideration of more sophisticated systems such as grain designs to increase propellant utilization and determination of optimum g-take-off conditions.