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High Altitude Engineering Laboratory

Interim Technical Report

SINGLE-STATION DOVAP—BALLISTIC CAMERA TRACKING REDUCTION FOR GRENADE-EXPERIMENT ROCKETS S.S. 12.50—S.S. 6.58

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ABSTRACT

A detailed description is given of the reduction of the tracking data for the nine USASRDL grenade-experiment rockets fired at Guam during November, 1958. Included in this description is the derivation of a mathematical solution for position in a single-station DOVAP—ballistic camera tracking system. A computer program embodying this solution together with corrections for the variation of doppler wavelength with altitude is discussed, and the grenade burst positions obtained from this program are listed together with the tracking data used to compute them. The observed errors in the horizontal coordinates and the estimated errors in the vertical coordinates satisfy the required tracking accuracy for estimation of temperature and winds from the grenade-experiment data.

1. INTRODUCTION

As a contribution to the International Geophysical Year effort, nine solid-propellant rockets were fired by the U. S. Army Signal Research and Development Laboratories from the island of Guam in November of 1958.[1] These vehicles carried aloft the rocket-grenade experiment to measure temperature and winds above 25 kilometers.

This series of experiments at Guam represents a logical continuation of the series involving ten grenade-experiment Aerobee rockets[1] fired at Ft. Churchill, Canada, during the I.G.Y. The flights at Ft. Churchill were also conducted by the USASRDL working together with the High Altitude Engineering Laboratory of The University of Michigan. In this series the U-M group was responsible for both the construction and operation of the rocket instrumentation, and subsequent reduction of the DOVAP tracking data.[9]

Tracking instrumentation for the Guam flights was constructed by personnel from the High Altitude Engineering Laboratory under the direction of Dr. Harold F. Allen and Mr. Elton A. Wenzel.[4] This same group was also responsible for operation of the tracking equipment at Guam, where complete data recovery was achieved

For each rocket, tracking data consisted of a single-station DOVAP[2] recording and two ballistic camera plates. Direction cosines for each grenade burst were obtained for both east and west camera plates by the Ballistic Reduction Section of the Physical Science Laboratory at the New Mexico State University.[3] These direction cosines, together with the corrected cycle counts obtained from film transcriptions of the DOVAP recordings, were used to obtain grenade burst positions. The reduction of these tracking data, done under contract DA-36-039-sc-64659 at the High Altitude Engineering Laboratory of The University of Michigan, is described in this report.

2. TRACKING INSTRUMENTATION

The tracking instrumentation at Guam[4] consisted principally of (Fig. 1):

(a) A 37-Mc transmitter radiating 1750 watts from a right-hand wound axial mode helix.
(b) A 37—74-Mc transponder in the vehicle with linearly polarized, transverse-mode loop antennas for receiving and transmitting.
(c) Two receivers with oppositely polarized helical antennas for the reradiated 74-Mc signal and stub antennas for the 37-Mc reference signal.
(d) A single receiver with a dipole antenna to record polarization nulls occurring with rotation of the rocket.
(e) Two ballistic cameras working in the visible region of light, one with an f:11 and the other with an f:16 stop setting.

At the DOVAP receivers the 74-Mc signal from the rocket-borne transponder is heterodyned with the doubled reference signal. The resulting difference frequency, recorded for both helices along with time references, constitutes the DOVAP tracking data.

The ballistic camera plates record the images of the grenade bursts and both pre-shoot and post-shoot exposures to obtain star trails which are interrupted at known times by opening and closing the shutter. These interruptions in the stellar images provide the plate calibration against which the burst directions are measured.

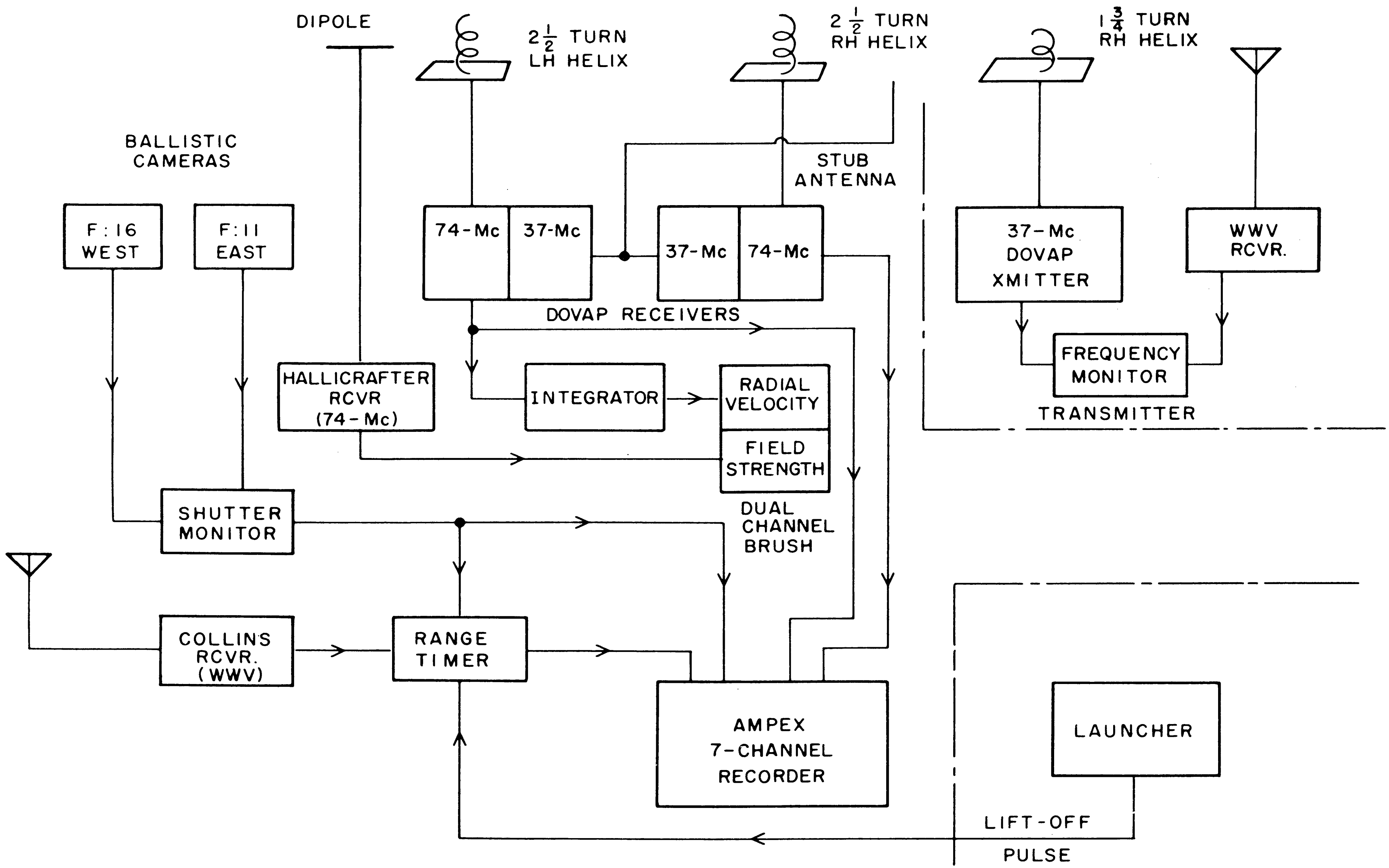

Fig. 1. The Guam tracking instrumentation.

3. MATHEMATICAL STATEMENT OF PROBLEM

3.1. BASIC SOLUTION FOR POSITION

The total change in the "transmitter to missile to receiver" path length during the interval from lift-off to grenade burst time is proportional to the spin-corrected cycle count accumulated over this interval. This spin-corrected count is obtained by differencing the total count of the two receivers, and then using this difference to correct the total count of one receiver. Using survey data, this same path length can be determined at lift-off and thus the path length at the instant of grenade burst is known.

Now we do not know the "transmitter to missile" or "missile to receiver" distances, but only their sum; however, this sum serves to define a prolate ellipsoid (with the transmitter at one focus and the receiver at the other) on whose surface the missile must be.

The position of the ballistic camera, together with the direction cosines of the grenade burst obtained from the ballistic camera plate reduction, define a ray through this surface and thus the grenade burst position is determined uniquely. This situation is represented in Fig. 2.

In the cartesian system indicated by Fig. 2, let the coordinates of the ballistic camera be:

$$\vec{X}_1 = (X_1, Y_1, Z_1) \quad ,$$

and let those of the transmitter and receiver be, respectively:

$$\vec{X}_2 = (X_2, Y_2, Z_2)$$

$$\vec{X}_3 = (X_3, Y_3, Z_3)$$

We are interested in solving simultaneously the equations defining a prolate ellipsoid and a ray through its surface. A simplification in the solution can be had by transforming the coordinate system to one in which the origin is at the center of the ellipsoid, and one axis (say the x-axis) is aligned with the major axis of the ellipsoid. By so doing, we will eliminate both the terms of first degree and the mixed terms of second degree in X, Y, and Z in the equation of the ellipsoid.

Suppose then that we define a new cartesian system (x, y, z) whose origin is at $(1/2)(\vec{X}_2 + \vec{X}_3)$, i.e., midway between the two foci determined by the trans-

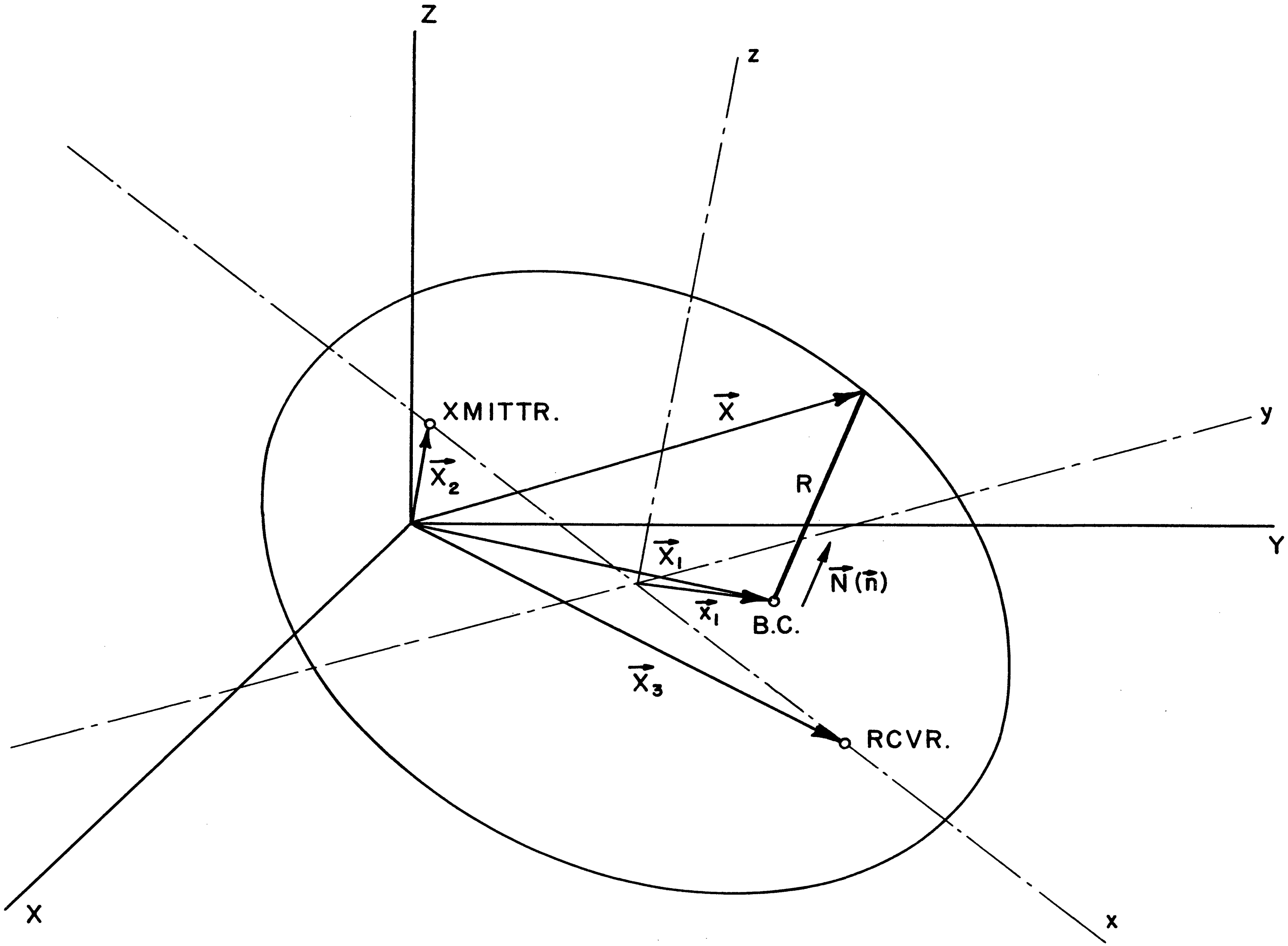

Fig. 2. The relation of the (X, Y, Z) and (x, y, z) systems.

mitter and receiver. Furthermore, let the x-axis be defined by the unit vector:

$$\vec{i} = \frac{\vec{X}_3 - \vec{X}_2}{|\vec{X}_3 - \vec{X}_2|} = \frac{\vec{X}_3 - \vec{X}_2}{2c} \tag{1}$$

where

$$\begin{aligned} 2c &= |\vec{X}_3 - \vec{X}_2| \\ &= \sqrt{(X_3 - X_2)^2 + (Y_3 - Y_2)^2 + (Z_3 - Z_2)^2} \end{aligned} \tag{2}$$

Because of the symmetry of the solution with respect to y and z, it is not necessary explicitly to define the unit vectors $\vec{j}$ and $\vec{k}$ corresponding, respectively, to the y and z axes. The coordinate transformation, defined for row-vectors, is then given by:

$$\vec{x} = (\vec{X} + \vec{T})S \quad , \tag{3}$$

where $\vec{T}$ is the translation:

$$\vec{T} = -\frac{1}{2}(\vec{X}_2 + \vec{X}_3) \quad , \tag{4}$$

and the rotation is

$$S = \begin{pmatrix} \vec{i} \\ \vec{j} \\ \vec{k} \end{pmatrix}' \tag{5}$$

with the prime indicating transposition.

Let us now define u as the "transmitter to missile to receiver" distance. In the (x, y, z) system the transmitter is at one focus (-c, 0, 0), while the receiver is at the other focus (c, 0, 0). Since the sum of distances to the two foci is equal to twice the length of the semi-major axis of the ellipsoid,[5] 2a, the semi-major axis is simply

$$a = u/2 \tag{6}$$

and the semi-minor axis, b, of the ellipsoid is then

$$b = \sqrt{\frac{u^2}{4} - c^2} \tag{7}$$

The ellipsoid is then defined by the canonical form (Ref. 5, p. 366):

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{b^2} - 1 = 0 \tag{8}$$

Rather than solving explicitly for the point of intersection of the ray on this surface and then applying the inverse transformation to the (X, Y, Z) system, we will determine instead an invariant under the orthogonal coordinate transformation, the distance, R, along the ray from the ballistic camera to the point of intersection. In the original cartesian system the point of intersection will then be

$$\vec{X} = \vec{X}_1 + R\,\vec{N} \ , \tag{9}$$

where the direction cosines L, M, and N, relative to the (X, Y, Z) system, form the unit vector

$$\vec{N} = (L, M, N) \ . \tag{10}$$

Let us rewrite Eq. (8) in the form

$$x^2 + \frac{a^2}{b^2}\,y^2 + \frac{a^2}{b^2}\,z^2 - a^2 = 0 \ . \tag{11}$$

This suggests that we can think of the ellipsoid as the affine image of the sphere[6]

$$x'^2 + y'^2 + z'^2 - a^2 = 0$$

under the transformation (Fig. 3)

$$(x', y', z') \xrightarrow{A} \left(x = x',\ y = \frac{b}{a}\,y',\ z = \frac{b}{a}\,z'\right)$$

Suppose now that the camera direction cosines relative to (x, y, z) are the components of the unit vector

$$\vec{n} = (\ell, m, n) = \vec{N}\,S \ ,$$

while the position vector for the ballistic camera is $\vec{x}_1$ in the (x, y, z) system.

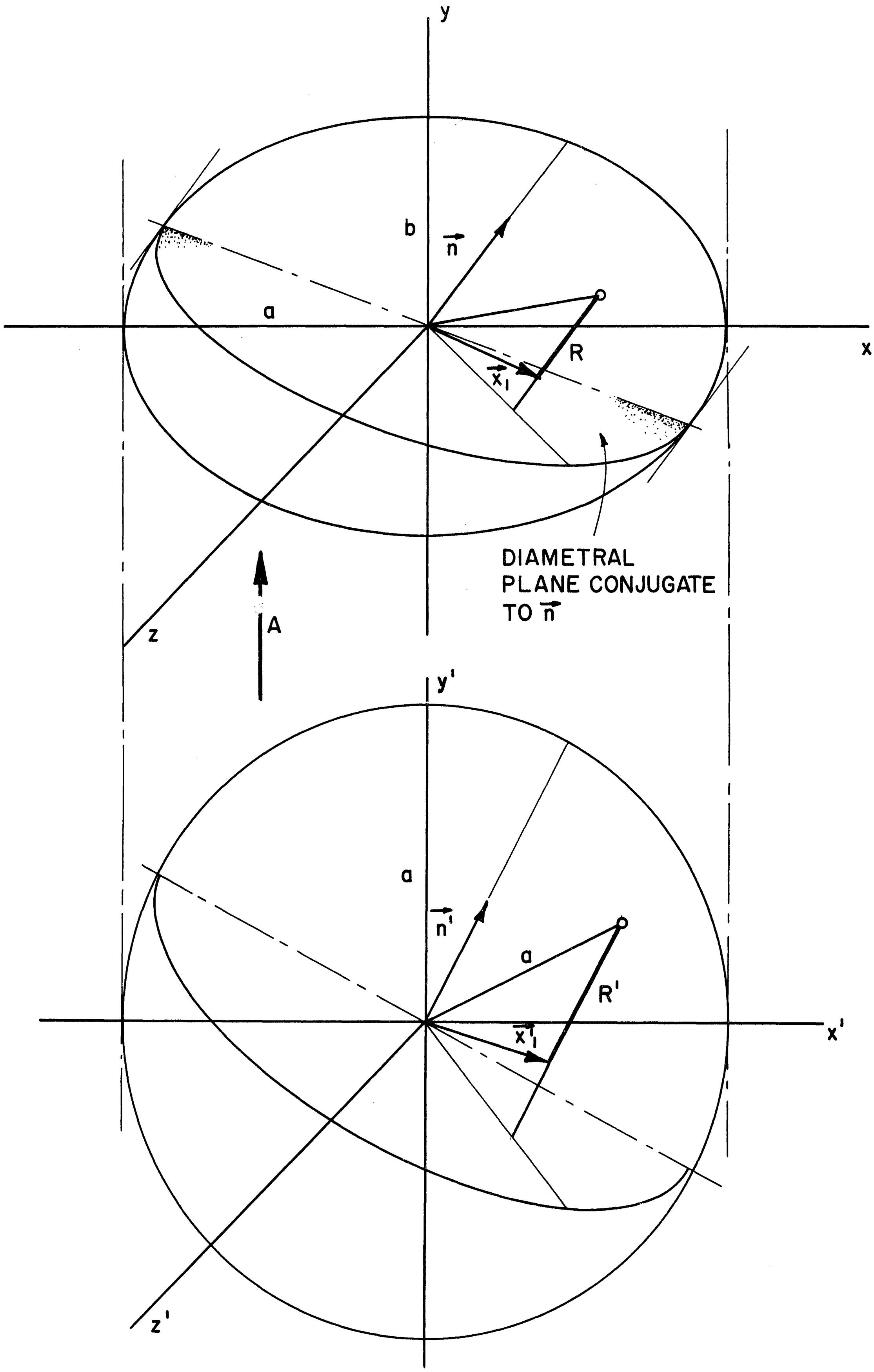

Fig. 3. The ellipsoid as the affine image of a sphere.

Then both $\vec{n}$ and $\vec{x}_1$ will have counter images $\vec{n}'$ and $\vec{x}_1'$ in the (x', y', z') space.

Consider now the plane section through the sphere containing $\vec{x}_1'$ and $\vec{n}'$ (Fig. 4). Let R' be the length of the line segment which is the counter image of the line segment of length R along the ray from $\vec{x}_1$ to the surface of the ellipsoid. Then

$$R' = -q + \sqrt{a^2 - p^2} , \tag{12}$$

where

$$q = \frac{\vec{n}' \cdot \vec{x}_1'}{|\vec{n}'|}$$

and

$$p^2 = \frac{|\vec{n}' \times \vec{x}_1'|^2}{|\vec{n}'|^2} ;$$

hence

$$\begin{aligned} R' &= -\frac{\vec{n}' \cdot \vec{x}_1'}{|\vec{n}'|} + \sqrt{a^2 - \frac{|\vec{n}' \times \vec{x}_1'|^2}{|\vec{n}'|^2}} \\ &= \frac{-\vec{n}' \cdot \vec{x}_1' + \sqrt{a^2(\vec{n}' \cdot \vec{n}') - |\vec{n}' \times \vec{x}_1'|^2}}{|\vec{n}'|} \end{aligned} \tag{12a}$$

Under the affine transformation we are considering (Ref. 6, pp. 161-162), a parallel family of lines is mapped into a parallel family of lines; furthermore, the ratio of the length of a line segment to the length of its image is a constant which has the same value for all members of a family of parallel lines. Consequently, to determine R we need only observe that

$$\frac{R}{R'} = \frac{|\vec{n}|}{|\vec{n}'|} = \frac{1}{|\vec{n}'|} ; \tag{13}$$

hence

$$R = \frac{-\vec{n}' \cdot \vec{x}_1' + \sqrt{a^2(\vec{n}' \cdot \vec{n}') - |\vec{n}' \times \vec{x}_1'|^2}}{\vec{n}' \cdot \vec{n}'} \tag{14}$$

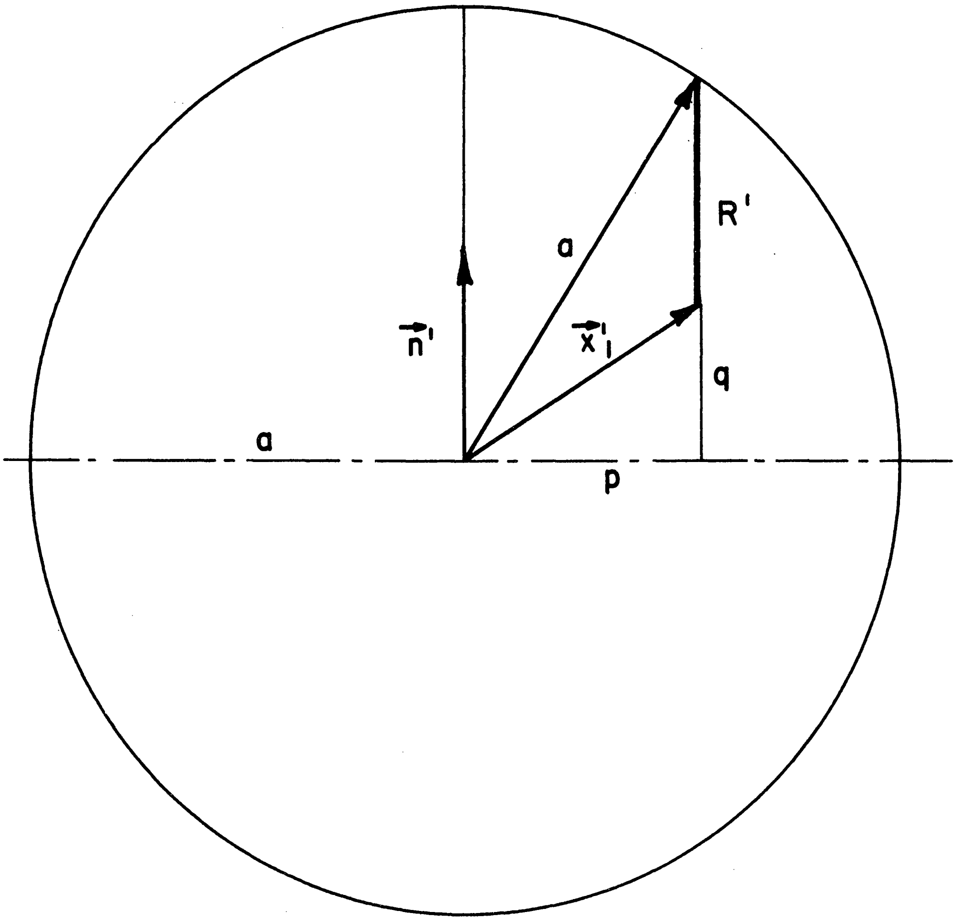

Fig. 4. Plane section containing $\vec{n}'$ and $\vec{x}_1'$.

Now we can write

$$\vec{n}' = \left(\ell, \frac{a}{b} m, \frac{a}{b} n\right) = \frac{a}{b}\left[\vec{n} - \left(1 - \frac{b}{a}\right)(\ell, 0, 0)\right] , \qquad (15)$$

and similarly

$$\vec{x}_1' = \left(x_1, \frac{a}{b} y_1, \frac{a}{b} z_1\right) = \frac{a}{b}\left[\vec{x}_1 - \left(1 - \frac{b}{a}\right)(x_1, 0, 0)\right] . \qquad (16)$$

Remembering than $\vec{n} \cdot \vec{n} = 1$, we then have

$$\begin{aligned} \vec{n}' \cdot \vec{n}' &= \frac{a^2}{b^2}\left[1 - 2\left(1 - \frac{b}{a}\right)\ell^2 + \left(1 - \frac{b}{a}\right)^2 \ell^2\right] \\ &= \frac{a^2}{b^2}\left[1 - \left(1 - \frac{b^2}{a^2}\right)\ell^2\right] \end{aligned} \qquad (17)$$

and similarly

$$\vec{n}' \cdot \vec{x}_1' = \frac{a^2}{b^2}\left[\vec{n} \cdot \vec{x}_1 - \left(1 - \frac{b^2}{a^2}\right)\ell x_1\right] \qquad (18)$$

After some manipulation we also obtain

$$|\vec{n}' \times \vec{x}_1'|^2 = \frac{a^2}{b^2}\left[|\vec{n} \times \vec{x}_1|^2 + \frac{a^2}{b^2}\left(1 - \frac{b^2}{a^2}\right)(\vec{n} \times \vec{x}_1)_x^2\right] , \qquad (19)$$

where

$$(\vec{n} \times \vec{x}_1)_x = (mz_1 - ny_1) .$$

Substituting (17), (18), and (19) in (14) and observing that

$$|\vec{n} \times \vec{x}_1|^2 = (\vec{x}_1 \cdot \vec{x}_1) - (\vec{n} \cdot \vec{x}_1)^2 ,$$

we then have

$$R = \frac{-\frac{a^2}{b^2}\left[\vec{n}\cdot\vec{x}_1 - \left(1 - \frac{b^2}{a^2}\right)\ell x_1\right]}{\frac{a^2}{b^2}\left[1 - \left(1 - \frac{b^2}{a^2}\right)\ell^2\right]} + \frac{\sqrt{a^2\,\frac{a^2}{b^2}\left[1 - \left(1 - \frac{b^2}{a^2}\right)\ell^2\right] - \frac{a^2}{b^2}\left[(\vec{x}_1\cdot\vec{x}_1) - (\vec{n}\cdot\vec{x}_1)^2 + \frac{a^2}{b^2}\left(1 - \frac{b^2}{a^2}\right)(\vec{n}\times\vec{x}_1)_x^2\right]}}{\frac{a^2}{b^2}\left[1 - \left(1 - \frac{b^2}{a^2}\right)\ell^2\right]},$$

which can be written as

$$R = \frac{-(\vec{n}\cdot\vec{x}_1) + \left(1 - \frac{b^2}{a^2}\right)\ell x_1}{\left[1 - \left(1 - \frac{b^2}{a^2}\right)\ell^2\right]} + \frac{a\sqrt{\frac{b^2}{a^2}\left[1 - \left(1 - \frac{b^2}{a^2}\right)\ell^2 - \frac{1}{a^2}(\vec{x}_1\cdot\vec{x}_1) + \frac{1}{a^2}(\vec{n}\cdot\vec{x}_1)^2\right] - \frac{1}{a^2}\left(1 - \frac{b^2}{a^2}\right)(\vec{n}\times\vec{x}_1)_x^2}}{\left[1 - \left(1 - \frac{b^2}{a^2}\right)\ell^2\right]} \tag{20}$$

We already have $a^2 = u^2/4$ and $b^2 = (u^2/4) - c^2$ so that

$$R = \frac{-(\vec{n}\cdot\vec{x}_1) + \frac{4}{u^2}c^2\,\ell x_1}{\left(1 - \frac{4}{u^2}c^2\,\ell^2\right)} + \frac{\frac{u}{2}\sqrt{\left(1 - \frac{4}{u^2}c^2\right)\left[1 - \frac{4}{u^2}\left\{c^2\ell^2 + \vec{x}_1\cdot\vec{x}_1 - (\vec{n}\cdot\vec{x}_1)^2\right\}\right] - \frac{16}{u^4}c^2\,(\vec{n}\times\vec{x}_1)_x^2}}{\left(1 - \frac{4}{u^2}c^2\,\ell^2\right)}. \tag{20a}$$

Now in this expression for R there are terms involving scalar products and vector components. Since scalar products are invariant under coordinate rotation, we can make the following substitutions. Let

$$\vec{X}_1' = \vec{X}_1 + \vec{T}\ ; \tag{21}$$

then

$$\vec{X}_1'\cdot\vec{X}_1' = \vec{x}_1\cdot\vec{x}_1\ .$$

Also

$$\vec{N} \cdot \vec{X}_1' = \vec{n} \cdot \vec{x}_1$$

In addition

$$\ell = \vec{i} \cdot \vec{N} \ ,$$

and

$$x_1 = \vec{i} \cdot \vec{X}_1'$$

We also have

$$(\vec{n} \times \vec{x}_1)_x = \vec{i} \cdot (\vec{N} \times \vec{X}_1')$$

$$= \vec{N} \cdot (\vec{X}_1' \times \vec{i})$$

Thus we can rewrite our solution for R in terms of the parameters

$$\begin{aligned} \beta &= \vec{i} \cdot \vec{X}_1' = \vec{x}_1 \\ \delta &= \vec{X}_1' \cdot \vec{X}_1' = \vec{x}_1 \cdot \vec{x}_1 \\ \vec{\nu} &= \vec{X}_1' \times \vec{i} \end{aligned} \qquad (22)$$

and the auxilliary variables

$$\begin{aligned} \eta &= \vec{i} \cdot \vec{N} = \ell \\ \sigma &= \vec{N} \cdot \vec{X}_1' = \vec{n} \cdot \vec{x}_1 \\ \tau &= \vec{N} \cdot \vec{\nu} = (\vec{n} \times \vec{x}_1)_x \end{aligned} \qquad (23)$$

as follows

$$R = \frac{-\sigma + \frac{4}{u^2} c^2 \beta \eta + \frac{u}{2} \sqrt{\left(1 - \frac{4}{u^2} c^2\right)\left[\left(1 - \frac{4}{u^2} c^2 \eta^2\right) - \frac{4}{u^2} (\delta - \sigma^2)\right] - \frac{4}{u^2} c^2 \cdot \frac{4}{u^2} \tau^2}}{1 - \frac{4}{u^2} c^2 \eta^2} \qquad (20b)$$

Equations (9) and (20b) then constitute the complete solution for position in the original cartesian system.

3.2. CORRECTION FOR VARIATION OF REFRACTIVE INDEX WITH ALTITUDE

It has been shown elsewhere[7] that systematic errors of the order of several doppler wavelengths in the "transmitter to missile to receiver" path length can result if the DOVAP propagation velocity is taken as the mean of the sea-level and vacuum velocities. Consequently a correction for the variation of the propagation velocity or the index of refraction is indicated.

An adequate approximation, particularly at radio frequencies, for the functional dependence of the index of refraction, μ, on atmospheric density, ρ, is given by

$$\mu - 1 = k\rho \tag{24}$$

for some constant k. Thus, if μ_o is the refractive index at the standard sea-level density, ρ_o, $k = (\mu_o - 1)/\rho_o$ and (24) becomes

$$\mu - 1 = (\mu_o - 1)\frac{\rho}{\rho_o} \tag{25}$$

Now we wish to determine an average refractive index $\overline{\mu}_1(Z)$ such that the average propagation velocity of radiation traversing the altitude interval from Z_1 to Z is

$$\overline{c}_1(Z) = \frac{c_o}{\overline{\mu}_1(Z)} \tag{26}$$

where c_o is the vacuum velocity. Since $\rho = \rho(Z)$, the propagation velocity at any altitude Z is

$$c(Z) = \frac{c_o}{1 + (\mu_o - 1)\frac{\rho(Z)}{\rho_o}} \tag{27}$$

Suppose that the ray defining the direction of propagation makes an angle of ϕ with the vertical; then for the infinitesimal altitude interval, dZ, the distance propagated is $ds = \sec\phi \, dZ$. Thus the total time to propagate from altitude Z_1 to Z is

$$t = \int_{Z_1}^{Z} \frac{\left[1 + (\mu \;\; - 1)\frac{\rho(Z)}{\rho_o}\right]}{c_o} \sec\phi \, dZ \tag{28}$$

$$= \frac{\sec\phi}{c_o}\left\{(Z - Z_1) + (\mu_o - 1)\int_{Z_1}^{Z} \frac{\rho(Z)}{\rho_o}\, dZ\right\} ,$$

whereas the total distance, s, propagated is simply sec $\phi(Z - Z_1)$. Thus the average index of refraction is

$$\bar{\mu}_1(Z) = \frac{c_o}{(s/t)}$$

$$= 1 + \frac{(\mu_o - 1)}{(Z - Z_1)} \int_{Z_1}^{Z} \frac{\rho(Z)}{\rho_o} dZ \qquad (29)$$

or

$$\frac{\mu_1(Z) - 1}{\mu_o - 1} = \frac{1}{Z - Z_1} \int_{Z_1}^{Z} \frac{\rho(Z)}{\rho_o} dZ \qquad (29a)$$

It should be noticed that the tacit assumptions of straight-line propagation and small earth curvature relative to the atmosphere's effective thickness have been made. We would now like to express (29a) in terms of the average refractive index relative to sea level, given by

$$\bar{\mu}(Z) = 1 + \frac{(\mu_o - 1)}{Z} \int_{o}^{Z} \frac{\rho(Z)}{\rho_o} dz \qquad (30)$$

To do this we rewrite (29a) as follows:

$$\frac{\bar{\mu}_1(Z) - 1}{\mu_o - 1} = \frac{1}{Z - Z_1} \left\{ \int_{o}^{Z} \frac{\rho(Z)}{\rho_o} dZ - \int_{o}^{Z_1} \frac{\rho(Z)}{\rho_o} dZ \right\}$$

$$= \frac{Z}{Z - Z_1} \left\{ \frac{1}{Z} \int_{o}^{Z} \frac{\rho(Z)}{\rho_o} dZ \right\} - \frac{Z_1}{Z - Z_1} \left\{ \frac{1}{Z_1} \int_{o}^{Z_1} \frac{\rho(Z)}{\rho_o} dZ \right\} ,$$

or

$$\bar{\mu}_1(Z) - 1 = \frac{1}{Z - Z_1} [Z(\bar{\mu}(Z) - 1) - Z_1(\bar{\mu}(Z_1) - 1)] . \qquad (31)$$

To evaluate the integral expression given by Eq. (30), it is necessary to know the functional relationship of density versus altitude; however, such a relationship is at best an average determined for one locality on the earth's surface. To make our equations of more general applicability, we will adopt a simplified static model of the earth's atmosphere.

At every point in this model there will be no acceleration and Euler's dynamical equation will require that the gradient of pressure be everywhere normal to the equipotential surfaces defined by the earth's gravitation field. Consequently, pressure will be constant along each equipotential surface. Furthermore, to insure hydrostatic balance, Euler's equation would also require that density (and hence also temperature) be constant along each equipotential surface. Thus it follows that by adopting such a model a single density versus geopotential altitude profile will be everywhere valid.

Now it has been shown elsewhere[8] that a very good approximation to geopotential altitude, H, in terms of the actual or geometric altitude, Z, is given by the expression

$$H = \left(\frac{g_o}{G}\right) \frac{rZ}{r + Z} \tag{32}$$

where g_o is the local acceleration due to gravity at sea level. G is a conversion factor depending on the units of H, and is numerically equal to the standard acceleration due to gravity when H and Z have the same units. Also, r is the earth's effective radius given by (Ref. 8, Appendix M):

$$r = -2g_o \Big/ \frac{\partial g}{\partial Z}\Bigg]_{Z = o} \tag{33}$$

If Z and Z' are altitudes at two separated locations on the earth, then the geopotential altitudes of these points will be the same when

$$\left(\frac{g_o}{G}\right) \frac{rZ}{r + Z} = \left(\frac{g_o^*}{G}\right) \frac{r^*Z'}{r^* + Z'} \tag{34}$$

Here g_o^* and r^* are values at the location which corresponds to the primed altitude. Thus $H(Z') = H(Z)$ when

$$Z' = \frac{(g_o/g_o^*)\,Z}{1 + \epsilon'\ Z/r}\ , \tag{35}$$

where

$$\epsilon' = 1 - \frac{g_o r}{g_o^* r^*}\ . \tag{36}$$

Similarly,

$$Z = \frac{(g_o^*/g_o)\,Z'}{1 + \epsilon\ Z'/r^*}\ , \tag{37}$$

where

$$\epsilon = 1 - \frac{g_o^* r^*}{g_o r} \quad . \tag{38}$$

Let us now evaluate the integral

$$\int_o^Z \frac{\rho(Z)}{\rho_o} \, dZ \quad ,$$

in terms of the primed altitude Z' which we will assume corresponds to some location relative to which the density-versus-altitude dependence has been established. From (37) we have

$$dZ = \frac{(g_o^*/g_o)\, dZ'}{[1 + \epsilon(Z'/r^*)]^2} \tag{39}$$

Also, because $H(Z') = H(Z)$, we have $\rho(Z') = \rho(Z)$, and as a result

$$\int_o^Z \frac{\rho(Z)}{\rho_o} \, dZ = \int_o^{Z'(Z)} \frac{\rho(Z')}{\rho_o} \cdot \frac{(g_o^*/g_o)}{[1 + \epsilon(Z'/r^*)]^2} \, dZ' \quad . \tag{40}$$

Now when $|\epsilon(Z'/r^*)| < 1$, we can write:

$$\begin{aligned} &\int_o^Z \frac{\rho(Z)}{\rho_o} \, dZ \\ &= \int_o^{Z'(Z)} (g_o^*/g_o) \frac{\rho(Z')}{\rho_o} \left\{ 1 - 2\epsilon\left(\frac{Z'}{r^*}\right) + 3\epsilon^2 \left(\frac{Z'}{r^*}\right)^2 - \ldots\ldots\ldots \right\} dZ' \\ &= (g_o^*/g_o) \left\{ \int_o^{Z'(Z)} \frac{\rho(Z')}{\rho_o} \, dZ' - 2\epsilon \int_o^{Z'(Z)} \frac{\rho(Z')}{\rho_o} \left(\frac{Z'}{r^*}\right) dZ' + \ldots\ldots \right\} \end{aligned} \tag{40a}$$

This expression is an alternating series so that the remainder after any truncation of this series is less in absolute value than the last term. Let us now evaluate each term of (40a) successively to establish where it can be truncated with sufficient accuracy. If we assume the atmosphere is approximately isothermal, then we can write

$$\frac{\rho(Z')}{\rho_o} = \frac{\rho_1}{\rho_o} e^{-(Z' - Z_1')/h} \quad , \tag{41}$$

where ρ_1 is the density at altitude Z_1' and h is the scale height. Since the term of nth order in (40a) contains the quantity $(Z'/r^*)^n$, the higher-order terms become more important at higher altitudes. Instead of evaluating the integrals of (40a) over some interval from sea level to the highest altitudes of interest, we will evaluate them over a small interval near the highest altitude to be considered, so that the relative importance of the higher-order terms will be more evident. Thus from (41) we have

$$\int_{Z_1}^{Z'} \frac{\rho(Z')}{\rho_o}\, dZ' = \frac{\rho_1}{\rho_o} \int_{Z_1}^{Z'} e^{-(Z' - Z_1')/h}\, dZ'$$

$$= - \frac{\rho_1}{\rho_o}\, h\, e^{-(Z' - Z_1')/h} \Big]_{Z_1'}^{Z'} \tag{42}$$

$$= h \left(\frac{\rho_1}{\rho_o} - \frac{\rho}{\rho_o} \right) .$$

Upon integration by parts we also obtain

$$\int_{Z_1'}^{Z'} \frac{\rho(Z')}{\rho_o} \left(\frac{Z'}{r^*} \right) dZ'$$

$$= - \frac{\rho_1}{\rho_o} \frac{h}{r^*} \left\{ Z'\, e^{-(Z' - Z_1')/h} + h\, e^{-(Z' - Z_1')/h} \right\} \Big]_{Z_1'}^{Z'} \tag{43}$$

$$= h \left\{ \frac{\rho_1}{\rho_o} \cdot \frac{(Z_1' + h)}{r^*} - \frac{\rho}{\rho_o} \cdot \frac{(Z' + h)}{r^*} \right\} .$$

Also upon solving (41) for scale height, h, we obtain

$$h = \frac{(Z' - Z_1')}{\log_e \rho_1/\rho_o - \log_e \rho/\rho_o} \tag{44}$$

Now let

$$Z_1' = 135 \text{ km}$$

$$Z' = 140 \text{ km}$$

Then from the ARDC model atmosphere we have

$$\rho/\rho_o = 3.5071 \times 10^{-9}$$

$$\rho_1/\rho_o = 5.8886 \times 10^{-9}$$

Substituting in (44), there results

$$h = \frac{5}{\log_e 1.67905} = 9.6484 \text{ km} \quad .$$

Let g_o and r be defined for 0° latitude, and g_o^* and $r*$ be defined at the reference latitude of 45°32'40". Then by differentiating the Lambert series (Ref. 8, Appendix N) for gravitational acceleration as a function of latitude, ϕ, and altitude, we have

$$-\left.\frac{\partial g}{\partial Z}\right]_{Z=o} = 3.085462 \times 10^{-6} + 2.27 \times 10^{-9} \cos 2\phi \ (\sec^{-2}) \qquad (45)$$

Using the values

$$g_o = 9.78039 \text{ m/sec}^2$$

$$g_o^* = 9.80665 \text{ m/sec}^2 \quad ,$$

we then obtain from Eqs. (45), (33), and (38)

$$\epsilon = 0.0061295$$

$$r* = 6{,}356.77 \text{ km}$$

Now

$$\left(\frac{\rho_1}{\rho_o} - \frac{\rho}{\rho_o}\right) = 2.3815 \times 10^{-9} \quad ,$$

and also

$$\left\{\frac{\rho_1}{\rho_o} \cdot \frac{(Z_1' + h)}{r*} - \frac{\rho}{\rho_o} \cdot \frac{(Z' + h)}{r*}\right\} = 0.051432 \times 10^{-9}$$

so that

$$2\epsilon \times 0.051432 \times 10^{-9} = 0.0006305 \times 10^{-9}$$

Thus in the altitude interval from 135 to 140 km the relative importance of the first two terms of the series expression (40a) is indicated by the rati

$$2.3815 \times 10^{-9} : 0.0006305 \times 10^{-9}$$

Since the relation of refractive index to density, given by (24), is not valid for ionospheric propagation, the resulting equation for average refractive index, $\bar{\mu}(Z)$, given by (30), is not valid for altitudes greater than ca. 100 km, and consequently this last result implies that we can truncate the series expression (40a) to the first term within the region of applicability of (30). Thus we will have

$$\int_0^Z \frac{\rho(Z)}{\rho_0}\, dZ \doteq (g_0^*/g_0) \int_0^{Z'(Z)} \frac{\rho(Z')}{\rho_0}\, dZ' \quad ,$$

or from (30) and (35)

$$\begin{aligned} \bar{\mu}(Z) &= 1 + \frac{\mu_0 - 1}{Z'[1 + \epsilon' \, (Z/r)]} \cdot \int_0^{Z'(Z)} \frac{\rho(Z')}{\rho_0}\, dZ' \\ &= 1 + \frac{\bar{\mu}[Z'(Z)] - 1}{1 + \epsilon' \, Z/r} \quad , \end{aligned} \qquad (46)$$

where $Z'(Z)$ is given by the relation (35). Equations (46), (35), and (31) now allow us to compute an average index of refraction at any location in terms of a single standard table of average indices of refraction at our reference latitude. For any latitude we need only first to evaluate the sea-level acceleration of gravity, g_0, and the earth's effective radius, r, via Eqs. (33) and (45).

3.3. GUAM GEOMETRY CORRECTION

At Guam the transmitting antenna is a right-hand helix; consequently the left-hand receiver will normally see a phase shift of ±1 cycle every missile rotation while the right-hand receiver will see a phase shift of ±3 cycles.[9] If we designate the total cycle counts from lift-off as N_{LH} and N_{RH}, respectively, for the left-hand and right-hand receivers, then the difference

$$N' = N_{RH} - N_{LH} \qquad (47)$$

will be entirely due to missile spin when the two receiving antennas are coincident, and will increase by ±2 cycles for each missile rotation. For this situation the spin-corrected cycle count is simply

$$N_1 = N_{LH} - \frac{1}{2} N' \quad . \tag{48}$$

If the antennas are at different locations, the separation of their phase centers will result in a phase difference which will contribute further to N'. To be quite precise, these helical antennas lack a fixed phase center because of the variation of phase over the lobe; however, if we assume that the phase center is located at the center of the ground plane, we will in no case make an error of more than a quarter cycle. Since both cycle counts are used to obtain a spin-corrected count, the counts from both receivers must be referred to a single location by making an equivalent phase correction. Suppose, then, that we refer both counts to the location of the left-hand antenna. Let $\vec{X}$ be the position of the missile and $\vec{X}_L$ and $\vec{X}_R$ be the positions of the left-hand and right-hand antennas, respectively. Because of the separation of the antenna phase centers, the left-hand count will be higher than the right-hand count by the amount

$$N^* = \Delta \left\{ \frac{|\vec{X} - \vec{X}_L| - |\vec{X} - \vec{X}_R|}{\bar{\lambda}} \right\} \quad , \tag{49}$$

where Δ indicates the total change in the bracketed quantity from lift-off, and

$$\bar{\lambda} = \lambda_o / \bar{\mu}_1(Z) \tag{50}$$

with λ_o as the vacuum wavelength.

For the Guam range geometry, a very good approximation is given by

$$|\vec{X} - \vec{X}_L| - |\vec{X} - \vec{X}_R| \cong (\vec{X}_R - \vec{X}_L) \cdot \vec{N} = \vec{s} \cdot \vec{N} \tag{51}$$

where $\vec{s} = (\vec{X}_R - \vec{X}_L)$ and, as before, $\vec{N}$ has as its components the direction cosines determined by the ballistic cameras. As a result

$$N^* = \frac{\vec{s} \cdot \vec{N}}{\bar{\lambda}} - \frac{\Delta R_o}{\lambda_o} \mu_1 \quad , \tag{52}$$

where $\Delta R_O = \{|\vec{X} - \vec{X}_L| - |\vec{X} - \vec{X}_R|\}$ at lift-off, and μ_1 is the index of refraction at the mean altitude of the ground array. Since the difference in phase between the left-hand and right-hand receivers due to the separation of their phase centers is then N^*, the right-hand count corrected to the location of the left-hand antenna is simply

$$N_{RH} + N^* \quad , \tag{53}$$

and consequently the difference

$$N_S' = (N_{RH} + N^*) - N_{LH} = N' + N^* \tag{54}$$

will be entirely due to spin, whereupon the spin-corrected count becomes

$$N = N_{LH} - \frac{1}{2} N_S' = (N_{LH} - \frac{1}{2} N') - \frac{1}{2} N^* = N_1 - \frac{1}{2} N^* \quad . \tag{55}$$

Equations (48), (52), and (55) are then the final form of our geometry correction. In particular, it should be noted that Eq. (55) is applicable only when the spin phase shifts are normal. When the spin phase shifts are anomalous over some interval, the correction is, in general, of the form (see Ref. 9, pp. 49-53):

$$\Delta N = \Delta N_{LH} + \frac{1}{2} \Delta N_S' + (\text{Corr.})$$

or

$$\begin{aligned} \Delta N &= [\Delta N_{LH} + \frac{1}{2} \Delta N' + (\text{Corr.})] + \frac{1}{2} \Delta N^* \\ &= \Delta N_2 + \frac{1}{2} \Delta N^* \qquad (\text{say}) \qquad (56) \\ &= (\Delta N_2 + \Delta N^*) - \frac{1}{2} \Delta N^* \quad , \end{aligned}$$

where Δ indicates the change in the quantity over the interval in question. The quantity ΔN_2, in particular, will be the result of the application of anomalous spin corrections to the cycle-count data over this interval. Thus if we wish to use the same form for the geometry correction as given by (55), we must correct the quantity ΔN_2 by the additional amount ΔN^* in this interval.

In general, the correction ΔN^* will be insignificant when anomalous corrections are applied over short intervals during all but the first few seconds of flight. This is especially true for the Guam range since the motion of the missile is almost entirely radial and consequently the direction represented by $\vec{N}$ is virtually constant.

3.4. COMPUTATION OF THE u VALUES

Suppose that at lift-off the sum or the "transmitter to missile" and "missile to receiver" distance is u_o; then the value of u at any other time will be

$$u = \left\{ \mu_1 \frac{u_o}{\lambda_o} + N \right\} \cdot \frac{\lambda_o}{\overline{\mu}_1(Z)} \quad . \tag{57}$$

where, as before, N is the corrected cycle count, λ_o is the vacuum wavelength at the doubled frequency, and μ_1 is the index of refraction at the average altitude of the ground array. Substituting Eqs. (52) and (55) into (57), there results

$$\begin{aligned} u &= \left\{ \mu_1 \frac{u_o}{\lambda_o} + N_1 - \frac{1}{2} \frac{\vec{s} \cdot \vec{N}}{\lambda_o} \overline{\mu}_1(Z) + \frac{1}{2} \mu_1 \frac{\Delta R_o}{\lambda_o} \right\} \frac{\lambda_o}{\overline{\mu}_1(Z)} \\ &= \left\{ \mu_1 (u_o + \frac{1}{2} \Delta R_o) + N_1 \lambda_o \right\} \frac{1}{\overline{\mu}_1(Z)} - \frac{1}{2} \vec{s} \cdot \vec{N} \quad , \end{aligned} \tag{58}$$

or writing this equation in terms of u/2, (= a):

$$u/2 = \frac{(U_o + N_1 \lambda_o)}{2\overline{\mu}_1(Z)} - \frac{1}{4} \vec{s} \cdot \vec{N} \quad , \tag{58a}$$

where

$$U_o = \mu_1 (u_o + \quad \Delta R_o) \tag{59}$$

and s has been previously defined as

$$\vec{s} = (\vec{X}_R - \vec{X}_L) \tag{60}$$

Equation (58a) will then be the expression used to compute geometry-corrected values for u/2 in terms of the spin-corrected counts N_1 given by Eq. (48), or the equivalent sum involving corrections of the form $(\Delta N_2 + \Delta N^*)$. The value of u/2 is, as a consequence, referred to the phase center of the left-hand helix.

4. THE LGP-30 PROGRAM FOR GRENADE BURST POSITIONS

4.1. TABLE OF AVERAGE REFRACTIVE INDICES

A major aspect of the computer program for burst positions is the inclusion of a standard table of average indices of refraction versus geometric altitude, Z, for the reference latitude of 45°32'40".

The actual tabular values are the average index of refraction less one, $\bar{\mu}(Z)-1$, as given implicitly by Eq. (30). The density ratio $\rho(Z)/\rho_0$ versus altitude profile used to evaluate this integral expression is taken from the ARDC model atmosphere (Ref. 8, metric table II). The value used for the sea-level index of refraction less one, (μ_0-1), is 2.882×10^{-4}, and represents a recent experimental determination at microwave frequencies.[10]

The integration of (30) is performed in a stepwise fashion by assuming the atmosphere is isothermal over each tabular interval of one kilometer in the ARDC table. This is equivalent to assuming that the scale height, h, in Eq. (41) is fixed over each altitude interval, in which case the contribution to the integral is given by Eqs. (42) and (44) with the integration being performed between the altitudes defining each interval.

The actual computation was performed by a separate machine program, and the resulting values are listed in Appendix A for altitudes from sea level to 127 km.

4.2. DESCRIPTION OF COMPUTATION FOR POSITION

To compute the grenade burst position from the spin-corrected cycle-count data and the ballistic camera data, we proceed as follows:

1. Input:
   a. The spin-corrected count, N_1.
   b. The camera direction cosines $\vec{N}^{(j)}$ (j = 1, 2 for the east and west cameras, respectively).
2. Bring the last computed value of $\bar{\mu}_1(Z)$ and compute u/2 from Eq. (58a).
3. With the good approximation $R \cong u/2$ compute the altitude Z as in Eq. (9), i.e., $Z = Z_0 + N^{(j)}R$ where Z_0 is the altitude of the camera.
4. Compute Z'(Z) from Eq. (35).
5. Look up the tabular value $\bar{\mu}[Z'(Z)]-1$.
6. Compute and store a new value for $\bar{\mu}_1(Z)$ using Eqs. (46) and (31).
7. Recompute u/2 from Eq. (58a) with this improved value of $\bar{\mu}_1(Z)$.
8. Compute the "camera to burst" slant range, R, from Eq. (20b).
9. Compute an improved value for Z as in step (3).
10. Repeat (4) through (8).

11. Compute the burst position $\vec{X}$ via Eq. (9) (relative to the center of the geophone array).
12. Translate $\vec{X}$ to the system with the launcher as origin.
13. Repeat computation for the remaining camera.
14. Compute the average positions relative to both origins.
15. Compute the standard deviations.
16. Print out results and return to (1).

The manner in which these steps are accomplished in the machine program is depicted by the flow chart (Fig. 5), and the actual coding of the fixed point program in machine language is reproduced in Appendix B.

Existing LGP-30 library routines are used in conjunction with this program for both data input and output. Routine 11.4 is used to input tracking data. This routine converts nine decimal digits to 30 binary bits with a maximum error of one in the lowest-order bit. Results are printed out using data output routine 12.4. The print-out format consists of eight (or possibly nine) decimal digits with the proper decimal point location followed by a minus sign if the number is negative.

4.3. PROGRAM CONSTANTS

With the exception of the vacuum doppler wavelength, λ_0, all the program constants depend upon a range survey. Such a survey was conducted for the Guam range by the Navy Public Works Center, and the results of this survey are listed in Table I for all relevant points. The elevations listed at all the stations are referred to mean sea level and were obtained by measurements relative to a station whose altitude is known very closely by barometric readings made over a long period of time.

The surveyed point on the antennas is the center of the ground plane. The top of the mounting pedestal was surveyed for the ballistic camera positions, and the actual position of the optical center of the cameras is 1.52' higher than the survey point. The position of the launcher was taken as the center of the blackened circular area made by the rocket exhaust on the concrete runway. Since all the launching angles were close to the vertical, the initial position of the missile antennas on the centerline or axis of the rocket is assumed to be directly above this point. This vertical distance is three meters for the Aerobee 75 rockets and six meters for the Nike-Cajun rockets.

Since the position data are used in conjunction with the sound-ranging data from the geophone array to compute temperature and winds above the array, it is convenient to refer the burst positions to some point in the geophone array. This point is taken to be the intersection of the two diagonal lines defined by opposing pairs of the four outermost microphones, and for this reason we list their positions in Table I. Since these lines are skew, we make use of the generally known solution for the least-squares intersection of two skew lines

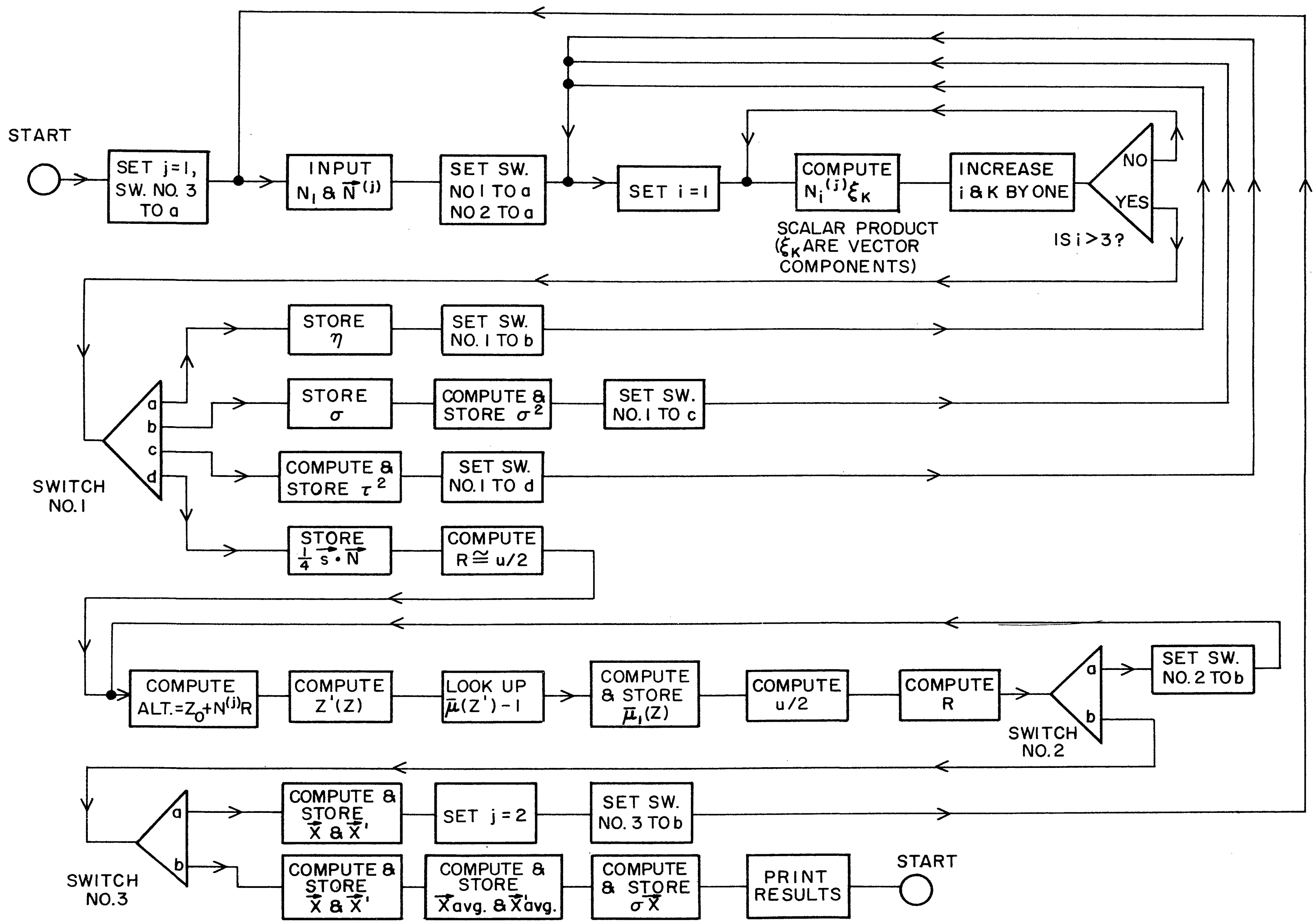

Fig. 5. BC/DOVAP program flow chart.

TABLE I

GUAM RANGE SURVEY DATA

Station	Elevation	Rectangular Coordinates		Geographical Coordinates	
		East	North	Longitude	Latitude
1*	488.21'	199369.85'	214216.23'	144°50'54"	13°36'38"
2*	488.31'	199365.06'	214213.21'	144°50'54"	13°36'38"
3	465.78'	201985.65'	214443.70'	144°51'20"	13°36'41"
4	484.34'	199219.42'	214213.40'	144°50'52"	13°36'38"
5	483.20'	199286.71'	214247.17'	144°50'53"	13°36'39"
6**	475.55'	199770.96'	214510.34'	144°50'58"	13°36'41"
7	491.40'	199472.34'	213531.51'	144°50'55"	13°36'32"
8	499.44'	196826.95'	214452.83'	144°50'28"	13°36'41"
9	519.92'	197898.62'	217195.72'	144°50'39"	13°37'08"
10	461.76'	200238.21'	216367.57'	144°51'03"	13°37'00"

1	f:11, east	Ballistic cameras
2	f:16, west	
3	Transmitting helical antenna	
4	Left-hand	Helical receiving antennas
5	Right-hand	
6	Launcher	
7	No. 1	Microphones (geophone array)
8	No. 2	
9	No. 3	
10	No. 4	

*Add 1.52' to elevation to obtain position of optical center.
**Add 9.84' for Aerobee 75, and 19.68' for Cajun, to obtain antenna ℄.

(identical in form to the solution for position from two fixed cameras). Let $\vec{X}^{(j)}$ $(j = 1, \ldots, 4)$ be the position vectors of the geophones, with the index counting clockwise from microphone 1.

Define

$$\begin{aligned} \vec{X}_{1,2} &= \vec{X}^{(2)} - \vec{X}^{(1)} \\ \vec{X}_{1,3} &= \vec{X}^{(3)} - \vec{X}^{(1)} \\ \vec{X}_{2,4} &= \vec{X}^{(4)} - \vec{X}^{(2)} \end{aligned} \tag{61}$$

and

$$p_1 = \frac{(\vec{X}_{1,2} \times \vec{X}_{2,4}) \cdot (\vec{X}_{1,3} \times \vec{X}_{2,4})}{|(\vec{X}_{1,3} \times \vec{X}_{2,4})|^2}$$

$$p_2 = \frac{(\vec{X}_{1,2} \times \vec{X}_{1,3}) \cdot (\vec{X}_{1,3} \times \vec{X}_{2,4})}{|(\vec{X}_{1,3} \times \vec{X}_{2,4})|^2} \tag{62}$$

then the point of closest approach on each line is

$$\vec{X}_1^* = p_1\vec{X}^{(3)} + (1 - p_1)\vec{X}^{(1)}$$

$$\vec{X}_2^* = p_2\vec{X}^{(4)} + (1 - p_2)\vec{X}^{(2)} \tag{63}$$

and the least-square solution for the point of intersection is

$$\vec{X}^* = \frac{1}{2} (\vec{X}_1^* + \vec{X}_2^*) \quad . \tag{64}$$

Using the positions from Table I and Eqs. (61) through (64), we have the result

$$\vec{X}^* = (198639.59', 215470.08', 492.96') \quad ,$$

which we will refer to as the center of the geophone array.

Let us establish a right-hand cartesian system by orienting the x-axis eastward, the y-axis northward, and the z-axis vertically. Then with the survey data from Table I and the relations developed in Section 3, we can evaluate the range parameters necessary for the machine program. These parameters, together with a reference to the equation used to compute them, are listed in Table II. The quantities have initially been computed in feet and then converted to kilometers using the conversion factors:

$$1 \text{ foot} = 3.04800610 \times 10^{-4} \text{ kilometer}$$

$$1 \text{ foot}^2 = 9.29034116 \times 10^{-8} \text{ kilometer}^2$$

In these computations the position of the ballistic cameras is taken as the position of the optical centers, and the initial missile position is taken to be the location of the antennas along the missile axis. The positions of the ballistic cameras, $\vec{X}_1^{(j)}$, are referred to the center of the geophone array so that if $\vec{X}_0^{(j)}$ are the survey coordinates of the cameras, then

$$\vec{X}_1^{(j)} = \vec{X}_0^{(j)} - \vec{X}^* \tag{65}$$

TABLE II

PROGRAM CONSTANTS FOR GUAM RANGE

		Vector Quantities (km)		
		X	Y	Z
$\vec{i}$	Eq. (1)*	-0.996530013	-0.082965213	0.006686211
$\vec{X}_1'^{(1)}$	Eqs. (4), (21)	-0.375725	-0.034235	0.004471
$\vec{X}_1'^{(2)}$		-0.377185	-0.035156	0.004502
$\vec{\nu}^{(1)}$	3rd Eq. of (22)	0.000142	-0.001944	-0.002944
$\vec{\nu}^{(2)}$		0.000138	-0.001964	-0.003741
$\vec{s}$	Eq. (60)	0.020510	0.010293	-0.000347
$\vec{X}_1^{(1)}$	Eq. (65)	0.222584	-0.382174	-0.000985
$\vec{X}_1^{(2)}$		0.221124	-0.383095	-0.000954
$\vec{\Delta X}$	Eq. (66)	-0.344842	0.292529	0.005307

		Scalar Quantities	
c^2	Eq. (2)	0.178964742	(km^2)
$\beta^{(1)}$	1st Eq. of (22)	0.377291	(km)
$\beta^{(2)}$		0.378823	(km)
$\delta^{(1)}$	2nd Eq. of (22)	0.142361066	(km^2)
$\delta^{(2)}$		0.143524457	(km^2)
Z_0	See text	0.149285	(km)
Z_1		0.146581	(km)
(g_0/g_0^*)		0.997609581	
(ϵ'/r)	Eqs. (33), (36), (45)	$0.85818269 \times 10^{-6}$	(km^{-1})
$[\bar{\mu}(Z_1)-1]$	Eq. (35) and Appendix A	0.00028622	
$U_0/2$	Eq. (59)	0.439007	(km) (Aerobee 75)
		0.439035	(km) (Cajun)

* $\vec{i}$ is dimensionless.

The vector $\vec{\Delta X}$ is the translation which transforms a position vector relative to the center of the geophone array to a position relative to the launcher. Thus if $\vec{X}_L$ is the survey position of the launcher, we define

$$\vec{\Delta X} = \vec{X}^* - \vec{X}_L \quad . \tag{66}$$

Z_o is the average altitude of both ballistic cameras used to compute altitudes above sea level for interpolation of $\overline{\mu}_1(Z)$, as in steps (3) and (9), Section 4.2. Z_1 is the lower limit of the integral expression for $\overline{\mu}_1(Z)$ and is obtained by taking the average of the mean sea-level altitudes of all the DOVAP antennas at lift-off, i.e., the average altitude of all the ground antennas taken together with the mean position of the missile antennas at lift-off. The quantity $\overline{\mu}(Z_1)-1$, used in expression (31) to compute $\overline{\mu}_1(Z)$, is also listed as a constant.

The values for g* and r* were taken from Ref. 8 and are identical to those used for the computation in Section 3.2. The sea-level acceleration due to gravity, g, at Guam was computed from the expression for the latitude variation of gravitational acceleration which appears in the Smithsonian Physical Tables:

$$g = 9.806160\ (1 - .0026373 \cos 2\phi + .0000059 \cos^2 2\phi)\ (\text{m sec}^{-2}) \quad .$$

At Guam we take $\phi = 13°36'38''$, in which case

$$g = 9.783208\ (\text{m sec}^{-2}) \quad .$$

In addition to the constants which appear in Table II, we also require a value of the vacuum wavelength λ_o for each missile flight. Since the reference frequency varies by less than 1 part in 10^7 during each flight, we can for all practical purposes assume it is a constant. Let f_{avg} be the reference frequency averaged over the period of the flight; then

$$\lambda_o = \frac{C_o}{2f_{avg}} \quad , \tag{67}$$

where the value for the vacuum light velocity is taken to be that given by Dumond and Cohen, i.e., $C_o = 299792.9 \pm 0.8$ km sec^{-1}. Table III lists the reference frequency readings logged from the frequency monitor during each of the flights at Guam, and Table IV lists the corresponding frequency averages and the values for $\lambda_o/2$.

TABLE III

REFERENCE FREQUENCY LOG FOR GUAM FLIGHTS (Mc)

X-Time	SS 12.50	SS 12.51	SS 6.52	SS 6.53	SS 6.54
0	36.939918	---	---	36.939725	---
+ 30	" 18	36.939764	36.939774	" 26	36.939711
+ 60	" 18	" 64	---	" 26	" 11
+ 90	" 19	" 65	36.939775	" 25	" 10
+120	" 19	" 65	" 74	" 26	" 10

X-Time	SS 6.55	SS 6.56	SS 12.57	SS 6.58
0	36.938531	36.939732	36.938542	36.938968
+ 30	" 29	" 33	" 41	" 68
+ 60	" 30	" 34	" 41	" 69
+ 90	" 30	" 33	" 42	" 68
+120	" 31	" 32	" 42	" 69

Note: Rockets numbered with 12 prefix are Aerobee 75's; others are Cajuns.

TABLE IV

AVERAGE FREQUENCY AND VACUUM WAVELENGTH
FOR GUAM FLIGHTS

Rocket	f_{avg} (Mc)	$\lambda_o/2$ (10^{-3} km)
SS 12.50	36.939918	2.02892234
SS 12.51	36.939764	2.02893080
SS 6.52	36.939774	2.02893025
SS 6.53	36.939726	2.02893289
SS 6.54	36.939711	2.02893371
SS 6.55	36.938530	2.02899858
SS 6.56	36.939733	2.02893250
SS 12.57	36.938542	2.02899792
SS 6.58	36.938968	2.02897452

5. TRACKING DATA

5.1. BALLISTIC CAMERA DATA

Reference 3 contains:

1. A description of the measurement of the Ballistic Camera plate coordinates of the grenade bursts and star calibration images.
2. A description of the method used to compute direction cosines from the plate coordinate data.
3. The resulting direction cosines of the grenade bursts, corrected for refraction.

The direction cosines in this report are listed, respectively, as:

cos E sin A,   cos E cos A, and   sin E.

Although no explicit identification is made for the angles A and E in Ref. 3, it has been established that E is the elevation angle, and A is the azimuth angle measured clockwise, from north, so that with the coordinate system we have defined at Guam we can identify the components of $\vec{N}$ as follows:

$$L = \cos E \sin A$$
$$M = \cos E \cos A$$
$$N = \sin E$$

It will be noted that direction cosines are listed for Cajun ignition in all the Nike-Cajun flights. It was discovered not only that the images of the ignition flash are well defined on the plates, but also that the sound returns from the ignition are of good quality. For this reason a position was obtained for each of the ignition points.

The direction cosines published in this report for rocket SS 6.54 are incorrectly identified and the designations "east" and "west" should be interchanged.

Because the west plate of SS 6.56 had fewer than six stars from which a plate calibration could be made using the Herget solution, no results for this plate appear in the report. Subsequently, however, a reduction of the west plate of SS 6.56 has been made, using a solution for which a plate calibration is defined by three constants for each standard coordinate. The results of this reduction were made available for computation of burst positions in a separate correspondence.

5.2. DOVAP DATA

For each of the missile flights the recordings of the doppler signal for both helical antennas along with the time reference signal were transcribed from the magnetic tape onto 35-mm film. These transcriptions were made by the Ballistic Research Laboratories, Aberdeen, Maryland, and are of excellent quality.

On the film the doppler signals appear as half of an amplitude-modulated envelope while the time reference pulses are recorded both as y- and z-axis deflections, i.e., as sharp spikes and dots. The time signal consists of two distinct pulses with repetition rates of 2 per second and 100 per second, respectively.

To establish grenade burst times, signals from two ground-based flash detectors were recorded, along with the time reference signals and the lift-off pulse, on a Consolidated Electrodynamics recorder (not shown in Fig. 1). The times of these events, which were read separately and recorded, serve to define the intervals over which the corresponding doppler cycle counts are to be made. Whenever possible, the event times are established by observing on the film transcription the modulation of the doppler signal caused by the grenade fireball, and the times read from the C.E.C. record are considered as supporting or back-up information.

All the cycle counts are made by hand on a light-box or film reader using a pair of dividers set at an interval corresponding to some convenient multiple of cycles, usually 5, 10, or 20. A detailed description of this counting process is given in Ref. 9, Section IV.

The cycle counts start from lift-off and the total counts are interpolated to the nearest hundreth of a cycle at each grenade burst time, including, in addition, ignition time for the Cajuns. Counts are also interpolated at each half-second to provide sufficient time resolution for the analysis of phase errors. Every record is read at least twice and the counts are then compared to detect reading errors, whereupon the two sets of readings are averaged.

Each successive pair of average half-second counts are then differenced independently for each receiver, and the resulting difference, ΔN, is plotted against time. Similarly, the pair of half-second counts corresponding to the left-hand and right-hand receivers are differenced, Eq. (47), to obtain N'. These values of N' are then corrected for the phase change due to the separation of the antenna phase centers by evaluating expression (49) in terms of the approximate trajectory given by the rough counts and the direction cosines of the lowest grenade burst. The corrected values of N', given by (54), are also plotted versus time.

On this same plot the times of each field strength maximum versus the total number of maxima accumulated to that time are plotted to the same scale. Because N_s' normally accumulates two cycles for each missile rotation while two field-strength maxima occur every rotation, the two plots should differ at any time by no more than one cycle. Any discrepancy which does occur (care being taken to account for change of rotation which happens often upon separation of a two-stage missile) indicates the presence of anomalous phase shifts at the doubled reference frequency. Anomalous phase shifts at the reference frequency, on the other hand, are evidenced by discontinuities in the slope of the ΔN plot. Consequently, careful examination of both plots will indicate where normal and anomalous spin corrections are to be applied. The method by which these corrections are applied are described fully in Ref. 9, Section VI. The final spin-corrected cycle counts are listed in Appendix C for each of the flights.

6. COMPUTED BURST POSITIONS

The grenade burst positions computed from the ballistic camera data and the cycle-count data by the computer program appear in Appendix D.

The point identified by A* is the Cajun ignition point.

Positions are listed with the center of the geophone array as origin, and with the survey position of the launcher as origin. These results are given in kilometers with the least significant figure corresponding to millimeters.

For each grenade, the east camera positions are printed on the first line of the group, and the west camera positions are printed on the second line with the average of these two positions printed on the third line. The corresponding standard deviations are printed on the third line in columns six through nine.

7. ERRORS

7.1. RESOLUTION OF HORIZONTAL AND VERTICAL COORDINATE ERRORS

Let $\partial\vec{X}/\partial\vec{N}$ mean the limit of the ratio of the change in $\vec{X}$ to the magnitude of an infinitesimal vector displacement $\Delta\vec{N}$ of $\vec{N}$. Since $\vec{N}$ is a unit vector, any infinitesimal change $\Delta\vec{N}$ must be at right angles to $\vec{N}$, and we can set $\Delta\vec{N} = \vec{K}\epsilon$ where $\vec{K}$ is a unit vector perpendicular to $\vec{N}$ and ϵ is some small scalar. By Eq. (9) $\Delta\vec{X} = R\Delta\vec{N}$, so that

$$\frac{\partial\vec{X}}{\partial\vec{N}} = \lim_{|\Delta\vec{N}|\to 0} \frac{\Delta\vec{X}}{|\Delta\vec{N}|}$$

$$= \lim_{\epsilon\to 0} \frac{R\vec{K}\epsilon}{\epsilon} = R\vec{K} \quad . \tag{68}$$

Furthermore,

$$\frac{\partial\vec{X}}{\partial u} = \frac{\partial\vec{X}}{\partial\vec{N}} \cdot \frac{\partial\vec{N}}{\partial u} + \frac{\partial\vec{X}}{\partial R} \cdot \frac{\partial R}{\partial u} \quad .$$

Now $\vec{N}$ and u are independent so that $\partial\vec{N}/\partial u = 0$, and if we set $u \cong 2R$, we have $\partial R/\partial u = 1/2$ and as a result

$$\frac{\partial\vec{X}}{\partial u} = \frac{\partial\vec{X}}{\partial R} \cdot \frac{1}{2}$$

$$= \frac{1}{2}\vec{N} \quad . \tag{69}$$

$\vec{N}$ is always close to the vertical so that relation (69) implies

$$\frac{\partial X}{\partial u} \cong 0 \; , \qquad \frac{\partial Y}{\partial u} \cong 0$$

and (70)

$$\frac{\partial Z}{\partial u} \cong \frac{1}{2} \quad .$$

Similarly, since $\vec{K} \perp \vec{N}$, $\vec{K}$ will be nearly horizontal in which case $\vec{K} = (\cos\alpha, \sin\alpha, 0)$, where α is the angle made by the direction of $\Delta\vec{N}$ and the X-axis. Thus (68) implies

$$\frac{\partial X}{\partial \vec{N}} \cong R \cos \alpha , \qquad \frac{\partial Y}{\partial \vec{N}} \cong R \sin \alpha$$

and (71)

$$\frac{\partial Z}{\partial \vec{N}} \cong 0 .$$

Thus we conclude that errors in altitude are primarily due to errors in the u-values obtained from DOVAP data, whereas errors in the horizontal coordinates arise mainly from errors in the direction cosines given by the ballistic camera data.

Survey errors will propagate errors into both the slant range, R, and the direction cosines, $\vec{N}$, in addition to affecting the position of the ballistic camera, $\vec{X}_1$, directly. For the type of survey conducted at Guam, one might reasonably expect an accuracy of one part in 5000, and since the maximum interval in the triangulation net containing all the relevant points is of the order of a kilometer, the maximum error in position due to survey errors would be 20 centimeters.

7.2. DOVAP ERRORS

As we have already indicated, errors in the slant range, R, are due almost entirely to errors in u while errors in the survey, not affecting u directly, and errors in the direction cosines both have only a second-order effect on R.

The value of u depends in turn on (Section 3.4):

(1) the initial value u_o of u determined by survey data;
(2) the spin-corrected doppler count; and
(3) the average doppler wavelength, $\bar{\lambda}(Z) = \lambda_o/\bar{\mu}_1(Z)$.

With regard to item (1), an assumed survey accuracy of one part in 5000 would result in an error no greater than 20 cm in u since u_o is less than a kilometer.

Item (2) represents the most serious source of error affecting the u-value. An error of one cycle, for example, would produce an error of one doppler wavelength, or 4 meters, in u.

Experience has shown that it is extremely improbable that counting errors involving an integral number of cycles will remain undetected. In every case at least two independent readings of the film are compared and rechecked until the agreement is within a tenth of a cycle. Consequently, it is felt that systematic counting errors can be disregarded, and that random errors are less than a quarter cycle.

An important error source affecting the doppler counts results from the phase shifts due to missile spin. Extreme care must be exercised in the examination of the ΔN and N' plots versus time, and an occasional inspection of the film transcription is necessary, to insure that an accurate determination of the phase corrections is made. In particular, anomalous phase shifts on a single antenna at 74 Mc, or anomalous phase shifts at 37 Mc, are often difficult to interpret. Because of such difficulties it is not unlikely to accumulate a total error of the order of a cycle in correcting for spin phase shifts. In addition to this systematic error, a random error of the order of a half cycle can be introduced into the data by both the normal and anomalous spin corrections, although the methods outlined in Ref. 9, Section VI, tend to minimize this error so that they are generally substantially smaller.

If we take into account both counting errors and errors resulting from spin, the total estimated error in the doppler count will be a systematic error of a cycle and a random error of three quarters of a cycle. The corresponding error in u will be a systematic error of the order of 4 meters and a random error of the order of 3 meters.

Wavelength errors, item (3), arise from errors in the quantities:

(a) the vacuum propagation velocity C_6;
(b) the sea-level index of refraction μ_0; and
(c) the average reference frequency f_{avg}.

The uncertainties in the vacuum propagation velocity and the sea-level index of refraction (or equivalently the propagation velocity at sea-level air density) are of the same order, which is about one part in 10^5.

The corresponding error in u would then be about 2 meters for an altitude of 100 km.

Uncertainty in the average reference frequency should be less than one part in 10^7 so that errors arising from this source are negligible.

One might consider at this point the possibility that the solution for the burst position has implicit assumptions which are as yet unjustified, and, as a result, the computation would be in error.

One assumption that has been tacitly made is that propagation is straight-line. A discussion of the errors incurred by this assumption is given in Ref. 7, Appendix B, where it is concluded that they are completely negligible.

It has also been assumed that the point in space to which the doppler count defines a u-value is coincident with the grenade burst. To be precise, this point is the common phase center of the missile antennas and is separated from the grenade bursts by an interval of approximately 8 meters. As a consequence, the slant range R can be in error by an amount which is less than or equal in absolute value to 8 meters. The actual magnitude of this error can be estimated only if the aspect of the missile is known. By examining spin phase shifts we can usually determine the aspect sufficiently to establish the sense of this error. In all the Guam flights, the true slant range for each grenade burst should be greater than the computed value by an amount not exceeding 8 meters. Excluding precessional effects, the magnitude of this error probably varies but little during each individual flight because of spin stability.

Excluding this last error, let us summarize the errors affecting u as follows:

(1) A systematic error of 20 cm caused by survey errors;
(2) A systematic error of 4 meters and a random error of 3 meters due to count errors;
(3) A systematic error of as much as 4 meters at the highest altitude due to errors in the doppler wavelength arising from uncertainties in the propagation velocity and refractive index.

Assuming each of these error sources operate independently, the total error in the slant range, R(=u/2), will be a systematic error of 4 meters and a random error of 1-1/2 meters.

7.3. BALLISTIC CAMERA ERRORS

Systematic errors in a plate calibration can arise from uncertainties in the local sidereal time and the elevation angle of the celestial pole. Universal time, on the other hand, can be established very accurately so that most of the uncertainties in these quantities are due to inaccuracies in the geographical position of the camera.

It is felt that the latitudes at Guam are known with good accuracy while the error in longitudes, though larger, can be no more than 30" of arc. This latter error implies a possible systematic east-west error of this magnitude in the direction cosines.

As a check on the accuracy of the plate calibration, the plate constants were used to compute positions of each of the calibration stars. These computed positions were then compared to the positions originally obtained from the star catalog, and the resulting residuals provide an estimate of the accuracy of the calibration (Ref. 3, p. 2). The maximum residual for all the plates, with the exception of SS 6.56 west, was 17.415" of arc, while the average was much less. Assuming that the calibration stars were distributed uniformly over each plate, it would be reasonable to conclude that the residuals for the grenade burst positions show the same statistical distribution as the residuals for the calibration stars. Consequently, the total systematic and random error in the direction cosines of the grenade bursts, due to calibration errors, should average much less than about 20" of arc. Since 20" of arc is equal to 9.7×10^{-5} radians, this would imply an error of less than 10 meters in the horizontal coordinates at altitudes of 100 km or less.

In general, the east plate calibration will be independent of the west plate calibration because of differences in lens distortion, camera orientation, and the selection of calibration stars. Thus the burst positions determined from the east and west plate reductions represent two relatively independent estimates of the same quantity, and the residuals of the burst coordinates will be indicative of the average error of each estimate.

A plot of the residuals of the horizontal coordinates of the grenade bursts appears in Appendix E for each of the Guam flights. Since the corresponding residuals for the two cameras are necessarily equal in magnitude and opposite in sign, only the east camera residuals have been plotted.

The angular error in radians associated with each of the residuals is simply the ratio of each residual to the altitude of the burst. Upon examination of these residuals we discover that, with the exception of the residuals for S.S. 6.56, the corresponding angular errors are all less than 20" of arc with the average considerably less. This corroborates our previous conclusions. The calibration for the west plate of S.S. 6.56 was obtained by a more simplified method involving fewer plate constants, and, as one would expect, the residuals are noticeably larger. Despite this fact, the corresponding angular errors are at worst very slightly greater than 20" of arc.

Examination of the residuals for all the flights suggests that there is a small systematic difference between the positions given by the east and west plates. In almost every case the position given by the east plate lies roughly in the northwest to northeast quadrant relative to the position given by the west plate.

To summarize, the ballistic camera direction errors are primarily:

(1) A systematic east-west error of 30" of arc or less;
(2) A random error of the order of 20" of arc or less;
(3) A small systematic north-south difference between the two cameras.

8. CONCLUSION

The ballistic camera—single-station DOVAP tracking system makes the best use of camera and doppler information since positions are determined by a near-normal intersection of a ray and an ellipsoidal surface. This is contrasted to an all-camera or all-DOVAP system where positions are determined, respectively, by the intersection of two rays or three surfaces. In both cases the included angles between the intersecting geometric elements are small, making the position sensitive to error.

The experience gained with this system leads us to conclude the following:

(a) As in any ballistic camera system, care must be exercised in making the plate exposures to insure that images of the grenade bursts are small and distinct, and to provide a sufficiently large number of identifiable stars on the pre-shoot and post-shoot exposures.

(b) A check on doppler errors should be provided by back-up or supporting data. This might be accomplished by providing an additional ground receiver with an antenna placed some distance from the other receiving antennas.

(c) With regard to grenade experiment, some method should be devised whereby a correction can be made for the uncertainty in the positions resulting from the separation of the bursts and the missile antenna phase center.

Considering (b), one might say that any modification of the DOVAP scheme which would serve to reduce systematic count errors due to spin or provide a check on such errors would be desirable. This, of course, can also be said of an all-DOVAP system.

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APPENDIX A

AVERAGE REFRACTIVE INDICES

TABLE OF AVERAGE REFRACTIVE INDICES

Altitude (km)	μ-1	Altitude (km)	μ-1	Altitude (km)	μ-1	Altitude (km)	μ-1
0	.28820 (10^{-3})	32	.07545 (10^{-3})	64	.03806 (10^{-3})	96	.02537 (10^{-3})
1	.27465	33	.07325	65	.03747	97	.02511
2	.26181	34	.07117	66	.03690	98	.02486
3	.24959	35	.06919	67	.03635	99	.02461
4	.23798	36	.06732	68	.03582	100	.02436
5	.22695	37	.06554	69	.03530	101	.02412
6	.21648	38	.06386	70	.03480	102	.02388
7	.20654	39	.06225	71	.03431	103	.02365
8	.19711	40	.06072	72	.03383	104	.02342
9	.18817	41	.05926	73	.03337	105	.02320
10	.17970	42	.05786	74	.03292	106	.02298
11	.17168	43	.05653	75	.03248	107	.02277
12	.16399	44	.05526	76	.03205	108	.02256
13	.15660	45	.05405	77	.03164	109	.02235
14	.14956	46	.05288	78	.03123	110	.02215
15	.14290	47	.05176	79	.03083	111	.02195
16	.13662	48	.05069	80	.03045	112	.02175
17	.13071	49	.04966	81	.03007	113	.02156
18	.12517	50	.04868	82	.02971	114	.02137
19	.11998	51	.04773	83	.02935	115	.02118
20	.11511	52	.04681	84	.02900	116	.02100
21	.11055	53	.04593	85	.02866	117	.02082
22	.10628	54	.04509	86	.02833	118	.02064
23	.10227	55	.04427	87	.02800	119	.02047
24	.09852	56	.04348	88	.02768	120	.02030
25	.09499	57	.04272	89	.02737	121	.02013
26	.09167	58	.04199	90	.02707	122	.01997
27	.08855	59	.04128	91	.02677	123	.01980
28	.08562	60	.04059	92	.02648	124	.01964
29	.08285	61	.03992	93	.02619	125	.01949
30	.08024	62	.03928	94	.02591	126	.01933
31	.07778	63	.03866	95	.02564	127	.01918

APPENDIX B

BC/DOVAP POSITION PROGRAM CODING SHEETS

LGP-30 CODING SHEET

PREPARED FOR: High Altitude Laboratory			PAGE 1 OF 9
JOB NO.	PROGRAM NO.	PROGRAM PREPARED BY: M. G. Whybra	PROGRAM CHECKED BY:
DATE			
PROBLEM: Ballistic Camera/Dovap Position			TRACK

PROGRAM INPUT CODES	STOP	LOCATION	OPERATION	ADDRESS	STOP	CONTENTS OF ADDRESS	NOTES
;0000500'	'						
/0000500'	'	☒					
		0000		b0403'	'	z0504	loc. of i_1
		01		y0018'	'		
		02		b0404'	'	z0539	loc. of δ
		03		y0137'	'	☒	
		04		b0405'	'	z0537	loc. of β
		05		y0204'	'		
		06		b0406'	'	z0215	
		07		y0214'	'	☒	
		08		r1008'	'		} Data Input
		09		u1000'	'		# 11.4
		10		b0407'	'	z0032	
		11		y0031'	'	☒	
		12		b0408'	'	z0209	
		13		y0208'	'		
		14		b0409'	'	z0501	loc. of L
		15		y0019'	'	☒	
		16		xc6300'	'		
		17		xc6300'	'		0 → 6300
		18		b0000'	'		[0504] etc.
		19		m0000'	'	☒	[0501] etc.
		20		xa6300'	'		
		21		xc6300'	'		
		22		b0018'	'		
		23		a0416'	'	☒ 1 @ 29	
		24		y0018'	'		
		25		b0019'	'		
		26		a0416'	'	1 @ 29	
		27		y0019'	'	☒	
		28		s0411'	'	m0504	
		29		t0018'	'		
		30		xb6300'	'		
		31		u0000'	'	☒	[0032] etc.

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☒ CARRIAGE RETURN

' = CONDITIONAL STOP CODE

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LGP-30 CODING SHEET

PREPARED FOR:				PAGE OF 2/9
JOB NO.	PROGRAM NO.	PROGRAM PREPARED BY:	PROGRAM CHECKED BY:	DATE
PROBLEM:				TRACK

PROGRAM INPUT CODES	STOP	LOCATION	INSTRUCTION OPERATION ADDRESS	STOP	CONTENTS OF ADDRESS	NOTES
	'					
	'	☒				
		0032	d0413'	'	1 @ 2	
		33	xc6305'	'		η @ 0
		34	r0031'	'		
		35	u0014'	'	☒	
		36	xh6303'	'		σ @ 8
		37	d0412'	'	1 @ 1	
		38	xm6303'	'	σ @ 8	
		39	xc6304'	'	☒	σ^2 @ 15
		40	r0031'	'		
		41	u0014'	'		
		42	d0412'	'	1 @ 1	
		43	xm6300'	'	☒ τ @ 7	
		44	xc6302'	'		τ^2 @ 15
		45	r0031'	'		
		46	u0014'	'		
		47	xc6312'	'	☒	(1/4)s . $\vec{N}$ @ 7
		48	b0500'	'	N_1 @ 16	
		49	m0529'	'	$\lambda_0/2$ @ -8	
		50	a0528'	'	$U_0/2$ @ 8	
		51	xh6310'	'	☒	
		52	d0533'	'	π_1 (Z)	@ 1
		53	xs6312'	'	(1/4)s . N @ 7	u/2 @ 7
		54	m0503'	'	N @ 1	
		55	d0412'	'	☒ 1 @ 1	
		56	a0530'	'	Z_0 @ 7	
		57	xh6308'	'		Z @ 7
		58	m0535'	'	ϵ'/r @ -6	
		59	a0412'	'	☒ 1 @ 1	
		60	xc6311'	'		1 + (ϵ'/r)Z @ 1
		61	b0531'	'	Z_1 @ 7	
		62	xd6308'	'	Z @ 7	
		63	xh6314'	'	☒	Z_1/Z @ 0

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PREPARED FOR:				PAGE OF 3/9
JOB NO.	PROGRAM NO.	PROGRAM PREPARED BY:	PROGRAM CHECKED BY:	DATE
PROBLEM:				TRACK

PROGRAM INPUT CODES	STOP	LOCATION	INSTRUCTION OPERATION ADDRESS	STOP	CONTENTS OF ADDRESS	NOTES
	/					
	/	☒				
		0100	m0532'	/	$\bar{\mu}(Z_1)-1$	@ 1
		01	xc6306'	/		$(Z_1/Z)(\bar{\mu}(Z_1)-1)$ @ 1
		02	b0412'	/	1/2 @ 0	
		03	xs6314'	/	☒ Z_1/Z	@ 0
		04	a0412'	/	1/2 @ 0	
		05	xc6307'	/		$1 - Z_1/Z$ @ 0
		06	xb6308'	/	Z @ 7	
		07	m0534'	/	☒ (g_0/g_0^*)	@ 1
		08	xd6311'	/		
		09	xh6301'	/		Z' @ 7
		10	m0415'	/	1 @ 22	Z' @ 29
		11	a0410'	/	☒ z 0800	
		12	y0121'	/		
		13	a0416'	/	1 @ 29	
		14	y0124'	/		
		15	xb6301'	/	☒ Z' @ 7	
		16	e0417'	/	WWWWWWW'	extract fraction
		17	d0414'	/	1 @ 6	
		18	xc6309'	/		fraction @ 1
		19	xs6309'	/	☒	
		20	a0412'	/	1 @ 1	
		21	m0000'	/		
		22	xc6308'	/		
		23	xb6309'	/	☒	
		24	m0000'	/		
		25	xa6308'	/		$\bar{\mu}(Z') - 1$ @ 2
		26	xd6311'	/	$1 + (\epsilon'/r)Z$	@ 1
		27	xs6306'	/	☒	
		28	xd6307'	/	$1 - Z_1/Z$	@ 0
		29	a0412'	/	1 @ 1	
		30	c0533'	/		$\bar{\mu}_1(Z)$ @ 1
		31	xb6310'	/	☒	

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PREPARED FOR:				PAGE OF 4/9
JOB NO.	PROGRAM NO.	PROGRAM PREPARED BY:	PROGRAM CHECKED BY:	DATE
PROBLEM:				TRACK

PROGRAM INPUT CODES	STOP	LOCATION	INSTRUCTION OPERATION	INSTRUCTION ADDRESS	STOP	CONTENTS OF ADDRESS	NOTES
	'						
	'						
		0132		d0533'	'	$\bar{\mu}_1(Z)$ @ 1	
		33		xs6312'	'	$s \cdot \vec{N}/4$ @ 7	
		34		h0550'	'		$u/2$ @ 7
		35		m0550'	'		
		36		xc6308'	'		$u^2/4$ @ 14
		37		b0000'	'	δ @ 15	[0539],[0540]
		38		xs6304'	'	σ^2 @ 15	
		39		xd6308'	'	$u^2/4$ @ 14	
		40		xc6311'	'		$(4/u^2)(\delta - \sigma^2)$ @ 1
		41		xb6302'	'	τ^2 @ 15	
		42		xd6308'	'	$u^2/4$ @ 14	
		43		xc6301'	'		$(4/u^2)\tau^2$ @ 1
		44		b0536'	'	c^2 @ 15	
		45		xd6308'	'	$u^2/4$ @ 14	
		46		xh6306'	'		$(4/u^2)c^2$ @ 1
		47		xm6305'	'	η @ 0	
		48		xh6313'	'		$(4/u^2)c^2\eta$ @ 1
		49		xm6305'	'	η @ 0	
		50		xc6308'	'		$(4/u^2)c^2\eta^2$ @ 1
		51		xb6306'	'		
		52		xm6301'	'		
		53		xc6318'	'		$(4/u^2)c^2\ (4/u^2)\tau^2$ @ 2
		54		b0412'	'	1 @ 1	
		55		xs6306'	'		
		56		xc6314'	'		$1-(4/u^2)c^2$ @ 1
		57		b0412'	'	1 @ 1	
		58		xs6308'	'		
		59		xh6317'	'		$1-(4/u^2)c^2\eta^2$ @ 1
		60		xs6311'	'		
		61		xm6314'	'		
		62		xs6318'	'		
		63		r1450'	'		

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☒ CARRIAGE RETURN

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PROBLEM:	TRACK

PROGRAM INPUT CODES	STOP	LOCATION	INSTRUCTION (OPERATION / ADDRESS)	STOP	CONTENTS OF ADDRESS	NOTES
	'					
	'	⊠				
		0200	u1400'	'	enter sq. root, #15.0	
		01	m0550'	'	u/2 @ 7	
		02	xc6321'	'		$u/2\sqrt{\quad}$ @ 8
		03	xb6313'	'	⊠ $\frac{4}{u^2}c^2\eta$	@ 1
		04	m0000'	'	β @ 7	[0537] , [0538}
		05	xs6303'	'	σ @ 8	
		06	xa6321'	'		
		07	xd6317'	'	⊠	R @ 7
		08	u0000'	'		[0209] , [0211]
		09	r0208'	'		
		10	u0054'	'		
		11	h0551'	'	⊠	R @ 7
		12	m0501'	'	L @ 1	
		13	d0412'	'	1 @ 1	
		14	u0000'	'		[0215]
		15	a0541'	'	⊠ $X_1^{(1)}$ @ 7	
		16	h0552'	'		X @ 7
		17	a0547'	'	ΔX @ 7	
		18	c0555'	'		X' @ 7
		19	b0551'	'	⊠ R @ 7	
		20	m0502'	'	M @ 1	
		21	d0412'	'	1 @ 1	
		22	a0542'	'	$Y_1^{(1)}$ @ 7	
		23	h0553'	'	⊠	Y @ 7
		24	a0548'	'	ΔY @ 7	
		25	c0556'	'		Y' @ 7
		26	b0551'	'	R @ 7	
		27	m0503'	'	⊠ N @ 1	
		28	d0412'	'	1 @ 1	
		29	a0543'	'	$Z_1^{(1)}$ @ 7	
		30	h0554'	'		Z @ 7
		31	a0549'	'	⊠ ΔZ @ 7	

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PROBLEM:				TRACK

PROGRAM INPUT CODES	STOP	LOCATION	INSTRUCTION OPERATION / ADDRESS	STOP	CONTENTS OF ADDRESS	NOTES
	'					
	'					
		0232	c0557'	'		Z' @ 7
		33	b0137'	'	b 0539	
		34	a0416'	'	1 @ 29	
		35	y0137'	'		b 0540
		36	b0204'	'	m 0537	
		37	a0416'	'	1 @ 29	
		38	y0204'	'		m 0538
		39	r0214'	'		
		40	u0008'	'		
		41	a0544'	'	$X_1^{(2)}$ @ 7	
		42	h0558'	'		X @ 7
		43	a0547'	'	ΔX @ 7	
		44	c0561'	'		X' @ 7
		45	b0551'	'	R @ 7	
		46	m0502'	'	M @ 1	
		47	d0412'	'	1 @ 1	
		48	a0545'	'	$Y_1^{(2)}$ @ 7	
		49	h0559'	'		Y @ 7
		50	a0548'	'	ΔY @ 7	
		51	c0562'	'		Y' @ 7
		52	b0551'	'	R @ 7	
		53	m0503'	'	N @ 1	
		54	d0412'	'	1 @ 1	
		55	a0546'	'	$Z_1^{(2)}$ @ 7	
		56	h0560'	'		Z @ 7
		57	a0549'	'	ΔZ @ 7	
		58	c0563'	'		Z' @ 7
		59	b0552'	'	X(E) @ 7	
		60	m0412'	'	1/2 @ 0	
		61	xc6315'	'		
		62	b0558'	'	X(W) @ 7	
		63	m0412'	'	1/2 @ 0	

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PROBLEM:				TRACK

PROGRAM INPUT CODES	STOP	LOCATION	INSTRUCTION OPERATION ADDRESS	STOP	CONTENTS OF ADDRESS	NOTES
	'					
	'	☒				
		0300	xa6315'	'		
		01	h0600'	'		X avg @ 7
		02	a0547'	'	ΔX @ 7	
		03	c0603'	'	☒	X' avg @ 7
		04	b0553'	'	Y(E) @ 7	
		05	m0412'	'	1/2 @ 0	
		06	xc6321'	'		
		07	b0559'	'	☒ Y(W) @ 7	
		08	m0412'	'	1/2 @ 0	
		09	xa6321'	'		
		10	h0601'	'		Y avg @ 7
		11	a0548'	'	☒ ΔY @ 7	
		12	c0604'	'		Y' avg @ 7
		13	b0554'	'	Z(E) @ 7	
		14	m0412'	'	1/2 @ 0	
		15	xc6301'	'	☒	
		16	b0560'	'	Z(W) @ 7	
		17	m0412'	'	1/2 @ 0	
		18	xa6301'	'		
		19	h0602'	'	☒	Z avg @ 7
		20	a0549'	'	ΔZ @ 7	
		21	c0605'	'		Z' avg @ 7
		22	b0552'	'	X(E) @ 7	
		23	s0558'	'	☒ X(W) @ 7	
		24	m0412'	'	1/2 @ 0	
		25	t0327'	'		
		26	u0329'	'		
		27	c0606'	'	☒	
		28	s0606'	'		
		29	c0606'	'		σX @ 7
		30	b0553'	'	Y(E) @ 7	
		31	s0559'	'	☒ Y(W) @ 7	

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PROBLEM:				TRACK

PROGRAM INPUT CODES	STOP	LOCATION	INSTRUCTION OPERATION ADDRESS	STOP	CONTENTS OF ADDRESS	NOTES
	'					
	'	☒				
		0332	m0412'	'	1/2 @ 0	
		33	t0335'	'		
		34	u0337'	'		
		35	c0607'	'	☒	
		36	s0607'	'		
		37	c0607'	'		σ-Y @ 7
		38	b0554'	'	Z(E) @ 7	
		39	s0560'	'	☒ Z(W) @ 7	
		40	m0412'	'	1/2 @ 0	
		41	t0343'	'		
		42	u0345'	'		
		43	c0608'	'	☒	
		44	s0608'	'		
		45	c0608'	'		σ Z @ 7
		46	xp1632'	'		C. Return
		47	xz3200'	'	☒	delay
		48	r1503'	'		Data output
		49	u1500'	'		#12.4
		50	z0552'	'		loc. of 1st word
		51	xz0607'	'	☒	no. and q
		52	xp1638'	'		C. R.
		53	xz3200'	'		delay
		54	r1503'	'		enter
		55	u1500'	'	☒	#12.4
		56	z0558'	'		loc. of 1st word
		57	xz0607'	'		no. and q
		58	xp1644'	'		C. R.
		59	xz3200'	'	☒	delay
		60	r1503'	'		enter
		61	u1500'	'		#12.4
		62	z0600'	'		loc. of 1st word
		63	xz0907'	'	☒	no. and q

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PROBLEM:	TRACK

PROGRAM INPUT CODES	LOCATION	INSTRUCTION (OPERATION / ADDRESS)	CONTENTS OF ADDRESS	NOTES
	0400	xp1650'		C. R.
	01	xz3200'		delay
	02	u0000'		transfer to start
	03	z0504'		loc. of i_1
	04	z0539'		loc. of δ
	05	z0537'		loc. of β
	06	z0215'		
	07	z0032'		
	08	z0209'		
	09	z0501'		loc. of L
	10	z0800'	loc. of 1st wd. of $\bar{\mu}$(Z) table	
	11	m0504'		
,0000006'	12	40000000'		1 @ 1
	13	20000000'		1 @ 2
	14	2000000'		1 @ 6
	15	200'		1 @ 22
	16	4'		1 @ 29
	17	wwwwww'		mask
	18			
	19			
	20			
	21			
	22			
	23			
	24			
	25			
	26			
	27			
	28			
	29			
	30			
	31			

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ROYAL McBEE, J13541X

Royal McBee Corporation
DATA PROCESSING DIV.
PORT CHESTER, NEW YORK

BC/DOVAP PROGRAM CONSTANTS

(Addresses are relative to 0500)

Location	Contents		Location	Contents		Location	Contents	
0500	N_1	@ 16	0532	$\bar{\mu}(Z_1)-1$	@ 1	0600	X_{avg}	
01	L		33	$\bar{\mu}_1(Z)$	@ 1	01	Y_{avg}	
02	M	@ 1	34	(g_o/g_o^*)	@ 1	02	Z_{avg}	
03	N		35	(ϵ'/r)	@ -6	03	X'_{avg}	
04	i_1		36	c^2	@ 15	04	Y'_{avg}	@ 7
05	i_2	@ 1	37	$\beta(1)$	@ 7	05	Z'_{avg}	
06	i_3		38	$\beta(2)$		06	σX	
07	X_1'		39	$\delta(1)$	@ 15	07	σY	
08	Y_1'		40	$\delta(2)$		08	σZ	
09	Z_1'	@ 7	41	$X_1^{(1)}$				
10	ν_1	(East Camera)	42	$Y_1^{(1)}$				
11	ν_2		43	$Z_1^{(1)}$	@ 7			
12	ν_3		44	$X_1^{(2)}$				
13	$s_1/4$		45	$Y_1^{(2)}$				
14	$s_2/4$	@ 7	46	$Z_1^{(2)}$				
15	$s_3/4$		47	ΔX				
16	i_1		48	ΔY	@ 7			
17	i_2	@ 1	49	ΔZ				
18	i_3		50	$u/2$	@ 7			
19	X_1'		51	R	@ 7			
20	Y_1'		52	X				
21	Z_1'	@ 7	53	Y				
22	ν_1	(West Camera)	54	Z	@ 7			
23	ν_2		55	X'	(East Camera)			
24	ν_3		56	Y'				
25	$s_1/4$		57	Z'				
26	$s_2/4$	@ 7	58	X				
27	$s_3/4$		59	Y				
28	$U_o/2$	@ 8	60	Z	@ 7			
29	$\lambda_o/2$	@ -8	61	X'	(West Camera)			
30	Z_o	@ 7	62	Y'				
31	Z_1	@ 7	63	Z'				

APPENDIX C

GRENADE TIMES AND DOPPLER COUNTS

GRENADE TIMES AND DOPPLER COUNTS

	S.S. 12.50		S.S. 12.51		S.S. 6.52	
Grenade	Time	Count	Time	Count	Time	Count
A*					17.238	4231.72
1	47.199	15629.10	32.383	10066.55	44.073	17428.00
2			38.472	12649.89		
3	57.414	18883.55			70.298	25023.63
4					88.347	28093.75
5					108.380	29664.70

	S.S. 6.53		S.S. 6.54		S.S. 6.55	
Grenade	Time	Count	Time	Count	Time	Count
A*	17.459	4236.50	18.770	4517.30	17.383	4348.95
1	39.650	15307.90	36.626	13670.00	49.086	18971.65
2					54.947	21063.45
3	51.950	20269.65				
4	59.140	22814.20	55.978	21586.82		
5	66.232	25075.57	63.082	24014.58	70.488	25790.92
6	75.330	27619.13	71.206	26484.15	78.449	27758.73
7	83.564	29577.15	79.252	28608.33	86.289	29396.97
8			88.253	30603.65	93.035	30568.78
9	101.700	32753.72	99.060	32485.85	102.899	31892.50
10	114.978	34096.55	109.888	33811.02		

GRENADE TIMES AND DOPPLER COUNTS

	S.S. 6.56		S.S. 12.57		S.S. 6.58	
Grenade	Time	Count	Time	Count	Time	Count
A*	17.678	4515.90			17.878	4708.68
1	42.930	17566.80	78.307	24840.22		
2	50.336	20766.14	87.234	26506.86	56.038	22972.72
3	58.922	24131.94	99.402	28173.43	60.966	24787.18
4	66.179	26693.85	108.222	28948.31	67.786	27104.95
5	74.373	29280.21	134.832	29099.97		
6	83.858	31870.21			80.804	30911.88
7					87.414	32534.50
8					95.140	34170.77
9	116.438	37498.03			109.090	36413.56
10	128.349	38326.77			122.680	37736.23

APPENDIX D

GRENADE BURST POSITIONS

SS 12.50 Grenade Burst Positions
(km)

(Origin Ctr. Geophone Array)			(Origin Launcher)					
x	y	z	x	y	z	Δx	Δy	Δz
3.047583-	.257401	31.926032	3.392425-	.549930	31.931339			
3.044369-	.255595	31.926433	3.389211-	.548124	31.931740			
3.045976-	.256498	31.926232	3.390818-	.549027	31.931539	.001606	.000903	.000200
4.023545-	.381063	38.459181	4.368387-	.673592	38.464488			
4.020260-	.377360	38.459645	4.365102-	.669889	38.464952			
4.021903-	.379212	38.459413	4.366745-	.671741	38.464720	.001642	.001851	.000231

(x is positive East, y is positive North, and z is elevation above the origin. Positions obtained from East and West camera plates are printed on lines one and two, respectively, of each group with the averages on the third line. Δx, Δy, and Δz are the corresponding standard deviations which, for two estimates, are equal to the absolute value of either residual. The position of the center of the geophone array is (198639.59', 215470.08', 492.96') relative to the Guam survey origin.)

SS 12.51 Grenade Burst Positions
(km)

	(Origin Ctr. Geophone Array)			(Origin Launcher)					
	x	y	z	x	y	z	Δx	Δy	Δz
	1.236779-	1.859000	20.653055	1.581621-	2.151529	20.658362			
1	1.236377-	1.857459	20.653255	1.581219-	2.149988	20.658562			
	1.236578-	1.858229	20.653155	1.581420-	2.150758	20.658462	.000201	.000770	.000099
	1.709457-	2.541324	25.830423	2.054299-	2.833853	25.835730			
2	1.708595-	2.538430	25.830823	2.053437-	2.830959	25.836130			
	1.709026-	2.539877	25.830623	2.053868-	2.832406	25.835930	.000431	.001447	.000200

(x is positive East, y is positive North, and z is elevation above the origin. Positions obtained from East and West camera plates are printed on lines one and two, respectively, of each group with the averages on the third line. Δx, Δy, and Δz are the corresponding standard deviations which, for two estimates, are equal to the absolute value of either residual. The position of the center of the geophone array is (198639.59', 215470.08', 492.96') relative to the Guam survey origin.)

SS 6.52 Grenade Burst Positions
(km)

	(Origin Ctr. Geophone Array)			(Origin Launcher)					
	x	y	z	x	y	z	Δx	Δy	Δz
	.267385-	.191148	8.950263	.612227-	.483677	8.955570			
A*	.266636-	.190721	8.950358	.611478-	.483250	8.955665			
	.267011-	.190934	8.950310	.611853-	.483463	8.955617	.000374	.000213	.000047
	2.297385-	2.545263	35.554302	2.642227-	2.837792	35.559609			
1	2.296746-	2.544102	35.554448	2.641588-	2.836631	35.559755			
	2.297066-	2.544683	35.554375	2.641908-	2.837212	35.559682	.000320	.000580	.000073
	3.886005-	4.449707	50.778033	4.230847-	4.742236	50.783340			
3	3.884703-	4.448812	50.778227	4.229545-	4.741341	50.783534			
	3.885354-	4.449259	50.778130	4.230196-	4.741788	50.783437	.000651	.000447	.000097
	4.887833-	5.672308	56.849681	5.232675-	5.964837	56.854988			
4	4.887628-	5.671816	56.849753	5.232470-	5.964345	56.855060			
	4.887731-	5.672062	56.849717	5.232573-	5.964591	56.855024	.000102	.000245	.000036
	5.999316-	7.039998	59.802935	6.344158-	7.332527	59.808242			
5	5.998789-	7.038772	59.803144	6.343631-	7.331301	59.808451			
	5.999053-	7.039385	59.803039	6.343895-	7.331914	59.808346	.000264	.000613	.000104

(x is positive East, y is positive North, and z is elevation above the origin. Positions obtained from East and West camera plates are printed on lines one and two, respectively, of each group with the averages on the third line. Δx, Δy, and Δz are the corresponding standard deviations which, for two estimates, are equal to the absolute value of either residual. The position of the center of the geophone array is (198639.59', 215470.08', 492.96') relative to the Guam survey origin.)

SS 6.53 Grenade Burst Positions
(km)

	(Origin Ctr. Geophone Array)			(Origin Launcher)					
	x	y	z	x	y	z	Δx	Δy	Δz
A*	.319129	.164430	8.998568	.025712-	.456959	9.003875			
	.319051	.163938	8.998592	.025790-	.456467	9.003899			
	.319090	.164184	8.998580	.025751-	.456713	9.003887	.000039	.000246	.000012
1	.360300-	1.608670	31.411616	.705142-	1.901199	31.416923			
	.359329-	1.606606	31.411776	.704171-	1.899135	31.417083			
	.359814-	1.607638	31.411696	.704657-	1.900167	31.417003	.000485	.001031	.000080
3	.779325-	2.386843	41.441860	1.124168-	2.679372	41.447167			
	.778717-	2.383982	41.442072	1.123559-	2.676510	41.447379			
	.779021-	2.385412	41.441966	1.123863-	2.677941	41.447273	.000304	.001430	.000106
4	1.032907-	2.837167	46.580552	1.377749-	3.129696	46.585859			
	1.031476-	2.834023	46.580814	1.376318-	3.126552	46.586121			
	1.032192-	2.835595	46.580683	1.377034-	3.128124	46.585990	.000715	.001572	.000131
5	1.280639-	3.275463	51.143542	1.625481-	3.567992	51.148849			
	1.280231-	3.273151	51.143715	1.625073-	3.565680	51.149022			
	1.280435-	3.274307	51.143628	1.625277-	3.566836	51.148935	.000203	.001155	.000086
6	1.604432-	3.842526	56.268229	1.949274-	4.135055	56.273536			
	1.605275-	3.838718	56.268480	1.950118-	4.131247	56.273787			
	1.604854-	3.840622	56.268355	1.949696-	4.133151	56.273662	.000421	.001903	.000125
7	1.905511-	4.351550	60.204810	2.250353-	4.644079	60.210117			
	1.904052-	4.348665	60.205091	2.248894-	4.641194	60.210398			
	1.904781-	4.350107	60.204950	2.249624-	4.642636	60.210257	.000729	.001442	.000141
9	2.566003-	5.467886	66.556526	2.910845-	5.760415	66.561833			
	2.567075-	5.464562	66.556765	2.911917-	5.757091	66.562072			
	2.566539-	5.466224	66.556645	2.911381-	5.758753	66.561952	.000535	.001661	.000119
10	3.045057-	6.281413	69.197364	3.389899-	6.573942	69.202671			
	3.045827-	6.276127	69.197830	3.390669-	6.568656	69.203137			
	3.045442-	6.278770	69.197597	3.390284-	6.571299	69.202904	.000385	.002643	.000233

(x is positive East, y is positive North, and z is elevation above the origin. Positions obtained from East and West camera plates are printed on lines one and two, respectively, of each group with the averages on the third line. Δx, Δy, and Δz are the corresponding standard deviations which, for two estimates, are equal to the absolute value of either residual. The position of the center of the geophone array is (198639.59', 215470.08', 492.96') relative to the Guam survey origin.)

SS 6.54 Grenade Burst Positions
(km)

	(Origin Ctr. Geophone Array)			(Origin Launcher)					
	x	y	z	x	y	z	Δx	Δy	Δz
	.157593	.000577	9.571398	.187249-	.293106	9.576705			
A*	.158357	.000299	9.571442	.186484-	.292829	9.576749			
	.157975	.000438	9.571420	.186866-	.292967	9.576727	.000382	.000138	.000022
	.771946-	.662643	28.112239	1.116788-	.955172	28.117546			
1	.771918-	.660602	28.112312	1.116760-	.953131	28.117619			
	.771932-	.661623	28.112276	1.116774-	.954152	28.117583	.000014	.001020	.000036
	1.800386-	1.358396	44.129549	2.145228-	1.650925	44.134856			
4	1.799438-	1.354838	44.129740	2.144280-	1.647367	44.135047			
	1.799912-	1.356617	44.129644	2.144754-	1.649146	44.134951	.000474	.001779	.000095
	2.180066-	1.608965	49.035981	2.524908-	1.901494	49.041288			
5	2.178805-	1.604744	49.036219	2.523647-	1.897273	49.041526			
	2.179436-	1.606855	49.036100	2.524278-	1.899384	49.041406	.000630	.002110	.000119
	2.614824-	1.903152	54.022105	2.959666-	2.195681	54.027412			
6	2.613979-	1.899911	54.022289	2.958821-	2.192440	54.027596			
	2.614402-	1.901531	54.022197	2.959244-	2.194060	54.027504	.000422	.001620	.000091
	3.043328-	2.194481	58.305418	3.388170-	2.487010	58.310724			
7	3.043012-	2.190307	58.305616	3.387854-	2.482836	58.310923			
	3.043170-	2.192394	58.305517	3.388012-	2.484923	58.310823	.000157	.002087	.000099
	3.528274-	2.513392	62.320837	3.873116-	2.805921	62.326144			
8	3.526543-	2.508967	62.321157	3.871385-	2.801497	62.326464			
	3.527409-	2.511180	62.320997	3.872251-	2.803709	62.326304	.000865	.002212	.000159
	4.660585-	3.294513	68.733478	5.005427-	3.587042	68.738785			
10	4.660831-	3.290723	68.733662	5.005673-	3.583252	68.738969			
	4.660708-	3.292618	68.733570	5.005550-	3.585147	68.738877	.000123	.001894	.000092

(x is positive East, y is positive North, and z is elevation above the origin. Positions obtained from East and West camera plates are printed on lines one and two, respectively, of each group with the averages on the third line. Δx, Δy, and Δz are the corresponding standard deviations which, for two estimates, are equal to the absolute value of either residual. The position of the center of the geophone array is (198639.59', 215470.08', 492.96') relative to the Guam survey origin.)

SS 6.55 Grenade Burst Positions
(km)

	(Origin Ctr. Geophone Array)			(Origin Launcher)					
	x	y	z	x	y	z	Δx	Δy	Δz
	.132180	.157343	9.220663	.212662-	.449872	9.225970			
A*	.132667	.157052	9.220702	.212174-	.449581	9.226009			
	.132423	.157198	9.220682	.212418-	.449727	9.225989	.000243	.000145	.000019
	.741977-	2.531822	38.792546	1.086819-	2.824351	38.797853			
1	.742067-	2.530680	38.792627	1.086909-	2.823209	38.797934			
	.742022-	2.531251	38.792586	1.086864-	2.823780	38.797893	.000045	.000571	.000040
	.909964-	2.956431	43.013761	1.254806-	3.248960	43.019068			
2	.909287-	2.954432	43.013941	1.254129-	3.246961	43.019248			
	.909626-	2.955432	43.013851	1.254468-	3.247961	43.019158	.000338	.000999	.000089
	1.372320-	4.071676	52.536879	1.717162-	4.364205	52.542186			
5	1.373205-	4.070100	52.536979	1.718047-	4.362629	52.542286			
	1.372762-	4.070888	52.536929	1.717604-	4.363417	52.542236	.000442	.000787	.000050
	1.618198-	4.642573	56.488751	1.963040-	4.935102	56.494058			
6	1.619351-	4.641692	56.488783	1.964194-	4.934221	56.494090			
	1.618775-	4.642132	56.488767	1.963617-	4.934661	56.494074	.000576	.000440	.000015
	1.858712-	5.204802	59.768495	2.203554-	5.497331	59.773802			
7	1.860334-	5.202163	59.768675	2.205176-	5.494692	59.773982			
	1.859523-	5.203482	59.768585	2.204365-	5.496011	59.773892	.000810	.001319	.000089
	2.067782-	5.681521	62.104871	2.412624-	5.974050	62.110178			
8	2.071427-	5.680224	62.104840	2.416269-	5.972752	62.110147			
	2.069605-	5.680872	62.104855	2.414447-	5.973401	62.110162	.001822	.000648	.000015
	2.369726-	6.380169	64.723278	2.714568-	6.672697	64.728584			
9	2.371798-	6.379149	64.723283	2.716640-	6.671678	64.728590			
	2.370762-	6.379659	64.723280	2.715604-	6.672188	64.728587	.001036	.000509	.000003

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SS 6.56 Grenade Burst Positions
(km)

	(Origin Ctr. Geophone Array)			(Origin Launcher)					
	x	y	z	x	y	z	Δx	Δy	Δz
	.183816-	.180962	9.538604	.528658-	.473491	9.543910			
A*	.185628-	.178882	9.538571	.530470-	.471411	9.543878			
	.184722-	.179922	9.538587	.529564-	.472451	9.543894	.000905	.001039	.000016
	1.944081-	1.989911	35.905073	2.288923-	2.282440	35.910380			
1	1.949038-	1.982346	35.905214	2.293880-	2.274875	35.910521			
	1.946559-	1.986129	35.905143	2.291401-	2.278658	35.910450	.002478	.003782	.000071
	2.462368-	2.515829	42.355547	2.807211-	2.808358	42.360854			
2	2.467854-	2.508051	42.355678	2.812696-	2.800580	42.360984			
	2.465111-	2.511940	42.355612	2.809953-	2.804469	42.360919	.002742	.003888	.000065
	3.069534-	3.114003	49.133532	3.414376-	3.406532	49.138839			
3	3.076692-	3.105743	49.133579	3.421534-	3.398272	49.138886			
	3.073113-	3.109873	49.133555	3.417955-	3.402402	49.138862	.003579	.004130	.000023
	3.585286-	3.619693	54.284492	3.930128-	3.912222	54.289799			
4	3.593039-	3.609001	54.284677	3.937881-	3.901530	54.289984			
	3.589163-	3.614347	54.284584	3.934005-	3.906876	54.289891	.003876	.005346	.000092
	4.167136-	4.184249	59.475053	4.511978-	4.476778	59.480360			
5	4.175615-	4.172684	59.475251	4.520457-	4.465213	59.480558			
	4.171376-	4.178467	59.475152	4.516218-	4.470996	59.480459	.004239	.005782	.000098
	4.840867-	4.841879	64.657017	5.185709-	5.134407	64.662324			
6	4.850606-	4.829463	64.657196	5.195448-	5.121992	64.662503			
	4.845736-	4.835671	64.657106	5.190578-	5.128200	64.662413	.004869	.006208	.000089
	7.140589-	7.074088	75.756252	7.485431-	7.366617	75.761559			
9	7.150051-	7.060703	75.756594	7.494893-	7.353232	75.761901			
	7.145320-	7.067396	75.756423	7.490162-	7.359925	75.761730	.004731	.006692	.000171
	8.041641-	7.881965	77.277066	8.386483-	8.174494	77.282373			
10	8.051121-	7.866343	77.277663	8.395963-	8.158872	77.282970			
	8.046381-	7.874154	77.277364	8.391223-	8.166683	77.282671	.004739	.007810	.000298

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SS 12.57 Grenade Burst Positions
(km)

	(Origin Ctr. Geophone Array)			(Origin Launcher)					
	x	y	z	x	y	z	Δx	Δy	Δz
	3.730513	4.810209	50.470150	3.385671	5.102738	50.475457			
1	3.731089	4.810159	50.470119	3.386247	5.102688	50.475426			
	3.730801	4.810184	50.470134	3.385959	5.102713	50.475441	.000288	.000024	.000015
	4.163173	5.489903	53.778067	3.818331	5.782432	53.783374			
2	4.162409	5.490904	53.778012	3.817567	5.783433	53.783318			
	4.162791	5.490404	53.778039	3.817949	5.782933	53.783346	.000382	.000500	.000027
	4.745076	6.417000	57.043638	4.400234	6.709529	57.048945			
3	4.744536	6.417081	57.043666	4.399694	6.709610	57.048973			
	4.744806	6.417040	57.043652	4.399964	6.709569	57.048959	.000270	.000040	.000014
	5.163004	7.084760	58.518592	4.818162	7.377289	58.523899			
4	5.159950	7.083170	58.519027	4.815108	7.375699	58.524334			
	5.161477	7.083964	58.518809	4.816635	7.376493	58.524116	.001527	.000795	.000217
	6.418576	9.080205	58.431407	6.073734	9.372734	58.436714			
5	6.418620	9.082666	58.431006	6.073778	9.375195	58.436313			
	6.418598	9.081436	58.431207	6.073756	9.373965	58.436514	.000022	.001230	.000200

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SS 6.58 Grenade Burst Positions
(km)

	(Origin Ctr. Geophone Array)			(Origin Launcher)					
	x	y	z	x	y	z	Δx	Δy	Δz
	.465442-	.277967	9.900413	.810284-	.570496	9.905720			
A*	.465314-	.277775	9.900440	.810156-	.570304	9.905747			
	.465378-	.277871	9.900426	.810220-	.570400	9.905733	.000064	.000095	.000013
	4.268112-	3.917662	46.593490	4.612954-	4.210191	46.598797			
2	4.266439-	3.916485	46.593298	4.611281-	4.209014	46.598605			
	4.267276-	3.917074	46.593394	4.612118-	4.209603	46.598701	.000836	.000588	.000096
	4.757927-	4.378795	50.216804	5.102769-	4.671324	50.222111			
3	4.755426-	4.377630	50.217180	5.100268-	4.670159	50.222487			
	4.756677-	4.378212	50.216992	5.101519-	4.670741	50.222299	.001250	.000583	.000188
	5.442285-	5.017310	54.833213	5.787127-	5.309839	54.838520			
4	5.439749-	5.015047	54.833713	5.784591-	5.307576	54.839020			
	5.441017-	5.016179	54.833463	5.785859-	5.308708	54.838770	.001267	.001131	.000250
	6.760444-	6.230092	62.373496	7.105286-	6.522621	62.378802			
6	6.753548-	6.227777	62.374550	7.098390-	6.520306	62.379857			
	6.756996-	6.228934	62.374023	7.101838-	6.521463	62.379330	.003448	.001157	.000527
	7.406463-	6.822767	65.567053	7.751305-	7.115296	65.572360			
7	7.405594-	6.821510	65.567297	7.750437-	7.114039	65.572604			
	7.406029-	6.822138	65.567175	7.750871-	7.114667	65.572482	.000434	.000628	.000121
	8.186701-	7.531182	68.756663	8.531543-	7.823711	68.761970			
8	8.184420-	7.531491	68.756919	8.529262-	7.824020	68.762226			
	8.185561-	7.531336	68.756791	8.530403-	7.823865	68.762098	.001140	.000154	.000128
	9.581136-	8.811387	73.040121	9.925978-	9.103916	73.045428			
9	9.575537-	8.807240	73.040768	9.920379-	9.099769	73.046075			
	9.578336-	8.809314	73.040444	9.923178-	9.101842	73.045751	.002799	.002073	.000323
	10.929251-	10.044161	75.415981	11.274093-	10.336690	75.421288			
10	10.931831-	10.044331	75.415561	11.276673-	10.336860	75.420868			
	10.930541-	10.044246	75.415771	11.275383-	10.336775	75.421078	.001290	.000084	.000209

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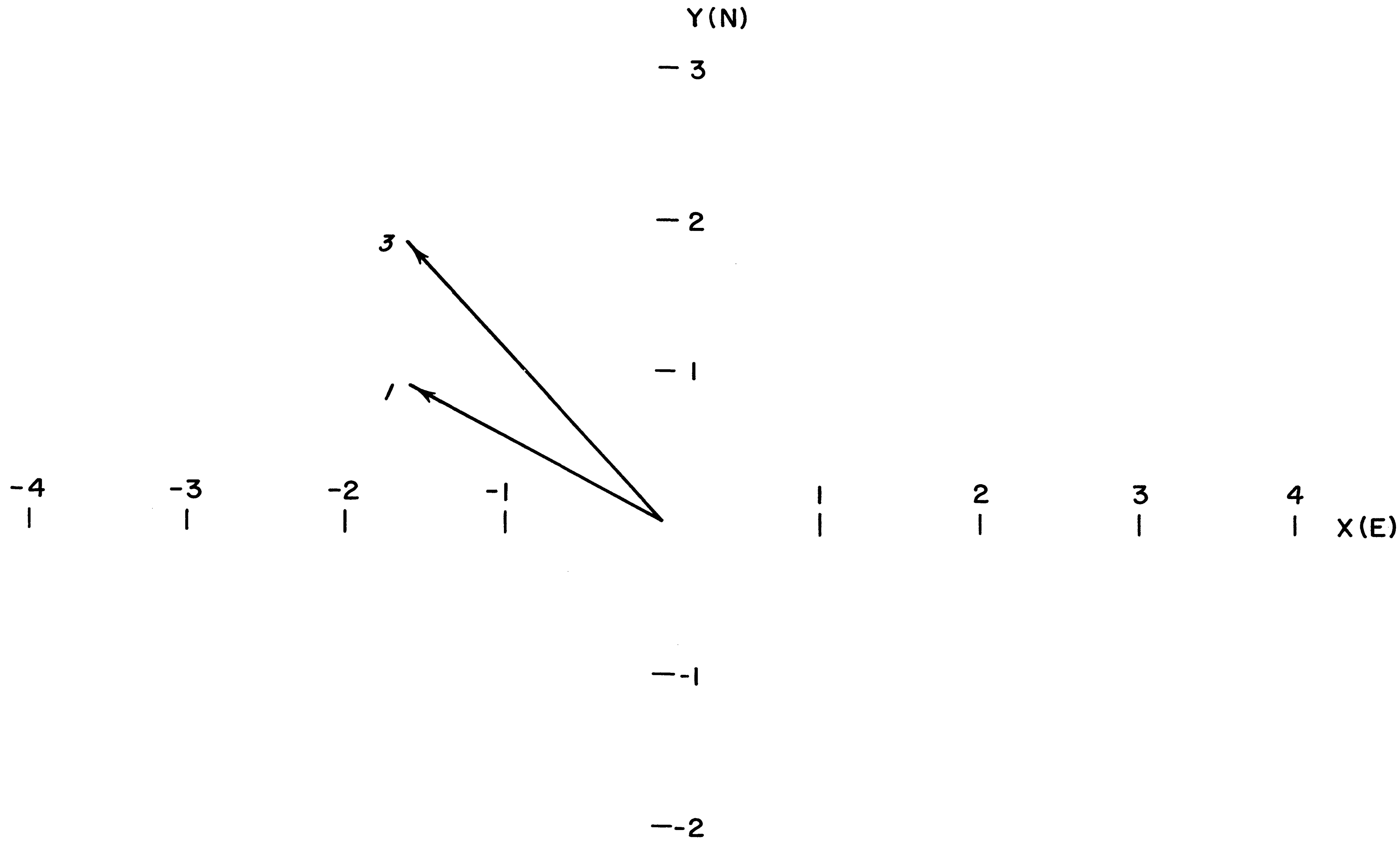
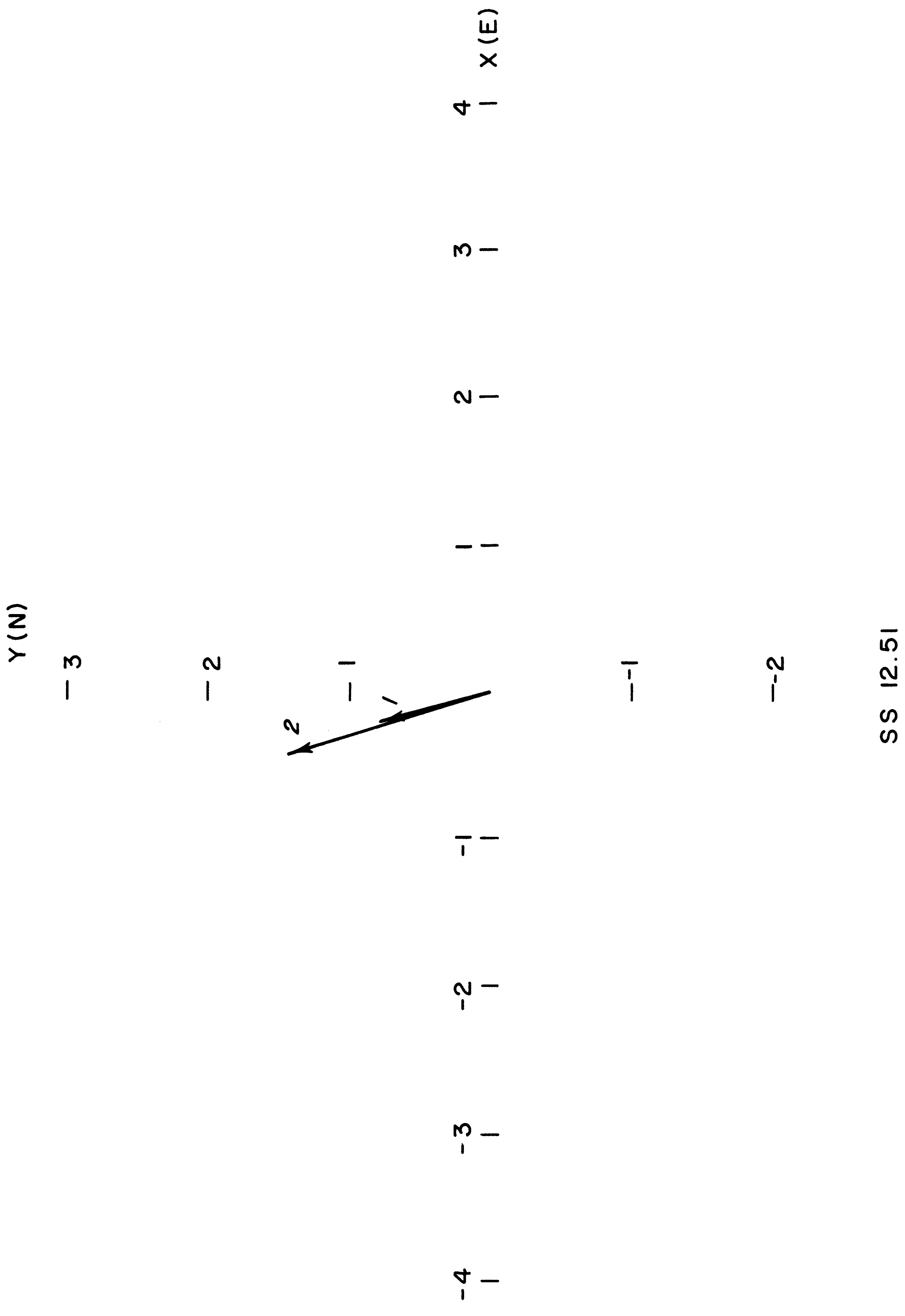
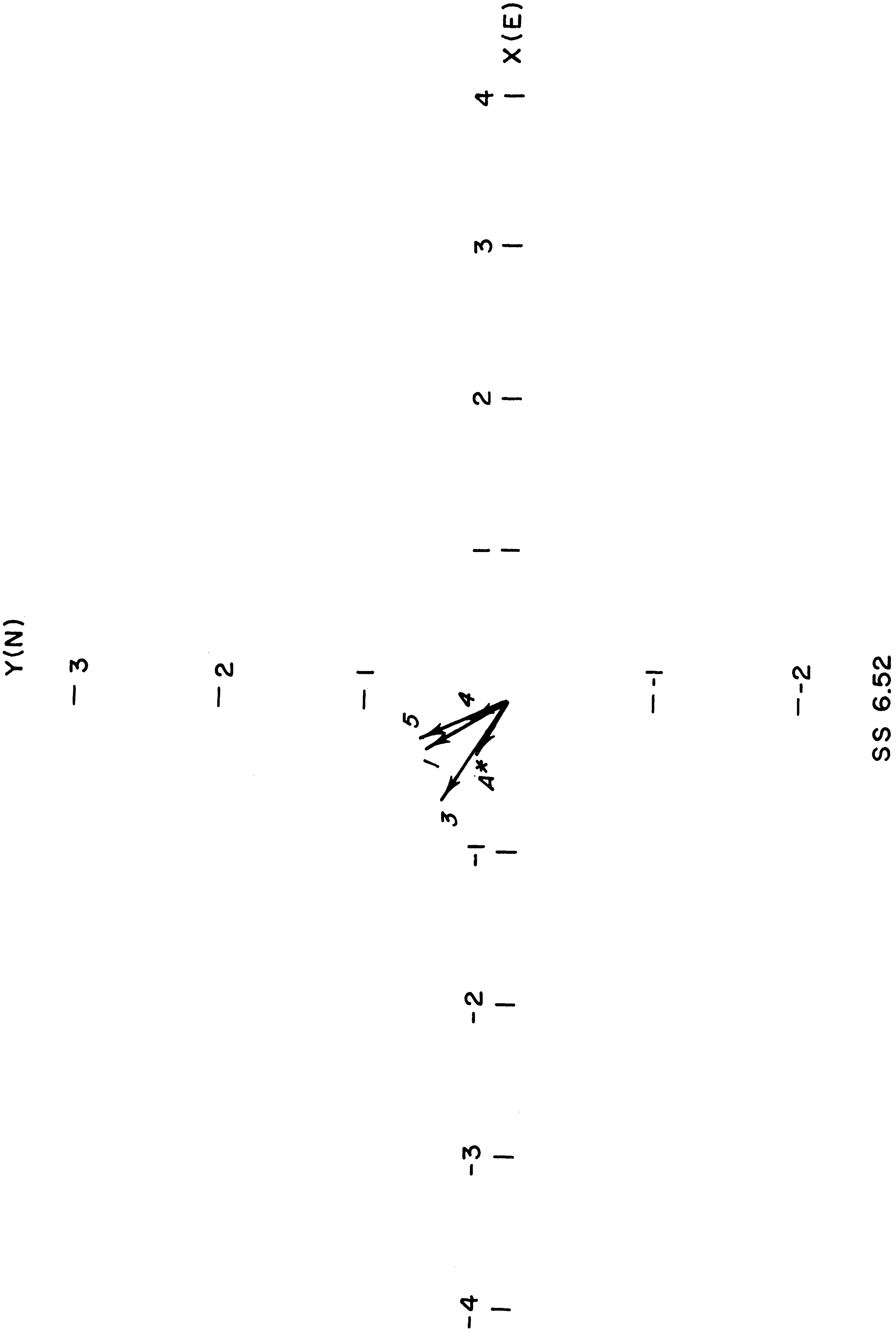
APPENDIX E

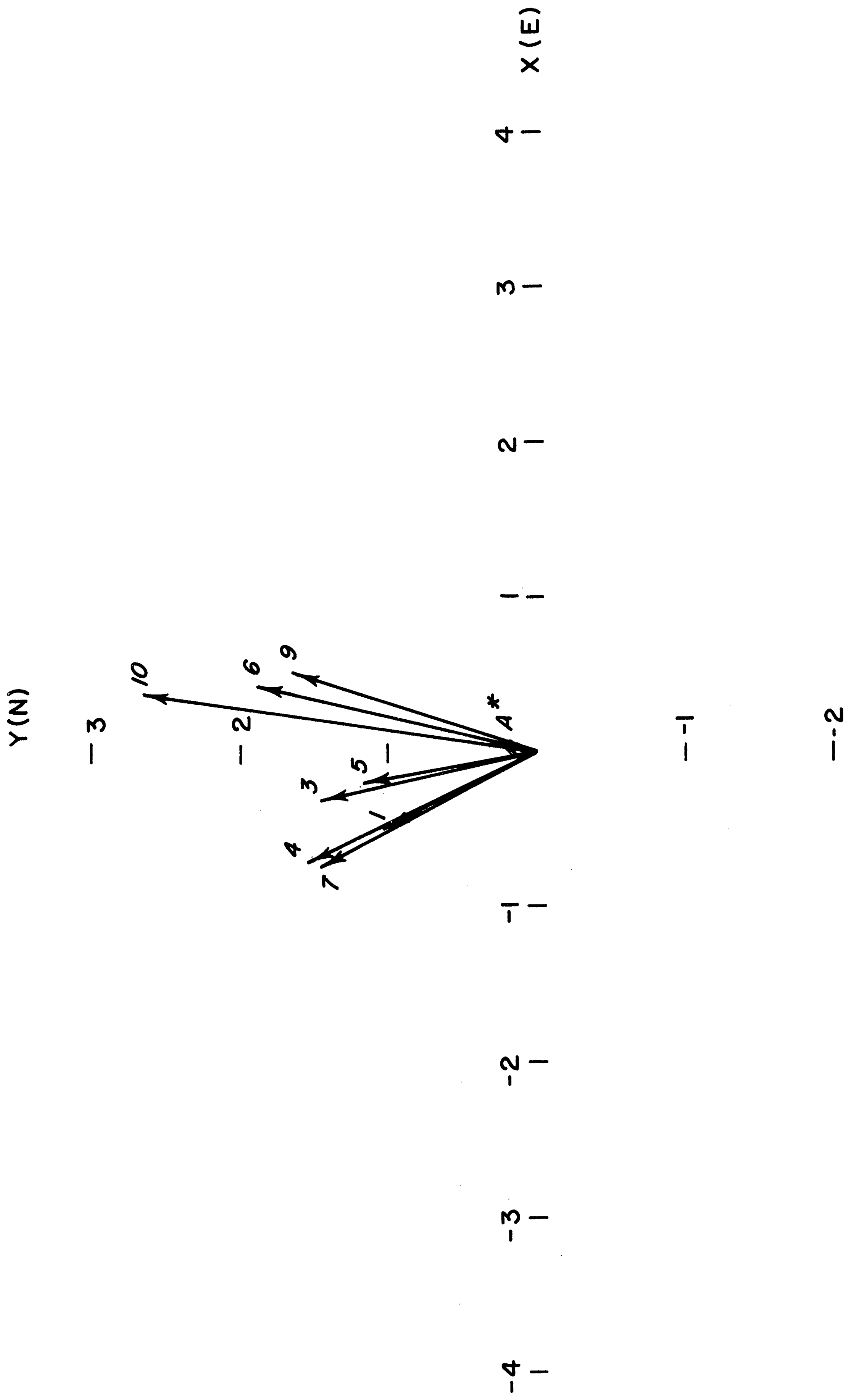
RESIDUAL VECTORS FOR GUAM FLIGHTS (METERS)



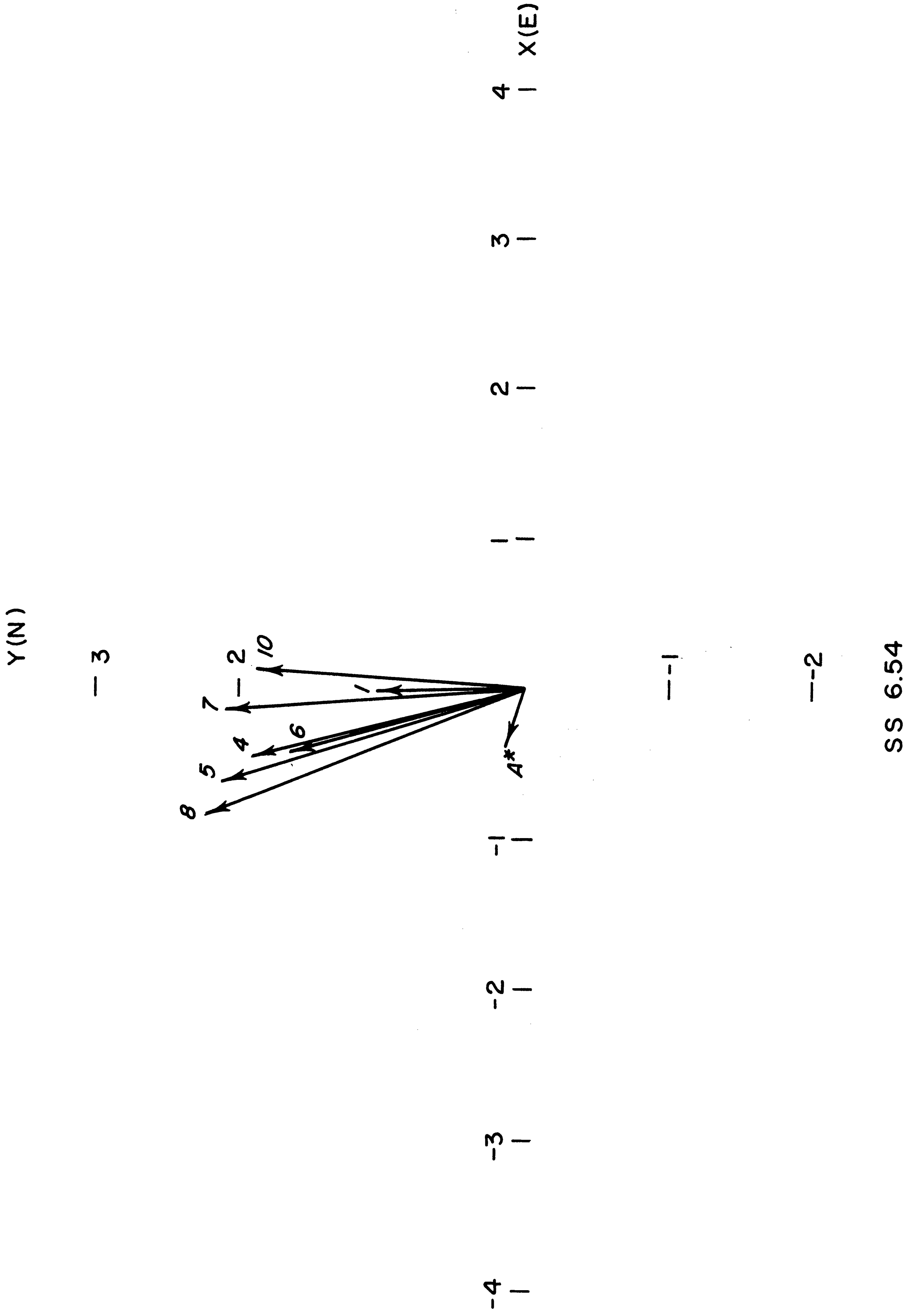
SS 12.50


X (E)
Y (N)
SS 12.51
4
3
2
1
-1
-2
-3
-4
1
2

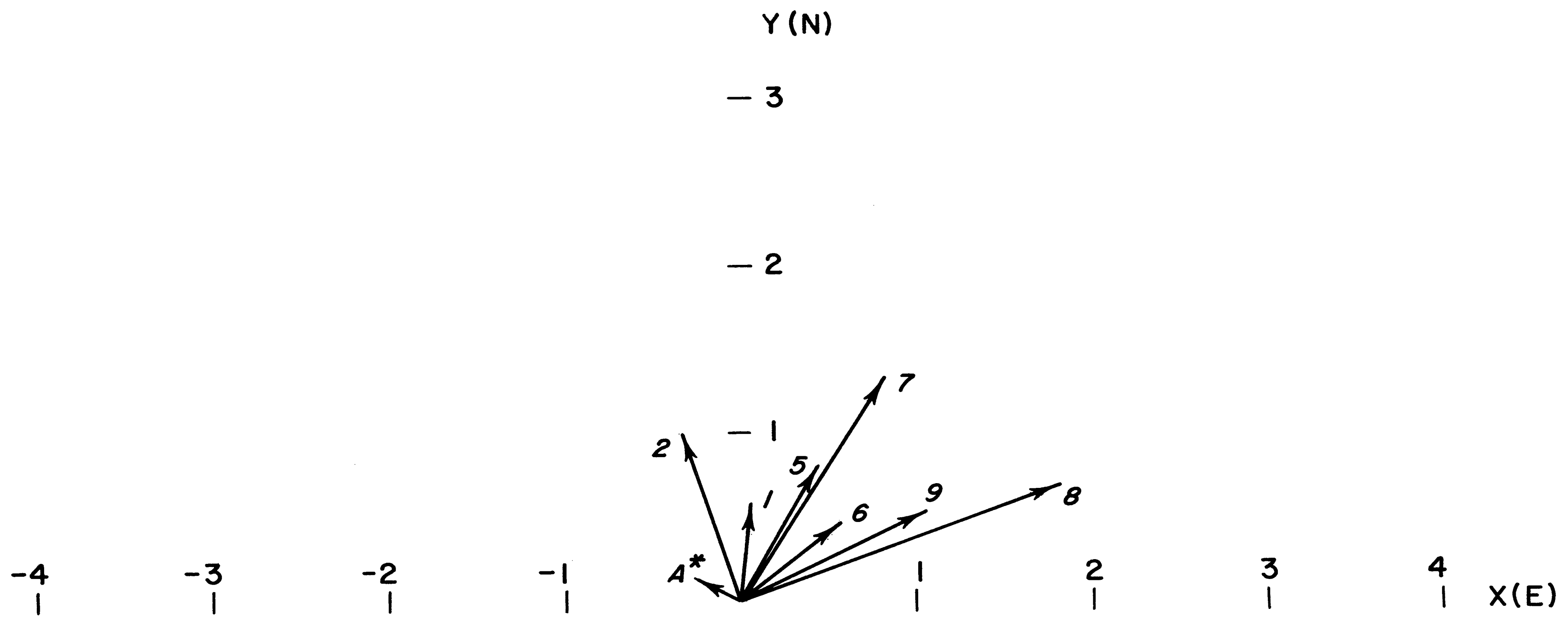

X(E)
Y(N)
SS 6.52
A*
1
3
4
5



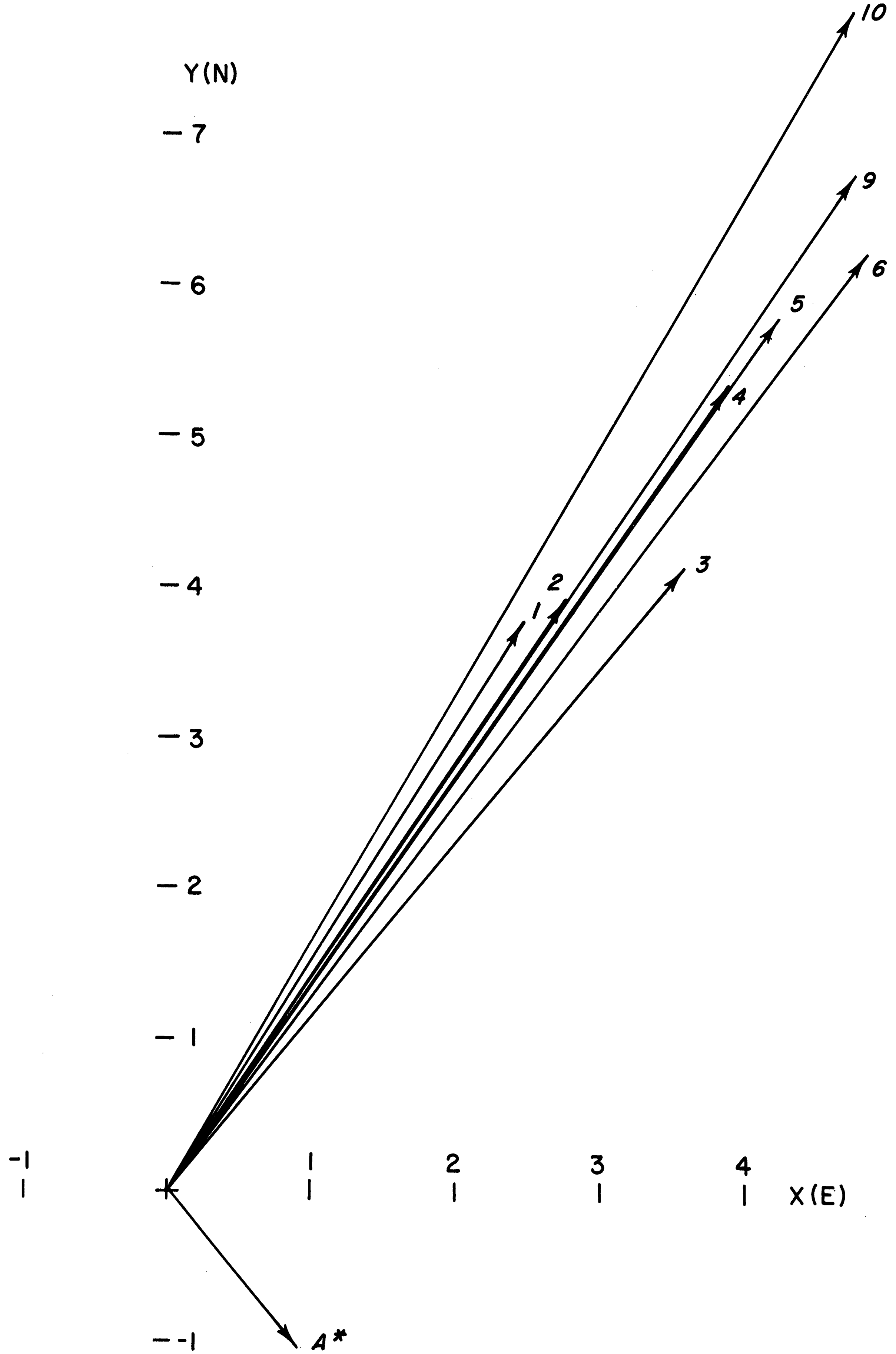
SS 6.53



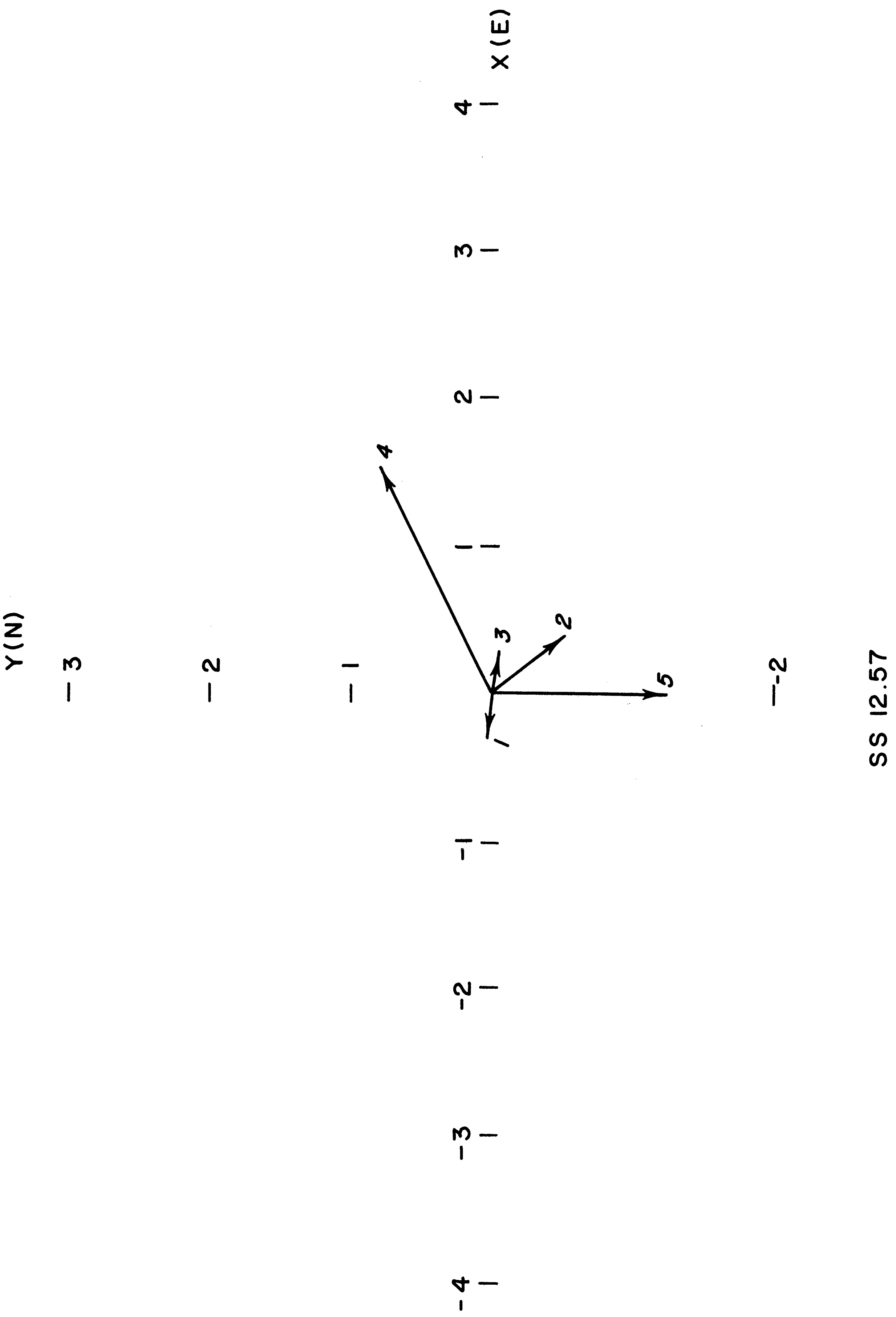
X(E)
Y(N)
A*
SS 6.54

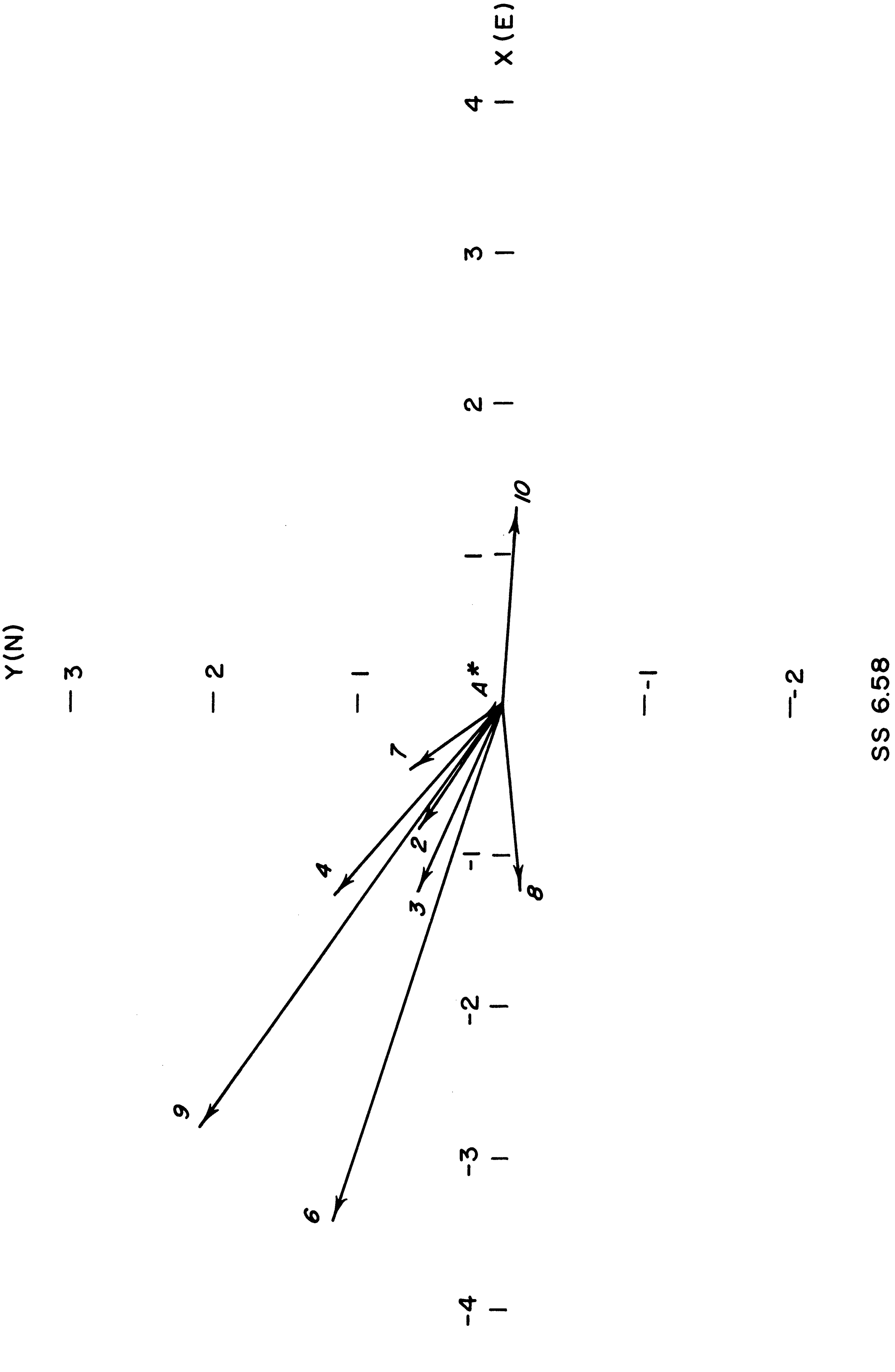


SS 6.55



SS 6.56


X (E)
Y (N)
SS 12.57


X (E)
Y(N)
A*
SS 6.58