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INVESTIGATION OF NUCLEAR-ENERGY LEVELS

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FOREWORD

Of the following eight articles, seven are reprints of papers published in The Physical Review or Physical Review Letters. The eighth, while not published in the journals, has proven to be useful and is being given wide private distribution to those working in the specialized field of angular correlation. The entire group of papers is submitted as a technical report for 1958.

M. L. Wiedenbeck

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Additional Data on the Radioactive Decay of Ho^{166} (27 hr), Nd^{147}, and Sm^{153}†

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(Received November 18, 1957)

By using chemically pure Ho^{165} and enriched isotopes of Nd^{146} and Sm^{152}, high-activity samples of Ho^{166}, Nd^{147}, and Sm^{153} were made by neutron capture. Studies of the strong sources by both magnetic and scintillation spectrometers showed the existence of some gamma rays not previously reported, yielded more exact energy values for the low-energy gamma rays, and made possible better resolution of the components of the beta spectra. In Ho^{166}, gamma rays occur at 80.25, 970, 1377, 1540, and 1620 kev, and the maximum beta energies are 1839, 1756, 869, 412, and 230 kev. Nd^{147} yields gamma rays at 91.3, 120.6, 198.2, 277, 321, 400, 441, 533, and 688 kev, with beta components at 812, 380, and 230 kev. In Sm^{153} gamma rays are found at 69.8, 103.5, 173.6, and 540 kev and beta components at 813, 710, and 640 kev. A level scheme in agreement with the observed data is presented for each nucleus.

A BETA activity with a half-life of 35 hours, ascribed to holmium Ho^{166}, was first reported[1] in 1935. Holmium exists in nature as a single isotope of mass 165; hence the radio-activity following neutron capture was assigned to mass 166. The beta emission to erbium is accompanied by gamma radiation. A long-lived isomer (>30 yr) also exists but is not discussed in this report. Earlier investigations of the short-lived activity using scintillation spectrometers have revealed[2] four gamma rays, and several reports on the beta spectrum have appeared.[3] In the present investigation a source of high specific activity was studied in both magnetic and scintillation spectrometers.

A resolution of the beta spectrum as currently observed by the double-focusing magnetic spectrometer shows five components. Three of these, with slightly altered energies, had been previously reported. The component with the highest energy (1839±5 kev) appears to have a large log*ft* value, namely, 8.2, yet it is probably an ordinary first forbidden transition. This has recently been discussed theoretically.[4] Another high-energy component differs from the first by approximately 80 kev, which is the energy of the well-established strong gamma transition. This second component has the shape of a unique first-forbidden spectrum. Figure 1 shows the resolution of the lower energy components, after the subtraction of the two higher energy transitions. A summary of the beta energies as previously reported[3] by Graham, Wolfson, and Clark, together with the data presently observed, is presented in Table I.

The decay of the irradiated sample was followed through ten octaves and the half-life found to be 26.9±0.1 hr. No indication of any contamination was observed. Strong electron conversion lines occur for the low-energy gamma ray. K, L_2, L_3, M, and N lines appear at 22.75, 70.94, 71.92, 78.44, and 80.02 kev, respectively, indicating an energy of 80.25±0.05 kev for the gamma ray. Corrected microphotometer traces of the photographic plates yield the relative intensities of the peaks. The values of the intensities for the K, L_2, L_3, M, and N lines, normalized to the K line, appear to be 100, 205, 225, 105, and 30, respectively. These values are not inconsistent with the predicted

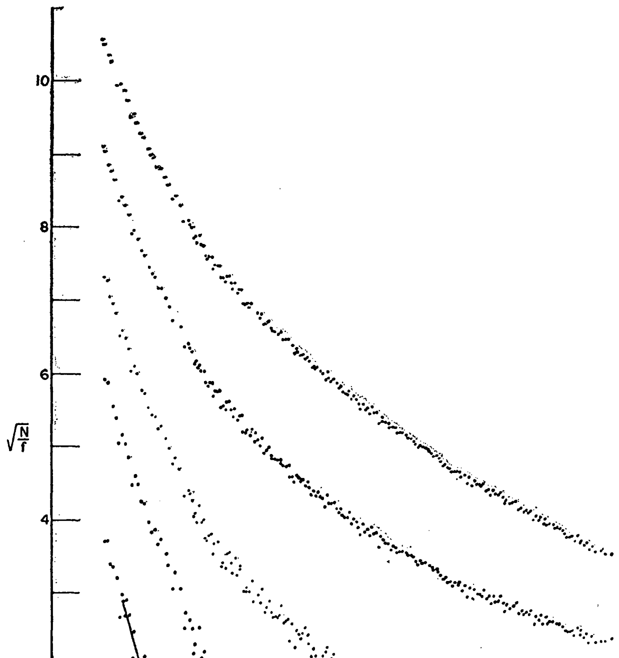

FIG. 1. Lower energy Fermi plot and analysis of the beta spectrum of Ho^{166}.

† This investigation received the joint support of the Office of Naval Research and the U. S. Atomic Energy Commission.
* Now deceased.
‡ Now at Atomic Energy Division, Phillips Petroleum Company, Idaho Falls, Idaho.
1 G. Hevesy and G. Levi, Nature **136**, 103 (1935); **137**, 185 (1936).
2 A. Sunyar, Phys. Rev. **93**, 1345 (1954).
3 K. Siegbahn and H. Slätis, Arkiv Fysik **1**, 559 (1950); Anton'eva, Bashilov, Dzhelepov, and Zolotavin, Doklady Akad. Nauk S.S.S.R. **70**, 397 (1950); Graham, Wolfson, and Clark, Phys. Rev. **98**, 1173 (1955); J. Frazer and J. Milton, Phys. Rev. **98**, 1173 (1955).
4 G. Alaga and B. Jakšić, Phys. Rev. **103**, 1441 (1956).

ratios for an $E2$ transition, as expected, leading to the ground state in an even-even nucleus.

Several high-energy gamma rays were observed only with the scintillation spectrometer. These were observed in a "singles" curve and with better resolution in coincidence traces. The results are in agreement with those reported by Sunyar, except that the energies are slightly larger and one unreported gamma ray at 970 kev is observed. There appeared to be no change in the spectrum as the sample decayed. The 80.2-kev gamma is in coincidence with one at 1378 and another at about 1540 kev which is the lower energy component of a composite peak in the "singles" curve. The other component then has an energy of 1620 kev. Figure 2 shows a level scheme which is in agreement with all observed data.

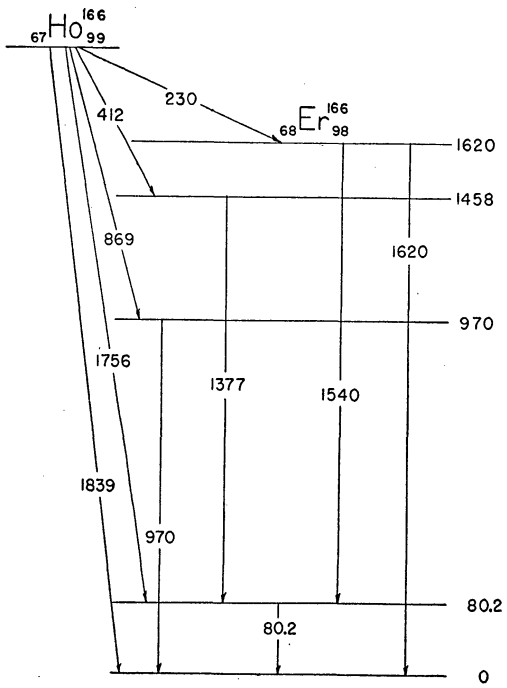

FIG. 2. Decay scheme for Ho^{166}.

Nd^{147}

Neodymium enriched in mass 146 up to 96% was irradiated in the Argonne CP-5 reactor, yielding a strong radioactivity in Nd^{147}. Following the early identification of this emitter with its half-life of 11.1 days, measurements[5] had been made of the energies of its radiations. A spectrometric study[6] by electron conversion using the separated isotope yielded eleven gamma rays. A subsequent investigation[7] of the coincidence relationships, without a precise evaluation of the gamma energies, has been made with the scintillation spectrometer.

TABLE I. Summary of results on the beta spectra of Ho^{166}.

Graham *et al.*[a] energy (kev)	Energy (kev)	Present intensity (%)	Logft
1854	1839±5	47	8.2
1771	1756±10	37	8.3
	869±20	9	7.8
393	412±20	5	7.2
	230±30	2	6.5

[a] See reference 3.

In the present work magnetic spectrometers have been used with the stronger sources now available to re-evaluate the gamma energies from electron conversion lines. Also coincidences have been observed with the scintillation spectrometer. While the results in general are in accord with previous reports, certain adjustments appear to be necessary. From electron conversion studies, three gamma rays previously reported[6] are not found, and one gamma ray at 688 kev not observed before is present.

The energies of the nine gamma rays observed are 91.3, 120.6, 198.2, 277, 321, 400, 441, 533, and 688 kev, all based on conversion electron measurements. As the sample decayed there was no noticeable change in the scintillation spectrum, indicating that these gammas are all in the same isotope. Also, the energy differences between the K and L conversion lines were observed for the 91.3-, 120.6-, 321-, and 533-kev transitions, indicating they are converted in promethium. The results of the coincidence observations are only slightly different from those shown[7] by Mandeville *et al.* The coincidence curve between the 91-kev gamma and all others was reported by those authors to show peaks at 120, 280, 320, 440, and 600 kev. There appears to be

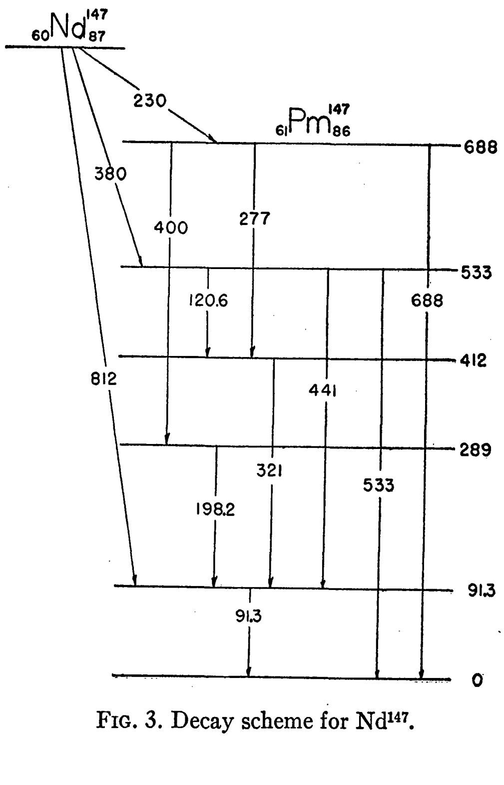

FIG. 3. Decay scheme for Nd^{147}.

[5] Hollander, Perlman, and Seaborg, Revs. Modern Phys. **25**, 469 (1953).
[6] Rutledge, Cork, and Burson, Phys. Rev. **86**, 775 (1952).
[7] Hans, Saraf, and Mandeville, Phys. Rev. **97**, 1267 (1955).

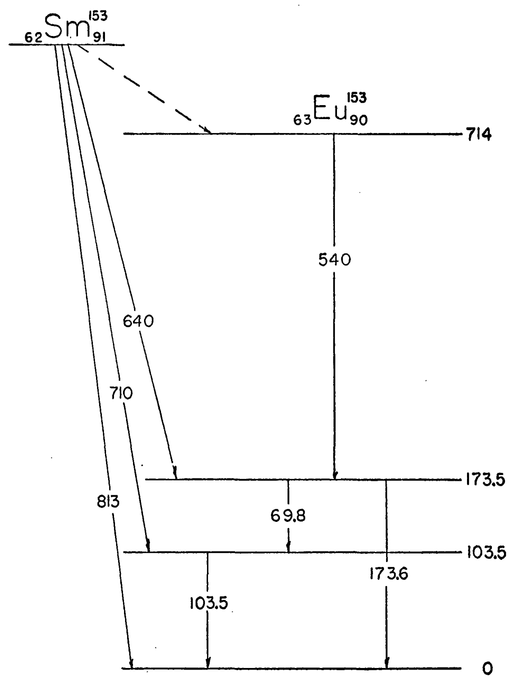

FIG. 4. Decay scheme for Sm[153].

an additional peak at 198 kev, and the one reported at 440 kev could be complex so as to include both the 400- and 441-kev gammas. These data are in complete agreement with the nuclear level scheme proposed in Fig. 3, which includes one more level than appears in previously reported schemes.

The beta decay is complex, with the maximum energy component at 812±30 kev having a relative abundance of 66%. Other components occur at 380±50 kev and 230±50 kev, with relative abundances of 18 and 16%, respectively. If other transitions occur at 500 or 600 kev, they are present to less than 2%.

Sm^{153}

Neutron capture in an enriched sample of Sm^{152} (98%) yields a strong source of radioactive Sm^{153}. The half-life was measured over 7 octaves for four samples, with a resulting value of 47.1±0.1 hours. No evidence of any short half-life contamination was found. An extremely weak long half-life contamination was found and identified by conversion electron measurements as Eu^{154}. This activity yields both beta and gamma rays, and many reports[5,8–10] have been made of their energies. A gamma ray at 84 kev has been reported. This is found, but the energy difference between its K and L conversion lines indicates that it is converted in gadolinium, so that it follows a decay from either europium or terbium rather than samarium. From the measurements on conversion electrons the more exact values of the low-energy gammas are 69.8, 103.5, and 173.6 kev. Energy differences between the K and L conversion lines indicate that the 69.8 and 103.5 transitions are in europium. A resolution of the beta spectrum on the double-focusing spectrometer yields components whose upper energies are 813±20, 710 ±20, and 640±20 kev with percentages of 20, 40, and 40, respectively, and log*ft* values of 7.3, 6.9, and 6.7, respectively. On the scintillation spectrometer another gamma ray is observed at 540 kev. The analysis of the beta spectrum indicates that the component that feeds this gamma ray is probably less than 150 kev. The coincidence measurements, similar to those of Mandeville *et al.*[9] confirm the level scheme as shown in Fig. 4.

[8] N. Marty, J. phys. radium **16**, 458 (1955).
[9] Dubey, Mandeville, and Rothman, Bull. Am. Phys. Soc. Ser. II, **1**, 164 (1956).
[10] R. L. Graham and J. Walker, Phys. Rev. **94**, 794(A) (1954).

II

DIRECTIONAL CORRELATION OF THE GAMMA RAYS IN Gd^{154}

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Directional Correlation of the Gamma Rays in Gd^{154}†

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Directional correlation measurements have been carried out on five cascades involving the gamma rays in Gd^{154}. The results of these measurements in conjunction with the work of Juliano and Stephens, and Cork *et al.*, show the existence of two rotational bands and possibly the start of two more such bands. The nature of these bands are explained in terms of the Bohr-Mottelson model for spheroidal nuclei. The levels are characterized by the quantum numbers (K, I, π) and are listed in order of increasing energy. The ground state rotational band consists of the ground state (0, 0, +) and the first and second excited states of (0, 2, +) and (0, 4, +), respectively. The 0.998-Mev and 1.130-Mev levels are assigned to a second rotational band with $K=2$ and spin and parity values 2+, 3+, respectively. The assignment of quantum numbers to the 1.400-Mev level can be made as either (2, 2, ±) with the mixture of (99.5±0.25% D, 0.5±0.25% Q) for the 1.277-Mev gamma ray, or (3, 3, ±) with the mixture of (85±2% D, 15±2% Q) for the 1.277-Mev gamma ray. The state at 1.723 Mev is best characterized by the quantum numbers (2, 2, −) with the mixture of (99.7±0.3% $E1$, 0.3±0.3% $M2$) for the 0.725-Mev gamma ray, or (3, 3, −) with the mixture of (78.5±6% $E1$, 21.5±6% $M2$) for the 0.725-Mev gamma ray.

INTRODUCTION

UNTIL rather recently, the exact nature of the decay of Eu^{154} has been obscured by the activity of Eu^{152}. Since the half-life of Eu^{154} (16±4 years) is of the same magnitude as that of Eu^{152} (13 years), separation of the two isotopes on the basis of their half-lives is virtually impossible. An investigation of the mode of decay of both of these isotopes has been carried out by Cork *et al.*[1] using sources obtained by neutron bom-

† Supported in part by the Michigan Memorial Phoenix Project and Office of Naval Research.
* Eastman Kodak Fellow.

[1] Cork, Brice, Helmer, and Sarason, Phys. Rev. **107**, 1621 (1957).

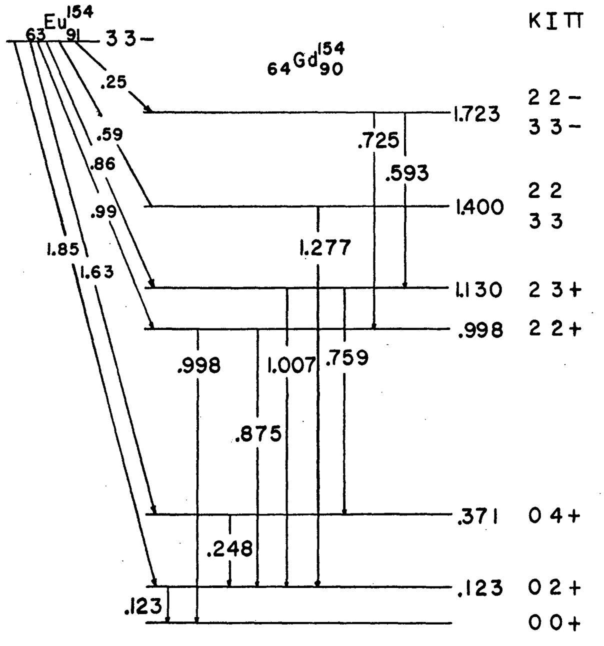

FIG. 1. Decay scheme of Eu^{154} proposed by Cork *et al.*, and by Juliano and Stephens. The proposed values of (K, I, π) are in best agreement with all existing data.

bardment of the enriched isotopes of europium. Juliano and Stephens[2] have also studied the decay of Eu^{154} using a very pure source of Eu^{154} which was obtained from fission products. The proposed level scheme of Juliano and Stephens for Eu^{154} is shown in Fig. 1. Except for a few minor discrepancies, the decay schemes which have been proposed by Cork *et al.* and Juliano and Stephens for the decay of Eu^{154} are in excellent agreement. A starting point is therefore established for the measurement of other properties of this interesting nucleus by angular correlation techniques.

The directional correlation of the 0.123-Mev gamma ray with the 0.248-Mev gamma ray has been reported by Grodzins[3] and supports a spin sequence of $4(Q)2(Q)0$ for the low-lying levels.

PROCEDURE

In the work to be described, measurements were made with a conventional "fast-slow" coincidence circuit with an effective resolving time of 3.5×10^{-8} sec. The scintillation counters consisted of 1-in.$\times1\frac{1}{2}$-in. NaI(Tl) crystals mounted on Dumont type 6292 photomultipliers. The counters were shielded frontally by $\frac{3}{16}$ in. of aluminum. Although differential discrimination was used to provide energy selection, lateral lead shielding was also employed in the measurements to eliminate coincidences due to scattering. The sources were prepared by dissolving Eu_2O_3 in a dilute solution of HCl. Although the europium was enriched in Eu^{154}, some Eu^{152} was present and prevented several of the possible correlations from being carried out. The gamma spectrum is shown in Fig. 2.

In all cases a least-squares fit of the correlation data was made to the function

$$F(\theta)=A_0'+A_2'P_2(\cos\theta)+A_4'P_4(\cos\theta). \qquad (1)$$

The error flags on the experimental points represent the root-mean-square statistical errors. The effect of finite angular resolution was computed by the Rose[4] method.

Data were obtained on five cascades and the results have been interpreted, with the aid of conversion data, in terms of the Bohr-Mottelson picture.

RESULTS

Directional Correlation of 0.123 Mev–0.248 Mev Gamma Rays

Discriminator (1) was set with a narrow window on the 0.123-Mev peak, while discriminator (2) was set with a narrow window on the 0.248-Mev peak. A least-squares fit of the data to the function $F(\theta)$ corrected for solid angle gives

$$W(\theta)=1+(0.059\pm0.007)P_2+(0.001\pm0.011)P_4.$$

This is shown by the dashed curve in Fig. 3. Nearly all the other gamma rays are in coincidence with the 0.123-Mev gamma ray, and therefore the degraded gamma rays which are produced from Compton scattering of these gamma rays will also be counted as real coincidences, and must be subtracted in order to give the true correlation function for this cascade.

This was carried out in the following manner. Discriminator (1) was kept on the 0.123-Mev peak while discriminator (2) was moved to the high side of

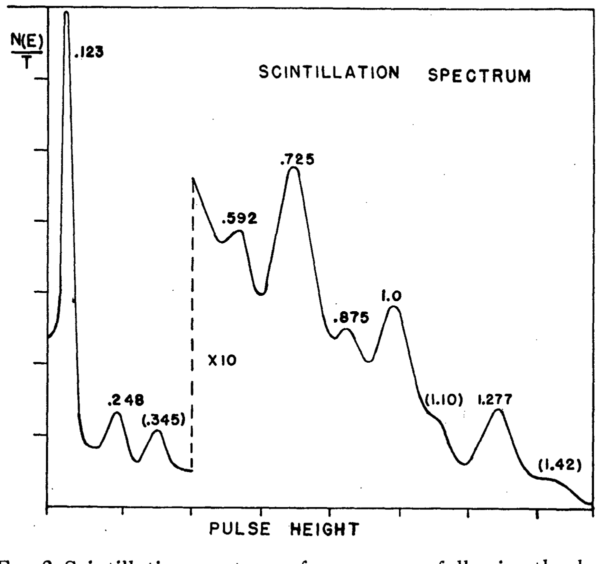

FIG. 2. Scintillation spectrum of gamma rays following the decay of Eu^{154}. The numbers listed on the peaks are the energies of the gamma rays given in Mev. The numbers in parentheses are the energies of the gamma rays which belong to the decay of Eu^{152}.

[2] J. O. Juliano and F. S. Stephens, Jr., Phys. Rev. **108**, 341 (1957).

[3] L. Grodzins (private communication) as reported by Cork *et al.*[1]

[4] M. E. Rose, Phys. Rev. **91**, 610 (1953).

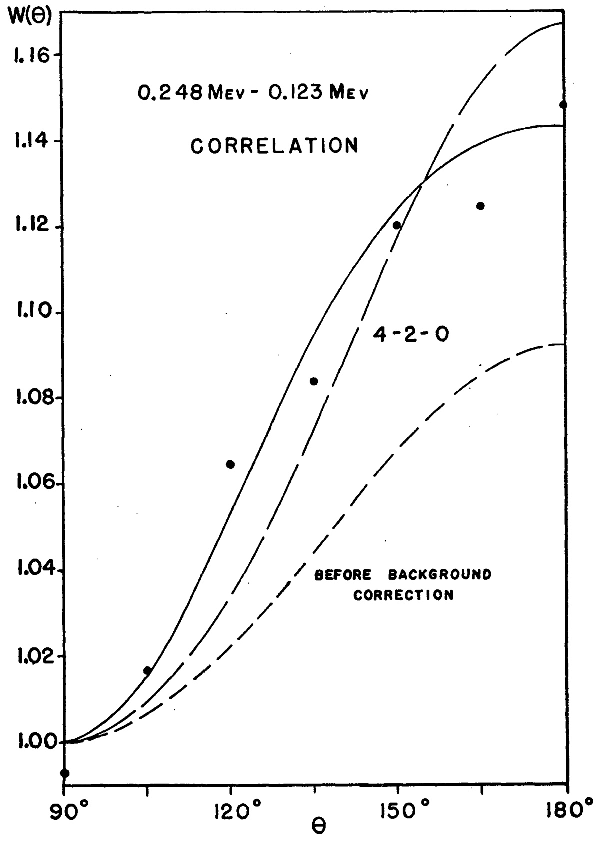

FIG. 3. Directional correlation of 0.248 Mev–0.123 Mev cascade is given by the solid curve. The experimental points are given without error flags, but are assumed to carry about 3% error. The correlation curve for this cascade before background subtraction is shown with small dashes, while the theoretical 4–2–0 curve is shown with large dashes.

the 0.345-Mev peak, which belongs exclusively to Gd^{152}. The correlation was obtained for this arrangement. The coincidences were due entirely to the Compton-scattered gamma rays from high-energy gammas, in coincidence with the 0.123-Mev gamma. This correlation function was nearly isotropic.

$$W(\theta)=1+(0.010\pm0.011)P_2+(0.023\pm0.014)P_4.$$

The result of this subtraction yields the corrected correlation function

$$W(\theta)=1+(0.098\pm0.018)P_2-(0.020\pm0.024)P_4$$

shown as the solid curve in Fig. 3, along with the corrected experimental points. The theoretical values for a $4(Q)2(Q)0$ cascade are $A_2=+0.102$ and $A_4=+0.009$. Within the experimental accuracy the data are in good agreement with $4(Q)2(Q)0$ assignment for the second and first excited states, respectively. (All even-even nuclei have a ground state spin of 0.) The conversion data[2] also supports the $E2$ character for both of these transitions. There is little doubt that this cascade is truly a $4(Q)2(Q)0$ cascade, all with even parity.

It should be noted that nearly all the gamma radiation goes through the 0.123-Mev gamma transition, while the 0.371-Mev level is fed very weakly by the other gamma rays. Therefore no correction was made for Compton-scattered gamma rays from higher energy radiations being in coincidence with the 0.248-Mev gamma ray.

Directional Correlation of 0.725 Mev–0.998 Mev Gamma Rays

One discriminator was set on the peak of the 0.725-Mev gamma with a narrow window, while the second discriminator was set with an equally narrow window to include only the 1-Mev peak. There are two gammas at this energy, but only one is in coincidence with the 0.725-Mev gamma. The result of the 0.725 Mev–0.998 Mev correlation is

$$W(\theta)=1+(0.213\pm0.025)P_2-(0.013\pm0.037)P_4.$$

The experimental points and least squares curve are shown in Fig. 4. Theoretical 6–2–0 and 5–2–0 curves are also shown in this figure. The coefficients for these two sequences are listed in Table I, along with those for a $2(D)2(Q)0$ sequence which is also in agreement with the experimental results.

The values of A_2 and A_4 obtained in this experiment are plotted on a mixture curve for a $2(D,Q)2(Q)0$ cascade in Fig. 5. Instead of plotting δ, the mixing parameter, as a function of the coefficients, a much

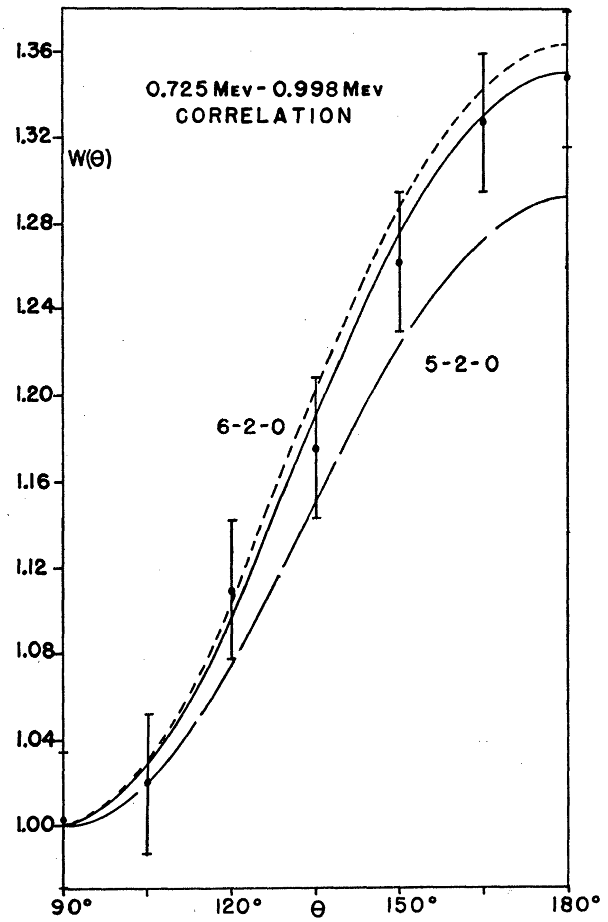

FIG. 4. Directional correlation of 0.725 Mev–0.998 Mev cascade. Theoretical 6–2–0 and 5–2–0 curves are given as broken lines.

TABLE I. "Pure" correlation functions.

Sequence	A_2	A_4
5–2–0	+0.179	−0.004
6–2–0	+0.221	−0.018
$2(D)2(Q)0$	+0.250	0

more convenient form is obtained if the substitution $Q=\delta^2/(1+\delta^2)$ is made. In this form Q represents the amount of quadrupole admixture, while $1-Q$ is the dipole admixture. The required mixture, which is in agreement with both the A_2 and A_4 data, is found to be $Q=0.3\pm0.3\%$. The 0.725-Mev gamma ray would be a mixture of 0.3% quadrupole and 99.7% dipole radiation.

The experimental data also support a $3(D,Q)2(Q)0$ sequence as shown in Fig. 6, with a mixture of $21.5\pm6\%$ Q. (It can also be fit with approximately 60% Q, but it is felt that with better statistics on the A_4 this would not be the case.)

If a spin of 4 is considered for the 1.723-Mev level, the 0.725-Mev gamma ray is required to have an octupole content of $6\pm3\%$ or $52\pm6\%$. These results are summarized in Table II.

If a spin of 2 is assumed for the 0.998-Mev level, from the correlation data alone spins of 2, 3, 4, 5, 6, can be assigned to the 1.723-Mev level. Spins of 5 and 6 are precluded on the basis of the conversion data, and will not be given further consideration. The only other assignment of spin for the 0.998-Mev level which was given consideration was a spin of 1. If this level is

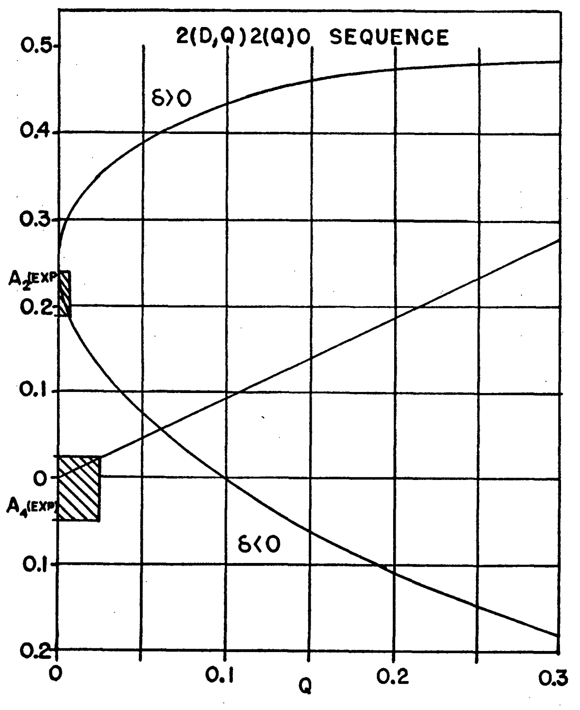

FIG. 5. A_2 and A_4 *versus* Q, the quadrupole intensity, for a $2(D,Q)2(Q)0$ sequence. The shaded areas show the values of Q consistent with the experimental values of A_2 and A_4 obtained for the 0.725 Mev–0.998 Mev correlation.

assumed to have a spin of 1, then the correlation data can be fit by spins of 1 or 2 for the 1.723-Mev level with mixture in the 0.725-Mev gamma transition. A spin of 3 to the 0.998-Mev level was not given consideration as this requires the 0.998-Mev radiation to be pure octupole. This is in disagreement with the conversion data. In addition if a spin 3 is assigned to the 0.998-Mev level, the transition to the 0.371-Mev (4+) level would be expected to compete with the 0.998-Mev radiation to the ground state (0+). The transition from the 0.998-Mev level to the 0.371-Mev level is not observed.

Directional Correlation of 0.725 Mev–0.875 Mev Gamma Rays

One discriminator was set on the 0.725-Mev peak, while the second discriminator was set on the 0.875-Mev peak. As in the other correlations, narrow windows were used in order to keep interfering radiations to a mini-

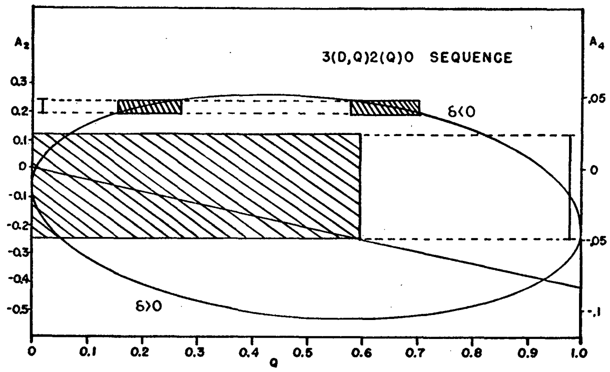

FIG. 6. A_2 and A_4 *versus* Q for a $3(D,Q)2(Q)0$ sequence. The shaded areas show the values of Q consistent with the experimental values of A_2 and A_4 obtained for the 0.725 Mev–0.998 Mev correlation.

mum. This correlation yields

$$W(\theta)=1-(0.029\pm0.016)P_2-(0.031\pm0.024)P_4.$$

These data must be corrected for Compton-scattered gamma radiation from the 0.998-Mev gamma being counted with the 0.875-Mev peak. This radiation is in coincidence with the 0.725-Mev radiation and yields the 0.725 Mev–0.998 Mev correlation. It is estimated that this interference is $30\pm10\%$. When a value of 30% interference is considered, the resulting correlation function for the 0.725 Mev–0.875 Mev correlation is

$$W(\theta)=1-(0.133\pm0.024)P_2-(0.039\pm0.037)P_4.$$

These correlation results along with 20% and 40% subtractions are shown in Fig. 7. It is seen that the interference which is caused by the 0.725 Mev–0.998 Mev coincidences changes the correlation function which is calculated for the 0.725 Mev–0.875 Mev correlation quite drastically. Although this is the case, it has been determined that the 0.875-Mev gamma ray requires an appreciable amount of quadrupole mixture, and possibly could be nearly 100% quadrupole which would be expected if a K value of 2 is assigned to the 0.998-Mev level.

Directional Correction of 1.277 Mev–0.123 Mev Gamma Rays

One discriminator was set to detect the 0.123-Mev radiation with a narrow window, while the other discriminator was set on the 1.277-Mev peak, also with a narrow window. This was done in order to minimize the amount of 1.415-Mev radiation, arising from the contamination of Eu^{152}. This interference was estimated

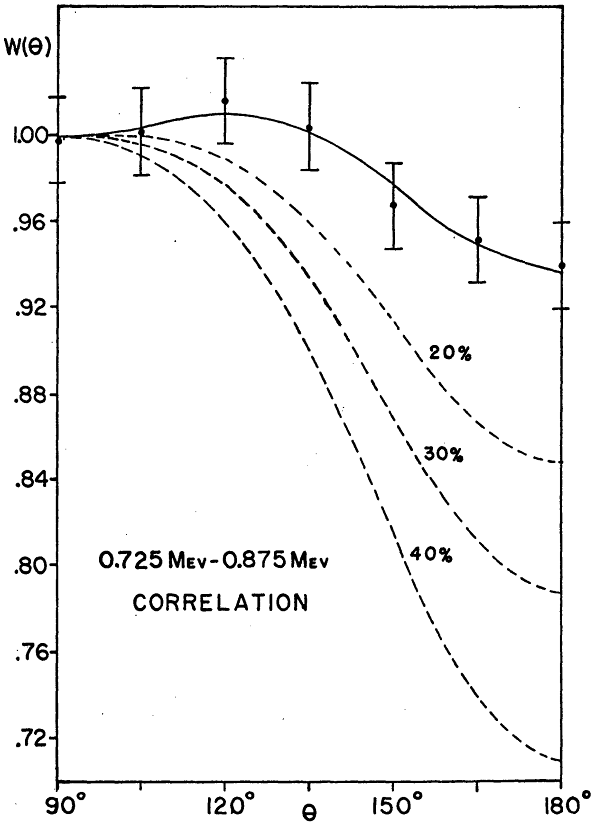

FIG. 7. Directional correlation of 0.725 Mev–0.875 Mev cascade. The dashed curves represent various amounts of interference from the 0.725 Mev–0.998 Mev correlation.

to be small and therefore no correction was made. In addition this correlation gave approximately the same correlation function as has been reported for the 1.415 Mev-0.123 Mev correlation[5] in Sm^{152}, and therefore a small contamination could not change the results to an appreciable extent. The correlation function was found to be

$$W(\theta)=1+(0.191\pm0.010)P_2-(0.007\pm0.015)P_4.$$

The experimental points along with the least-squares curve are shown in Fig. 8. The theoretical 6–2–0 and 5–2–0 curves are shown on the same figure. Reference should be made at this time to Table I. As with the correlation between the 0.998 Mev–0.725 Mev gammas, the data can also be fit by $2(D,Q)2(Q)0$, $3(D,Q)2(Q)0$ and $4(Q,0)2(Q)0$ sequences. These are shown in Figs. 9, 10, and 11, respectively. The results are summarized in Table III.

According to the directional correlation measurements, therefore, the various spin assignments to the 1.400-Mev level can be given as 2, 3, 4, 5, and possibly 6. These are based on a spin of 2 for the 0.123-Mev level. Spins of 5 and 6 are precluded on the basis of the conversion data, and will not be given further consideration.

TABLE II. Mixture content of higher order pole radiation for the 0.725-Mev gamma ray for various spin sequences.

Spin sequence	Higher order content	Percentage of higher order content
$2(D,Q)2(Q)0$	Q	0.3±0.3%
$3(D,Q)2(Q)0$	Q	21.5±6% 60% (?)
$4(Q,0)2(Q)0$	0	6 ±3% 52 ±6%

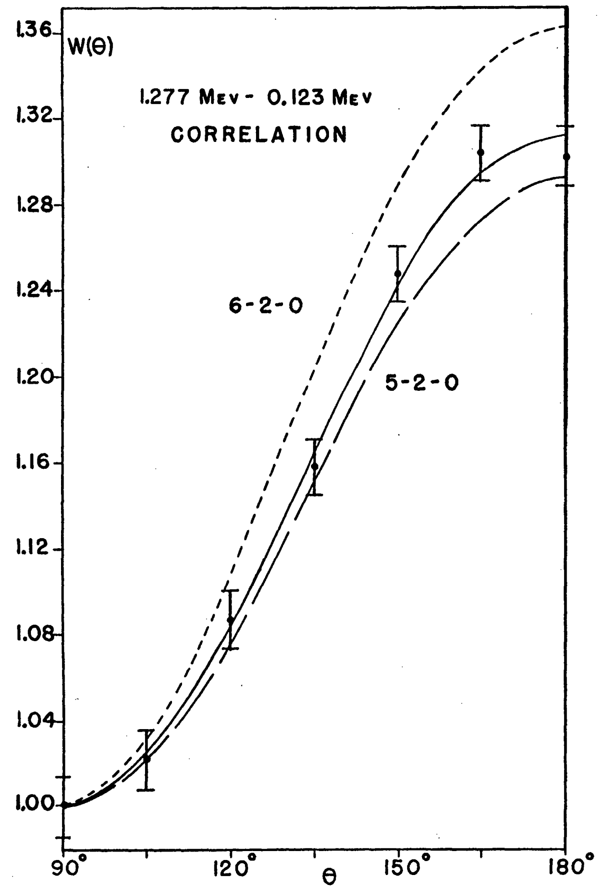

FIG. 8. Directional correlation of 1.277 Mev-0.123 Mev cascade. The dashed curves represent theoretical 6–2–0 and 5–2–0 sequences.

[5] S. Ofer, Nuclear Phys. 4, 477 (1957); R. W. Lide (private communication).

Directional Correlation of 0.592 Mev–1.007 Mev Gamma Rays

One discriminator was set on the 1-Mev peak, while the other was set on the 0.592-Mev peak. The correlation function is

$$W(\theta)=1+(0.047\pm0.016)P_2+(0.005\pm0.023)P_4.$$

This correlation had to be corrected for Compton-scattered gamma radiation produced from the 0.725-Mev gamma which are in coincidence with the 0.998-Mev gamma. This was found to contribute about 25% of the total number of true coincidences. When the subtraction was carried out, the resultant correlation function was isotropic within the errors of the coefficients.

$$W(\theta)=1-(0.008\pm0.023)P_2+(0.011\pm0.032)P_4.$$

No effort was taken to fit these results to a particular sequence. Even a 10% error assigned to the subtraction process changes the correlation function to the extent that the sign of the asymmetry becomes uncertain.

DISCUSSION

Gd^{154} may be characterized as a spheroidal nucleus although admittedly it is on the very edge of the strongly deformed nuclei with its 64 protons and 90 neutrons. Nuclei whose equilibrium shape deviates strongly from spherical symmetry can be distinguished by two types of excitation.[6–10] The first is associated with a collective motion of the nucleons. This affects only the orientation of the nucleus in space while preserving the internal structure of the nucleus. The second mode of excitation can be associated with the excitation of individual particles or with collective vibration of the nuclear shape. When one leaves the spherical nucleus and goes over to a spheroidal nucleus, the vibrational quantum number cannot be used. Another quantum number is available, however. In addition to the total angular momentum (I), and parity (π), a third quantum number labeled (K) which

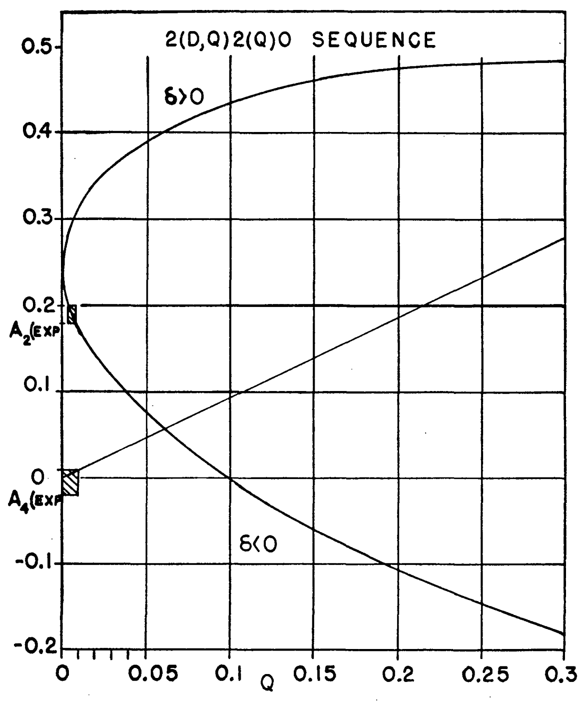

FIG. 9. A_2 and A_4 *versus* Q for $2(D,Q)2(Q)0$ sequence. The shaded areas represent the values of Q consistent with experimental A_2 and A_4 obtained for the 1.277 Mev–0.123 Mev correlation.

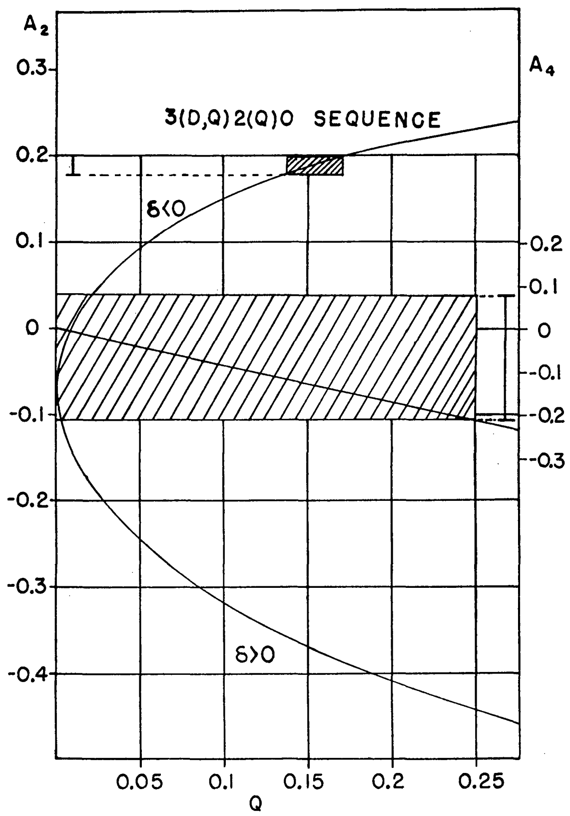

FIG. 10. A_2 and A_4 *versus* Q for a $3(D,Q)2(Q)0$ sequence. The shaded areas represent the values of Q consistent with the experimental results for A_2 and A_4 obtained for the 1.277 Mev–0.123 Mev correlation.

represents the projection of the total angular momentum on the symmetry axis may be considered. From symmetry conditions it is seen that $K=I=0$ for the ground state of even-even nuclei, and $I=0, 2, 4, \cdots$ even parity, for the ground state rotational band. The

[6] Alaga, Alder, Bohr, and Mottelson, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. 29, No. 9 (1955).
[7] Aage Bohr, *Rotational States of Atomic Nuclei* (Ejnar Munhsgaard, Copenhaven, 1954).
[8] Alder, Bohr, Huus, Mottelson, and Winther, Revs. Modern Phys. 28, 432 (1956).
[9] S. A. Moszkowski, *Handbuch der Physik* (Springer-Verlag, Berlin, 1957), Vol. 39, p. 476.
[10] A. Bohr and B. R. Mottelson, Kgl. Danske Videnskab. Selskab, Mat-fys. Medd. 27, No. 16 (1953).

energy levels in a rotational band are given by

$$E_{\rm rot}=\frac{\hbar^2}{2\tau}[I(I+1)+a(-)^{I+\frac{1}{2}}(I+\tfrac{1}{2})\delta_{K,\frac{1}{2}}], \qquad (2)$$

where τ = moment of inertia. The selection rule that is forced on K is

$$|K_i-K_f|\equiv\Delta K\leqq L. \qquad (3)$$

This rule on K is not a rigorous one, but depends upon how well the deformed nucleus can be described by a rotational wave function. Deviations from this description relax the above selection rule which then acts to retard rather than completely forbid the transition. This is called K forbiddenness:

$$\nu=\Delta K-L, \qquad (4)$$

where ν = degree of forbiddenness.

Besides the low-lying rotational states occurring in spheroidal nuclei, vibration about the equilibrium shape might be expected. The lowest order vibrations can be classified as quadrupole vibrations.

There are two basically different types of quadrupole vibration. The first is called beta (β) vibrations and

TABLE III. Mixture content of higher order pole radiation for the 1.277-Mev gamma ray for various spin sequences.

Spin sequence	Higher order content	Percentage of higher order content
2(D,Q)2(Q)0	Q	0.5±0.25%
3(D,Q)2(Q)0	Q	15 ±2%
4(Q,0)2(Q)0	0	2.5±1.5%

represents oscillation of the eccentricity about its equilibrium value, but with a preservation of axial symmetry. These vibrations carry no angular momentum about the symmetry axis. The lowest mode of β vibrations is expected to have $K=0$, 2, 4, $\cdots$, even parity. The second type of quadrupole vibration is called gamma (γ) vibrations. This type of vibration involves the oscillation of the nuclear shape about axial symmetry. These vibrations carry two units of angular momentum parallel to the symmetry axis. The rotational band corresponding to the first excited state of γ vibrations can be identified by $K=2$ and a spin sequence $I=2, 3, 4\ \cdots$, even parity. For higher modes, the initial value of I depends on the value of K since

$$|K|\leqq I.$$

Higher order vibrations, for example, octupole, may also occur. These vibrations can have $K=0, 1, 2, 3$ and may be identified by their negative parity.

Gd^{154} is on the edge of the lower side of the rotational region. The rotational, or strong-coupling region is the region where the neutrons and protons are far away from closed shells. If Eq. (2) is assumed to hold in this

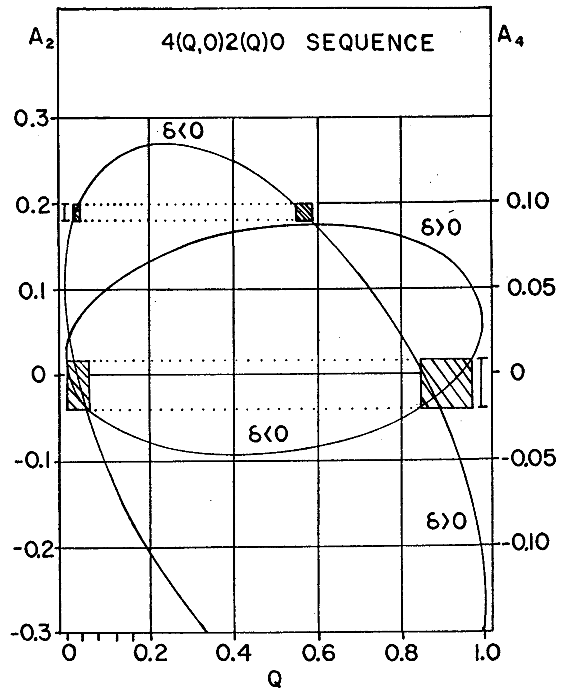

FIG. 11. A_2 and A_4 *versus* Q for a 4(Q,0)2(Q)0 sequence. The shaded areas represent the values of Q consistent with the experimental A_2 and A_4 obtained for the 1.277 Mev–0.123 Mev correlation.

case, it is found that the level at 0.371 Mev is 10% below the expected energy for the 4+ level belonging to the ground state rotational band. The 4+ character of the 0.371-Mev level is fairly certain as was seen from the correlation and conversion data.

Some of the 10% energy discrepancy between a 4+ level predicted by Eq. (2) and that observed for the 0.371-Mev level may be due to "vibrational-rotational" interaction. This "vibrational-rotational" interaction is evidenced by a vibrational energy of around 1 Mev. This is the magnitude of energy that is observed for the third excited state in Gd^{154}. From conversion data, quantum numbers (2, 2, +) and (2, 3, +) were assigned to the 0.998-Mev and 1.130-Mev levels, respectively. The quantum numbers refer, respectively, to K, I, and π.

The gamma transition probability can be expressed as

$$T=\frac{8\pi(L+1)}{L[(2L+1)!!]^2}\frac{1}{\hbar}\left(\frac{\omega}{c}\right)^{L+1}B(L), \qquad (5)$$

where $B(L)$ = reduced transitional probability. When a comparison of $B(L)$ for the emission of given multipole radiation from a state, i, to different numbers f, f' $\cdots$ of a rotational family is made, the factor in $B(L)$ involving the intrinsic wave functions is the same. A ratio which depends only on the geometrical factors is obtained:

$$\frac{B(L,I_i\rightarrow I_f)}{B(L,I_i\rightarrow I_{f'})}=\frac{\langle I_iLK_iK_f-K_i|I_iLI_fK_f\rangle^2}{\langle I_iLK_iK_f-K_i|I_iLI_{f'}K_f\rangle^2}. \qquad (6)$$

TABLE IV. Comparison of the intensities of the transitions from the third excited state to the second and first excited states, $B(2\rightarrow I_f)$, with the intensity of the transition from the third excited state to the ground state, $B(2\rightarrow 0)$, for various values of K_i.

Final state I_f	Experimental $B(2\rightarrow I_f)/B(2\rightarrow 0)$	Theoretical $B(2\rightarrow I_f)/B(2\rightarrow 0)$ $K_i=0$	$K_i=1$	$K_i=2$
2+ (123 kev)	1.94±0.27	1.43	0.357	1.43
4+ (371 kev)	not observed	2.57	1.14	0.071

By using the experimental relative intensities it is possible to compare the experimental reduced transition ratio with the theoretical values.[11] It should be noted that the Clebsch-Gordan coefficient vanishes when $\Delta K>L$.

The relative intensities which are used to compute the experimental reduced transition ratios which appear in Tables IV, VI, and VII are those reported by Juliano and Stephens.[2] The errors on the experimental reduced transition ratios were obtained assuming the intensities of the gamma transitions correct to within 10%.[2]

It is seen from Table IV that a spin of 2 and K value of 2 will fit the 0.998-Mev level very well. The theoretical reduced transition probability divided by the experimental value is equal to 74±12%. This may be considered as a rather good agreement with experimental data.

The conversion data[2] for a few of the gamma rays in Gd^{154} are given in Table V. These conversion coefficients are reported to be good to within 20–30%.

A comparison between theoretical and experimental reduced transition ratios has been carried out for the gamma rays originating from the level at 1.130 Mev. The results are shown in Table VI. Here again it appears that a value of $K=2$ is most consistent with the experimental data. The rather high experimental value compared to the theoretical value is not surprising considering the lack of information on the 759-kev gamma ray.

In all the above calculations it was assumed that all transitions occurring were $L=2$ type radiation.

Of the four gamma rays which have been under consideration, the 0.998-Mev gamma ray certainly appears to be of $E2$ character which would demand an assignment of 2+ to the 0.998-Mev level. The conversion data also support an $E2$ character to the 0.759-Mev gamma ray, and although both the 0.875-Mev and 1.007-Mev gamma rays appear, from conversion data, to be closer to $M1$ type radiation, the experimental conversion coefficients would have to be changed by less than a factor of 2 in both cases to make them compatible with $E2$ radiation. If K is considered a good quantum number, $L=1$ radiation is forbidden with a resultant increase of $E2$ radiation. It thus appears that the 0.998-Mev and 1.130-Mev levels may be considered to be members of a second rotational band which is due to a vibrational-rotational interaction of approximately 1 Mev. This band would be a γ-vibrational band with $K=2$, $I=2$, 3, and even parity. The energy difference of 0.132 Mev between these two levels is very close to the 0.123-Mev separation of the first excited state from the ground state. This would imply a value for the moment of inertia for this second band close to the ground state moment of inertia. The transition from the 1.130-Mev level to the 0.998-Mev level is calculated to be too weak to be observed. This transition has not been found. The state at 0.998 Mev could possibly be assigned a spin of 1 although the conversion data do not seem to fit either an $M1$ or $E1$ as well as they do an $E2$. Based on the observed intensities of the gamma rays originating from the level at 1.130 Mev, the spin assignment of 3 to the 1.130-Mev level seems to be favored. The conversion data is in best agreement with a positive parity assignment. The assignment of 2+ to the 0.998-Mev level and 3+ to the 1.130-Mev level might be expected from the systematics in this rotational region. A few other deformed nuclei in this region have been found to exhibit two states lying about 1 Mev above the ground state and having the same energy separation as exists between the first excited state and the ground state.[12] The classification of these two states has been made as 2+, 3+, respectively, in the order of increasing energy. The possibility of spins of 2 or 4 to the 1.130-Mev level cannot be completely eliminated. Because of the contamination of Eu^{152} which was present in the source used in the correlation study, the directional correlation of the 1.007 Mev-0.123 Mev gamma rays was not attempted. If this correlation were done with a "pure" source of Eu^{154}, a more unique assignment of quantum numbers to the 1.130-Mev level might be made.

The correlation data are consistent with spins of 2, 3, and 4, for the 1.400-Mev state. The results are given in Table III. A 4− interpretation is not in agreement with the conversion data. A 4+ interpretation would require an $E2$-$M3$ mixture. This would require an $M3$ transition probability greatly enhanced over the single-

TABLE V. K-conversion coefficients for gamma rays in Gd^{154}.

Energy (kev)	α_K(exp)	$\alpha_K(E1)$	$\alpha_K(E2)$	$\alpha_K(M1)$	$\alpha_K(M2)$
759	$>5.0\times10^{-3}$	1.9×10^{-3}	4.9×10^{-3}	9×10^{-3}	2.4×10^{-2}
875	5.8×10^{-3}	1.4×10^{-3}	3.3×10^{-3}	5.8×10^{-3}	1.5×10^{-2}
998	2.1×10^{-3}	1.1×10^{-3}	2.4×10^{-3}	4.2×10^{-3}	1.0×10^{-2}
1008	4.4×10^{-3}				

[11] A. Simon, Oak Ridge National Laboratory Report ORNL-1718, 1954 (unpublished).

[12] Murray, Boehm, Marmier, and DuMond, Phys. Rev. 97, 1007 (1955); Bjørnholm, Nathan, Nielsen, and Sheline, Nuclear Phys. 4, 313 (1957); Rasmussen, Stephens, and Strominger, Phys. Rev. 99, 47 (1955); O. Nathan, Nuclear Phys. 4, 125 (1957).

particle estimate. Such a mixture, while not common, cannot be eliminated completely.

It appears that the level at 1.400 Mev is in best agreement with the directional correlation data, the observed intensities, and the conversion data, if spins of 2 or 3 are considered for this level. The directional correlation results (Table III) show that with either the assignment of $I=2$ or 3 to the 1.400-Mev level, the 1.277-Mev gamma ray is mainly dipole radiation (less than 17% quadrupole). If the most probable assignment of K and I to this state is given as (2, 2) or (3, 3), it is seen that in either case dipole radiation should be K-forbidden for the 1.277-Mev gamma ray. It appears that there is at least a partial breakdown of K as being a good quantum number.

From the correlation data, it has been shown that spins of 2, 3, and 4 should be considered as possibilities for the 1.723-Mev level (Tables I and II). Spins and K values of (2, 2), (3, 3) and (4, 4) are in agreement with the experimental data. This is shown in Table VII.

The correlation data show that in order for spin 4 to be assigned to the level at 1.723 Mev, an appreciable octupole content would have to be included in the 0.725-Mev radiation. This is not in agreement with the conversion data which show the 0.725-Mev radiation to be either $E1$ or $E2$. With a spin 2 or 3 for this level, K can be no larger than 2 or 3, respectively. Dipole radiation to the levels at 0.998 Mev and 1.130 Mev is not K-forbidden. Dipole radiation is expected to predominate. This is confirmed from the correlation results (Table II). Since the conversion data on the 0.725-Mev radiation favors either $E1$ or $E2$ over that of $M1$ or $M2$, the negative parity seems to be favored for the 1.723-Mev level. This requires the "unfavored" mixture between $E1$ and $M2$. For a 2− assignment to this level, the mixture will be (99.7%$E1$-0.3%$M2$) for the 0.725-Mev gamma ray. The 3− assignment requires a mixture of (78.5%$E1$-21.5%$M2$) for the 0.725-Mev gamma ray.

From the high logft values, it appears that K forbiddenness is of great importance. It has been predicted that Eu^{154} has (3, 3, −) for the quantum numbers describing the ground state.[1] Recently the ground state has been measured to be 3.[13] This could account for the lack of a beta transition to the (0, 0, +) member and low percentages to the other two members of the $K=0$ band. The logft values get smaller and the percentages of the beta transitions get larger for the higher excited states. A value of $K=2$ for the 1.130-Mev and 0.998-Mev levels is in good agreement with this. The beta-decay data give a percentage of 42% to the 1.400-Mev level, and 28% to the 1.723-Mev level. This suggests a K value of 2 or larger for these states, and therefore agrees with the possible assignments that have been listed for these two levels. If the negative parity is maintained for either of these states, the level might be interpreted as lying in an octupole vibration band.

It has been pointed out by Juliano and Stevens[2] that K forbiddenness could explain the high logft values to ground state rotational band members as ΔK would be equal to 3. Hence, so far as K is a good quantum number, these transitions would be second or higher forbidden. However, such is not the case for the beta transitions leading to the higher excited states. The values of ΔI and ΔK for these transitions indicate that the beta transitions should be either allowed or first forbidden. Although the logft values do get smaller for these higher excited states, the smallest logft value observed is around 9, instead of 7 or less. The quantum number K appears to be a good quantum number for some of the data obtained on this radioactive nucleus, since some of those data seem to involve at least a partial breakdown of the K selection rules.

TABLE VI. Comparison of the intensity of the transition from the fourth excited state to the second excited state, $B(3\rightarrow 4)$, with the intensity of the transition from the fourth excited state to the first excited state, $B(3\rightarrow 4)$, for various values of K_i.

Final state I_f	Experimental $B(3\rightarrow I_f)/B(3\rightarrow 2)$	Theoretical $B(3\rightarrow I_f)/B(3\rightarrow 2)$ $K_i=0$	$K_i=1$	$K_i=2$
4 (371 kev)	<0.85±0.12	$B(3\rightarrow 4)=0$ $B(3\rightarrow 2)=0$	2.50	0.40

TABLE VII. Comparison of the intensity of the radiation from the 1.723-Mev level to the 0.998-Mev level with that to the 1.130-Mev level, assuming various values of K and I for the 1.723-Mev level. The transitions were assumed to be either both dipole or both quadrupole radiation.

K_i	I_i	L_1	L_2	$B(I_i\rightarrow 2)/B(I_i\rightarrow 3)$ Theoretical	Experimental
4	4	2	2	1.79	1.91±0.27
3	3	1	1	2.86	2.86±0.40
3	3	2	2	0.86	1.91±0.27
3	4	2	2	7.14	1.91±0.27
2	4	2	2	0.45	1.91±0.27
2	3	2	2	∞	1.91±0.27
2	3	1	1	0.71	2.86±0.40
2	2	1	1	2.00	2.86±0.40
2	2	2	2	0.57	1.91±0.27

[13] Abraham, Kedzie, and Jeffries, Phys. Rev. **108**, 58 (1957).

III

DIRECTIONAL CORRELATION OF GAMMA RAYS IN Ge^{72}

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Directional Correlation of Gamma Rays in Ge^{72}†

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Directional correlation measurements have been made on the 0.63–0.835 Mev, 2.20–0.835 Mev, 2.49 and 2.51–0.835 Mev, and 1.88–1.46 Mev gamma-gamma cascades in Ge^{72} following beta decay of 14.2-hour Ga^{72}. The level at 0.835 Mev has a spin and parity assignment of 2+ and proceeds to the 0+ ground state by pure electric quadrupole radiation. The 0.63-Mev transition proceeds from a 2+ level at 1.46 Mev to the 0.835-Mev level by radiation which is largely electric quadrupole with a small magnetic dipole admixture. The correlation data are consistent with spin assignments of 2 or 3 for the levels at 3.04, 3.32, and 3.34 Mev.

I. INTRODUCTION

THE decay of Ga^{72} has been the subject of many investigations.[1] A portion of the decay scheme proposed by Kraushaar *et al.* is shown in Fig. 1. Important features of the level structure of Ge^{72} were confirmed by studies of the decay of As^{72}.[2]

The first excited state of ${}_{32}Ge_{40}{}^{72}$ has spin zero and even parity. This represents a departure from the usual 2+ first excited state found in even-even nuclei. The only other known exceptions (O^{16}, Ca^{40}, Zr^{90}, Pb^{208}) result when both neutrons and protons form closed shells. In view of this peculiarity it was felt that direct measurement of the spins and parities of some of the higher levels might be of value in establishing the relation of Ge^{72} to the systematics of other nuclei in this region.

II. EXPERIMENTAL METHOD

A conventional fast-slow coincidence circuit with a resolving time of 1×10^{-8} second was employed in the angular correlation measurements.[3] A 4-in.×5-in. NaI(Tl) crystal and DuMont 6364 photomultiplier were used to detect the 1.88-Mev gamma ray in the weak 1.88 Mev–1.46 Mev correlation. The detectors in all other cases consisted of 2-in.×2-in. NaI(Tl) crystals mounted on RCA 6342 photomultipliers. Energy selection was provided by differential analyzers.

The Ga^{72} sources were obtained from Oak Ridge National Laboratory as gallium chloride in a dilute HCl solution. No interfering activities were present in the source material.

The half-life of the 0.835-Mev level has been measured to be $(3.2\pm0.8)\times10^{-12}$ sec[4] and the half-life of the 1.46-Mev level is expected to be short. This, together with the fact that the source was in dilute solution, eliminates the possibility of attenuation of the correlation function due to extranuclear fields.

Data were taken in a double quadrant sequence. The real coincidence rate was normalized to correct for source decay and electronic drift. A least-squares fit of the data was made to the function

$$W'(\theta)=\alpha_0+\alpha_2P_2(\cos\theta)+\alpha_4P_4(\cos\theta).$$

The resultant expansion coefficients were then normalized and corrected for finite angular resolution by a collimated beam method.[5] This yielded a correlation function of the form

$$W(\theta)=1+(A_2\pm\sigma_2)P_2(\cos\theta)+(A_4\pm\sigma_4)P_4(\cos\theta).$$

The σ_2 and σ_4 are the root-mean-square errors as defined by Rose, Eq. (30).[6]

† Supported in part by the Michigan Memorial Phoenix Project and the Office of Naval Research.
* Dow Chemical Company Fellow in Physics.
[1] Kraushaar, Brun, and Meyerhof, Phys. Rev. **101**, 139 (1956), and references cited therein.
[2] Brun, Kraushaar, and Meyerhof, Phys. Rev. **102**, 808 (1956).
[3] Stewart, Scharenberg, and Wiedenbeck, Phys. Rev. **99**, 691 (1955).
[4] F. R. Metzger, Phys. Rev. **101**, 286 (1956).
[5] A. M. Feingold and S. Frankel, Phys. Rev. **97**, 1025 (1955).
[6] M. E. Rose, Phys. Rev. **91**, 610 (1953).

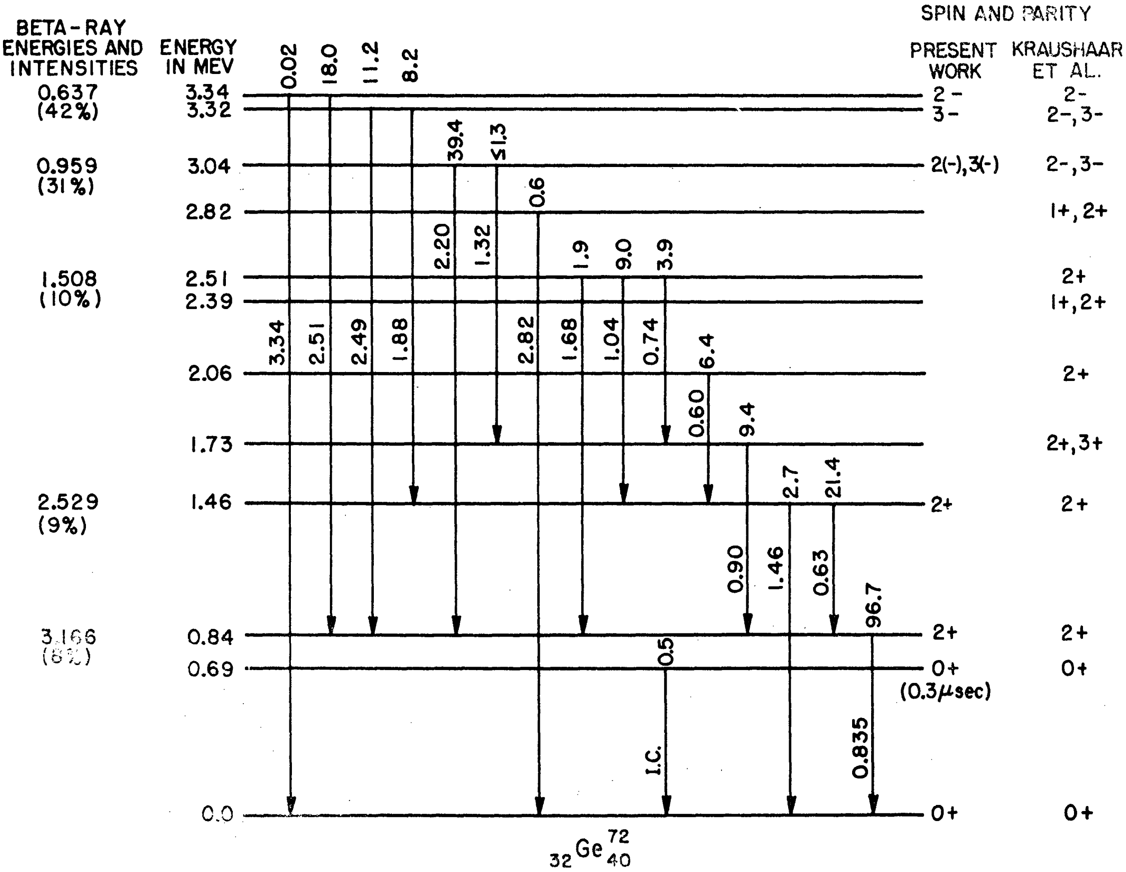

FIG. 1. Partial decay scheme of Ga^{72} according to Kraushaar *et al.*[1] Energies are given in Mev and intensities in percent. Some of the uncertain gamma rays and transitions involving the 0+ first excited state have been omitted. The spin and parity assignments of Kraushaar *et al.* are based on relative intensity considerations.

III. RESULTS

0.63 Mev–0.835 Mev Correlation

This correlation was measured by accepting all coincidences in a narrow portion of the scintillation photopeak for each of the gamma rays. However, owing to the complexity of the decay scheme, a fair percentage of these coincidences result from other interfering cascades. It was found that coincidences between the 0.835-Mev gamma ray and the Compton distribution of higher energy gamma rays resulted in 20.5±2.1% of the total real coincidences. Similarly, coincidences between the 0.63-Mev gamma ray and interfering Compton distributions were found to comprise 23.4 ±2.5% of the total coincidences. The interfering correlations were measured and subtracted from the main correlation function. The resulting expansion coefficients were found to be $A_2=-0.145\pm0.040$ and $A_4=+0.314 \pm0.065$. It should be noted that the corrected expansion coefficients still contain contributions from coincidences between the 0.835-Mev gamma ray and the weak 0.60-Mev and 0.74-Mev gamma rays.

Metzger has shown that the 0.835 level has a spin of 2 by studying the angular distribution of resonance radiation from this level.[4] It was found that the present experimental data could not be fit by sequences of the form $4(Q)2(Q)0$, $3(D,Q)2(Q)0$, or $1(D,Q)2(Q)0$. The data are in agreement with a $2(D,Q)2(Q)0$ sequence in which the $2(D,Q)2$ transition (0.63-Mev gamma ray) is mostly quadrupole with a small dipole admixture. The experimental coefficients correspond to a quadrupole intensity for the 0.63-Mev transition of $Q\geqslant0.975$. If reasonable values are assumed for the maximum contribution from the interfering 0.60 Mev–0.835 Mev and 0.74 Mev–0.835 Mev cascades, the amended coefficients still demand a quadrupole intensity of $Q\geqslant0.90$.

2.49 and 2.51 Mev–0.835 Mev Correlation

Hedgran and Lind have successfully resolved the 2.51- and 2.49-Mev gamma rays and assigned a relative intensity of 8 to 5.[7] The directional correlation was measured by accepting the full-energy peak of the combined 2.49-Mev and 2.51-Mev gamma rays in one differential analyzer and the 0.835-Mev gamma ray in the other. The experimental expansion coefficients were found to be $A_2=+0.163\pm0.031$ and $A_4=-0.133 \pm0.046$. As has been pointed out,[1] the relative weakness of the ground-state transition (although assigned an energy of 3.34 Mev, the crossover gamma ray may originate from the other level) precludes the possibility of a spin 1 assignment for the levels at 3.32 Mev and 3.34 Mev. Assuming reasonable errors for the relative intensities of these cascades, all possible combinations of spins 2, 3, or 4 were considered as assignments for the 3.32-Mev and 3.34-Mev levels. The experimental data were found to be consistent only with spin 2 for

[7] A. Hedgran and D. Lind, Arkiv Fysik 5, 177 (1952).

one level and spin 3 for the other (in either order), with spin 3 for both levels, or with spin 3 for one level and spin 4 for the other. Relative intensities require that the ground-state transition be quadrupole radiation. Hence it must originate from a state of spin 2 and the correlation data then require a spin of 3 for the other state.

2.20 Mev–0.835 Mev Correlation

A portion of the real coincidences accepted in this correlation was due to the Compton and escape peaks of the 2.49-Mev and 2.51-Mev gamma rays. A fair estimate of the contribution ($23.7\pm2.4\%$) was obtained by moving the window of the differential analyzer from the 2.20-Mev photopeak to the valley between it and the 2.50-Mev photopeak and comparing coincidence rates. Although the assumptions involved are crude, the resultant coefficients are not very sensitive to the actual contribution. After correction for the interfering cascades, the experimental coefficients were found to be $A_2=+0.250\pm0.024$ and $A_4=-0.008\pm0.032$. These are in excellent agreement with the theoretical coefficients for a $2(D)2(Q)0$ sequence. The experimental coefficients will also fit a $3(D,Q)2(Q)0$ sequence with $0.23\leqslant Q\leqslant 0.32$ for the mixed transition.

1.88 Mev–1.46 Mev Correlation

This correlation was measured by accepting the full-energy peak of each gamma ray in separate differential analyzers. There were no interfering cascades. The resultant coefficients, $A_2=+0.010\pm0.051$ and $A_4=-0.055\pm0.074$ are consistent with sequences of the form $1(D,Q)2(Q)0$, $2(D,Q)2(Q)0$, or $3(D,Q)2(Q)0$. (The spin of the 1.46-Mev level has been assumed to be 2 from the results of the 0.63 Mev-0.835 Mev correlation.) The absence of a strong crossover transition eliminates the first possibility.

IV. DISCUSSION

The ground-state nuclear angular momentum of Ga^{72} has been shown to be 3.[8] An odd-parity state is to be expected. Kraushaar *et al.* have summarized the log*ft* values of the five known beta transitions to Ge^{72}.[1] The spins inferred from the beta decay are in excellent agreement with values obtained above from angular correlation measurements.

Van Patter has recently summarized the systematics of even-even nuclei in this region.[9] The energy ratio E_2/E_1 of the second to the first excited state was shown to be less than 2 over the range $30\leqslant N\leqslant 40$. If the 0+ first excited state in $_{32}Ge_{40}{}^{72}$ is omitted, the energy ratio of the next two states is 1.75. Over the range of $32<N<50$, the spin of the second excited state is 2+ in all known cases, omitting the low-lying 0+ states in Ge^{70} and Ge^{72}. The enhancement of the electric quadrupole transition probability in the $2-2$ transition which was found in the present case is also characteristic of the $2-2-0$ cascades in other nuclei in this region.

The present work did not include any direct measurements involving the 0+ first excited state in Ge^{72}. However, it has been shown that various features of the Ge^{72} level structure are in good agreement with the systematics of even-even nuclei in this region if the 0+ state is ignored. The probable absence of a low-lying state of spin 4 would argue against the possibility that the 0+ state arises from the splitting of a degenerate level. One might speculate that the $p_{\frac{1}{2}}$ neutron shell and the $p_{\frac{3}{2}}$ proton shell are filled in Ge^{72} and the excitation to the 0+ state may result from raising a proton pair or a neutron pair to a higher configuration.[10]

ACKNOWLEDGMENTS

The authors wish to acknowledge the help of E. G. Funk, Jr., and R. E. Sund in recording some of the data.

[8] L. S. Goodman and W. J. Childs, Bull. Am. Phys. Soc. Ser. II, 3, 21 (1958).
[9] D. M. Van Patter, Bull. Am. Phys. Soc. Ser. II, 3, 212 (1958).
[10] G. Scharff-Goldhaber, Phys. Rev. 90, 587 (1953).

IV

ANALYSIS OF GAMMA-GAMMA ANGULAR CORRELATIONS INVOLVING MULTIPOLE MIXTURES

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Analysis of Gamma-Gamma Angular Correlations Involving Multipole Mixtures*

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A method is outlined which provides for rapid analysis of directional correlations involving gamma rays of mixed multipolarity. Specific examples are given to illustrate the application to cascades in which one or both transitions involve a mixture of dipole and quadrupole radiation.

I. INTRODUCTION

WITH the application of angular correlation techniques to more complex decay schemes, gamma rays which are not of pure multipole order have come to be expected. The interpretation of angular correlation data becomes more complex in these cases. Various methods have been suggested to facilitate the analysis when one transition is pure and the other mixed.[1,2] The present graphical method of analysis has been extended to include the case in which neither gamma ray is pure.

II. MIXED MULTIPOLARITY IN GAMMA-GAMMA ANGULAR CORRELATION

Denote j_1, j, and j_2 as the initial, intermediate, and final momenta of a cascade in the gamma decay of a nucleus. Let the transition between j_1 and j be a mixture of 2^{L_1} and 2^{L_1+1} poles. Similarly describe the second transition (from j to j_2) as a mixture of 2^{L_2} and 2^{L_2+1} poles. The angular correlation function will then be of the form

$$W(\theta)=\sum_{k\text{ even}} \alpha_k^{(1)}\alpha_k^{(2)}P_k(\cos\theta), \tag{1}$$

in which the constant $\alpha_k^{(\nu)}$ for the νth transition is dependent only upon the spins and multipolarities associated with that transition. The $\alpha_k^{(\nu)}$ are of the form

$$\alpha_k^{(\nu)}=F_k(L_\nu,L_\nu,j_\nu,j)+2\delta_\nu F_k(L_\nu,\, L_\nu+1,\, j_\nu,\, j) +\delta_\nu^2 F_k(L_\nu+1,\, L_\nu+1,\, j_\nu,\, j). \tag{2}$$

The summation over k in (1) runs from zero to the least of $2j$, $2(L_1+1)$, or $2(L_2+1)$. The $F_k(L,L',j',j)$ are the F-coefficients as defined and tabulated by Ferentz and Rosenzweig.[3,4] δ_ν^2 is the ratio of the intensity of the $L_\nu+1$ radiation to the L_ν radiation in the νth transition of the cascade.

The least-squares fit of experimental data in an angular correlation experiment[5] is ordinarily made to the function

$$W'(\theta)=\alpha_0+\alpha_2P_2(\cos\theta)+\alpha_4P_4(\cos\theta). \tag{3}$$

Upon normalization, this takes the form

$$W(\theta)=1+B_2A_2P_2(\cos\theta)+B_4A_4P_4(\cos\theta). \tag{4}$$

The B_k are attenuation factors depending upon the geometry of the experiment, especially detector solid angle, and

$$A_k=(\alpha_k^{(1)}/\alpha_0^{(1)})(\alpha_k^{(2)}/\alpha_0^{(2)}). \tag{5}$$

Owing to the properties of the F-coefficient for $k=0$, Eq. (5) becomes

$$A_k=A_k^{(1)}A_k^{(2)}=[\alpha_k^{(1)}/(1+\delta_1^2)][\alpha_k^{(2)}/(1+\delta_2^2)]. \tag{6}$$

If Q_ν is defined as the fraction of the $L_\nu+1$ multipole in the νth (mixed) transition, Q_ν then bears the following relation to δ_ν:

$$Q_\nu=\delta_\nu^2/(1+\delta_\nu^2). \tag{7}$$

Then $1-Q_\nu$ is the fraction of the L_ν multipole in the same transition and the $A_k^{(\nu)}$ become

$$A_k^{(\nu)}=a_k^{(\nu)}(1-Q_\nu)+b_k^{(\nu)}[Q_\nu(1-Q_\nu)]^{\frac{1}{2}}+c_k^{(\nu)}Q_\nu, \tag{8}$$

where

$$\begin{aligned} a_k^{(\nu)}&=F_k(L_\nu,L_\nu,j_\nu,j),\\ b_k^{(\nu)}&=2F_k(L_\nu,\, L_\nu+1,\, j_\nu,\, j),\\ c_k^{(\nu)}&=F_k(L_\nu+1,\, L_\nu+1,\, j_\nu,\, j). \end{aligned} \tag{9}$$

III. CASCADES WITH DIPOLE-QUADRUPOLE MIXTURE IN ONE TRANSITION

Equation (6) takes on an especially simple form if one transition is a mixture of dipole and quadrupole radiation and the other is of pure multipole order. If, for example, the mixture is in the first transition, one has

$$A_2=A_2^{(1)}A_2^{(2)}=\{a_2^{(1)}(1-Q_1)+b_2^{(1)}[Q_1(1-Q_1)]^{\frac{1}{2}} +c_2^{(1)}Q_1\}a_2^{(2)}, \tag{10}$$

$$A_4=A_4^{(1)}A_4^{(2)}=\{c_4^{(1)}Q_1\}a_4^{(2)}. \tag{11}$$

A plot of $A_2^{(1)}$ *vs* Q_1 from Eq. (10) is an ellipse. Equation (11) is a straight line in the variables $A_4^{(1)}$ and Q_1. Single-transition mixture curves of this type have been tabulated for all transitions involving integer spins up

* Supported in part by the Michigan Memorial Phoenix Project and the Office of Naval Research.
† Dow Chemical Company Fellow in Physics.
[1] Jastram, Wood, and Hurley, Bull. Am. Phys. Soc. Ser. II, 3, 65 (1958).
[2] C. F. Coleman, Nuclear Phys. 5, 495 (1958).
[3] M. Ferentz and N. Rosenzweig, Argonne National Laboratory Report ANL-5324, 1955 (unpublished).
[4] These are related to the definition of L. C. Biedenharn and M. E. Rose [Revs. Modern Phys. 25, 729 (1953)] as follows:

$$F_k(Lj'j)=F_k(L,L,j',j),$$
$$(-1)^{j'-j-1}[(2L+1)(2L'+1)(2j+1)]^{\frac{1}{2}}G_k(LL'j'j)=F_k(L,L',j',j).$$

[5] M. E. Rose, Phys. Rev. 91, 610 (1953).

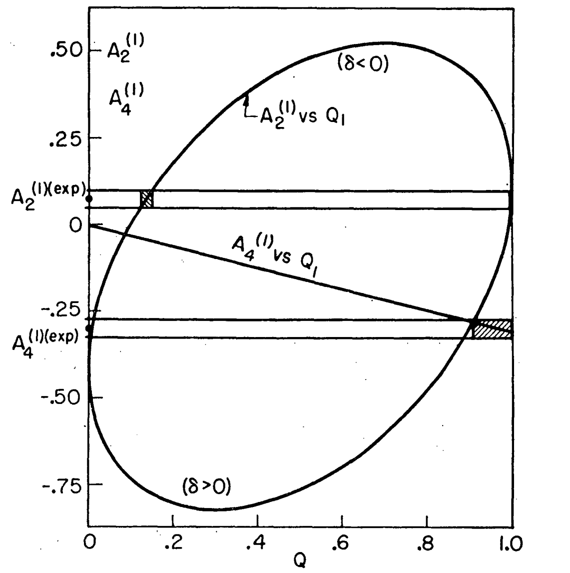

FIG. 1. $A_2^{(1)}$ and $A_4^{(1)}$ (same scale) *vs* Q_1 for a $2(D,Q)2$ transition in Se^{76}. The experimental coefficients have been divided by the appropriate F-coefficients for a $2(Q)0$ transition.

to 6 and half-integer spins up to 11/2.[6] The least-squares fit will yield values of $A_2^{(\exp)}\pm\sigma_2$ and $A_4^{(\exp)}\pm\sigma_4$. The σ_k's are the mean square errors defined by Rose, Eq. (30).[5] In order to determine the quadrupole content of the mixed transition it becomes convenient to divide the $A_2^{(\exp)}\pm\sigma_2$ and $A_4^{(\exp)}\pm\sigma_4$ by the corresponding $a_2^{(\nu)}$ and $a_4^{(\nu)}$ from the unmixed transition. The resultant numbers are then to be compared with the single-transition mixture curves described above in order to find the range of Q consistent with the experiment. Often it is desirable to know the mixing in terms of δ. In each case where $b_k^{(\nu)}$ [as defined in Eq. (9)] is positive, the upper portion of the ellipse corresponds to δ positive and vice versa.

This method is illustrated for a $2(D,Q)2(Q)0$ sequence in Fig. 1. The experimental coefficients, $A_2=-0.042\pm0.015$ and $A_4=0.319\pm0.023$, were measured by Funk and Wiedenbeck[7] for the 650 kev–550 kev cascade in Se^{76}. The graphical solution indicates a $Q_1=99.7\pm0.2\%$ for the $2(D,Q)2$ transition.

IV. CASCADES WITH MIXTURES IN BOTH TRANSITIONS

If both steps of a cascade are mixtures of dipole and quadrupole radiation, both $A_2^{(1)}$ and $A_2^{(2)}$ have the form of Eq. (8). A_4 reduces to

$$A_4=A_4^{(1)}A_4^{(2)}=(c_4^{(1)}Q_1)(c_4^{(2)}Q_2). \quad (12)$$

The set of values of the $A_k^{(\nu)}$ satisfying

$$A_k^{(1)}A_k^{(2)}=A_k^{(\exp)}\pm\sigma_k \quad (13)$$

[6] R. G. Arns and M. L. Wiedenbeck, University of Michigan Engineering Research Institute, Technical Report 2375-3-T, January, 1958 (unpublished).

[7] E. G. Funk, Jr., and M. L. Wiedenbeck, Phys. Rev. **109**, 922 (1958).

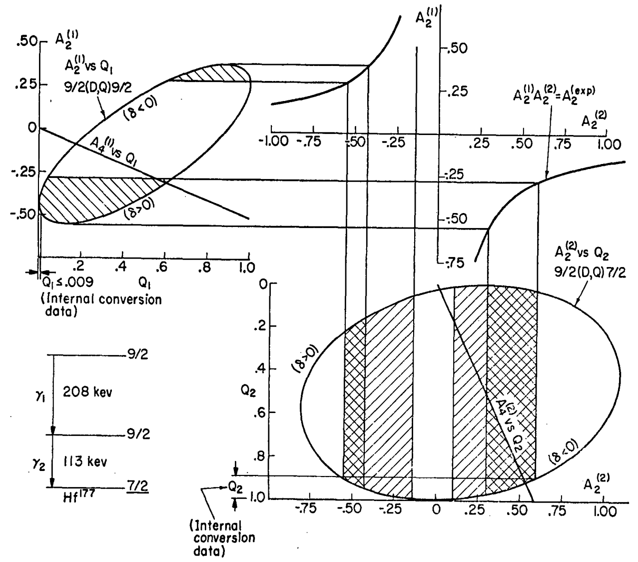

FIG. 2. Analysis of angular correlation data for a $9/2(D,Q)9/2(D,Q)7/2$ sequence in Hf^{177}. The quadrupole content for the second step of the cascade is known (entire cross-hatched area, lower right). The doubly cross-hatched area in the $9/2(D,Q)7/2$ curve corresponds to the values of Q_2 allowed by the correlation (within the Q_2 conversion data limits) provided that the sequence is of the form $9/2(D,Q)9/2(D,Q)7/2$. The experimental angular correlation data are represented by equilateral hyperbolae. (Because of the unusually small experimental error, the area between the hyperbolas is not evident in this case.)

can be calculated. These can be plotted as an area between equilateral hyperbolas. Various single transition mixture curves can be placed with scales to coincide with the $A_k^{(\nu)}$ axes of the experimental graph [i.e., graph of Eq. (13)]. Then a range of Q_1 consistent with the first transition will correspond to a range of values of Q_2 for the second transition required by the experimental graph and vice versa. Additional information is always needed in order to obtain unique assignments from angular correlations involving double mixtures. Often limits can be placed on the mixture in one transition from internal conversion data or from other angular correlation measurements. This information can then be incorporated into the graphical analysis.

Application of this method to a typical doubly mixed sequence is shown in Fig. 2. The experimental coefficient, $A_2=-0.1614\pm0.0015$, was measured by Klema for the 208 kev–113 kev cascade in Hf^{177}.[8] Auxiliary information is available in the form of K-conversion coefficients measured by McGowan *et al.*,[9] and Marmier and Boehm.[10] These are shown in Table I. The ground-state spin is known to be 7/2.[11] The 113-kev level has been reached by Coulomb excitation and a spin of 9/2 was assigned.[12] Thus (as has been assumed in Fig. 2) the 113-kev transition is of the form $9/2(D,Q)7/2$ with known mixing ratio. If the experimental K-conversion

[8] E. D. Klema, Phys. Rev. **109**, 1652 (1958).

[9] McGowan, Klema, and Bell, Phys. Rev. **85**, 152 (1952).

[10] P. Marmier and F. Boehm, Phys. Rev. **97**, 103 (1955).

[11] D. R. Speck and F. A. Jenkins, Phys. Rev. **101**, 1831 (1956).

[12] N. P. Heydenburg and G. M. Temmer, Phys. Rev. **100**, 150 (1955).

coefficients of McGowan are interpreted using Sliv's[13] calculated K-conversion coefficients, the 208-kev transition is found to be an $E1+M2$ mixture with $Q_1 \leqslant 0.009$, and the 113-kev transition is an $M1+E2$ mixture with $0.896 \leqslant Q_2 \leqslant 0.994$. These are in agreement with conversion ratio measurements by Wiedling.[14] Figure 2 illustrates a $9/2(D,Q)9/2(D,Q)7/2$ cascade. It is seen that the experimental A_2 coefficient is consistent with a $9/2(D,Q)9/2$ assignment for the 208-kev gamma with any value of Q_1. However, the absence of an A_4 coefficient from the angular correlation requires a small value for Q_1. The angular correlation is then clearly consistent with the conversion data. Both the angular correlation and the conversion data will also fit a $7/2(D,Q)9/2(D,Q)7/2$ sequence, but this possibility has been eliminated by Wiedling on the basis of other data.[14] Thus the angular correlation confirms the spin assignment of 9/2 for the 321-kev level in Hf^{177} and the predominantly dipole character of the 208-kev transition.

TABLE I. Measured K-conversion coefficients for the 208-kev and 113-kev gamma rays in Hf^{177}.

E_γ (kev)	McGowan *et al.*	Marmier and Boehm
208	0.042 ± 0.015	0.044
113	0.81 ± 0.08	0.75

ACKNOWLEDGMENT

The authors wish to acknowledge helpful discussions with Robert W. Lide.

[13] L. A. Sliv and I. M. Band, Leningrad Physico-Technical Institute Report, 1956 [translation: Report 57 ICC Kl, issued by Physics Department, University of Illinois, Urbana, Illinois (unpublished)]. Klema did not use Sliv's tables and also omitted the limits of error from his interpretation.

[14] Tor Wiedling, *Directional Correlation Measurements and Some Other Related Investigations of Excited Nuclei* (Almquist and Wiksells Boktryckeri AB, Uppsala, 1956).

V

DECAY OF Ag^{110m}

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Decay of Ag^{110m}

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The gamma rays from Ag^{110m} were studied in magnetic spectrometers. Twenty-two electron lines corresponding to 14 gamma transitions were observed. These gamma rays were arranged in a plausible decay scheme. Directional correlation measurements were carried out on six gamma cascades in Cd^{110} resulting in spin assignments for 4 levels and multipolarity assignments for the strong gamma transitions.

INTRODUCTION

The decay of Ag^{110m} (253 days) has been the subject of numerous investigations.[1–7] The presently accepted decay scheme, which was first proposed by Siegbahn,[1] is shown in Fig. 1. In addition to the gamma rays shown in Fig. 1, five weak transitions have been found in internal conversion work. These are at approximately 575 kev,[7] 618 kev,[2,6,7] 740 kev,[2] 1480 kev,[2,7] and 1560 kev.[8]

The 656-kev transition has been definitely established as *E*2 by conversion coefficient measurements[1,7] and Coulomb excitation results.[9,10] Therefore the first excited state in Cd^{110} has a spin of 2+.

The two high-energy beta transitions of 2.86 and 2.12 Mev are of allowed type requiring a 1+ assignment for the ground state of Ag^{110}. Antoneva *et al.*[2] have reported an additional weak beta transition at about 1.4 Mev but no confirmation of this has been made. Using the measured beta branching ratios,[2] approximate log *ft* values for the low-energy beta transitions can be found from Moszkowski's graphs.[11] This analysis gives a log *ft* value of 8.4 for the 530-kev beta and 5.6 for the 87-kev beta. These values indicate that the 530-kev beta transition is first forbidden while the 87-kev beta transition is allowed.

The spin of Ag^{110m} has been measured as 6 by Ewbank *et al.*[12] using an atomic beam method. Measurements of the *K*/*L* ratio for the 116-kev transition and lifetime considerations indicate that the 116-kev transition is of *M*4 character. Since there is a spin difference of 5 between the Ag^{110m} level and the ground state of Ag^{110}, there may be a low-energy dipole transition in cascade with the 116-kev transition,[13] somewhat similar to the case of Cs^{134m}.[14]

The ground state of Ag^{110} most certainly is formed from a configuration with the odd proton in a $g_{9/2}$ state and the odd neutron in a $g_{7/2}$ state. The 1+ assignment is then in agreement with Nordheim's strong coupling rule. The isomeric state probably results from the odd proton in a $g_{9/2}$ state and the odd neutron in a $d_{5/2}$ state giving even parity for the spin 6 level.

Gamma-gamma directional correlation measurements on five cascades in Cd^{110} have been reported by Knipper.[15] The results are shown in Table II. Knipper based his interpretation on the Siegbahn decay scheme and a spin of 5− for Ag^{110m}. He assigned spins and parities of 3 or 4 to the 1415-kev level, 4+ to the 1541-kev level, 4+, 5+, or 6+ to the 2476-kev level, and 5− to the 2920-kev level, with the 1389-kev transition consisting of *E*1+*M*2 radiation, the 935-kev transition *M*1 or *E*2, the 885-kev transition *E*2+*M*3, and the 1516- and 759-kev transitions both mixtures with their

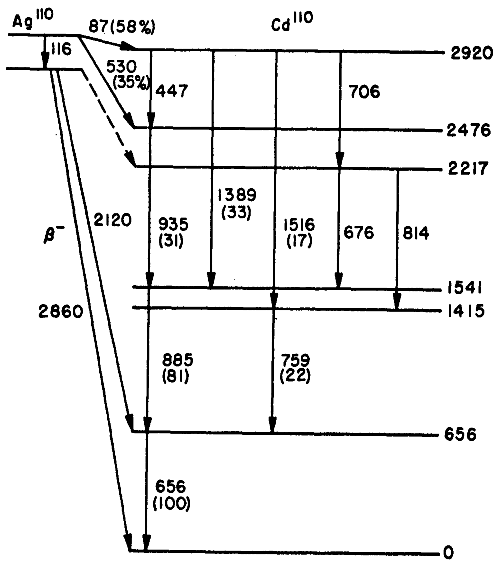

Fig. 1. Decay scheme of Ag^{110m} proposed by Siegbahn. All energies are in kev.

* Now at the University of Notre Dame, Notre Dame, Indiana.
[1] K. Siegbahn, Phys. Rev. **77**, 233 (1950).
[2] Antoneva, Bashilov, and Dzhelepov, Doklady Akad. Nauk S. S. S. R. **77**, 41 (1951).
[3] V. S. Shpinel and N. V. Forafontov, J. Exptl. Theoret. Phys. U. S. S. R. **21**, 1376 (1951).
[4] Dzhelepov, Zhukovskii, and Kholnov, Doklady Akad. Nauk S. S. S. R. **77**, 597 (1951).
[5] S. Johansson and S. Almquist, Arkiv Fysik **5**, 427 (1952).
[6] T. Azuma, Phys. Rev. **94**, 638 (1954).
[7] Thomas, Whitaker, and Peacock, Bull. Am. Phys. Soc. Ser. II, **1**, 86 (1956).
[8] Dzhelepov, Zhukovskii, and Kondakov, Izvest. Akad. Nauk S. S. S. R. Ser. Fiz. **21**, 973 (1957).
[9] G. M. Temmer and N. P. Heydenburg, Phys. Rev. **104**, 967 (1956).
[10] P. H. Stelson and F. K. McGowan, Oak Ridge National Laboratory ORNL-2076, 1956 (unpublished), p. 6.
[11] S. A. Moszkowski, Phys. Rev. **82**, 35 (1951).
[12] Ewbank, Nierenberg, Shugart, and Silsbee, Bull. Am. Phys. Soc. Ser. II, **2**, 317 (1957).
[13] For comments on this matter, see Nuclear Data Sheets, Sheet No. 58-5-52, C. L. McGinnis, editor, National Research Council.
[14] Sunyar, Mihelich, and Goldhaber, Phys. Rev. **95**, 570 (1954).
[15] A. C. Knipper, Proc. Phys. Soc. (London) **71**, 77 (1958).

character depending on whether the spin of the 1415-kev level is 3 or 4.

The investigation described in this paper consists of precise energy measurements of the gamma rays in the decay of Ag^{110m} and angular correlation measurements on six gamma cascades in Cd^{110}. A revised decay scheme is proposed.

GAMMA-RAY ENERGY MEASUREMENTS

The source material was obtained from Oak Ridge and was in the form of $AgNO_3$ in HNO_3. The source was studied in 180° permanent magnet spectrometers employing photographic detection (resolution$\sim$0.1%). A total of 22 electron lines was observed. These corresponded to 14 gamma transitions. The energies and interpretation of the electron lines are listed in Table I.

All of the observed transitions can be placed in a decay scheme in a consistent manner if one assumes that the 764-kev transition precedes the 1504-kev transition. This scheme is shown in Fig. 2. The 1504-kev gamma ray is known to be in coincidence with both the 764-kev and 657-kev gamma rays but the order of the 1504- and 764-kev transitions has never been definitely established. Further evidence for the inverted order of these transitions is found in the angular correlation measurements which are discussed in the following sections.

The energy sum of the 657- and 814-kev gammas is close to the 1473-kev gamma and therefore a level is placed at 1473 kev. The existence of a state in Cd^{110} at this energy has been verified by Stelson and McGowan[16] by means of Coulomb excitation. The 1473-kev level must then necessarily have a spin of 2 and positive parity. The 1560-kev transition observed by Dzhelepov *et al.*[8] would fit between the levels at 657 and 2219 kev and the 1400-kev beta transition reported by Antoneva[2] would be the expected transition between the 1+ Ag^{110} level and the 2+ level at 1473 kev in Cd^{110}.

In Fig. 2, the spins listed beside the Cd^{110} levels and the multipolarity of the gamma rays are those determined by the angular correlation measurements. The beta energies are those of Antoneva *et al.*[2]

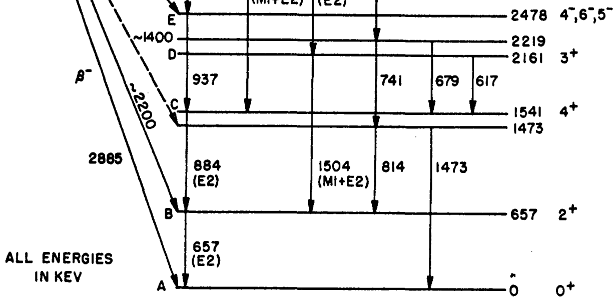

FIG. 2. Revised decay scheme of Ag^{110m}.

[16] P. H. Stelson and F. K. McGowan, Oak Ridge National Laboratory ORNL-2430, 1958 (unpublished), p. 19.

RESULTS OF DIRECTIONAL CORRELATION MEASUREMENTS

The correlation measurements were carried out with a conventional fast-slow coincidence circuit having an effective resolving time of 1×10^{-8} sec.[17] The scintillation counters consisted of 2 in.×2 in. NaI(Tl) crystals mounted on RCA type 6342 photomultipliers. The counters were shielded frontally by $\frac{3}{16}$ in. of aluminum. Lateral lead shielding was employed in some measurements to eliminate coincidences due to scattering.

The liquid source was contained in a cylindrically-shaped Lucite holder, $\frac{1}{8}$ inch in diameter and $\frac{3}{8}$ inch in length. In all correlation measurements the source was centrally mounted at a distance of 10 cm from the front face of each crystal. The data were taken in the double-quadrant sequence at intervals of 15°, running 5 minutes at each angle. The real coincidence rates were normalized by dividing by the single counting rates, and after all data for a given angle were combined, a least-squares fit was made to the function

TABLE I. Conversion electron energies in kev and their interpretation. The energies are accurate to about 0.2%.

Electron energy	Interpretation	Energy sum	Electron energy	Interpretation	Energy sum
91.2	K^1	116.7	787.6	K^9	814
113.4	L^1	116.8	814	L^9	817
116.2	M^1	116.6	857.6	K^{10}	884
420.0	K^2	447	879	L^{10}	883
590.2	K^3	617	910.4	K^{11}	937
630.7	K^4	657	932	L^{11}	936
652.6	L^4	656	1355.8	K^{12}	1383
and	K^5	679	1379	L^{12}	1383
678.7	K^6	705	1446.7	K^{13}	1473
714.3	K^7	741	1476.9	K^{14}	1504
737.2	K^8	764	1494	L^{14}	1498
760.8	L^8	764			

$$W'(\theta)=A_0'+A_2'P_2(\cos\theta)+A_4'P_4(\cos\theta),$$

where P_k is the Legendre polynomial of order k. The annihilation radiation method[18] was used to correct for the effect of finite angular resolution, giving the normalized coefficients A_2 and A_4.

The scintillation spectrum is shown in Fig. 3. The arrows and letters refer to discriminator settings used for various correlations.

For the 1383−884 kev cascade the differential discriminators were set at positions *a* and *b* as shown in Fig. 3. With these settings there is no interference from any other cascades. The corrected correlation function shows an asymmetry of about 39% negative and is given by

$$W(\theta)=1-(0.308\pm0.013)P_2+(0.009\pm0.020)P_4.$$

The 937−884 kev correlation cannot be obtained

[17] Stewart, Scharenberg, and Wiedenbeck, Phys. Rev. **99**, 691 (1955).

[18] E. L. Church and J. J. Kraushaar, Phys. Rev. **88**, 419 (1952).

without interference from the 1383−884 kev cascade. Since the interfering correlation has a large asymmetry it can appreciably alter the true 937−884 kev correlation. However, a fairly accurate subtraction process was carried out in this case. The correlation was run with the discriminators set at positions c and d, and background correlations were run with the discriminators set first at positions c and e and then at d and e. After proper normalization, the background correlation was subtracted point for point from the total correlation. Since some uncertainty was introduced here, a ±10% error was assigned to the background rate before subtraction. The background amounted to approximately 25% of the total coincidence rate. After correction for finite geometry the correlation function is given by

$$W(\theta)=1+(0.150\pm0.027)P_2-(0.006\pm0.036)P_4.$$

For the 884−657 kev correlation the discriminators were set at positions f and g. It was estimated that about 10 to 15% of the total coincidence counts were due to interfering correlations, but no correction could be made for this interference. The 884−657 kev correlation function is given by

$$W(\theta)=1+(0.073\pm0.014)P_2+(0.009\pm0.020)P_4.$$

The 1504−657 kev and 764−1504 kev correlations are complicated by the fact that the 657- and 764-kev peaks are not resolved by the scintillation counter and the 1504- and 1383-kev peaks are only partially resolved. With the discriminators set at positions h and j, the measured correlation function was mainly composed of the 764−1504 kev correlation but contained some interference from the 1504−657 correlation. The result was

$$W_A(\theta)=1-(0.202\pm0.021)P_2-(0.006\pm0.030)P_4.$$

With the discriminators set at positions h and g, the correlation function was composed mainly of the 1504−657 kev correlation but contained interference from the 764−1504 kev correlation. For these settings the result was

$$W_B(\theta)=1-(0.328\pm0.023)P_2-(0.022\pm0.031)P_4.$$

If W_1 is the true 764−1504 kev correlation function and W_2 is the true 1504−657 correlation function, then W_1 and W_2 are given in terms of the observed functions W_A and W_B by

$$W_1=[(1-X_B)W_A-(1-X_A)W_B]/(X_A-X_B),$$
$$W_2=(X_AW_B-X_BW_A)/(X_A-X_B),$$

where X_A and X_B are the fractions of W_A and W_B which are composed of the 764−1504 kev correlation. An estimate of the quantities X_A and X_B was obtained graphically. The values found were $X_A=0.85$ and $X_B=0.32$. Since uncertainty is introduced here it was considered reasonable to assign maximum errors of ±0.10 to X_A and X_B. The result for the 764−1504 kev correlation was

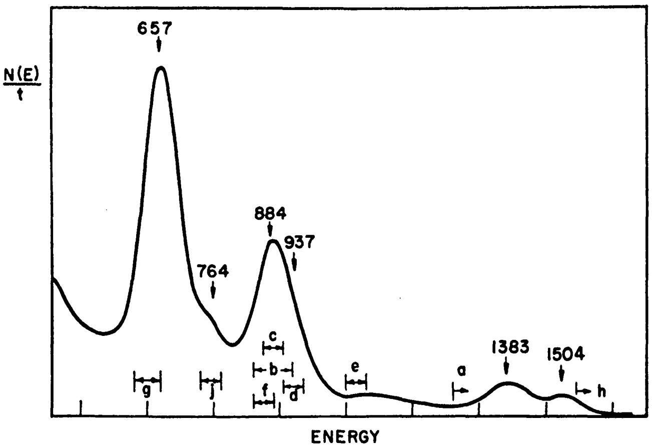

FIG. 3. Scintillation spectrum of Ag^{110m}. Letters and arrows refer to discriminator settings used in the correlation measurements.

$$W_1(\theta)=1-(0.17_{-0.10}^{+0.05})P_2-(0.002_{-0.066}^{+0.035})P_4.$$

The result for the 1504−657 kev correlation was

$$W_2(\theta)=1-(0.40_{-0.07}^{+0.13})P_2-(0.03_{-0.04}^{+0.09})P_4.$$

The coefficients here are the values found for $X_A=0.85$ and $X_B=0.32$ and the errors quoted for the coefficients correspond to the maximum errors assigned to X_A and X_B.

In an attempt to gain information on the 1383−657 kev correlation, a correlation was run with the discriminators set at positions a and g. This correlation consisted mainly of a composite of the 1504−657 kev and 1383−657 kev correlations. It showed an asymmetry of 40% negative and a negligible value for the coefficient of the P_4 term. It was estimated that about 45% of the coincidences were due to the 1383−657 kev correlation and about 25% were due to the 1504−657 kev correlation. Other interfering cascades were the 764−1504 (~10%) and the 1383−884 (~20%). Assuming these estimates for the interference it was concluded that the 1383−657 kev correlation has an asymmetry of about 40% negative (with negligible P_4 contribution). One can definitely say that the asymmetry is greater than 25% negative.

The results of the directional correlation measurements are listed in Table II together with the results published by Knipper.[15]

INTERPRETATION OF CORRELATION DATA

The Cd^{110} levels pertinent to the correlation data are designated by letters in Fig. 2 and henceforth will be referred to by these letters. Since the spin of Ag^{110m} is known to be 6, the possible spin values considered for levels E and F are 4, 5, and 6. No crossover transitions from levels C and D occur to the ground state of Cd^{110}, and no beta transitions leading to these states have

TABLE II. Directional correlation results for Ag^{110m}.

Correlation	Results of Knipper A_2	Results of Knipper A_4	Results of this investigation A_2	Results of this investigation A_4
1383−884	−0.288±0.012	−0.018±0.014	−0.308±0.013	+0.009±0.020
937−884	+0.14 ±0.03	−0.02 ±0.03	+0.150±0.027	−0.006±0.036
884−657			+0.073±0.014	+0.009±0.020
1383−657	−0.37 ±0.05	+0.06 ±0.06	Asymmetry∼−40% (A_4 small)	
1504−657	−0.34 ±0.07	−0.12 ±0.09	$-(0.40_{-0.07}^{+0.13})$	$-(0.03_{-0.04}^{+0.09})$
764−1504	−0.16 ±0.02	+0.01 ±0.03	$-(0.17_{-0.10}^{+0.05})$	$-(0.002_{-0.066}^{+0.035})$

been observed. Therefore no spin lower than 2 or higher than 4 is considered for levels C and D.

Since the 1383−884 kev correlation is free of interference from other cascades it is the logical correlation with which to begin the interpretation. Considering pure transitions only, no combination of the possible spins for levels B, C, and F is consistent with the data. If one considers dipole-quadrupole mixtures in one or both of the transitions, three sequences are compatible with the data. These are $5(D,Q)4(Q)2$, $6(0)3(D,Q)2$, and $4(D,Q)3(D,Q)2$. Even though the 1383−657 kev and 884−657 kev correlations are not too accurate due to interference, the results of these correlations are sufficient to eliminate the sequences $6(0)3(D,Q)2$ and $4(D,Q)3(\mathrm{D},Q)2$ as possibilities for the 1383−884 kev cascade.

The discrepancy between the experimental coefficients for the 884−657 kev cascade and the theoretical coefficients $A_2=0.1020$ and $A_4=0.0091$ for a $4(Q)2(Q)0$ sequence is not too great and can easily be explained by interfering correlations. The 1383−657 kev correlation results are in good agreement with the spin sequence of $5(D,Q)4(Q)2(Q)0$. For this sequence the 1383−657 kev correlation should be identical with the 1383−884 kev correlation, and the experimental data agree with this.

Therefore, $5(D,Q)4(Q)2(Q)0$ is the only sequence of spins for levels F, C, B, and A, respectively, which simultaneously satisfies the results of the 1383−884 kev, 884−657 kev, and 1383−657 kev correlations. With this sequence the 1383−884 kev correlation data require the 1383-kev gamma transition to be a mixture of $(13.5\pm1.5)\%$ quadrupole and $(86.5\mp1.5)\%$ dipole radiation ($\delta>0$). Since the spin and parity of Ag^{110m} are 6 and +, respectively, and the 87-kev beta transition is allowed, level F must have positive parity. An assignment of positive parity to level C would be in agreement with the fact that no beta transition to this level has been observed and the empirical fact that in even-even nuclei, low-lying levels with even spin normally have even parity. In addition this would require the 1383-kev transition to be an $M1-E2$ mixture which is more likely than an $E1-M2$ mixture. Therefore, assignments of 5+ and 4+ are made to the 2924-kev and 1541-kev levels, respectively, with the 884-kev gamma being pure $E2$ and the 1384-kev gamma being a mixture of $(13.5\pm1.5)\%$ $E2$ and $(86.5\mp1.5)\%$ $M1$ radiation ($\delta>0$).

Since the spins of levels C and B have been established as 4 and 2, the spin combinations to be considered for the 937−884 kev correlation are 6-4-2, 5-4-2, and 4-4-2. The theoretical coefficients for a pure $6(Q)4(Q)2$ sequence are $A_2=0.1020$ and $A_4=0.0091$, and for a pure $4(D)4(Q)2$ sequence they are $A_2=0.1965$ and $A_4=0$. Neither of these sequences is quite consistent with the experimental values of $A_2=0.150\pm0.027$ and $A_4=-0.006\pm0.036$ for the 937−884 kev correlation, but they are both close enough to warrant consideration as possibilities. They cannot be ruled out on the basis of the correlation data alone because the subtraction process introduces uncertainties in the data. A pure $5(D)4(Q)2$ sequence can definitely be discarded since it would require a negative A_2. If dipole-quadrupole mixtures are considered, both the $4(D,Q)4(Q)2$ and $5(D,Q)4(Q)2$ sequences are compatible with the data. With a spin of 4 for level E, the 937-kev transition would be a mixture of $(98.0\pm1.8)\%$ dipole and $(2.0\mp1.8)\%$ quadrupole ($\delta<0$) and with a spin of 5 for level E, it would be a mixture of $(86\pm4)\%$ dipole and $(14\mp4)\%$ quadrupole radiation ($\delta<0$).

Since the beta-decay data indicate that the 520-kev transition is first forbidden or unique first forbidden, level E must have negative parity. A 5− assignment then requires an $E1-M2$ mixture (∼14% $M2$) for the 937-kev gamma. This is unlikely, and therefore a spin of 5 for level E is not probable. An assignment of 4− requires almost pure $E1$ radiation for the 937-kev transition while an assignment of 6− requires $M2$ radiation. These are the most likely possibilities.

The 1504−657 kev and 764−1504 kev correlations are not too accurate because of interfering correlations. However, some information about level D can be obtained from them since the spins of all the other states involved in these correlations have been determined. Considering spins of 2, 3, or 4 for level D, it is found that only a $3(D,Q)2(Q)0$ sequence will explain the 1504−657 kev correlation data. Spins of 2 and 4 can definitely be ruled out. One should thus be able to explain the 764−1504 kev results by a $5(Q)3(D,Q)2$ sequence with a mixture in the 1504-kev gamma which is consistent with the result obtained from the 1504−657 kev correlation. This is not the case, but the interference from the highly negative 1383−657 and 1383−884 kev correlations could account for the discrepancy. If it is conceded that this is possible, a spin of 3 for level D is consistent with the data. This requires a mixture of about 10% quadrupole and 90% dipole radiation in the 1504-kev transition.

If the 1504- and 764-kev transitions are reversed in order as shown in Fig. 1, the sequences which must be considered for the 1504−657 kev correlation are 5(0)2(D,Q)2(Q)0, 5(Q)3(D,Q)2(Q)0, and 5(D,Q)4(Q)2(Q)0. Neither of the first two sequences can give a negative asymmetry greater than 6% no matter what the mixing parameter. From the experimental data it is certain that the 1504−657 kev correlation must have a large negative asymmetry and these sequences can be ruled out. If the spin sequence were 5(D,Q)4(Q)2(Q)0, the correlation function for the 1504−657 kev cascade would have to be identical with the 1504−764 kev correlation. The data are not in agreement with this, and any interference which might be present due to the interfering cascades cannot possibly explain the discrepancy. Since the 1504−764 kev and 1504−657 kev correlations are not at all in agreement with any reasonable spin sequences if the 1504-kev gamma precedes the 764-kev gamma, it is apparent that the data favor the new ordering of these transitions.

In the interpretation of the 1504−657 kev and 764−1504 kev correlations, the possibility of both the 764- and 1504-kev gammas being mixed multipoles was not considered. This would require a quadrupole-octupole mixture in one gamma transition. Relative intensity considerations and conversion coefficient data[8] tend to rule out this possibility. Therefore it is concluded that the 764-kev gamma is $E2$ and the 1504-kev gamma is an $M1-E2$ mixture ($\sim$10% $E2$) with the 2160-kev level having a spin of 3 and positive parity.

DISCUSSION

The revised decay scheme is in better agreement with the systematics of medium weight even-even nuclei in the region around Cd^{110}. In the old decay scheme the second excited state occurs at 1421 kev and would necessarily have a spin of 3 or 4. This is not in agreement with the systematics of nuclei in this region since they all have a 2+ second excited state. In the new scheme, the spin of the second excited state is 2+.

From Coulomb excitation of Cd^{110}, Stelson and McGowan[16,19] found that the $E2$ transition probability for the 657-kev transition is 32 times the single-particle estimate, and the $E2$ transition probability for the 1473-kev transition is 0.88 times the single-particle value. These results together with the fact that the ratio of the energy of the second excited state to that of the first excited state is 2.24 indicate that the low-lying levels in Cd^{110} follow the near-harmonic vibrational level structure. The 2+ and 4+ states at 1473 and 1541 kev could then be members of the expected close-lying triplet of states of the type 0+, 2+, and 4+.

[19] P. H. Stelson and F. K. McGowan, Phys. Rev. **110**, 489 (1958).

VI

NUCLEAR RESONANCE FLUORESCENCE IN Mg^{24}

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NUCLEAR RESONANCE FLUORESCENCE IN Mg^{24}†

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(Received December 22, 1958)

A gamma ray emitted from a nucleus initially at rest has an energy which is less than the energy difference between the levels by an amount equal to the recoil energy of the emitting nucleus. In addition, if the gamma ray is to be resonantly scattered by a nucleus of the same kind, a like amount of recoil energy must be given to the scattering nucleus. Thus the nuclear resonance scattering cross section is extremely small unless this energy deficit is restored.

The 15-hour Na^{24} decays to Mg^{24} by beta emission with an endpoint of 1.39 Mev followed by a gamma cascade with energies of 2.76 Mev and 1.38 Mev. Pollard and Alburger[1] attempted to restore the resonance condition for scattering of the 1.38-Mev gamma ray by making use of the recoil from the previous radiations. No resonant scattering was observed and it was concluded that the 1.38-Mev level had a width of less than 0.01 ev. Recently Burgov and Terekhov[2] successfully used a coincidence method to observe the resonant scattering of those 1.38-Mev gamma rays which have the resonance condition restored by recoil due to the preceding 2.76-Mev gamma ray. The resonance effect was small and a direct cross-section measurement was not possible. However a lower limit (1.6×10^{-4} ev) was placed on the width of the 1.38-Mev level.

A large resonance effect has now been observed and preliminary measurements of the level width have been made. If the emitting nucleus is assumed to be at rest before emission of the gamma cascade, it is not difficult to show that the resonance condition for the second gamma ray can be restored by the recoil of the preceding gamma ray when the angle between the emission direction is given by $\cos\theta = -E_{\gamma 2}/E_{\gamma 1}$. In the particular case of Mg^{24}, $E_{\gamma 1} = 2.76$ Mev, $E_{\gamma 2} = 1.38$ Mev, and it is possible to observe resonant scattering of the 1.38-Mev gamma ray when the angle between it and the 2.76-Mev gamma ray is about 120°.

Pure sodium hydroxide was irradiated for 4-hour periods in the Ford Nuclear Reactor of the University of Michigan. The sources consisted of dilute (1.3 normal) aqueous solutions of NaOH sealed in Lucite containers.

A conventional fast-slow coincidence circuit with an effective resolving time of 15 millimicroseconds was employed in the present measurements. The 2.76-Mev gamma ray was detected by a 5 in. diameter × 4 in. long NaI(Tl) crystal mounted on a DuMont 6364 phototube. A lead collimator was used to restrict the angular width of the gamma ray beam entering this crystal and a pulse-height analyzer was set integrally to accept only the 2.76-Mev gamma ray. A 1 in. high by 7/16 in. wide rectangular hole was cut between the parallel flat faces of a 2 in. diameter × 2 in. long cylindrical NaI(Tl) crystal. A bar of magnesium metal was placed within this well and the crystal was mounted on an RCA 6342 phototube. The resultant scintillation counter had a pulse height resolution (for direct radiation) of 10% at 1.38 Mev. A lead collimator limited the gamma rays entering this crystal to those which are scattered from the magnesium bar in the center. A differential analyzer was set to accept a narrow range of pulses above the center of the 1.38-Mev photopeak. Thus pulses due to Compton scattering from the magnesium were largely eliminated.

The coincidence rate was observed as a function of angle over a small range of angles centered at 120°. A sharp peak was observed, which, within the limits of experimental error, had an angular width as narrow as that of the collimating system (2.6° as measured by annihilation radiation). The coincidence rate at the peak was consistently two or three times the rate measured at neighboring angles off the peak (depending mainly on the sharpness of the pulse height discrimination).

Since the observed resonance effect was so large, it was possible to perform a self-absorption experiment[3] in order to measure the resonance scattering cross section. The coincidence rate was observed as a function of angle as an aluminum and a magnesium absorber were alternately inserted between the source and the resonance scattering detector. In this way it was possible to calculate the selective attenuation of the resonance radiation by the magnesium absorber. The ratio of the coincidence rates on resonance ($\theta = 120°$) to off resonance ($\theta = 114°$) for a magnesium absorber of 1.96 cm thickness was found to be 1.39 ± 0.26. For an aluminum absorber which attenuated the off-resonance coincidence rate by an amount similar to the magnesium absorber, the ratio of the coincidence rates at 120° and 114° was found to be 2.03 ± 0.3. The level width then follows from the attenuation upon inclusion of the dependence of the resonance effect on the spins involved and the thermal Doppler width. Preliminary data indicate a level width of 7×10^{-4} ev. This corresponds to a mean life of $\tau_\gamma = 0.95 \times 10^{-12}$ sec for the 1.38-Mev level. The statistical uncertainty in these measurements is about 90%. Helm[4] has estimated the mean life of this level from electron scattering data. His value, $\tau_\gamma = 1.9 \times 10^{-12}$ sec, is probably correct to within one order of magnitude and agrees well with the present work.

Experiments are continuing in order to improve the precision of the data and to study the apparent absence of beta recoil effects. A complete report will be published as soon as the measurements are concluded.

It is a pleasure to acknowledge our indebtedness to R. R. Lewis for many helpful discussions.

[†]Supported in part by the Michigan Memorial Phoenix Project and the Office of Naval Research.

[1]E. Pollard and D. E. Alburger, Phys. Rev. 74, 926 (1948).

[2]N. A. Burgov and Yu. V. Terekhov, Soviet J. Atomic Energy 2, 629 (1957).

[3]F. R. Metzger, Phys. Rev. 103, 983 (1956).

[4]R. H. Helm, Phys. Rev. 104, 1466 (1956).

VII

DIRECTIONAL CORRELATION OF GAMMA-RAYS FOLLOWING THE DECAY OF Eu^{152}

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DIRECTIONAL CORRELATION OF GAMMA-RAYS FOLLOWING THE DECAY OF Eu^{152}. R. W. Lide,* and M. L. Wiedenbeck, Department of Physics, The University of Michigan, Ann Arbor, Michigan (Received October 2, 1958).

Directional correlation measurements have been made on the 245-kev—122-kev, 969-kev—122-kev, 1118-kev—122-kev, 1416-kev—122-kev, 872-kev—245-kev, and 1170-kev—245-kev cascades in Sm^{152}, and on the 782-kev—345-kev cascade in Gd^{152}. The spins of Sm^{152} are found to be 0, 2, 4, 2, 3, 3 for the ground, 122-kev, 367-kev, 1092-kev, 1240-kev, and 1538-kev levels. The 245-kev gamma ray is found to be pure quadrupole; the 872-kev gamma ray is 98% quadrupole, 2% dipole; the 969-kev gamma ray is 98% quadrupole, 2% dipole; the 1118-kev gamma ray is 99.8% quadrupole, 0.2% dipole; the 1170-kev gamma ray is 98% quadrupole, 2% dipole; the 1416-kev gamma ray is 15% quadrupole, 85% dipole. In Gd^{152} the spin of the 1127-kev level is found to be 3 and the 782-kev gamma ray is found to be pure dipole. A spin assignment of 1, 2, or 3 for the 757-kev level would not be inconsistent with the data. For any of these cases the 412-kev gamma ray must be mostly dipole; the quadrupole content must be less than 15%.

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VIII

GEOMETRICAL CORRECTIONS IN DIRECTIONAL CORRELATION EXPERIMENTS

GEOMETRICAL CORRECTIONS IN DIRECTIONAL CORRELATION EXPERIMENTS*

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The relationship between the point-point angular correlation, $W(\Theta) = \sum_{\ell} A_{\ell} P_{\ell} (\cos \Theta)$, and the measured correlation function, $W'(\Theta) = \sum_{\ell} A_{\ell}' P_{\ell} (\cos \Theta)$, has been discussed by Feingold and Frankel[1] for finite detectors of arbitrary shape and efficiency. Their results are applied to the special case of gamma-gamma angular correlation experiments with cylindrical scintillation detectors and axially extended sources. The inherent limitations of the method and effect of detector shielding are discussed. The resulting correction factors have been determined for various energies and configurations and verified experimentally.

I. INTRODUCTION

The expansion functions ordinarily encountered in the angular correlation of successive nuclear radiations vary slowly and smoothly as a function of angle. Because of this a successful measurement will require "good statistics" but can be made in a "poor geometry" detector system. In the case of a gamma-gamma angular correlation experiment with NaI(Tl) scintillation detectors, the maximum source strength is dependent upon the resolving time of the coincidence circuit which is inherently limited by the response of the phosphor. Often the counting rates will be further limited by detector efficiency, the branching ratio of the cascade being studied, and the necessity of using a liquid source to avoid perturbations. In order to perform the experiment in a reasonable time, it then becomes imperative to increase the solid angle subtended by the detectors and the physical size of the source without impairing the accuracy of the measurement.

The effect of a constant efficiency detector has been treated to first approximation by Walter *et al.*[2] for various source and detector configurations.

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*Supported in part by the Michigan Memorial—Phoenix Project and the Office of Naval Research.

[1] A. M. Feingold and S. Frankel, Phys. Rev. 97, 1025 (1955).

[2] Walter, Huber, and Zunti, Helv. Phys. Acta 23, 697 (1950).

Rose[3] has calculated the corrections in the case of a point source of gamma radiation incident on an unshielded cylindrical scintillation detector. Lawson and Frauenfelder[4] calculated the same correction from a measured detector-efficiency function. For gamma ray energies near .5 Mev, Church and Kraushaar[5] developed a practical method of making this correction from the measured correlation for annihilation radiation. Feingold and Frankel[1] have treated theoretically the cases of axially extended sources and detectors of arbitrary shape and efficiency. Several of their equations have been reproduced in the following application of their results to the gamma-gamma angular correlation experiment.[6]

II. ARBITRARY DETECTORS WITH A POINT SOURCE

The efficiency, E, of a general gamma ray detector is a function of (Θ,ϕ), the coordinates specifying a point on the detector surface; and also of the coordinates which give the angle of incidence of the radiation at that point. If the efficiency of the detector is independent of incident angle, and a point source of radiation defines the center of the (Θ,ϕ) coordinate system, the efficiency can be expressed as a series of the spherical harmonics, $Y_\ell^m(\Theta,\phi)$. If we further assume that the two detectors, 1 and 2, are sensitive to radiations b and c, respectively, the expansions may be given as:

$$E_{b_1}(\Theta_1,\phi_1) = \sum_{\ell' m'} \left(\frac{2\ell'+1}{4\pi}\right)^{1/2} b_{\ell' m'}\, Y_{\ell'}^{m'}(\Theta_1,\phi_1) \tag{1}$$

$$E_{c_2}(\Theta_2,\phi_2) = \sum_{\ell'' m''} \left(\frac{2\ell''+1}{4\pi}\right)^{1/2} c_{\ell'' m''}\, Y_{\ell''}^{m''}(\Theta_2,\phi_2) \tag{2}$$

in which the coordinates are defined as shown in Fig. 1. The experimental angular correlation,

$$W'(\Theta) = \sum_{\ell} A'_\ell P_\ell(\cos\Theta) \tag{3}$$

can now be related to the point-point correlation,

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[3]M. E. Rose, Phys. Rev. 91, 610 (1953).
[4]J. S. Lawson, Jr., and H. Frauenfelder, Phys. Rev. 91, 649 (1953).
[5]E. L. Church and J. J. Kraushaar, Phys. Rev. 88, 419 (1952).
[6]We have made slight changes in notation and have corrected several errors which appeared in reference 1.

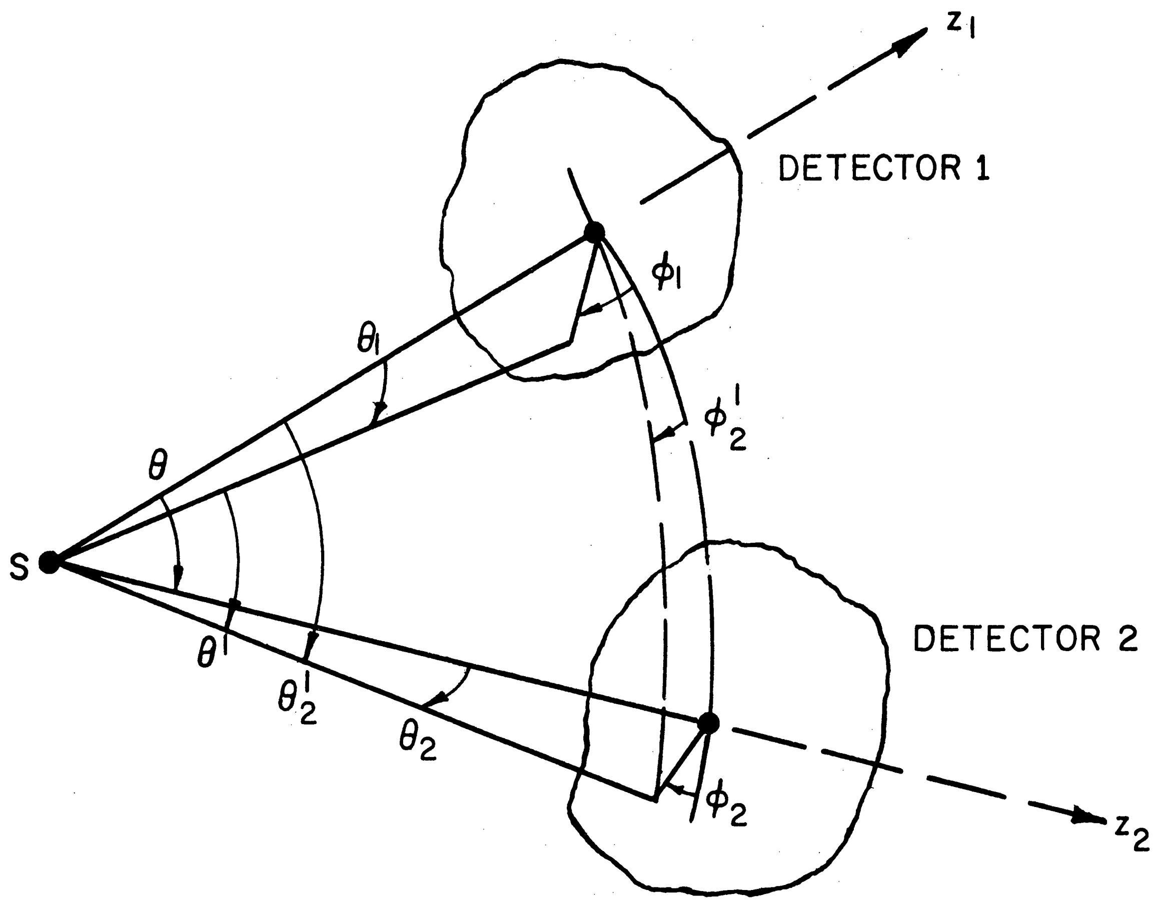

Fig. 1. Geometry of the angular correlation experiment.

$$W(\Theta') = \sum_{\ell} A_{\ell} P_{\ell} (\cos \Theta') \tag{4}$$

by the relation:

$$W'(\Theta) = \iint E_{b_1}(\Theta_1,\phi_1)\, W(\Theta')\, E_{c_2}(\Theta_2,\phi_2)\, d\Omega_1\, d\Omega_2 \quad . \tag{5}$$

$P_{\ell}(\cos \Theta')$ may be expressed in terms of Θ_1,ϕ_1 and Θ_2',ϕ_2' by the addition theorem. Also $Y_{\ell''}^{m''}(\Theta_2,\phi_2)$ may be expressed in terms of Θ_2',ϕ_2' by a rotation of the coordinate system through the angle Θ:

$$Y_{\ell''}^{m''}(\Theta_2,\phi_2) = \sum_{n'} d_{n'm''}^{\ell''}(\Theta)\, Y_{\ell''}^{n'}(\Theta_2',\phi_2') \quad . \tag{6}$$

Now define:

$$g_{mm'}^{\ell\ell'} = \frac{2\ell+1}{4\pi} \int_{-1}^{+1} d_{mm'}^{\ell}(\Theta)\, P_{\ell'}(\cos \Theta)\, d(\cos \Theta) \tag{7}$$

By combining the above equations and making use of the orthonormality relation for the spherical harmonics,[7] one obtains:

$$A'_{\ell} = \sum_{\ell' mm'} (-1)^{m+m'+\ell+\ell'} \frac{2\ell+1}{4\pi} A_{\ell'} b_{\ell' m} c_{\ell' m} g^{\ell\ell'}_{mm'} \quad . \tag{8}$$

Feingold and Frankel[1] have tabulated the $g^{\ell\ell'}_{mm'}$ for $\ell \leq 6$ and m,m' even and have discussed the effect of various detector symmetries on the experimental angular correlation. In the special case of the ϕ-symmetric detector, the relation between the coefficients reduces to:

$$A'_{\ell} = A_{\ell} b_{\ell o} c_{\ell o} \quad . \tag{9}$$

III. CYLINDRICAL SCINTILLATION DETECTOR WITH AN AXIAL SOURCE

An axial source is a line source centered at the origin perpendicular to the plane of the angular correlation measurement. The axial source can be looked upon as a sum of point sources displaced from the origin. Let $b_{\ell m}(\epsilon')$ be the $b_{\ell m}$ for a displacement ϵ' from the origin. For a source of length ϵ_o and density of activity $n(\epsilon')$, placed symmetrically about the origin:

$$b_{\ell m}(\epsilon_o) = \frac{\int_{-\epsilon_o/2}^{\epsilon_o/2} b_{\ell m}(\epsilon') \, n(\epsilon') \, d\epsilon'}{\int_{-\epsilon_o/2}^{\epsilon_o/2} n(\epsilon') \, d\epsilon'} \tag{10}$$

where the $b_{\ell m}(\epsilon_o)$ contains both source and counter corrections. The $b_{\ell m}(\epsilon')$ is difficult to determine unless the detector efficiency is independent of incident angle. This requirement is not satisfied for the ordinary scintillation counter employed in gamma-gamma angular correlation experiments. However, it will be shown below that the errors involved are not significant as long as the source extension is small compared to detector size and source-to-detector distance.

Assume that the face of the detector is circular and that $n(\epsilon')$ is constant. We may write

$$b_{\ell m} = \left(\frac{4\pi}{2\ell+1}\right)^{1/2} \int Y^{m}_{\ell}(\theta,\phi) \, E(\theta,\phi) \, d\Omega \quad . \tag{11}$$

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[7] The normalization convention is that given by L. I. Schiff, Quantum Mechanics (McGraw-Hill Book Co., New York, 1955).

Upon transforming to Cartesian coordinates at the center of the detector (cf. Fig. 2), Eq. (11) becomes:

$$b_{\ell m}(\epsilon) = \left(\frac{4\pi}{2\ell+1}\right)^{1/2} \int_{-\infty}^{\infty} dy \int_{-\infty}^{\infty} dx \; \frac{Y_{\ell}^{m}(x,y-\epsilon)\; E(x,y)}{[1+x^2+(y-\epsilon)^2]^{3/2}} \qquad (12)$$

in which

$$x = \frac{x'}{r_o}, \qquad y = \frac{y'}{r_o}, \qquad \epsilon = \frac{\epsilon'}{r_o} .$$

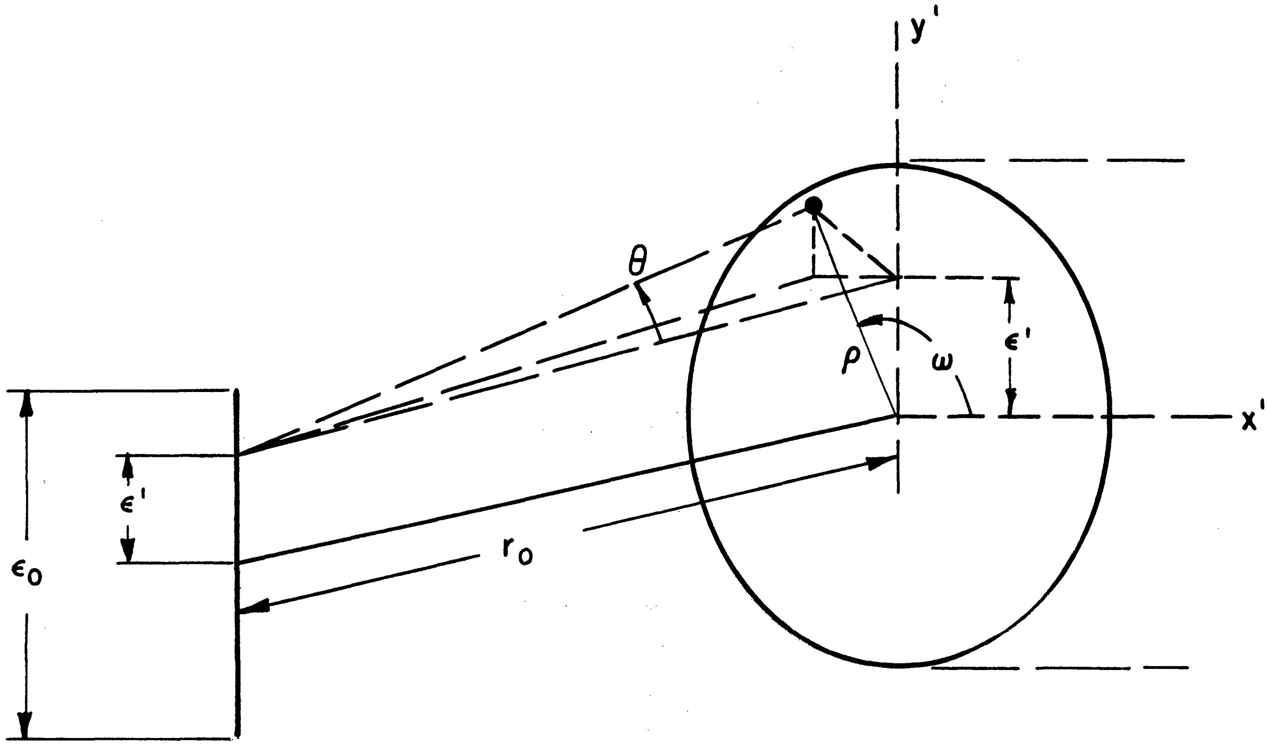

Fig. 2. Geometry for circular detector with axial source.

Transforming to polar coordinates on the face of the detector:

$$b_{\ell m}(\epsilon) = r_o \sqrt{\frac{4\pi}{2\ell+1}} \int_0^{\infty} rdr \int_0^{2\pi} d\omega \; \frac{Y_{\ell m}(r,\omega,\epsilon)\; E(r,\omega)}{[1+r^2+\epsilon^2-2\epsilon r \sin\omega]^{3/2}} \qquad (13)$$

in which $r = \rho/r_o$ and $\epsilon = \epsilon'/r_o$. Equation (13) is not introduced into (10), defining $\gamma = \epsilon_o/(2r_o)$:

$$b_{\ell m}(\gamma) = \frac{1}{2\gamma} \sqrt{\frac{4\pi}{2\ell+1}} \int_{-\gamma}^{\gamma} d\epsilon \int_0^{\infty} rdr \int_0^{2\pi} d\omega \; \frac{Y_{\ell m}(r,\omega,\epsilon)\; E(r,\omega)}{[1+r^2+\epsilon^2-2\epsilon r \sin\omega]^{3/2}} . \qquad (14)$$

If the efficiency is independent of incident angle, $E(r,\omega) = E(r)$, only the $b_{\ell 0}$ terms will enter the expansion. The $Y_\ell^0(r,\omega,\epsilon) = \sqrt{(2\ell+1)/4\pi}\, P_\ell(r,\omega,\epsilon)$ can be expanded if we note that $\cos\Theta = [1+r^2+\epsilon^2-2\epsilon r\sin\omega]^{-1/2}$ and use the binomial expansion. Upon changing the order of the r and ϵ integrations, the $b_{\ell 0}$ reduce to a series of r integrals. The coefficients b_{00}, b_{20}, and b_{40} which occur in the usual gamma-gamma angular correlation experiment are then:

$$b_{00} = 2\pi\alpha_0 \left[1 - \frac{3}{2}\left(\alpha_2 + \frac{\gamma^2}{3}\right) + \frac{5}{8}\left(3\alpha_4 + 4\alpha_2\gamma^2 + \frac{3}{5}\gamma^4\right) - \dots\right] \tag{15}$$

$$b_{20} = 2\pi\alpha_0 \left[1 - 3\left(\alpha_2 + \frac{\gamma^2}{3}\right) + \frac{15}{8}\left(3\alpha_4 + 4\alpha_2\gamma^2 + \frac{3}{5}\gamma^4\right) - \dots\right] \tag{16}$$

$$b_{40} = 2\pi\alpha_0 \left[1 - \frac{13}{2}\left(\alpha_2 + \frac{\gamma^2}{3}\right) + \frac{25}{4}\left(3\alpha_4 + 4\alpha_2\gamma^2 + \frac{3}{5}\gamma^4\right) - \dots\right] \tag{17}$$

in which:

$$\alpha_0 = \int_0^\infty r\, E(r)dr \tag{18}$$

$$\alpha_2 = \frac{\int_0^\infty r^3\, E(r)dr}{\int_0^\infty r\, E(r)dr} \tag{19}$$

$$\alpha_4 = \frac{\int_0^\infty r^5\, E(r)dr}{\int_0^\infty r\, E(r)dr} \quad . \tag{20}$$

If $E(r)$ is measured by means of a collimated beam experiment, α_0, α_2, and α_4 can be obtained by numerical integration. The assumption that the scintillation counter is independent of incident angle introduces an error of order γ^4 and this method is therefore not valid if terms of this order contribute significantly to the $b_{\ell 0}$.

IV. APPLICATION TO TYPICAL GEOMETRIES

The efficiency functions, $E(r)$, were determined for a 2-in.-by-2-in. cylindrical NaI(Tl) crystal for various energies and at source to detector distances of 7.00 and 9.90 cm. The crystal was mounted on an RCA 6342 phototube and the counting rate was observed as the scintillation counter was moved through small angles about the center of a coordinate system defined by a well-collimated beam of gamma radiation. A differential analyzer was set to accept only the photopeak of the gamma ray involved. The measurement was repeated for gamma ray energies of .065 Mev, .129 Mev, .511 Mev, and 1.33 Mev with and without lateral lead shielding on the crystal. The integrands of α_0, α_2, and α_4 were plotted vs. r and integrated by means of a compensating polar planimeter. Plots of α_2 and α_4 vs. gamma ray energy for unshielded detectors are shown in Figs. 3 and 4.

The effect of lateral shielding was not found to be significant except at low energies (< 100 Kev) where x-rays from the shielding material can enter the crystal. It should be noted, however, that the shielding can make an important contribution to the correction at higher energies if the differential analyzer is allowed to accept gamma rays which are compton-scattered from the shield.

The results of the present method for a gamma energy of .511 Mev were compared to results obtained by the annihilation correlation method of Church and Kraushaar.[5] The normalized attenuation coefficients, $(b_{\ell 0}/b_{00})^2$, for both methods are shown in Table I.

An examination of the results makes it clear that the method is valid for axial extensions of up to 1 to 2 cm (depending on the energy) for the source-to-detector distances studied. This is sufficient for most applications.

TABLE I

THE NORMALIZED ATTENUATION COEFFICIENTS,
$(b_{20}/b_{00})^2$ AND $(b_{40}/b_{00})^2$, FOR A GAMMA RAY ENERGY
OF .511 MEV AND $r_0 = 9.90$ CM

	$(b_{20}/b_{00})^2$	$(b_{40}/b_{00})^2$
Present Method	.931 ± .006	.787 ± .016
Method of Church and Kraushaar	.930 ± .006	.782 ± .015

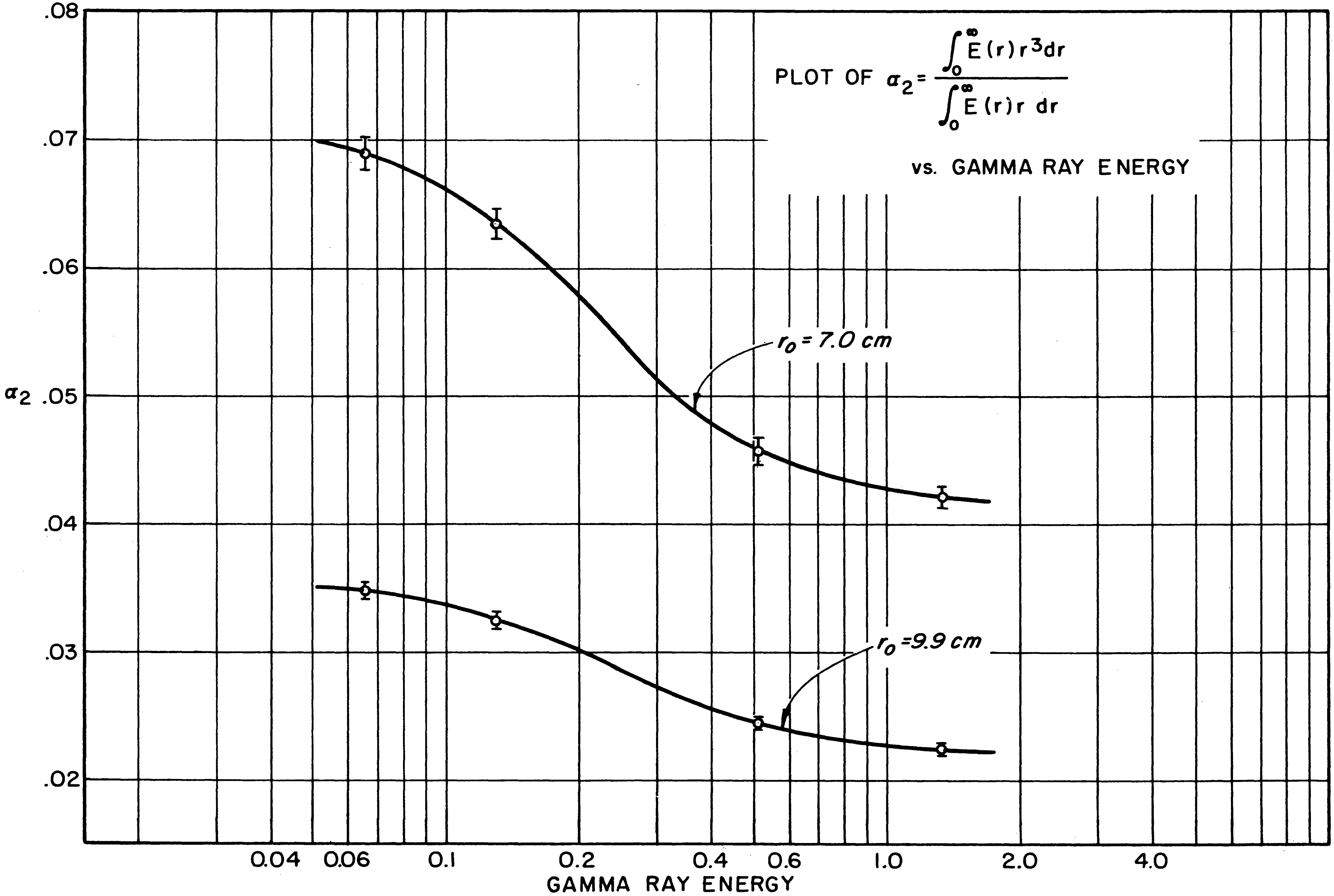

Fig. 3. Plot of α_2 vs. gamma ray energy for r_o = 7.00 cm and r_o = 9.90 cm. The error limits represent our best estimate of the uncertainty in these measurements due to both random and systematic sources of error.

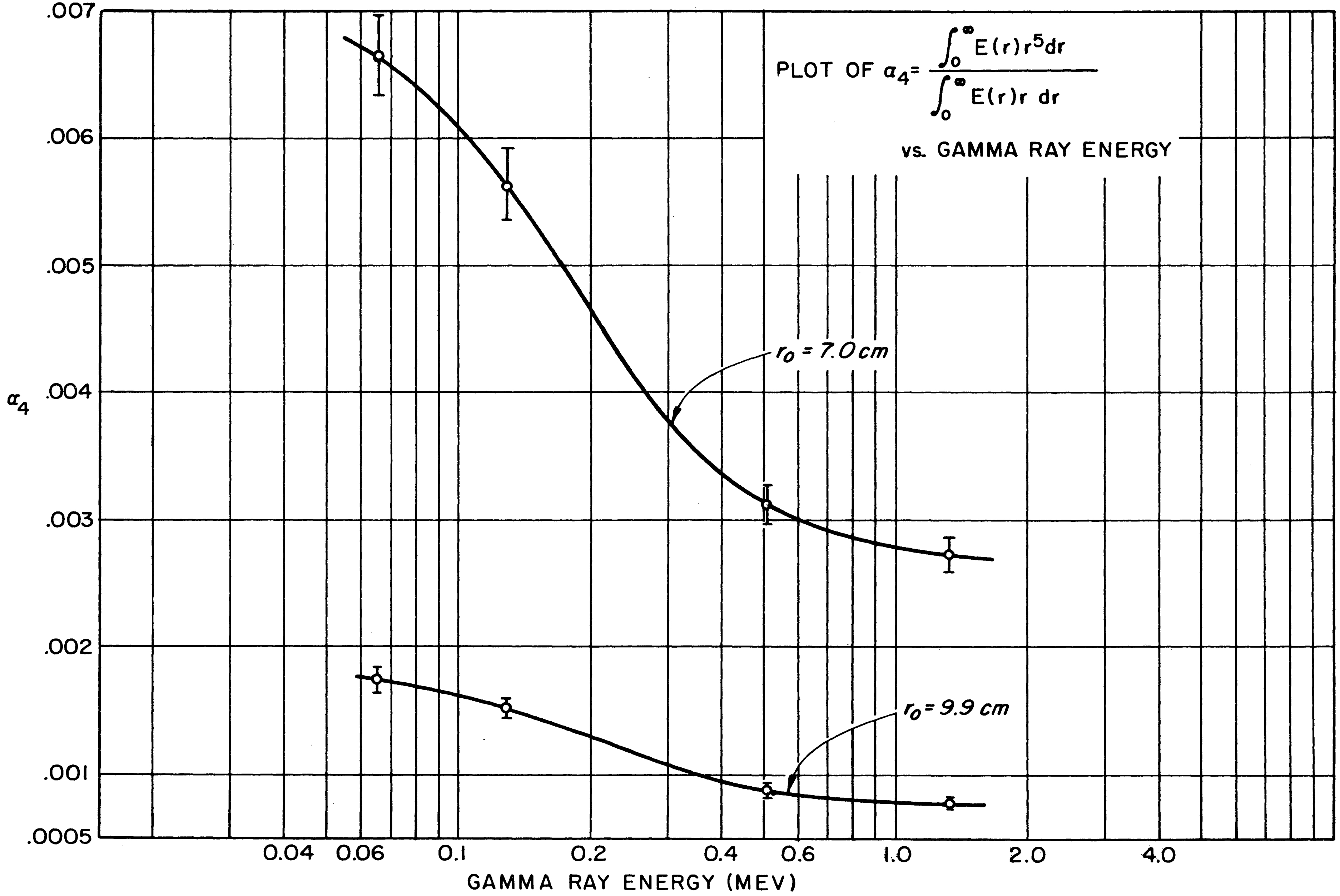

Fig. 4. Plot of α_4 vs. gamma ray energy for r_o = 7.00 cm and r_o = 9.90 cm.