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Equilibrium Temperature of A Ram-Jet Burner Shell

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SYMBOLS

I_{ic} = Heat Transfer to Shell by Convection from Inside, (Btu per ft^2 sec)

I_{ir} = Heat Transfer by Internal Radiation (Btu per ft^2 sec)

I_{oc} = Heat Loss from Wall by Convection Outside, (Btu per ft^2 sec)

I_{or} = Heat Transfer by Outside Radiation (Btu per ft^2 sec)

M = Mach Number

Nu = Nusselt Number

R = Gas Constant (ft lb per slug ^oR)

Re = Reynolds Number

T_b = Boundary Layer Temperature (^oR)

T_i = Absolute Temperature of the Flame Gases (^oR)

T_{ib} = Boundary Layer Temperature, Inside (^oR)

T_{ob} = Boundary Layer Temperature, Outside (^oR)

T_s = Static Temperature (^oR)

T_t = Total Temperature (^oR)

T_w = Shell Temperature (^oR)

h_i = Convective Heat Transfer Coefficient for Inside Surface (Btu per ft^2 sec)

h_o = Convective Heat Transfer Coefficient for Outside Surface (Btu per ft^2 sec) (Used in Appendix for Enthalpy of Ambient Air)

h_{st} = Stagnation Enthalpy (Btu per lb)

h_1 = Enthalpy at Point 1 (Btu per lb)

v = Air Speed (ft per sec)

α_i = Absorptivity of Gas

γ = Ratio of Specific Heats of Air

ε_i = Emissivity of the Gases

ε_w = Wall Emissivity

ε_{wo} = External Wall Emissivity

η = Diffuser Efficiency

μ = Coefficient of Viscosity (lb sec per ft)

ρ = Density (slug per cu ft)

σ = Black Body Radiation Constant (Btu per sq ft sec $^{o}R^{4}$)

SUMMARY

A procedure is described for calculating the equilibrium temperature of a ram jet burner shell made of thin metal. The temperature calculated for a particular ram jet is compared with that measured on the same vehicle in flight and good agreement is found. The same method of calculation is applied to predict ram jet burner shell temperatures at various altitudes and flight speeds. The special case of a burner shell with non-conducting liner is considered briefly.

INTRODUCTION

One of the problems anticipated in the design of ram jets for supersonic speeds is that of keeping the combustion chamber wall cool enough to retain its rigidity. At very high flight velocities, the stagnation temperature of air is so high, even without the burning of fuel, that ordinary metals would soften. The absolute stagnation temperature for Mach number 6 is about seven times the static temperature. At lower flight speeds, the airflow outside the ram jet helps to keep the wall cooled as heat is transmitted to it from the flame inside.

A number of special ways of constructing a ram jet burner could be devised to keep some part of the burner wall cool (and hence rigid) in the presence of a hot flame. An insulating ceramic layer might be put inside to reduce heat flow to the metal shell or an extended surface of some sort might be used on the outside.

The simple and obvious way to build a ram jet burner shell is to make it of thin metal. It is the purpose of the calculation here reported to find the equilibrium temperature of such a thin ram jet burner shell at various speeds

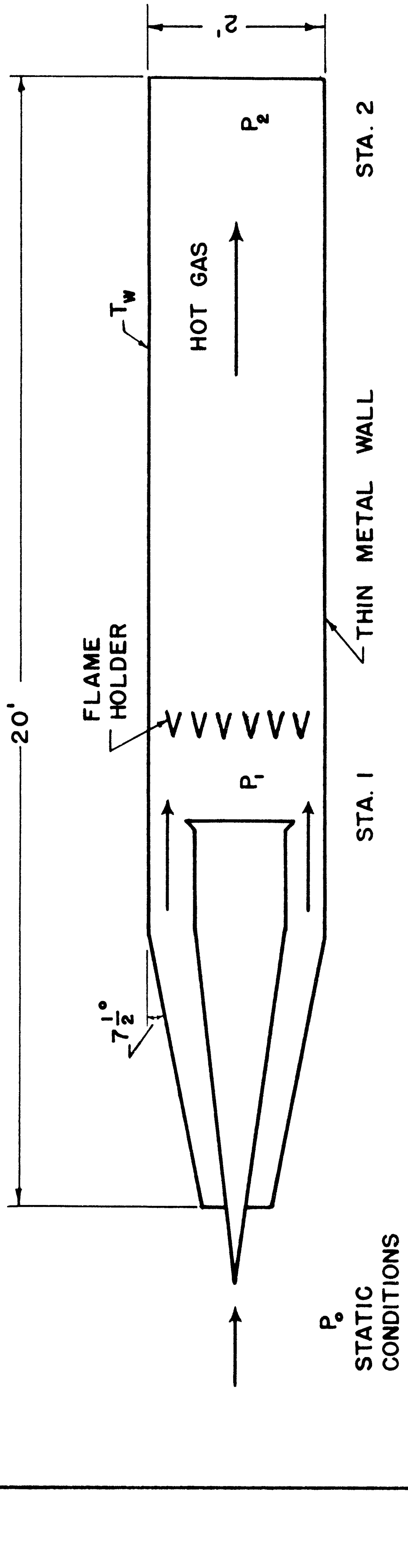

FIG. I - DIAGRAM OF RAM JET USED IN COOLING CALCULATIONS
FUEL IS OCTANE AND AIR.

speeds and altitudes. The results of the calculation, once they are checked by experiment, will indicate the conditions of flight that will call for the more complicated burner shell designs.

DISCUSSION

1. Theory

Consider the ram jet burner shell as a thin metal wall separating two parallel air flows. Figure 1 is a sketch of the ram jet as assumed. Heat transfer occurs to the wall from inside and away from it outside by two different means. One means is forced convection and the other is radiation.

To consider first the forced convection processes, define T_{ib} and T_{ob} as the boundary layer temperatures inside and outside the burner shell respectively. If T_w is the shell temperature, the rate of heat transfer to the shell by convection is

$$I_{ic} = h_i (T_{ib} - T_w) \qquad (1)$$

where h_i is a heat transfer coefficient that is fixed by the flow conditions at the inside wall surface. The rate of heat loss from the wall by convection is

$$I_{oc} = h_o (T_w - T_{ob}) \qquad (2)$$

where h_o is a heat transfer coefficient fixed by the flow conditions at the outside wall surface.

Radiant heat is transmitted to the wall by the inside gases at the rate given by the expression $\sigma \varepsilon_i \alpha_w T_i^4$ where ε_i is the emissivity of the gases, α_w is the absorptivity of the wall, T_i is the absolute temperature of the gases inside the chamber, and σ is the black body radiation constant. Likewise heat is given up by radiation from the wall to the interior gases in an amount expressed by $\sigma \varepsilon_w \alpha_i T_w^4$ Btu per sec. The net radiative exchange rate internally is

$$I_{ir} = \sigma (\varepsilon_i \alpha_w T_i^4 - \varepsilon_w \alpha_i T_w^4). \qquad (3)$$

ε_w and α_w, the wall emissivity and absorptivity, respectively, were assumed equal to unity, as the inside surface was considered to be black. The gas emissivity ε_i and absorptivity α_i are functions of T_i and T_w, respectively, as well as gas pressure, combustion chamber shape, etc.

The rate of heat loss by radiation from the chamber externally is

$$I_{or} = \sigma \varepsilon_{wo} T_w^4 \qquad (4)$$

since the heat absorbed by the wall due to radiation from the surroundings is negligible. The external wall emissivity ε_{wo} is set equal to unity.

In the steady state, the heat transferred to the wall is equal to that lost from the wall to the ambient air. That is

$$I_{ic} + I_{ir} = I_{oc} + I_{or}$$

Or, from Equation 1 through 4

$$h_i (T_{ib} - T_w) + \sigma \varepsilon_w (\varepsilon_i T_i^4 - \alpha_i T_w^4) = h_o (T_w - T_{ob}) + \sigma \varepsilon_{wo} T_w^4 \qquad (5)$$

This equation can be solved for T_w in terms of the other quantities. The equation is not solvable for T_w explicity but the determination is relatively simple by a graphical procedure.

Equation 5 has the form

$$a T_w^4 + T_w = b \qquad (6)$$

where $$a = \frac{\sigma(1 + \varepsilon_w \alpha_i)}{h_o + h_i} \tag{7}$$

and $$b = \frac{h_i T_{ib} + h_o T_{ob} + \sigma \varepsilon_w \varepsilon_i T_i^4}{h_o + h_i} \tag{8}$$

Once a and b are determined, the burner shell temperature T_w is found from Equation 6. As a convenience in the calculation, the function

$$b = aT_w^4 + T_w$$

is plotted as a function of T_w for various values of a in Figure 2; the value of T_w can be read from one of the figures directly when a and b are known.

2. The Heat Transfer Parameters

The parameters a and b in Equation 6, as defined in Equations 7 and 8 are calculable by use of theory and experiments taken from an extensive literature dealing with heat transfer and aerodynamics. As is to be expected, the results of the experiments may not be independent of the experimental apparatus in some cases. Furthermore, the initial conditions of the calculation are in some cases quite different from those in the experiments. It follows that some direct experimental verification of the method is desirable before too much reliance is placed on the results. One experimental check point will be presented after the method has been described.

3. Internal Gas Flow

As will be evident later, it is necessary to know the velocity, density, and temperature of the gas flowing over the portion of the shell whose temperature is to be calculated. Consider first the internal flow. The air flows through a diffuser in the ram jet, then through the burner, and

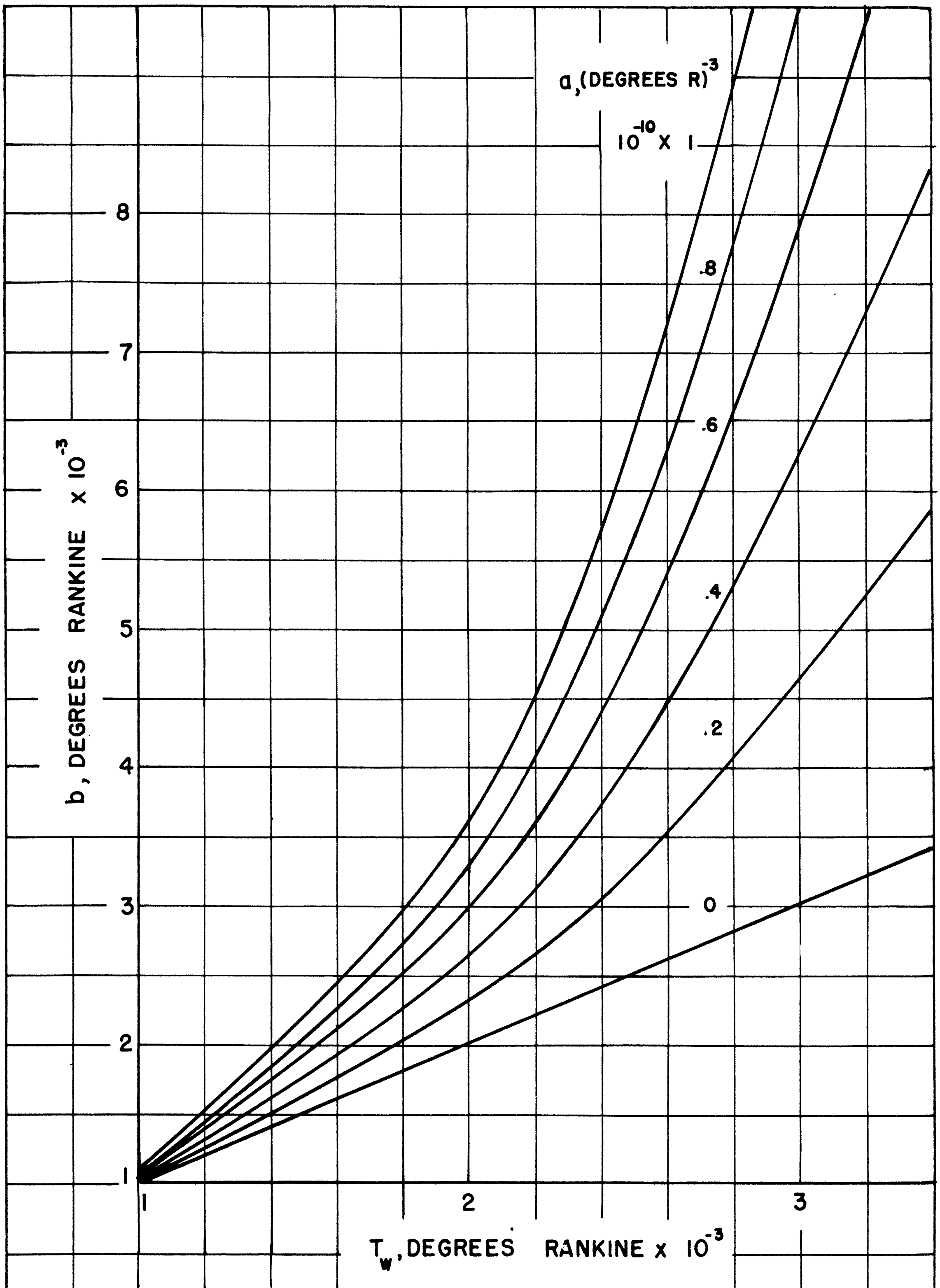

FIG. 2 VARIATION OF $b = aT_w^4 + T_w$ WITH T_w

finally, in some cases, through a nozzle. The heating by forced convection is most intense at the point where the air speed is highest. At the lower supersonic flight speeds this point is the down-stream end of the combustion chamber, since in that range of speed there is no advantage in a nozzle throat, from the standpoint of performance. Furthermore, a properly designed supersonic ram jet would supposedly have the flow limited at that point by the choking condition (internal flow Mach number equal to 1). A stoichiometric air-fuel mixture with 100% combustion efficiency is chosen as involving the most severe heating condition; a 90% diffuser efficiency is assumed as the proportion of dynamic head transformed into pressure energy.

The manner in which the temperature, density, and velocity are calculated for critical flow at station 2 (see Figure 1) is considered sufficiently novel to warrant a description in Appendix 1.

4. <u>External Gas Flow</u>

The pressure, density, and temperature must be known in the calculation of h_o as will be discussed later. In finding those quantities, the air is assumed to pass through an oblique compressibility shock due to the simple divergent diffuser, and thereafter through an expansion shock at the end of the diffuser. In the calculations, the shocks were assumed to be the same as those in two-dimensional flow as described by Liepmann and Puckett, Reference 2, page 51. While the shocks are actually three-dimensional, this two-dimensional flow is much simpler to calculate and is believed to be sufficiently accurate for these calculations. The nose shock is assumed to be the same as that on the nose of a wedge of 15^{o} total included angle. The expansion shock is calculated for a $7\frac{1}{2}^{o}$ change in direction and the Prandtl-Meyer theory of two-dimensional flow around a corner is used. Conditions behind these two shocks are calculated for a series of Mach numbers up to $M = 5$, and for various altitudes. The NACA standard atmosphere is assumed.

5. Calculation of Radiation from Internal Gas Flow

In the calculation of a and b in Equation 6, it is necessary to evaluate σ, ε_w, α_i, and ε_i. The Stefan-Boltzmann constant is $\sigma = .48 \times 10^{-12}$ Btu per sq ft sec $^oR^4$. The remaining constants are functions of the gas composition and temperature, wall temperature, and chamber shape. The manner of calculating the parameters has been worked out and is described in McAdams "Heat Transmission", Chapter II (see Reference 3). The combustion chamber is assumed for the purposes of the calculation to have an emissivity both inside and outside of unity and to have an inside diameter of two feet. The concentrations of water vapor and carbon dioxide are assumed to be those for a stoichiometric mixture of octane and air, burning to completion. A plausible estimate of wall re-emission to the gas based on a guess as to the wall temperature is sufficient for this part of the calculation.

6. Convective Heat Transfer Outside

The heat transfer problem outside the ram jet burner is relatively new. Most data have been calculated to predict the heat transfer to tubes or other bodies in flows of relatively low speeds of the fully developed type. Our problem here, however, is concerned with flows of supersonic velocity, and there is no direct information as to the nature of the boundary layer. Some recent data published by Eber (Reference 4) on heat transfer in an air jet of Mach numbers up to 3 and more indicates that the Mach number can be eliminated as an independent variable by a proper manipulation of the data. The Nusselt number was found to be a function of the Reynolds number alone, if the Reynolds number is calculated with air density ρ back of the oblique shock but outside the boundary layer. The air speed v was calculated for the same condition, μ was taken at the "temperature of the boundary layer", and the scale factor was the length of the generating lines of the cones used as probes in the test. The Nusselt number itself was calculated by use of the conductivity of air at the same boundary layer temperature, the same scale factor, and heat transfer coefficient directly measured. The heat transfer measurement was made with the boundary layer temperature used in place of the temperature of the free air back of the oblique shock. The boundary layer temperature used was considered to be equal to the static temperature of the air, plus .85 times the dynamic temperature. That is,

$$T_b = T_s + .85\,(T_t - T_s) \qquad (9)$$

Here T_b is the boundary layer temperature, T_t is the total temperature, and T_s is the static temperature. The relation Eber found to exist between the Nusselt number Nu and Reynolds number Re was

$$Nu = 0.0107\,Re^{0.82} \qquad (10)$$

In this research the length of the tested cones was of the order of 5 cm and the Reynolds numbers were of the order of 10^6. In the present calculation the range of Reynolds numbers requires some extrapolation of Eber's results, but not so much as to cast much doubt on the predictions of the calculation. The length of the ram jet was assumed to be 20 feet for most of the calculations.

Figure 3 shows the variation of external convective heat transfer coefficient h_o as a function of altitude for four Mach numbers, for this 20 ft ram jet.

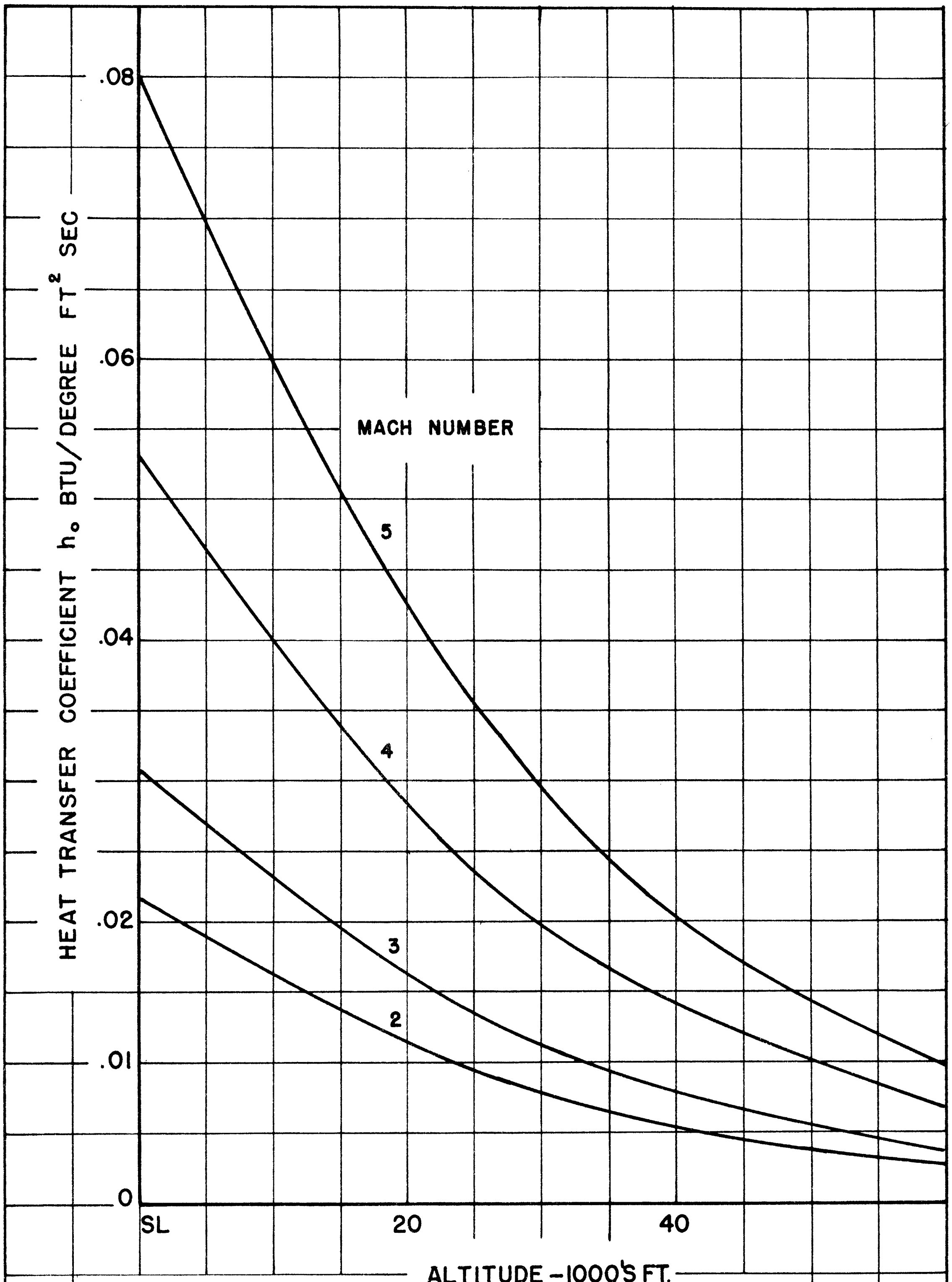

FIG. 3 VARIATION OF EXTERNAL HEAT TRANSFER COEFFICIENT WITH ALTITUDE FOR SEVERAL MACH NUMBERS. 20 FT. RAM JET.

7. Convective Heat Transfer Inside

There is considerable information in the literature on heat transfer inside tubes. Such information is presumably based on tests with more or less fully developed flow and with reasonably low flow speeds. Heat transfer coefficients h_i for the internal flow were calculated by use of Reference 3, the particular equation being

$$Nu = .025\ Re^{.8} \qquad (11)$$

An alternative equation for calculation of the same coefficient is Equation 10 with the boundary layer temperature given by Equation 9. This alternative method was the one principally used. The choice of method for calculation of h_i was made for the reasons that the flow is not likely to be so fully developed in the ram jet as in more usual heat transfer applications, and the velocities are subsonic but high. The single test point available to check the calculation appeared to lie closer to the prediction made with the Eber data, Equation 10.

Figure 4 shows the variation of internal convective heat transfer coefficient h_i as a function of altitude for various Mach numbers for the 20 ft ram jet.

8. Experimental Check of Method

A basis for estimating the reliability of the method is a calculation of burner shell temperature for a particular ram jet flown recently, as reported in Bumblebee Reports 57 and 58. This vehicle had an observed equilibrium temperature of 1350°F when its speed at sea level was 1600 ft per sec, air-fuel ratio 18. The fuel was ANF-32 kerosene; the burner was 6 inches in diameter.

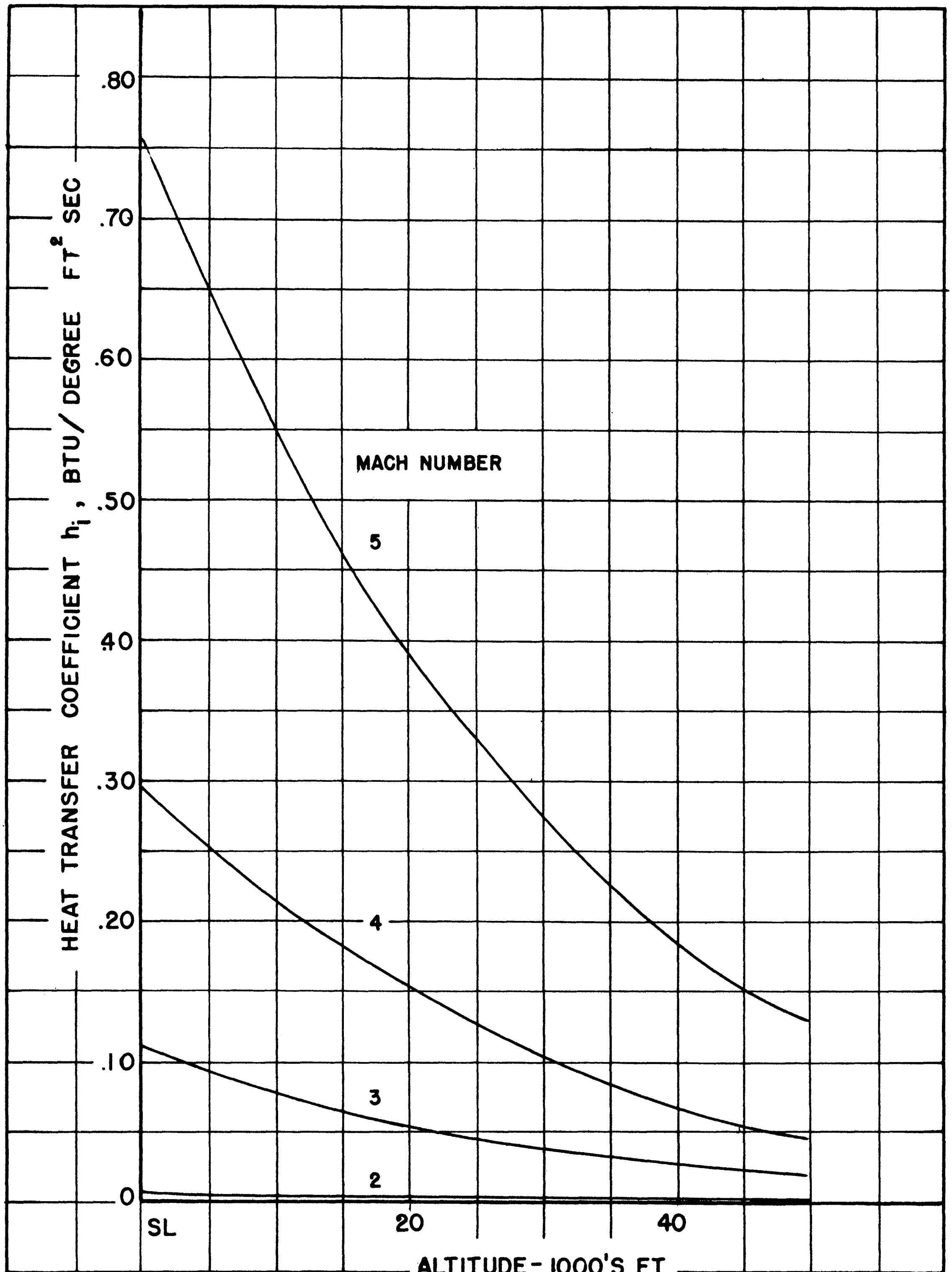

FIG. 4 VARIATION OF INTERNAL HEAT TRANSFER COEFFICIENT WITH ALTITUDE FOR VARIOUS MACH NUMBERS EBER DATA, 20 FT. RAM JET.

A number of assumptions were made in place of data missing in the reports. The ram jet length of 10 ft was assumed, with the system of oblique shocks, as previously mentioned, for Mach number 1.5 due to a 15° wedge followed by expansion around a $7\frac{1}{2}°$ corner. The heat of combustion of 19,810 Btu per lb was assumed for the fuel. The carbon-hydrogen ratio was assumed .45 by mols, characteristic of nonane. These assumptions are believed to be rather good. The Eber correlation was used for calculating the convective heat transfer both inside and outside the burner.

The result of the calculation was T_w = 1465°F, which is 115 degrees higher than the observed temperature, 1350°F. The agreement is remarkably good in view of the fact that a burner efficiency of 100% was assumed in the calculation, and that there is no assurance that the thermocouple and transmitting apparatus were free of error. This agreement tends to give one some confidence in the predictions as calculated.

9. Results of Calculations

Figure 5 shows the results of the calculations of the 20 ft long, 2 ft diameter ram jet in the form of curves of burner shell temperature T_w as a function of the altitude for various Mach numbers. For M = 2 the shell temperatures are from 1825 to 1575°R, that is, red heat, or hotter. At Mach number 3, the temperatures range from 3460 to 2700°R. The upper temperature is above the melting points of various ferrous alloys.

By way of showing the effect of length and diameter of the ram jet on the cooling problem, Figures 6 and 7 show the wall temperature for ram jet burners changed as indicated. The length of the ram jet can apparently be halved without affecting the wall temperature perceptibly. The effect of decreasing the burner diameter is somewhat greater. The temperature is

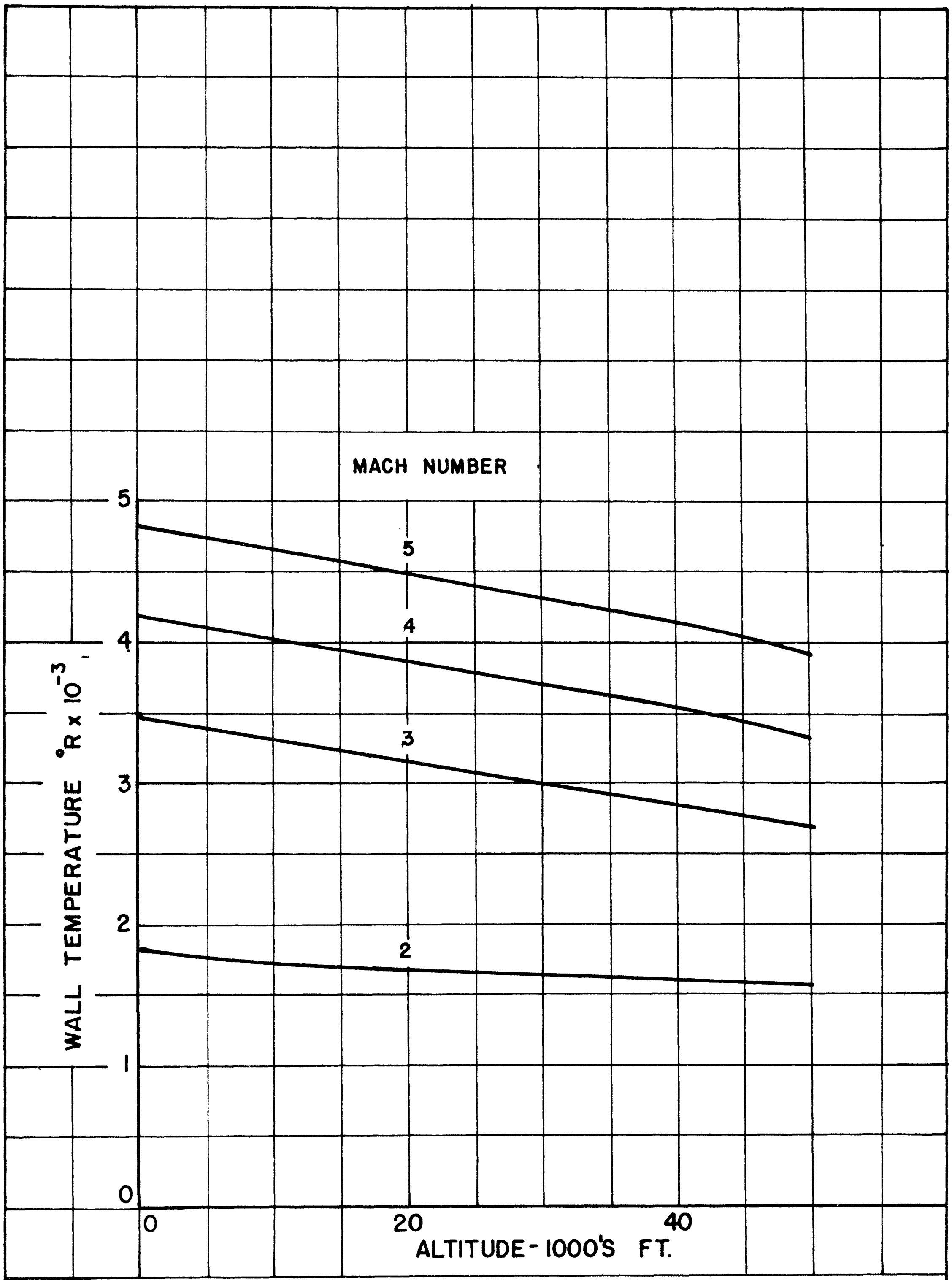

FIG. 5 – VARIATION OF WALL TEMPERATURE WITH ALTITUDE FOR VARIOUS MACH NUMBERS 20 FT. RAM JET 2 FT. DIAM. A/F STOICHMETERIC.

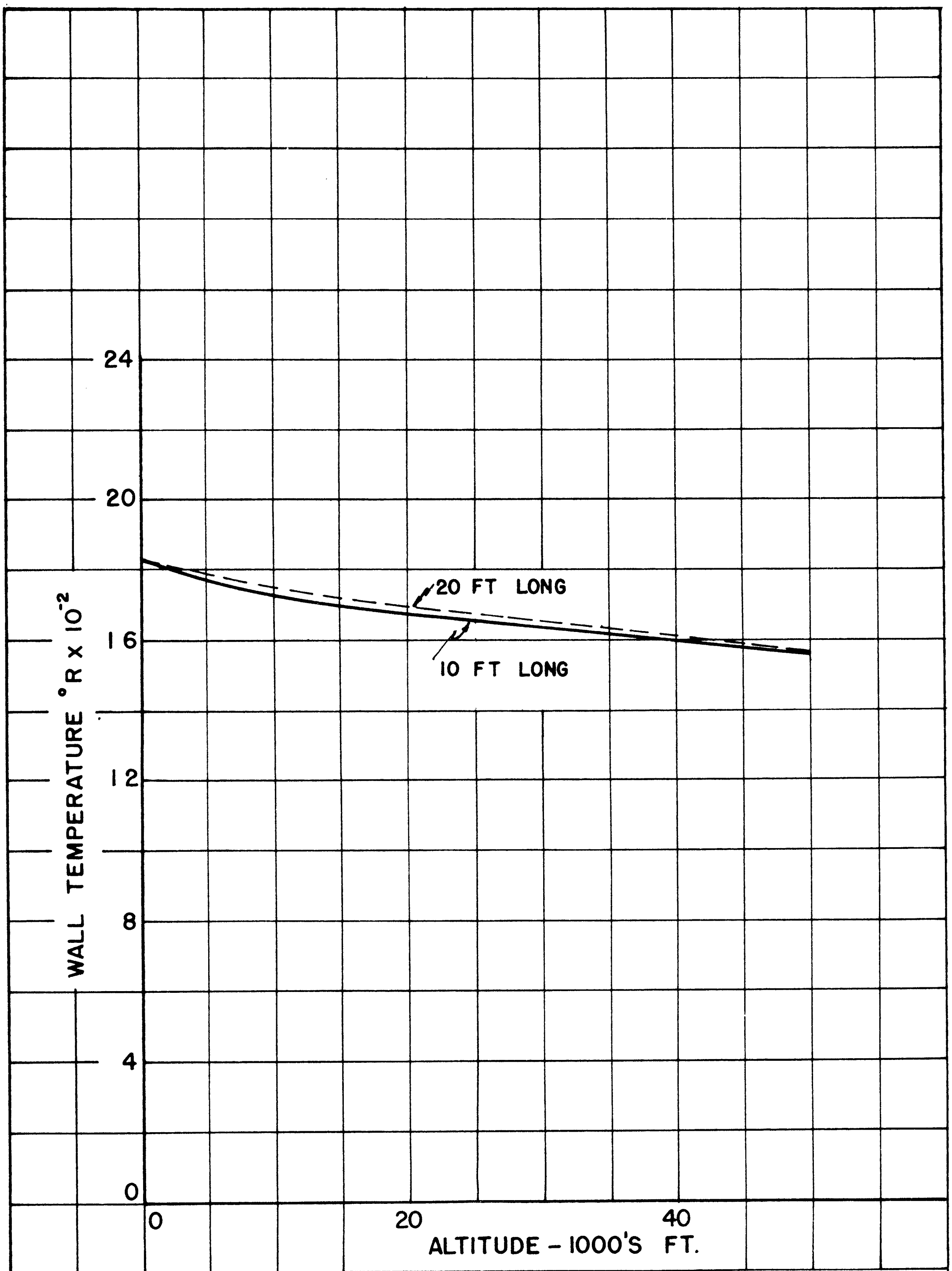

FIG. 6 — VARIATION OF WALL TEMPERATURE WITH ALTITUDE FOR MACH NUMBER 2 A/F STOICHIOMETRIC, 2 FT. DIAMETER RAM JET, FOR TWO LENGTHS

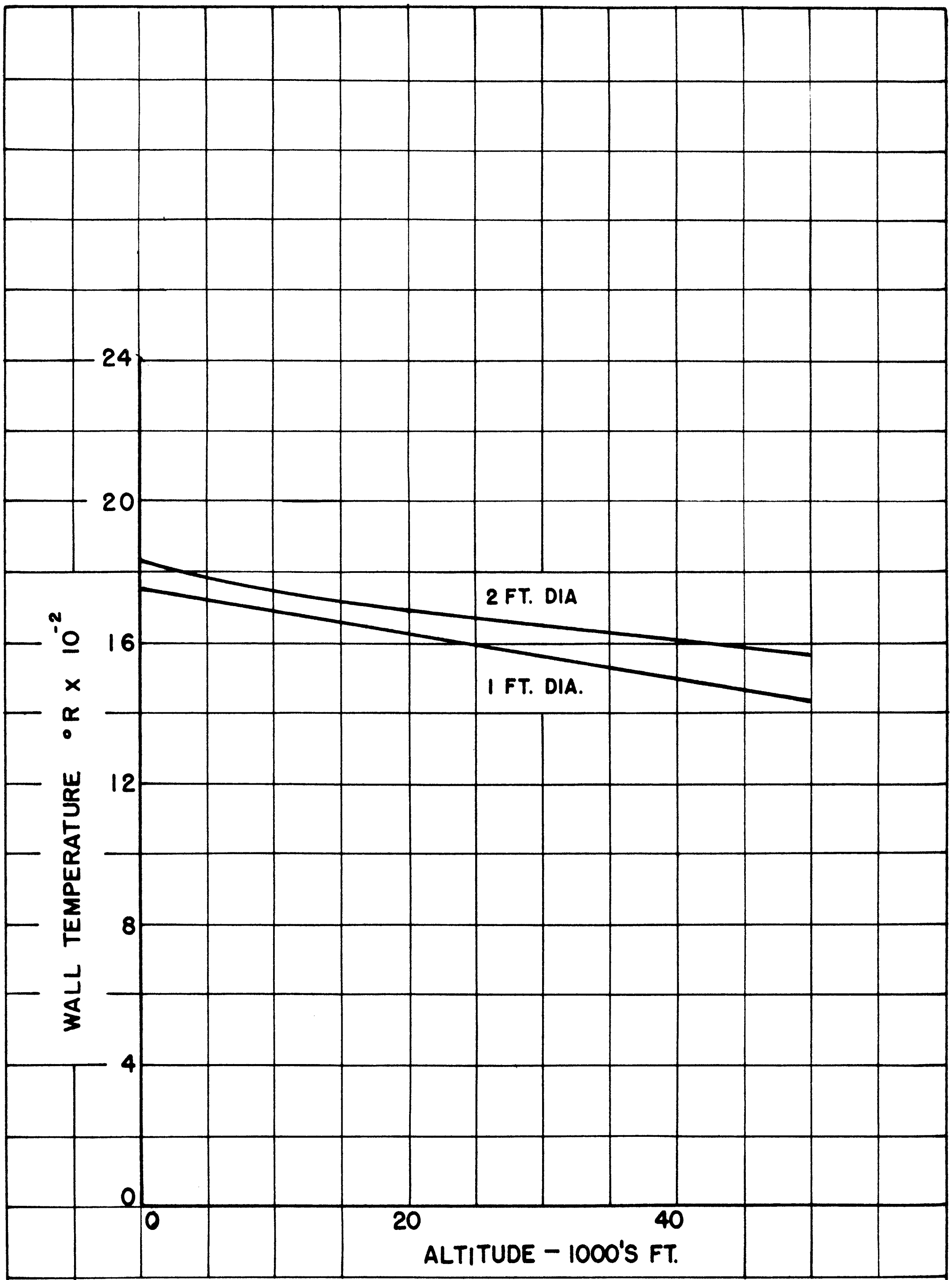

FIG.7 – VARIATION OF WALL TEMPERATURE WITH ALTITUDE FOR MACH NUMBER 2 A/F STOICHIOMETRIC, 20 FT. RAM JET, FOR TWO DIAMETERS

lowered from 80 to 130 degrees by a reduction of the diameter from 2 ft to 1 ft.

For steady flight at Mach numbers above 3 under the conditions assumed, some kind of liner would be required to protect the shell from the burner flame, for any but extreme altitudes. The problem of calculating the shell temperature is fairly complicated for that case. However, suppose that a very effective liner is used, so that, in the limit, no heat is transmitted through the shell at all. The outer shell would then be heated by the boundary layer and cooled by radiation. In such cases the shell temperature would be found as the solution of the equation

$$a\,T_w^4 + T_w = b$$

where, if the shell is black

$$a = \frac{\sigma}{h_o}$$

and

$$b = T_{ob}$$

Figure 8 is a curve of the ram jet wall temperature as a function of altitude for Mach number 5. As would be anticipated, the wall temperature decreases steadily as altitude increases. The ram jet shell has a temperature of 1480°R at 80,000 ft altitude according to the calculation.

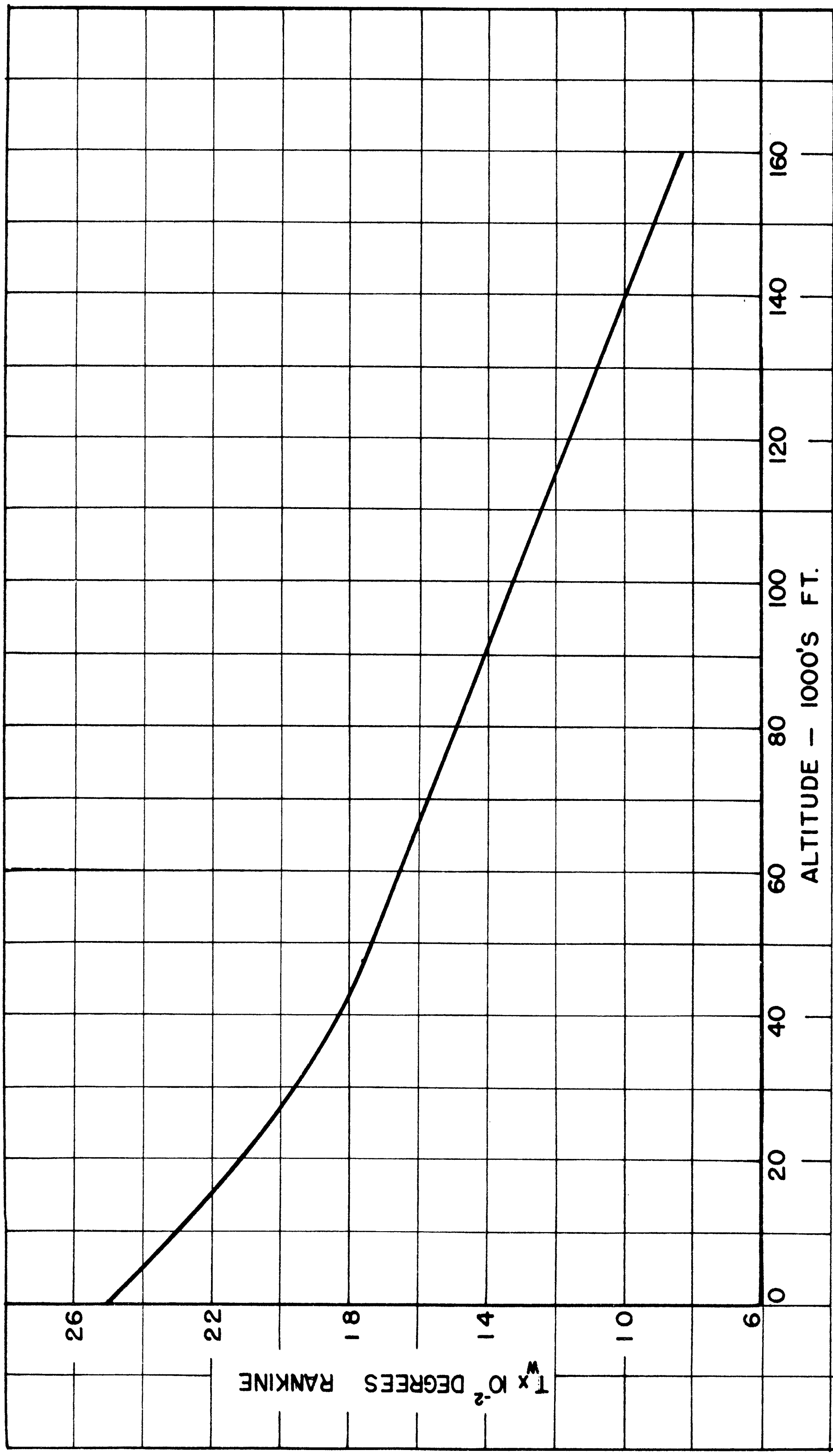

FIG.8 – WALL TEMPERATURE OF A RAM JET WITH AN INSULATED COMBUSTION CHAMBER FLYING AT MACH NUMBER 5. RAM JET LENGTH = 20 FT. DIAM. = 2 FT.

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APPENDIX

Method for Determining the Critical Flow Conditions in a Combustion Chamber of Uniform Section

Consider the flow in a uniform tube in which heating is taking place. At entrance the pressure, density, temperature and enthalpy are p_1, ρ_1, T_1 and h_1. At the downstream end, the ccrresponding quantities are p_2 ρ_2, T_2 and h_2. Assume that the heat addition in the tube is Q. The equations describing the heating process are the conservation laws for mass and energy and the force equation. These are now written in terms of the static pressure increase $\delta p = p_2 - p_1$ and the velocities v_1 and v_2 :

$$\rho_1 v_1 = \rho_2 v_2 \qquad \text{A - 1}$$

$$\delta p = \rho_1 v_1^2 - \rho_2 v_2^2 \qquad \text{A - 2}$$

and

$$\tfrac{1}{2} v_1^2 + h_1 + Q = \tfrac{1}{2} v_2^2 + h_2 \qquad \text{A - 3}$$

For h_2 we write

$$h_2 - (h_2)_p = c_{p_2} \delta T \qquad \text{A - 4}$$

Here $(h_2)_p$ is the value of h_2 for a constant pressure combustion. That is

$$(h_2)_p = h_1 + Q \qquad \text{A - 5}$$

It is apparent that δT is always a negative quantity, as is δp. It follows from A - 3 that

$$\tfrac{1}{2} (v_2^2 - v_1^2) = - c_{p_2} \delta T$$

or $$M_2^2 \left(\frac{\gamma_2 - 1}{2}\right)\left(1 - \frac{\rho_2^2}{\rho_1^2}\right) = -\frac{\delta T}{T_2} \qquad A - 6$$

Also Equation A - 2 can be written

$$\frac{\delta p}{p_1}\frac{p_1}{p_2} = \gamma_2 M_2^2 \left(\frac{\rho_2}{\rho_1} - 1\right)$$

or, from the definition of p/2

$$\frac{\delta p}{p_1} = -\frac{\gamma_2 M_2^2 \,(1_2 - \rho_2/\rho_1)}{1 + \gamma_2 M_2^2 \,(1 - \rho_2/\rho_1)} \qquad A - 7$$

Again by the gas law,

$$\frac{\rho_2}{\rho_1} = \frac{R_1 T_1}{R_2 T_2}\frac{p_2}{p_1} \text{ and } \frac{p_2}{p_1} = \frac{R_1 T_1}{R_2 T_2}\left(1 - \frac{\delta p}{p_1}\right) \qquad A - 8$$

For the starting point of the calculation, the enthalpy h_1 should be found as that for stagnation at station one.

The enthalpy h_{st} of the stagnant air is found by the relation

$$h_{st} = h_o + \tfrac{1}{2} v^2$$

where h_o is the enthalpy of the ambient air, found as a function of T_o in the table of Reference 5; v is the airspeed of the craft. The stagnation pressure p_{st} is obtained from the same table by use of the assumption of isentropic compression from enthalpy h_o to h_{1st} where

$$h_{1st} = h_o + \tfrac{1}{2} v^2 \eta$$

Here η is the efficiency of the diffuser, chosen in the present calculations as equal to .9.

The stagnation temperature is that corresponding to h_{st} .

As an alternative method for finding stagnation conditions, if v is not too large, the conventional relations can be used.

$$\frac{p_{st}}{p_o} = \left(1 + \eta \frac{\gamma - 1}{2} M_o^2\right)^{\frac{\gamma}{\gamma - 1}}$$

$$\frac{T_{st}}{T_o} = 1 + \frac{\gamma - 1}{2} M_o^2$$

$$\frac{\rho_{st}}{\rho_o} = \frac{p_{st}}{p_o} \frac{T_o}{T_{st}}$$

Here M_o is the flight Mach number.

The procedure in finding T_2, ρ_2 and p_2 is now a straightforward series of approximations as follows:

a. The enthalpy $(h_2)_p$ is found by use of Equation A - 5 from h_1 and Q. For this calculation the thermodynamic charts of Hershey, Eberhardt, and Hottel are required, as published in Reference 1. From the same charts γ_2 is found as the ratio of an enthalpy increment to the corresponding internal energy increment in the general region of $(h_2)_p$. Likewise $(T_2)_p$, the temperature of the air for constant pressure burning, and the effective gas constant R_2 based on one pound of air are taken from the charts.

b. A first approximation of ρ_2/ρ_1 is found by use of Equation A - 8 in which T_2 is chosen equal to $(T_2)_p$ and $\delta p/p_1$ is neglected.

c. With this first approximation to ρ_2/ρ_1, a calculation of $\frac{\delta p}{p_1}$ is made by use of Equation A - 7, and of $\frac{\delta T}{T_2}$ with Equation A - 6, M_2 is taken equal to 1.

d. A second approximation to T_2 is found by the relation

$$T_2 = (T_2)_p + \delta T.$$

e. With this value of T_2, a second approximation of ρ_2/ρ_1 is found from Equation A - 8, and the process is repeated until no change is observed in the results of successive approximations.

If desired, the values of M_1, ρ_1, p_1, and T_1 can be found readily as soon as $\delta p/p_1$ and ρ_2/ρ_1 have been found. Equation A - 2 can be written

$$\frac{\delta p}{p_1} = \gamma M_1^2 \ (1 - \rho_1/\rho_2)$$

from which M_1 can be determined. Then if the subscript $/_o$ refers to the ambient air,

$$\frac{p_1}{p_o} = \left\{\frac{1 + \frac{\gamma-1}{2} M_o^2 \ \cdot 9}{1 + \frac{\gamma-1}{2} M_1^2 \ \cdot 9}\right\}^{\frac{\gamma}{\gamma-1}}$$

is a good approximation, for M_1 small. Also

$$\frac{T_1}{T_o} = \frac{1 + \frac{\gamma-1}{2} M_o^2}{1 + \frac{\gamma-1}{2} M_1^2}$$

and $$\frac{\rho_1}{\rho_o} = \frac{p_1}{p_o} \frac{T_o}{T_1}$$

Again the velocity can be found from the Mach number and the absolute temperature at any point by

$$v = M\sqrt{\gamma RT}$$

Here R is the gas constant per unit mass of the mixture. For v in ft per sec, R is in ft lb per slug ^{o}R.

The error in ρ_2/ρ_1, $\frac{\delta p}{p_1}$ and $\frac{\delta T}{T_2}$ caused by the assumption of total enthalpy in place of local static enthalpy at the beginning of the calculation is expected to be negligible. The calculations can be carried out quite rapidly, and only a minimum number of approximations are necessary.

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