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INVESTIGATIONS WITH A MATHEMATICAL MODEL OF THE LAKE BREEZE

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ABSTRACT

Although the lake breeze is considered an important mesometeorological phenomenon, there has not been much work done to mathematically describe it. Many studies of sea breezes have been carried out; some of the results are applicable to the lake breeze, but others are not. Moroz in 1965 modelled the lake breeze using modifications of some of the sea breeze ideas, and a computer program is now available to serve as the model.

This study perturbs this model in three ways, each different in concept. The first perturbation is simply a data change: the maximum temperature of the land is increased by 3.2°C. The model correctly predicts the increase of the lake breeze circulation. The next modification involves trying to model a land breeze, as yet not attempted with the model. No circulation characteristic of a land breeze is produced, but reasons are presented to possibly explain why. In the last case the model itself is changed by substituting an eddy diffusivity profile which decreases from the top of the boundary layer to the top of the model as the square of the height, instead of linearly as previously used. The movement of the lake breeze is slowed, but its strength remains as in the unperturbed model.

ACKNOWLEDGMENTS

The author is indebted to Dr. Alan L. Cole for his suggestions and advice during this study, and to Dr. William J. Moroz for his guidance in using the computer program which serves as the model under consideration in this paper.

1. INTRODUCTION

The Great Lakes are without a doubt very influential in determining local meteorological situations within many miles of their shorelines. The influence of Lake Michigan has been shown both during the winter, in relation to the snow belt (Thomas, 1964), and in the summer affecting thunderstorms (Moroz and Hewson, 1966; Lyons, 1966; Lyons and Wilson, 1967). Air pollution is becoming an increasingly important problem, and the effects of Lake Michigan upon pollutants have been pointed out by both Lyons (1966) and Lyons and Wilson (1967).

Many of the above effects can be linked to the lake breeze circulation along the shore of the lake. If models of the lake breeze were available these effects could be studied in a more mathematical way, and perhaps give some insight into their mechanisms. Moroz (1965) has developed such a model, and an extension of his work will be discussed in detail in this paper.

Three types of perturbations are introduced, one at a time, into the model. The first involves increasing the maximum temperature of the land by 3.2°C. This should produce a more intense lake breeze. For the second case, the land temperatures are changed so as to represent a nighttime situation, which should produce a land breeze. This has not before been tried using the Moroz model. As a last modification, the model itself is modified by changing the eddy diffusivity profile from the top of the boundary layer to the upper boundary of the model.

2. THE MOROZ LAKE BREEZE MODEL

The mathematical model of Moroz is summarized here, but for a more complete description the reader is referred to the origional work.

The model uses the differential equations of motion and heating, the equation of continuity, and the hydrostatic equation, and computes fields of wind (both onshore and alongshore), temperature, diffusivity, pressure, and potential temperature. Finite differencing of the equations is done on an IBM 7090 computer. Some of the relations are taken from Estoque's work on the unbounded sea breeze model.

The lake breeze model is of the semi-bounded type; i.e. the grid extends normal to the shoreline a finite difference into the lake but an infinite distance over land. The tangential component extends to infinity both up and down the coastline. An expanding grid is used in the horizontal, with close spacing near the shore and increasing distance between grid lines as the distance from the shoreline increases in both directions.

Vertical spacing of the grid is linear above the top of the constant flux layer, with a spacing of 100 m. This extends to 3050 m, where all influences from the lake are assumed to vanish. The depth of the constant flux (or surface boundary) layer is taken as 50 m.

The result of the gridding technique is a 16 x 32 grid, set up in the x-z plane. Distances along the y (North-South) axis are not considered, since the lake breeze is viewed as being homogeneous in this direction.

The inputs to the computer model are the initial surface pressure and temperature at the shoreline, prevailing lapse rate, grid spacing (by changing this the model is applicable to other lakes), and hourly temperatures for a station far enough inland to be considered not under lake effects. This is taken to be Grand Rapids, 53 km from Lake Michigan.

Initial conditions dictate that land and water surface temperatures are equal at the beginning of the calculations (called time zero; in reality about 0600 LST). There can be no geostrophic wind blowing through any of the model boundaries. This is the only other initial restriction.

Calculations are performed at time intervals of five minutes, with hourly results being printed. After twelve hours, computational instabilities become large due to the manner in which the equations are handled, and the results become masked by the instability waves. They become even worse after sixteen hours. Twenty-five minutes of computer time are required to model sixteen hours of real time.

The Appendix contains the results of an unperturbed case, as taken from the Moroz paper, for comparison with results obtained in this study.

3. THREE PERTURBATIONS ON THE MODEL

3.1 Maximum temperature increase

The purpose of this change was to see if a stronger lake breeze circulation would be generated by the model as intuitively predicted for reality. Table 1 gives the hourly Grand Rapids temperatures as used by Moroz, and Table 2 gives the increased values used in this study. Since the model requires that land and water be at equal temperature at time zero, this was preserved but the maximum for the day was increased by 3.2°C. Because of the increased heating one would expect to see a stronger lake breeze, but not until several hours had passed.

Figure 1 shows the onshore component of the wind, u, for the perturbed case four hours after time zero. By comparing it with Figure a in the Appendix, one can see that the two patterns are almost identical. The lake breeze has a velocity of 1 m sec^{-1} about 150 m above the shoreline, and it extends 7-8 km inland. The return flow is centered at 500 m in the unperturbed case, but almost 700 m in the new one.

The lake breeze two hours later is shown in Figures 2 and b. No major change is yet evident; the maximum velocity has increased to 2 m sec^{-1}, but has shifted 2-3 km inland and stayed at a height of 150 m. The offshore flow aloft is now 0.6 m sec^{-1} in the case with augmented heating, and has increased its height to 1000 m (800 m in the original case). The warmed case does show that the boundary of the lake breeze (the zero isotach) has moved 100 m higher than in the unwarmed case. A small return flow at 200 m is beginning to appear in both figures, but it is somewhat larger in Figure 2.

Vertical velocity structures for the same time are represented by Figures 3 and c. In Figure 3 it is more apparent that a change is occurring in the flow. Both upward motion over the land and downward motion over the water have intensified by 2 cm sec^{-1}.

The time of maximum heating is eight hours after time zero. One hour after maximum heating or nine hours after time zero the flow patterns are as they appear in Figures 4 and d. Computationally introduced instability waves are beginning to form over the shoreline. The perturbed case definitely has a more well-developed lake breeze. Over the shoreline it extends to a height of almost 1000 m, whereas in Figure d it is only 550 m high. Increased heating of the air seems to have increased the onshore maximum from 3 to 4 m sec^{-1}, and the offshore maximum from 1 to 3 m sec^{-1}. Notice that in Figure 4 the return flow core is situated almost directly above the onshore maximum, whereas in Figure d there is a definite tilt to the west with height. A secondary onshore cell at 3 km over the lake is present in both cases, but stronger in the heated one. At 18 km over the land a strong offshore flow with a return at 1400 m is much more evident than in Figure d. The same pattern, only smaller, is evident 19 km over the lake in Figure 4, but entirely lacking in Figure d.

Vertical velocities, presented in Figures 5 and e, show just how strong the lake breeze has become by additional heating. The upward motion over the land is now 27 m sec^{-1}, compared with 9 cm sec^{-1} without the temperature increase. Over the water the vertical velocities differ by a factor of more than four. These new values are closer to Estoque's (1960) results for his sea breeze model. Also present in his work and in the perturbed case studied here is the secondary maximum of sinking motion far inland from the rising air maximum.

By twelve hours after time zero the instability waves have become rather large and are distorting the pattern considerably. The lake breeze developed with a warmer air perturbation has penetrated 25 km inland, and the maximum onshore flow has moved to 18 km inland and strengthened to 6 m sec^{-1}. The return flow has not progressed as rapidly; it is centered at 9 km and is weakening. Smaller cells are developing over the lake, but the flow in these cells is fairly weak. Figures 6 and e illustrate these flow situations.

TABLE 1

AIR TEMPERATURE OVER THE LAND AT A LOCATION WELL INLAND FROM THE LAKESHORE*

Time Hrs.	Temperature °K	Time Hrs.	Temperature °K
0**	294.0	10	301.3
1	295.8	11	299.9
2	297.3	12	298.1
3	298.9	13	296.7
4	299.9	14	295.5
5	300.8	15	294.3
6	301.4	16	293.3
7	301.9	17	292.4
8	302.1	18	291.8
9	301.9	19	291.2

* After Moroz (1965)

** Time zero corresponds to the time when over land air temperature equals water temperature.

TABLE 2

AIR TEMPERATURE OVER THE LAND AT A LOCATION WELL INLAND FROM THE LAKESHORE MAXIMUM TEMPERATURE PERTURBATION

Time Hrs.	Temperature °K	Time Hrs.	Temperature °K
0	294.0	10	304.5
1	296.2	11	302.0
2	297.7	12	300.8
3	299.0	13	298.8
4	302.0	14	296.9
5	303.5	15	295.4
6	304.5	16	294.5
7	304.9	17	293.6
8	305.2	18	292.4
9	305.0	19	292.0

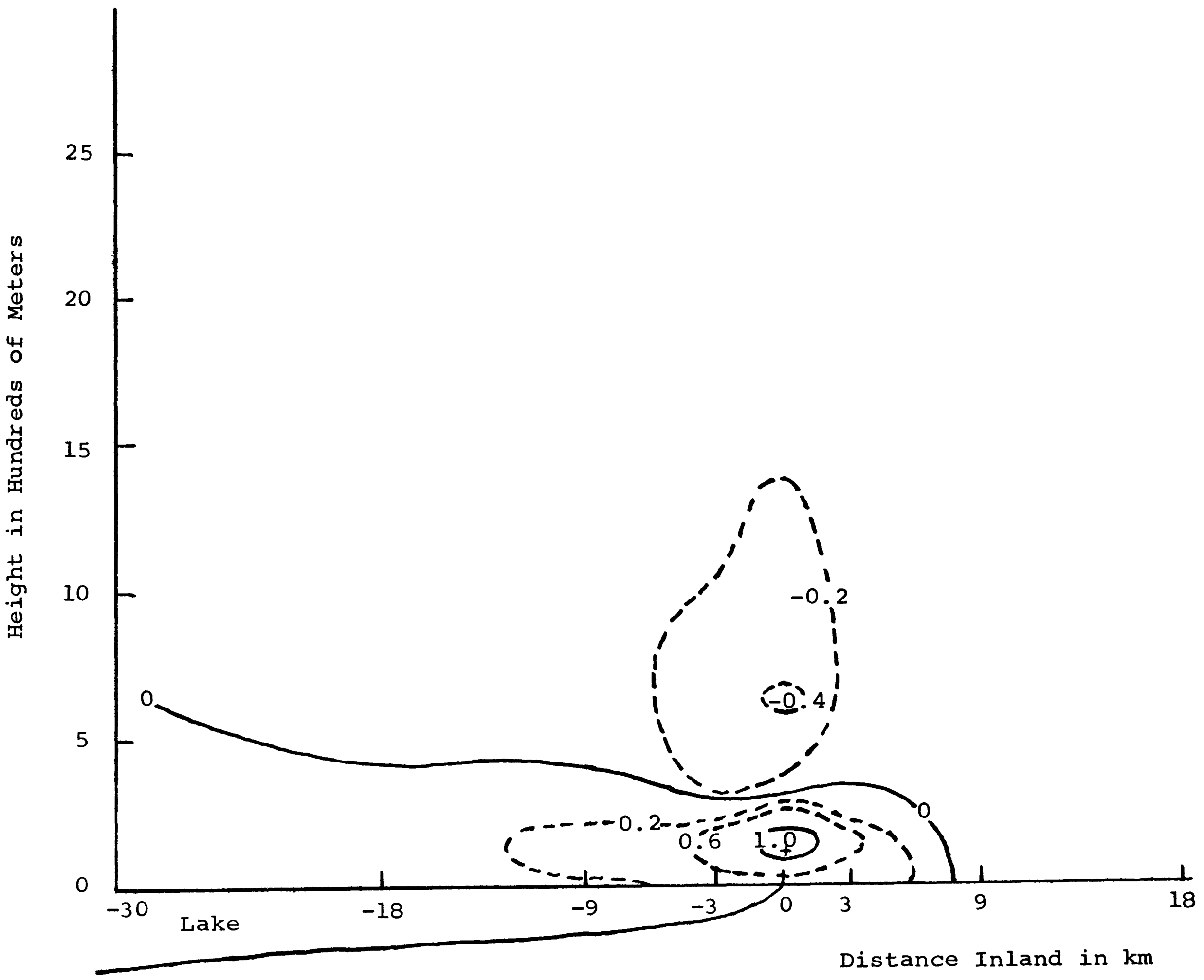

Fig. 1. The across shore wind component (u, in m sec^{-1}) in the model plane 4 hrs. after time zero, for the increased land temperature perturbation.

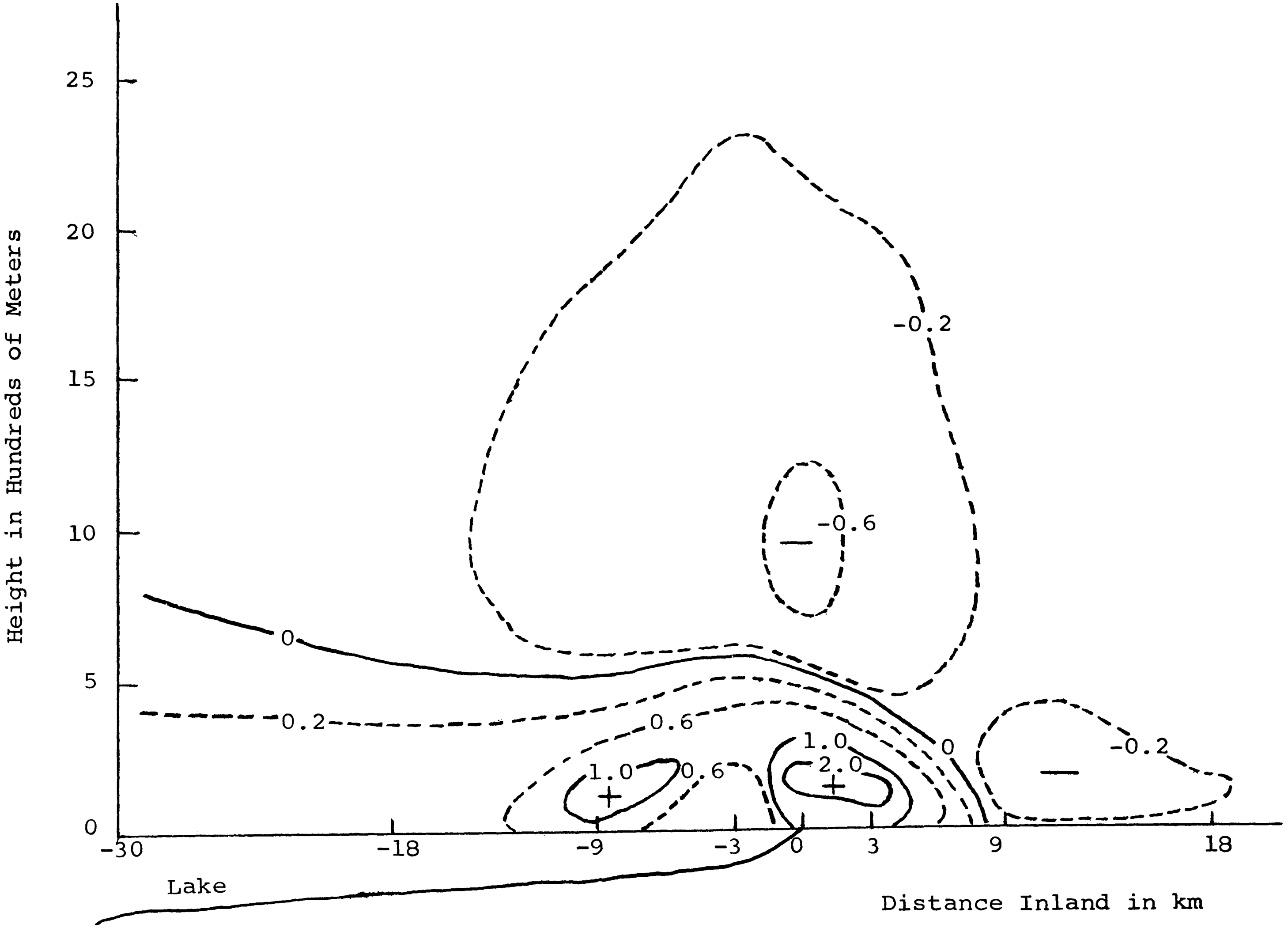

Fig. 2. The across shore wind component (u, in m sec^{-1}) in the model plane 6 hrs. after time zero, for the increased land temperature perturbation.

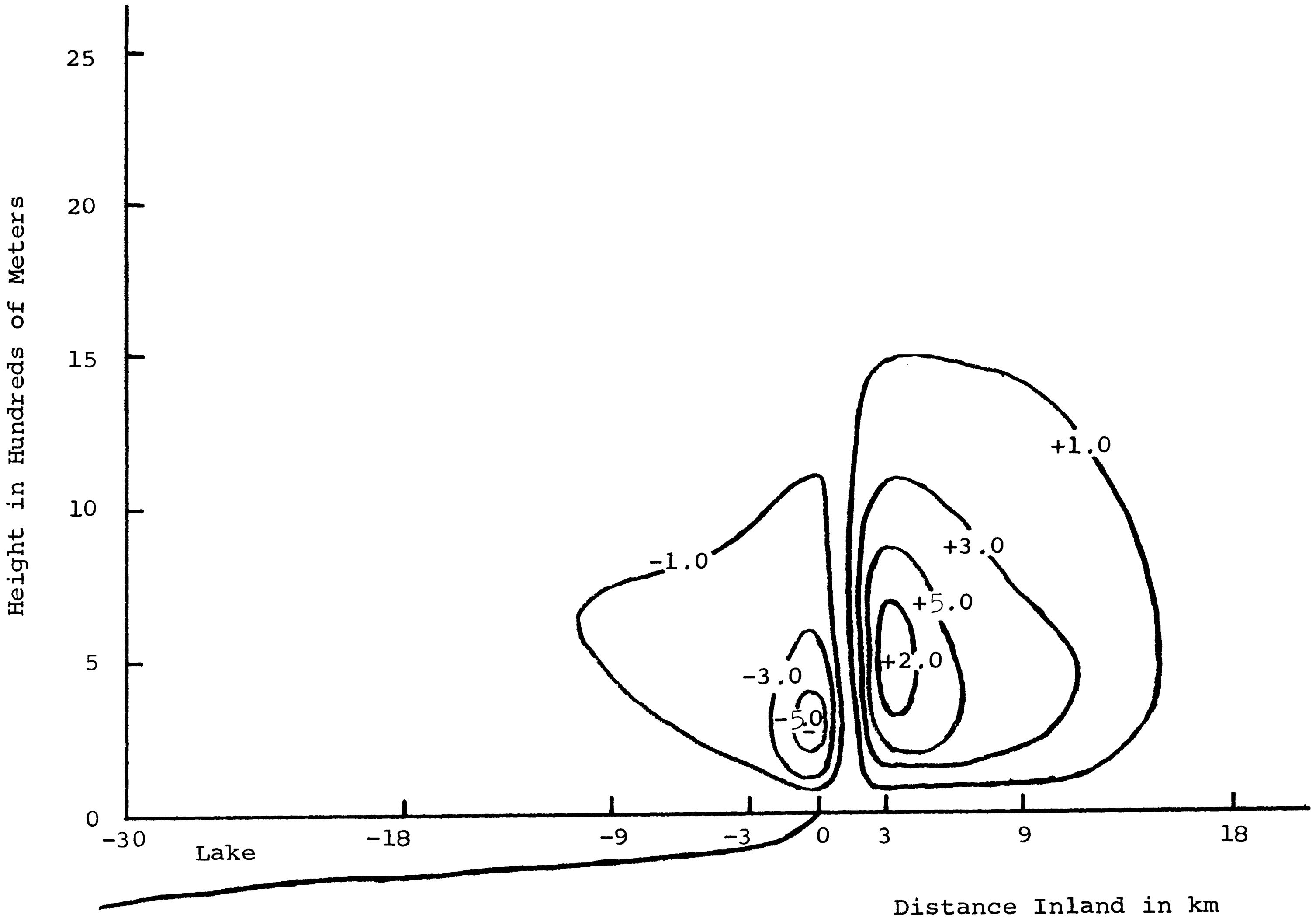

Fig. 3. The vertical wind component in cm sec^{-1} in the model plane 6 hrs. after time zero, for the increased land temperature perturbation.

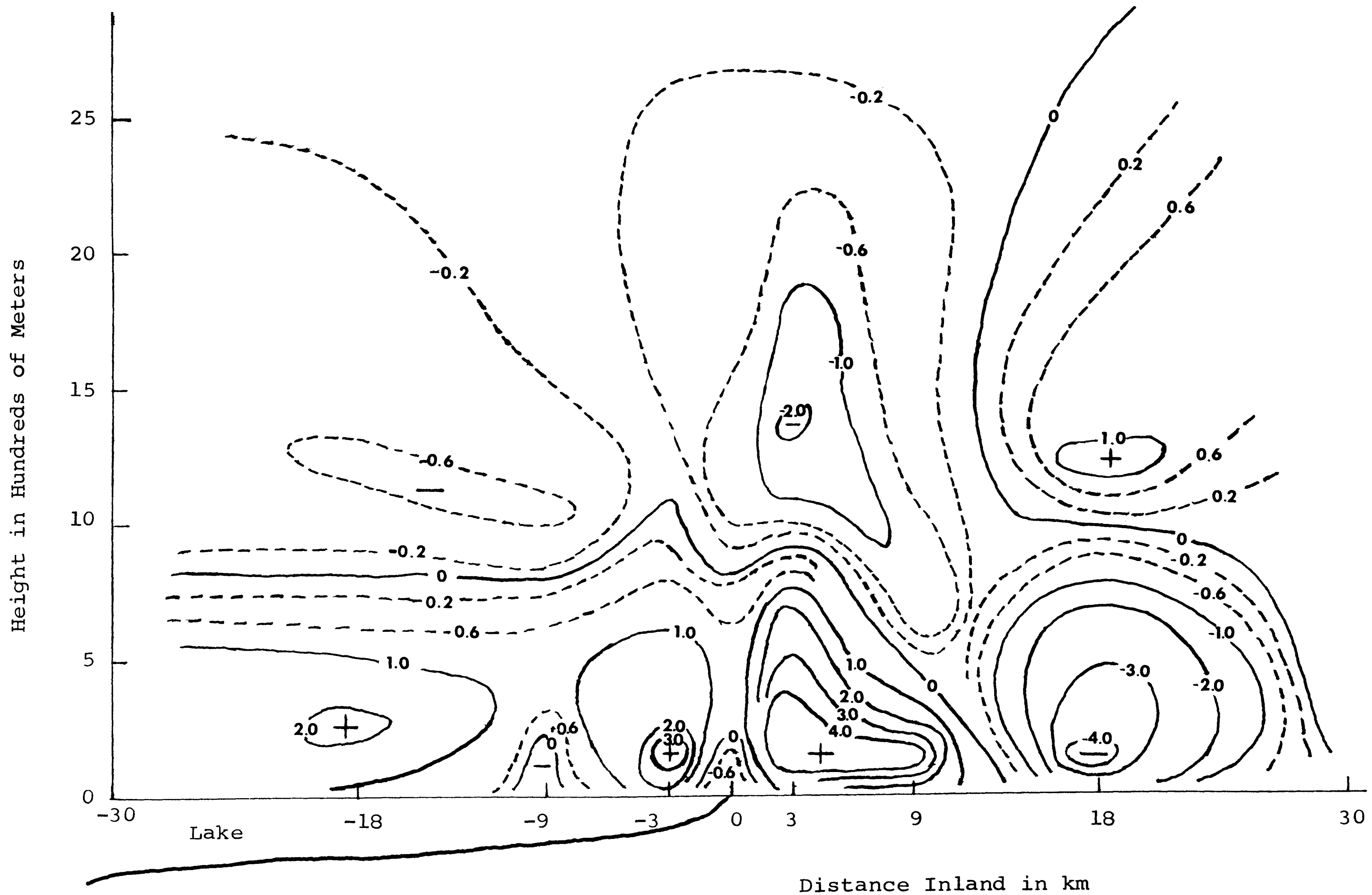

Fig. 4. The across shore wind component (u, in m sec^{-1}) in the model plane 9 hrs. after time zero, for the increased land temperature perturbation.

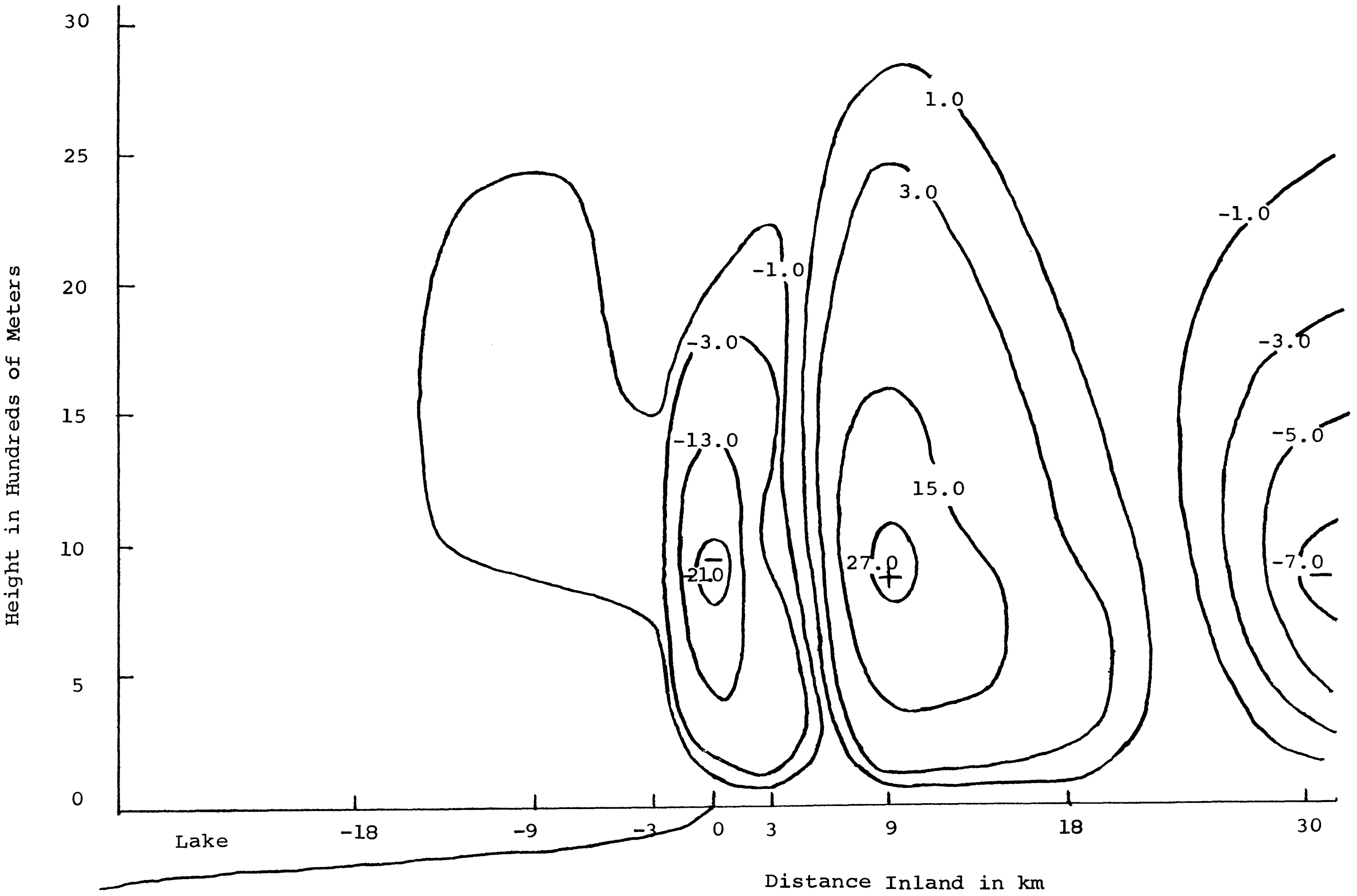

Fig. 5. The vertical wind component in cm sec^{-1}) in the model plane 9 hrs. after time zero, for the increased land temperature perturbation.

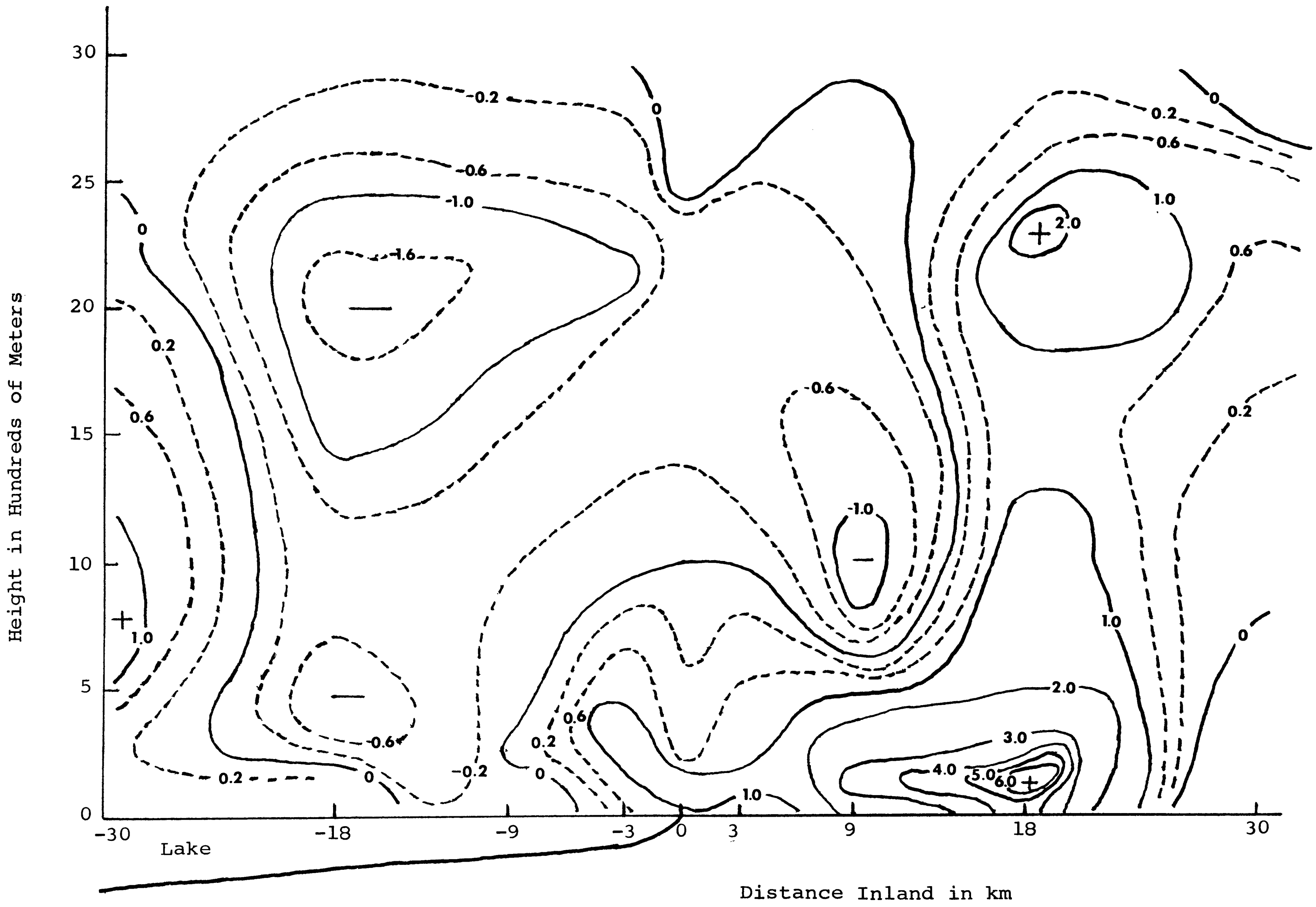

Fig. 6. The across shore wind component (u, in m sec^{-1}) in the model plane 12 hrs. after time zero, for the increased land temperature perturbation.

3.2 Land breeze

The Moroz model is primarily a lake breeze model, and until the present there have been no published attempts to produce a land breeze with it, or for that matter with other lake or sea breeze models. For a first approximation to the land breeze, the input data for the Moroz model were modified to force the over land air temperature to decrease from a value of 21°C at time zero, 2200 LST, to a minimum of 12°C nine hours later at 0700 LST. This is fairly strong cooling, with a temperature change of 9°C, but it was chosen to accent any land breeze which might develop. The hourly temperatures are given in Table 3.

There should be some nocturnal inversion present over the land; unfortunately its strength is not known, and local effects (sand, sand dunes, and trees) make estimating it with any accuracy very risky. It was decided to run the model without the inversion, partly for this reason and partly because the model would have to be extensively modified unless the inversion were to go to 3050 m, which does not seem very realistic.

The last problem when considering the use of the Moroz model for a land breeze, is the eddy diffusivity profile. With the inversion mentioned, the Richardson number

$$Ri = \frac{g}{T} \frac{\partial\theta/\partial z}{(\partial v/\partial z)^2} \qquad (1)$$

will be positive, and in the forced convection regime. Estoque (1959) has said that the eddy diffusivity for momentum in this regime can be expressed as

$$K_m = k^2 z^2 (1+\alpha Ri)^2 \frac{\partial v}{\partial z} \qquad (2)$$

where k is von Karman's constant ≈ 0.4, and $\alpha = -3$. Because the Richardson number is different at night than during the day the value for the diffusivity at any height in the model will be different. Again, because the land breeze has not been investigated in the light of the model, values for the K_m profile are not known. As a first approximation, the daytime profile was used to see what type of flow patterns the model would produce.

With the approximations discussed above, no land breeze

TABLE 3

AIR TEMPERATURE OVER THE LAND AT A LOCATION WELL INLAND FROM THE LAKESHORE

LAND BREEZE

Time Hrs.	Temperature °K	Time Hrs.	Temperature °K
0	294.0	10	285.2
1	293.3	11	287.2
2	292.4	12	289.0
3	291.8	13	291.0
4	291.2	14	292.5
5	290.0	15	294.0
6	287.3	16	295.3
7	286.4	17	296.0
8	285.5	18	297.0
9	285.0	19	298.5

was produced by the model. The onshore wind was positive and of the order of 10^{-4} m sec^{-1} all night. When heating started again in the morning some semblance of order returned to the patterns, but computational instability had masked any quantitative values by that time. It thus appears that more realistic modifications will be required before the Moroz model can represent the land breeze.

3.3 Diffusivity profile modification

Moroz mentions that the least acceptable feature of his lake breeze model is the method used to evaluate the turbulent transfer of heat and momentum. He arbitrarily specifies a value of 500 cm^2 sec^{-1} for the eddy diffusivity K_m at the top of the constant flux layer, and then decreases it linearly to zero at the top of the model. Figure 7 shows the profile.

If this profile could be changed so as to weight the lower heights with a steeper curve, the entire model would be affected because of the increased transport of heat and momentum in the lower levels. This was chosen as the third perturbation for consideration; the new profile (and hence what will be called the new model) is shown in Figure 7. The function used was a simple square of the height.

Figure 8 shows the across shore wind component isotachs for four hours after time zero. By comparing this with the unperturbed results, Figure a, one can see only two changes. The return flow above the lake breeze is not as strong as before, and onshore flow weakens sooner as it moves out over the lake.

Six hours after the inception of the lake breeze the differences between the new and old models are becoming more pronounced, as a comparison between Figure 9 and Figure b illustrates. Maximum onshore velocity seems to be moving inland slower than in the original model, and it is slightly weaker. The return flow is spread over a larger area, and has almost reached the ground ahead of the zero isotach marking the leading edge of the lake breeze.

Figure 10 shows the vertical velocity pattern; comparison with Figure c in the Appendix shows almost no difference. The two zero isotachs are probalby a consequence of the weak flow surrounding the regions in the lake breeze flow.

Nine hours after time zero the area of maximum onshore flow is still lagging the unperturbed results. Figure 11 shows the remainder of the pattern to be fairly compatible with Figure d, except for one feature. A small wave has been introduced at a mean height of 2500 m.

The vertical velocities under the new model appear in Figure 12. They are generally smaller than those of Figure e, and have several smaller cells developing, mostly over the water.

Three hours later, the wave on the 9 hour across shore wind pattern has become large enough to slightly distort the return flow over the shoreline at 2200 m. An offshore flow of 1 m sec^{-1} is evident 9 km into the lake; this is not present in the unmodified model. At 18 km inland and 1500 m a 3 m sec^{-1} onshore core appears; this is on the other model, but much weaker.

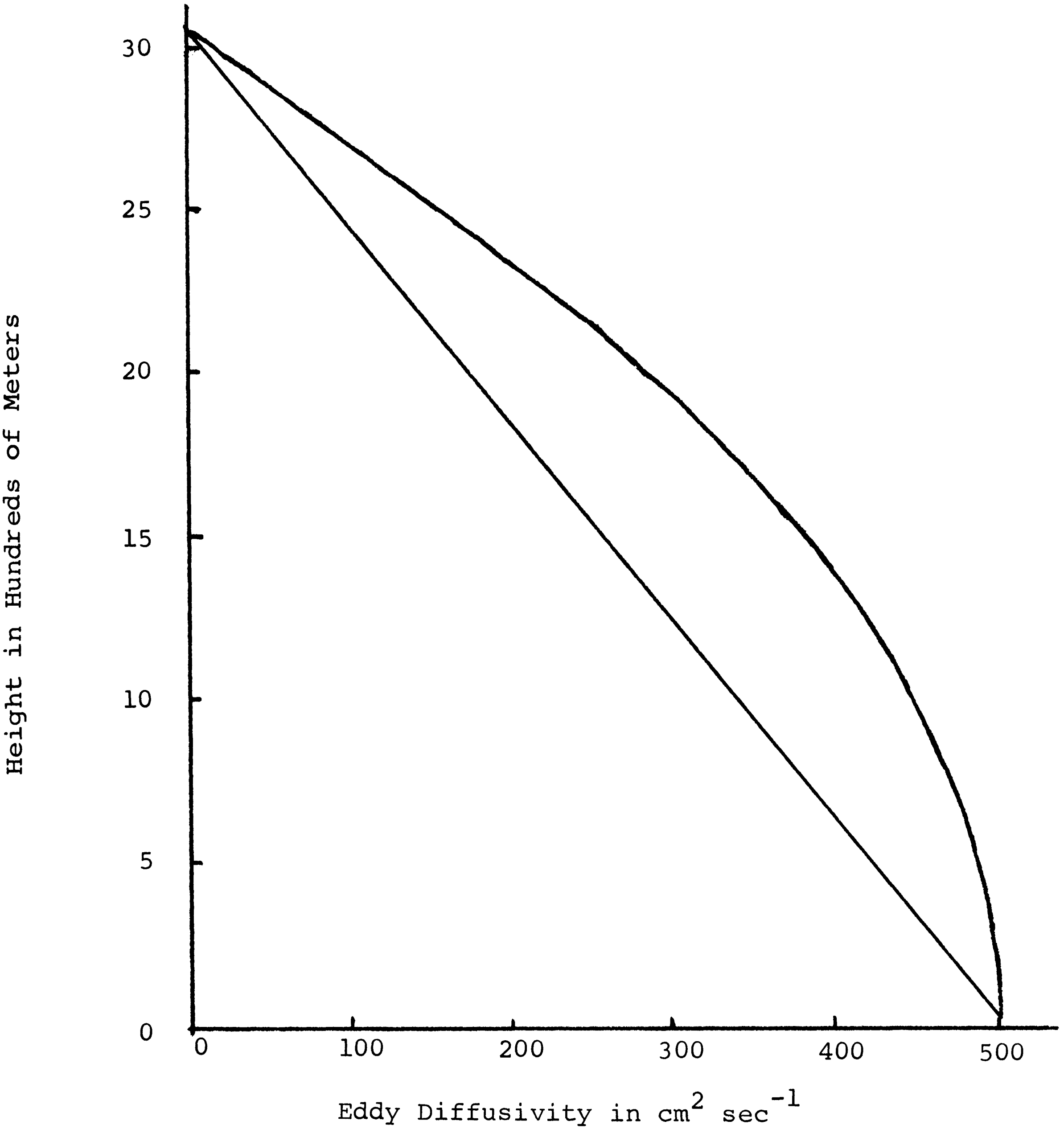

Fig. 7. Diffusivity profile used by Moroz (straight line) and by Wilson in the perturbed Moroz model (curved line).

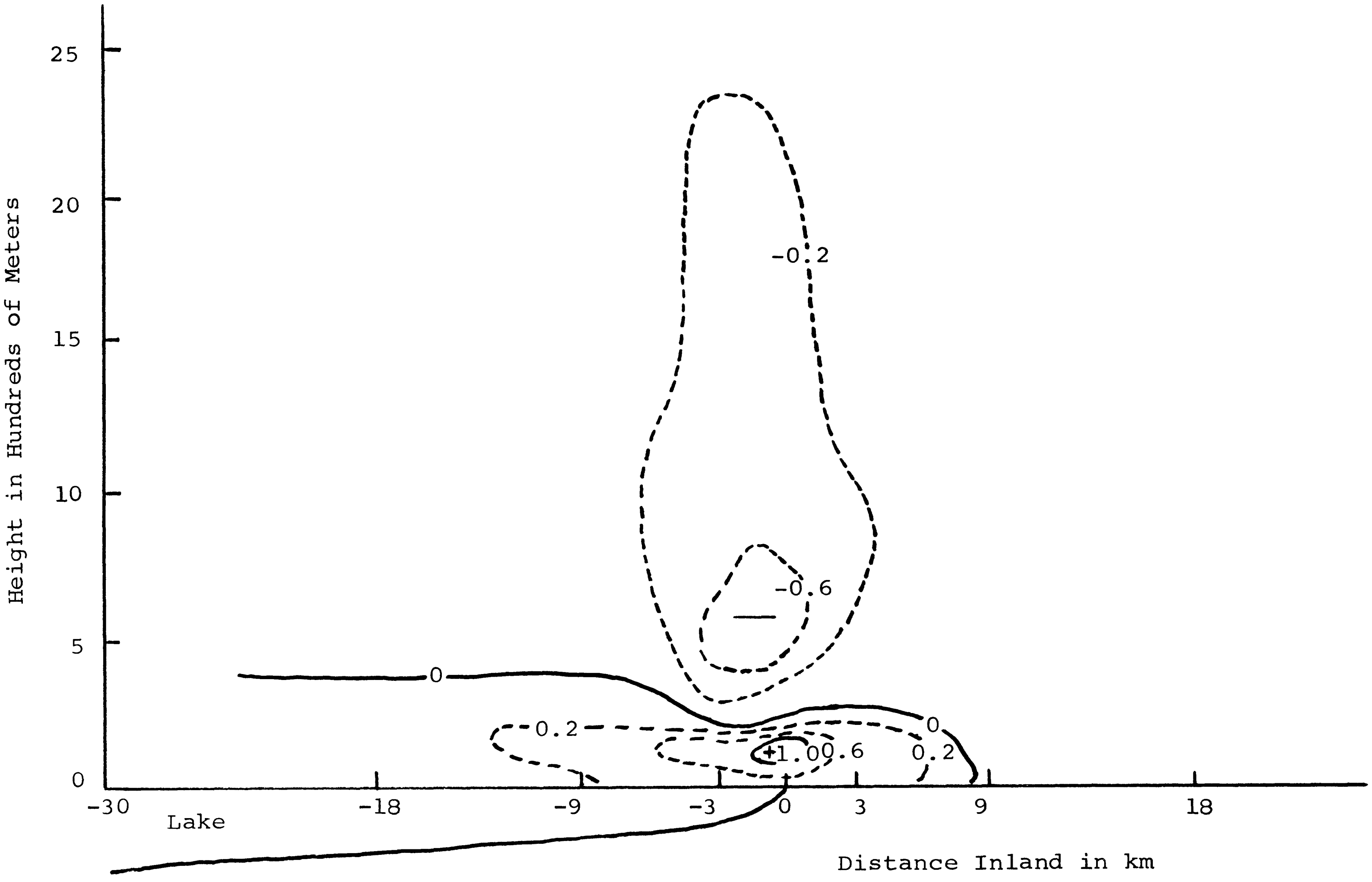

Fig. 8. The across shore wind component (u, in m sec^{-1}) in the model plane 4 hrs. after time zero, for the diffusivity profile modification.

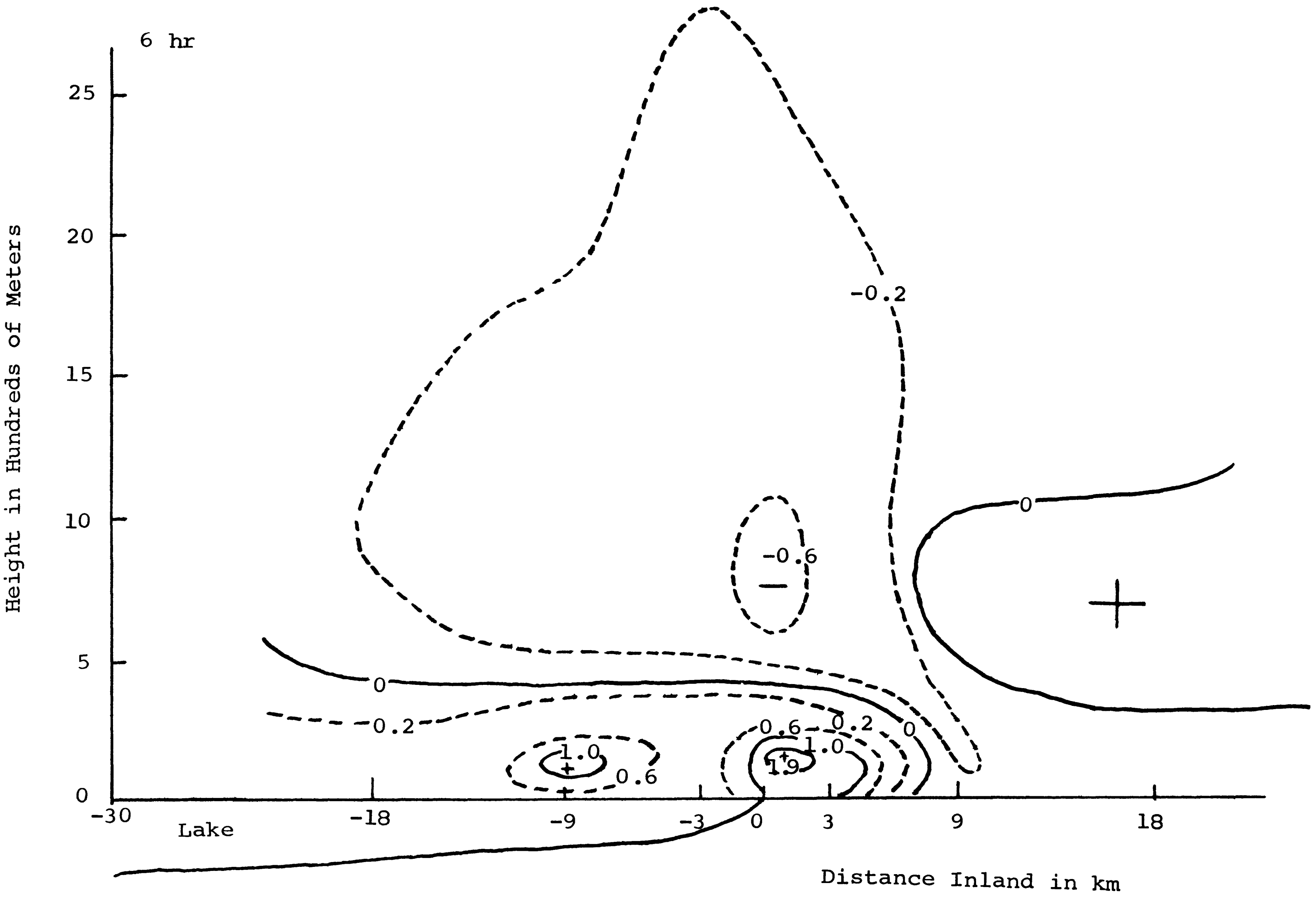

Fig. 9. The across shore wind component (u, in m sec^{-1}) in the model plane 6 hrs. after time zero, for the diffusivity profile modification.

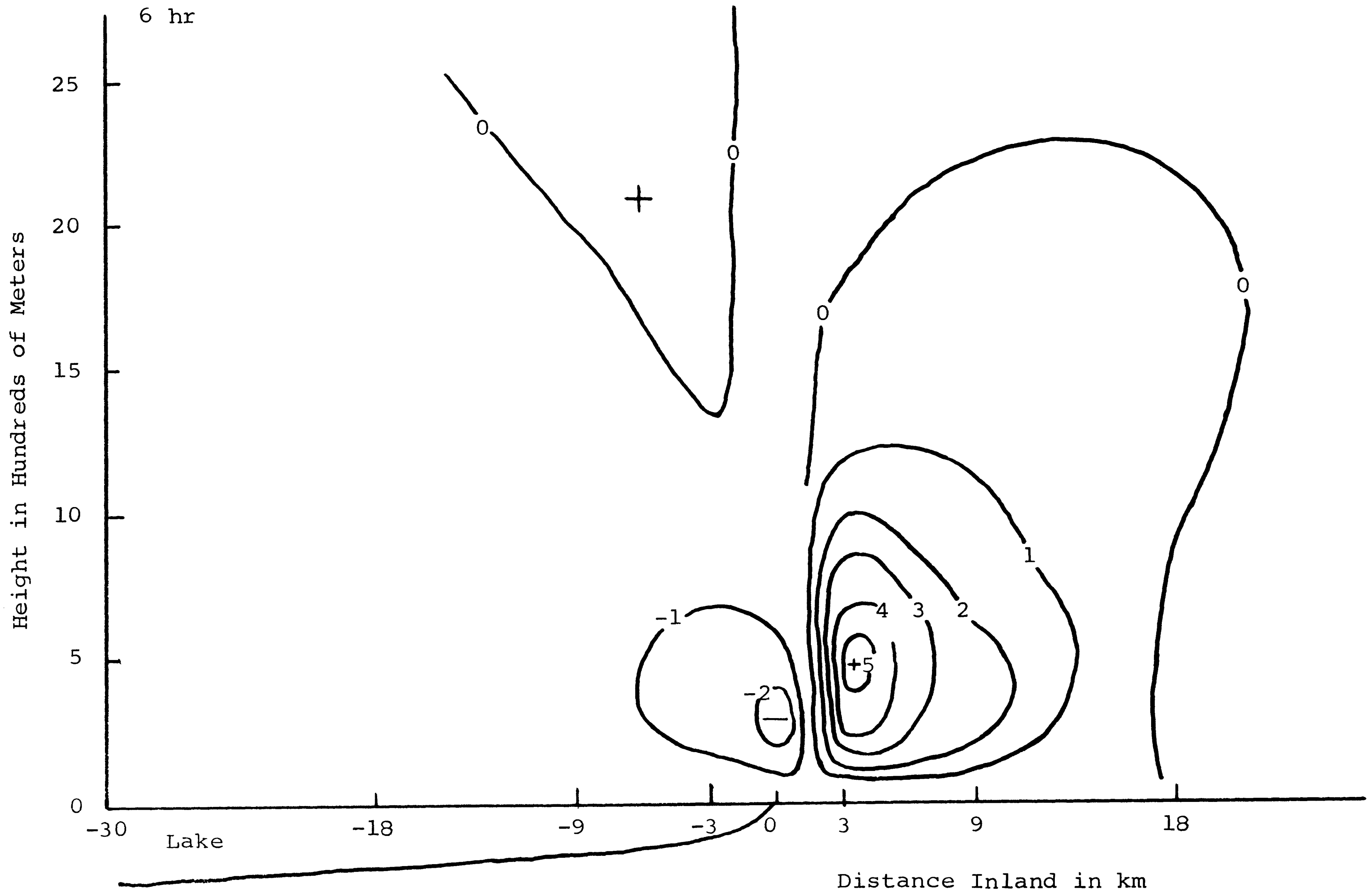

Fig. 10. The vertical wind component in cm sec^{-1} in the model plane 6 hrs. after time zero, for the diffusivity profile modification.

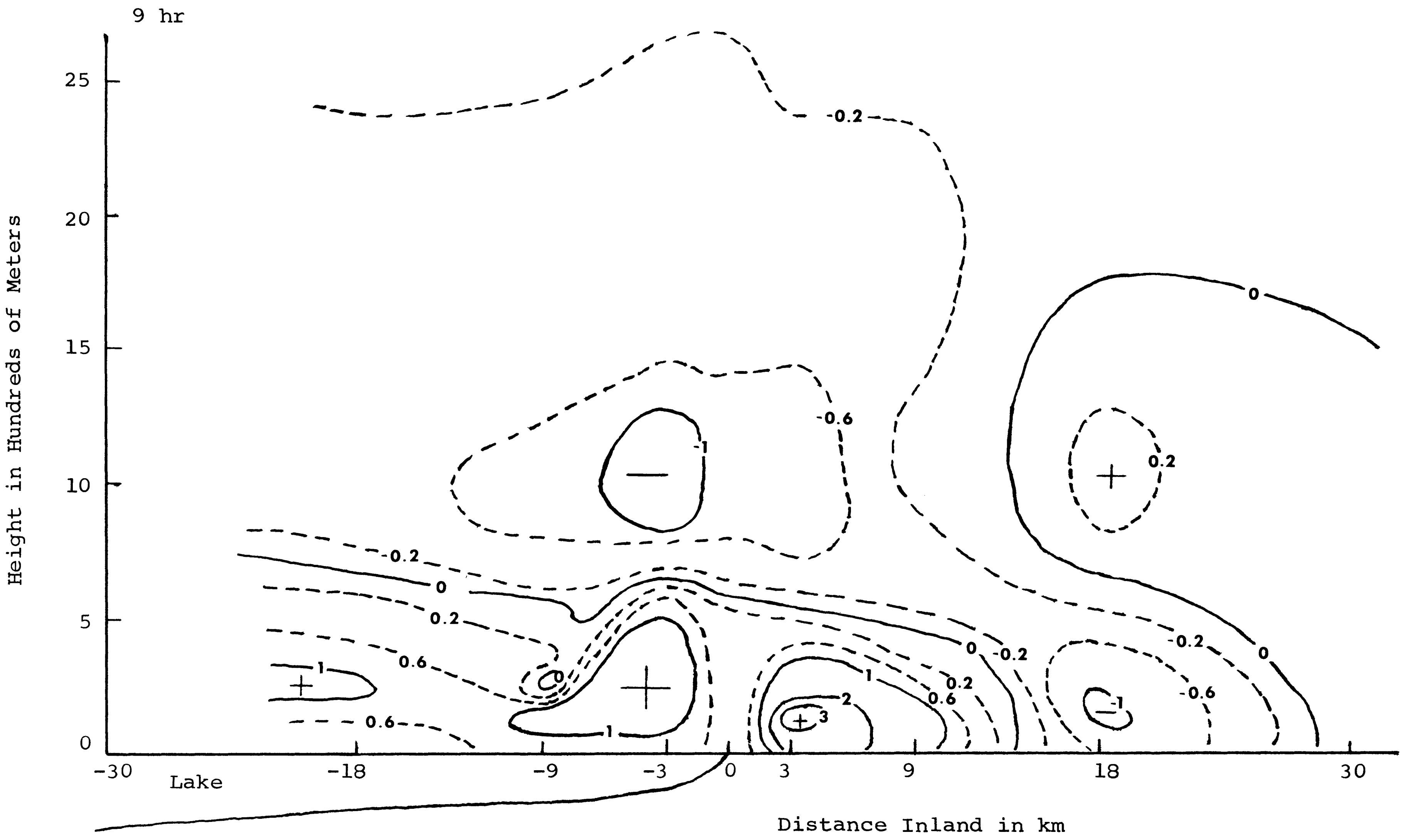

Fig. 11. The across shore wind component (u, in m sec^{-1}) in the model plane 9 hrs. after time zero, for the diffusivity profile modification.

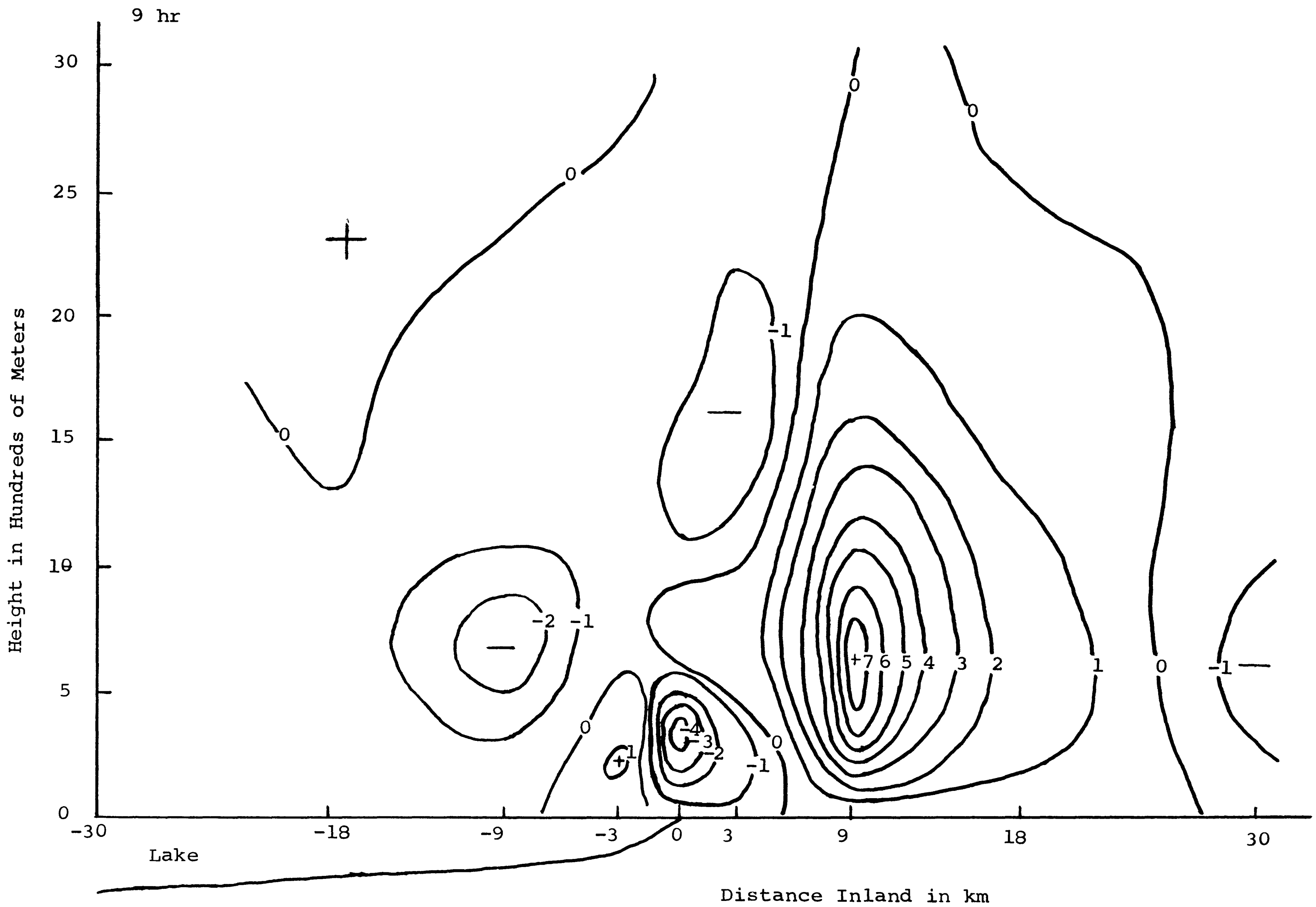

Fig. 12. The vertical wind component in cm sec^{-1} in the model plane 9 hrs. after time zero, for the diffusivity profile modification.

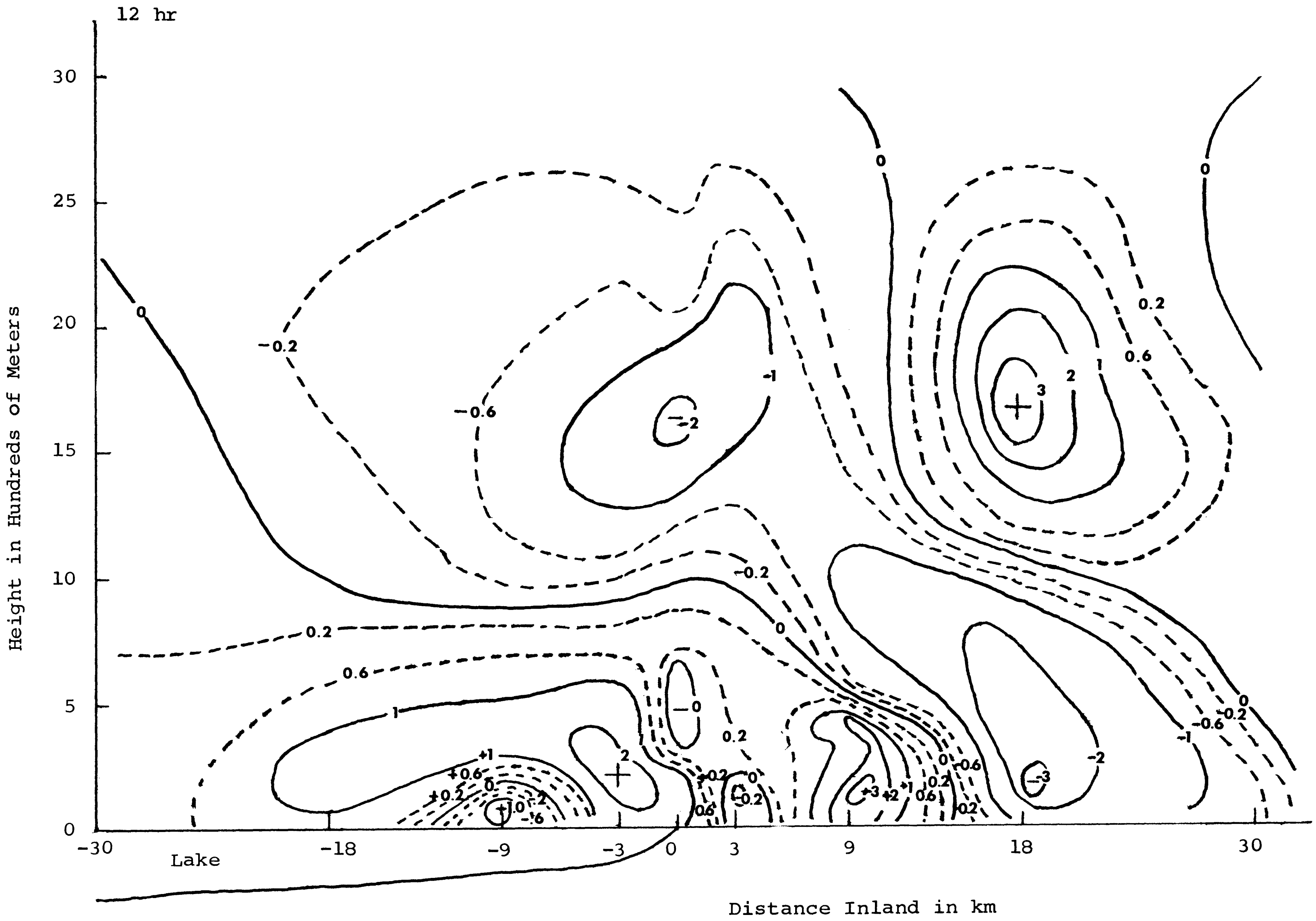

Fig. 13. The across shore wind component (u, in m $\sec^{-1}$) in the model plane 12 hrs. after time zero, for the diffusivity profile modification.

4. SUMMARY AND CONCLUSIONS

Of the three types of perturbations imposed on the Moroz model, each produced its own characteristic results.

Increasing the maximum temperature intensifies the lake breeze and moves it further inland. An hour after maximum heating, vertical velocities are triple the values with no excess heating, and the maximum onshore velocity is greater, but not by as much.

Trying to introduce a land breeze into the model was not successful, but some additional work with nocturnal temperature and diffusivity profiles might correct this.

Substituting a new diffusivity profile that allowed K_m to decrease as the square of the height decreased the movement of the lake breeze, but not its strength. After the time of maximum heating, however, computationally introduced instability waves developed on the isotach pattern and distorted it.

5. SUGGESTIONS FOR FURTHER WORK

Perturbations of all three kinds presented here (new data, new situation, or change in the model) are possible, but in the last one lies the widest range of possibilities. The two biggest problems are the edddy diffusivity profile and the computational instability introduced after the model has run through several hours of meteorological time. If solutions to these problems were available the model would be more versatile, and could possibly remain accurate through lake breeze decay and land breeze development. Another interesting modification would be to incorporate a prevailing geostrophic wind into the model, and test its effect on the lake breeze. Haurwitz (1947) has modelled a sea breeze with a geostrophic wind involved; his results provide an interesting staring point from which to work.

APPENDIX

SUMMARY OF UNPERTURBED RESULTS

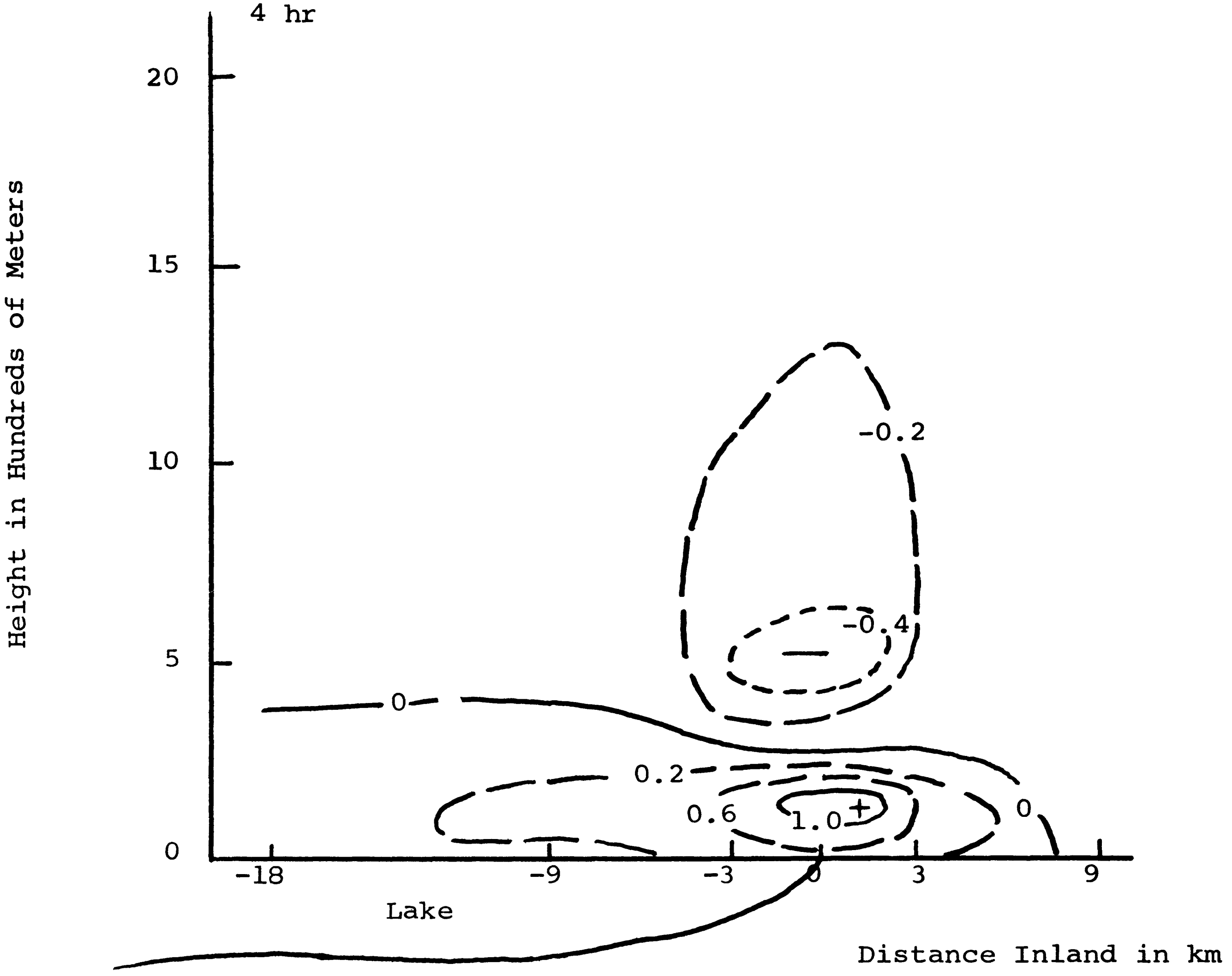

Fig. a. The across shore wind component (u, in m sec^{-1}) in the model plane 4 hrs. after time zero. (After Moroz)

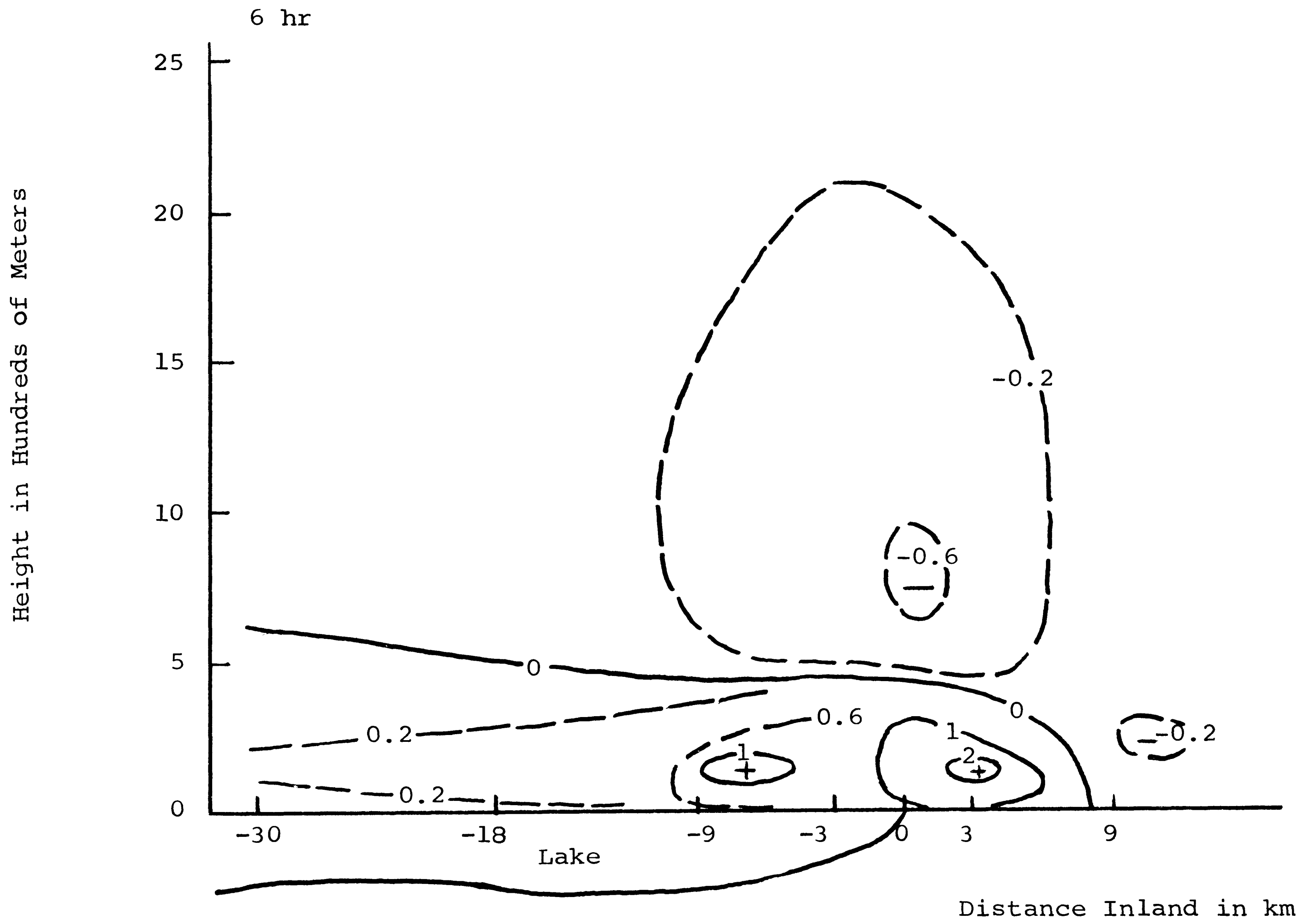

Fig. b. The across shore wind component (u, in m $\sec^{-1}$) in the model plane 6 hrs. after time zero. (After Moroz)

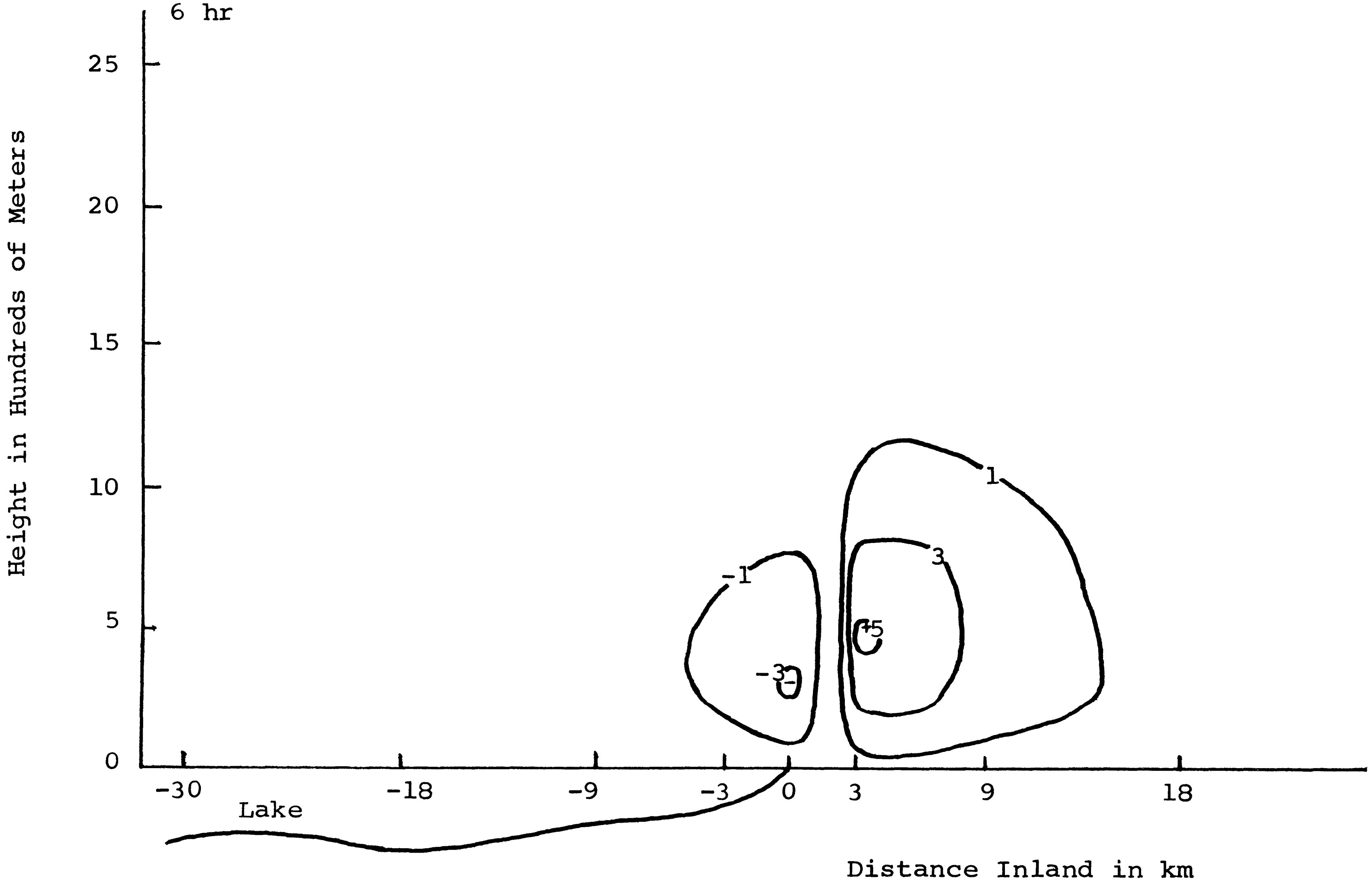

Fig. c. The vertical velocity wind component in cm sec^{-1} in the model plane 6 hrs. after time zero. (After Moroz)

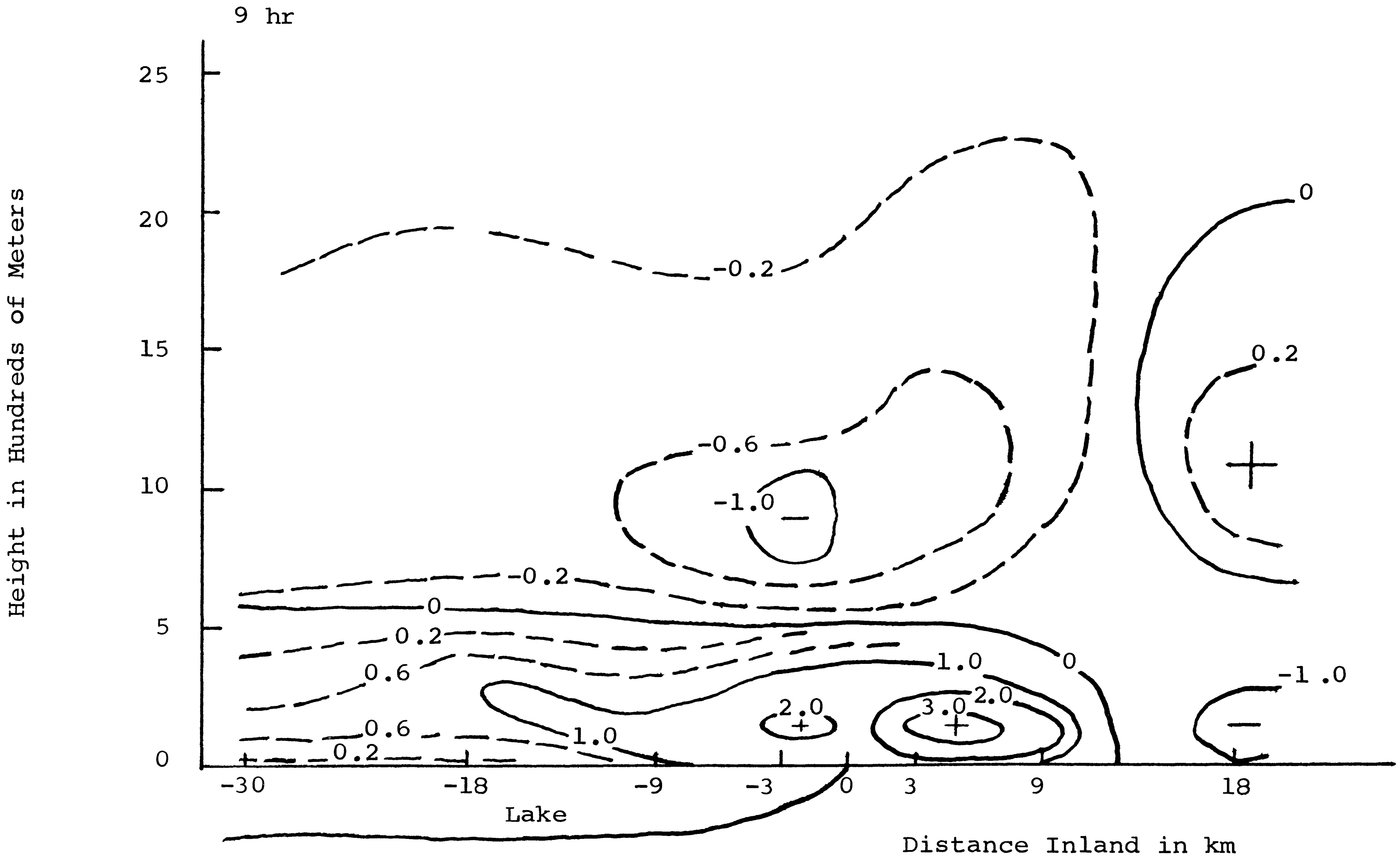

Fig. d. The across shore wind component (u, in m sec^{-1}) in the model plane 9 hrs. after time zero. (After Moroz)

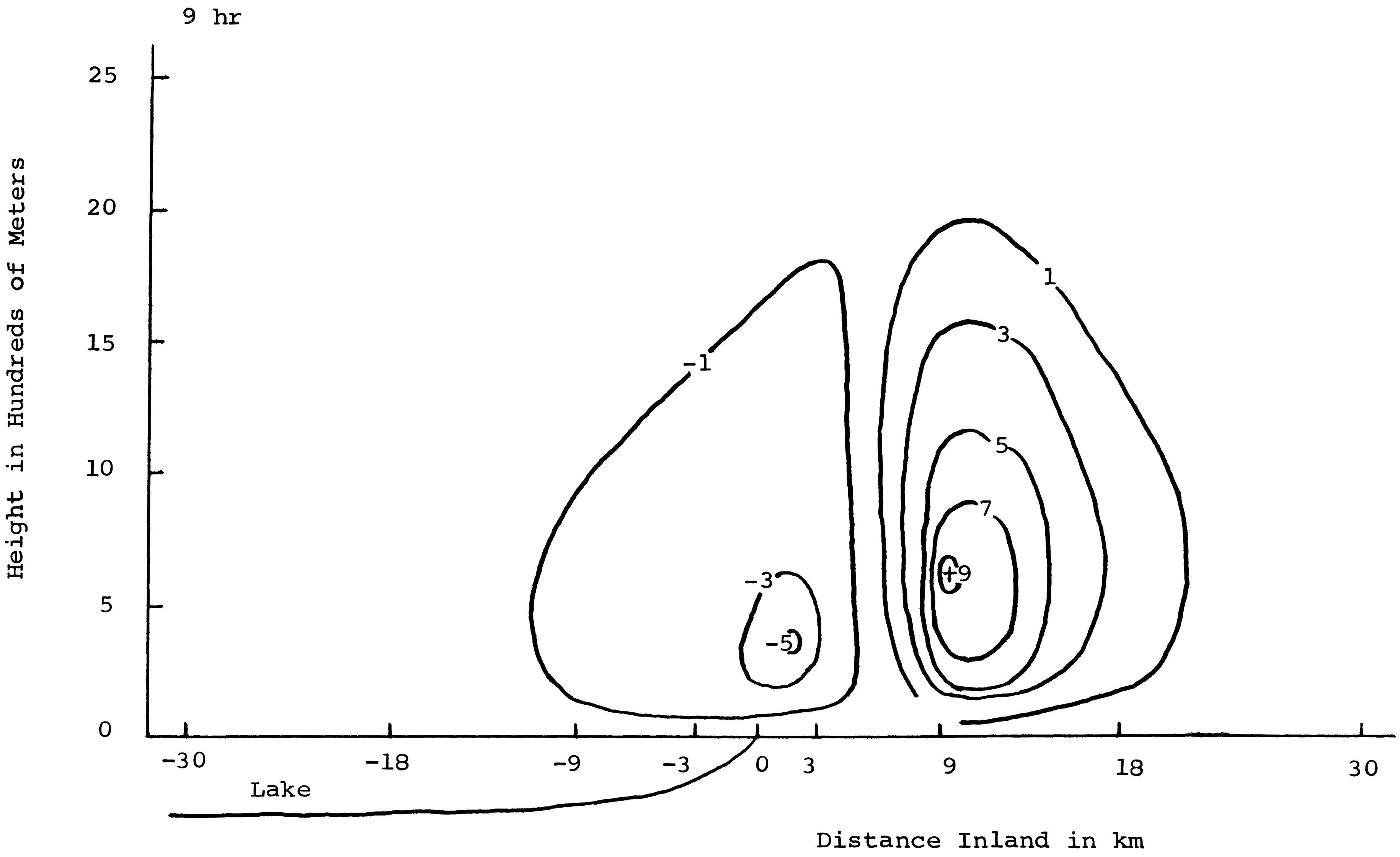

Fig. e. The vertical velocity wind component in cm sec^{-1} in the model plane 9 hrs. after time zero. (After Moroz)

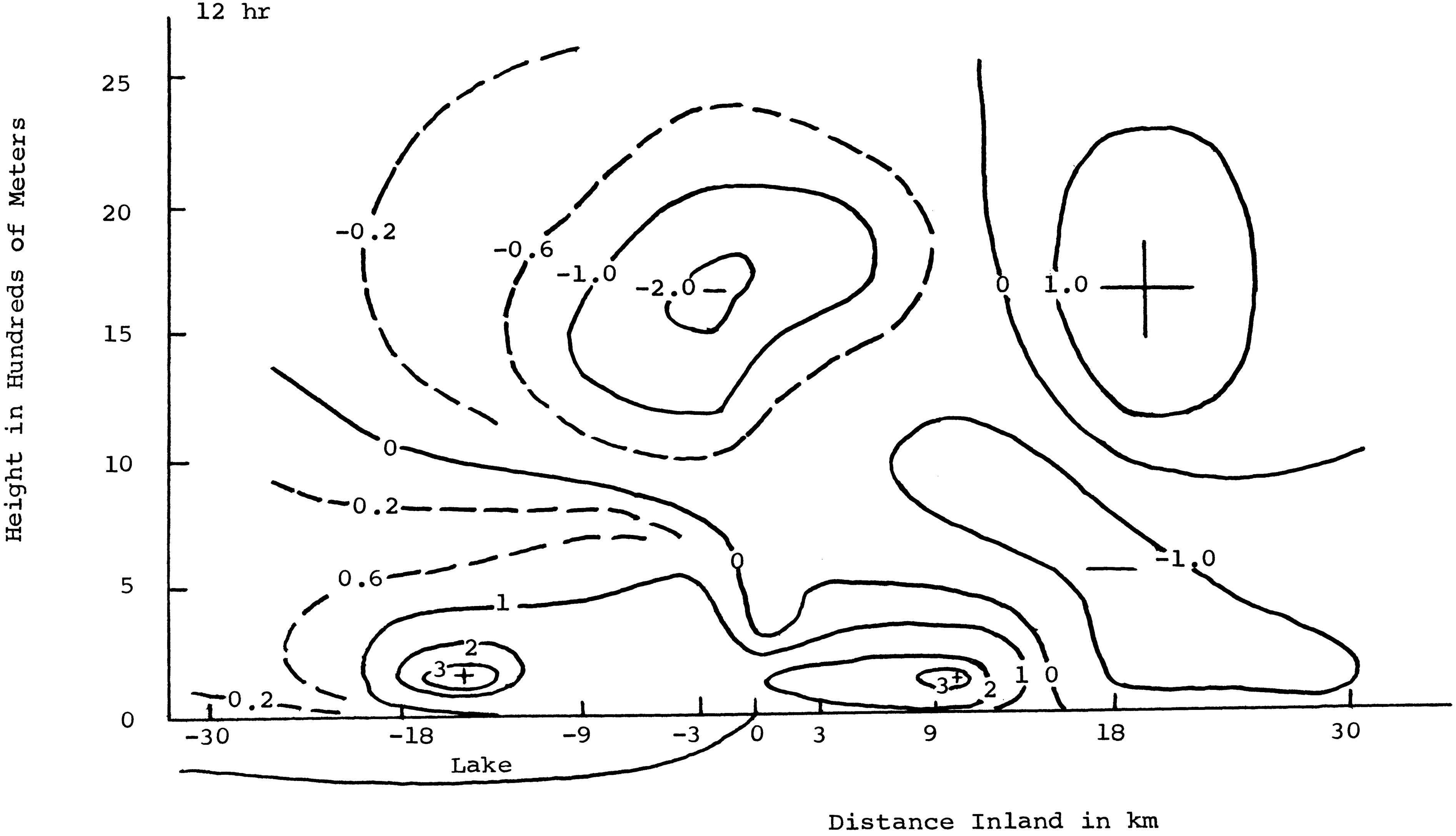

Fig. f. The across shore wind component (u, in m sec^{-1}) in the model plane 12 hrs. after time zero. (After Moroz)

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