SOME AIR WAR GAMES

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SUMMARY

This paper gives an expansion of the air
war game explained in a RAND document
"Some War Games" by John Nash and R.M.
Thrall. Several variations of the basic
games are introduced. These games are
designed with the view of use for screen-
ing and training purposes, and include
enough variations so as to test the adpta-
bility as well as the ability of the
players.

SOME AIR WAR GAMES

1. <u>Introduction</u>. These games are descendants of a game originally investigated by A. Mood and are played on a hexagonal (honeycomb) pattern board. Many of the basic concepts of the ground war games were suggested by O. Helmer, A. Mood, John Nash, L.S. Shapley, R.M. Thrall and others at Rand. The air war games, which came later, and grew out of the ground war games were suggested and developed by R.M. Thrall. Some suggestions for variations in the air war games came from various members of Projects M-720-1 and M-965 at the University of Michigan. Some interesting observations were made while experimenting with these games, and will be discussed after the rules have been explained.

All games described here are two person games with incomplete information. Each player has a board marked off into hexagonal fields (see model on following page) and there is a barrier which can be placed between the boards when secrecy is desired. Each board is divided into three parts— home territory and enemy territory with a no-man's-land between. Disks of various colors are used as markers for the fighting units and cities. At the beginning of a game the number of fighting units is roughly ten times the number of cities. The object of the game is to "kill" all the units of the opponent.

2. <u>Rules</u>. After the initial stage the play goes in cycles, each of which consists of the following stages:

        1. Simultaneous moves with barrier in place.

        2. Barrier removed to give complete information about

           opponents' position.

3. Aim fire.

4. Remove dead.

5. Receive new units at cities which have been designated production centers.

2.0. <u>Initial Stage.</u> The boards are placed on a table with a space between for the barrier. The boards are lined up so that the field label al is at the lower left of one player and at the upper right for the other player. The barrier is placed between the boards to start the game. Each player determines the location of the opponent's cities (except in certain variations in which the city positions are preassigned) subject to the following conditions: all cities must be in the opponents section (enemy territory) of the board, two cities are separated by at least four fields, and each city and the adjacent airport is separated from any boundary by at least two fields. In most of the games played thus far the board is divided into two equal parts by a heavy dashed line in the middle. This line is considered a boundary, called the <u>middle boundary</u>. The fields adjacent to the middle boundary belong to the territory of neither player and make up a region called <u>no-man's-land</u>. (Cities are separated from no-man's-land by at least one field so units can be placed directly in front of the cities).

After the cities have been placed the barrier is removed and each player duplicates on his own board the positions assigned to his cities by the opponent. Again the barrier is placed between the boards and each player places his units at will in his home territory provided that he places no units in <u>no-man's-land</u> and no field has more than three units on it. The barrier is removed and each player fills in on his board the position of his

opponent's units. The barrier is then put in place and the first cycle begins.

2.1. <u>Moves</u>. Each unit may be moved to an adjacent field, provided that at the end of the move there are no more than three units of the same color on any field. Two fields are said to be <u>adjacent</u> if they coincide or have a boundary in common. A field is said to be <u>contested</u> if it is occupied by units belonging to both players or if it is an airport and the adjacent city is contested. The field immediately behind (away from the center of the board) each city is an <u>airport</u>. A city is <u>controlled</u> by a player until the opponent has uncontested fighting units in the city. An airport is controlled by the player who controls the adjacent city. In case no fighting units are in a city it is controlled by the player last in control of that city. A city is <u>possessed</u> by whichever player last had uncontested occupation of it. Any airport belongs to that player who possesses the adjacent city. If an airport is uncontested one unit may be transported (flown) to any field on the board with this exception; it can not be placed on any city, airport or fields adjacent to a city or airport controlled by the opponent. Two units may be flown from an uncontested airport to another airport controlled by the same player.

2.2. <u>Information</u>. After both players have moved the barrier is removed and each player justifies his position by playing his units on the opponent's board. If a player cannot justify a move the misplaced units may be relocated in any legal position designated by the opponent.

2.3. <u>Aim Fire</u>. All units on a contested field must direct fire to

that same field (except in the case of an airport which is contested only because the adjacent city is contested). Any unit on an uncontested field may direct fire on any adjacent field but no further. It takes two directed units of fire power to kill one enemy unit. A unit can fire only once in each cycle.

2.4. **Remove Dead.** When the players have justified their moves and the position on the two boards agree, the killing begins. Each player kills on his own board while the other player checks to see that his kills are possible. If either player requests secrecy the aiming fire must be done in writing. The dead are removed from both boards.

2.5. **Production.** After the dead units are removed the final stage of the cycle is the addition of a new unit at certain designated cities provided three fighting units are not on the city. The new unit has the color of the player in possession of the city. There is no production at a contested city. Whenever a city possessed by one player is possessed by the other player that city is said to be captured. Generally in unsymmetric games (see section 3.2), a player who captures a productive city receives production at that city in each cycle after he first possesses the city uncontested for five complete cycles. The same rule applies to recaptured cities. In symmetric games captured cities do not receive production, but recaptured cities receive production in the next cycle after recapture (provided the city possessed belongs to the original owner).

After production the barrier is put in place and the second cycle starts. The play continues until one player concedes defeat or until all his units are removed by kills.

3. <u>Games</u>. Two different size boards are used. The smaller board has 25 rows of fields and 21 columns of fields. The larger board has 49 rows and 21 columns. Two fundamentally different type games are played on each board. One type is played with symmetric situations, i.e. each player has the same number of cities, the same number of units, and the same number of productive cities. The other type is played with unsymmetric situations. For example, one player may get more cities and more men, but fewer production. Games played on the large board will be explained first.

3.1. <u>Symmetric games</u>.

3.1.1. <u>Game with production in six cities</u>. The players place six opponent cities. There must be one city separated from each boundary by either two or three fields. Each player gets 60 fighting units to start the game. Each city not contested and not captured at the end of each cycle gains one fighting unit as reinforcement.

3.1.2. <u>Game with production in three cities</u>. This game is the same as 3.1.1 except that exactly three cities must be placed so as to be separated three or four fields from the back boundary. The boundary nearest a player is called his <u>back boundary</u>. These are the only cities receiving production.

3.2. <u>Unsymmetric games</u>. In the symmetric games each player may start an attack on the first move. However, in some of the unsymmetric games the rules require that one player shall wait a few cycles before he starts his attack. These games are designed so that the player who has the initial attack must win within a few cycles, say 10 to 15, if he can expect to win at all.

3.2.1. Game with production in three and four cities. One player, say A, has 6 cities; the other player, say B, has 7 cities. All the cities are placed by the opponents. Player A receives production in 4 cities which must be placed so that they are separated from his back boundary by three or four fields. Player B receives production in 3 cities which must be placed so that they are separated from his back boundary by three fields. All cities have airports. Player A gets 60 units and player B 70 units to start the game. Production starts at the beginning of the first cycle. The production rule explained in section 2.5 applies to captured cities.

3.2.2. Game with one city in no-man's-land and unequal production. One city is placed in no-man's-land near the center of the board. This city is considered captured by one player, say C, and must have exactly 10 fighting units adjacent to it . The airport of this city is opposite C. The players place the opponents six cities with those nearest the players separated by three fields from the back boundary. The three cities closest to C are his only productive cities and the four cities closest to the other player, say D, are his only productive cities. These cities get production at the start of the first cycle. All cities have airports. D can not attack for 15 cycles. D cannot place units closer than 3 moves from the city in no-man's-land, but he may attack this city, provided he does not place any units in C's territory. In defending his cities D is allowed two double transportations to any airport provided one unit immediately moves onto the adjacent city. Also D is allowed one unit production even if he has three units on a productive city.

Games played on the small board. These games are similar to those played on the large board. The game corresponding to 3.1.1 is played with 3 cities and 30 disks. The game corresponding to 3.1.2 is played with 4 cities and 40 disks where the two back cities are the only ones gaining production.

For an unequal game the boards are rotated one right angle. The pattern of the boards is now different. The airports are beside the cities on the inside (toward the center of the board). Player A has territory in the middle nine horizontal rows of fields, and player B has his territory divided, with six horizontal rows of fields on each end of the board. The 4 horizontal rows one on each side of the two common boundary lines are no-man's-land. Each player gets 4 cities and 40 disks. Cities are placed by opponents behind the barrier. A has only three productive cities. All of B's cities are productive. The production rules of section 2.5 apply. At the beginning of the first cycle A announces which city is not productive. A is the initial attacker. B cannot attack A's cities for 5 complete cycles.

There are many more possible games on both boards. Most of the experimenting so far has been done with the games explained above.

4. Some observations. The strategy of play is different on these two boards. Games on the larger board offer more of a challenge because of a larger variety of possibilities. However, the smaller board games are useful. For example, the symmetric game with three cities is easier to learn and faster to play.

There is a possibility of a stalemate in the symmetric game. In the unsymmetric game one player is almost certain to win within a fixed number of

cycles, but the game is set up so that it is possible for either player to win. A stalemate is impossible in the unsymmetric game. This game offers a real challenge to the skillful and daring player.

Time limits have not been set on the length of moves in each cycle, but it seems desirable that they should be made within one minute when playing on the larger board. In this case, games should seldom last longer than two hours. Time limits for moves and length of games played on the small board should be half as long.

All these games have distinguishing features, but they also have much in common. Methods of attacking cities are very similar. Methods of defense are almost the same. Here are some general principles of strategy which seem good. Choose the battleground if possible. Avoid fighting against superior numbers except in defense of a city. Do not let isolated units keep a stationary position (especially when the fighting is light). When attacking, do so from as many directions as possible as long as the attacking force is not spread too thin. Do not attack unless you outnumber the enemy except when the attack is a diversionary action or intended as a surprise.

It is hard to get a strong surprise attack directed on a city in three city games. The way the units are placed initially may give one player a decided advantage for the attack. In this case the other player is almost forced to defend for a few cycles. But he can almost always defend this first city under attack successfully if he has his units about equally divided among and near (4 or 5 moves) his three cities. At this stage of the game the attacker had better try to get his men pulled around on one side of the city under attack so as to get an attack started on another near city in

case it seems desirable. The timing at this stage of the game makes a big difference. If the defender is thrown out of position, even though he saves the city under fire, he still may easily lose the nearby city. The attacker, because of the kill rules, can't build up much of a threat by flying in three units unless these units are strongly supported by ground units. In defending a city a player can fly four units in each cycle while the attacker can fly in only three units. Thus the defender can kill two units from flights while the attacker can kill only one. Because of kill rules, it is good to distribute fighting units so as to have an even number of units firing in any battle.

On the other hand, the element of surprise plays a more important role in the symmetric six city games. Six units can appear any place on the board (see flight rules). A player can generate a strong attack using just flight units. However, it is very unlikely that a player can capture a city using only flight units unless his ground units are at the same time pressing an attack elsewhere. Further, when playing on the large board, it is more difficult to determine just where the enemy will attack next. The flight units, if well placed, may easily cause the opponent to wonder which of three or four cities will shortly be under attack. Since this is the case quick and wise action is necessary on the part of the player whose territory is invaded. Generally he should try to kill as many units as possible, that is, he should attack the attacker. For the larger the force the enemy builds up the more danger there is to all cities. But if a player only defends in this way he will usually lose. The player under attack should start an attack in enemy territory if he possibly can. It appears that if a player can

press his attack each cycle so that the opponent must always defend the defender will lose the game.

In the unsymmetric games, the player with fewer units is usually forced to defend early in the game. If he can defend his cities successfully, after a few cycles the extra production he receives will allow him to build up his forces so that he is almost sure to win. But defending is a difficult job. The ability to meet the enemy so as to kill equally or better is necessary. Since the enemy has the initiative this is hard to do. For suppose the enemy puts a long arc of units around the fields adjacent to the city and airport, then the defender should concentrate his defense on one side. If the attacking force should move on the fields adjacent to the city and airport and fly units in to back the units up, then the defender should try to put an arc of fighting units in front of and at the ends of the advancing force. Since the defender doesn't know in advance where the attacker is going to strike it is difficult to know how to defend so as to kill off as many advanced units as possible and still kill equally or better.

It is evident that at least one of the games, 3.2.1 or 3.2.2, must be unfair. It appears that 3.2.1 is biased in favor of the player with 6 cities since he is allowed to start a diversionary attack early in the game. In fact, 3.2.2 was developed because it was thought that 3.2.1 is slightly unfair. But it is included here because games with slight handicaps are sometimes interesting and useful.

The location of cities can make a difference in all these games. For example, if D in game 3.2.2 has four cities along the back boundary the game seems to be biased in his favor. Because these cities are so near each other

units can quickly be moved from one city. On the other hand, if D has one productive city isolated from the others the bias seems to favor C.

When the enemy moves in a line toward a city with two or three units in each field the defender may place one unit on each of the fields where he expects the enemy to be and back them up with three units. If the enemy moves in the defender kills twice as many units as the attacker. On the other hand, if the enemy does not move in the defender loses about twice as many units as the attacker. This sacrifice play should be used with caution.

In defending a city it is highly desirable to keep as many of the fields adjacent to the city and airport free of enemy units as possible. So moves and kills should be made with this in mind. As long as the defender can have two fields (one being the airport) adjacent to the city which he is sure will not have enemy units on them in the next cycle he can be reasonably certain of defending this city assuming, of course, that he is not hopelessly outnumbered. In most of these games a player is hopelessly outnumbered in a battle if the opponent has twice as many units firing in that battle.

It is easy, while concentrating on an attack or defense in one part of the board, to slight play in another part. For example, in an effort to save one city, most of the units may be flown or moved from another city, thus leaving that city weak and vulnerable. Or while attacking most of the units may move out of one city to fortify the attack. In either case the player is encouraging a diversionary attack. So whenever a city is left weakened in this way the player should be mindful of the sacrifice and ask if he can afford to make such a sacrifice. Generally, his decision will depend on what he expects the enemy to do.

5. <u>Applications</u>. These games are designed with the view of use for screening and training purposes. The equipment and rules have been kept simple so that the games are inexpensive and easily learned. They have been tried out on grade school children; they were quite popular with them.

Some of the games are definitely unfair in the sense that one player has a definite advantage. In others the players operate under non-symmetric conditions. The purpose of this variability is to test adaptability. Some games require boldness and others caution. The terms used to describe the play are military and an effort has been made to make the situations as realistic as possible within the bounds set by the requirement of simplicity.

The work on these games is not finished. It would be helpful to have an opportunity to try them out with military personnel, but there is still room for quite a bit of work before "field" testing becomes essential.