FORTRAN IV PROGRAM FOR ANALYSIS OF SATURN V PROPELLANT SYSTEMS

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PROPELLANT LINE DYNAMICS

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I. INTRODUCTION

The digital computer program to be used to analyze the propellant system dynamics of Saturn V is presented in this report. The basic theory underlying the method of analysis is developed and the program details are described.

The objective of the program is to provide an analysis of the transient flow condition that may develop in the propellant feed system of a liquid fueled rocket when subjected to longitudinal structural vibrations. The details described herein refer to Stage I, Saturn V, but the program is equally applicable to the other stages of the missile, or to other liquid-propelled missiles of similar configuration.

In this report the missile propellant system is first described, the basic theory behind the method of analysis is set forth, then the program is described and details are presented. The report is concluded with the results from an example problem.

II. DESCRIPTION OF MISSILE SYSTEM

A schematic view of the propellant feed system to one engine of Stage I, Saturn V, is shown in Figure 1. Only the component parts considered to be an important influence on the fluid system response are shown. The turbine gas generator and turbine are assumed to have no affect on an oscillation in the feed system. Stage I has five engines, each with separate supply systems leading from the lox and fuel tanks. These are assumed to act independently in parallel without any fluid flow or pressure interaction at the feed tanks. Any differences that exist between the outboard and inboard systems can be treated by using the data appropriate to the system being considered.

Figure 1 shows the component parts that are believed to be important in the dynamic response study, namely, the feed tank, the suction lines, the PVC, the pump, the parallel discharge lines, and the engine which includes the injector plate and combustion chamber. The fuel and lox systems are separate units with the combustion chamber beyond the orifice injector plate being the only point in the fluid flow and pressure field that is common to both systems. With this configuration a meaningful analysis can be performed in either system alone, assuming the other steady, or in the two systems together wherein an interaction is permitted at the combustion chamber. If a single-fueled-engine missile were being analyzed only one system would be considered.

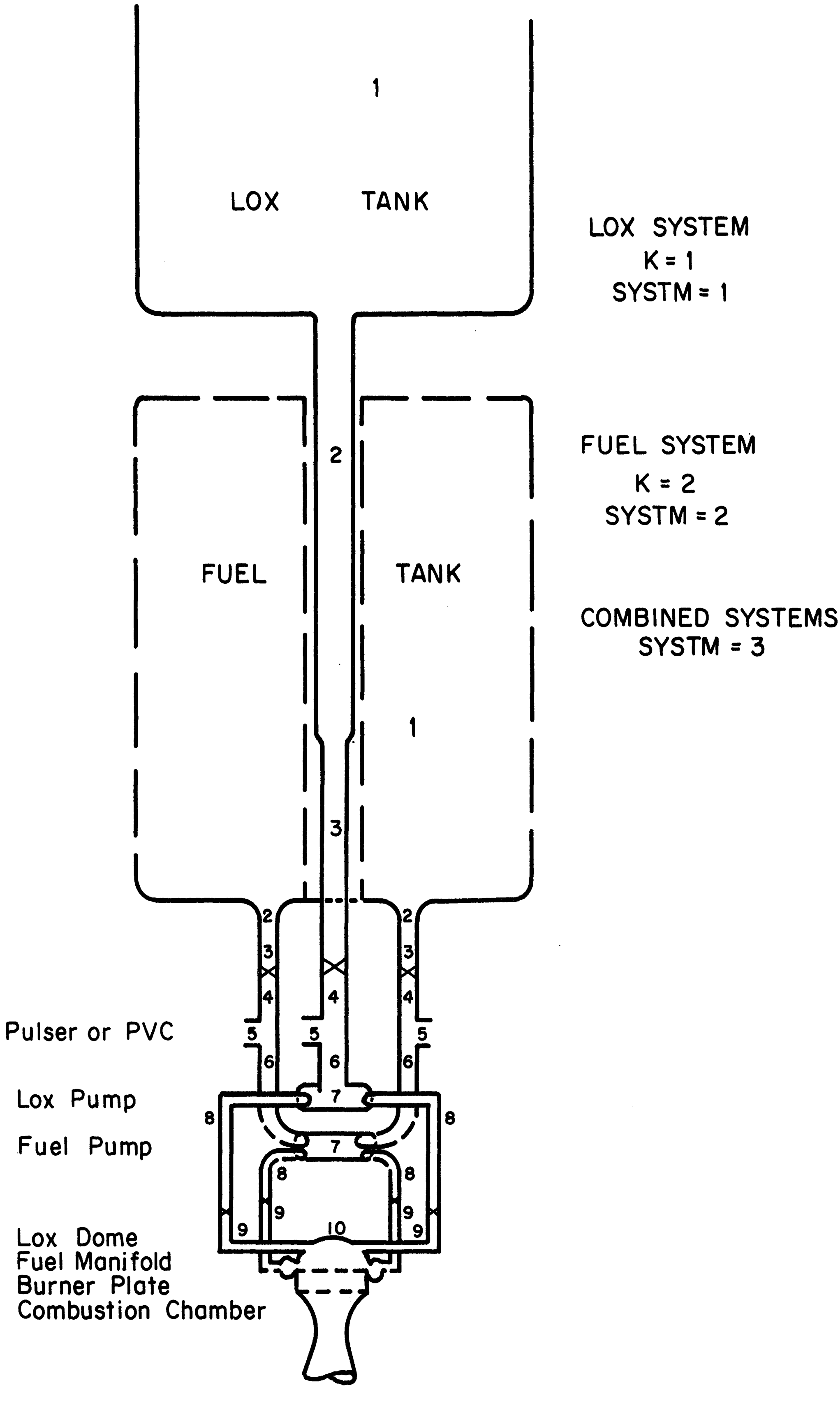

Figure 1. Schematic Diagram of Saturn V

III. BASIC THEORY

The equation of motion and continuity equation that describe one-dimensional unsteady flow of a compressible fluid in an elastic pipeline are first developed. The equations involve the actual distributed parameters that describe the system and include the nonlinear viscous terms. These partial differential equations are transformed into particular total differential equations by use of the method of characteristics. A finite difference method is then used to place the total differential equations in a form suitable for numerical solution on a digital computer.

The transient conditions in each pipeline in the system are treated in accordance with the above description. Boundary conditions are used to transfer the effect of one pipeline to another, and to introduce the effect of the pump, the PVC, the engine, etc. In a typical computer analysis of a specific problem, an initial steady-state operating condition is imposed upon the system; then a numerical solution is obtained at finite points throughout the entire system at finite time increments during the life of the transient.

The equation of motion is developed by examining a control volume that is moving with the vehicle, Figure 2.

$$p_x A \Delta x + \tau_o \pi D \Delta x - \rho g \Delta x A \sin \alpha + \rho A \Delta x \frac{dV}{dt} = 0$$

The density of the fluid is given by ρ and the pressure is given by p. The angle α is measured between a plane normal to the flight direction

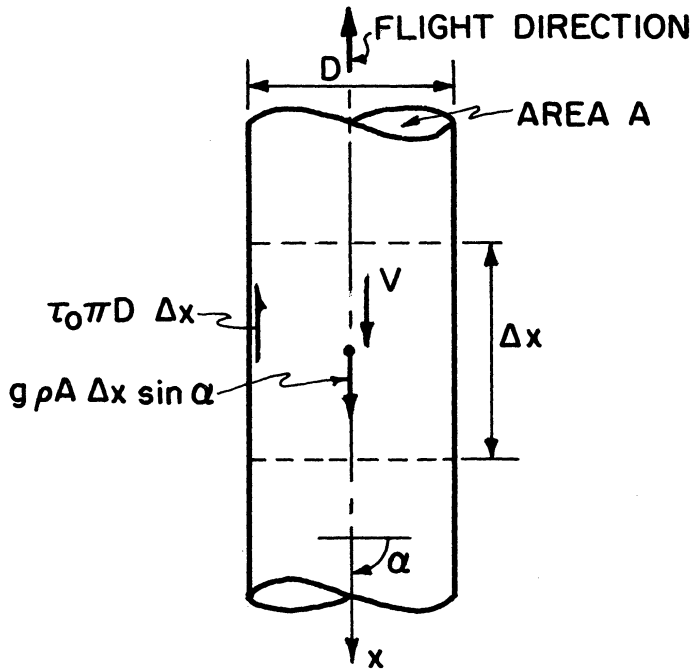

Figure 2. Control Volume

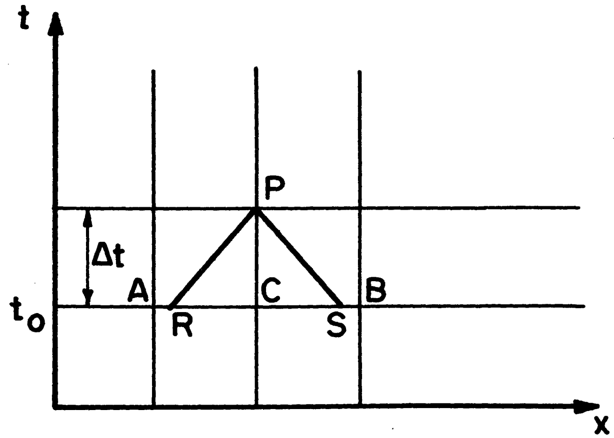

Figure 3. Characteristics on the XT Plane

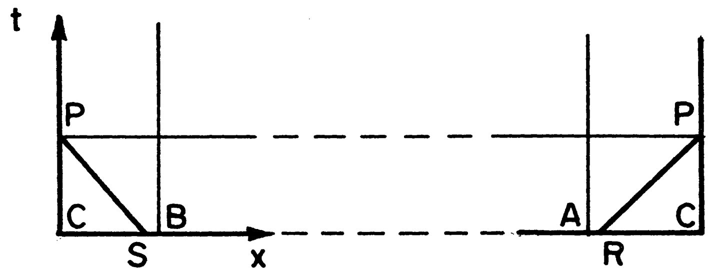

Figure 4. Characteristics at the Boundary Conditions

and the positive x axis, and g is the acceleration at the particular time in flight. Other variables are defined in Figure 2. The equation can be simplified to the following form:[1]

$$\frac{p_x}{\rho} + \frac{fV^2}{2D} + VV_x + V_t - g \sin \alpha = 0$$

The subscripts x and t indicate partial differentiation.

The condition of continuity applied to the same control volume at any instant yields:

$$(\rho AV)_x \, \Delta x + (\rho A \, \Delta x)_t = 0$$

which can be simplified to

$$V_x + \frac{1}{A}\frac{dA}{dt} + \frac{1}{\rho}\frac{d\rho}{dt} = 0$$

The introduction of the elasticity of the pipe walls and compressibility of the fluid allows the equation to be rewritten in the form:[1]

$$a^2 \, V_x + \frac{Vp_x}{\rho} + \frac{p_t}{\rho} = 0$$

The pressure pulse wave speed in the pipeline is denoted by a, and, for liquid flow in an elastic tube, is defined by

$$a = \sqrt{\frac{K/\rho}{1 + \frac{KD}{Ee}}}$$

The bulk modulus of elasticity of the liquid is given by K, the modulus of elasticity of the pipe wall by E, and the pipe wall thickness by e.

The equation of motion and continuity equation can be combined in a linear manner using an unknown multiplier, λ. Thus, by multiplying the continuity equation by λ, adding the simplified equation of motion, and rearranging, we have

$$\frac{\lambda}{\rho}\left[p_x\left(V+\frac{1}{\lambda}\right)+p_t\right]+\left[V_x\left(V+a^2\lambda\right)+V_t\right]+\frac{fV^2}{2D}-g\sin\alpha=0$$

If $\frac{dx}{dt}$ is restricted to the values $V+\frac{1}{\lambda}$ and $V+a^2\lambda$, then this equation can be written as a total differential equation.

$$\frac{\lambda}{\rho}\frac{dp}{dt}+\frac{dV}{dt}+\frac{fV}{2D}-g\sin\alpha=0$$

The restriction imposed upon $\frac{dx}{dt}$ requires that $\lambda=\pm\frac{1}{a}$. When these values of λ are substituted, the following set of equations results

$$\left.\begin{aligned}&\frac{1}{a\rho}\frac{dp}{dt}+\frac{dV}{dt}+\frac{fV^2}{2D}-g\sin\alpha=0 &(1)\\ &\frac{dx}{dt}=V+a &(2)\end{aligned}\right\}\ C+$$

$$\left.\begin{aligned}&-\frac{1}{a\rho}\frac{dp}{dt}+\frac{dV}{dt}+\frac{fV^2}{2D}-g\sin\alpha=0 &(3)\\ &\frac{dx}{dt}=V-a &(4)\end{aligned}\right\}\ C-$$

These are the characteristic equations. The first equation is valid only if the second is satisfied, and the third equation is valid only if the last is satisfied. By use of the first order finite difference approximation to these equations, they can be placed in a form which is suitable for numerical solution on the digital computer.

The independent variable (xt) plane is shown in Figure 3. If conditions (p,V) are considered known at points R and S, then they can be determined at point P at time Δt later. In Eqs. (2) and (4), the velocity V is generally much less than the wave speed a, and can be neglected. Since a is a constant in any given pipeline, Eqs. (2) and (4) are straight lines on the xt-plane, RP and SP, respectively.

It is also convenient to write the equations in terms of mass flow rate ($Q = \rho AV$, slugs/sec.). In finite difference form, Eqs. (1) to (4) become

$$Q_P - Q_R + \frac{A}{a}(p_P - p_R) + \frac{f\,\Delta t}{2D\rho A} Q_R^{\,2} - g\rho A\,\Delta t \sin\alpha = 0 \qquad (5)$$

$$x_P - x_R = a\,(t_P - t_R) \qquad (6)$$

$$Q_P - Q_S - \frac{A}{a}(p_P - p_S) + \frac{f\,\Delta t}{2D\rho A} Q_S^{\,2} - g\rho A\,\Delta t \sin\alpha = 0 \qquad (7)$$

$$x_P - x_S = -a\,(t_P - t_S) \qquad (8)$$

These four equations enable a solution to be obtained for the four unknowns, Q_P, p_P, x_P, and t_P, if conditions are known at R and S.

An orderly computer solution is obtained if each pipe length is divided into N equal reaches, $\Delta x = L/N$. This specifies the time increment that can be used since for stability and convergence of the solution it is necessary that $\Delta t\, a \le \Delta x$ in each pipe. A specified time interval grid is therefore established in each pipeline, Figure 3. If conditions are known along the system at the initial time, t_o, they can be determined at time Δt later by a systematic application of Eqs. (5) and (7) at each point P. Simultaneous solution of Eqs. (5) and (7) yields

$$Q_P = \frac{1}{2}\left[Q_R + Q_S + \frac{A}{a}(p_R - p_S) - \frac{f\,\Delta t}{2D\rho A}\left(Q_R^{\,2} + Q_S^{\,2}\right)\right] + g\rho A\,\Delta t \sin\alpha \qquad (9)$$

$$p_P = \frac{1}{2}\left[p_R + p_S + \frac{a}{A}(Q_R - Q_S) - \frac{af\,\Delta t}{A^2 2D\rho}(Q_R^2 - Q_S^2)\right] \qquad (10)$$

Conditions can be found at points R and S by use of a linear interpolation(1) between known conditions at points on the specified time interval grid A, C, and B.

Equations (9) and (10) permit the calculation of all interior points of the grid for all time, however, the influence of the pipe end conditions must be introduced to yield the complete solution.

At the inflow end (left side), Fig. 4, of the pipeline, Eq. (7) can be written in the form

$$Q_P = Q_S - \frac{A}{a}p_S - \frac{f\,\Delta t}{2DA\rho}Q_S^2 + g\rho A\,\Delta t\,\sin\alpha + \frac{A}{a}p_P \qquad (11)$$

or

$$Q_P = C_1 + C_2\,p_P \qquad (12)$$

where C_1 is a variable that can be evaluated in terms of known quantities at each new time step, and $C_2 = A/a$. Similarly, at the outflow end (right end) of the pipeline, Eq. (5) can be written

$$Q_P = Q_R + \frac{Ap_R}{a} - \frac{f\,\Delta t}{2DA\rho}Q_R^2 + g\rho A\,\Delta t\,\sin\alpha - \frac{A}{a}p_P \qquad (13)$$

or

$$Q_P = C_3 - C_2\,p_P \qquad (14)$$

where C_3 can be evaluated in terms of known values. When an external condition that relates Q_P and p_P, or defines one of the variables, is applied to a pipe end a solution can be obtained in combination with either Eq. (12) or (14) for the inflow or outflow end, respectively.

By combining the entire system in an orderly manner with the appropriate boundary conditions, a complete solution is obtained for points throughout the entire system for all time during the life of the transient condition.

IV. PROGRAM ORGANIZATION AND DETAILS

Figure 1 displays a schematic diagram of the entire propellant system from feed tanks to combustion chamber. The component parts of the systems have been numbered to enable simple identification in the program.

The results from any given execution of the program show the complete system response to a particular excitation. That is, an initial steady state operating condition is defined for the system; then, the transient condition produced by the exciter is followed for as long as may be desired.

A summarized discussion of the options in the program are presented below, followed by an itemized account of the major steps in the program. The details involved in the establishment of an initial steady operating condition are then presented. In general, the order of discussion of the program details follows the order of computation in the actual program.

A. Program Options

The following major options, pertaining mainly to the system to be analyzed and the type of exciter to be used, must be decided upon by the program user and specified in the data.

(a) System to be analyzed.

Both lox and fuel systems can be analyzed in the same run, in which case the variable SYSTM = 3. Excitation can be in either or both of the systems. If the lox system only is to be analyzed, the variable SYSTM = 1; and if the fuel system only is to be analyzed, the variable SYSTM = 2.

(b) Combustion chamber.

If one system is excited, it may be desirable to isolate that system from the other in order to study its response alone. This is accomplished by setting the variable SINGLE = 1, which isolates the systems at the combustion chamber and essentially assumes steady-state flow from the second system. The more common situation, wherein one system may influence the other through the combustion chamber, is obtained by the variable SINGLE = 0. By using this option, one system can be excited, and the extent to which it influences the other system can be studied. Or, both systems can be excited and the resulting oscillations in the combustion chamber can be observed.

(c) Pressure volume compensator.

The PVC is included in the fuel and lox systems if the variable PVC = 1. If PVC = 0, the PVC is not in the system. Thus the effectiveness of the PVC for any given excitation can be evaluated.

(d) Points of excitation.

There are three possible excitation points in each system and one disturbance mechanism which is common to both fuel and lox systems. The desired amplitude and phase angle can be given for each of these elements at the frequency OMEGA. The table summarizes the variables involved. The variables pertaining to a particular location must be set equal to one to use the exciter and an amplitude must be given. If the exciter is wanted in only one of the systems, the variable controlling the amplitude in the second system must be set to zero. In order to remove any exciter from the system the variable controlling that element is set to zero.

Location	Variable	Lox Amplitude	Lox Phase	Fuel Amplitude	Fuel Phase
Junction of supply tank to suction pipeline	TANK = 1	AM1(1)	ALPHA(1)	AM1(2)	ALPHA(2)
Pulser in suction line	PULSER = 1	AM5(1)	ALPHA5	AM5(2)	ALPHA5
Pump	PUMP = 1	AM7(1)	ALPHA7	AM7(2)	ALPHA7
Burner Plate Orifice	ORIFIC = 1	AM9	ALPHA9	AM9	ALPHA9

B. Program Organization

The computing procedure followed in solving a particular problem involving a particular set of data is as follows:

a) Read in the data pertaining to the system.

b) Calculate steady-state conditions throughout the system and initialize the various variables and constants required for the transient analysis.

c) Set up the various column headings for the print out of the results of the calculations and print out the initial steady-state conditions.

d) Increment the time variable, T, by Δt and the integer counter, U, by one. If the lox system is to be analyzed, proceed. If the fuel system only is to be analyzed, the variable K is set equal to 2.

e) Calculate the interior points of all the pipelines in the system being analyzed.

f) Calculate the boundary conditions of each pipe in the system. This includes all the series pipeline connections, the pump, the pulser or PVC, and the feed tank.

g) If the lox system was analyzed and the fuel system is going to be analyzed, the program returns to step (e) and repeats the steps for the fuel system. If the fuel system is not to be analyzed the program continues with the next step.

h) Calculate the engine boundary condition.

i) Substitute all newly computed values of pressure and discharge to a second storage location so they can be reused during the next time increment.

j) Compute the pump characteristics and constants to be used for the computations of the next time increment.

k) If a print out is desired, transfer to the print statement, otherwise, return to step (d) and repeat the calculations at the new time. A check is made to see if the time exceeds the desired duration of the transient calculations. If so, the program is stopped.

C. Initial Steady State Conditions

Data are given for the number of reaches in one particular pipeline, as well as its length and pressure pulse wave speed. These data are used to calculate the time increment, DT, to be used for the transient calculation. The number of reaches in each of the remaining pipes in the system are then calculated, based on this DT. A number of other constants that are needed in later calculations are then established and stored for future use.

At any location in the system where identical parallel pipes exist, they are treated as a single pipeline having an area equal to the combined area of the pipes but the actual pipeline diameter is maintained. An example of such a situation would be the two pipes labeled number eight in the pump discharge section of the lox system, Fig. 1. With this representation the velocities are correctly represented in the pipe and the frictional losses are also correctly described. This substitution provides a significant simplification in the program.

The given data, in addition to the physical description of the system, that define the initial operating conditions are: the flow in each system, the pressure in each feed tank, and the pressure in the combustion chamber. The factor DPF, which corresponds to the change in hydraulic gradeline in each reach of each pipeline, is computed; then the pressure at the pump outlet is computed by beginning at the combustion chamber and working through the orifice and discharge piping (see Figure 1). All steady state operating conditions (Q,p) are now stored for each point at which transient computations will be performed in the system. The pressures in the suction side are computed by beginning at the feed tank pressure and proceeding towards the pump. When the pump suction pressure is calculated, the pressure rise through the pump (PPUMP) is computed as the difference between the suction and discharge pressures. Since it is unlikely that this value of pressure head rise and given initial discharge will agree exactly with the given pump characteristic curve data, the amount (DPO) which the head rise vs. suction pressure should be shifted to produce the desired agreement is computed. That is, the pump speed is assumed to be changed slightly to obtain the desired agreement. Also the value of pressure rise, PO, corresponding to the initial discharge value is computed from the given head rise vs. discharge data. This is discussed in more detail under the pump boundary condition.

Constants pertaining to the engine operation are also computed. These are discussed in detail under the engine boundary condition.

Information dealing with the print out of results is next established in the program. This includes column headings and the print out of some of the newly stored initial conditions.

The balance of the program involves the calculations during the life of the transient condition. It involves repetitive calculations which are performed every Δt. This is accomplished by performing all calculations to the end of the program, incrementing time and repeating until sufficient results are obtained.

The first repetitive calculations involve the interior points along each pipeline. Pressures and flow rates are computed at each time increment by use of the interpolation equations and Eqs. (9) and (10) as described in Section III. At the same time increment as calculations are made at the interior points, computations are also performed at the various boundary conditions. These are discussed in the next section.

V. BOUNDARY CONDITIONS

The various boundary conditions are now treated individually in some detail.

A. Series Pipe Connection

At a junction of two pipes, as pipes 2 and 3, Figure 1, $Q_{P_2} = Q_{P_3}$ and $H_{P_2} = H_{P_3}$ at any instant. In order to handle all similar series connections the numbers of all adjoining pipes are stored in pairs in the linear array IND2. A simple iteration statement then takes each series connection in sequence and obtains the solution at the new time.

Figure 5 shows a typical series connection. Equation (14) is written for pipe X and Eq. (12) is written for pipe J.

$$Q_{PX} = C_3 - C_X \, p_P$$

$$Q_{PJ} = C_1 + C_J \, p_P$$

Since $Q_{PX} = Q_{PJ}$, the solution for p_P is

$$p_P = \frac{C_3 - C_1}{C_J + C_X}$$

With p_P known, Q_P can be found from either of the previous two equations.

The pump inlet and discharge connections are treated as special series connections. These are discussed under the pump boundary condition. Also if the excitation mechanism in the system is

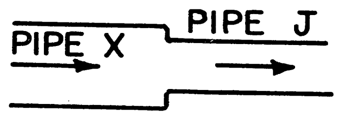

Figure 5. Series Connection

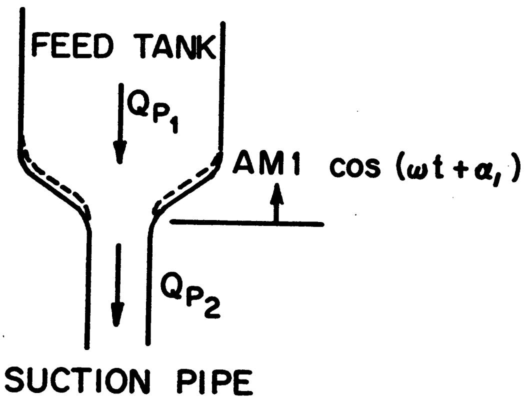

Figure 6. Connection at Feed Tank

either a structural displacement of the feed tank or pump, an alteration is made to those particular series connections.

B. System Excitation at the Feed Tanks

A structural motion of the tank with respect to the feed line is assumed if the variable TANK is set equal to one. The amplitude of the sinusoidal motion is AM1, the frequency is OMEGA, and the phase angle is ALPHA1, Fig. 6. The continuity condition at the junction becomes

$$Q_{P_1} + QP1 = Q_{P_2}$$

where $QP1 = \rho\ (PROP * A_1 - A_2)\ (AM1)\ \omega \sin (\omega t + \alpha_1)$. The variable PROP can be used to describe the amount of the tank area that actually contributes additional flow. It would depend upon the shape of the pipe entrance and upon the number of suction lines from each tank. When this continuity condition is used with the earlier boundary condition equations, the pressure at the junction can be determined.

$$p_P = \frac{C_3 - C_1 + QP1}{C_J + C_X}$$

C. Turbopump Representation

In the analysis the pump is considered to be a pipe with a length equal to the mean particle flow path through the pump. The head rise through the pump is considered a linear rise along its length and frictional losses are charged to the pump head rise. A pressure pulse wave speed is assumed through the pump which gives rise to a small time delay. Thus, the pump is treated as any other pipe in the system with interior point computations performed as described above, and it is connected to the suction and discharge lines using the same procedures as described above for series connections.

The equation of motion takes a slightly different form when it includes the head rise produced by the pump. If the total pressure head developed by the pump is PPUMP, then an additional force $\frac{PPUMP}{L} A \Delta x$ can be considered to act in the direction of flow on the control volume shown in Figure 2. Then the equation of motion becomes

$$p_x A \Delta x - \frac{PPUMP}{L} A \Delta x - \rho g A \Delta x \sin \alpha + \rho A \Delta x \frac{dV}{dt} = 0$$

which reduces to the following form:

$$\frac{p_x}{\rho} - \frac{PPUMP}{\rho L} - g \sin \alpha + V V_x + V_t = 0$$

When this equation is combined with the original continuity equation in the same manner as in section III, and placed in finite difference form, the following equations are obtained.

$$Q_P - Q_R + \frac{A}{a} (p_P - p_R) - A \Delta t \left(g\rho \sin \alpha + \frac{PPUMP}{L}\right) = 0 \qquad (15)$$

$$x_P - x_R = a (t_P - t_R) \qquad (16)$$

$$Q_P - Q_S - \frac{A}{a} (p_P - p_S) - A \Delta t \left(g\rho \sin \alpha + \frac{PPUMP}{L}\right) = 0 \qquad (17)$$

$$x_P - x_S = -a (t_P - t_S) \qquad (18)$$

These equations are used for the pump in place of Eqs. (5) to (8).

In order to use this set of equations it is necessary that PPUMP be evaluated. The procedure used for this evaluation is next described. It is assumed that the steady state characteristic curves for the pump are valid during the transient condition. It is further assumed that the shape of the curves remains the same although the pump speed may vary slightly during the unsteady flow.

Tabular data of pressure rise vs. suction pressure (PNP vs. P_S) are given, starting at $P_S = P_i$, at equal increments of P_S, Figure 7. Since initial steady state conditions in the system are not likely to match the given pump curve exactly, the pump speed is assumed to be altered slightly so that the curve is shifted DP_o. The initial suction pressure and pressure rise conditions in the system then fall on the newly positioned curve. This shifted curve is used throughout the analysis. At any suction pressure, the constants for a parabola (B_1, B_2, B_3), passing through three adjacent points of the given curve, can be determined. The equation of the adjusted parabola is

$$P_{RISE} = B_1 + DP_o + B_2\ (P_S - P_i) + B_3\ (P_S - P_i)^2$$

Tabular data are also given for the second pump characteristic curve, pressure-head rise vs. discharge at a constant speed, Figure 8 (PQ vs. Q, data given at equal increments of discharge, DQ). Again for any discharge Q, the constants can be determined for a parabola (Z_1, Z_2, Z_3) that passes through three adjacent given data points. Thus for the initial discharge Q_o, the parabolic equation is

$$P_o = Z_1 + Z_2\ (Q_o - Q_1) + Z_3\ (Q_o - Q_1)^2$$

A slight change in the assumed pump speed is likely to be necessary to shift the curve so the actual initial operating conditions fall on the

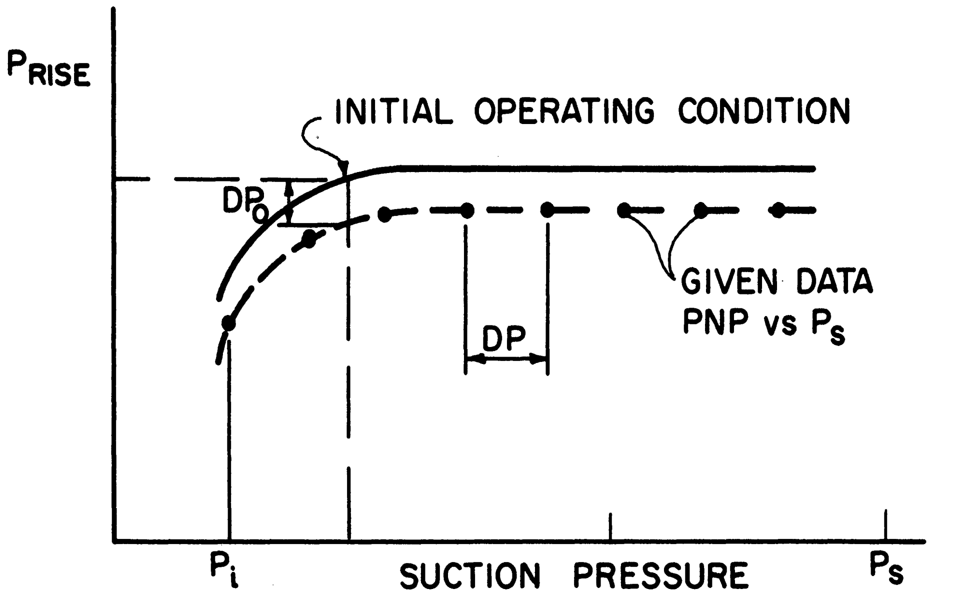

Figure 7. Pump Characteristic Curve. Pressure Rise vs Suction Pressure

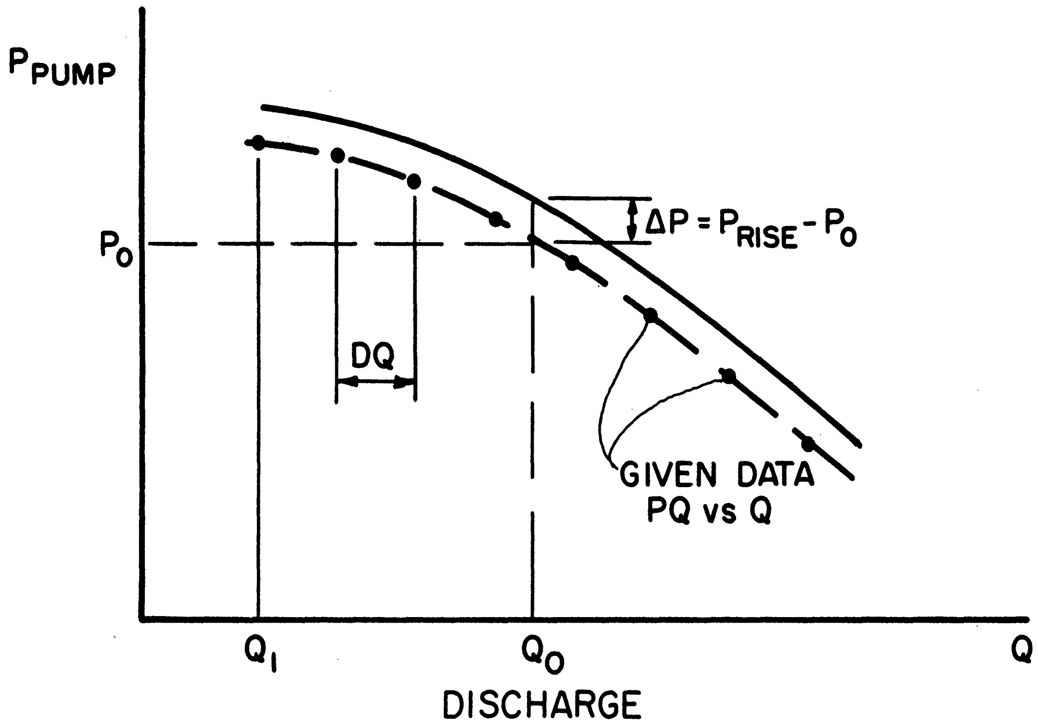

Figure 8. Pump Characteristic Curve. Pressure Rise vs Discharge

pump curve. The amount the curve is displaced is $\Delta P = P_{RISE} - P_o$.

During the transient calculations, for a given suction pressure, B_1, B_2, and B_3 are found and P_{RISE} can be evaluated. The difference $P_{RISE} - P_o$ is the amount that the pressure rise-discharge curve must be shifted. With values of Z_1, Z_2, and Z_3 determined with the aid of the most current value of discharge, the relationship between the current head rise and discharge is given.

$$\text{PPUMP} = Z_1 + Z_2 (Q_P - Q_1) + Z_3 (Q_P - Q_1)^2 + P_{RISE} - P_o$$

It can be noted that this analysis assumes that the pressure rise vs. suction pressure curve is a constant discharge curve as well as a constant speed curve. For convenience and simplicity in the analysis, values of suction pressure and discharge from the previously computed time increment are used to evaluate PPUMP.

D. <u>System Excitation at the Pump</u>

A structural motion of the pump with respect to the suction line is assumed if the variable PUMP is set equal to one. The assumed sinusoidal motion has an amplitude, frequency and phase angle of AM7, OMEGA, and ALPHA7, respectively.

The continuity condition at the suction line junction with the pump is

$$Q_{P6} + QP6 = Q_{P7}$$

where

$$QP6 = \rho A_6 \, \omega(AM7) \sin(\omega t + \alpha_7)$$

When this continuity condition is used with the boundary equations presented under A, the pressure at the pump inlet can be determined.

$$p_P = \frac{C_3 - C_1 + QP6}{C_J + C_X}$$

E. Pulser

If the variable PULSER is set equal to one in the input data, a sinusoidal pulser is assumed at location 5 of Fig. 1. A piston is assumed in a branch at the junction of pipes 4 and 6 as shown in Fig. 9. The motion of the piston is controlled in amplitude, frequency, and phase by the variable AM5, OMEGA, and ALPHA5, respectively.

The continuity condition at the junction is given by

$$Q_{P4} + Q_{P5} = Q_{P6}$$

where Q_{P4} is defined by Eq. (14)

$$Q_{P4} = C_3 - C_4 \; p_P$$

Q_{P6} is defined by Eq. (12)

$$Q_{P6} = C_1 + C_6 \; p_P$$

Q_{P5} is defined by

$$Q_{P5} = \rho \mathbf{A}_5 (AM5) \; \omega \sin (\omega t + \alpha_5)$$

The resulting pressure at the junction is given by the relation

$$p_P = \frac{C_3 - C_1 + Q_{P5}}{C_4 + C_6}$$

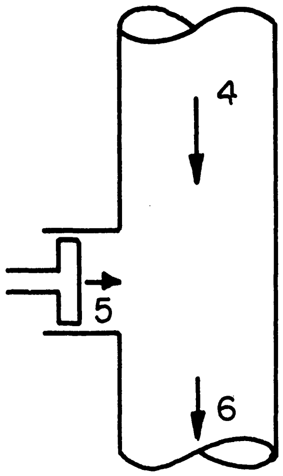

Figure 9. Location of Pulser

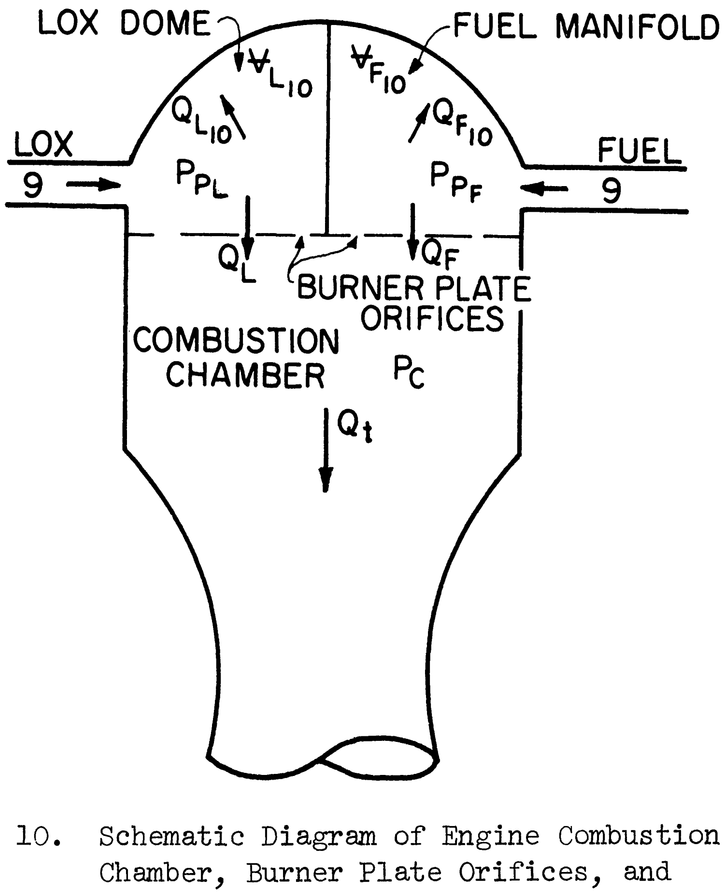

Figure 10. Schematic Diagram of Engine Combustion Chamber, Burner Plate Orifices, and Pump Discharge Lines

F. Pressure Volume Compensator

If the variable PVC is set equal to one in the input data, the volumetric response of the PVC is assumed at location 5 of Fig. 1. This volumetric response can be visualized as a piston in a branch, as shown in Fig. 9, that moves in response to the physical displacement of the pipe. The variables that control the PVC motion are the same as those controlling the pump motion, namely AM7, OMEGA, and ALPHA7. The equations at the junction are identical to those for the pulser except Q_{P5} is defined

$$Q_{P5} = - \rho A_6 (AM7) \ \omega \sin (\omega t + \alpha_7)$$

It is not possible to use the pulser and PVC in the program at one time.

G. Feed Tank Surface

The pressure in the fuel and lox supply tanks is given by PTANK and is assumed constant during the transient analysis. With the pressure defined at a boundary the discharge can be computed directly from Eq. (12).

H. Engine for Combined System Analysis

The action of the engine beyond the burner plate orifices has been greatly simplified. It is assumed that the combined mass flow rate of fuel and lox, Q_t, to the engine nozzle can be related to the combustion chamber pressure, P_c, by the relation

$$Q_t = C_{30} P_c \tag{19}$$

The variable C_{30} is dependent upon the nozzle throat area and the characteristic velocity $C*$, which in turn is a function of the mixture ratio M_r (flow rate of lox over flow rate of fuel).[2] Since small variations in the mixture ratio may produce significant changes in the characteristic velocity, the variable C_{30} is evaluated during the transient as the mixture ratio changes.

Tabular data, characteristic velocity vs. mixture ratio, are given for the particular fuel and oxidizer used in the system. Values of C*, CSTAR, are given at equal increments, DMR, of mixture ratio beginning at MR1. For the initial combustion chamber pressure, total mass flow rate, and mixture ratio, the constant ATH is evaluated.

$$ATH = C_{30}\ C* = C*\ Q_t/P_c$$

During transient conditions C_{30} is evaluated from the constant ATH and the C* that corresponds to the current mixture ratio.

Figure 10 displays a schematic view of important component parts of the engine that are needed to describe its dynamic response in connection with the fluid pipeline. The notation used to describe the variables in the engine is shown. The fluid in each of the feed lines passes into a volume, then through the burner plate orifices to the combustion chamber. The volume $\forall$ and effective compliance K' of the space at the orifice entrance are specified by the data, VOL and KP, respectively. The effective compliance, which represents the combined effect of the fluid compressibility and elasticity of the container, can be expressed $K' = \Delta p/(\Delta\forall/\forall)$. For a small time interval the mass flow rate going into storage in the volume is $Q_{10} = \rho \frac{\Delta\forall}{\Delta t}$. By combining these two expressions

$$Q_{10} = C_{10}\ \Delta p$$

where $C_{10} = \rho\, \forall/(K'\Delta t)$. As a first order approximation, $\Delta p = p_P - p$, where p_P is the unknown value of the pressure at the end of the time increment and p is the known value of the pressure at the beginning of the time increment. This expression is used in Eqs. (24) and (28) below.

In addition to Eq. (19) and the continuity condition in the combustion chamber

$$Q_L + Q_F = Q_t \tag{20}$$

an equivalent set of equations must be written for the fuel and lox sides of the engine. For the fuel side these equations are as follows:

The orifice relationship:

$$Q_F = (A_o C_D)_F \sqrt{2\rho_F (p_{PF} - P_c)} \tag{21}$$

Continuity at the volume:

$$Q_F + Q_{F10} = Q_{F9} \tag{22}$$

Characteristic equation from pipe 9:

$$Q_{F9} = C_{3F} - C_{9F}\, p_{PF} \tag{23}$$

Volume-compliance relationship:

$$Q_{F10} = C_{10F}\, p_{PF} - C_{10F}\, p_F \tag{24}$$

The corresponding equations for the lox system are

$$Q_L = (A_o C_D)_L \sqrt{2\rho_L (p_{PL} - P_c)} \tag{25}$$

$$Q_L + Q_{L10} = Q_{L9} \tag{26}$$

$$Q_{L9} = C_{3L} - C_{9L}\, p_{PL} \tag{27}$$

$$Q_{L10} = C_{10L}\, p_{PL} - C_{10L}\, p_L \tag{28}$$

At each new time increment Eqs. (19) to (28) must be solved for the ten unknowns, Q_t, P_c, Q_F, Q_{F10}, Q_{F9}, Q_L, Q_{L10}, Q_{L9}, p_{PF}, p_{PL}. Inasmuch as two of the equations involve quadratic terms, a direct solution is not possible.

The equations are first combined, then an iteration procedure is used to obtain a solution. Equations (19) and (20) are combined to eliminate Q_t

$$C_{30}\, P_c = Q_F + Q_L \tag{29}$$

Equations (26) to (28) are combined to eliminate Q_{L9} and Q_{L10}

$$Q_L = L_3 - L_4\, p_{PL} \tag{30}$$

where $L_3 = C_{3L} + C_{10L}\, p_L$ and $L_4 = C_{9L} + C_{10L}$. Equations (22) to (24) are combined to eliminate Q_{F9} and Q_{F10}

$$Q_F = F_3 - F_4\, p_{PF} \tag{31}$$

where $F_3 = C_{3F} + C_{10F}\, p_F$ and $F_4 = C_{9F} + C_{10F}$. Equations (21), (25), (29), (30) and (31) now involve five unknowns P_c, Q_F, Q_L, p_{PF}, and p_{PL}. The method of solution involves the assumption of a small perturbation on each of the three pressures, a linearization of the orifice equations, then iterating and correcting the pressures until the perturbation is arbitrarily small. The final values are true solutions of the original nonlinear equations.

Each pressure is expressed as the sum of a base pressure plus a variation from the base pressure.

$$P_c = \bar{P}_c + P_c'$$

$$p_{PF} = \bar{P}_{PF} + P_{PF}'$$

$$p_{PL} = \bar{P}_{PL} + P_{PL}'$$

The orifice equation for the lox system, Eq. (25) can be written

$$Q_L = OR_L \sqrt{p_{PL} - P_c} = OR_L \sqrt{\bar{P}_{PL} - \bar{P}_c + P_{PL}' - P_c'}$$

where $OR_L = (C_D A_o) \sqrt{2\rho_L}$. If the first two terms of a binominal expansion are used the orifice equation can be written

$$Q_L = L_1 \left(1 + \frac{P_{PL}' - P_c'}{2(\bar{P}_{PL} - \bar{P}_c)} \right)$$

or

$$Q_L = L_1 + L_2 P_{PL}' - L_2 P_c' \qquad (25a)$$

where $L_1 = OR_L \sqrt{\bar{P}_{PL} - \bar{P}_c}$ and $L_2 = L_1/(2(\bar{P}_{PL} - \bar{P}_c))$. Equations (25a) and (30) can be combined and P_{PL}' eliminated.

$$Q_L = \frac{L_1 L_4 + L_2(L_3 - L_4 \bar{P}_{PL})}{L_2 + L_4} - \frac{L_2 L_4}{L_2 + L_4} P_c' \qquad (32)$$

A similar manipulation of the equations for the fuel system produces a comparable equation.

$$Q_F = \frac{F_1F_4 + F_2(F_3 - F_4\,\bar{P}_{PF})}{F_2 + F_4} - \frac{F_2F_4}{F_2 + F_4}\,P_c' \qquad (33)$$

Equations (29), (32), and (33) are combined to give the solution for P_c'.

$$P_c' = L_6/L_7 \qquad (34)$$

where

$$L_6 = \frac{L_1L_4 + L_2(L_3 - L_4\,\bar{P}_{PL})}{L_2 + L_4} + \frac{F_1F_4 + F_2(F_3 - F_4\,\bar{P}_{PF})}{F_2 + F_4} - C_{30}\,\bar{P}_c$$

and

$$L_7 = C_{30} + \frac{L_2L_4}{L_2 + L_4}\;\frac{F_2F_4}{F_2 + F_4}\;.$$

Equations (25a) and (30) are combined to find P_{PL}'; and P_{PF}' is found similarly. New base values for each of the pressures are now found by combining the previous base value and the newly computed variation. The process is repeated until the variations are very small. In the program, four iterations are used to establish the new pressures. The four unknown discharges can be evaluated directly when the pressures are known.

I. Engine when Systems are Isolated at the Combustion Chamber

When the variable SINGLE is set equal to one in the input data, either the lox or fuel system can be analyzed alone without the interference of one upon the other. The fuel system analysis assumes that the lox flow rate to the combustion chamber is constant, QLB. When this assumption is made, Eqs. (19), (20), (21), and (31) can be combined and a direct solution can be obtained for Q_F.

$$Q_F = (-\,F_1 + \sqrt{F_1^2 + 4F_2})/2.$$

where

$$F_1 = OR_F^{\,2}\left(\frac{1}{F_4} + \frac{1}{C_{30}}\right)$$

and

$$F_2 = OR_F^{\,2}\left(\frac{F_3}{F_4} - \frac{Q_{LB}}{C_{30}}\right)$$

With Q_F known, a direct solution is available for the other variables, Q_t, P_c, p_{PF}, and Q_{F9}.

A similar procedure is used when it is desired to isolate the lox system. The fuel flow rate, QFB, is assumed constant for this condition.

J. System Excitation at the Burner Plate Orifice

A structural motion of the engine with respect to the pump discharge lines is assumed if the variable ORIFIC is set equal to one. The amplitude of the sinusoidal motion is AM9, the frequency is OMEGA, and the phase angle is ALPHA9. In the fuel system, the continuity condition between the end of the pump discharge line and the orifice plate becomes

$$Q_{F9} = Q_F + Q_{F10} - Q_{PF9} \tag{22a}$$

where

$$Q_{PF9} = \rho\ A_9\ \omega(AM9)\ \sin\ (\omega t + \alpha_9)\ .$$

Similarly, for the lox system, Eq. (26) becomes

$$Q_{L9} = Q_L + Q_{L10} - Q_{PL9} \tag{26a}$$

where

$$Q_{PL9} = \rho\ A_9\ \omega(AM9)\ \sin\ (\omega t + \alpha_9)\ .$$

VI. THE COMPUTER PROGRAM

A copy of a listing of the program is shown in Figure 11. It is written in FORTRAN IV-G Level and was executed on the IBM 360/67 system at the Computing Center, The University of Michigan. The storage required by the program is approximately 42,000 bytes. The execution time of the program is dependent upon the particular data being processed.

The data that is necessary for execution of the program is listed. In all double subscripted data, the first subscript refers to the system (LOX = 1, FUEL = 2) and, except for the pump characteristics, the second subscript refers to the pipe number.

D(1,1)...D(1,9), D(2,1)...D(2,9)

L(1,1)...L(1,9), L(2,1)...L(2,9)

A(1,1)...A(1,9), A(2,1)...A(2,9)

F(1,1)...F(1,9), F(2,1)...F(2,9)

NO(1,1)...NO(1,9), NO(2,1)...NO(2,9)

SLOPE(1,1)...SLOPE(1,9), SLOPE(2,1)...SLOPE(2,9)

RHO(1), RHO(2), FLOW(1), FLOW(2), CDORIF(1), CDORIF(2),

KP(1), KP(2), PTANK(1), PTANK(2), PCO, DQ, Q1(1), Q1(2),

VOL(1), VOL(2), PQ(1,1)...PQ(1,20), PQ(2,1)...PQ(2,20), DP,

PI(1), PI(2), PNP(1,1)...PNP(1,20), PNP(2,1)...PNP(2,20),

IND2(1)...IND2(12), J2, DMR, MR1, CSTAR(1)...CSTAR(20),

TANK, PULSER, PUMP, ORIFIC, OMEGA, AM1(1), AM1(2),

AM5(1), AM5(2), AM7(1), AM7(2), AM9, ALPHA1(1), ALPHA1(2),

ALPHA5, ALPHA7, ALPHA9, SYSTM, SINGLE, N(2,7), G, TSTOP,
UU, PVC, PROP

The program prints out pressure and discharge information at the points indicated by P and Q in Fig. 12. The pressures are given in psi although all pressure computations are performed in psf in the program. The discharge unit is mass flow rate, slugs/sec.

Definitions of all program variables are given in Appendix A.

```
FORTRAN IV G LEVEL 0, MOD 0              MAIN

              CSATURN 5 PROPELLANT SYSTEMS INCLUDING PUMPS AND ENGINE
              CSUBSCRIPT K OR KK=1 REFERS TO THE LOX SYSTEM
              CSUBSCRIPT K OR KK=2 REFERS TC THE FUEL SYSTEM
              CSYSTM=1 LOX SYSTEM ONLY IS ANALYZED
              CSYSTM=2 FUEL SYSTEM ONLY IS ANALYZED
              CSYSTM=3 BOTH SYSTEMS ARE ANALYZED
              CSINGLE=1 SYSTEMS ARE ISOLATED AT COMBUSTION CHAMBER
              CPVC=1 THE PVC IS INCLUDED IN THE ANALYSIS
              CALL PRESSURES ARE IN PSFA, ALL FLOWS ARE MASS FLOW RATES, SLUGS/SEC
              CPTANK(K) IS PRESSURE IN SUPPLY TANKS IN PSF
              CRHO (K) IS THE MASS DENSITY
              CTHE VARIABLES FOR THE AMPLITUDES AND PHASE ANGLES AT THE VARIOUS
              CEXCITATION POINTS ARE AS FOLLOWS (FREQUENCY OF OMEGA)
              CLOX TANK   TANK=1, AM1(1), ALPHA(1)
              CFUEL TANK   TANK=1, AM1(2), ALPHA(2)
              CPUMP   PUMP=1, AM7(1), AM7(2), ALPHA7
              CORIFICE   ORIFIC =1, AM9, ALPHA9
              CLOX PULSER   PULSER=1, AM5(1),ALPHA5
              CFUEL PULSER   PULSER=1, AM5(2), ALPHA5
0001                PARAB(DC1,DC2,DC3,DTH)=DC2+.5*DTH*(DC3-DC1+DTH*(DC3+DC1-2.*DC2))
0002                QRR(DQ1,DQ2,DTH)=DQ2-DTH*(DQ2-DQ1)
0003                PRR(DP1,DP2,DTH)=DP2-DTH*(DP2-DP1)
0004                QSS(DQ2,DQ3,DTH)=DQ2-DTH*(DQ2-DQ3)
0005                PSS(DP2,DP3,DTH)=DP2-DTH*(DP2-DP3)
0006                INTEGER U,UU,X,PULSER,Z,ORIFIC,PUMP,TANK,SYSTM,SINGLE,PVC
0007                REAL KP,L1,L2,L3,L4,L6,L7,L,MR1,  NO
0008                DIMENSION D(2,10),F(2,10),L(2,10),A(2,10),AR(2,10),THA(2,10),C(2,1
                   20),N(2,10),FF(2,10),Q0(2,10),DPF(2,10),NO(2,10),SLOPE(2,10),ACCEL(
                   32,10),P4(2,10),PQ(2,20),PNP(2,20)
0009                DIMENSION RHO(2), FLOW(2), AM5(2), AM1(2), AM7(2), CDORIF(2), ORIF
                   2(2), VOL(2), KP(2), C10(2),Q1(2),PO(2), PI(2),PPUMP(2), DPO(2), PT
                   3ANK(2), ALPHA1(2), IND2(20), CSTAR(20)
0010                DIMENSION Q(2,10,40), QP(2,10,40), P(2,10,40), PP(2,10,40)
0011                NAMELIST/INSET/D,L,A,F,NC,SLOPE,RHO,FLOW,CDORIF,KP,PTANK,PCO,DQ,Q1
                   2,VOL,PQ,DP,PI,PNP,IND2,J2,DMR,MR1,CSTAR,TANK,PULSER,PUMP,ORIFIC,OM
                   3EGA,AM1,AM5,AM7,AM9,ALPHA1,ALPHA5,ALPHA7,ALPHA9,SYSTM,SINGLE,N,G,T
                   3STOP,UU,PVC,PROP
0012          10    READ(5,INSET)
0013                WRITE (6,INSET)
0014                U=0
0015                T=0.
0016                DT=L(2,7)/(N(2,7)*A(2,7))
              CINITIAL STEADY STATE CONDITICNS IN SYSTEM
0017                DO 60 K=1,2
0018                PC=PCO
0019                DO 20 J=1,9
0020                N(K,J)=L(K,J)/(DT*A(K,J))
0021                IF(J.NE.5) THA(K,J)=N(K,J)*DT*A(K,J)/L(K,J)
0022                AR(K,J)=.7854*D(K,J)** 2*NO(K,J)
0023                C(K,J)=AR(K,J)/A(K,J)
0024                FF(K,J)=F(K,J)*DT/(2.*D(K,J)*AR(K,J)*RHO(K))
0025                Q0(K,J)=FLOW(K)
0026                IF(J.EQ.5)Q0(K,5)=0.
0027                IF(J.NE.5)DPF(K,J)=F(K,J)*L(K,J)*Q0(K,J)**2/(2.*D(K,J)*N(K,J)*AR(
                   2K,J)**2*RHO(K))-RHO(K)*G*L(K,J)*SLOPE(K,J)/N(K,J)
```

Figure 11. Listing of Saturn V Program

```
20    ACCEL(K,J)=G*RHO(K)*DT*SLOPE(K,J)*AR(K,J)
      DPF(K,5)=0.
      ORIF(K)=CDORIF(K)*SQRT(2.*RHO(K))
      C10(K)=VOL(K)*RHO(K)/(DT*KP(K))
      P(K,8,1)=PC+(FLOW(K)/ORIF(K))**2+DPF(K,9)*N(K,9)+DPF(K,8)*N(K,8)
      P(K,1,1)=PTANK(K)
      PP(K,1,1)=PTANK(K)
      WRITE (6,311)DT,(N(K,I),I=1,9),(AR(K,I),I=1,9)
311   FORMAT (1H0,4H DT=,F8.5,8H N(K,I)=,9I5,11H AREA(K,I)=,9F6.3)
      DO50 J=1,9
      NN=N(K,J)+1
      DO 30 I=1,NN
      P(K,J,I)=P(K,J,1)-(I-1)*DPF(K,J)
30    Q(K,J,I)=Q0(K,J)
      IF(J-6) 50,40,50
40    PPUMP(K)=P(K,8,1)-P(K,6,N(K,6)+1)
      DPF(K,7)=-PPUMP(K)/N(K,7)
50    P(K,J+1,1)=P(K,J,N(K,J)+1)
      PP(K,9,N(K,9)+1)=P(K,9,N(K,9)+1)
CEVALUATION OF PUMP CHARACTERISTIC CURVE CONSTANTS
      Z=(Q(K,6,N(K,6)+1)-Q1(K))/DQ
      P1=PQ(K,Z)
      P2=PQ(K,Z+1)
      P3=PQ(K,Z+2)
      Z3=(P1+P3-2.*P2)/(2.*DQ*DQ)
      Z2=(P2-P1)/DQ-Z3*DQ*(2.*Z-1.)
      Z1=P2-DQ*Z*(Z2+Z3*DQ*Z)
      Z=(P(K,6,N(K,6)+1)-PI(K))/DP
      P1=PNP(K,Z)
      P2=PNP(K,Z+1)
      P3=PNP(K,Z+2)
      B3=(P1+P3-2.*P2)/(2.*DP*DP)
      B2=(P2-P1)/DP-B3*DP*(2.*Z-1.)
      B1=P2-DP*Z*(B2+B3*DP*Z)
      P0(K)=Z1+Z2*(Q(K,6,N(K,6)+1)-Q1(K))+Z3*(Q(K,6,N(K,6)+1)-Q1(K))**2
60    DP0(K)=PPUMP(K)-B1-B2*(P(K,6,N(K,6)+1)-PI(K))-B3*(P(K,6,N(K,6)+1)-
     2PI(K))**2
      QL=FLOW(1)
      QF=FLOW(2)
      QT=QL+QF
      Z=(QL/QF-MR1)/DMR+1
      TH=(QL/QF-MR1-Z*DMR)/DMR
      CSTA=PARAB(CSTAR(Z-1),CSTAR(Z),CSTAR(Z+1),TH)
      ATH=CSTA *QT/PC
      C30=ATH/CSTA
      IF(SINGLE-1)80,70,80
70    QFB=QF
      QLB=QL
      CF30=C30
      CL30=C30
80    F4=C10(2)+C(2,9)
      L4=C10(1)+C(1,9)
      JJ2=2*J2
      WRITE (6,630)
630   FORMAT (1H0,81H                          LIQUID OXYGEN SYSTEM
     2                       FUEL SYSTEM/111H   TIME     TANK OUT   PULSER  P
     3UMP IN  PUMP OUT  ORIFICE TANK OUT    PULSER  PUMP IN   PUMP OUT ORIF
     4ICE COMBUSTION)
```

Figure 11. (Concluded)

```
 320   DO 321 K=1,2
       DO 321 J=1,9
 321   P4(K,J)=P(K,J,N(K,J)+1)/144.
       PC4=PC/144.
       NN=N(1,4)+1
       I=N(2,4)+1
       WRITE (6,640)T,(P4(K,1),P4(K,4),P4(K,6),P4(K,7),P4(K,9),K=1,2),PC4
      2,Q(1,2,1),Q(1,4,NN),Q(1,7,1),Q(1,8,1),QL,Q(2,2,1),Q(2,4,I),Q(2,7,1
      3),Q(2,8,1),QF,QT
 640   FORMAT (1H0,F6.4,3H P=,11F9.1/7X,3H Q=,11F9.2)
 330   T=T+DT
       U=U+1
       IF(T.GT.TSTOP) GO TO 240
       DO 120 K=1,2
       IF (SYSTM.EQ.2) K=2
CINTERIOR POINT COMPUTATIONS
       DO 100 J=1,9
       NN=N(K,J)
       DO 100 I=2,NN
       QR=QRR(Q(K,J,I-1),Q(K,J,I),THA(K,J))
       PR=PRR(P(K,J,I-1),P(K,J,I),THA(K,J))
       QS=QSS(Q(K,J,I),Q(K,J,I+1),THA(K,J))
       PS=PSS(P(K,J,I),P(K,J,I+1),THA(K,J))
       QP(K,J,I)=.5*(QR+QS+C(K,J)*(PR-PS)-FF(K,J)*(QR*ABS(QR)+QS*ABS(QS))
      2)+ACCEL(K,J)
       PP(K,J,I)=.5*(PR+PS+(QR-QS-FF(K,J)*(QR*ABS(QR)-QS*ABS(QS)))/C(K,J)
      2)
 100   IF (J.EQ.7)QP(K,7,I)=QP(K,7,I)+PPUMP(K)*AR(K,7)*DT/L(K,7)
CSERIES PIPE CONNECTIONS INCLUDING PUMP INLET AND DISCHARGE
       DO110 M=1,JJ2,2
       X=IND2(M)
       I=N(K,X)+1
       QR=QRR(Q(K,X,I-1),Q(K,X,I),THA(K,X))
       C3=QR+C(K,X)*PRR(P(K,X,I-1),P(K,X,I),THA(K,X))-FF(K,X)*QR*ABS(QR)+
      2ACCEL(K,X)
       IF(X.EQ.7)C3=C3+PPUMP(K)*AR(K,7)*DT/L(K,7)
       J=IND2(M+1)
       QS=QSS(Q(K,J,1),Q(K,J,2),THA(K,J))
       C1=QS-C(K,J)*PSS(P(K,J,1),P(K,J,2),THA(K,J))-FF(K,J)*QS*ABS(QS)+AC
      2CEL(K,J)
       IF(J.EQ.7) C1=C1+PPUMP(K)*AR(K,7)*DT/L(K,7)
       QP6=0.
       IF(PUMP.EQ.1.AND.X.EQ.6)QP6=RHO(K)*AR(K,6)*AM7(K)*OMEGA*SIN(OMEGA*
      2T+ALPHA7)
       QP1=0.
       IF(TANK.EQ.1.AND.X.EQ.1)QP1=RHO(K)*(PROP*AR(K,1)-AR(K,2))
      2*AM1(K)*OMEGA*SIN(OMEGA*T+ALPHA1(K))
       PP(K,X,I)=(C3-C1+QP6+QP1)/(C(K,J)+C(K,X))
       PP(K,J,1)=PP(K,X,I)
       QP(K,X,I)=C3-C(K,X)*PP(K,X,I)
 110   QP(K,J,1)=QP(K,X,I)
CPULSER OR PVC JUNCTION
       I=N(K,4)+1
       QR=QRR(Q(K,4,I-1),Q(K,4,I),THA(K,4))
       C3=QR+C(K,4)*PRR(P(K,4,I-1),P(K,4,I),THA(K,4))-FF(K,4)*QR*ABS(QR)+
      2ACCEL(K,4)
```

Figure 11. (Continued)

```
      QS=QSS(Q(K,6,1),Q(K,6,2),THA(K,6))
      C1=QS-C(K,6)*PSS(P(K,6,1),P(K,6,2),THA(K,6))-FF(K,6)*QS*ABS(QS)+AC
     2CEL(K,6)
      QP(K,5,1)=0.
      IF(PULSER.EQ.1)QP(K,5,1)=RHO(K)*AR(K,5)*AM5(K)*OMEGA*SIN(OMEGA*T+A
     2LPHA5)
      IF(PULSER.EQ.0.AND.PVC.EQ.1)QP(K,5,1)=-RHO(K)*AR(K,6)*AM7(K)*OMEGA
     2*SIN(OMEGA*T+ALPHA7)
      PP(K,4,I)=(C3-C1+QP(K,5,1))/(C(K,4)+C(K,6))
      PP(K,6,1)=PP(K,4,I)
      PP(K,5,1)=PP(K,6,1)
      QP(K,4,I)=C3-C(K,4)*PP(K,4,I)
      QP(K,6,1)=C1+C(K,6)*PP(K,6,1)
CBOUNDARY CONDITION AT FUEL AND LOX TANKS
      QS=QSS(Q(K,1,1),Q(K,1,2),THA(K,1))
      C1=QS-C(K,1)*PSS(P(K,1,1),P(K,1,2),THA(K,1))-FF(K,1)*QS*QS+ACCEL(K
     2,1)
      QP(K,1,1)=C1+C(K,1)*PP(K,1,1)
  120 IF (SYSTM.EQ.1) K=2
CBOUNDARY CONDITION AT ENGINE
      I=N(1,9)+1
      QR=QRR(Q(1,9,I-1),Q(1,9,I),THA(1,9))
      C3=QR+C(1,9)*PRR(P(1,9,I-1),P(1,9,I),THA(1,9))-FF(1,9)*QR*QR+ACCEL
     2(1,9)
      J=N(2,9)+1
      QR=QRR(Q(2,9,J-1),Q(2,9,J),THA(2,9))
      C2=QR+C(2,9)*PRR(P(2,9,J-1),P(2,9,J),THA(2,9))-FF(2,9)*QR*QR+ACCEL
     2(2,9)
      L3=C3+C10(1)*P(1,9,I)
      F3=C2+C10(2)*P(2,9,J)
      IF(ORIFIC-1)140,130,140
  130 QPL9=RHO(1)*AR(1,9)*AM9*OMEGA*SIN(OMEGA*T+ALPHA9)
      QPF9=RHO(2)*AR(2,9)*AM9*OMEGA*SIN(OMEGA*T+ALPHA9)
      GO TO 150
  140 QPL9=0.
      QPF9=0.
  150 IF(SINGLE-1)190,160,190
  160 IF(SYSTM.EQ.2) GO TO 170
      L1=ORIF(1)**2*(1./L4+1./CL30)
      L2=ORIF(1)**2*((L3+QPL9)/L4-QFB/CL30)
      QL=.5*(-L1+SQRT(L1*L1+4.*L2))
      QT=QL+QFB
      PC=QT/CL30
      PP(1,9,I)=(-QL+L3)/L4
      QP(1,9,I)=C3-C(1,9)*PP(1,9,I)
      Z=(QL/QFB-MR1)/DMR+1
      TH=(QL/QFB-MR1-Z*DMR)/DMR
      CL30=ATH/(PARAB(CSTAR(Z-1),CSTAR(Z),CSTAR(Z+1),TH))
  170 IF(SYSTM.EQ.1) GO TO 210
      F1=ORIF(2)**2*(1./F4+1./CF30)
      F2=ORIF(2)**2*((F3+QPL9)/F4-QLB/CF30)
      QF=.5*(-F1+SQRT(F1*F1+4.*F2))
      QT=QF+QLB
      PC=QT/CF30
      PP(2,9,J)=(-QF+F3)/F4
      QP(2,9,J)=C2-C(2,9)*PP(2,9,J)
      Z=(QLB/QF-MR1)/DMR+1
```

Figure 11. (Continued)

```
      TH=(QLB/QF-MR1-Z*DMR)/DMR
      CF30=ATH/(PARAB(CSTAR(Z-1),CSTAR(Z),CSTAR(Z+1),TH))
      GO TO 210
190   DO 200 M=1,4
      L1=ORIF(1)*SQRT(PP(1,9,I)-PC)
      L2=L1/(2.*(PP(1,9,I)-PC))
      F1=ORIF(2)*SQRT(PP(2,9,J)-PC)
      F2=F1/(2.*(PP(2,9,J)-PC))
      L6=-C30*PC+(L1*L4+L2*(L3-L4*PP(1,9,I)+QPL9))/(L2+L4)+(F1*F4+F2*(F3
     2 -F4*PP(2,9,J)+QPF9))/(F2+F4)
      L7=C30+L2*L4/(L2+L4)+F2*F4/(F2+F4)
      PCP=L6/L7
      PLP=(L3-L4*PP(1,9,I)-L1+L2*PCP+QPL9)/(L2+L4)
      PFP=(F3-F4*PP(2,9,J)-F1+F2*PCP+QPF9)/(F2+F4)
      PP(1,9,I)=PP(1,9,I)+PLP
      PP(2,9,J)=PP(2,9,J)+PFP
200   PC=PC+PCP
      QP(1,9,I)=C3-C(1,9)*PP(1,9,I)
      QP(2,9,J)=C2-C(2,9)*PP(2,9,J)
      QL=ORIF(1)*SQRT(PP(1,9,I)-PC)
      QF=ORIF(2)*SQRT(PP(2,9,J)-PC)
      QT=QF+QL
      Z=(QL/QF-MR1)/DMR+1
      TH=(QL/QF-MR1-Z*DMR)/DMR
      C30=ATH/(PARAB(CSTAR(Z-1),CSTAR(Z),CSTAR(Z+1),TH))
CSUBSTITUTION STATEMENTS FOR PRESSURE AND DISCHARGE
210   DO 230 K=1,2
      IF (SYSTM.EQ.2) K=2
      DO 220 J=1,9
      NN=N(K,J)+1
      DO 220 I=1,NN
      Q(K,J,I)=QP(K,J,I)
220   P(K,J,I)=PP(K,J,I)
CPUMP CHARACTERISTIC CONSTANTS AND PUMP PRESSURE RISE
      Z=(Q(K,6,N(K,6)+1)-Q1(K))/DQ
      P1=PQ(K,Z)
      P2=PQ(K,Z+1)
      P3=PQ(K,Z+2)
      Z3=(P1+P3-2.*P2)/(2.*DQ*DQ)
      Z2=(P2-P1)/DQ-Z3*DQ*(2.*Z-1.)
      Z1=P2-DQ*Z*(Z2+Z3*DQ*Z)
      Z=(P(K,6,N(K,6)+1)-PI(K))/DP
      P1=PNP(K,Z)
      P2=PNP(K,Z+1)
      P3=PNP(K,Z+2)
      B3=(P1+P3-2.*P2)/(2.*DP*DP)
      B2=(P2-P1)/DP-B3*DP*(2.*Z-1.)
      B1=P2-DP*Z*(B2+B3*DP*Z)
      PPUMP(K)=Z1+Z2*(Q(K,6,N(K,6)+1)-Q1(K))+Z3*(Q(K,6,N(K,6)+1)-Q1(K))
     2**2+B1+DPO(K)+B2*(P(K,6,N(K,6)+1)-PI(K))+B3*(P(K,6,N(K,6)+1)-PI(K
     3))**2-PO(K)
230   IF (SYSTM.EQ.1) K=2
      IF(U/UU*UU-U) 330,320,330
240   STOP
      END
```

Figure 11. (Continued)

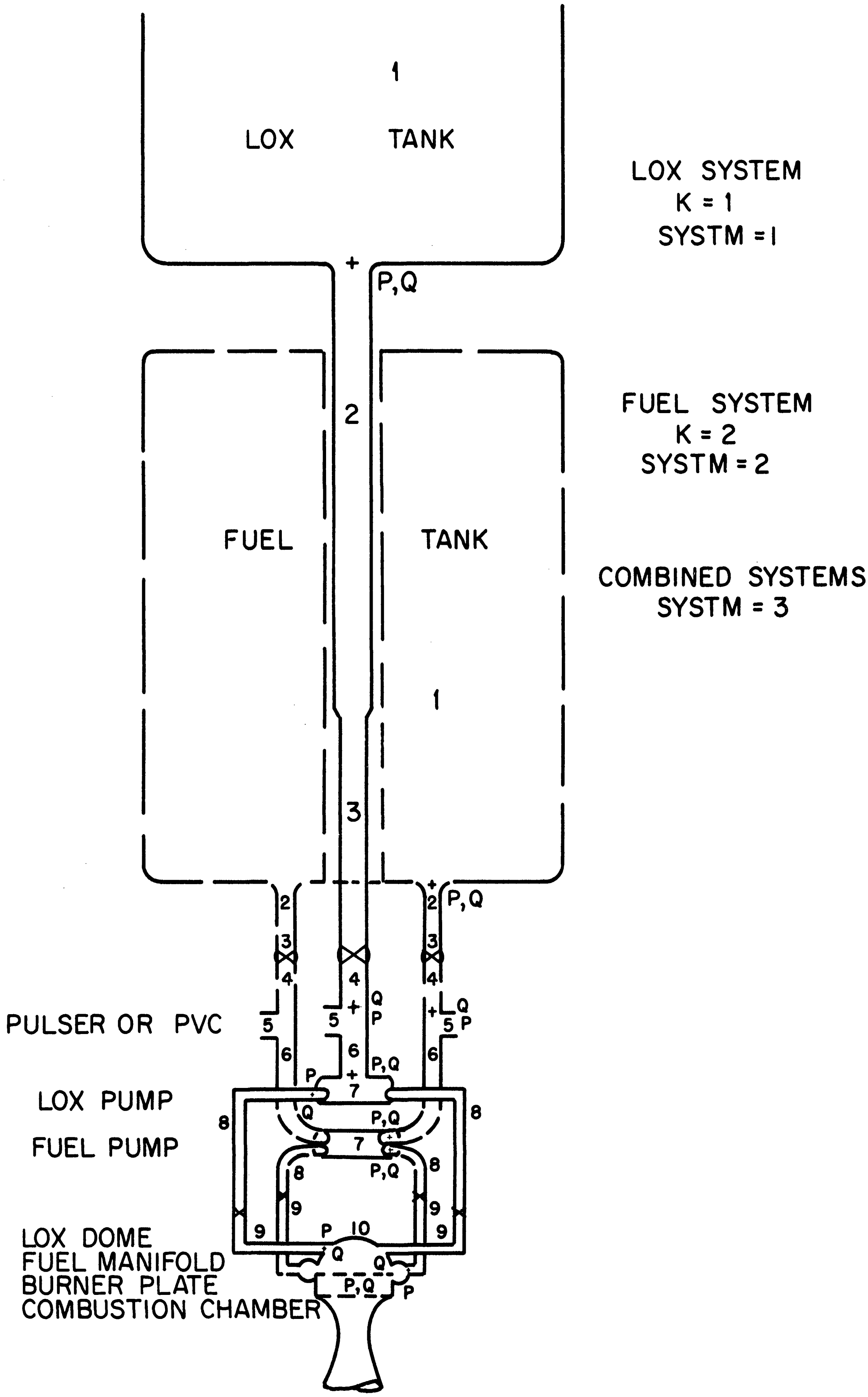

Figure 12. Sketch of System Showing Location of Program Output Quantities P, Q.

VII. SAMPLE PROBLEM

In order to demonstrate the use of the program, a sample problem is included which analyzes a transient condition in Titan II missile. The program analyzes the transients that develop in the oxidizer system as a result of a forced motion of the pump. The motion is assumed to be sinusoidal at a frequency of 62.8 rad/sec and half amplitude equal to 0.007 ft.

Figure 13 gives a copy of the listing of the input data and Figure 14 shows a portion of the output from the calculations.

The effect of the addition of a PVC in the system can be seen in Fig. 15, where a comparison is made between two runs. Both runs are subjected to the same excitation at the pump, one includes the PVC, the other does not.

```
EXECUTION BEGINS
 &INSET
 D=  10.000000    ,  10.000000    , 0.57499999    , 0.50000000    , 0.57499999    , 0.50000000    , 0.57499999    , 0.50000000    ,
  0.41700000    , 0.41700000    , 0.57499999    , 0.50000000    , 0.57499999    , 0.50000000    , 0.41700000    , 0.42999995    ,
  0.41700000    , 0.42999995    , 0.0           , 0.0           ,L=  8.0000000  ,  8.0000000    ,  23.000000    , 0.39999998    ,
   3.5999994    , 0.42999995    ,  1.5000000    ,  1.6699991    , 0.0           , 0.0           , 0.50000000    , 0.50000000    ,
   2.0000000    ,  1.5000000    ,  2.5000000    ,  6.0000000    ,  3.0000000    ,  14.000000    , 0.0           , 0.0           ,A=
   483.00000    ,  985.00000    ,  1750.0000    ,  138.00000    ,  1100.0000    ,  138.00000    ,  650.00000    ,  138.00000    ,
   1750.0000    ,  3640.0000    ,  650.00000    ,  138.00000    ,  2980.0000    ,  3000.0000    ,  2980.0000    ,  4230.0000    ,
   2980.0000    ,  4230.0000    , 0.0           , 0.0           ,F= 0.79999976E-02, 0.79999976E-02, 0.11000000E-01, 0.11000000E-01,
  0.11000000E-01, 0.11000000E-01, 0.72999954E-01, 0.12099999    , 0.11000000E-01, 0.11000000E-01, 0.72999954E-01, 0.12099999    ,
  0.0           , 0.0           ,  1.0000000    , 0.35999998E-01,  1.0000000    ,  1.4799995    , 0.0           , 0.0           ,
 NO=  1.0000000    ,  1.0000000    ,  1.0000000    ,  1.0000000    ,  1.0000000    ,  1.0000000    ,  1.0000000    ,  1.0000000
   1.0000000    ,  1.0000000    ,  1.0000000    ,  1.0000000    ,  1.0000000    ,  1.0000000    ,  1.0000000    ,  1.0000000    ,
   1.0000000    ,  1.0000000    ,  1.0000000    ,  1.0000000    ,SLOPE=  1.0000000    ,  1.0000000    ,  1.0000000    ,
   1.0000000    ,  1.0000000    ,  1.0000000    ,  1.0000000    ,  1.0000000    , 0.0           , 0.0           ,  1.0000000    ,
   1.0000000    , 0.0           , 0.0           , 0.0           , 0.0           , 0.0           , 0.0           , 0.0           ,
  0.0           ,RHO=  2.8199997    ,  1.7599993    ,FLOW=  17.000000    ,  8.1699991    ,CDORIF= 0.36699999E-01, 0.20999998E-01,
 KP=  100000000.   ,  100000000.   ,PTANK=  10500.000    ,  2460.0000    ,PCO=  115000.00    ,DQ=  1.0000000    ,Q1=  12.000000
   5.0000000    ,VOL= 0.0       , 0.0           ,PQ=  197000.00    ,  231000.00    ,  193000.00    ,  229000.00    ,
   189000.00    ,  224000.00    ,  184000.00    ,  218000.00    ,  179000.00    ,  210000.00    ,  173000.00    ,  200000.00    ,
   166000.00    ,  188000.00    ,  159000.00    ,  174000.00    ,  150000.00    ,  170000.00    ,  141000.00    ,  165000.00    ,
   1.0000000    ,  1.0000000    ,  1.0000000    ,  1.0000000    ,  1.0000000    ,  1.0000000    ,  1.0000000    ,  1.0000000    ,
   1.0000000    ,  1.0000000    ,  1.0000000    ,  1.0000000    ,  1.0000000    ,  1.0000000    ,  1.0000000    ,  1.0000000    ,
   1.0000000    ,  1.0000000    ,  1.0000000    ,  1.0000000    ,DP=  250.00000    ,PI=  10500.000    ,  2000.0000    ,PNP=
   176000.00    ,  190800.00    ,  176000.00    ,  194500.00    ,  176000.00    ,  196800.00    ,  176000.00    ,  198800.00    ,
   176000.00    ,  200000.00    ,  176000.00    ,  201000.00    ,  176000.00    ,  201900.00    ,  176000.00    ,  202500.00    ,
   176000.00    ,  203000.00    ,  176000.00    ,  203400.00    ,  176000.00    ,  203700.00    ,  176000.00    ,  203900.00    ,
   176000.00    ,  204000.00    ,  176000.00    ,  204100.00    ,  1.0000000    ,  1.0000000    ,  1.0000000    ,  1.0000000    ,
   1.0000000    ,  1.0000000    ,  1.0000000    ,  1.0000000    ,  1.0000000    ,  1.0000000    ,  1.0000000    ,  1.0000000    ,
 IND2=          1,          2,          2,          3,          3,          4,          6,          7,          7,          8,
          8,          9,          0,          0,          0,          0,          0,          0,          0,          0,J2=
          6,DMR= 0.99999964E-01,MR1=  1.7999992    ,CSTAR=  5775.0000    ,  5756.0000    ,  5730.0000    ,  5695.0000    ,
   5650.0000    ,  5610.0000    ,  5565.0000    , 0.0           , 0.0           , 0.0           , 0.0           , 0.0           ,
  0.0           , 0.0           , 0.0           , 0.0           , 0.0           , 0.0           , 0.0           , 0.0           ,
 TANK=          0,PULSER=          0,PUMP=          1,ORIFIC=          0,OMEGA=  62.799988    ,AM1= 0.0           , 0.0           ,
 AM5= 0.0           , 0.0           ,AM7= 0.29999998E-02, 0.0           ,AM9= 0.0           ,ALPHA1= 0.0           , 0.0
 ALPHA5= 0.0           ,ALPHA7= 0.0           ,ALPHA9= 0.0           ,SYSTM=          1,SINGLE=          1,N=          0,
          0,          0,          0,          0,          0,          0,          0,          0,          0,          0,
          0,          0,          1,          0,          0,          0,          0,          0,          0,G=  32.199997    ,
 TSTOP= 0.63999951E-01,UU=          8,PVC=          0,PROP= 0.19999999
 &END
```

Figure 13. Input Data for Example Problem

DT= 0.00050 N(K,I)= 33 26 6 4 0 1 1 1 2 AREA(K,I)=78.540 0.260 0.260 0.260 0.137 0.260 0.260 0.137 0.137

DT= 0.00050 N(K,I)= 16 5 6 24 0 7 1 2 6 AREA(K,I)=78.540 0.196 0.196 0.196 0.137 0.196 0.196 0.145 0.145

		LIQUID OXYGEN SYSTEM					FUEL SYSTEM					
TIME		TANK OUT	PULSER	PUMP IN	PUMP OUT	ORIFICE	TANK OUT	PULSER	PUMP IN	PUMP OUT	ORIFICE	COMBUSTION
0.0	P=	78.0	92.0	92.0	1314.4	1062.8	20.2	19.8	19.6	1401.2	1097.2	798.6
	Q=	17.00	17.00	17.00	17.00	17.00	8.17	8.17	8.17	8.17	8.17	25.17
0.0040	P=	78.0	92.3	92.4	1314.2	1062.7	20.2	19.8	19.6	1401.2	1097.2	798.6
	Q=	17.00	16.98	16.97	17.00	17.00	8.17	8.17	8.17	8.17	8.17	25.17
0.0080	P=	78.0	92.9	92.9	1314.0	1062.6	20.2	19.8	19.6	1401.2	1097.2	798.5
	Q=	17.00	16.96	16.95	17.00	17.00	8.17	8.17	8.17	8.17	8.17	25.17
0.0120	P=	78.0	93.5	93.5	1314.0	1062.5	20.2	19.8	19.6	1401.2	1097.2	798.5
	Q=	17.00	16.94	16.93	17.00	17.00	8.17	8.17	8.17	8.17	8.17	25.17
0.0160	P=	78.0	94.1	94.1	1314.1	1062.5	20.2	19.8	19.6	1401.2	1097.2	798.5
	Q=	17.00	16.92	16.92	17.00	16.99	8.17	8.17	8.17	8.17	8.17	25.16
0.0200	P=	78.0	94.7	94.7	1314.2	1062.6	20.2	19.8	19.6	1401.2	1097.2	798.5
	Q=	16.99	16.91	16.91	17.00	17.00	8.17	8.17	8.17	8.17	8.17	25.17
0.0240	P=	78.0	95.1	95.1	1314.4	1062.7	20.2	19.8	19.6	1401.2	1097.2	798.6
	Q=	16.97	16.91	16.91	17.00	17.00	8.17	8.17	8.17	8.17	8.17	25.17
0.0280	P=	78.0	95.4	95.4	1314.6	1062.8	20.2	19.8	19.6	1401.2	1097.2	798.6
	Q=	16.94	16.91	16.91	17.00	17.00	8.17	8.17	8.17	8.17	8.17	25.17
0.0320	P=	78.0	95.5	95.5	1314.8	1062.9	20.2	19.8	19.6	1401.2	1097.2	798.7
	Q=	16.91	16.92	16.92	17.00	17.00	8.17	8.17	8.17	8.17	8.17	25.17
0.0360	P=	78.0	95.4	95.3	1315.0	1063.1	20.2	19.8	19.6	1401.2	1097.2	798.7
	Q=	16.88	16.93	16.93	17.01	17.00	8.17	8.17	8.17	8.17	8.17	25.17
0.0400	P=	78.0	94.8	94.8	1315.1	1063.2	20.2	19.8	19.6	1401.2	1097.2	798.8
	Q=	16.86	16.94	16.95	17.01	17.01	8.17	8.17	8.17	8.17	8.17	25.18
0.0440	P=	78.0	93.6	93.6	1314.7	1063.1	20.2	19.8	19.6	1401.2	1097.2	798.8
	Q=	16.85	16.96	16.97	17.00	17.01	8.17	8.17	8.17	8.17	8.17	25.18
0.0480	P=	78.0	92.2	92.1	1314.3	1062.9	20.2	19.8	19.6	1401.2	1097.2	798.7
	Q=	16.84	16.97	16.99	17.00	17.00	8.17	8.17	8.17	8.17	8.17	25.17
0.0520	P=	78.0	90.6	90.5	1313.7	1062.6	20.2	19.8	19.6	1401.2	1097.2	798.5
	Q=	16.84	16.99	17.01	17.00	17.00	8.17	8.17	8.17	8.17	8.17	25.17
0.0560	P=	78.0	88.9	88.8	1313.1	1062.2	20.2	19.8	19.6	1401.2	1097.2	798.3
	Q=	16.84	17.00	17.02	16.99	16.99	8.17	8.17	8.17	8.17	8.17	25.16
0.0600	P=	78.0	87.3	87.2	1312.5	1061.8	20.2	19.8	19.6	1401.2	1097.2	798.2
	Q=	16.86	17.02	17.03	16.98	16.98	8.17	8.17	8.17	8.17	8.17	25.15
0.0640	P=	78.0	85.9	85.9	1312.0	1061.5	20.2	19.8	19.6	1401.2	1097.2	798.0
	Q=	16.90	17.03	17.05	16.98	16.98	8.17	8.17	8.17	8.17	8.17	25.15

Figure 14. Output from Analysis with Oscillating Pump

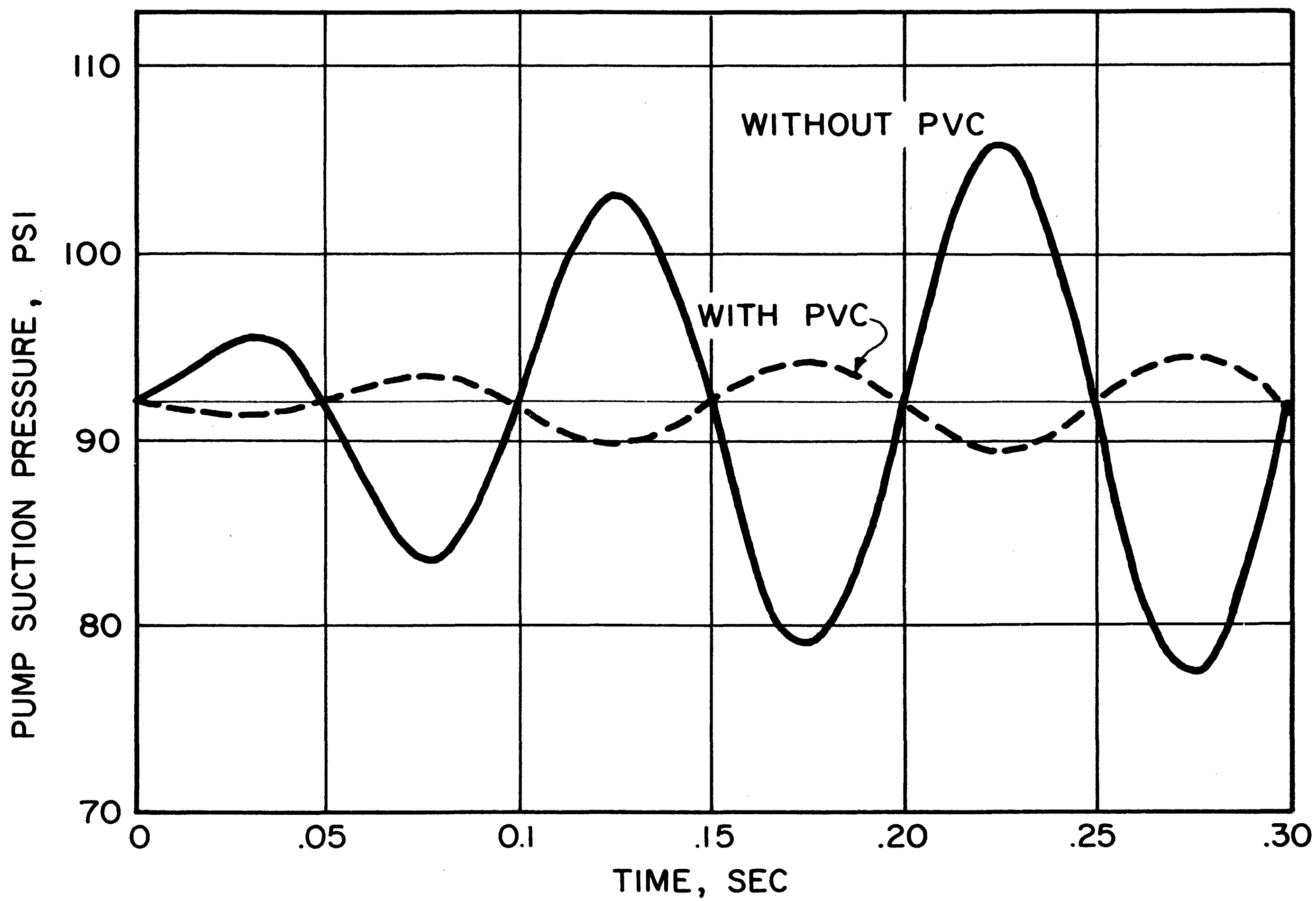

Figure 15. Pressure at Pump Inlet of Oxidizer System of Titan II Due to Pump Oscillation, $\omega = 62.8$ rad/sec.

VIII. SUMMARY

This report includes the digital computer program for the analysis of the propellant systems of Saturn V. Much of the background material necessary to understand the computations performed by the computer during execution of the program is included.

Data for Saturn V have not been used in any trial runs of the program as complete data on the geometric properties of the missile are not available. The program was cleared using Titan II data. Computed results have been checked against experimental results obtained on Titan II and reported in the literature,(3) and favorable comparisons have been made.

It is our intention that the program can be used directly for any one of the three stages of Saturn V without further modification. However, since the exact configurations are not available, minor changes may be necessary to accommodate the particular data.

APPENDIX A

DEFINITION OF PROGRAM VARIABLES

ACCEL(k,j)	Constant associated with vehicle acceleration in pipe j of system k
ALPHA1(k)	Phase angle associated with the feed tank motion in system k
ALPHA5	Phase angle associated with the pulser motion
ALPHA7	Phase angle associated with the pump motion
ALPHA9	Phase angle associated with the engine burner plate motion
AM1(k)	Amplitude associated with the feed tank motion in system k
AM5(k)	Amplitude associated with the pulser motion in system k
AM7(k)	Amplitude associated with the pump motion in system k
AM9	Amplitude associated with the engine burner plate motion
AR(k,j)	Cross-sectional area of pipe j in system k
A(k,j)	Pressure pulse wave speed in pipe j in system k
ATH	Constant associated with the engine
B1,B2,B3	Variables associated with the pump head rise-suction pressure curve
C10(k)	Constant in system k describing the capacitance of fluid and chamber at the engine orifice plate inlet
C1	Variable associated with the C- characteristic
C2	Variable associated with the C+ characteristic
C30	Variable relating combustion chamber pressure and total propellant mass flow rate
C3	Variable associated with the C+ characteristic
CDORIF(k)	Constant used to describe the discharge coefficient and area of opening of the engine orifice in system k

CF30,CL30	Variables used with combustion chamber boundary condition
C(k,j)	Constant for system k, pipe j, AR(k,j)/A(k,j)
CSTAR(i)	Variable used for storage of tabular data of the engine characteristic velocity vs. the mixture ratio
CSTA	Variable identifying the characteristic velocity at a particular mixture ratio
DMR	Interval of mixture ratio values at which the characteristic velocities are stored beginning at MR1
DPO(k)	Adjustment of the tabulated pump data, head rise vs. suction pressure, to match initial flow conditions
DPF(k,j)	Steady-state frictional head loss per section in pipe j in system k
DP	Interval of suction pressure values at which the pump head rise values are stored beginning at PI(k)
DQ	Interval of discharge values at which the pump head rise values are stored, beginning at Q1(k)
D(k,j)	Pipe diameter, pipe j, system k
DT	Time interval at which computations are performed
F1,F2,F3,F4	Variables used in connection with the fuel supply to the engine
FF(k,j)	Constant associated with the friction losses in pipe j of system k
FLOW(k)	Given steady-state mass flow rate in system k
F(k,j)	Darcy-Weisbach friction factor in system k, pipe j
G	Acceleration in the missile axial direction at the time in flight when the transient is being investigated
IND2(i)	Set of integer constants that identify the pipes, in pairs, by number, that are connected in series
I	Integer variable, generally refers to the pipe section number under consideration
J2	Constant identifying the number of series connections in each system

JJ2	Constant = 2(J2)
J	Integer variable, generally refers to pipe under consideration
KP(k)	Constant defining the effective bulk modulus of volume at the engine orifice plate entrance
K	Integer variable, generally refers to the system, K = 1 refers to lox system, K = 2 refers to fuel system
L1,L2,L3,L4	Variables used in connection with the oxidizer supply to the engine
L6,L7	Variables used with the boundary condition at the engine
L(k,j)	Pipe length, pipe j in system k
MR1	Lowest value of mixture ratio at which characteristic velocities are stored
M	Integer variable, used in series connections and engine boundary condition
NO(k,j)	Integer constant identifying the number of identical pipes connected in parallel that can be treated as a single pipe j in system k
N(k,j)	Number of reaches into which pipe j of system k is divided
NN	Integer variable used in place of N(k,j)+1
OMEGA	Angular frequency of excitation
ORIFIC	Integer used to include or exclude excitation at the engine orifice plate
ORIF(k)	Constant used with the orifice in system k
PO(k)	Pump head rise associated with initial steady state discharge, as determined from the given head rise vs. discharge data in system k
P1,P2,P3	Variables associated with the pump head rise-discharge curves
P4(k,j)	Pressure at outflow end of pipe j, system k, in psi; used for print out

PCO	Initial steady state pressure in the combustion chamber
PCP	Variation in combustion chamber pressure at new time increment
PC	Pressure in combustion chamber
PFP	Variation in pressure upstream of orifice plate in fuel system at new time increment
PI(k)	Lowest value of suction pressure at which pump head rise values are given in system k
PLP	Variation in pressure upstream of orifice plate in oxidizer system at new time increment
PNP(k,i)	Values of pump head rise stored for equal increments of suction pressure, DP, in system k
PP(k,j,i)	Newly computed values of pressure at section i, pipe j, system k
PPUMP(k)	Pressure head produced by the pump in system k
PQ(k,i)	Values of pump head rise stored for equal increments of discharge, DQ, in system k
PROP	Constant used to define the proportion of the feed tank area to be used when the system excitation is occurring at the tank
PR,PS	Interpolated value of pressure head in the pipeline
P(k,j,i)	Pressure head at section i, pipe j, system k
PTANK(k)	Pressure level maintained in feed tank of system k
PULSER	Integer used to include or exclude excitation of the system using the pulser
PUMP	Integer used to include or exclude excitation of the system using the pump motion
PVC	Integer used to include or exclude the pressure volume compensator in the system
QO(k,j)	Initial steady state flow in pipe j of system k
Ql(k)	Lowest value of discharge at which pump head rise values are given in system k

QFB	Constant flow rate of fuel to combustion chamber when lox system alone is being analyzed
QF	Flow rate of fuel to combustion chamber
QLB	Constant flow rate of lox to combustion chamber when fuel system alone is being analyzed
QL	Flow rate of lox to combustion chamber
QP1	Variable used when the system excitation is considered to be at the supply tank
QP6	Variable used when the system excitation is considered to be at the pump
QPF9	Variable used in the fuel system when the system excitation is considered to be at the engine orifice
QPL9	Variable used in the lox system when the system excitation is considered to be at the engine orifice
QP(k,j,i)	Newly computed values of flow rate at section i, pipe j, system k
QR,QS	Interpolated value of flow rate
Q(k,j,i)	Flow rate at section i, pipe j, system k
QT	Total flow rate to the combustion chamber
RHO(k)	Fluid mass density in system k
SINGLE	Integer constant used to isolate the systems at the combustion chamber
SLOPE(k,j)	Constant used to indicate whether a pipe is in the axial direction of the missile or normal to the axial direction
SYSTM	Integer constant used to specify what systems are being studied
TANK	Integer used to include or exclude excitation of the system using the motion of the supply tank
THA(k,j)	Constant used in pipe j, system k; pertains to the characteristics grid-mesh ratio
TH	Variable used with parabolic interpolation
T	Time

TSTOP	Constant identifying the time at which the transient calculation should stop
U	Integer counter, incremented by one when time is incremented by DT
UU	Integer constant controlling the print out
VOL(k)	Constant defining the volume at the engine orifice plate entrance
X	Integer used to identify the inflow pipe number at series connections
Z1,Z2,Z3	Variables associated with pump head rise-discharge curve
Z	Integer used with the pump characteristics

Function References

PARAB	Function used for parabolic interpolation
PRR	Function used for linear interpolation of pressure at R
PSS	Function used for linear interpolation of pressure at S
QRR	Function used for linear interpolation of discharge at R
QSS	Function used for linear interpolation of discharge at S

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