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Physical - mathematical bases of the principle of independence of cavity expansion

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ABSTRACT

For the first steps of cavitation researches the very important general peculiarity of supercavitating flow was found which discovered practical independence of cavity sections expansion in motionless fluid. This peculiarity gave the possibility for practical estimation of the cavities in the most part of applications. The paper presents the system of the simple dependencies for practical calculations of axisymmeric and near to one supecavitation flows with account of its perfection on the base of modern achievements of the theoretic and experimental research which based on the property of independence of the cavity section expansion. Main attention is paid to asymptotic dependencies on the base of Slender Body Theory and heuristic models. The calculations examples of steady and unsteady cavities for motion under gravity, axelration, harmonic oscillation of pressure are given. The problems of ventilated cavities and possible ways of drag reduction for motion with supercavitation are considered.

Key words: high speed hydrodynamics, supercavitation, cavity, drag reduction.

INTRODUCTION

The supercavitation application give the possibility isolated body against water avoids the viscid losses of boundary layer and considerably reduces resistance at high speed motion in water. Application of supercavitation regime give the possibility to reach very small drag coefficients and the smaller - the smaller is cavity aspect ratio. These values can reach values ~ 0.01 and less and are restricted by maximal body aspect ratios from point of view of its strength. The most important for realization of motion the problems of hopeful defining of the form, main sizes of the cavity and cavitation drag are together with possibilities of the drag decreasing.

The base of the theory for the most typical range of high speeds $100 \div 200$ м/с and over is the problem of mathematical statement for potential flow of ideal incompressible fluid with unknown before solution free boundaries. Basic parameter defining cavitation flow is Cavitation Number $\sigma = 2(P_\infty - P_c)/\rho U_\infty^2$, where: $\Delta P = P_\infty - P_c$ is pressure difference in the flow and cavity, ρ - water mass density, U_∞ - flow speed at infinity. Bases of supercavitation are expounded in a number of known publications [3, 7-9, 13, 14 , 31]. Under the first steps of researches for 1940-50 these problems were solved in the most part on the base of simplest heuristic approaches and integral conservation laws with basic on experimental dates. It was found here: ellipsoidal steady axisymmetric cavity form $R = R(x)$ (1a) (r, x - cylindrical coordinates system connected with cavitator), first order dependence for maximal cavity radius (1b R_k (H. Reichardt, G. Logvinovich, L. Epshtein [7, 8, 17] and first order dependence for aspect ratio of slender axisymmetric cavity λ (1c) (P. Garabedian) [11], known asymptotic for expanding of the cavity section for $\sigma = 0$ at infinity was found (1d) (M. Гуревич, N. Levinson) [13, 16]:

$$
\begin{aligned}
&\text{a)}\quad \bar{R}^2 = (1/\lambda^2)\bar{x}(2-\bar{x}),\quad \bar{x} = x/L_k,\ \bar{R} = R/L_k \\
&\text{b)}\quad R_k = R_n\sqrt{\frac{c_d}{k\sigma}},\quad \text{c)}\quad \lambda = \sqrt{\frac{\ln 1/\sigma}{\sigma}}, \\
&\text{d)}\quad \bar{R}^2 = \frac{2\sqrt{c_{do}}\,\bar{x}}{\sqrt{\ln(x^2/R^2)}} \approx \frac{2\sqrt{c_{do}}\,\bar{x}}{\sqrt{\ln\bar{x}}}\left(1 - \frac{1}{4}\frac{\ln\ln\bar{x}}{\ln\bar{x}}\right), \\
&\qquad \bar{x} = x/R_n,\ \bar{R} = R/R_n,
\end{aligned}
\tag{1}
$$

where: R_n, c_d, c_{do} - accordingly: radius, drag coefficient of cavitator and it's value for $\sigma = 0$. L_k - semi-length of steady for $\sigma = \text{const}$ cavity.

LINEARIZED THEORY - BASIC RESULTS

It is need to note that the most important for applications namely the maximally slender cavities are as the most important for providing the minimal cavitation drag. With account of this fact further more deep advancing of supercavitation researches was achieved under next steps in the fields of development of linearized theory of axisymmtric supercavitation [18, 20-29,33] on the base of known Slender Body Hydrodynamics (SBH) [2, 10], theory of small perturbation of axisymmetric cavities [34, 36] and development of nonlinear numerical methods for prediction of flows with free boundaries [4-6, 12, 15, 35]]. Below the attempt to analyze and precise existing system of the simplest dependencies (1) on the base of results of linearized theory with ground on known nonlinear numerical calculations and experimental date are undertaken. Idea to develop linearized theory of axisymmetric supercavitation was proposed by F. Francel and E. Karpovich, 1948 in their known book on gas dynamics of slender bodies [10]. Note here the results of the linear 2D supercavitation by M. Tulin [31] stimulated researches in axisymmetric case.

Base of linearized theory of axisymmetric supercavitation on the base of Matched Asymptotic Expansion Method (MAEM) [32] under approximation of the Slender Body Hydrodynamics (SBH) [2, 10] were developed in works [18, 20-29, 33]. Basis here is integer-differential equation for slender axisymmetric cavity $r = R(x)$ which in the case of slender cavitator $r = r_1(x)$ is:

$$\underset{(\ln 1/\delta^2)^{-1}}{\frac{1}{2R^2}\left(\frac{dR^2}{dx}\right)^2} + \underset{(1)}{\frac{d^2R^2}{dx^2}\ln\frac{R^2}{4x(L-x)}} - \underset{(\ln 1/\delta^2)^{-1}}{\int_0^{x_o}\frac{\frac{d^2r_1^2}{dx^2}\big|_{x=x_1} - \frac{d^2R^2}{dx^2}}{|x_1-x|}dx_1} - \underset{(\ln 1/\delta^2)^{-1}}{\int_{x_o}^{L}\frac{\frac{d^2R^2}{dx^2}\big|_{x=x_1} - \frac{d^2R^2}{dx^2}}{|x_1-x|}dx_1} - \underset{(\ln 1/\delta^2)^{-1}}{\frac{\frac{dr_1^2}{dx}\big|_{x=0}}{x}} + \underset{(\ln 1/\delta^2)^{-1}}{\frac{\frac{dR^2}{dx}\big|_{x=L}}{L-x}} = \underset{(1)}{2\sigma} . \quad (2)$$

Here δ is slenderness parameter with value $\delta = O(1/\lambda_*)$, λ_* - cavitator and cavity as whole aspect ratio. Below for each term of equations it's order for $\delta \to 0$ is indicated. In the case of small disc type cavitator with radius $R_n = \delta^2\sqrt{\ln 1/\delta}$ the cavitator action is described by pressure sources instead of integral term for case of slender cavitator. For development of asymptotic theory outer second order solution for largest middle part of the cavity for given cavity semi-length and $\sigma = \text{const}$ was found and defined second order dependence for cavity aspect ratio (2a) [21] which have been précised first order dependence (1d.) [11]. Further researches advancing [18, 20, 26, 33] given the possibility to define third terms in asymptotic (2a.) and, matched it with the outer solution [21], to find second order dependencies for cavity maximal radius R_k (2b) and cavity semi length L_k (2c.):

$$\text{a.}\quad \sigma = \frac{2\ln\lambda/m\sqrt{e}}{\lambda^2},$$

$$\text{b.}\quad R_k^2 = R_n^2\frac{c_d}{\sigma}\left[1 + 2\frac{\ln 2/\sqrt{e}}{\ln\lambda^2/m^2}\right], \quad (3)$$

$$\text{c.}\quad L_k = R_n\frac{\sqrt{c_d\ln\lambda^2/m^2}}{\sigma}\left[1 - \frac{\ln e/2}{\ln\lambda^2/m^2}\right],$$

which are applicable also for subsonic flow, $m = \sqrt{1 - M^2}$, M - Mach Number. Comparison accuracy of dependencies (3) with results of nonlinear numerical calculations [12] is given in in Tab.1.

\ σ	0.03	0.04	0.05
λ dependence (3a.)	11.33	9.291	7.924
λ numerical calculation [12]	11.46	9.453	8.132
R_k dependence (3b.)	5.537	4.834	4.357
R_k numerical calculation [12]	5.554	4.845	4.362
L_k dependence (3c.)	63.497	45.583	36.109
L_k numerical calculation [12]	63.518	45.8	35.473

Table 1: Comparison of calculations results on the base of linearized theory (3) as compared to nonlinear numerical calculations date [12]

Табл. 1 and also experimental date discover stricken good co-inciding theory with numerical and experimental date along very wide range of the cavity aspect ratios essential for applications. So these results are applied for further consideration as base for receiving more simple and convenient dependencies for practical calculations of supercavitation flows. The significations in essential part of paper are similar as in known book by G. Logvinovich [17].

STEADY AXISYMMETRICAL CAVITIES

Simplest flow model

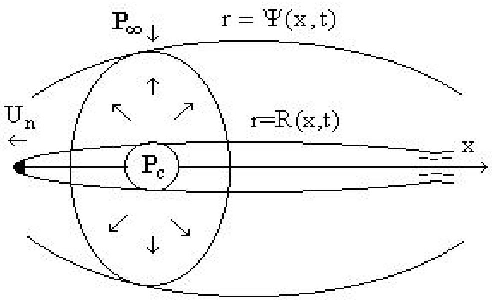

Figure 1: Radial flow model.

Creating a slender axisymmetric cavity can be explained with help of a simple model of radial flow, Fig. 1. In the case of prolate cavities the cavitator size is small and its drag is practically independent on the cavity form, additionally the cavity form is independent of the cavitator form which is defined by the cavitator drag only. The moving cavitator pushed the motionless fluid aside and its work is transformed into kinetic energy of mainly radial near cavity flow in the each motionless section which the cavitator has passed. In the main perturbed zone the main part of energy and impulse of flow is concentrated in finite region limited with a boundary $r = \psi(x,t)$ with extension of some more as compared to the semi-length of the cavitator and the cavity surface. This fact makes the cavity

alike as a wake of definite type. Further the expansion of the cavity section together with the radial flow near the cavity is controlled by inertia and the pressure difference in the undisturbed flow and inside the cavity. In doing so the expansion process depends weakly enough on the surface $r = \psi(x,t)$ form (x, t – axial coordinate, time) and the less the more slender is surface of cavitator and cavity as whole. The cavity section reaches the maximal radius in the middle part and further starts to decrease by the action of the external pressure. Shrinking of the cavity section is leads to an unstable closing regime with chaotic flow where the energy of radial flow is transformed into energy of the wake behind cavity.
Some elements of this simplest model on mention of G. Birkhoff (1971) [7] were offered by him in a coauthor with R. Isaacs as early as 50th. The completed type of this model [18, 22] is developed on the base of SBH and is one of the bases of practical methods for calculation of supercavitation.

Cavitators drag

Drag D of disc type cavitators: disc, blunted cone and i.e. with fixed section of separation practically depend not on fluid viscosity and are defined by known dependence:

$$\text{a)}\ \ D = D(\sigma) = c_d(\sigma)\pi R_n^2 \frac{\rho U_\infty^2}{2}, \qquad \text{b)}\ \ c_d = c_{do}(1+\sigma), \qquad \text{c.}\ \ c_d \sim c_{do} + \sigma, \tag{4}$$

where R_n cavitator radius in the section of separation, in case of disc: $c_{do} \approx 0.82 \div 0.83$. Dependence for drag coefficient in the case of cone for $c_{do}(\gamma)$ at $\sigma = 0$ where γ - cone semi-angle is illustrated by Fig. 2. Here the most hopeful date for $c_{do}(\gamma)$ applied on the base of date of nonlinear numerical date of numbers of authors and with account of also experimental date in particular presented in [17] and some different as compared to nonlinear numerical calculations.

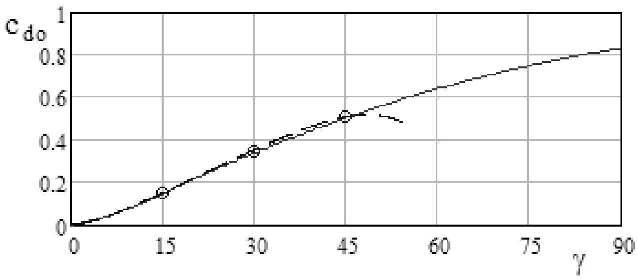

Figure 2: Dependence for cone drag coefficient $c_{do}(\gamma)$
——— nonlinear numerical calcullations [12,15]
∘∘∘∘ asymptotic approximation (5)

For small cone angles the preference is given to the date of specialized for this case of nonlinear numerical calculation [15] and also with account of date of linearized theory. For considerable angles including disc the preference is given to date of nonlinear numerical calculations [12] which are turned out to be near enough to date of specialized calculations for small cone angles. For estimation of c_{do} till semi-angles $2\gamma < 90^o$, $\varepsilon = \tan\gamma$ the asymptotic approximation can be used:

$$c_{do} \approx 2\varepsilon^2 \ln\left[\frac{3}{2e}\frac{(1+(4/3)\varepsilon)}{m\varepsilon}\right] \tag{5}$$

In the case of disc type cavitators dependence $c_d = c_d(\sigma)$ is described good enough by formula (4b). At present there are here more accurate approximate formulas for disc on the basic of nonlinear numerical calculation, however dependence (4b) is applicable enough for practical estimations. In the case of the slender cavitators the formula (4c) is valid in case of cavitator essentially more slender as compared to cavity only.

Disc cavitator is the most effective for applications from point of view of the motion stability. For deflection of the disc plane for angle β: $\beta = 90^o - \alpha$, α - attack angle normal component of drag coefficient on the base of experimental date of [17] is described good enough by dependence til $\sim\beta < 45^o$:

$$c_{don}(\beta) = c_{do}\cos\beta$$

Here α is attack angle, (without of disc deflection: $\alpha = 90^o$, $\beta = 0$). In doing so the components of longitudinal c_{dox} and lateral c_{doy} forces relay to motion direction are defined accordingly by dependencies:

$$c_{dox}(\beta) = c_{do}(\cos\beta)^2, \ c_{doy}(\beta) = c_{do}(\cos\beta)(\sin\beta). \tag{6}$$

In the case of slender cavitators, included slender cone, lateral force F_y and inductive drag F_x can be estimated by known aerodynamic dependence on the base of lateral added mass in the section of the flow separation:

$$F_y \approx 2\alpha S_n \frac{\rho U_\infty^2}{2}, \ \ F_x \approx \alpha^2 S_n \frac{\rho U_\infty^2}{2},$$

where S_n cavitation cavity square in the separation section, α cavitator attack angle.

Simplest solutions for the cavity form

The most simple form of slender axisymmetric cavity for $\sigma = \text{const}$ is known ellipsoidal one $\bar{R} = R/L_k$, $\bar{x} = x/L_k$:

$$\bar{R}^2 = (1/\lambda^2)\bar{x}(2-\bar{x}).$$

Developing of heuristic approach of G. Birkhoff and R. Isaacs [7] for prediction of steady cavities of constant pressure only gave the possibility to generalize this approach on the base of (SBH) for calculation of the most general case of unsteady cavities under changing on x, t pressure difference $\Delta P = (P_\infty - P_c) = \Delta P(x,t)$ between outer pressure and pressure on the cavity surface 1972 [21-24]. Elementary to the limit variant of these equations for steady cavity is:

$$\text{a)}\ \ \mu\frac{d^2R^2}{dx^2} + \sigma(x) = 0, \qquad \text{b)}\ \ \left.\frac{dR^2}{dx}\right|_{x=0} = 2\sqrt{\frac{D}{k\pi\mu\rho U_\infty^2}}, \quad \left.R^2\right|_{x=0} = 0 \tag{7}$$

Here in the first initial condition the energy conservation law is used for the energy which passed by cavitator into radial flow behind cavitator at initial moment. In doing so in the second conditions (7b) for cavitator sizes $R_n \sim O(\delta^2 \ln 1/\delta)$ is neglected for $\delta \to 0$, $\delta \sim 1/\lambda$ For $\sigma = \text{const}$ these equations define ellipsoidal cavity and known dependencies for it's sizes:

$$R^2 = R_n \sqrt{\frac{2c_d}{k\mu}} x - \frac{\sigma}{2\mu} x^2 \tag{8}$$

$$R_k = R_n \sqrt{\frac{c_d}{k\sigma}}, \quad L_k = R_n \frac{\sqrt{c_d 2\mu / k}}{\sigma}, \quad \lambda^2 = \frac{2\mu}{\sigma} \tag{9}$$

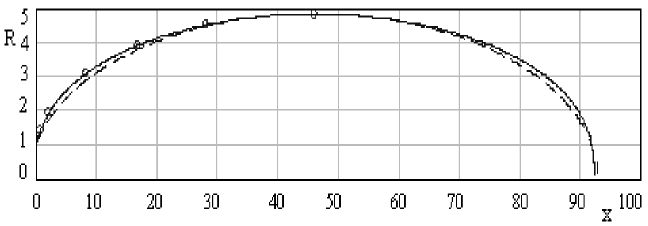

Figure 3: Ellipsoidal cavity form

-- -- -- dependence (14), ∘∘∘∘ numerical calculation [12]

The equations (7-9) can be used for calculations of the cavities of alternate pressure and in particular steady vertical cavities. The equations include 2 typical values which have clean physics. Key idea to obtain these equations system is following. We approximate cavity form by equation of first approximation of type (7a), but in doing so we find the coefficients μ, k as defined on the base of more accurate second order theory in particular on the base of integer- differential equations (IDE) for slender cavities. Initial base for defining μ value is dependence (2a, 10a). Value of μ characterizes inertial properties of the cavity sections and is alike as definite inertial coefficient similar added mass value and defined by dependencies (10):

$$\text{a)}\ \ \mu = \ln \frac{\lambda}{\sqrt{me}}$$

$$\text{b)}\ \ \mu \simeq \ln \sqrt{\frac{\ln 2 / m^2\sigma}{em^2\sigma}} \Bigg|_{\sigma \sim 0.04 \div 0.02} \approx 0.5 \ln \frac{1.5}{m^2\sigma}, \tag{10}$$

$$\mu\big|_{\sigma \sim 0.04 \div 0.02} \sim 1.8 \div 2.2$$

Initial base for defining of k value is dependence (2b). Howe ever notes that experiments [8, 17] discover not high increase of k $\sim$ 2-3% as compared to theory on the base of ideal fluid due to viscose loss.

$$k = 1 - \frac{2 \ln 2 / \sqrt{e}}{\ln 4 / m^2\sigma} \Bigg|_{\sigma \sim 0.04 \div 0.02} \sim 0.92 - 0.93 \tag{11}$$

Dependencies (10, 11) (especially the first of its) give the possibility to obtain the solutions with very accurate significances of main cavity sizes in the case of prolate cavities which are close enough to steady cavity for $\sigma = \text{const}$. They are applicable in very wide range including even case of super slender cavities under super high speeds in water compared with sonic speed. It's more rough dependencies and significance correspond to range of $\sigma \sim 0.04 - 0.02$. In the case of cavities of alternate pressure which are not strong differenced of $\sigma = \text{const}$ this values μ and k can be applied as for $\sigma = \text{const}$. Under considerable difference the values μ and in less measure k should be found on the base of (IDE).

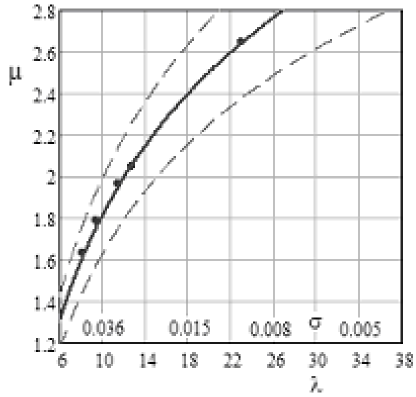

Figure 4 : Dependence of inertial coefficient $\mu(\lambda)$, $\mu(\sigma)$

——— Eq. (10), -- -- -- values of 5% deflections for μ,

• • • • - numerical calculations $\sigma\lambda^2/2$ [12]

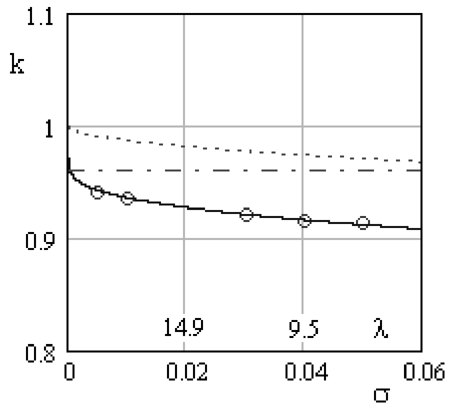

Figure 5: Dependence for coefficient $k = k(\sigma)$

——— Eq. (11), - - - - - H. Reichardt dependence,

∘∘∘∘ - numerical calculations [12]

The calculations results of the dependencies (10, 11) for μ and k values as compared to nonlinear numerical calculation date [12] are illustrated by Figs. 4, 5.

Equation (7) for $\sigma = \text{const}$ have clear energetic physics expressed law of full energy kinetic E_{kx} and potential E_{px} conservation in the each section which passed by cavitator at the initial moment.

$$E_{kx} + E_{px} = \pi \frac{\mu\rho}{4} U_\infty^2 \left(\frac{dR^2}{dx} \right)^2 + \pi R^2 \Delta P = D / k \tag{12}$$

$k \setminus \sigma$	0.01	0.02	0.025	0.03	0.04
Dependence (11)	0.936	0.927	0.924	0.921	0.916
Numerical calculation [12]	0.936	-	-	0.920	0.916
Approximate dependence [12]	0.96	0.942	0.936	0.931	0.924
H. Reichardt [12]	0.987	0.981	0.979	0.977	0.974
G. Logvinovich [17]	$0.96 \div 1$				

Table 2: Comparison values k with dates of different authors

$\lambda \setminus \sigma$	0.01	0.02	0.025	0.03	0.04	0.0002 $M = 0.7$
Numerical calculation [12]	23.00	-	12.82	11.46	9.45	-
Dependence (17a)	22.69	14.90	12.91	11.46	9.47	220.65
Dependence (17b)	22.38	14.69	12.80	11.55	9.52	211.22
P. Garabedian (1c)	21.46	13.99	12.15	10.81	8.97	206.36
Dependence (17c)	20.00	14.14	12.65	11.55	10	141.42
H. Reichardt [12]	21.69	14.00	12.17	10.83	8.96	618.02
G. Logvinovich [17]	20.52	14.21	12.58	11.34	9.63	148.07
L. Epshtain [8]	16.58	11.67	10.41	9.48	8.17	117.84

Table 3: Comparison values λ with dates of different authors

Tables 2, 3 illustrate results of comparison of key for prediction of the steady cavities values λ, k of different authors.

Elementary variant of equations (7) is the most convenient for theoretical consideration giving accurate enough significances for main cavity sizes ant it's volume. More accurate equations system roughly applicable also near disc is:

$$\mu_c \frac{d^2R^2}{dx^2} + \sigma = 0, \quad \left.\frac{dR^2}{dx}\right|_{x=0} = R_n\sqrt{\frac{2(c_d - k\sigma)}{k\mu_c}}, \quad R^2\big|_{x=0} = R_n^2, \tag{13}$$

where value $\mu_c = \eta\mu$ some precise solutions with account finiteness of back closer. For $\sigma = \text{const}$ we have here also ellipsoidal cavity, but starting from cavitator of finite sizes.

$$R^2 = R_n^2 + R_n\sqrt{\frac{2(c_d - k\sigma)}{k\mu_c}}x - \frac{\sigma}{2\mu_c}x^2 \tag{14}$$

Calculation results Eq. (14) on the base of Eq. (13) are illustrated in Fig. 3 as compared to nonlinear numerical date [12].

Steady axisymmetric cavity main sizes

Systems of very accurate dependencies for maximal cavity radius R_k, cavity semi length L_k and it'aspect ratio λ for steady $\sigma = \text{const}$ cavity on the base of approximations of second order solutions of linearized theory together with approximation of this values for the range of $\sigma \sim 0.04 \div 0.02$ are:

$$R_k = R_n\sqrt{\frac{c_d}{k\sigma}}, \quad k = 1 - \frac{2\ln 2/\sqrt{e}}{\ln 4/m^2\sigma} \sim 0.92 - 0.93 \tag{15}$$

$$\text{a)}\ L_k = \frac{R_n}{\sigma}\sqrt{\frac{c_d}{k}\ln\frac{\ln(2/m^2\sigma)}{em^2\sigma}}$$
$$\text{b)}\ L_k \approx \frac{R_n}{\sigma}\sqrt{\frac{c_d}{k}\ln\frac{1.5}{m^2\sigma}}, \quad \text{c)}\ L_k \sim \frac{2R_n}{\sigma} \tag{16}$$

$$\text{a)}\ \lambda = \frac{1}{\sqrt{\sigma}}\sqrt{\ln\frac{\ln(2/m^2\sigma)}{em^2\sigma}},$$
$$\text{b)}\ \lambda \approx \frac{1}{\sqrt{\sigma}}\sqrt{\ln\frac{1.5}{m^2\sigma}} \qquad \text{c)}\ \lambda \sim \frac{2}{\sqrt{\sigma}} \tag{17}$$

Here for $M < 0.7 - 0.8$, $m^2 = 1 - M^2$, M is Mach Number.

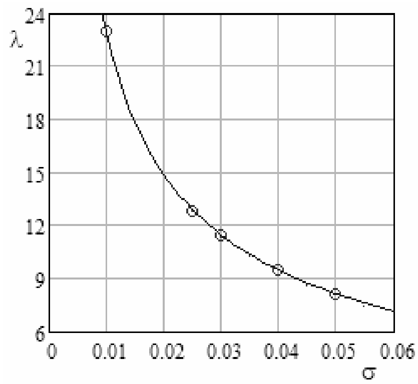

Figure 6: Dependence for cavity aspect ratio $\lambda = \lambda(\sigma)$

——— Eq. (18a), ∘∘∘∘ numerical calculations [12].

Fig. 6 illustrates calculation results for aspect ratio for $\sigma = \text{const}$ cavity as compared of nonlinear numerical date [12}. With account of excellent coinciding date of asymptotic solutions for slender cavities with date of nonlinear numerical calculations and experimental date presented dependencies can be considered at present as the most hopeful ones for very wide range of σ. The form of stable cavity form usually is good enough described as fixed one for more ~ 2/3 of its length.

Nonlinear approximation of steady axisymmetric cavity

It is possible to note as it can see on Fig. 3 that ellipsoidal approximation rough enough describes the cavity near to type disc cavitator. However more important here is the fact that real cavity far enough from cavitator is in considerable more long region near to known asymptotic M. Gurevich, N. Levinson (1d) which is essentially different of paraboloidal cavity form for ellipsoidal approximation of the cavity. Approach of linearized theory give the possibility to construct accurate enough asymptotic approximation of the form of steady for $\sigma = \text{const}$ cavity behind disc $R_n = 1$:

$$R^2 = R_n^2 + \frac{L_k^2}{\lambda_c^2}\sqrt{\frac{\mu_m}{\mu_{xx}}}\frac{x}{L_k}\left(2-\frac{x}{L_k}\right), \quad \lambda_c^2 = \eta\lambda_k^2,$$

$$\mu_{xx} = 0.5\ln\left\{\frac{(x+\sqrt{e}\Delta)^2\left[\left(2+\sqrt{e}\frac{\Delta}{L_k}\right)-\frac{x}{L_k}\right]^2}{em^2\left[R_n^2+\frac{L_k^2}{\lambda_c^2}\sqrt{\frac{\mu}{\mu_*}}\frac{x}{L_k}\left(2-\frac{x}{L_k}\right)\right]\left(1+\sqrt{e}\frac{\Delta}{L_k}\right)^4}\right\}$$

$$\Delta = 0.5R_n\left(\sqrt{c_d}+\frac{1}{\sqrt{c_d}}\right), \quad \mu_m = \mu_x|_{x=L_k}. \qquad (18)$$

For: $\sigma \sim 0.01 \div 0.05$: $\mu_* \sim 2$

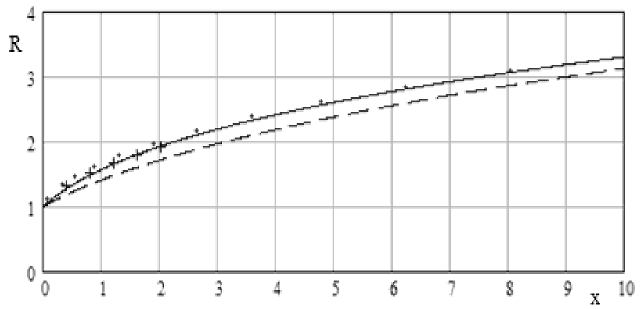

Figure 7: Nonlinear approximation of the cavity form $\sigma = 0.04$

– – – – ellipsoidal cavity form,

——— nonlinear approximation Eq. (18),

+ + + + G. Logvinovich experimental date [17],

• • • • nonlinear numerical calculation [12]

Approximation (18) give good enough results start from $\sigma \sim 0.05$ and till range of super slender cavities for subsonic speeds $M < 0.7 - 0.8$. Fig. 7 illustrates calculation results on the base of approach (18) as compared to ellipsoidal approximation and non linear numerical prediction results [12].

UNSTEADY CAVITIES PREDICTION: *"PRNCIPLE OF INDEPENDENCE OF CAVITY EXPANSION"*

For prediction of unsteady cavities application of the coordinate system r, x, t (t-time) connected with motionless fluid is the most effective. One of the most important properties of axisymmetric cavitation flows in the case of slender cavities is essential independence of expansion of the cavity section depend on expansion of neighboring cavity sections and also cavitator and cavity form as whole. This property was discovered in experiments long ago by H. Reichardt what gave the possibility for G.Logvinovich to formulate this fact with help known "*Principle of independence of the cavity expansion":* *"Each lateral section of the cavity is expanded relay to trajectory of the body center near independently on the body motion before and after passing of this section along definite law which depend on pressure difference at infinity and cavity, speed, sizes and drag of body only at the moment of passing by it the plane of the section under consideration. This formulation is approximated and can not be proved rigorously but is acted the more accurate the more near body motion to the direct line and with const speed."*

On the first steps of caviitation researches the property of cavity expansion independence was applied on the base of this principle along following way. Defined by some or another way form of steady for $\sigma = \text{const}$ cavity is presented in the motionless coordinate system and expansion of the sections of unsteady cavity is defined on the base of the dependence for this steady cavity depend on date for initial moment of passing by cavitator of this section. This way conserve some of its value and even till now day.

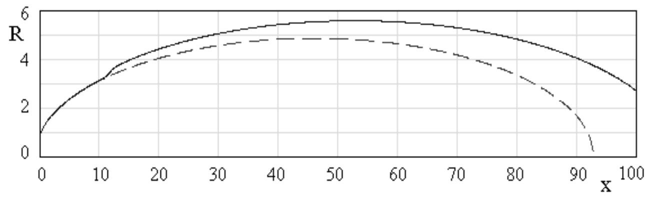

Figure. 8: Cavity under step increase of the cavitator size

– – – – undisturbed cavity under $\sigma_o = 0.04$

——— cavity under step type decrease of R_n

However this way gave not the possibility to calculate the cavities in the important for applications case of cavities under alternate pressure difference. Generalization of the steady system (13) for prediction of wide range of unsteady steady cavities with changing pressure difference is:

$$\mu(x)\frac{\partial^2 R^2}{\partial t^2} + \frac{2\Delta P(x,t)}{\rho} = 0,$$

$$\left.\frac{\partial R^2}{\partial t}\right|_{t=t_n(x)} = R_n(x)U(x)\sqrt{\frac{2[c_d(x)-k(x)\sigma(x)]}{k(x)\mu(x)}}$$

$$\left.R^2\right|_{t=t_n(x)} = R_n^2(x) \qquad (19)$$

By the first primitive way the number of cavitation flows in particular for enter into water and motion with acceleration was calculated by H. Abelson, 1970 [1], Yu. Zhuravlev, 1970, [34], A. Boldurev 1973 and other ones. All these solutions can be found also on the base of equations (19) too as some special cases. The system of equations (19) what was obtained on the base of lineariszd theory in (SBH) approximation is really as modern presentation of the "*Principle of independence of the cavity expansion"* and has very clear physics. Cavitator for general case with alternate size $R_n(t)$ and drag coefficient $c_d(t)$ moving in motionless fluid with alternate speed on law $x = x_n(t)$ at the moment of time $t = t_n(x)$ passes motionless coordinate x. In doing so the cavitator create initial section of the cavity and energy of his resistance is transformed with some correction $k(x)$ to the radial flow at initial section cavity behind cavitator. After that the process of independent expansion of cavity section under pressure difference $\Delta P(x,t)$ without action of neighboring sections is started. All initial values in the system (19) are applied for the initial moment when cavitator passed given motionless section of fluid :

$$R_n(x) = R_n(t)\big|_{t=t_n(x)},\ c_d(x) = c_d(t)\big|_{t=t_n(x)},\ \sigma(x) = \sigma(x,t)\big|_{t=t_n(x)},$$

where function $t = t_n(x)$ is defined as reversed to function $x = x_n(t)$. For $c_d(x)$ its quasi steady significances are used. The solution of the system (19) in general case $R_n(t)$, $c_d(t)$, $U(t)$, $\Delta P(x,t)$ is defined by integral:

$$R^2 = R_n^2(x) + 2R_n(x)U(x)\sqrt{\frac{c_d(x) - k(x)\sigma(x)}{2k(x)\mu(x)}}\left[t - t_n(x)\right] - \\ -\frac{2}{\rho\mu(x)}\int\limits_{t_n(x)}^{t}\int\limits_{t_n(x)}^{t}\Delta P(x,t)dtdt \tag{20}$$

In particular case $\Delta P = \Delta P(x)$ universal dependence is:

$$R^2 = R_n^2(x) + 2R_n(x)U(x)\sqrt{\frac{c_d(x) - k(x)\sigma(x)}{2k(x)\mu(x)}}\left[t - t_n(x)\right] - \\ -\frac{\Delta P(x)}{\rho\mu(x)}\left[t - t_n(x)\right]^2 \tag{21}$$

and in particular for $\mu = 2$ express solution for the case presented in Fig. 11. This integral for constant cavitator speed and pressure difference define ellipsoidal cavity and includes really all obtained earlier by traditional way simplest solutions on the base of principle of independence. With account of week change of the values of μ , k simplest variant for estimation of cavities, which are not very strongly different as compared ordinary steady cavity, is to apply this values in the range of $\sigma \sim 0.04 \div 0.02$ as universal constants $\mu \sim 2$, k~0.93-0.96 .

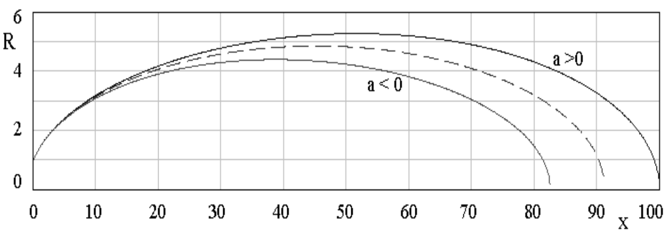

Figure 9: Cavity under acceleration - desecration of speed

---- undisturbed cavity under $\sigma_o = 0.04$

——— $a > 0$ - acceleration , $a < 0$ - deceleration

More accurate solutions are obtained for using of this values as functions $\mu(x)$, $k(x)$ defining characterize significances of this values for each expanding section which the most easy to define on the base of steady solutions. Application of $\mu(x)$, $k(x)$ is some averaging of this values along time for each cavity section what can limit applicability of this solution by conditions of the type $U_* T_* / L_* = O(1)$ for typical values of speed, cavity length, time length of this process. More precisely construction of the values for $\mu(x,t), k(x,t)$ provided more accurate solutions we can provide if will calculate these values with some delay defined on the base of dependence for semi-length of steady cavity L_K . Here dependence for steady $\sigma = \text{const}$ cavity (16) can be used calculated on the base of values for considered section as: $\mu([x - L_k(x)], t)$, $k[(x - L_k), t]$. Here calculation until $x < L_k$ started on the base of values without delay and for $x > L_k$ delay correction is included. For very strong differences cavity form of steady $\sigma = \text{const}$ cavity for example in the case of vertical cavity under gravity with sharp end the values μ, k should be found separately on the base second order equations. One of the way to precise μ here is prediction of this value on the base of steady dependency but with more close to reality cavity typical aspect ratio of the cavity values.

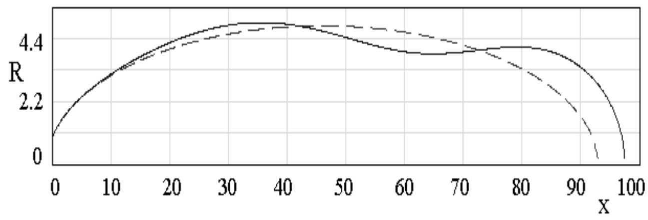

Figure: 10: Cavity - harmonic oscillations of pressure

---- undisturbed cavity $\sigma_o = 0.04$

——— cavity for harmonic oscillations of pressure on time

Solutions (20-21) give the possibility to analyze typical cavity form for different cases of flow. Below for calculations of the solutions the values of μ, k for first approximation are accepted as for $\sigma = \sigma_o \sim 0.04$. Cavity form is presented finally in the coordinate system connected with cavitator.

Figure 11: Cavity for water entry, experimental date [34] Points corresponds calculation by Eg. (19, 21) for $\mu \sim 2$

Step change of $R_n(t)$, $c_d(t)$, and cavitator speed $U_n(t)$ under const pressure difference in particular close to step type change found similar typical disturbances of the cavity form of local type. Fig. 8 illustrates change of the cavity form in the case of step change of cavitator size. Acceleration action is calculated for the case when accelerated and decelerated motion is started from the same speed under not changing of other parameters.

Fig. 9 illustrates the cavity form at the moment when cavitator reach distance which equal of the length of undisturbed cavity at initial moment.

Analogously on the base of equations (21) the cavity under alternate pressure difference are estimated. Fig. 10 illustrates action of pressure difference corresponding harmonic oscillations of the pressure in the cavity. Like this forms are observed in particular for the case of cavity oscillations in known theory by E. Paryshev [19] of ventilated axisymmetric cavities oscillations based on equation system (19). Accuracy of prediction of unsteady cavity form is illustrated by Fig.11 with date of experimental study performed by Yu. Zhuravlev [34] in the case f vertical enter body into water. Calculated points corresponds estimation on the base of system (19) – universal solution (21) for $\mu \sim 2$.

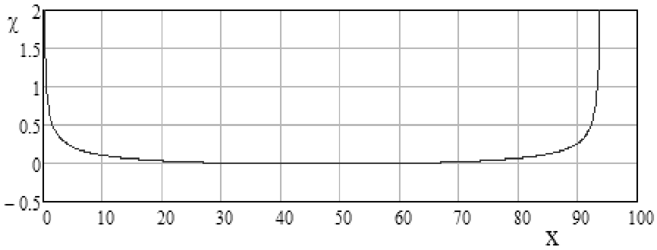

Figure 12: Correction coefficient $\chi = \chi(x)$ distribution in case of steady cavity for $\sigma = 0.04$

Linearized theory gives the possibility correct solutions (20, 21) near cavitator zone in the case of blunt disc type cavitators. This correction in general case can be made for final step of solutions on the base of Eq . (20) with help précising.

Steady case: It is supposed disc with radius $R_n = 1$, coordinate system is connected with cavitator, R_F is corrected solution applied also near cavitator zone:

$$R_F^2(x) = 1 + \chi(x)\left[R^2(x) - 1\right], \quad \chi(x) = \frac{\sqrt{\mu} - \sqrt{\mu_x(x)}}{\sqrt{\mu\mu_x(x)}}, \tag{22}$$

$$\mu_x = 0.5\ln\left[1 + x(2 - x/(L_* + 1)\right]$$

Correction result of simplest solution in the form of ellipsoidal cavity (14) which was received on the base of dependencies (22) is presented in Fig. 3 by solid line and practically coincides with date of nonlinear numerical calculations.

Unsteady case, coordinates system is connected with motionless fluid; corrected solution R_F is defined by dependencies:

$$R_F^2(x,t) = R_n^2(x) + \chi(x,t)\left[R^2(x,t) - R_n^2(x)\right]$$

$$\chi(x,t) = \frac{\sqrt{\mu_x(x)} - \sqrt{\mu_{xt}(x,t)}}{\sqrt{\mu_x(x)\mu_{xt}(x,t)}} \tag{23}$$

$$T_*(x) = [L_c(x) + R_n(x)]/U_n(x)$$

$$\mu(x,t) = 0.5\ln\left\{1 + \frac{U_n(x)}{R_n(x)}[t - t_n(x)]\left[2 - \frac{[t - t_n(x)]U_n(x)/R_n(x)}{1 + T_*(x)U_n(x)/R_n(x)}\right]\right\}$$

where L_*, T_* the cavity semi-length and semi-term until section cavity with radius what is equal nose cavitator radius. For steady $\sigma = \text{const}$ $L_* = L_k$, $T_* = T_k = L_k / U_\infty$. Correction dependencies (21, 23) are essential for not large forward and back parts of cavity only, so these corrections are applicable for very wide range of different cavities what is used in applications. Fig. 12 illustrates values of correction coefficient $\chi = \chi(x)$ for case of steady cavity for $\sigma = 0.04$ Dependencies (22-23) can be used for correction in the cases when nose radius are not the same value as cavity back closer. For such case correction dependencies are applied separately for forward and back part of the cavity. The radius of equivalent disk of cavity closer can be found on the base of value of $\frac{\partial R^2}{\partial t}$ at the back of the cavity.

3-D PERTURBATIONS OF AXISYMMETRIC CAVITIES

The most interesting here is possibility to estimate action of lateral gravity and attack angle of cavitator on cavity form. The main part of deformations here is lift of the cavity axis $h = h(x)$. Approach for estimation of these deformations was proposed by G. Logvinovich 1969, [17]. It is supposed that cavity section is near to circle and impulse theorem is used for laterally moving sections of the cavity. Dependence for cavity axis lift under action of lateral gravity for steady flow in the system coordinates connected with cavitator is:

$$h_g = \frac{g}{\pi U_0^2}\int_0^x\left(\frac{1}{R_0^2(x)}\int_0^x R_0^2(x)dx\right)dx \tag{24}$$

Elementary solution for $\sigma = \text{const}$ on the base of undisturbed ellipsoidal cavity $R = R_0(x)$ is:

$$\text{a) } h_g = \frac{g L_k^2}{U_0^2}\frac{1}{3}\left[\frac{\bar{x}^2}{2} - \bar{x} - 2\ln\left(1 - \frac{\bar{x}}{2}\right)\right], \text{ b) } h_g \approx \frac{g L_k^2}{U_0^2}\frac{\bar{x}^2}{3}. \tag{25}$$

Here g is gravity. Dependence (25b) proposed by Yu. Zhuravlev, 1972, [34] can be defined by Tailor expansion on the base of dependence (25a).

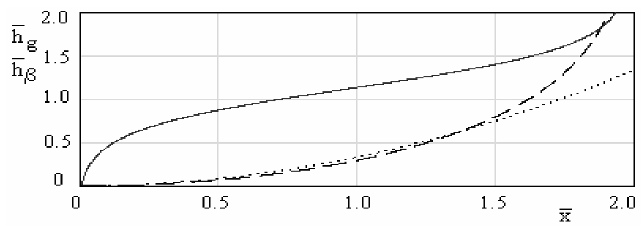

Figure 13: Axis lift of cavity under gravity and cavitator attack angle actions

---- $\bar{h}_g$ - dependence (26a),

------ $\bar{h}_g$ - dependence (26b)

——— $\bar{h}_\beta$ - estimation on the basic of undisturbed ellipsoidal cavity form

Analogously dependence for axis curvatures under cavitator attack angles is obtained:

$$h_\beta = -\frac{D_y}{\pi\rho U_o^2}\int_0^x \frac{dx}{R_o^2(x)}. \qquad (26)$$

However application here R_o on the base of ellipsoidal form gives more rough results so main effects here as distinguished from gravity case are concentrated namely in the nonlinear zone near cavitator. For more corrected prediction here dependencies for more accurate cavity form (18) can be used. Fig. 13 illustrates action of the gravity on cavity axis h_g, action of deflection of the cavitator plane on cavity axis h_β (on the base of ellipsoidal form of undisturbed cavity) in the form of the dependencies:

$$\bar{h}_g = \frac{h_g}{L_k}\frac{U_o^2}{gL_k}, \quad \bar{h}_\beta = \frac{h}{L_k}\frac{c_{dx}}{\sigma c_{dy}}, \quad \bar{x} = x / L_k \qquad (27)$$

Dependency (25) indicates lift increasing for decreasing of the Froude Number relay to cavity semi-length. Axis curvatures under action of the deflection of the cavitator are near linear dependence on lateral force coefficient and cavitator size. Dependencies of type (25-26) analogously can be written in the coordinate system connected with motionless fluid and can be applied for prediction of this type deformations of unsteady cavities too.

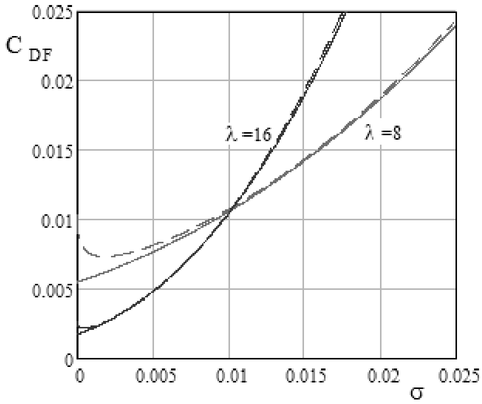

Figure 14: Cavitation drag coefficient $C_{Df}(\lambda,\sigma)$ Eqs. (29, 29a) depend on given λ_f, σ, for $\lambda_f = 8,\ 16$, $\kappa_p = 1$

——— - dependencies (29)

– – – – – - dependence (29a) for $\mu_x = \mu$

CAVITATION DRAG

More informative the drag coefficients per definite cavity section are. This is drag coefficient for forward part of the cavity C_{D0} under $\sigma = 0$ for motion in the forward part of the cavity and drag coefficient per middle cavity section C_D :

$$C_{D0} = \frac{1}{8}\frac{\ln(4\lambda_f / \sqrt{e})}{\lambda_f^2}, \quad C_D = k\sigma = \frac{2}{\lambda^2}\ln\frac{\lambda}{2}, \qquad (28)$$

More universal is dependence for C_{DF} per interstitial forward part of cavity until maximal body section touched with cavity which have aspect ratio λ_f :

$$\begin{aligned} &D = C_{DF}\pi R_N^2 \frac{\rho U_\infty^2}{2}, \\ &C_{DF} = k\mu_x \frac{[1+2\lambda_f^2\sigma/\xi\mu]^2}{8\lambda_f^2}, \quad \xi = \sqrt{\frac{\mu_x}{\mu}}, \\ &\mu_x = 0.5\ln\left\{\frac{16}{e}(\lambda_f^2+1)\left[1-\frac{2\lambda_f^2\sigma/\mu}{1+2\lambda_f^2\sigma/\mu}\right]^2\right\}, \end{aligned} \qquad (29)$$

where R_N - cavity radius in the touched by cavity body maximal section, λ_f aspect ratio of this forward part of cavity, $e \sim 2.72....$ For $\mu_x = \mu,\ \xi = 1$ C_{DF} expression [10] is defined on the base of ellipsoidal cavity form. Simplest variant of Eq. (29) defined on the basis of ellipsoid cavity form is:

$$C_{DF} = \frac{k\mu}{8}\frac{[1+2\lambda_f^2\sigma/\mu]^2}{\lambda_f^2} \qquad (29a)$$

Here λ_f is aspect ratio of part of body to his maximal section, rounded a cavity. For rough understanding of sizes the values $k \sim 0.93-0.95$, $\mu \sim \ln 0.7\lambda \sim 0.5\ln 1/\sigma \sim 2.2$, are suitable. More general is presentation of body drag D and drag of unit of his volume D/V_b by dependencies:

$$\begin{aligned} &D = C_{VF}V_b^{2/3}\frac{\rho U_\infty^2}{2}, \quad \frac{D}{V_b} = \frac{C_{VF}}{\sqrt[3]{V_b}}\frac{\rho U_\infty^2}{2}, \\ &C_{VF} = \sqrt[3]{\frac{\pi}{\kappa_p^2}}\frac{k\mu}{8}\frac{[1+2\lambda_f^2\sigma/\mu]^2}{(\lambda_f^2)^{4/3}}, \end{aligned} \qquad (30)$$

where C_{VF} - coefficient of volume, κ_p - relation volume of body to the volume of the paraboloid to be entered in a cavity . These expressions are received on the base of simplest variant of dependencies (29a) on the base of ellipsoidal cavity form for paraboloidal form of body in cavity.

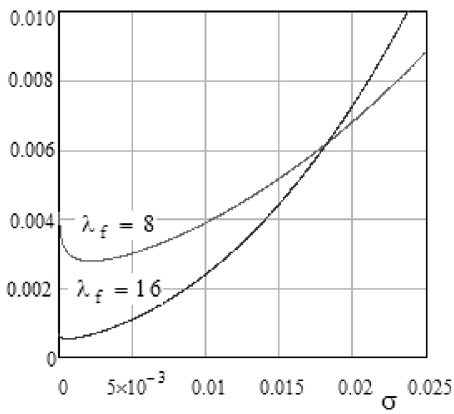

Figure 15: Coefficient of volume $C_{Vf}(\lambda,\sigma)$ Eq. (31) depend on given λ_f σ, for $\lambda_f = 8,\ 16$, $\kappa_p = 1$

Figures 14, and 15 illustrate values of cavitational drag coefficients C_{Df} and coefficient of volume C_{Vf} depend on cavitation number under given body aspect ratios $\lambda_f = 8,\ 16$. Using ellipsoidal cavity form for receiving of these dependencies make them not applicable close to $\sigma \sim 0$ .Most typical values of volume coefficients at motion in front area of cavity $\sigma \to 0$ and at the dense inscribing in a cavity are:

$$C_{V0} = \sqrt[3]{\frac{\pi}{\kappa_p^2}\frac{k}{8}\frac{\ln 4\lambda_f/\sqrt{e}}{\lambda_f^2}} \approx \sqrt[3]{\frac{\pi}{\kappa_p^2}\frac{k}{8}\frac{\mu_*}{\lambda_f^2}},$$

$$C_V = \frac{3}{2}\sqrt[3]{\frac{4\pi}{3\kappa_c^2}\frac{\ln\lambda/2}{(\lambda^2)^{4/3}}} = \frac{3}{4}\sqrt[3]{\frac{2\pi}{3\kappa_c^2\mu}}k\sigma^{4/3},$$

where: κ_p - relation of real body motion per inserted in the forward part of cavity parabolic form, κ_c - relation of real body volume per finite cavity volume, $\mu = \mu_*$ some typical values for range of σ under consideration, usually $\mu_* \sim 2.2$ is used practical estimations.

MAIN WAYS OF DRAG REDUCTION FOR MOTION WITH SUPERCAVITATION

There are 2 typical cases of body motion in cavity.

It can be motion of body maximally closely inserted in cavity. For this case body λ_f and cavity λ aspect ratios are near to be coinciding. The larger is aspect ratio – the less is cavitation drag coefficient C_D and coefficient of volume C_V. Under given body aspect ratio λ_f what can not be too large over $\lambda_f \sim 15-17$ from the point of view of strength restrictions considerable less value of the drag coefficients C_{DF} coefficients and coefficients of volume C_{VF} can be reached for body motion in the forward part of cavity. In doing so cavity aspect ratio λ can be considerable larger as compared to body aspect ratio λ_f . As it follow from Fig. 14, 15 the values of both coefficients C_{DF}, C_{VF} are the less - the larger aspect ratios of body and cavity are. In doing so minimal energy expenses per unit of distance for motion in cavity under given pressure difference are realized for motion of body in ahead half of the cavity for condition $\lambda_f / \lambda = 0.5$. In both cases considerable drug reduction can be arrived at the considerable increase of motion speed or gas injection. In doing so gas loss very quickly grows at the increase of the cavity aspect ratio and hydrostatic pressure. Physically energy of cavitation resistance is lost in dulled end of cavity where a loss of stability of flow is with formation of chaotic flow of liquid of transforming energy cavitation flow in energy of wake after a cavity.

One of basic ways of diminishing of resistance at motion of body densely entered in a cavity there is the use of the proper closers in the back-end of body. In an ideal model the law of Dalamber - Euler takes a place during complete indemnification of resistance. However in reality even partial realization of a compensate force is related to considerable difficulties in connection with instability of flow in the blunted back-end of cavity.

At the changing along cavity length number of cavitation, in particular in the case of vertical cavities, the possibility of existence of cavities with sharpening in a back point are defined by dependence 31a [36] . Fig. 16 illustrates vertical cavity form with sharpening at the cavity end as compared to form of ordinary $\sigma = \text{const}$ cavity.

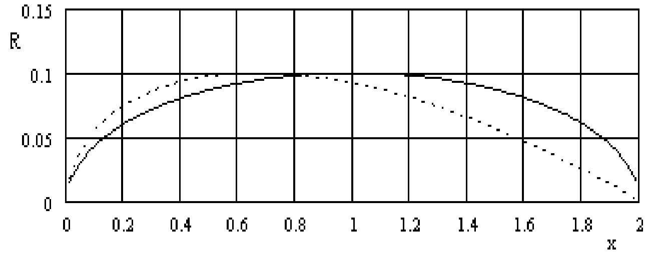

Figure 16: Vertical cavity with sharp end – dash as compared to ordinary cavity – solid

From one side the sharpening of back-end of cavity diminishes possibility of origin of instability in this area. From other side dependence (32a) defined conditions of sharp end existence means possibility of the cavity form without the losses of energy in wake. It was found out analogical possibility also in the case of ordinary cavities with a partition, separating the back-end of cavity with the more high pressure $\sigma_2 < 0$ as compared to the forehand of cavity $\sigma_1 > 0$ [37]. The solutions in both cases of vertical cavity and cavity of part by a partition with different pressures in each of parts easily are defined on the basis of equations (7). Conditions of existence of points of sharpening in the back-ends of vertical cavity Eq. (31a) and in the case of more high pressure in the back section of ordinary cavity (31b) are defined by dependencies:

$$\text{a)}\ \sigma Fr_L^2 = 4/3, \qquad \text{b)}\ \sigma_2 = -\frac{\sigma_1}{2}\frac{(\bar{L}_1 - 1)^2}{\bar{L}_1(1-\bar{L}_1/2)}, \qquad (31)$$

where: $Fr_L = U_\infty / \sqrt{gL}$, $\bar{L}_1 = L_1 / L_k$ define distance until section where second cavity part under $\sigma_2 < 0$ is started, $L = 2L_k$ - cavity length. More general is conditions for the cavity back part what provides wake with zero kinetic energy both behind some body or first section of the cavity . In these cases back cavity is closed on cylinder with given radius r_c. For $r_c = 0$ dependence (32b) is transformed into Eq. (31b) corresponds to point closer. Here γ is cavity angle at the back initial section with radius R_s.

$$\text{a)}\ \sigma_2 = -\frac{2\mu(\tan\gamma)^2}{(1 - r_c/R_s)},$$
$$\text{b)}\ \sigma_2 = -\frac{\sigma_1}{2}\frac{(\bar{L}_1 - 1)^2}{\bar{L}_1(1-\bar{L}_1/2) - r_c^2/R_k^2} \qquad (32)$$

It is need to note the closer of the forward cavity part with help of dividing of the cavity into two parts with more high pressure in the back part in the case of sharp closure defines surface form what is very close to the cone surface. Fo cavity smooth closing on some cylinder this surface is essentially different as compared to cone. The μ values for back cavity part with more high pressure can be considerable different as compared its value for forward cavity part. So here second order asymptotic solutions for the back cavity form were found. The solution is looking for on the base of IDE (2) in the form of asymptotic expansions:

$$R^2 = \delta^2\left(R_o^2 + \frac{R_1^2}{\ln 1/\delta^2}\right), \ \sigma = \delta^2 \ln\frac{1}{\delta^2}\left(\sigma_o + \frac{\sigma_1}{\ln 1/\delta^2}\right),$$

where cavity length is supposed as given $L_c = 1$, for $\delta = R_s / L_c$ and initial and boundary conditions:

$$\left.\frac{dR^2}{dx}\right|_{x=1} = 0\,,\ \left.R^2\right|_{x=0} = 1\,,\ \left.R^2\right|_{x=1} = r_c / R_s\,,$$

In doing so asymptotic solution is transformed to the series of problems. First order problem is:

$$\frac{d^2R_o^2}{dx^2} = 2\sigma_o\,,\ \left.\frac{dR_o^2}{dx}\right|_{x=1} = 0\,,\ \left.R_o^2\right|_{x=0} = 1\,,\ \left.R_o^2\right|_{x=1} = \left(\frac{r_c}{R_s}\right)^2.$$

Second order problem:

$$\frac{d^2R_1^2}{dx^2} = \frac{1}{2R_o^2}\left(\frac{dR_o^2}{dx}\right)^2 + \frac{d^2R_o^2}{dx^2}\ln\frac{R_o^2}{4x(1-x)} + 2\sigma_1 -$$

$$-\int_{-Lo}^{0}\frac{\left.\frac{d^2r^2}{dx^2}\right|_{x=x_1} - \frac{d^2R^2}{dx^2}}{|x_1 - x|}dx_1 - \frac{\left.\frac{dr^2}{dx}\right|_{x=-Lo}}{L_o + x}\,,$$

$$\left.\frac{dR_1^2}{dx}\right|_{x=1} = 0\,,\ \left.R_o^2\right|_{x=0} = 0\,,\ \left.R_1^2\right|_{x=1} = 0\,.$$

Here $r = r(x)$ is form of body or the forward cavity, L_o - body or forward cavity part length until section with radius R_s.

After each problems solution the asymptotic solution of the equation for defining of δ is required on the base of equation:

$$\left.\frac{dR^2}{dx}\right|_{x=0} = 2R_s \mathrm{tg}\gamma$$

The second order solution of the problem in the case of sharpening point closure and neglected body influence is:

$$R^2 = \delta^2\Big\{(1-x^2) +$$

$$+\left[(1-x^2)\left(\frac{1}{4}+\frac{1}{2}\ln(1-x)\right) - \frac{1}{2}x^2\ln x + \frac{1}{4}x^2 - \frac{1}{4}\right]\frac{1}{\ln 1/\delta^2}\Big\},$$

$$\delta^2 = (\tan\gamma)^2\left(1 - \frac{1}{\ln 1/(\tan\gamma)^2}\right),\ \sigma = \delta^2\left(\ln\frac{1}{\delta^2}\right)\left(1 + \frac{\ln 4/\sqrt{e}}{\ln 1/\delta^2}\right)$$

In the case of back sharpening the back cavity form was occurred as very close to the cone surface, but in doing so the values of σ for the back part are essentially corrected as compared to using of the first approximation dependencies only. Realization of pressure value in the back-end of cavity greater as compared to hydrostatical pressure in a stream can be related to the considerable problems. One of possible ways for realization of flowing around of back-end of cavity with sharpening and accordingly with indemnification of resistance there can be application of back hard sharp cowling the surface of which is insulated from water by flowing around of high speed flow of gas a thin layer. Simplest approximate approach on the base of model of ideal gas can be defined by the system of equations:

$$\text{a)}\ \left(\frac{2k}{k-1}\right)\left(\frac{P_{c1}}{\rho_{c1}}\right)\left(\frac{P_c}{P_{c1}}\right)^{\left(\frac{k-1}{k}\right)} + U_c^2 = \left(\frac{2k}{k-1}\right)\left(\frac{P_{c1}}{\rho_{c1}}\right) + U_{c1}^2\,,$$

$$\text{b)}\ \left(R^2 - B^2\right)\rho_{c1}U_{c1} = \left(R_1^2 - B_1^2\right)\rho_{c1}U_{c1}\,, \qquad (33)$$

$$\text{c)}\ \rho_c = \rho_{c1}\left(\frac{P_c}{P_{c1}}\right)^{1/k},\quad \text{d)}\ P_c = P_\infty + \mu\frac{\rho U_\infty^2}{2}\frac{d^2R^2}{dx^2}\,.$$

Here: $r = B(x)$ is the rigid surface in the cavity equation, k is the adiabatic coefficient, ρ_c, ρ_{c1}, U_c, U_{c1}, P_c, P_{c1} are gas mass density, speed, pressure at arbitrary and initial locations. System (33) for a given cavity form $r = R(x)$ is transformed to the equation defining the surfaces $r = B(x)$. For a given $r = B(x)$, the system solution is transformed to the ODE for the cavity form. The system of equations (33) approximately describe the gas layer flow where prolong speed and pressure is as constant along radius and is defined by pressure on the cavity surface.

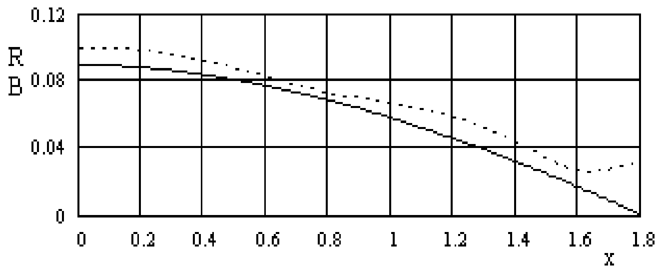

Figure 17: Cavity with account of gas layer

- - - - - $r = R(x)$ - cavity form

——— $r = B(x)$ - rigid surface form

For very high speeds $P_{c1} / \rho_{c1}U_{c1}^2 \to 0$ the nonlinear equation is simplified to the equation (σ_1 correspond to the initial section):

$$\frac{d^2R^2}{dx^2} + \frac{\sigma_1}{\mu} - \frac{2}{\mu}\frac{P_{c1}}{\rho U_\infty^2}\left[\left(\frac{R_1^2 - B_1^2}{R^2 - B^2}\right)^k - 1\right] = 0 \qquad (34)$$

Analogous equation based on the incompressible gas model flow for comparison is:

$$\frac{d^2R^2}{dx^2} + \frac{\sigma_1}{\mu} - \frac{1}{\mu}\frac{\rho_c U_{c1}^2}{\rho U_\infty^2}\left[1 - \left(\frac{R_1^2 - B_1^2}{R^2 - B^2}\right)^2\right] = 0 \qquad (35)$$

Here R_1, B_1 cavity and body radiuses in initial section. It is need to note the qualitative nature of equations (33-35). Significant influence of viscosity and centrifugal forces on lateral pressure gradients is possible here. Limited is adiabatic approximation. The ideal gas model has limited applicability for super overheated vapor only and heat-mass transfer with phase changes can be significant here. The perfection of the model is required.

Fig. 17 illustrates calculation results based on equation (34) in the case of air for: $k \sim 1.4$, $\sigma_1 \sim 0.04$, $2P_{c1} / \rho U_\infty^2 \sim 0.02$, $R'\big|_{x=0} = 0$, where $\mu \sim 2$ is used for a rough estimation. As follow from calculation results for sufficiently high gas speeds: it is possible to significantly control cavity form, however due to gas compressibility we have an elastic system where high frequency oscillations and waves on the cavity appearance are possible and can disturb flow at the cavity end with violation of flow stability. Thus, oscillations can appear even in the case of cylindrical rigid surfaces, but it can suppose that in the case of rigid surface form similar as cavity forms under constant pressure for $\sigma < 0$ (32b) these oscillations can be minimized.
Another ways of drag reduction can be realized with account of work of definite mover as whole propulsive system only.

CAVITIES VITH GAS INJECTIONS

Possibilities for calculation of ventilated axisymmetric cavities are depended on 2 key factors: possibility of gas loss prediction by end part of the cavity prediction; possibility of calculation of unsteady supercavitation under alternating pressure in the cavity.

Gas loss problems

In reality supercavity is under action of gravity cavitator attack angle and body gliding in cavity lift and here 2 main forms of gas loss can be realized realized. Under strong gravity action strong and sometimes catastrophic gas loss along vortex tubes are realized. This phenomenon firstly was studied in the work [38] and after that in the works [8]. Under high enough speeds gravity influence become as not essential and gas loss are realized by unsteady chaotic flow at the cavity end. In reality for the most part cases the mixed form of gas loss is realized depend on cavity volume value and values of lateral forces. For the first steps the experimental dependencies for prediction of the gas loss including mixed gas loss form were proposed [8, 17]. However the prediction results by these dependencies for the flows along different conditions not always gave satisfied enough results. Number of main experimental researches in this field are reflected in the works [8,17,41] and others. One of the hopeful is approach developed by J. Spurk [39] and verified by semi-natural scales experiments is for high speed with chaotic flow in the cavity end. Dependence for volume gas loss Q corresponding pressure in cavity is defined by dependence:

$$\overline{Q} = Q / c_{do} d_n^2 U_\infty = 0.013 \frac{1+\sigma}{\sigma\sqrt{\sigma}} \sqrt{\ln\frac{1}{\sigma}}\,, \tag{36}$$

were d_n - disc cavitator diameter. Main physics idea of this approach is that gas involved by boundary layer on cavity walls only is evacuated from cavity. In doing so boundary layer in small scale experiments in cavitastional tubes is laminar but for nature scales and speeds is turbulent one what provides very strong scale effects. This idea is very fruitful but it would rather developed approach is for definite interstitial range of $\sigma > 0.01 - 0.02$ only.

Possibilities of ventilated cavities prediction

For calculation of unsteady ventilated cavity the equations system to obtain alternating pressure in cavity $Pc(t)$ can be used. In general case this system should take into account heat transfer process what can be essential [40] with using of ideal gas law. However the researches found for considerable part of cases the possibility of more simple approaches on the base of polytrophic (adiabatic – isothermal) approximation of ideal gas equation what give the possibility to apply the gas mass conservation equation only. Simplest system of equations for predicting unsteady cavities with gas injection is shown below:

$$\begin{gathered} R^2\big|_{x=x_e(t)} = 0, \quad \Delta V = \int_{x_n(t)}^{x_e(t)} \pi R^2 dx - V_b \\ \frac{d}{dt}\left(\frac{\Delta V P_c}{\alpha_T}\right) = (Q_{m\_in} - Q_{m\_out}), \qquad \frac{\Delta V P_c}{\alpha_T}\bigg|_{t=0} = M_{co} \end{gathered} \tag{37}$$

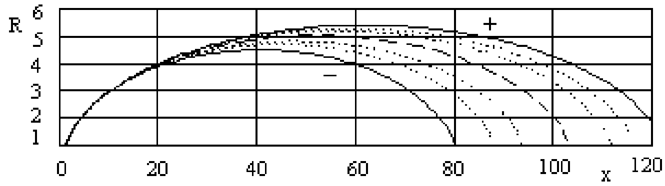

Figure 18: Cavity form evolution
——— steady cavity for $\sigma = 0.035$
------- gas injection increasing, decreasing $\overline{t} = 50,\ 100$,
——— + gas injection increasing, – decreasing , $\overline{t} = 400$

This system of equations should be solving together with simplest variant of the equations system for defining of the unsteady cavity form (19). Here $x_n(t)$, $x_e(t)$ are laws of motion for the cavitator and the end of the cavity; $P_c = \alpha_T \rho_c$ is the isothermal dependence of pressure on gas mass density ρ_c in the cavity; $\Delta V P_c / \alpha_T = M_c(t)$, M_c, $\Delta V = (V - V_b)$ are the mass and volume of gas in the cavity, respectively, V_b is the volume of the body. Q_{m_in} , Q_{m_out} are the gas injection mass flow in and losses out of the cavity end, respectively.

Linear variants of the similar system were used by Paryshev [19] for development known theory of slender cavities pulsations. He firstly used similar (37) systems for prediction of unsteady ventilated cavity. Ventilated cavity for enter disc and cone into water were calculated in the work [42]. Experimental study of unsteady cavity with gas injection is presented in particular by work [41]. On the base of this statement the quasi steady approach for prediction of unsteady axisymmetric cavity with gas injection was perfected too in the form the most suitable for practical calculations [29]. Below modified equations of quasi steady approach are presented with account date for gas loss (36) in the case not essential gravity influence:

$$\begin{gathered} \frac{d}{dt}\left\{\left[a_v (c_{do}\overline{d}_n^2)^{3/2} \frac{(1+\sigma)^{3/2}}{\sigma^2} - \overline{V}_b\right]\left[\overline{P}_\infty(1-\sigma/E)\right]^{1/n}\right\} + \\ + \left\{\left[a_q c_{do}\overline{d}_n^2 \overline{U}_n \frac{(1+\sigma)}{\sigma\sqrt{\sigma}}\right]\left[\overline{P}_\infty(1-\sigma/E)\right]^{1/n}\right\} = Q_{m\_in}, \\ a_v = \frac{\pi}{6}\frac{\sqrt{2\mu}}{k^{3/2}}\bigg|_{\sigma\sim 0.02} \sim 1.26, \quad a_q \simeq 0.011\frac{\sqrt{2\mu}}{k}\bigg|_{\sigma\sim 0.02} \sim 0.026 \end{gathered} \tag{38}$$

Simple equation with averaging of weakly changing values is:

$$1.29\frac{d}{d\bar{t}}\left\{\left[\frac{(c_{do}\bar{d}_n^2)^{3/2}}{\sigma^2}-\bar{V}_b\right]\left[\bar{P}_\infty(1-\sigma/E)\right]^{1/n}\right\}+ \\ +0.27\left\{\left[\frac{c_{do}\bar{d}_n^2\bar{U}_n}{\sigma\sqrt{\sigma}}\right]\left[\bar{P}_\infty(1-\sigma/E)\right]^{1/n}\right\}=\bar{Q}_{m\_in} \quad (39)$$

Here c_{do} - of disk type cavitator drag coefficient under zero cavitation number, $\bar{d}_n = d_n/\sqrt{0.82}d_{n*}$, d_n cavitator diameter and its typical value d_{n*}, $\bar{U}_n = U_n/U_{n*}$, U_n - cavitator speed; $\bar{P}_\infty = P_\infty/P_a$, P_∞ - hydrostatic pressure, $E = (P_\infty)/\frac{\rho U_n^2}{2}$, n -polytrophic coefficient ($n = 1$ - isotherm), $\bar{Q}_{m\_in} = Q_{m\_in}/\rho_a U_{n*} 0.82 d_{n*}^2$, Q_{m_in} -mass gas injection, $\bar{V}_b = V_b/(\sqrt{0.82}d_n)^3$, V_b - body in cavity volume. Dimensional values $\sqrt{0.82}d_{n*}$, ρ_a - characteristic gas mass density under 1 Atm. pressure $P = P_a$, U_{n*} - characteristic speed present characteristic scales for non dimensional values. Under given alternating as function of $\bar{t} = t\sqrt{0.82}d_{n*}/U_{n*}$ the equation (39) solutions are found for the function $\sigma = \sigma(\bar{t}) = (P_\infty - P_c)/\frac{\rho U_n^2}{2}$ and initial condition $\sigma(\bar{t})\big|_{t=0} = \sigma_o$. Obtained $\sigma = \sigma(\bar{t})$ defines dependence $P_c = P_c(\bar{t})$. Unsteady cavity form after that can be estimated by the Eq. (19-21) or by more accurate equations. Fig. 18 illustrates calculation results based on equation (39) (applied to conditions for cavitational tube at $U_\infty \sim 8 m/s$, $P_\infty \sim 1.025 atm.$) for the evolution of the cavitation number and cavity form given steady variations in mass gas injection rate leading to instantaneous values of gas injection of 50% to 150% that of values of mass gas injection for a steady cavity at $\sigma = 0.035$. For a more pronounced illustration, a considerably larger value of k_q as compared to given in Eq.(36) $k_q \sim 0.013$ is used. Considered quasi steady approach is essentially different as compared to usual presentation of unsteady cavity as steady one and is considerable more accurate. Calculation results based on equations (39 are compared with experiments for unsteady cavities with changing gas injection [41] and others dates. Instead of essential restrictions this approach can be very convenient for practical estimations thank simplicity and are occurred as applicable enough for the most part of real cases for not very fast changing of the motion speed and gas injections. It is need to note that presented consideration of ventilated cavities is very particular only and can not fully enough reflect situation in this very wide field. More details here can be fund in current publications in this field.

CONCLUSIONS

The most part of practical approaches for estimation of supercavitation is connected with property of independence cavity section expansion. In doing so all dependencies (7, 13, 19-21, 24-27) can be considered as the most general presentation of the "*Principle of independence of the cavity expansion*". This approach have stricken universality together with simplicity and clear physics and give hopeful estimation of the forms and cavity sizes in the most part important for application cases. Possibility to correct solutions near disk type cavitators and also obtaining of the solutions in the cases of considerable difference the solution as compared usual cavity for $\sigma = const$ by linearized theory approach make as possible accurate enough calculations of different steady and unsteady cavities in very wide range of possible applications. Calculations here even by traditional way of "principle of independence" application always gave hopeful results multiply verified by numerous experimental dates. It is need to note also that at present this principle is not pure empirical and fully based and are as consequence of the linearized theory and is as one of it's results. Accuracy of this approach for 3-D distribution of the cavity form is limited by linear theory. In the case of axisymmetric forms the independent of expansion is limited by not very small values $O(1/\ln\lambda_*)$ where λ_* is some typical aspect ratio of the surface of the cavitator and cavity as whole. So here it is possible to say on not independence but it would rather on almost independence of cavity section expansion. In doing so this dependence is very weak and give the possibility even in the case of the cavities considerable different as compared to usual steady cavity to apply steady dependencies for μ, k . However in doing so sometime accounting of aspect ratio change existing in specific concrete cases become as essential for estimation of μ as compared to it's values for steady cavity.

NOMENCLATURE

r, x, t	Cylindrical coordinates, time
$r = R(x,t)$	Axisymmetric cavity form
R_k, L_k, λ_k	Maximum radius, semi-length, aspect ratio of ordinary cavity for $\sigma = const$
U_∞, P_∞, ρ_∞	Speed, pressure, mass density at infinity
$\sigma = \frac{\Delta P}{\rho_\infty U_\infty^2/2}$	Cavitation Number (ΔP -pressure difference hydrostatic and in cavity)
$D; c_{do}, c_d$	Drag, cavitator drag coefficients
C_{D0}, C_D	Cavitating drag coefficients for $\sigma = 0$, $\sigma > 0$ per forward and maximal cavity section

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