<table>
<tr><th colspan="2">To explain a concept (or a proposition)</th><th>such as “the number of diagonals of a polygon of N sides is N(N-3)/2”</th></tr>
<tr><th>A teacher</th><th>Which means</th><th>For example</th></tr>
<tr><td>Problematizes the concept</td><td>• He or she presents a problem that students can understand and accept with their existing knowledge, and yet the new concept can help solve better (faster, more elegantly, more comprehensively, etc.) the problem.</td><td>Before even hinting that there is a formula, she shows students a polygon of many sides, say 10 sides and asks—how many diagonals do you think this polygon has?
Note that students could actually find out by drawing diagonals and counting, but such method is not as efficient as using the formula.</td></tr>
<tr><td>Connects to prior knowledge</td><td>• Draws individual students’ prior knowledge that is relevant to the new concept</td><td>Before posing the problem, she asks—who can remind us what a diagonal is? Or she draws a hexagon and asks—can you come up here and draw a diagonal of this hexagon?</td></tr>
<tr><td></td><td>• Uses that prior knowledge with the class as a collective to construct and connect to the new concept.</td><td>After verifying that the class knows what a diagonal is, asks—how many diagonals does a quadrilateral have? What about a pentagon? And when the class says “more”, she asks, how do you know it will have more?</td></tr>
<tr><td>Represents the concept</td><td>• Uses multiple representations of a same concept while explaining a new idea, going back and forth between them to build understanding of connections.</td><td>Starts a table that charts the number of diagonals d(N) for each number of vertices N. The table starts with N=3 (triangle) and d(N)=0, then for N=4, d(N)=2, etc. She connects (with arrows) each of the rows of the table to the diagrams of each of those polygons and their diagonals have been drawn.
She might add a column to the table, to chart the number of diagonals through one vertex. She might also note the first and second differences in the table.
She would help students see a pattern (“predict how many diagonals will an octagon have”) from the first differences. Then she will note that the total number of diagonals depends on the number of diagonals through one vertex times the number of vertices and write the formula d(N)=N*(N-3)/2.
<table>
<tr><th>N</th><th>d(N)</th><th>First difference</th><th>Diagonals through one vertex</th></tr>
<tr><td>3</td><td>0</td><td></td><td>0</td></tr>
<tr><td></td><td></td><td>2</td><td></td></tr>
<tr><td>4</td><td>2</td><td></td><td>1</td></tr>
<tr><td></td><td></td><td>3</td><td></td></tr>
<tr><td>5</td><td>5</td><td></td><td>2</td></tr>
<tr><td></td><td></td><td>4</td><td></td></tr>
<tr><td>6</td><td>9</td><td></td><td>3</td></tr>
<tr><td></td><td></td><td>5</td><td></td></tr>
<tr><td>7</td><td>14</td><td></td><td>4</td></tr>
</table></td></tr>
<tr><td>Exemplifies the concept</td><td>• Provides examples and non examples chosen to emphasize different features of the concept.</td><td>She might calculate the number of diagonals for a 10-sided polygon or show how the formula predicts the number of diagonals for a quadrilateral and pentagon.
She might also show how to handle the questions, draw a polygon that has 54 diagonals or is there a polygon that has 50 diagonals?</td></tr>
<tr><td>Identifies core principles of the concept</td><td>• Defines the concept in general, explains what each component of the concept means and why the various components of the concept</td><td>She would clarify that the factor (N-3) in the formula names the number of diagonals through one vertex
She would say that since a polygon has N vertices and (N-3) diagonals through a vertex, the total number of diagonals is related to the product N*(N-3)</td></tr>
</table>

	are relevant (or true).	She would also note that since a diagonal connects two vertices, the product above is counting every one diagonal exactly twice, and that is the reason one divides by 2.
Identifies key errors	• Creates contexts for <u>conceptual errors to appear and be discussed</u>. Shows connections between the new concept and reasonable but errorful ways of dealing with situations that the concept clarifies. Helps students rectify and justify their work.	Instead of just telling students why one divides by 2, she could say, "Bubba thinks that there are 70 diagonals in a decagon, because there are 10 vertices and 7 diagonals from each vertex. What do you think of Bubba's argument?" She might also show that when applying the formula, one needs to subtract 3 from N before multiplying by N. That is, d(10)=10*(10-3)/2 and not d(10)=(10*10)-3/2
Establishes the range/ boundaries of the new concept	• Solicits and asks questions that the new concept will motivate in the future. • When the concept does not apply • Investigates with students some of those questions making connections to the concept.	She might anticipate problems students will get, such as **"given the number of vertices, calculate the number of diagonals"** or **"given the number of diagonals, identify the polygon."** She might also have them discuss the case of concave polygons or pose the question of how many points of intersection do all the diagonals of a polygon make. She might anticipate that they would use similar ideas to figure out the sum of the angles of a polygon and prompt them to conjecture that formula. She could ask students whether they can use this to find the number of diagonals in a cube.
Assesses and holds students accountable	• Asks questions of students to gauge what they understand of the new concept. • Expects students to think about those questions and participate in the explanation.	She would give a short written quiz with questions like "what is the number of diagonals through a vertex" and "what is the total number of diagonals" for two different polygons of not too large number of vertices. She would ask students to hand those in or to correct each other. She would be on the lookout for which students drew to find out, which students used the product without dividing, which students thought that the number of diagonals through a vertex equals the number of sides, etc.

Note: This rubric builds on the extensive work on instructional explanations by Gaea Leinhardt (see Leinhardt, G. and Steele, M. (2005). Seeing the Complexity of Standing to the Side: Instructional Dialogues. Cognition and Instruction. Cognition and Instruction, 23, 87-163).