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FATIGUE CRACK FRACTURE AND ARREST IN FIBER REINFORCED CONCRETE UNDER INTERFACIAL BOND DEGRADATION

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Abstract
Fatigue crack growth of fiber reinforced concrete is analyzed with a fracture mechanics based model that accounts for the effect of cycle-dependent crack bridging of aggregates and fibers. Fatigue life of fiber reinforced concrete is defined by the number of cycles at which final unstable fracture takes place subsequent to stable fatigue crack growth. The resulting theoretical S-N diagram is compared with experimental data reported in literature and shows general agreement. Furthermore, it is shown that the bond degradation at the fiber-matrix interface influences fatigue crack growth and, in turn, fatigue life. Fatigue crack arrest is predicted for the case of less degradable interface, resulting in infinite fatigue life. The model demonstrates the existence of fatigue limit in the standard S-N tests and suggests directions for the development of high-fatigue resistant fiber reinforced concrete.
Key words: Fiber reinforced concrete, Fatigue, Bond degradation

1 Introduction

Stress-life and fracture mechanics approaches complement each other to achieve the goal of fatigue research, which is to establish a basis for fatigue life prediction, improvement, and design of fiber reinforced concrete (FRC) for successful structural applications by fulfilling the expected service life in a cost efficient manner. To date, fatigue research of FRCs has relied mainly on stress-life approach. This approach measures an S-N diagram for a given FRC through experimental data collection, and the resulting diagram is essential for material evaluation and structural design. On the other hand, fracture mechanics approach focuses on fatigue crack growth of FRCs. This approach yields information on fatigue cracking mechanisms with the presence of crack bridging and is suitable for material development. Hence, it is appropriate to deal with subproblems with a proper approach towards the goal.

However, the two approaches have not been actively integrated so far. For example, although there are a large number of experimental S-N diagrams, few of them are reproduced by a theoretical fracture mechanics based model. Also, the mechanism of fatigue limit, which is conventionally determined in stress-life approach, has never been addressed theoretically in a fracture mechanics sense. The integration could fill the gap between the two approaches, including these two issues, and, furthermore, would make it possible not only to predict the S-N relation of a given FRC, but also to design the microstructure of an FRC to achieve a required S-N relation.

The objective of this paper is to present that a fracture mechanics based fatigue life model enables the integration of the two approaches. With the model, it is demonstrated below that theoretical S-N diagrams can be generated for four different FRCs and that fatigue limit in stress life approach is related to fatigue crack arrest in fracture mechanics approach. Also the validity of crack bridging degradation assumed in the model is assessed by the comparison to experimental data reported in literature. Design implications for developing high fatigue resistant FRCs are mentioned as well.

2 Fracture mechanics based fatigue life model of FRC

The fracture mechanics based fatigue model of randomly distributed discontinuous fiber composites previously developed (Li and Matsumoto, 1997) is summarized. The fatigue life model is based on fracture mechanics of fatigue crack growth of FRCs. FRCs fail in fatigue when final unstable fracture takes place subsequent to stable crack growth under fatigue loading. Thus, fatigue life of FRCs is defined by the number of cycles to final failure and, in turn, is controlled by fatigue crack growth behavior. Fatigue crack growth is governed by three factors: matrix fatigue crack growth law, crack bridging, and cycle-dependent degradation of crack bridging. These three factors are reviewed below.

2.1 Matrix fatigue crack growth law

Matrix fatigue crack growth law for concrete has been observed to obey a Paris law type equation (*Baluch et al., 1987*). According to Paris law, the current number of load cycles, N, at a given crack length, a, can be obtained by

$$N = \int_{a_i}^{a} \frac{1}{C(\Delta K_{tip})^n} da \tag{1}$$

where a_i = initial crack length, C = Paris constant, ΔK_{tip} = crack tip stress intensity factor amplitude, and n = Paris constant. Fatigue life, N_f, is defined by final unstable fracture subsequent to fatigue crack growth and is obtained by calculating the number of load cycles spent for growing the crack from the initial length, a_i, to the final length, a_f. This final unstable fracture condition is met when crack tip stress intensity factor at maximum load level, K_{max}, surpasses matrix fracture toughness, K_c, and this determines a_f at fracture failure.

Hence, fatigue life can be computed, if ΔK_{tip} and K_{max} are obtained for a given FRC with body geometry and loading mode given. ΔK_{tip} and K_{max} can be obtained in a similar manner, and only the steps to obtain ΔK_{tip} are summarized below. ΔK_{tip}, of FRCs is attributed to external applied loading and crack bridging, so ΔK_{tip} can be decomposed into two parts:

$$\Delta K_{tip} = \Delta K_a + \Delta K_b \tag{2}$$

where ΔK_a = stress intensity factor amplitude due to external applied loading and ΔK_b = stress intensity factor amplitude due to crack bridging. ΔK_a for a beam in flexural bending that has a crack on the tension face is given by

$$\Delta K_a = 2\int_0^a G(x,a,w)\Delta\sigma_a(x)dx \tag{3}$$

where w = beam depth, $\Delta\sigma_a(x) = \Delta\sigma_o(1-2x/w)$ (linear gradient for bending), $\Delta\sigma_o$ = applied external flexural stress amplitude, and x is measured from the tension face of the beam (Cox and Marshall, 1991). G(x,a,w) is a weight function that represents the contribution of a unit force on the crack surface to the crack tip stress intensity factor. Similarly, ΔK_b is a function of the relation between the crack bridging stress amplitude, $\Delta\sigma_b$, and the crack opening displacement amplitude, $\Delta\delta$, (hereafter referred to as the cyclic bridging law) and is given by

$$\Delta K_b = -2\int_0^a G(x,a,w)\Delta\sigma_b(\Delta\delta(x))dx \qquad (4)$$

where $\Delta\sigma_b(\Delta\delta)$ = cyclic bridging law and $\Delta\delta(x)$ = crack opening displacement amplitude at a point, x, on the crack surface. The computation of ΔK_{tip} and K_{max} requires a numerical scheme to solve an integral equation on $\Delta\delta(x)$ and crack opening displacement at maximum load level, $\delta_{max}(x)$, at a position, x, on the crack surface, respectively. Details on the numerical scheme can be found elsewhere (Cox and Marshall, 1991).

2.2 Cyclic bridging law

Crack bridging in FRCs is exerted by fibers and aggregates. Thus, the cyclic bridging law, $\Delta\sigma_b(\Delta\delta)$, is given by the superposition of crack bridging stress-crack opening displacement relation due to fibers and aggregates under cyclic loading:

$$\Delta\sigma_b = \Delta\sigma_f + \Delta\sigma_m \qquad (5)$$

where $\Delta\sigma_f$ = fiber bridging stress change under cyclic loading and $\Delta\sigma_m$ = aggregate bridging stress change under cyclic loading. The cyclic bridging law due to fibers, $\Delta\sigma_f(\Delta\delta)$, is also dependent on δ_{max} in case of short fiber composites. $\Delta\sigma_f(\Delta\delta)$ as well as $\sigma_f(\delta)$ are expressed by an explicit function of microstructural parameters such as fiber dimensions, fiber modulus, interfacial bond strength, and so on. The cyclic aggregate bridging law, $\Delta\sigma_m(\Delta\delta)$, is modeled based on an empirical monotonic aggregate bridging law, $\sigma_m(\delta)$ (Stang, 1992). The cyclic bridging law assumes that the aggregate bridging stress, σ_m, and the crack opening displacement, δ, decrease to zero when unloading takes place. Details on the fiber and aggregate bridging laws can be found elsewhere (Li and Matsumoto, 1997).

2.3 Cycle-dependent degradation of crack bridging

Cycle-dependent degradation of crack bridging is shown to exist by experimental observations in fiber reinforced concrete (Zhang and Stang, 1997) and concrete (Hordijk and Reinhardt, 1990, Reinhardt et al., 1986). Degradation of fiber- and aggregate-matrix interface bond is the main mechanism when no fiber or aggregate rupture is observed. Based on this observation, a bilinear degradation function for fiber bridging is suggested (Li and Matsumoto, 1997). The current fiber-matrix interfacial frictional bond strength, τ, is assumed to change as follows:

$$\frac{\tau}{\tau_i} = \max \text{ of}\left(1.0 + D_1 \sum_{i=1}^{N} \Delta\delta_i(x), B + D_2 \sum_{i=1}^{N} \Delta\delta_i(x)\right) \tag{6}$$

where τ_i = initial bond strength, D_1 = degradation coefficient for the early trend (negative for degradation), $\Delta\delta_i$ = crack opening displacement change at i-th cycle, B = intercept for the long trend, and D_2 = degradation coefficient for the long trend (negative for degradation) (see Fig. 1). Similarly, the current degraded aggregate bridging stress, σ_m, is scaled to the original undegraded aggregate bridging stress, σ_{mo}, as follows:

$$\frac{\sigma_m}{\sigma_{mo}} = \max \text{ of}\left(1.0 + D_{m1} \sum_{i=1}^{N} \Delta\delta_i(x), B_m + D_{m2} \sum_{i=1}^{N} \Delta\delta_i(x)\right) \tag{7}$$

where (D_{m1}, D_{m2}, B) = (D_1, D_2, B) for aggregate bridging. Here, the degradation is measured with

$$\sum_{i=1}^{N} \Delta\delta_i(x) = \text{accumulated crack opening displacement change at x.} \tag{8}$$

where x is the position on the crack surface. For a growing crack of non-uniform crack profile, this parameter takes the maximum value near the crack mouth and the minimum at the crack tip in response to the number of cycles and the crack opening displacement change, meaning that crack bridging degradation is the most severe near the crack mouth and minimal

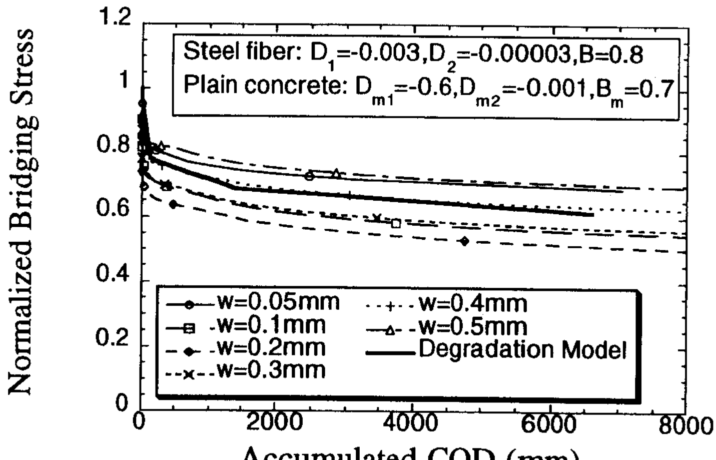

Fig. 1. Comparison between the experimental data of crack bridging degradation (Zhang and Stang, 1997) and the degradation model assumed in this study

at the crack tip.

3 Fatigue life analysis of FRCs

Theoretical crack bridging degradation model, which is the superposition of fiber and aggregate bridging degradation determined by Eqns 6 and 7, is compared with experimental crack bridging degradation curves measured by Zhang and Stang for a 1% smooth steel fiber FRC under uniaxial fatigue loading (Fig. 1). The experimental curves are obtained by fitting to the experimental data for various initial crack opening

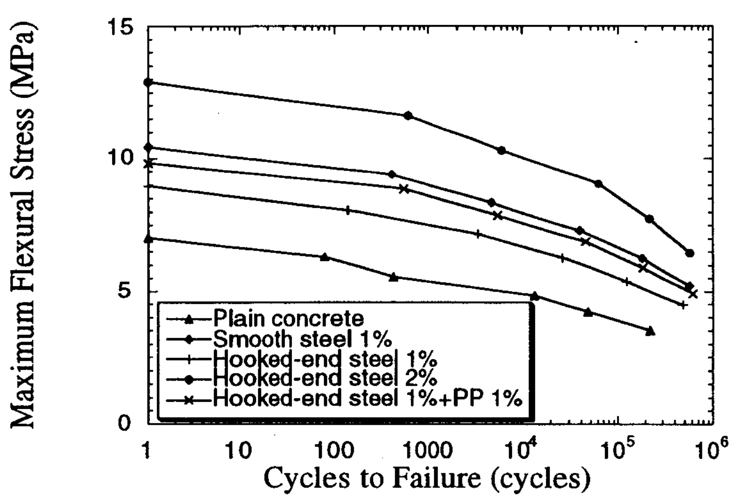

Fig. 2. Theoretical S-N diagram (Matsumoto and Li, 1998)

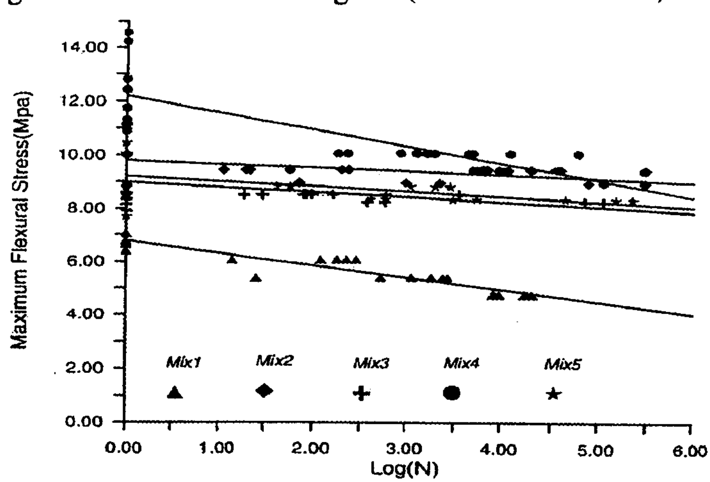

Fig. 3. Experimental S-N diagram (Zhang and Stang, 1996)

displacement values (w = 0.05, 0.1, 0.2, 0.3, 0.4, 0.5 mm) (Zhang and Stang, 1997). The coefficients in Eqns 6 and 7 shown in Fig. 1 are obtained from the theoretical S-N analysis below, namely they are determined independently of this uniaxial fatigue test. The validity of the degradation model is supported by the fact that it falls within the range of the experimental curves, although there is a need to include the effect of initial crack opening displacement values.

Theoretical S-N diagrams (Fig. 2) are generated and compared with the experimental ones (Fig. 3) measured by Zhang and Stang (Zhang and Stang, 1996). Materials studied are Mix1/plain concrete, Mix2/smooth steel 1% FRC, Mix3/hooked end steel 1% FRC, Mix4/hooked end steel 2% FRC, and Mix5/hooked end steel 1% + polypropylene 1% FRC. The

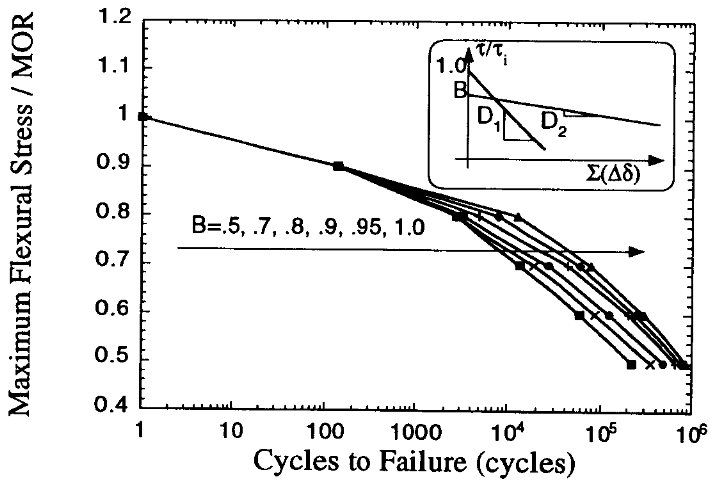

Fig. 4. Effect of B on fatigue life of Mix3

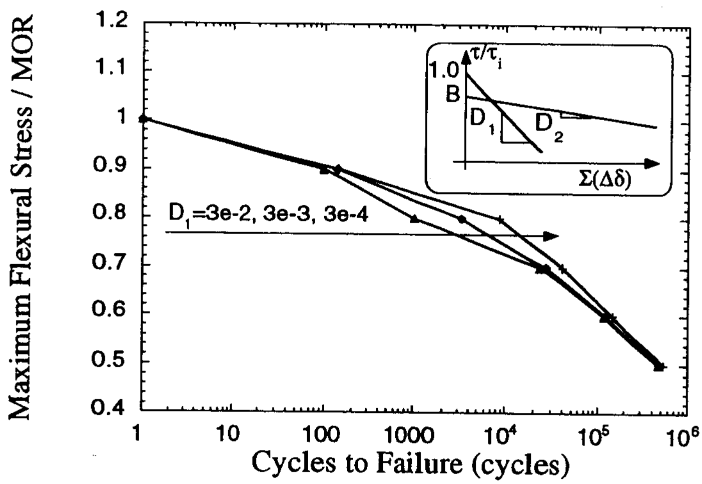

Fig. 5. Effect of D_1 on fatigue life of Mix3

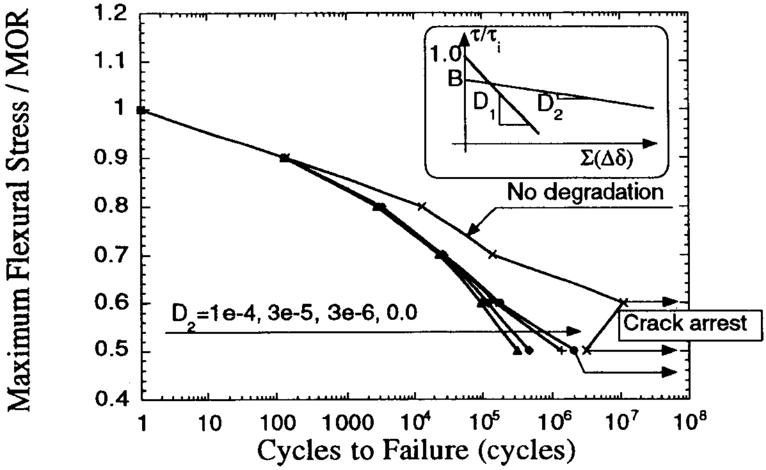

Fig. 6. Effect of D_2 on fatigue life of Mix3 and no degradation case

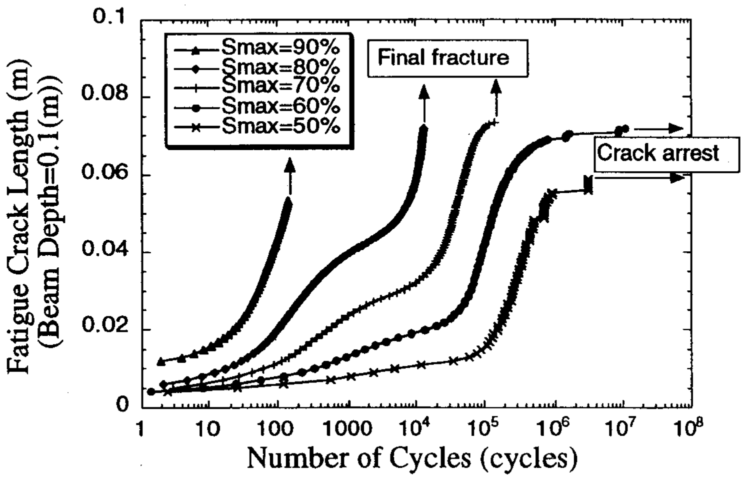

Fig. 7. Fatigue crack growth of Mix3 without interfacial bond degradation

beams with depth 100 mm, thickness 50 mm, and length 350 mm were loaded in three point flexural fatigue with a span of 345 mm. Theoretical diagrams are obtained with the same degradation parameters for all fiber types (see Fig. 1) because of the lack of experimental measurements. The comparison between theoretical and experimental diagrams show a general agreement, and it also shows that a single set of degradation parameters can explain a wide range of experimental data.

Effects of interfacial bond degradation on fatigue life are examined with Mix3/hooked end steel 1% FRC, and the results are shown in Figs. 4 through 6. The effect of B on fatigue life is prominent at stress levels lower than 80%, and the fatigue life improvement is about 4~5 times (Fig.

4). The effect of D_1 is limited to 80% stress level, therefore the effort to change this parameter is not meaningful, although fatigue life is improved by about one order (Fig. 5). The effect of D_2 appears at 50 and 60% stress levels (Fig. 6). It is observed that fatigue crack is arrested at 50% and that fatigue life becomes infinite. This fatigue crack arrest actually corresponds to fatigue limit in the conventional sense of stress-life approach, so the fatigue limit theoretically obtained in this analysis is about 50 to 60% of the MOR.

The limit of fatigue life improvement can be obtained as well with this parametric study. The fatigue life improvement is shown in Fig. 6, when no interfacial bond strength degradation ($B = 1.0$ and $D_2 = 0.0$) is assumed. Fatigue crack growth behavior with no degradation is shown in Fig. 7. It is clearly shown that crack growth behavior exhibits a transition. At 90% stress level, the fatigue crack is always accelerated throughout the life, whereas, at 80%, the crack is decelerated initially, but accelerated near the final moment. The fatigue crack at 70% is similar to that at 80%, but shows a noticeable deceleration near the final failure. This final deceleration is manifest at 60 and 50%: the fatigue crack is arrested as mentioned above.

4 Conclusions

This study showed that fracture mechanics and stress-life approaches can be effectively linked. The fracture mechanics based fatigue life model demonstrated that theoretical S-N diagrams can be generated and that fatigue limit is related to fatigue crack arrest. Fatigue crack arrest is influenced significantly by interfacial bond degradation behavior. The parametric study clearly showed that the suppression of the long-term decay of interfacial bond strength leads to the improved fatigue limit.

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