33 MICROMECHANICS OF FIBER EFFECT ON THE UNIAXIAL COMPRESSIVE STRENGTH OF CEMENTITIOUS COMPOSITES

V. C. LI and D. K. MISHRA
Advanced Civil Engineering Materials Research Laboratory,
Department of Civil Engineering, University of Michigan,
Ann Arbor, MI, USA

Abstract
A micromechanical model is presented for the uniaxial compressive strength of fiber reinforced cementitious composites (FRCC). The model is based on the classical models of compressive failure of brittle solids containing sliding microcracks that induce wing-crack growth under compressive loading. The concepts of increased microcrack sliding resistance and wing-crack growth retardation associated with fiber bridging are exploited to produce a strengthening effect of fibers on composite strength. The concept of defect introduction associated with fiber volume fraction is included to produce a composite strength degradation. The combined effects result in a composite compressive strength which increases initially and subsequently drops with increasing fiber content, as has been observed in FRCs reinforced with a variety of fibers. This paper represents an extension of a preliminary study of the influence of fibers on compressive strength of FRCC by Li (1991). More accurate stress intensity factor calibrations for wing-crack growth and interaction are employed in the present paper. The predicted general trends of compressive strength change with fiber parameters remain unchanged from the original work.
Keywords: Fiber Reinforced Cementitious Composites, Concrete, Compressive Strength, Fiber Bridging, Micromechanics, Model.

1 INTRODUCTION

Early experimental studies of compressive strength of fiber reinforced cementitious composites using steel, glass and polypropylene fibers (e.g., Shah and Rangan, 1971; Fannela and Naaman, 1983) suggested that the influence of fibers on the compressive strength was insignificant at low volume fractions. Both increase and decrease of compressive strength with different fiber types have been experimentally observed (e.g., Akihama et al., 1986a; Ward and Li , 1990; Zhu, 1990). Even for the same material, there is mounting evidence that compressive strength may first rise followed by a drop with increasing fiber volume fraction. These observations suggest that the addition of fibers in a cement composite using conventional mixing procedure leads to a competing process of strength improvement as well as degradation. Some recent research, however, suggest that compressive strength can be enhanced substantially even at high fiber volume fraction, when special processing techniques, presumably leading to reduced matrix defects, are employed (Tjiptobroto, 1991; Naaman et al., 1991). Li (1991) proposed a simplified model based on micromechanics of fiber reinforcement to explain some of these effects. In this brief paper the previous model is modified by utilizing more accurate formulations for the stress intensity factors for initiation, propagation and interaction of sliding microcracks in the composite.

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The proposed model is fundamentally based on well known micromechanical models of compressive failure in brittle solids (Horii and Nemat-Nasser, 1986; Ashby and Hallam, 1986; Kemeny and Cook, 1991). The influence of fibers on microcrack sliding and extension is based on crack bridging studies carried out in recent years (Li and co-workers, 1991, 1992). The model is kept to be as simple as possible in order to obtain close form solutions which elucidate the micromechanical parameters controlling the strengthening and the weakening mechanisms. It is found that depending on the effectiveness and amount of fiber bridging, and the degree to which fibers introduce defects to the composite, both increase and decrease of compressive strength can be derived from increasing fiber content. Furthermore, it is identified that the fiber/matrix bond strength and a snubbing coefficient can lead to higher compressive strength for a given fiber type and aspect ratio, and for a given composite fabrication process. A summary of compressive strength change with fiber volume fraction is presented in Figure 1, the details of which can be found in Li (1991).

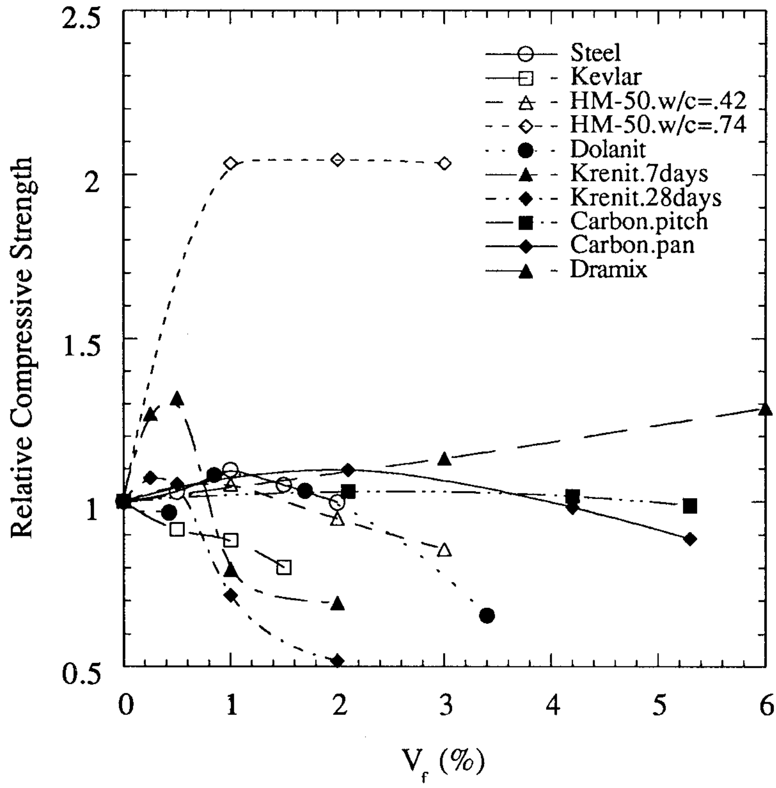

Figure 1. Compressive Strength of Various Fiber Reinforced Cementitious Composites Relative to the Matrix Compressive Strength, as a Function of Fiber Volume Fraction.

2 A MICROMECHANICAL MODEL

Unstable propagation of a critical tensile crack accounts for the failure of brittle solids under tension. However, under compressive loading the microcracks in the solid come under a local tensile field at their tips causing initiation of "wing-cracks". The extension of wing-cracks under such a local tension has been demonstrated to be unstable initially and becomes stable as the crack length increases. However presence of other microcracks and the interaction between them induces instability resulting in final failure. When fibers are present in such a body, they affect the crack propagation by increasing the resistance to sliding of the initial microcracks and opening of the wing cracks by crack-bridging. Thus the models used for fiber bridging developed for tensile strength can also be used in case of compressive strength. The proposed model can be divided into the following distinct parts that take into account different simultaneously occurring mechanisms:

(i) Crack propagation due to sliding and opening
(ii) Crack-crack interaction
(iii) Crack bridging
(iv) Fiber induced damage

The fundamental ideas behind each of the above concept has been discussed by Li (1991) and will only be summarized here.

2.1 Crack sliding and wing-crack propagation

For the microcrack of length $2a$ oriented at an angle β (Figure 2) in a brittle solid subjected to the uniaxial compressive load σ, a shear stress τ is generated which causes frictional sliding on the crack faces. The shear sliding in turn creates a singular stress field with tensile components on opposite quadrants, causing the initiation of tensile "wing-cracks". A simplified approximate expression for the stress intensity factor of straight wing-cracks of length ℓ parallel to the loading axis for the most critical sliding crack with orientation $\beta = 45°$ has been obtained by Horii and Nemat-Nasser (1986) and is given by:

$$K_I = \frac{2\tau\sqrt{\pi a}}{\pi\sqrt{2(\ell / a + 0.27)}} \tag{1}$$

where

$$\tau = \frac{1}{2}\sigma(1-\mu) \tag{2}$$

and μ is a coefficient of friction against shear sliding of the crack faces. The normalized crack driving force $K_I / (\sigma \sqrt{\pi a})$ drops rapidly as the wing-crack length (ℓ / a) extends. This means that wing-cracks are inherently stable, requiring increasing load to continue its growth. These wing-cracks lose their stability by crack-crack interaction, which raises the stress intensity factor and is a function of the crack density (i.e. how close the interacting cracks are to each other). Interaction between wing-cracks, therefore, is critical in leading to a critical load in compression -- the compressive strength of the solid with many microcracks.

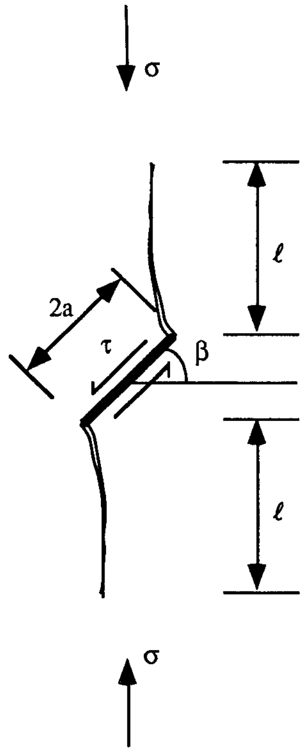

Figure 2. Wing-Crack Growth Induced by Sliding of Microcrack as the Basic Mechanism of Compressive Failure in Brittle Solids.

2.2 Crack interactions

Under overall compressive stress fields, Ashby and Hallam (1986) assumed crack interactions to result in the buckling of material columns formed by subparallel wing-crack growth. For this model, the stress intensity factor due to interaction between wing-cracks under uniaxial compression loading is given by:

$$K_I' = \frac{\sqrt{2}}{\pi}\sqrt{D_o}\left(\ell_o + \frac{1}{\sqrt{2}}\right)^{1/2} \sigma\sqrt{\pi a} \tag{3}$$

where

$$D_o = N_A \pi\, a^2 \tag{4}$$

and N_A is defined as the number of cracks per unit area. ℓ_o is the normalized wing-crack length ℓ / a. The intensity of crack interaction is dictated by D_o,which may therefore be regarded as an initial damage parameter of the solid in this model. For example, a poorly compacted concrete may be expected to have a large D_o value.

Assuming that linear elastic fracture mechanics holds on the scale of micro-defects, the condition for crack propagation is given by:

$$K_I + K_I' = K_{IC} \tag{5}$$

and $K_{IC} = K_m$ is the fracture toughness of the cementitious material without fibers. Inverting (5) the normalized compression load (σ_o) required to propagate the crack to length (ℓ / a) may be calculated:

$$\sigma_o \equiv \frac{\sigma\sqrt{\pi a}}{K_m} = \frac{1}{A[\ell_o, \mu, D_o]} \tag{6}$$

where

$$A[\ell_o, \mu, D_o] \equiv \frac{1-\mu}{\pi\sqrt{2(\ell_o + 0.27)}} + \frac{\sqrt{2}}{\pi}\sqrt{D_o}\left(\ell_o + \frac{1}{\sqrt{2}}\right)^{1/2}$$

This solution is illustrated in Figure 3 for different initial damage levels (D_o). The compressive strength is given by the peak of these curves, and is shown to decrease with the amount of initial damage. It is interesting to note that for tensile loading, $(\sigma\sqrt{\pi a}) / K_m$ is of the order of unity. Since for cementitious materials, the compressive strength is typically one order of magnitude higher, Figure 3 suggests that the typical initial damage level must be of the range of 0.0001 to 0.0015. For the calculations to follow, we choose D_o= 0.0005 as the natural flaw density of typical cementitious materials without fibers.

3 STRENGTHENING EFFECT OF FIBER ADDITION

3.1 Resistance to crack-sliding

Based on frictional resistance to the pull out of randomly oriented fibers bridging across a crack, Li (1991) developed the following equation for reduction in shear stress acting on a sliding crack.

$$\tau_B = \frac{1}{2} s V_f \left(1 - \frac{2a\sigma}{L_f G}(1-\mu)(1-\nu)\right) \tag{7}$$

where G and ν are the composite shear moduli and Poisson's ratio respectively, V_f is the fiber volume fraction and L_f is the fiber length. The reinforcement index s is defined as

$$s = g\tau_f\left(\frac{L_f}{d_f}\right) \tag{8}$$

where d_f is the diameter of the fiber and τ_f is the interfacial shear strength. For typical values of the snubbing coefficient f (associated with inclined fiber pull-out, Li 1992) ranging from 0 to 1, the snubbing factor g ranges from 1 to 2.3.

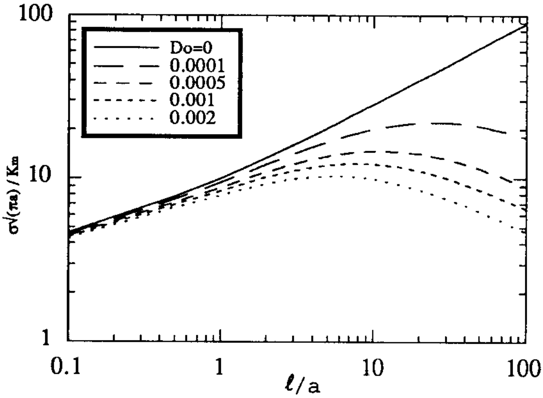

Figure 3. Normalized Compression Load Required to Drive a Wing-Crack of Length ℓ, for Four Different Initial Damage Level D_o. Calculated for $\mu = 0.5$.

The net shear stress acting on the sliding microcrack is therefore given by

$$\tau = \frac{1}{2}\sigma(1-\mu) - \tau_B \tag{9}$$

3.2 Resistance to wing-crack growth

The wing-crack, like the sliding microcrack, will also be bridged by fibers. As the wing-crack grows, increasing amount of bridging fibers will lead to an increase in crack closing pressure in an enlarging 'process zone'. Based on an R-curve concept associated with fiber bridging, Li (1991) developed an expression for the total toughness against which the wing-cracks have to propagate against:

$$K_{IC} = K_m + \sqrt{\frac{\ell}{\ell_o^*} E G_o} \tag{10}$$

where E and G_o are the composite Young's moduli and the composite fracture energy, respectively and ℓ^* defines the wing-crack length extension at which the linearized R-curve reaches the plateau value for the bridging fracture energy (Li, 1991). In the case of FRC, Li (1992) found that G_o can be related to the fiber and interfacial properties:

$$G_o = \frac{1}{12} s L_f V_f \tag{11}$$

3.3 Assessment of fiber strengthening effect

Combining eqns. (1), (3), (7) - (11) we obtain an expression relating the normalized compressive load σ_o required to maintain the normalized wing-crack length ℓ_o:

$$\sigma_o \equiv \frac{\sigma\sqrt{\pi a}}{K_m} = \left\{ \frac{B[\ell_o,\mu,V_f,c]}{A[\ell_o,\mu,D_o]} + C[K_o,s_o,V_f] \right\} \frac{1}{D[s_o,V_f]} \tag{12}$$

where

$$B[\ell_o,\mu,V_f,c] \equiv (1-\mu)\left\{1+\sqrt{\frac{\bar{a}}{\ell_o^*}c\ell_o V_f}\right\};$$

$$C[K_o,s_o,V_f] \equiv \frac{s_o V_f}{K_o};\text{ and}$$

$$D[s_o,V_f] \equiv (1-\mu)\left(1+4\left(1-\nu^2\right)s_o V_f\,\bar{a}\right)$$

and the non-dimensional parameters are defined as:

$$\ell_o \equiv \frac{\ell}{a};\ \ell_o^* \equiv \frac{\ell^*}{L_f};\ \bar{a} \equiv \frac{a}{L_f};\ s_o \equiv \frac{s}{2E};\ c \equiv \frac{1}{12}\frac{sL_fE}{K_m^2};\ K_o \equiv \frac{K_m}{E\sqrt{\pi a}}.$$

We are now in a position to access the positive influence of fibers on the load bearing capacity as the wing-cracks grow (Figure 4a). In this figure, the initial damage magnitude is fixed ($D_o = 0.0005$), and a family of curves is generated for various fiber volume fraction. (Other fixed parameters in this and subsequent calculations are $\ell_o^* = 20$; $\bar{a} = 0.1$; $c = 800$; $K_o = 0.0002$; $s_o = 0.01$. They are chosen to represent typical FRCs but can be adjusted for specific material systems.) Because σ_o scales linearly with V_f and s (eqn. 11), this family of curves may also be interpreted as a result of the influence of the interfacial bond strength or fiber aspect ratio. Figure 4b shows the monotonic increase in compressive strength σ_c with V_f. For the present set of parameters, compressive strength is shown to increase by more than 100 % for V_f up to 2%.

4 WEAKENING EFFECT OF FIBER ADDITION

As mentioned before, the addition of fibers beyond a certain optimal level may adversely affect the compressive strength due to introduction of additional defects and difficulties in processing. To account for this effect Li (1991) has suggested a simple modification in the initial flaw density or damage index parameter D_o following experimental observations. By introducing the fiber induced damage index k, we redefine D_o as given by equation (13) and replace the previous definition in equation (4).

$$D_o = (N_A \pi a^2)\, e^{kV_f} \tag{13}$$

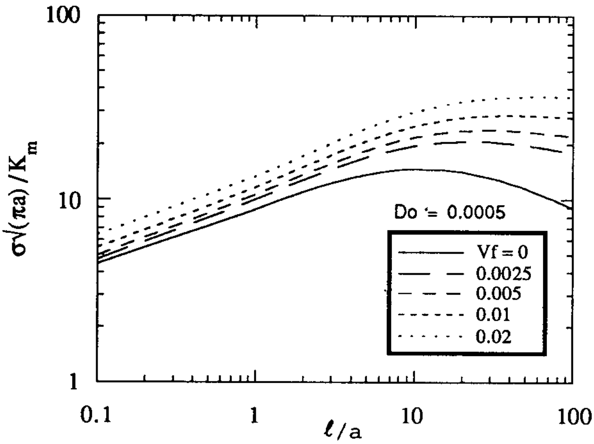

Figure 4a. Strengthening Effect of Fibers: Normalized Compression Load Required to Drive a Wing-Crack of Length ℓ, for Five Different Fiber Volume Fractions. Parametric Values Used are $\ell_o{}^* = 20$; $D_o = 0.0005$; $\bar{a} = 0.1$; $c = 800$; $K_o = 0.0002$; $s_o = 0.01$.

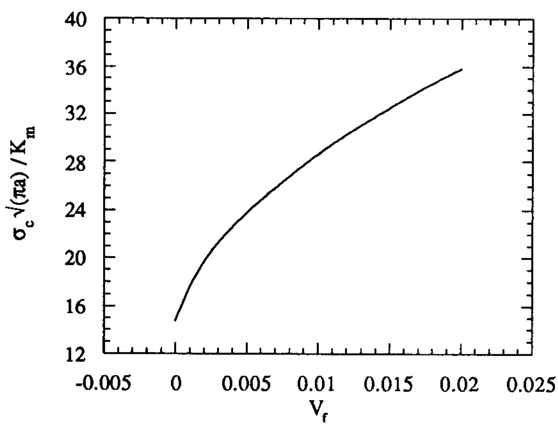

Figure 4b. Strengthening Effect of Fibers: Predicted Compressive Strength Increases with Fiber Volume Fraction, when No Fiber Induced Damage Effect is Included. Parametric Values Used are $\ell_o{}^* = 20$; $D_o = 0.0005$; $\bar{a} = 0.1$; $c = 800$; $K_o = 0.0002$; $s_o = 0.01$.

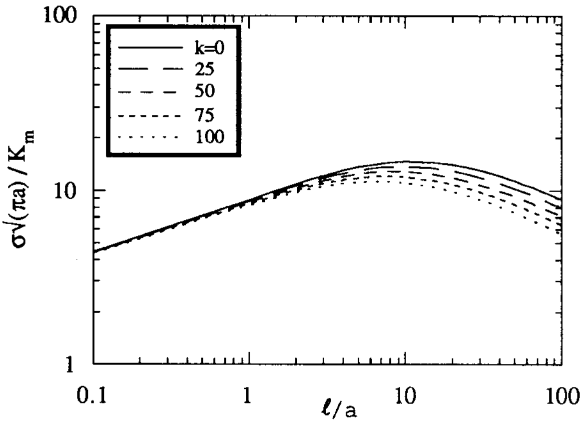

Figure 5a. Damage Effect of Fibers: Normalized Compression Load Required to Drive a Wing-Crack of Length ℓ, for Five Different Fiber Induced Damage Index k ($V_f = 0.01$). Parametric Values Used are $\ell_o{}^* = 20$; $D_o = 0.0005$; $\bar{a} = 0.1$; $c = 0$; $K_o = 0.0002$; $s_o = 0$.

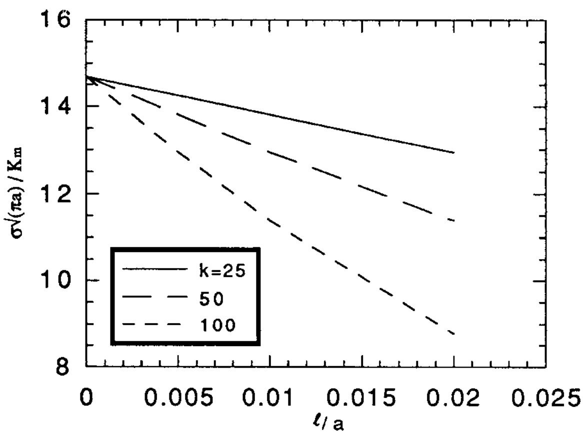

Figure 5b. Damage Effect of Fibers: Predicted Compressive Strength Decreases with Fiber Volume Fraction Due to Fiber Induced Damage Effect. Parametric Values Used are $\ell_o{}^* = 20$; $D_o = 0.0005$; $\bar{a} = 0.1$; $c = 0$; $K_o = 0.0002$; $s_o = 0$.

The parameter k is probably dependent upon the fiber type and processing techniques and has to be evaluated by experimental investigation for particular fiber-matrix systems.

Now we can evaluate the damage effect of the fibers. For the sake of clarity, we assume the fiber reinforcement index $s = 0$ (resulting in $c = s_o = 0$) in equation (12) so that the beneficial effects of fibers are suppressed. Figure 5a illustrates the damage effect of the fiber on the stress-crack length response for different values of k for a given volume fraction $V_f = 0.01$. As expected with increasing k, strong crack interaction reduces the compressive strength. Figure 5b indicates the negative influence of fiber addition on compressive strength.

5. COMBINED STRENGTHENING AND WEAKENING EFFECT OF FIBER ADDITION

Eqns. (12) and (13) may be used to study the effect of fiber on compressive strength in FRCs, when microcrack sliding resistance, wing-crack growth resistance, and damage introduction are operational simultaneously, as is suggested by experimental data such as that shown in Figure 1. Figure 6a shows the normalized compression load required to drive a wing-crack of length ℓ, for various fiber volume fractions. In Figure 6b, we show that the compressive strength may continue to rise even beyond 4% when the fiber damage index is small (e.g. $k = 25$), but rapidly drops beyond 0.4% when the fiber damage index is large (e.g. $k = 100$). In between these extremes, compressive strength is seen to rise initially with fiber volume fraction, and then decreases with additional amount of fibers. These predictions of fiber induced compressive strength changes are in qualitative agreement with the experimental data shown in Figure 1.

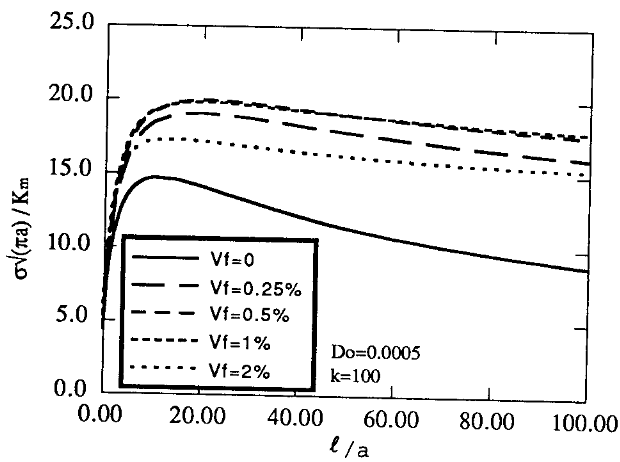

Figure 6a. Combined Strengthening and Damage Effect of Fibers: Normalized Compression Load Required to Drive a Wing-Crack of Length ℓ, for Five Different Fiber Volume Fractions. Parametric Values Used are $\ell_o^* = 20$; $D_o = 0.0005$; $\bar{a} = 0.1$; $c = 800$; $K_o = 0.0002$; $s_o = 0.01$, and k=100.

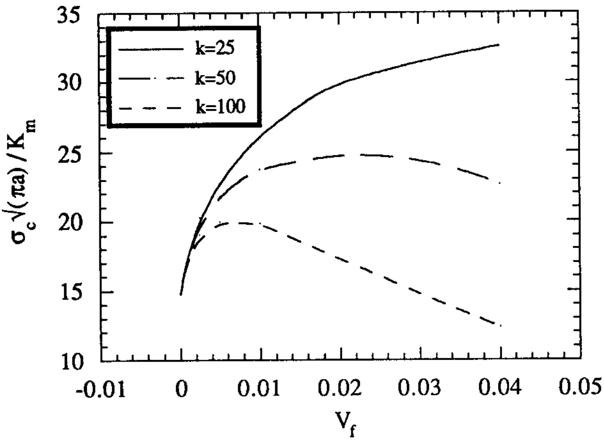

Figure 6b. Combined Strengthening and Damage Effect of Fibers: Predicted Compressive Strength Change with Fiber Volume Fractions, for Different Fiber Induced Damage Index k. Parametric Values Used are $\ell_o^* = 20$; $D_o = 0.0005$; $\bar{a} = 0.1$; $c = 800$; $K_o = 0.0002$; $s_o = 0.01$.

Figure 7 shows model predictions for the Krenit (a polypropylene) fiber reinforced concrete data (also shown in Figure 1). The original compressive strength data shows a higher value for the 28 days composite in comparison to the 7 days composite. When the data is normalized with respect to the plain concrete ($V_f = 0$) compressive strength, the relative strength for the 7 days composite lies above that of the 28 day composite. Common parametric values chosen for both sets of data are: $\ell_o^* = 40$; $D_o = 0.002$; a = 6mm; L_f/d_f = 12mm/.08mm = 150 (an effective diameter is used for the thin film-like fiber); τ = 2 MPa, μ = 0.5, and k = 250. The compressive strength of the plain concrete at 7 days is 23.1 MPa, and at 28 days is 38.8 MPa. Based on this, the elastic modulus and matrix fracture energy are estimated at 20 GPa, 150 N/m, and 30 GPa, 200 N/m for the composites of the two ages, respectively. The corresponding K_m values are then 1.75 MPa $\sqrt{m}$ and 2.5 MPa $\sqrt{m}$. All the non-dimensional parameters needed as model input can be calculated from this set of parameteric values. Reasonable comparisons can be found between experimental data and theoretical predictions (Figure 7). However, it should be mentioned that there is plenty of uncertainty in the exact parameteric values (since they are not measured), although the numbers used should not be too far off.

6. FIBER/MATRIX INTERACTION AND FIBER GEOMETRY EFFECT

It is interesting to note that while excessive amounts of fiber for a given fabrication process can lead to compressive strength degradation, alteration of fiber/matrix interaction property, or the fiber geometry, can lead to beneficial effect without the attendant damage introduction. For example, it is conceivable that fiber/matrix bond strength or the snubbing factor could be increased without causing a rise in the initial amount of damage. This is in fact one of the assumptions behind eqn. (12), and we illustrate this idea with Figure

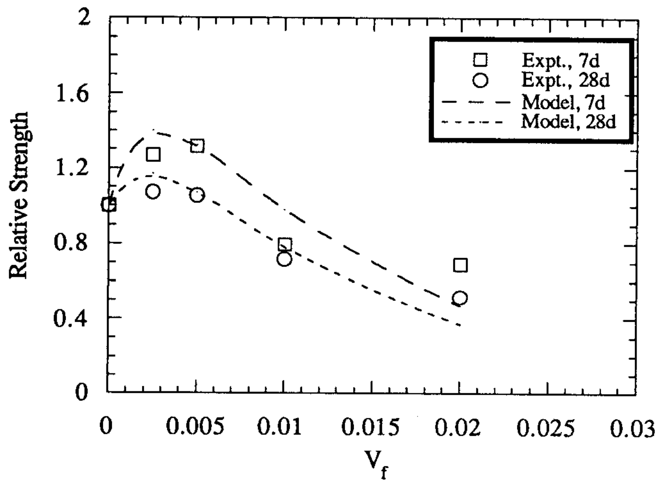

Figure 7. Model predictions for the Krenit fiber reinforced composite.

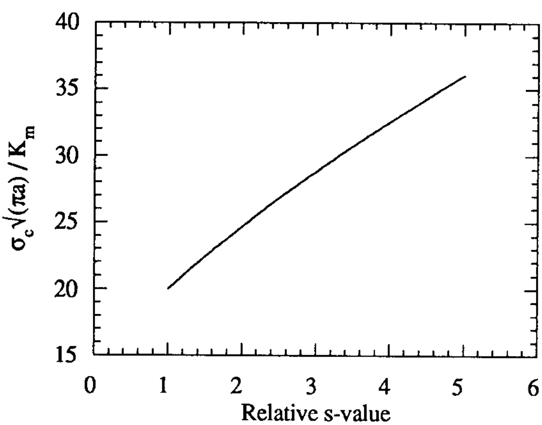

Figure 8. Compressive Strength Increase with Fiber Reinforcement Index s. The s -values have been normalized by the reference magnitude of s such that $s_o = 0.01$ and $c = 800$ as used in all the preceding calculations. Other Parametric Values Used are $\ell_o^* = 20$; $D_o = 0.0005$; $\bar{a} = 0.1$; $c = 800$; $K_o = 0.0002$; $s_o = 0.01$.

8, which shows the relationship between compressive strength σ_c and the reinforcement index s, at a fixed fiber volume fraction and fiber damage index ($V_f = 0.01$ and $k = 100$).

Note that the reinforcement index s as defined in eqn. (8), is directly proportional to g, τ_f, and L_f/d_f. Therefore variation in s may be interpreted as variation in any one of these parameters, with the others fixed. Figure 8 indicates a monotonic increase in the compressive strength with the s - value, suggesting the importance of these parameters in controlling the compressive strength in FRCs. However, it should be pointed out that the amount of initial damage may be expected to increase with fiber aspect ratio, even though this notion has not been incorporated in the present model.

7 FURTHER DISCUSSIONS AND CONCLUSIONS

Although the microcrack sliding model of compressive strength in brittle material has been discussed (eqn. (6)) in the context of uniaxial loading in the present paper, extensive studies (Horii and Nemat-Nasser, 1986; Ashby and Hallam, 1986; Kemeny and Cook, 1991) have shown that the compressive strength is very sensitive to confining stresses. This can be seen in the sensitivity of the stress intensity factor of the wing-cracks to normal compressive load. This notion is in accord with experience in cementitious materials, for which confinements are general prescriptions to derive higher compressive load bearing capacity. The present model of compressive strength for FRC shows that fibers can be exploited to increase the compressive strength and may therefore act as a passive confining pressure. This passive confinement idea was first proposed by Yin et al, (1990), who discovered this beneficial effect of fiber in a series of biaxial steel FRC tests.

The positive effect of fibers on the elastic modulus of composites has received extensive attention (e.g. Tandon and Weng, 1986; Wakashima and Tsukamoto, 1991). However, it is also well known, at least for some cementitious composites, that fiber can degrade the composite modulus to below that of the matrix modulus (e.g. Naaman et al, 1991). It is likely that fibers would induce a competing process of modulus improvement and degradation in cementitious composites, in the same manner that it influences compressive strength. In fact, many of the basic elements of the present work could be applied to analyze FRC elastic modulus.

The present work represents a preliminary look at how fibers in FRC contributes or degrades composite mechanical properties. The results based on the present model appear to capture much of what has been experimentally observed in compressive strength change due to fiber addition. These modelling results (particularly Figures 4 and 8) indicate that fibers can significantly improve compressive strength of FRCC if the weakening effect of fiber is controlled via novel processing routes. A difficulty in applying the current model, however, lies in the lack of knowledge in some micromechanisms and micromechanical parameters. These include, for example, the detail micromechanisms in the way fiber resist microcrack sliding, and the general unavailability of parameteric values of a, D_o, and k. Additional research is required to tackle these issues. The present work provides a framework for which these future research should be organized.

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