Computer-Aided Learning in the Mechanics of Solids and Structures

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Introduction

The mechanics of solids and structures is a fundamental subject in most undergraduate curricula in civil and mechanical engineering. Although the emphasis may vary somewhat from school to school and from department to department, it should not be unfair to say that one of the most important objectives in teaching such a course is to instill in our students an appreciation of how solids and structures behave under loads, for therein lie the building blocks for more sophisticated analysis and design of engineering structures. Therefore we have approached all topics of discussion, whenever appropriate, by asking the question: How would the behavior of the solid or structure alter by varying the loads (magnitude and/or configuration), the geometry of the structure, or the material of the structure? By behavior, we mean the stress distribution and the deformation field which are often related to the failure mode and the rigidity of the structure. After studying the basic notions of force equilibrium, geometric compatibility and stress-strain relations, methods of analysis (such as Airy stress functions, superposition techniques etc.) are introduced to get the behavioral response of certain structures and loading conditions. The implication of behavioral response is then studied in relation to engineering practice.

How can the computer help in teaching the behavior of solids and structures? The first level of computer usage may be the graphical illustration of the deformed shape of the structure, the distribution of stresses and strains in the body and how these parameters change with input load and selection of geometry and material (linear elastic). A second and higher level at which to use the computer is to build a tutorial system which incorporates elements of artificial intelligence and which provides more individual coaching to the students. We have so far concentrated on the first level, with the understanding that the library thus built would be directly exploited to advantage in the second level development.

We have built the library with these specific purposes in mind: 1) To graphically display the stress and deformation fields as a function of position in the body. This is aimed as an alternative to extracting information from complicated algebraic expressions. 2) Flexibility in alteration in load, structural geometry, and material specification to allow and encourage experimentation with these parameters in order to build an intuitive feel of the behavioral response. 3) To allow extension of studying simple cases to cases with more complicated loading and geometry, especially in homework problems, without inordinate amounts of mathematical manipulations, thus keeping the focus of the exercises on the behavioral responses and engineering implications.

To achieve these purposes, we have found important to meet several criteria of user-friendliness. The interactive programs must be easy to use so that users with no familiarity of computers would still be able to operate with the instructions provided in the programs. The amount of information provided by each picture generated must not be excessive so that the attention of the student is tightly tied to the essence of the problem. This is also related to the response time of the computer to provide a fresh drawing each time a parameter is varied. It is also for this reason that only problems with analytic solutions are incorporated in the present library. Although such a selection limits the flexibility of geometry or load changes, we were still able to cover most of the materials taught to a one-semester undergraduate class. Certainly, the availability of greater computer power should relax such limitations. We shall mention more of this in the concluding section of this paper.

At the present time we have chosen to display most of the results in a 3-D space, with two axes representing an appropriately selected plane in the structure and the third axis representing the magnitude of the parameter (say, the shear stress component). A hidden line removal program has been used to improve clarity of the computer-image generated.

In the following paragraphs, we shall describe briefly some of the programs which we have developed, giving the purpose of the programs together with the input parameters and output variables available for selection by the user. We have chosen to discuss in more detail two of these programs, in order to give a flavor of how such a library has been used to supplement and expand on class lectures.

Brief Description of the Programs Developed

i) TORSIO: (Figures 1-4)

Purpose: Torsion of unrestrained prismatic members.

Input variables: Cross sectional geometries, including solid and hollow circular, equilateral trianglar, rectangular and elliptical; intensity of applied torque; constitutive material.

Output: 3-D display of shear stress components and warping displacements as a function of position on the cross-section of the member.

ii) HOLE: (Figures 5-8)

Purpose: Study of stress concentration near the edge of a circular hole in a large plate (plane stress) or a circular cavity in a large body (plane strain) subjected to biaxial stressing.

Input variables: Type of model (plane strain or plane stress), constitutive material, loading magnitude in each direction (zero in one direction means uniaxial loading).

Output: 3-D display of all stress and strain components as a function of position in the plate (plane stress) or cross-section of the body (plane strain).

iii) CYLIN: (Figures 9-12)

Purpose: Analysis of thick-walled cylinder (plane strain) or disk (plane stress) under internal and external pressure.

Input variables: Type of model (plane strain or plane stress), constitutive material, applied pressures--internal and/or external, and the inner radius of the cylinder.

Output: 3-D display of stress components and radial displacement as a function of position in disk or on cross-section of thick-walled cylinder.

iv) KPERI: (Figures 13-15)

Purpose: Study of stress concentration factor along the perimeter of an elliptical hole in a large plate subjected to remote in-plane normal load directed along one axis on the ellipse.

Input variables: Aspect ratio of elliptical hole (aspect ratio=1 reduces to a circle).

Output: 2-D plot of concentration factor as a function of angular position along edge of hole.

v) CRACK: (Figures 16-18)

Purpose: Comparison of hoop stress magnitudes ahead of a mode I crack with that ahead of an elliptical hole subjected to remote load directed along minor axis.

Input variables: Aspect ratio of elliptical hole (large aspect ratio approaches a flat crack).

Output: 2-D plot of normalized stress as a function of distance from tip of crack or elliptical hole, superimposed display for comparison upon request.

vi) CRTIP: (Figures 19-22)

Purpose: Stress distribution around the tip of a crack under mode I or mode II loading.

Input variables: Fracture mode, corresponding loading magnitude (mode I: in-plane normal load, mode II: Shear-load) semi-crack length, and constitutive material (stress cut-off level).

Output: 3-D display of stress components surrounding the crack tip.

We next describe in some detail the use of the programs TORSIO and HOLE and their relation to classroom discussions, to give a representative view of how computer graphics may be exploited to advantage in the learning environment.

In the discussion of torsion of members, after the stress and displacement fields have been derived, we focus on how the structure spreads the torsional load across the cross-section. In particular, the location of maximum and zero stresses are related to efficiency of use of material. The expressions for the stresses and displacements are often so complicated that a physical "feel" of the behavior of the member is lost to the student. For example, the warping displacement of a member with rectangular cross-section may be derived from the Prandtl stress function given by Timoshenko and Goodier[1]:

$$w(x,y) = \Theta \left\{ xy - \frac{32\,a^2}{\pi^3} \sum_n \frac{(-1)^{\frac{n-1}{2}}}{n^3} \left\{ \frac{\sinh(\frac{n\pi y}{2a})}{\cosh(\frac{n\pi b}{2a})} \right\} \sin(\frac{n\pi x}{2a}) \right\} \tag{1}$$

where n = 1,3,5,...

Θ ; angle of twist per unit length.

The origin is at the centroid of the section; x is measured along the width (2a) and y along the height (2b).

Figure 2 provides a visual aid, showing the saddle shape of the warped cross-section. To many students, such a picture provides a better impression of the deformed cross-section of the member than equation (1). Further, the student can "experiment" with members of various slenderness ratios, going from a very thin rectangle to a square, and watch the change of the warped cross-section. He may also observe that the warping diminishes with increasing material rigidity (e.g. in comparing wood with steel) for a given applied torque. In checking the shear stresses, (Figure 3) the student may observe that the corners carry no stress at all, something that he has already expected, as he recalls that the boundary condition on the traction free lateral surface of the member requires this to be so. He concludes that these corners are "dead" material which do not contribute to the rigidity of the structure. The computer graphic display also confirms his intuition that the maximum shear stresses occur on the outermost fibers parallel to the perimeter of the cross-section (and at the mid-point on each edge). The student realizes that the membrane analogy discussed in class would have predicted this observation.

Of course it would be too much to expect all students to have already acquired a good intuition when they start to use the mechanics/graphics library, or even to be able to ask the right questions. It is here where homework problems may help, in which the questions are directed to focus the students' attention on particular aspects of the picture they are seeing on the screen, as well as encouraging the students to explore through interaction with the program. As indicated by Figures 1-4, TORSIO provides plenty of opportunities for exploration into effects of structural behavior through control of various parameters.

The second program we shall look at in some detail is HOLE, which studies the effect of geometric discontinuity on stress concentrations. This problem can represent that of a thin plate with a circular cavity undergoing plane strain deformation. Except for the out-of-plane component the stress field is the same for both deformation modes, while the strain field is slightly different and depends on the elastic constants. Apart from the effect of the deformation mode, this problem affords the study of the effect of a variety of loading configurations on the distribution of stresses and strains. Uniaxial or biaxial loading of any magnitude and sign (tension or compression) may be applied. Indeed for that matter, the effect of uniform shear loading may be studied by applying a tensile load in one direction and a compressive load of the same magnitude in the perpendicular direction and then rotating the body by 45 degrees.

In Figure 7, e.g., the student studies the tangential stress in the body subjected to uniaxial loading of ten units in the x-direction. He observes that the tangential stress got amplified by a factor of three at two points on the edge of the hole and a compressive stress induced (the depressed area) also near the hole. The stress concentration is seen to be confined to within an area a couple of diameters of the hole. The radial stress (Figure 8) rises from zero at the hole (required by the traction free boundary condition there) to the applied load level along a line parallel to the loading axis and through the center of the circular hole. Along a perpendicular line, however, the radial stress reaches a peak a couple of diameters from the hole and then rapidly dies to zero. After some thinking, the student figures that at some distance from the hole, the body "forgets" that it has a hole and so must experience a simple uniaxial stress field. Figure 6 shows the out-of-plane thinning (ϵ_{zz} negative) and thickening (ϵ_{zz} positive) of a slab of rock under the same loading condition.

Conclusion

Our experience in the use of this library by our students (mostly seniors) indicates that these graphics programs are useful in capturing some of the physical behaviors that might have been lost in complicated algebraic equations. For some students, the computer generated pictures stimulate thinking which sometimes induce them to check on the results of analysis presented in class. We have also found that good homework questions often help students focus on particular aspects of the problem and hence get more out of the interaction with the program. It is conceivable that such questions may eventually be built into the system and so enhance the interactivity.

The work presented in this report must of necessity be evolutionary in nature, as improvements in computer hardware and advance graphics software become available. The programs mentioned above have therefore been written in modular form, in expectation of further improvement. We also see the possibility of including a much larger class of problems, summarized in Appendix A, some of which are currently under development. It seems appropriate to conclude this paper with what we see as future outlook in software development for teaching mechanics of solids and structures.

Project ATHENA is an experiment at MIT in the effective use of computer for significant advances in engineering education. An extensive and coherent network of advanced microcomputers from Digital and IBM are being installed on campus. The most important impact on our software development is that computer power will be increased tremendously and that advanced workstations (VS100) provide opportunities unavailable to us previously. For example, color and split window features can be used to advantage. Also, we are no longer bound to present only problems which have analytic solutions. Rather, we foresee the use of discrete element techniques to incorporate problems whose geometry or loading configuration may deviate from those that can be handled by analytic methods. This flexibility allows the exploration of more realistic structures and make a closer tie to engineering design. We have also set up plans to develop a computer-aided tutorial system for many of our undergraduate courses. The computer-aided tutorial system utilizes elements of artificial intelligence and share resources between various modules. Its purposes include providing our students with an effective learning environment in fundamental concepts, principles and analysis techniques of mechanics applicable to physical systems encountered in Civil Engineering. It also aims at developing the intuitive feel for the mechanical behavior and the ability to reduce complex physical systems to models for engineering analysis for our students. Teaching of engineering design is part of the tutorial system. This ambitious program is a subject of research in the next several years by several faculty members in our department. We envision that the work presented in this paper represents a starting point of an effective computer-aided tutorial system in the teaching of mechanics of solids and structures, and becomes part of a larger effort in integrating advanced computer technology and software as an effective tool in our education program.

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Reference

1. Timoshenko and Goodier, Theory of Elasticity, McGraw Hill, 1970.

Appendix A--Some further problems to be incorporated into the Computer-aided-learning software.

The problems mentioned below represent some on-going programming activities which should be completed in the near future. They do not represent any limitation of what is needed to be built. The concluding section of the main text presents briefly some visions of the future in using computers to improve the learning environment.

1. Stress transformation: Display stresses acting on a differential element at arbitrary orientation. Compute and display principal stresses in principal directions. Act as a service module.

2. Half-space problems: Display stress and deformation fields associated with Boussinesq and Cerruti problems. Extension to general surface loaded half-space by superposition technique.

3. Some fundamental solutions: Display stress and deformation fields associated with a point force in an infinite elastic medium (Kelvin's solution), and the same with a dislocation.

4. Plastic yielding in thick-walled cylinder under internal pressure: Display stresses and location of boundary between elastic and plastic region.

5. Stress distribution in thin shell: Membrane stresses in spherical tanks, cone roofs etc..

6. Plate bending: Stresses and deformed shapes of flat plates.

VICTOR C. LI

Victor C. Li has been an assistant professor in the Department of Civil Engineering at the Massachusetts Institute of Technology since 1981. His research interests include mechanics of construction materials and mechanics of earthquake ruptures. He teaches courses in Engineering Mechanics at both the undergraduate and graduate levels. In 1983 and 1984, Victor C. Li was named the Esther and Harold E. Edgerton Professor. He has a BA (Econ., 1977), a BS (M.E., 1977), an MS (M.E., 1978) and a PhD (Solids & Structures, 1981), all from Brown University.

Claude Poirier is a 1980 civil engineering graduate from École Polytechnique de Montréal, Québec, Canada. In 1982, he obtained a Master's degree (M.Sc.A.) from the same institution. He entered M.I.T. the same year and is currently a graduate research assistant in the Constructed Facilities Division of the Department of Civil Engineering. His areas of interest are in computer applications in Structural mechanical engineering including computer graphics, and in the development of computer-aided learning tools in the same field.

Figure 1: GENERAL INSTRUCTIONS

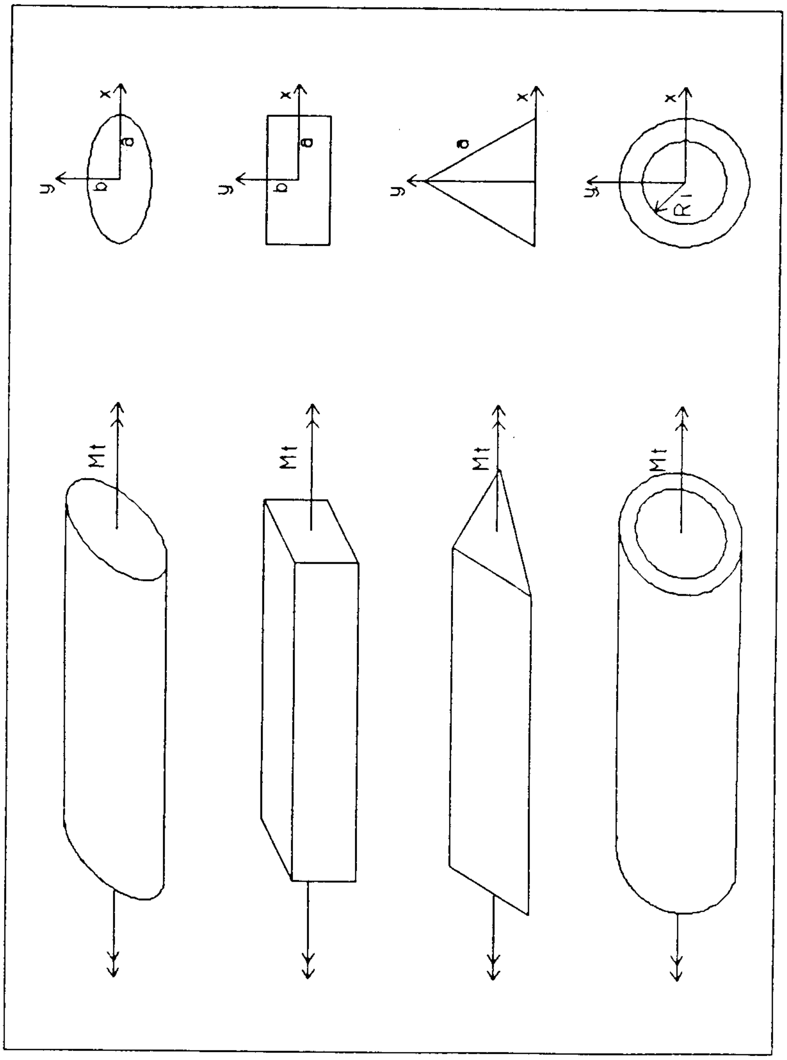
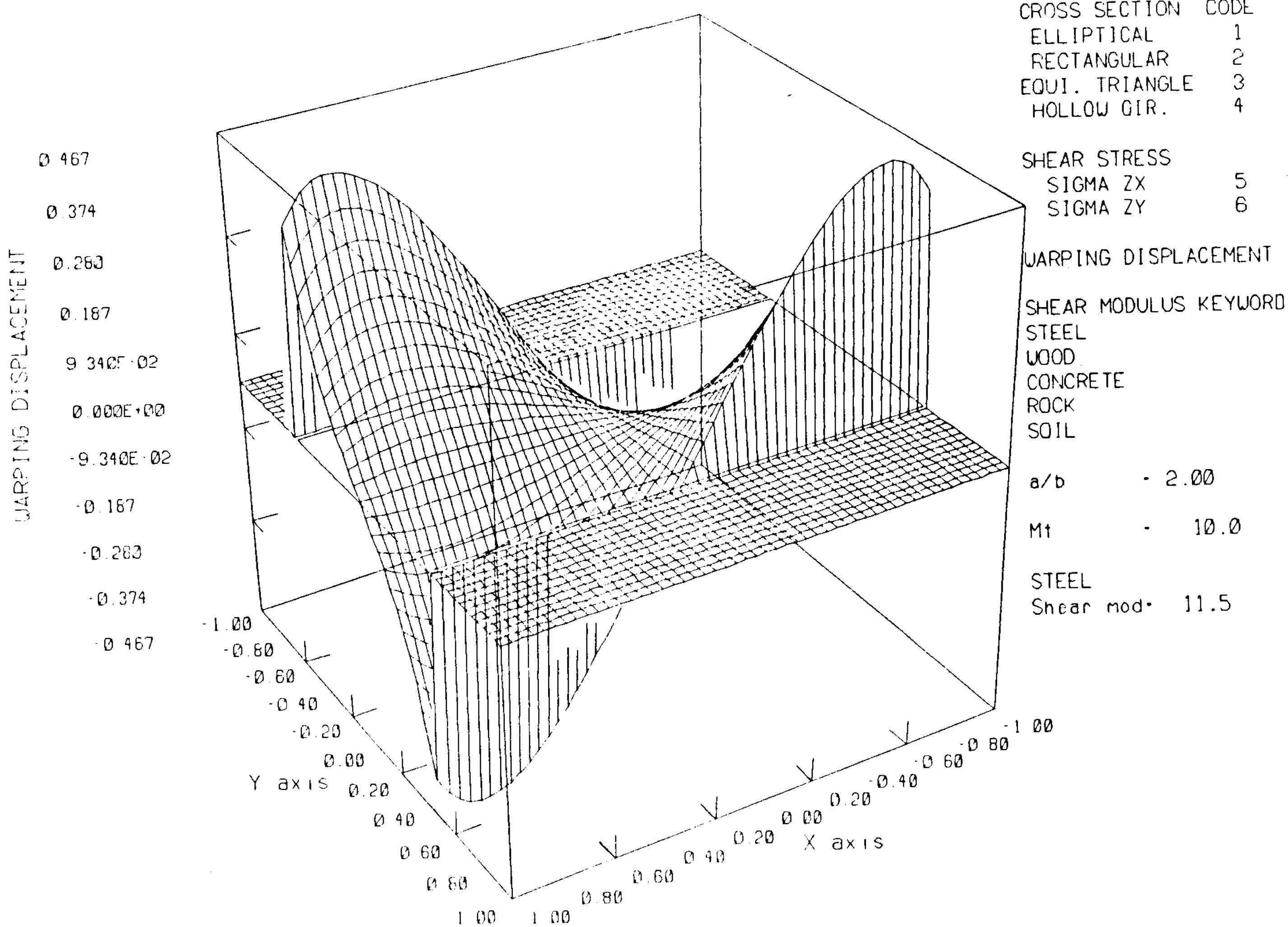


Figure 2: Warping displacement of a rectangular section



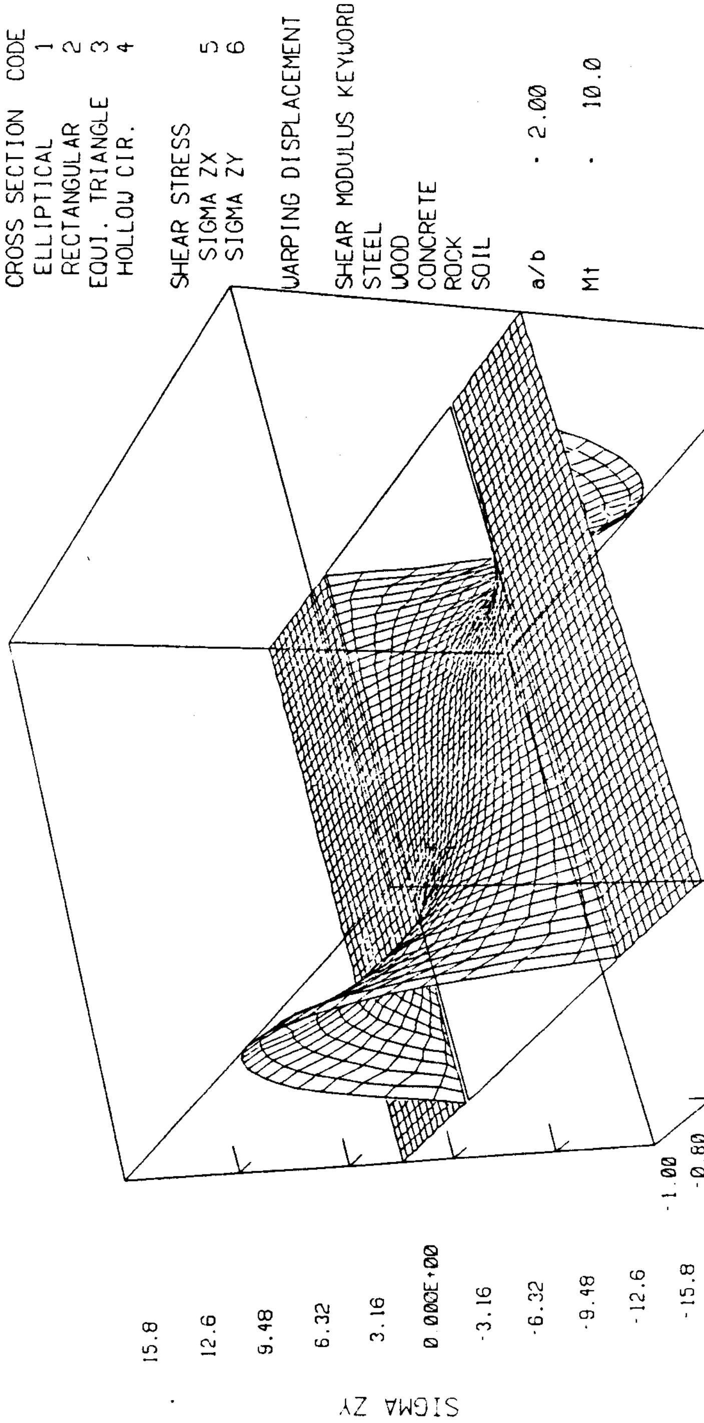


Figure 3: Shear stress in a rectangular section

Figure 4: Warping displacement of an elliptical section

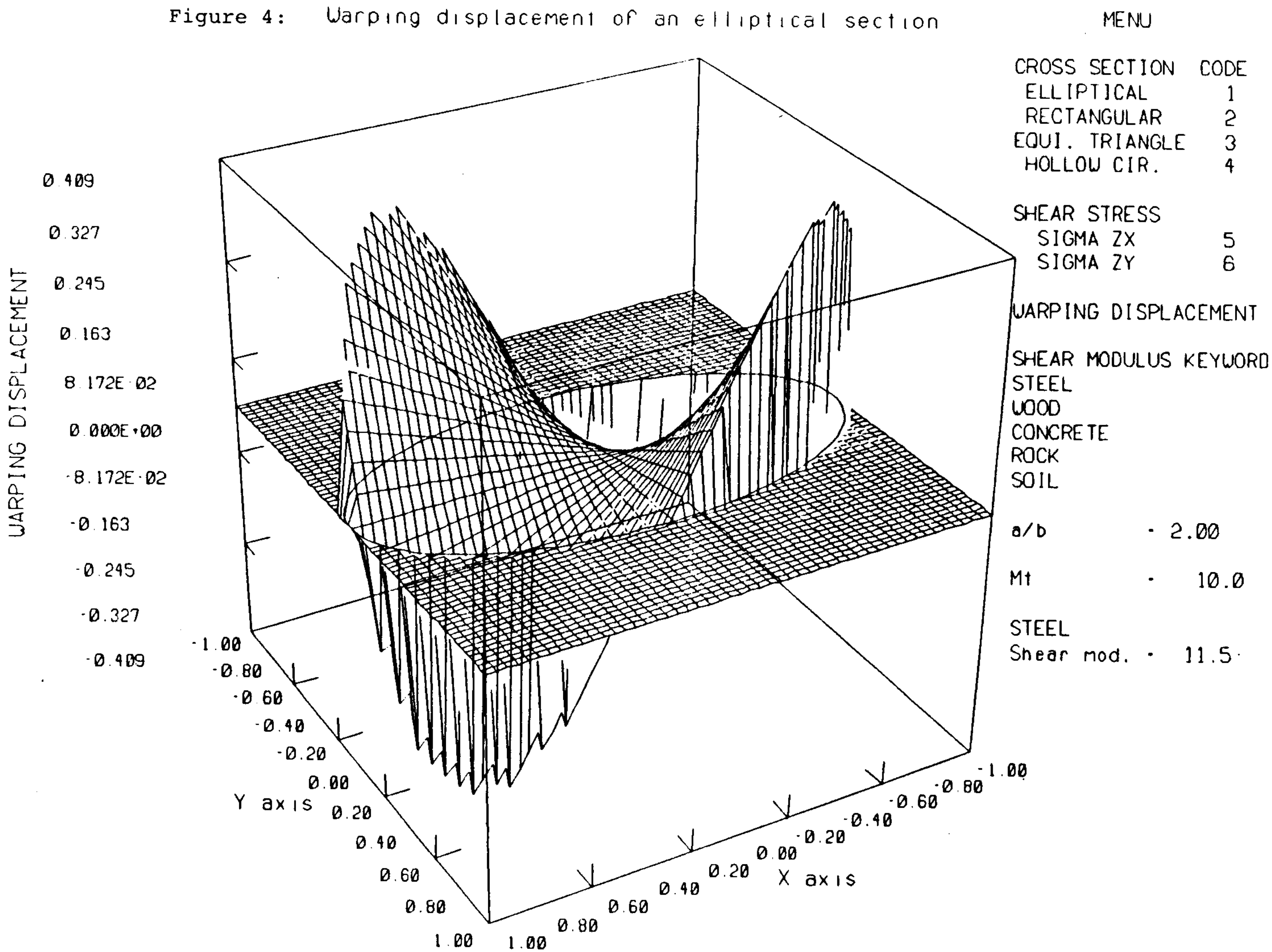


Figure 5: General instructions: Strain convention

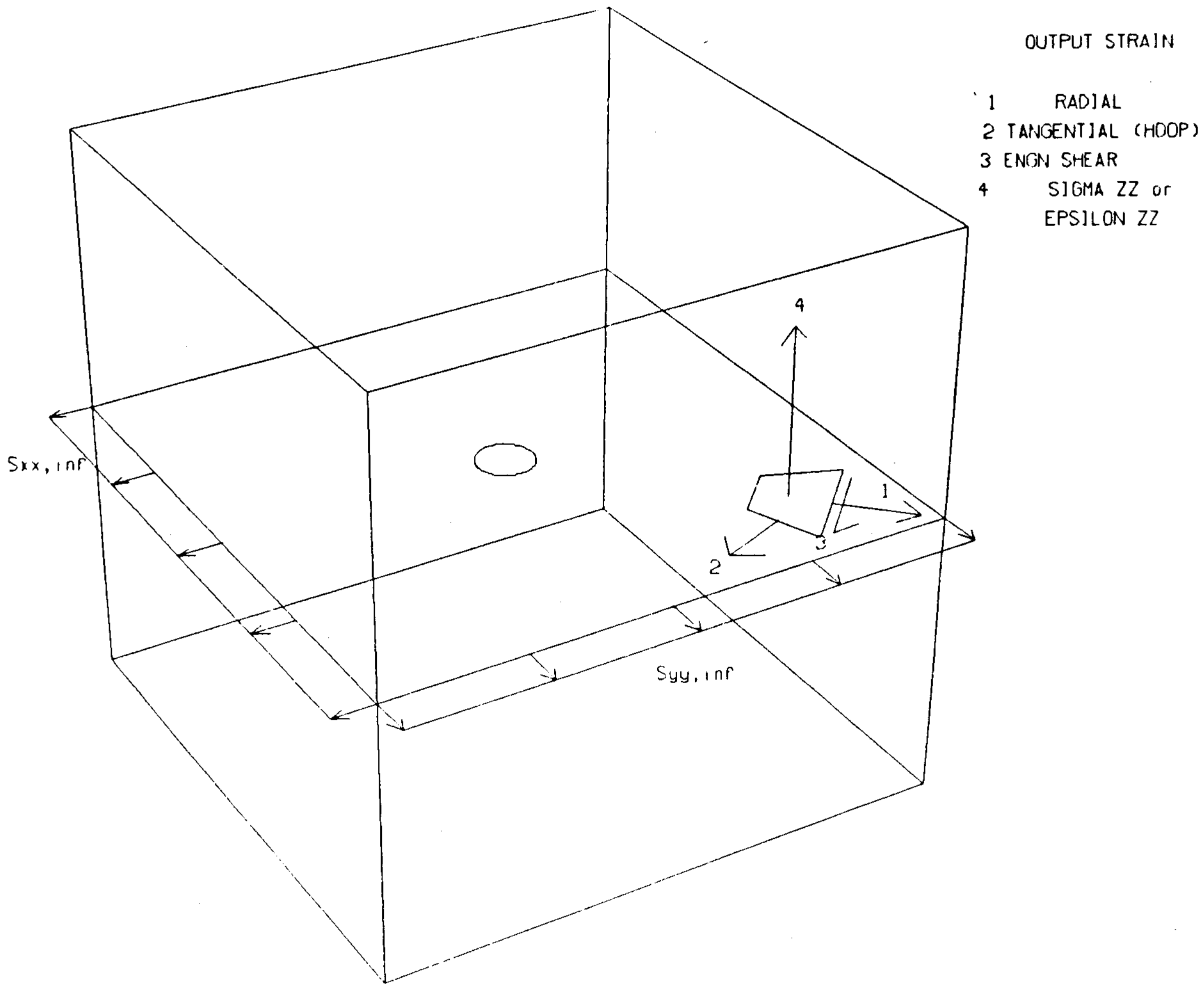


Figure 6: Epsilon ZZ Due To Sigma X

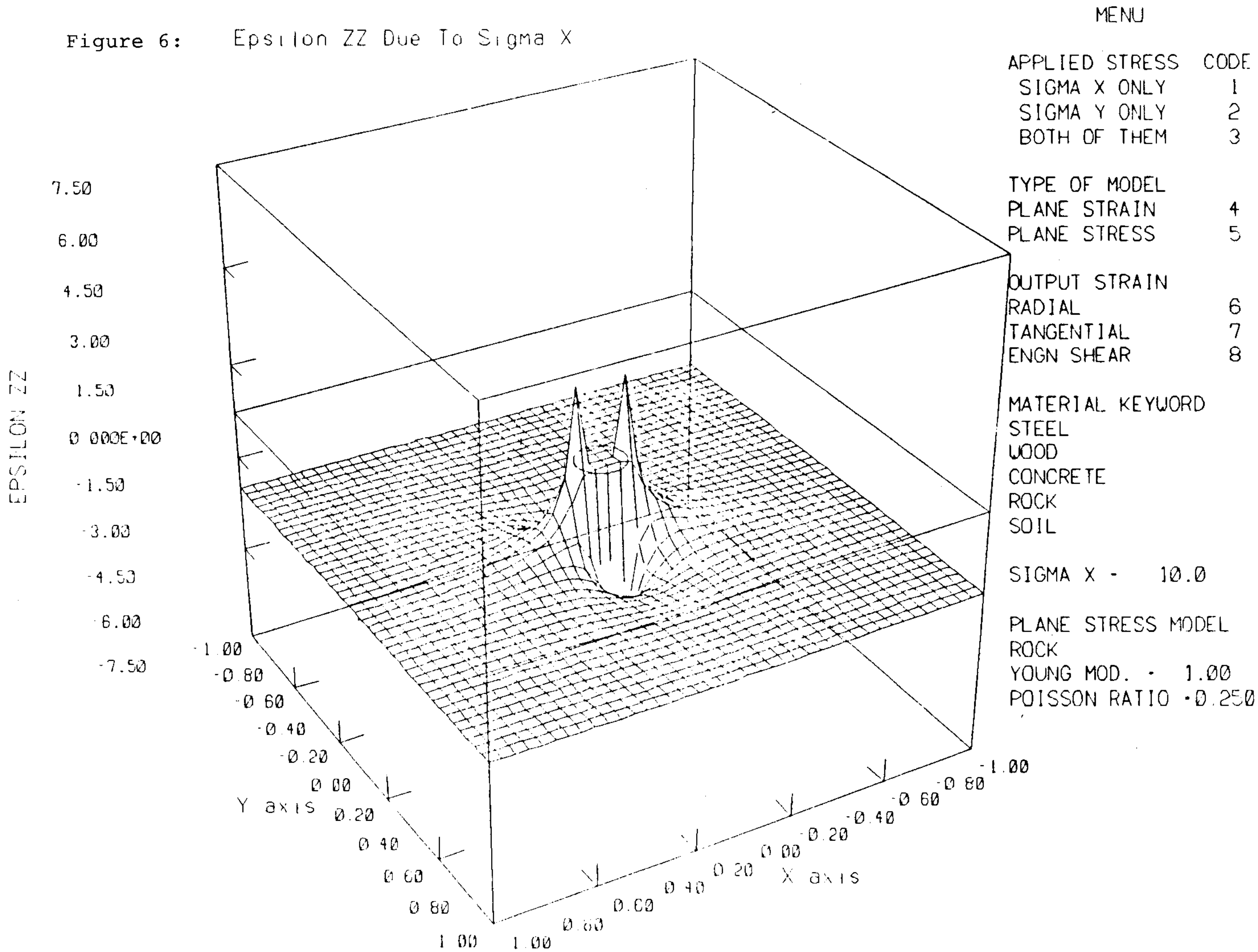


Figure 7: Tangential Stress Due To Sigma X

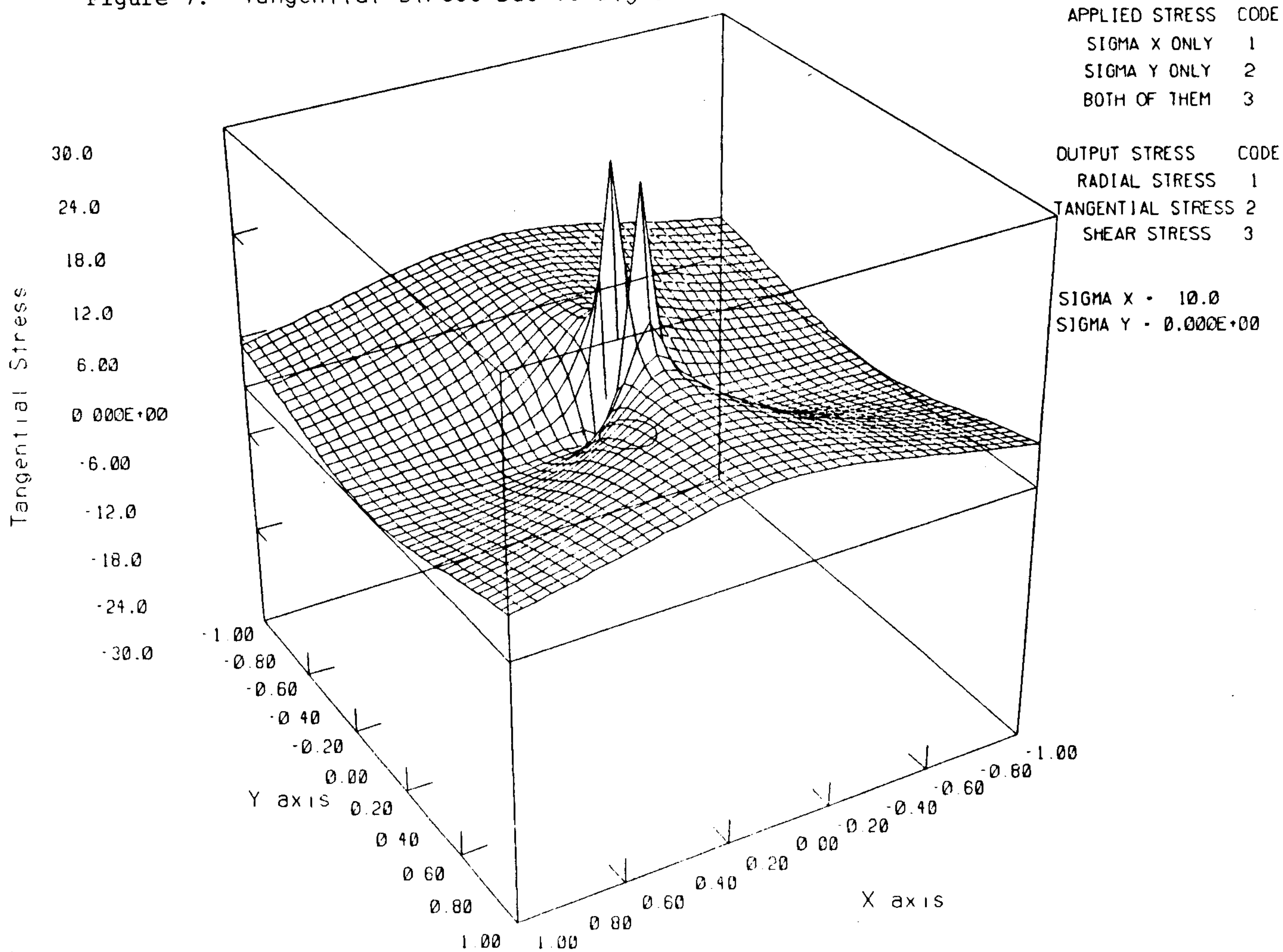


Figure 8: Radial Stress Due To Sigma X

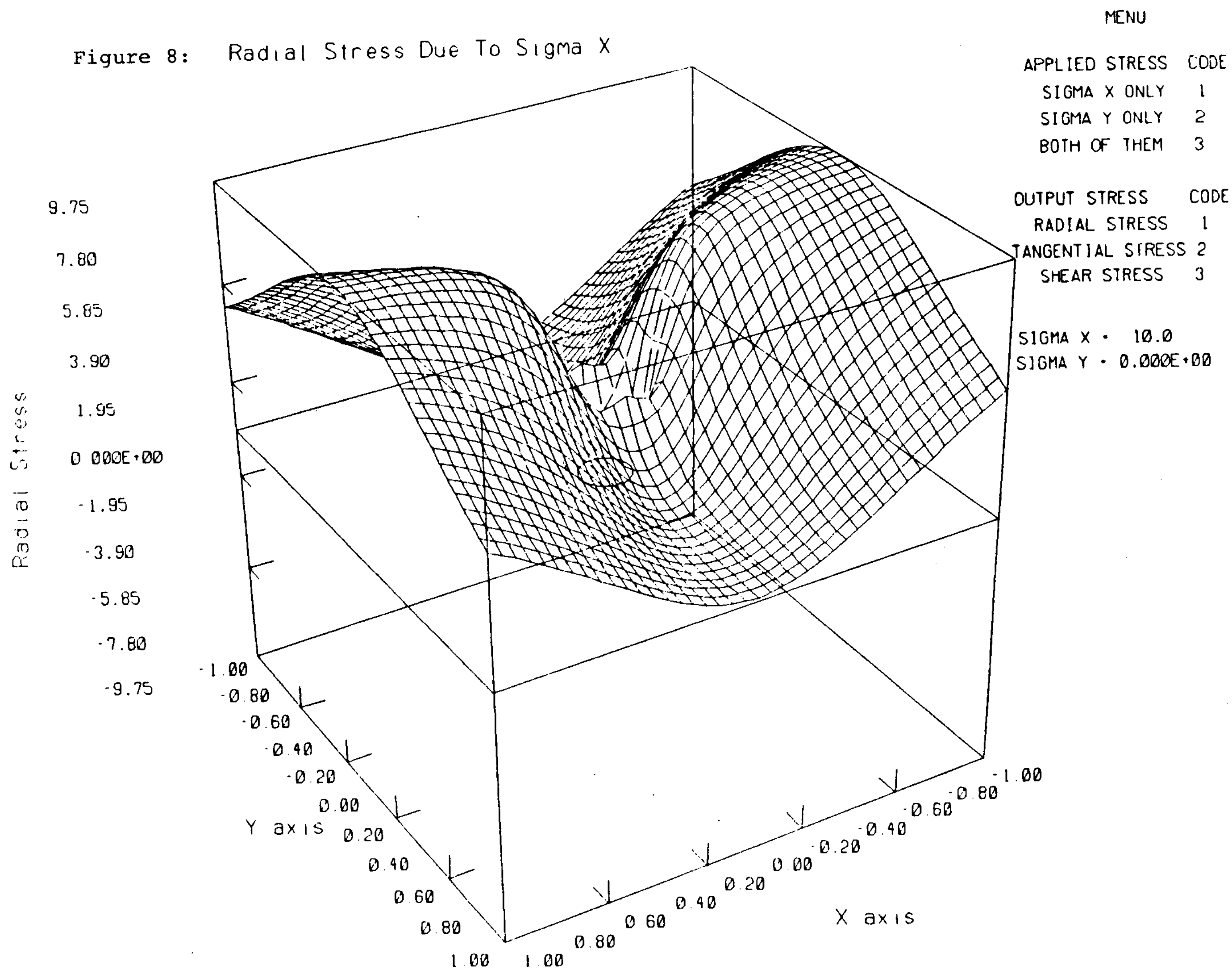


Figure 9

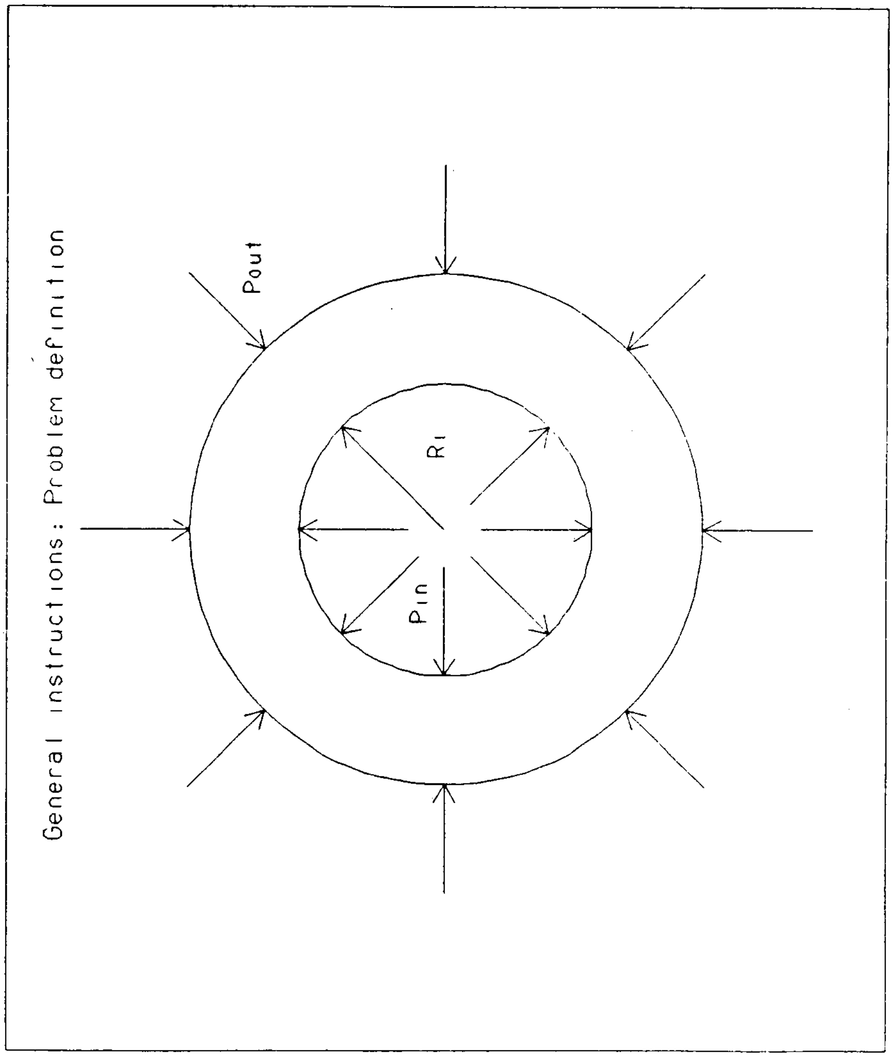


Figure 10: RADIAL STRESS DUE TO INNER PRESSURE

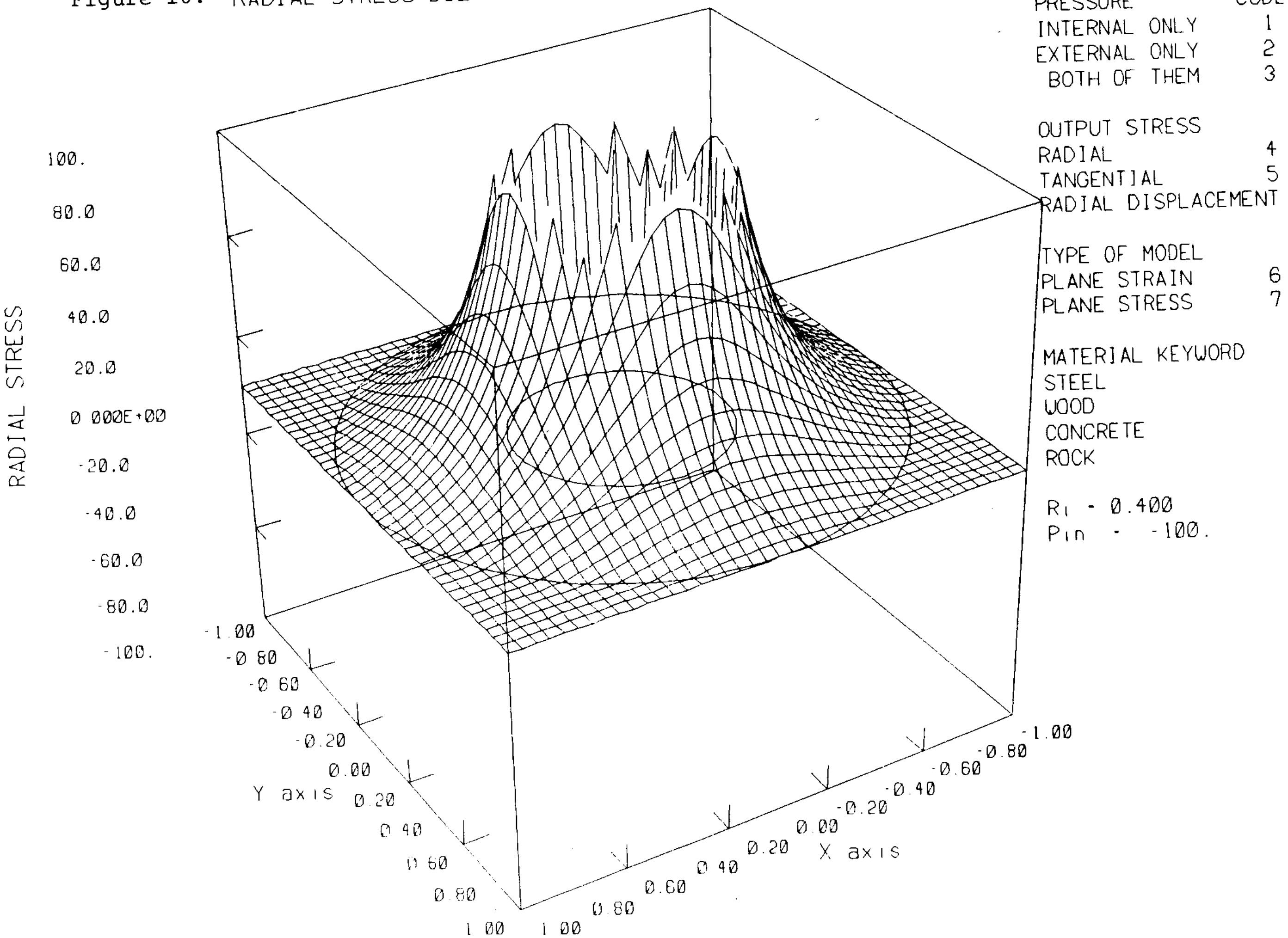


Figure 11: TANGENTIAL STRESS DUE TO INNER PRESSURE

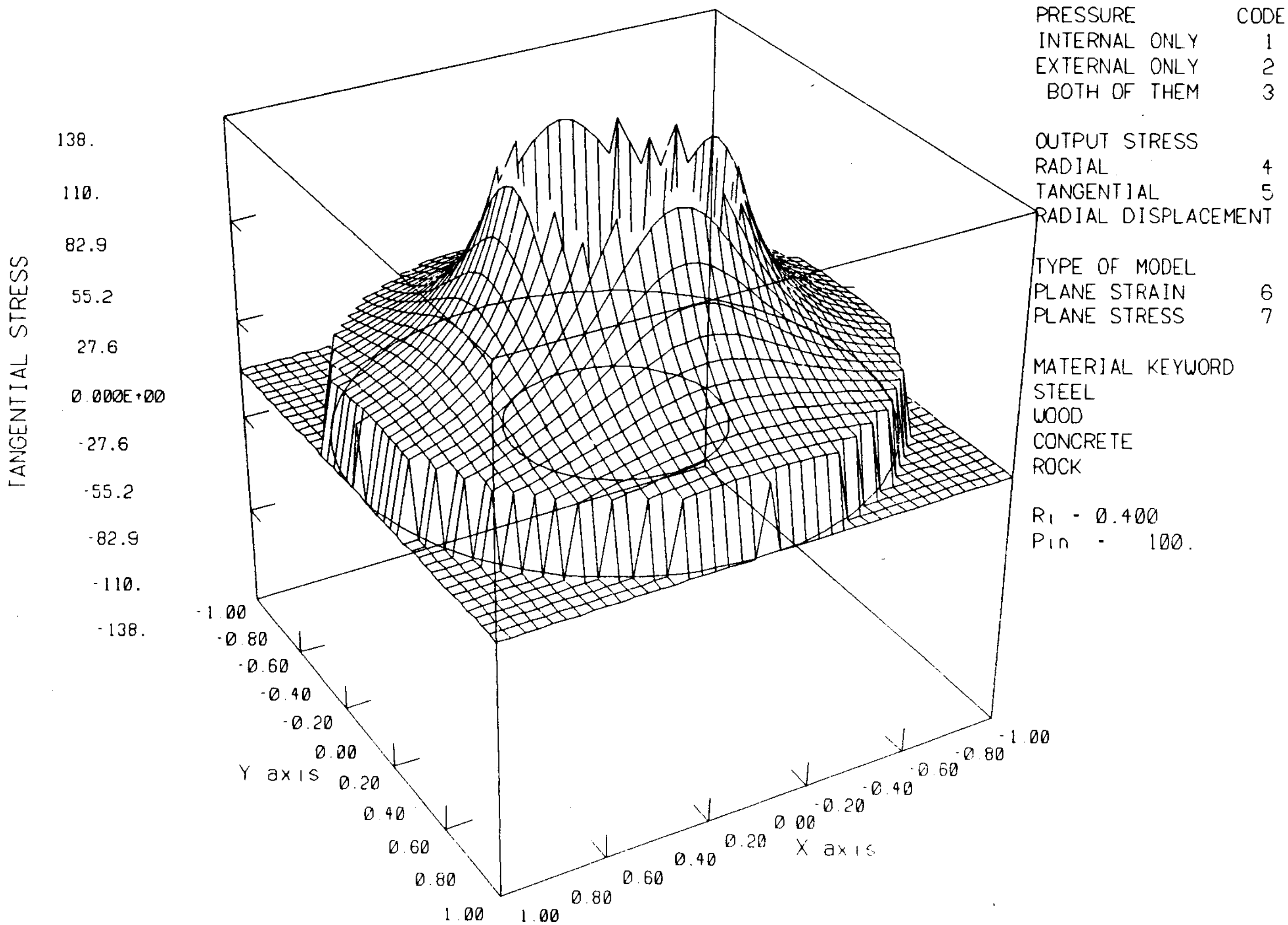


Figure 12: RADIAL DISPLACEMENT DUE TO INNER PRESSURE

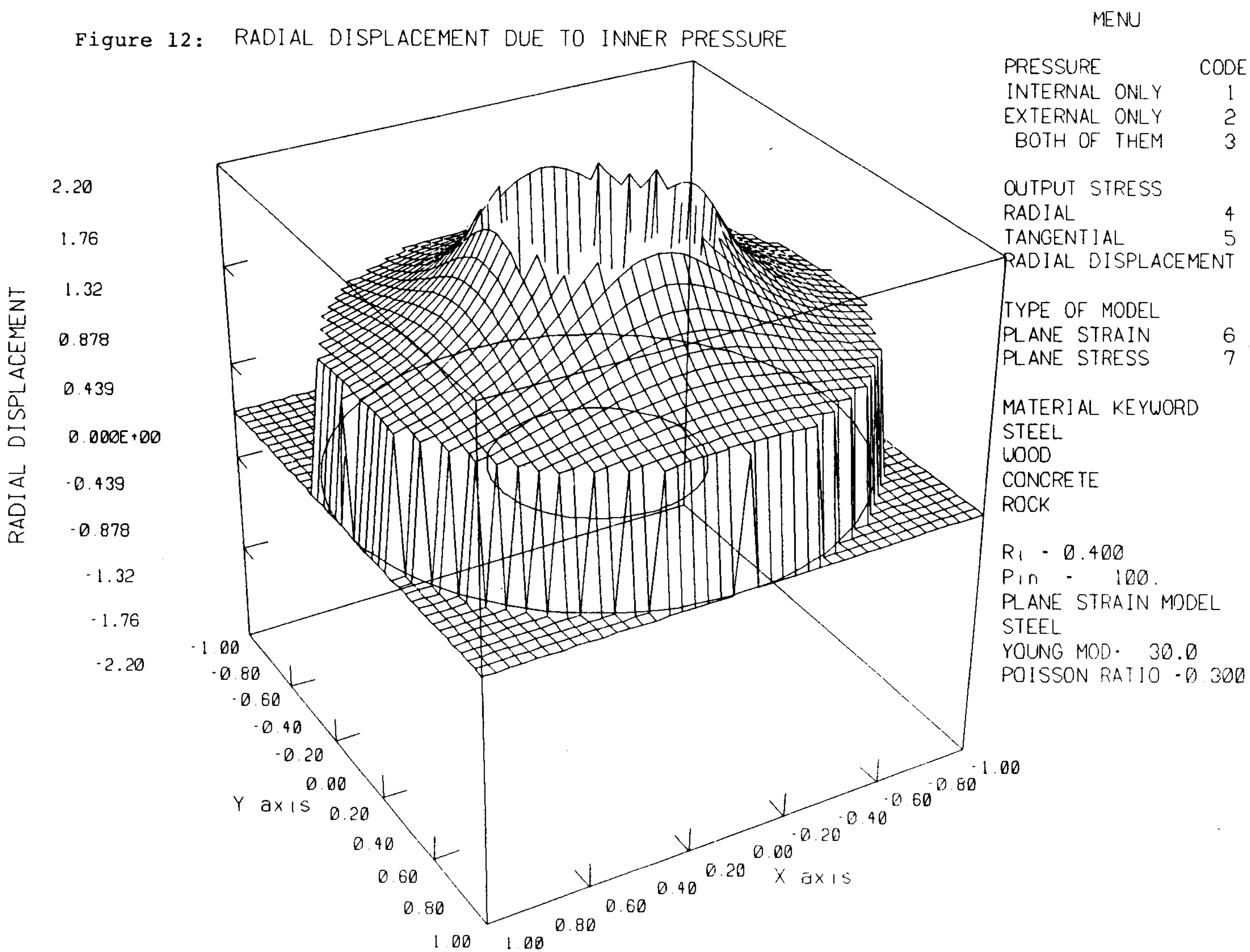
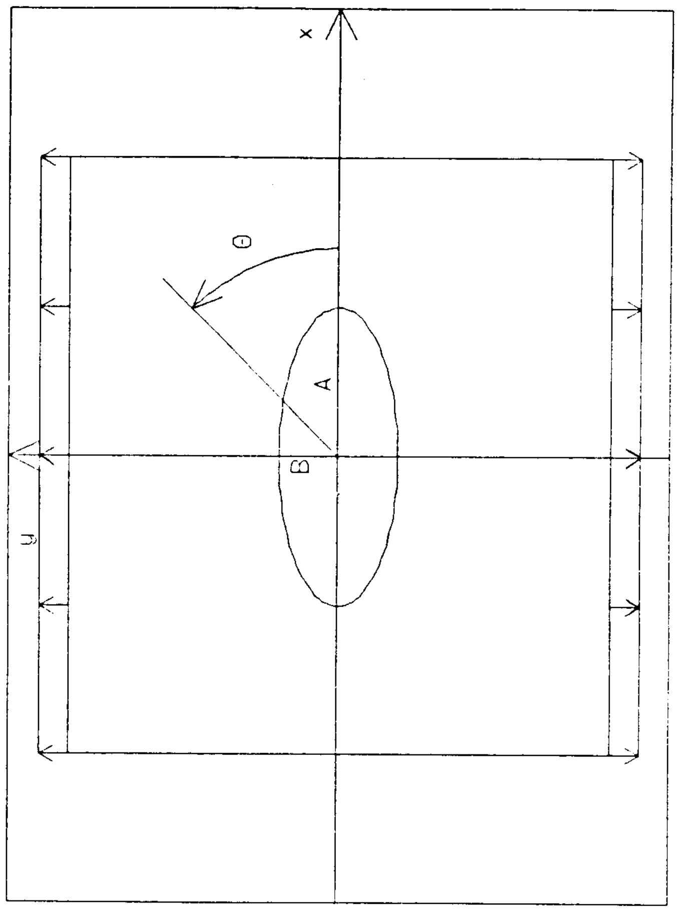


Figure 13

GENERAL INSTRUCTIONS : PROBLEM DEFINITION



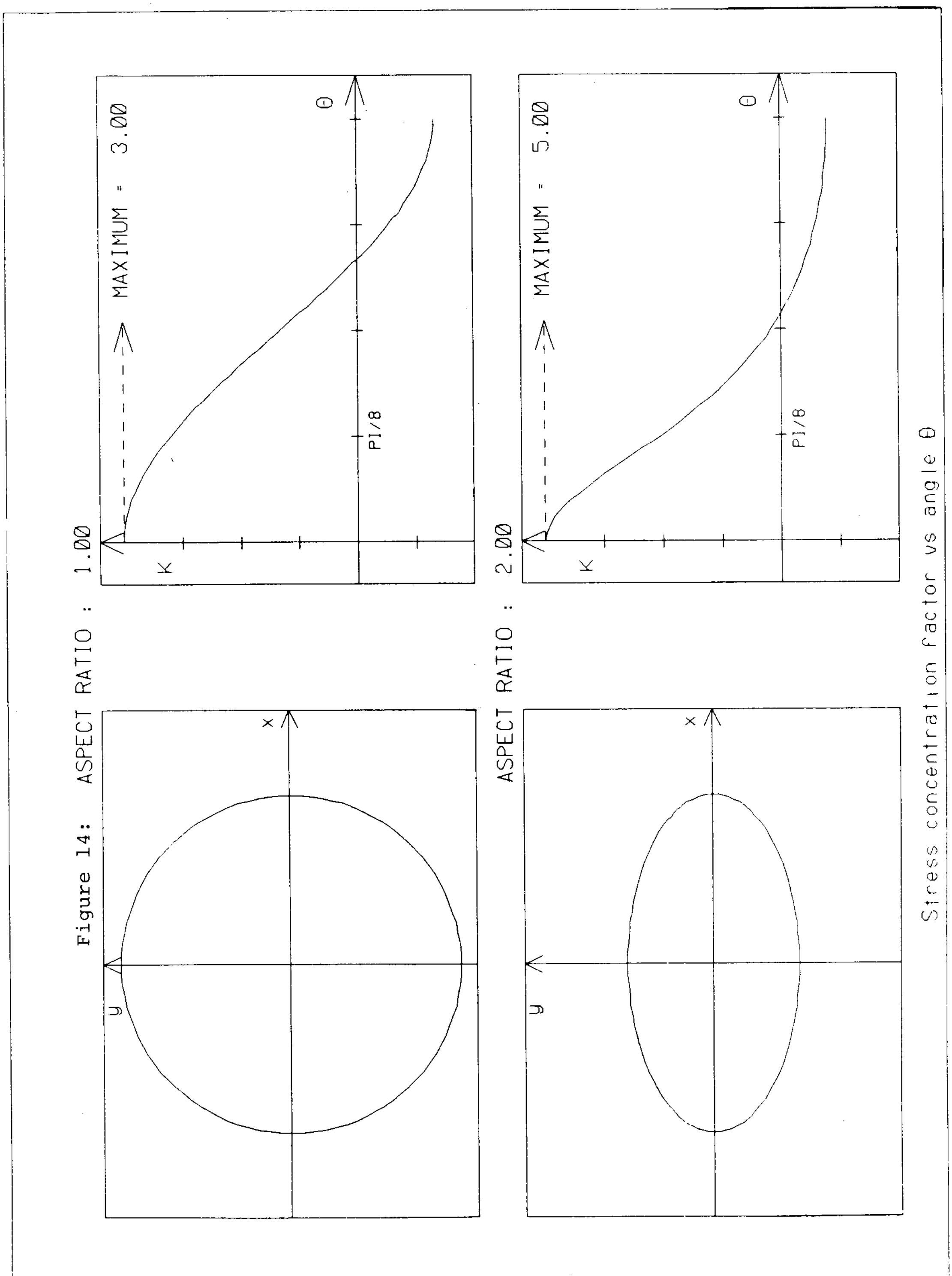
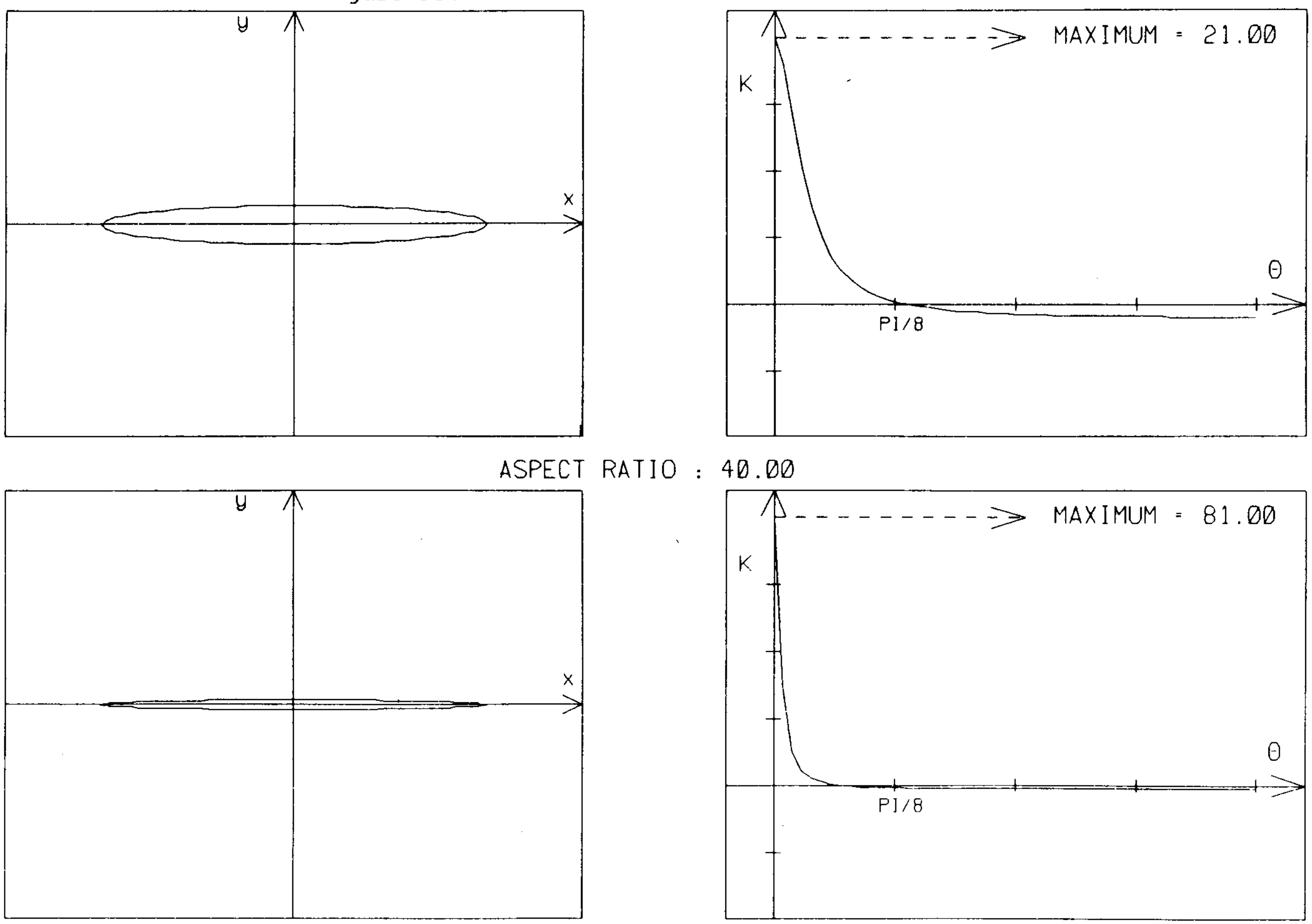


Figure 14: ASPECT RATIO : 1.00
MAXIMUM = 3.00
K
θ
PI/8
y
x
ASPECT RATIO : 2.00
MAXIMUM = 5.00
K
θ
PI/8
y
x
Stress concentration factor vs angle θ

Figure 15: ASPECT RATIO : 10.00



Stress concentration factor vs angle θ

Figure 16: GENERAL INSTRUCTIONS: PROBLEM DEFINITION

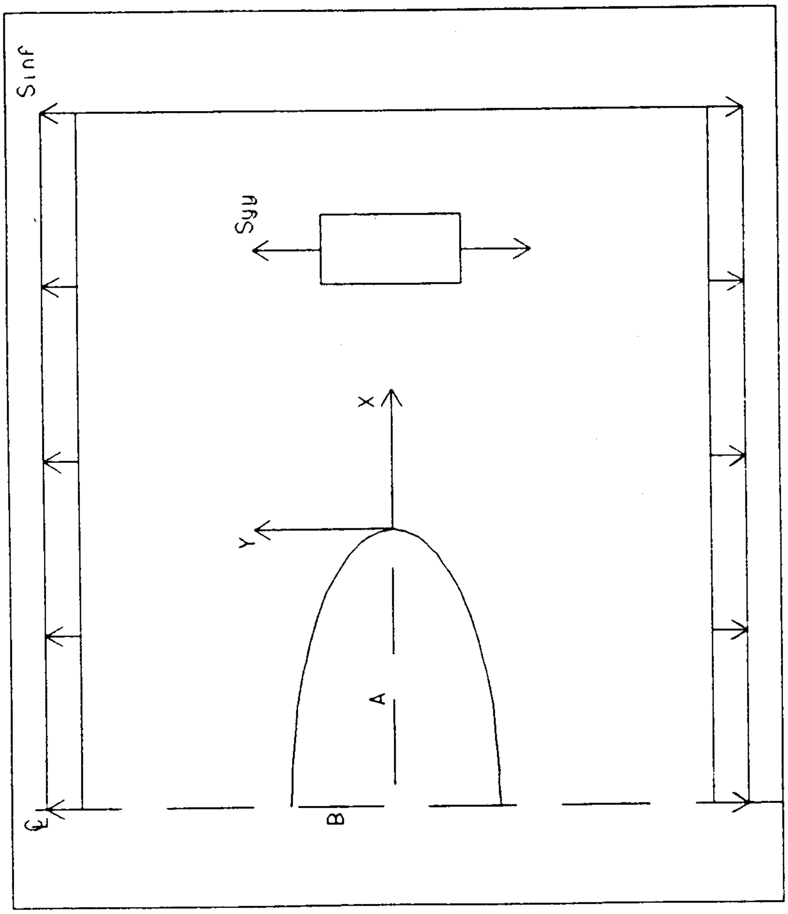


Figure 17: Aspect ratio of the elliptical hole : 1.0

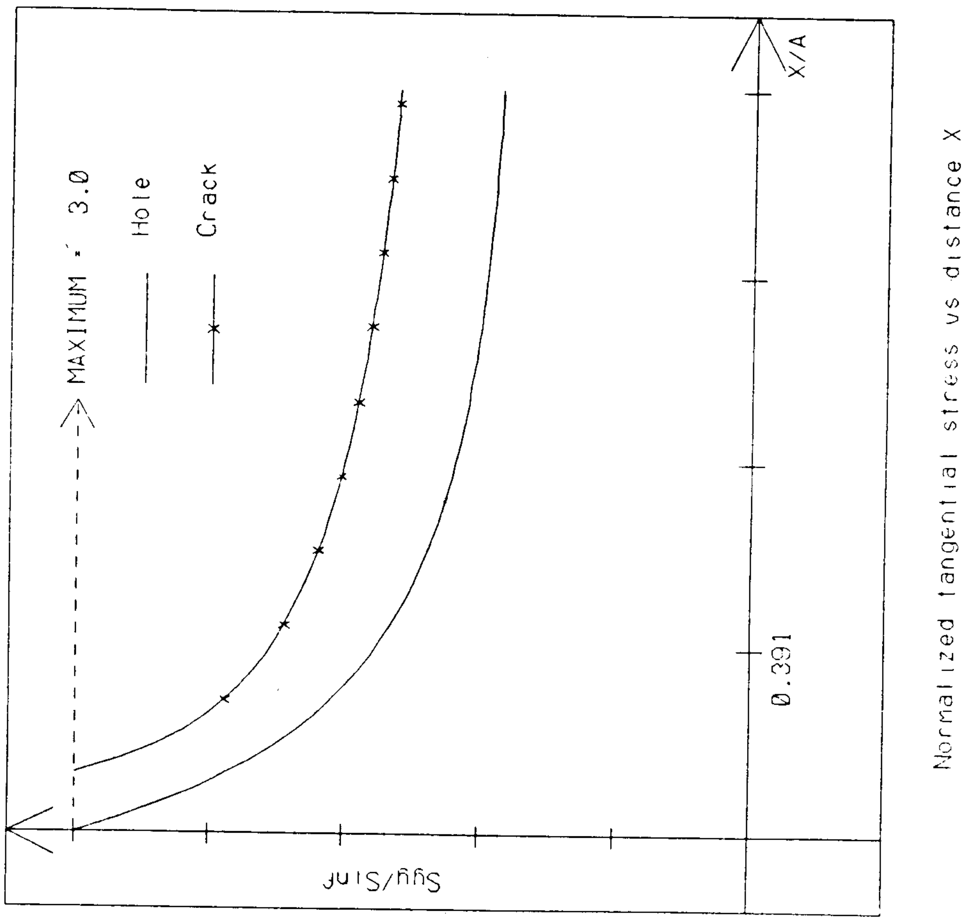
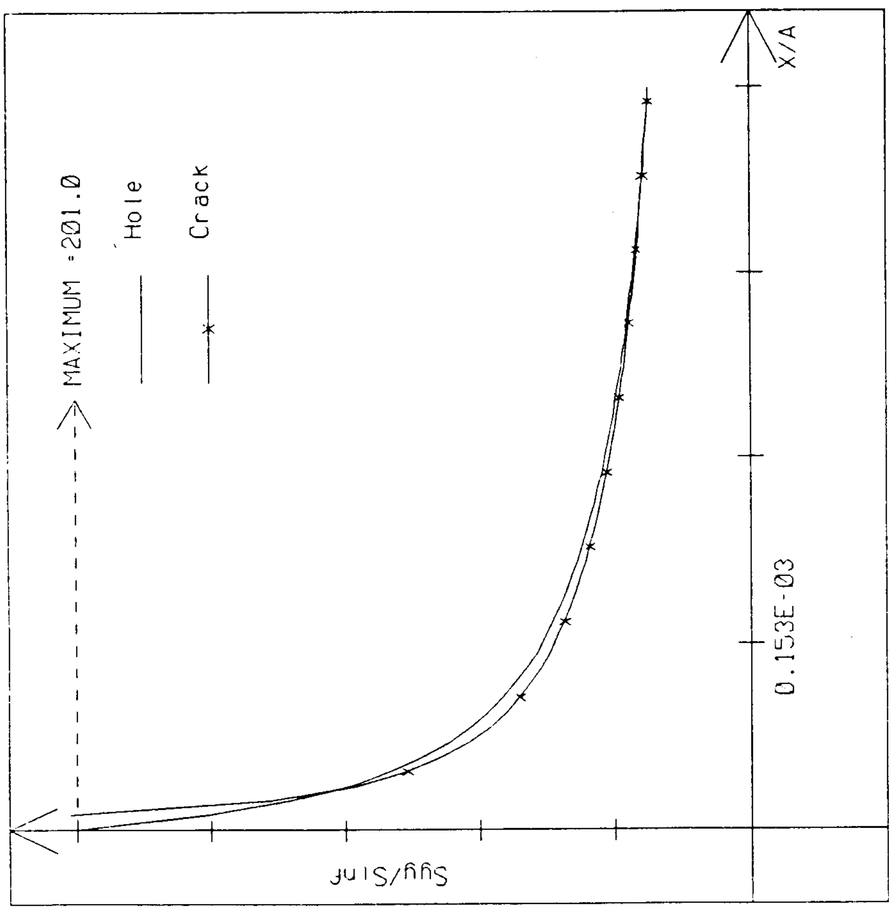


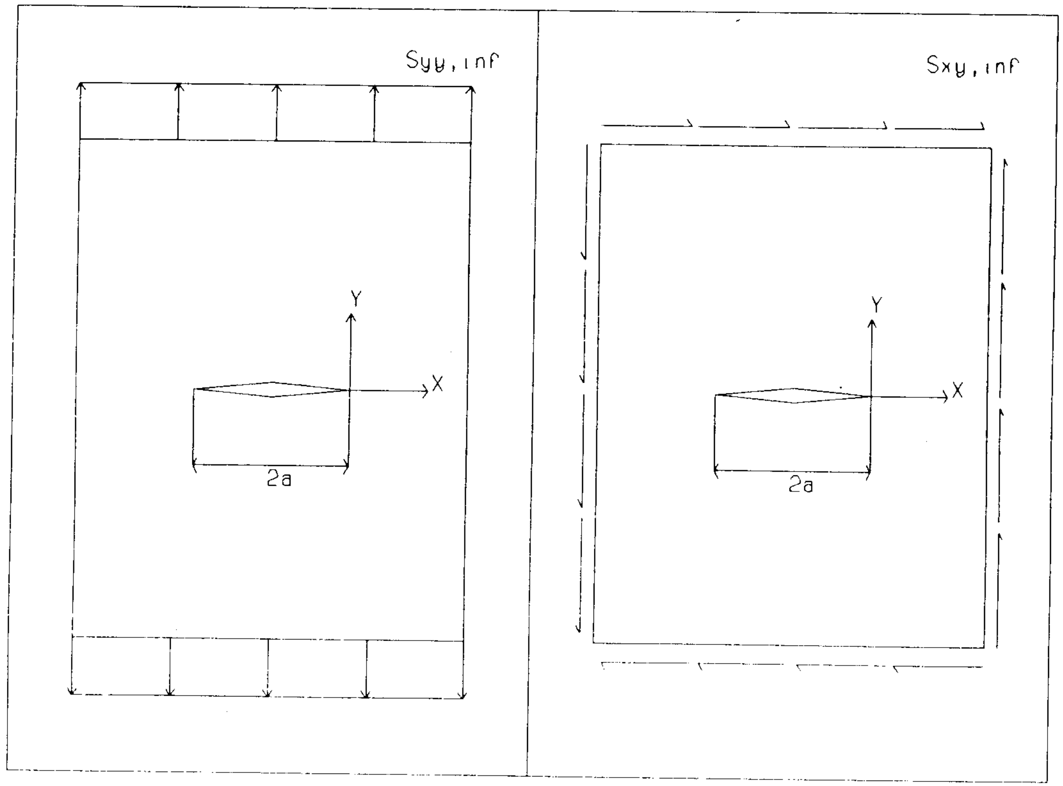
Figure 18: Aspect ratio of the elliptical hole : 100.0



Normalized tangential stress vs distance X

Figure 19

General Instructions: Problem Definition



MODE I

MODE II

Figure 20: SIGMA X NEAR CRACKTIP MODE I FRACTURE

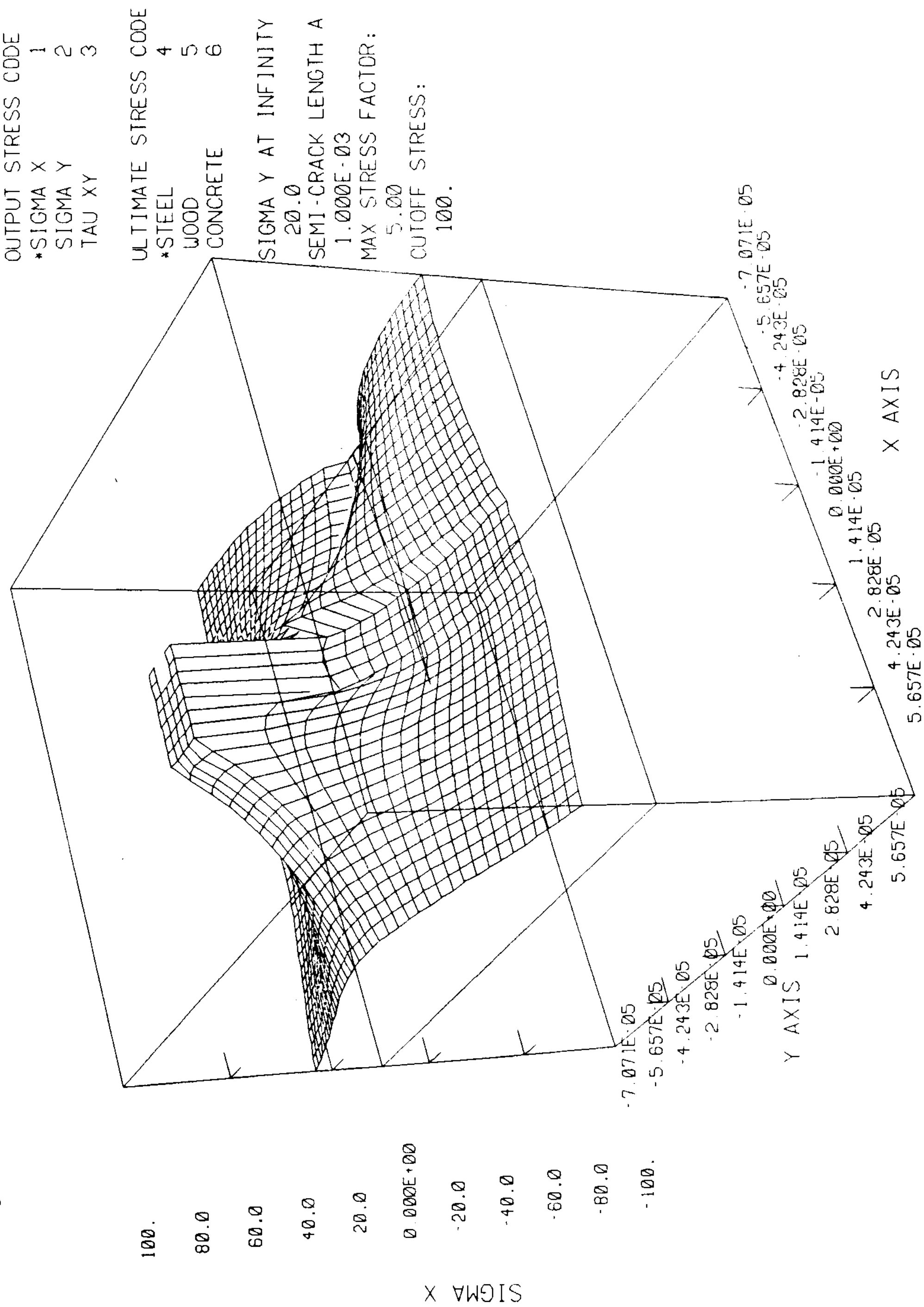


Figure 21: SIGMA Y NEAR CRACKTIP MODE II FRACTURE

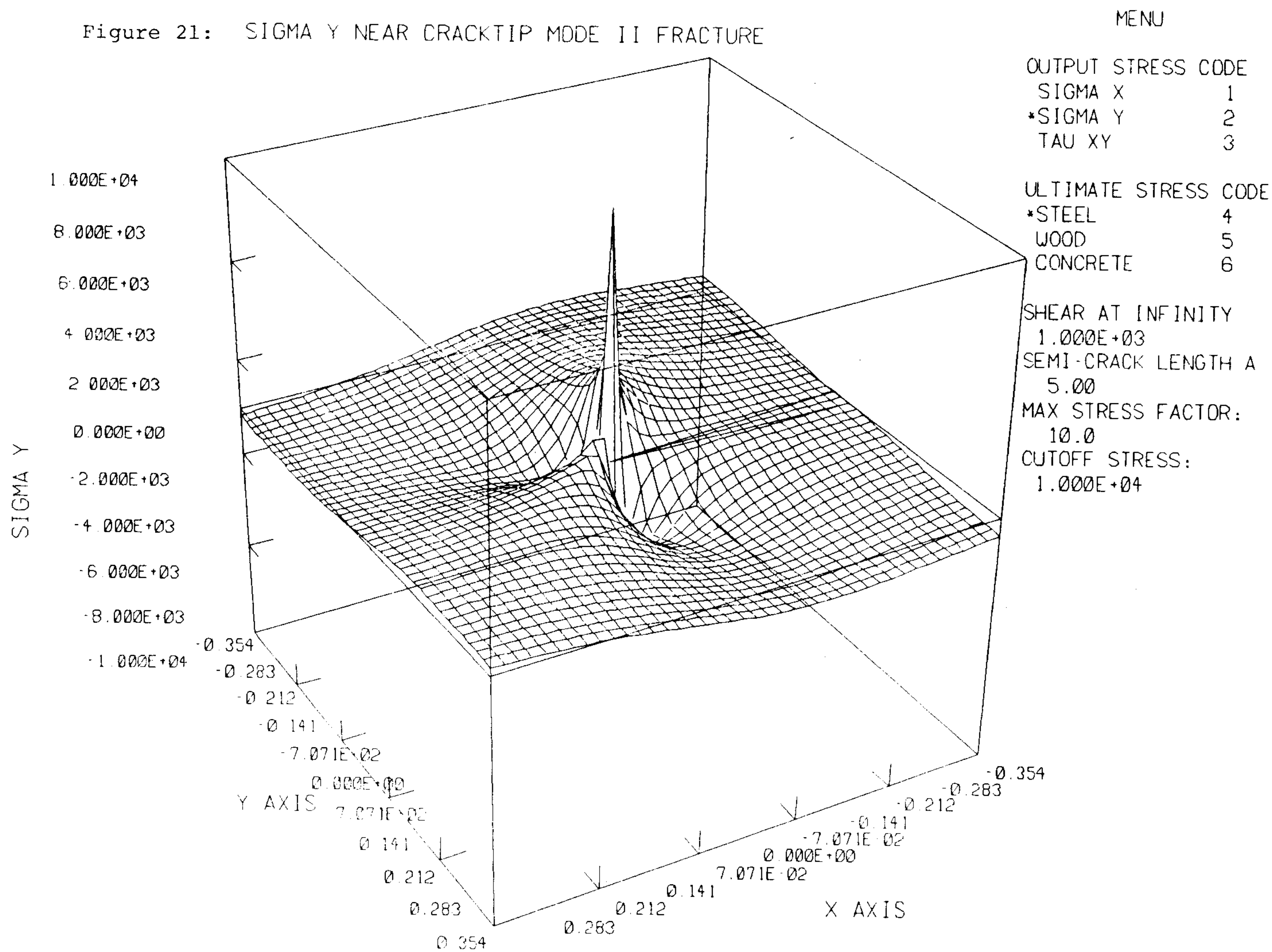


Figure 22: TAU XY NEAR CRACKTIP MODE II FRACTURE