Predictions from String Theory

by

Eric Kuflik

A dissertation submitted in partial fulfillment
of the requirements for the degree of
Doctor of Philosophy
(Physics)
in The University of Michigan
2011

Doctoral Committee:

Professor Gordon L. Kane, Chair
Professor Charles R. Doering
Professor Bing Zhou
Associate Professor Leopoldo A. Pando-Zayas
Assistant Professor Kathryn Zurek

To my family and friends.

ACKNOWLEDGEMENTS

I owe my deepest gratitude to my advisor, Gordon Kane. My development as a scientist and my understanding of physics would not have been possible without his encouragement and guidance.

It is a pleasure to thank those who made this thesis possible through collaboration on my many projects: Bobby Acharya, Lian-Tao Wang, Kathryn Zurek, Ran Lu, Timothy Cohen, Joseph Marsano, Aaron Pierce, Brent Nelson, Phill Grajek, and Kerim Suruliz

I would like to show my gratitude to Professors Ratindranath Akhoury, Charles Doering, Henriette Elvang, Jim Liu, Finn Larsen, James Wells, Leo Pando Zayas, and Bing Zhou for all I have learned from their classes and conversations.

I am indebted to my many of my friends and fellow graduate students at the University of Michigan, Ibrahima Bah, Alberto Faraggi, Sunghoon Jung, Brian Karrer, Piyush Kumar, Daniel Phalen, Jing Shao, Phillip Szepietowski, and Brooks Thomas for whatever it is that they did.

TABLE OF CONTENTS

LIST OF FIGURES

Figure

LIST OF TABLES

Table

LIST OF APPENDICES

Appendix

CHAPTER I

Introduction

The era of the Large Hadron Collider has begun. Currently, high energy proton beams are being collided with a total energy of 7 TeV–an energy frontier being explored for the first time. Out of these collisions will come already-discovered Standard Model particles, but also (hopefully) new particles such as the Higgs boson, dark matter and superpartners. The discovery of new particles, along with measurements of their properties, will allow us to answer questions that have vexed physicists for a long time. Dark matter is also being searched for in land-based and satellite experiments, with an unprecedented ability to discover dark matter over an enormous range of possible dark matter properties.

It is essential to have an appropriate theoretical framework within which to attempt to answer these vexing questions. Although it is the most successful model of particle physics, the Standard Model is nonetheless inadequate. It cannot address, let alone explain, why it has the particle content that is does or the scale at which electroweak symmetry is broken. It also gives an incomplete picture of particle physics since it cannot explain important properties of the universe, such as the baryon asymmetry and the dark matter abundance. Given the limits and attendant issues with the Standard Model, and the ramifications of new discoveries, the Large Hadron Collider data and its analysis is extraordinarily important. The analysis will take physicists beyond the SM description towards a much richer, deeper understanding of the universe.

Supersymmetry, which relates elementary particles of different spin, has become the leading, and most studied, candidate for a Beyond the Standard Model theory. It directly solves many problems of the Standard Model, such as the hierarchy problem–the sizeable gap between the electroweak scale (or the Higgs mass) and Planck scale, and gauge unification–the unification of the strong, weak and electromagnetic forces. Supersymmetry also offers solutions to the dark matter problem, and naturally builds a bridge to a high scale underlying theory. However, the inclusion of supersymmetry leads to many new questions, for by itself it cannot explain the origin of supersymmetry breaking and consequently the masses of the new (super)particles.

String theory is the leading candidate for the underlying theory, as it aims to unify the quantum forces with gravity while providing a framework to address the critical questions left unanswered by the Standard Model and Supersymmetry. A number of predictions from string constructions can be empirically tested at the Large Hadron Collider and dark matter experiments. In this work I aim to make generic predictions of string theory, i.e. predictions characteristic of string theory as a whole and not dependent on the specific string construction. At the same time string theory motivation will be combined with bottom-up approaches to fill in the gaps of our understanding of string theory and make predictions for current and upcoming experiments.

The first generic prediction of string theories is moduli – scalar fields with no classical potential and Planck scale (gravitationally) suppressed couplings to matter. The expectation values of the moduli classically describe the size and configuration of the curled up extra dimensions. In order to have a meaningful model describing phenomena below the string scale, the moduli must be “stabilized”, i.e., must have a potential with a minimum that determines their value in the vacuum. Otherwise, observable coupling strengths and masses would not have meaningful values when calculated from the theory, making it impossible to compare with data.

Early in the universe, the moduli fields begin to oscillate in their potential. One can estimate the energy stored in the oscillating moduli, which has long been known [73, 79, 87] to be large and to dominate the energy density of the Universe. Then the universe cools and moduli decay, reheating the Standard Model thermal bath, and washing out any relic abundances. This is an alternative to the standard thermal explanation for the relic abundances empirically verifiable at the LHC and in dark matter experiments, as it requires the dark matter to have vastly different properties than those it would have presuming a thermal history. This will further be discussed in Chapter II.

If the moduli decay after about 10^{-2} seconds, their decay products will inject additional photons, hadrons and leptons during Big Bang Nucleosynthesis (BBN), ruining its successful predictions. Lifetimes shorter than about 10^{-2} seconds require moduli masses larger than about 30 TeV [97, 134, 179], which then implies a similar bound on the gravitino mass, which typically sets the scale for all of the scalar masses in the theory. Then the scalar superpartners will be heavy, motivating light gauginos as the leading signal for beyond Standard Model physics at the Large Hadron Collider. Chapter III will be devoted to the discovery of these signals.

Starting from string theory and moving to its predictions–the "top-down" approach taken in the above paragraphs–results in the conclusion that string theories generically predict a particle spectrum and properties that are testable at today's experiments. On the other hand, with the LHC online it is also important to think about the relation between particle physics and string theory from a "bottom-up" approach–that is, by working backwards from what we know, find a consistent string theory and make new predictions.

Physicists have long suspected that the Minimal Supersymmetric Standard Model (MSSM) unifies the strong and electroweak forces [142] into a single $SU(5)$ grand unified group [106]. Key to approaching string theory from "bottom-up" is to understand the unification of these forces.

In an $SU(5)$ grand unified theory, each family of quarks and leptons is organized into a $\mathbf{10} \oplus \bar{\mathbf{5}}$ representation. The remaining MSSM fields, the Higgs doublets, do not form a complete $SU(5)$ representation. Minimally, the Higgs doublets can be assigned to a $\mathbf{5} \oplus \bar{\mathbf{5}}$ representation, but require the introduction of a pair of Higgs color triplets. The Higgs triplets can mediate baryon and lepton violating processes, and thus should be very heavy, $m_T \gtrsim 10^{14}$ GeV, to avoid rapid proton decay [161]. Additionally, they should be heavy to ensure gauge coupling unification in the minimal model.

In the Minimal Supersymmetric Standard Model, the only low energy supersymmetric parameter with mass dimension is the μ parameter. Through the μ-term in the superpotential (W), $W \supset \mu H_u H_d$, it gives mass to Higgsinos and also generates scalar potential couplings for Higgs fields. The size of μ plays an important role in phenomenology. In particular, it affects properties of potential dark matter particles. Searches for the charged Higgsino require $\mu \gtrsim 100$ GeV, while arguments against fine tuning of the mass of the Z-boson suggest that μ should not be too large. On the other hand, one might expect, with ignorance of the high scale theory, that $\mu \sim m_{GUT}$, the natural Ultraviolet (UV) cutoff.

In fact, an $SU(5)$ symmetric theory would require that the Higgs doublets masses be the same as the triplet mass, $\mu = m_T \sim M_{GUT}$, but it was just argued that is this a factor 10^{13} too large. A string theoretic solution to the μ-problem is inevitably related to the solution of the doublet-triplet problem of grand unified theories. Solving the μ-problem [136] presumably requires an understanding of the fundamental theory that generates the scale of the μ parameter. Thus the μ-problem is exceptionally important–a high scale theory cannot be qualitatively complete without addressing it, and its solution will have significant implications for dark matter, Higgs physics, and fine-tuning issues.

Within string theory, many explanations for the small value of μ have been proposed. In most scenarios the μ-term is forbidden at the high scale, then, it is somehow dynamically generated at a lower scale. In many cases, the μ-term is forbidden by a continuous or discrete symmetry, which is spontaneously broken at a smaller, dynamically generated scale ($\ll m_{GUT}$), and perhaps related to supersymmetry breaking [31, 163]. Some examples of the above include NMSSM scenarios [75, 77, 143, 169, 170, 175] and approximate R-symmetric models [55, 133]. Others scenarios have the μ-term forbidden by stringy selection rules, and are broken by non-perturbative instanton effects that produce exponentially suppressed mass scales [76, 113, 124, 125].

Therefore, it is paramount that the symmetry that protects the μ-term not forbid the triplet masses if both problems are to be solved. This restriction leads to an elegant, perhaps unique solution to the μ-problem in M theory; the symmetry which protects μ from being generated at the unification scale was originally proposed by Witten [184]. Although Witten did not discuss how this symmetry would be broken, we argue that the symmetry would–indeed must–be broken by moduli stabilization. The theory and dark matter phenomenology of this scenario is discussed in Chapter IV.

Not long after the development of $SU(5)$ GUTs, flipped $SU(5)$ emerged as a natural alternative [30, 40, 83]. Based on gauge group $SU(5) \times U(1)_\chi$, flipped $SU(5)$ is not a model of unification per se, but can accommodate the near unification of couplings that is observed by experiment while overcoming difficulties of minimal $SU(5)$ models that emerged as lower bounds on the proton lifetime increased. These successes center on the breaking of $SU(5) \times U(1)_\chi$ by nonzero vevs for components of "GUT-Higgs" fields that arise as a $\mathbf{10}/\overline{\mathbf{10}}$ pair. The degrees of freedom in these fields are just what is needed to lift leptoquarks and Higgs triplets from the low energy spectrum in a simple and elegant way.

In string theory, flipped $SU(5)$ models are of interest for a variety of reasons. It provides a mechanism for breaking the GUT group in 4 dimensions while solving doublet-triplet splitting without using large GUT representations. Such representations are typically unavailable in string theories. In weakly coupled Heterotic models, flipped $SU(5)$ gives one the flexibility to achieve gauge coupling unification at the string scale ($\sim 10^{17}$ GeV) if extra vector-like particles are added as in [128]. In perturbative type II GUT constructions based on intersecting branes, flipped $SU(5)$ is a natural goal [58, 59, 62, 63, 78] because one of the two MSSM Yukawa couplings is forced to be generated nonperturbatively there [48] and hence is strongly suppressed. In perturbative type II constructions with ordinary $SU(5)$, the top Yukawa is the small one but in flipped $SU(5)$ it is the down Yukawa that is suppressed, allowing the top Yukawa to be large.

However, explicit construction of flipped $SU(5)$ models in F-theory suffer several phenomenological pitfalls. The most significant challenges are related to the μ problem, whose severity depends on one's attitude toward fine-tuning, although it should be noted that flipped $SU(5)$ was partially motivated to solve tuning problems. In addition to this, there appears to be some tension between the μ problem and generation of neutrino masses. Finally, the prevention of rapid (dimension 4-induced) proton decay requires discrete symmetries that do not have their origin as an unbroken subgroup of a continuous $U(1)$ symmetry that preserves the ordinary MSSM Lagrangian. The model building issues of embedding flipped $SU(5)$ in F-theory will be discussed in Chapter V.

This thesis reports significant new results and progress toward relating string theories to experimental and cosmological phenomena.

CHAPTER II

Moduli Stabilization and Non-Thermal Cosmological Histories

In recent years, it has been realized that models with moduli which decay before BBN can have virtues which are comparable to, or improvements upon, models which have a 'thermal cosmological history'. There can be a 'non-thermal weakly interacting massive particle (WIMP) miracle' which is equally compelling as the thermal case [15, 160] and requires larger WIMP annihilation cross-sections (which happen to be better suited for explaining the PAMELA data [20, 99, 112, 120, 132]). Further, the entropy released from the moduli decays dilutes potential axion relic abundances and allows for much less fine-tuned cosmological axion physics than is the case in a 'thermal cosmological history', thereby relieving the tension between cosmological bounds and GUT scale axion decay constants [10, 104]. These virtues of a 'non-thermal cosmological history' indicate that the cosmological moduli problem is, perhaps, less of a problem and more likely part of a solution. Given the potential impact of such an indication, it is of importance to investigate in more detail the claim that generically the moduli masses will be of order $m_{3/2}$, the mass of the gravitino, the superpartner of the graviton.

In this chapter, we sharpen the existing arguments that realistic vacua arising from a compactified string theory will, generically, have moduli or moduli-like fields (such as hidden sector matter scalars, or axions), which dominate the energy density of the Universe prior to BBN. With moduli stabilized, the moduli F-terms and those of the moduli-like fields contribute to supersymmetry breaking, so the scalar goldstino will have significant moduli components. I will argue that one or more moduli or moduli-like fields will have masses of order the gravitino mass. These results can apply regardless of the value of the gravitino mass and, hence, can give a strong constraint on the model of mediation of supersymmetry breaking. I also discuss the moduli spectrum in a variety of different classes of vacua in which moduli stabilization is fairly well understood and demonstrate that there is always a modulus-like field which dominates the cosmic energy density prior to BBN. These latter results are described in Appendix C.

From the string theory point of view, the Universe is expected to have a partly non-thermal history – an important claim for considerations of cosmology, especially dark matter. The gravitino mass is required to be greater than about 30 TeV implying that gauge mediated supersymmetry breaking will be difficult to realize in a phenomenologically consistent string vacuum, unless one can dilute the moduli energy density through late inflation [80, 159].

2.1 Supergravity and Moduli Masses

The conjecture: In any string/M theory vacuum with observationally consistent energy density of the Universe, there exists at least one modulus-like field whose mass is such that it dominates the vacuum energy up to the BBN era.

Here, the term modulus-like refers not only to the geometric moduli fields of string theory, but includes other scalar fields whose couplings to Standard Model particles are suppressed by a high scale such as the Grand Unification scale or Planck scale. Examples include axions and other hidden sector fields.

Typically, the conjecture follows from the fact that the masses of these moduli-like fields are of order, or less than, the gravitino mass $m_{3/2}$; in fact this would be a more general version of the conjecture. As is shown below, there can be examples in which the relevant mass scale is much less than $m_{3/2}$ because of "large volume" effects in the extra dimensional theory [36]. These examples do not violate either conjecture.

Since the moduli fields are stabilized by assumption, they have non-trivial potentials and will most likely participate in supersymmetry breaking. If this is true, i.e., at least one moduli has a non-vanishing F-term, then one can make progress towards proving the conjecture by considering the scalar potential of the effective supergravity theory.

The scalar potential in a supergravity theory in four dimensions can be written in terms of a single real function G. In terms of the Kahler potential and superpotential $G = K + m_{pl}^2 ln(W\bar{W}/m_{pl}^6)$, though we will only consider G here. G is taken to have mass dimension two and all scalar fields are taken to be dimensionless in order to exhibit correctly the scaling of various operators with the reduced Planck mass m_{pl}. The scalar field potential, assuming that the D-term contributions are negligible is

$$V = m_{pl}^2 e^{G/m_{pl}^2} \left(G^i G_i - 3m_{pl}^2\right) \tag{II.1}$$

Critical points of V satisfy

$$\langle \nabla_i V \rangle = \left\langle m_{pl}^{-2} G_i V + e^{G/m_{pl}^2} \left(G_i + G^k \nabla_i G_k\right) \right\rangle = 0. \tag{II.2}$$

where subscripts on G denote derivatives with respect to the moduli fields ϕ_i or their conjugates $\phi_{\bar{i}}^*$.

One can calculate the complex scalar mass matrix in the vacua which satisfy the above conditions and, additionally, have zero cosmological constant. Assuming the kinetic terms have been properly normalized, the mass matrix for all the scalar fields in the theory is

$$\begin{aligned} M_{i\bar{j}}^2 &= e^{G/m_{pl}^2} \left(\nabla_i G_k \nabla_{\bar{j}} G^k - R_{i\bar{j}k\bar{l}} G^k G^{\bar{l}} + G_{i\bar{j}}\right) \\ M_{ij}^2 &= e^{G/m_{pl}^2} \left(2\nabla_i G_j + G^k \nabla_i \nabla_j G_k\right) \end{aligned} \tag{II.3}$$

Notice that there is no factor of m_{pl}^2 because the quantities in the brackets all have mass dimension two.

Since we are only interested in minima of the potential, the mass matrix is positive definite by assumption. Hence, we use the theorem that its smallest eigenvalue, m_{min}^2, is less than $\xi^\dagger M \xi$ for any unit vector ξ. Extending the work of [74, 82, 110], we take $\xi = (G^{\bar{j}} \quad c\, G^j)/\sqrt{3(1+|c|^2)}$ for

$c \in \mathbb{C}$, which is aligned in the (moduli components of the) two sGoldstino directions $\eta = G^i\phi_i$ and $\bar{\eta} = G_i\phi^i$. This gives a one (complex) parameter class of constraints on the upper bound of the lowest mass eigenvalue

$$
\begin{aligned}
m^2_{\min} &\leq \tfrac{1}{3(1+|c|^2)} \begin{pmatrix} G^i & c^\dagger G^{\bar{i}} \end{pmatrix} \begin{pmatrix} M^2_{i\bar{j}} & M^2_{ij} \\ M^2_{\bar{i}\bar{j}} & M^2_{\bar{i}j} \end{pmatrix} \begin{pmatrix} G^{\bar{j}} \\ cG^j \end{pmatrix} \\
&\leq m^2_{3/2} \left(2\frac{|1-c|^2}{1+|c|^2} + \mathrm{Re}\{\frac{2c}{1+|c|^2}\frac{u}{m^2_{pl}}\} - \tfrac{r}{m^2_{pl}} \right)
\end{aligned}
\tag{II.4}
$$

where $u \equiv \frac{1}{3}G^iG^jG^k\nabla_i\nabla_jG_k$, $r \equiv \frac{1}{3}R_{i\bar{j}k\bar{l}}G^iG^{\bar{j}}G^kG^{\bar{l}}$ and $m^2_{3/2} = m^2_{pl}e^{G/m^2_{pl}}$. r is the holomorphic sectional curvature of the scalar field space, evaluated in the sgoldstino directions in field space. I have extended the previous work to include the effects of the curvature r as well as u.

To understand the constraint given by Eq. (II.4) we rewrite this equation by taking $u = |u|e^{i\theta_u}$

$$
m^2_{\min} < m^2_{3/2}\left(2 - 2\alpha\cos\theta + \alpha\frac{|u|}{m^2_{pl}}\cos(\theta+\theta_u) - \frac{r}{m^2_{pl}}\right) \tag{II.5}
$$

for any $\alpha \equiv \frac{2|c|}{1+|c|^2} \in [0,1]$ and $\theta \in [0, 2\pi]$.

It therefore follows that

$$
m^2_{\min} < m^2_{3/2}\left(2 - \frac{r}{m^2_{pl}}\right) \tag{II.6}
$$

and for $u \in \mathbb{R}$

$$
m^2_{\min} < m^2_{3/2}\left(\min\{\frac{u}{m^2_{pl}}, 4 - \frac{u}{m^2_{pl}}\} - \frac{r}{m^2_{pl}}\right). \tag{II.7}
$$

So, as long as $|r|, |u| \leq \mathrm{O}(m^2_{pl})$, the upper limit on the lightest modulus mass is of order the gravitino mass $m_{3/2}$. (See Appendix B for a simple model illustrating the bound given by Eq. (II.6) and Eq. (II.7)).

In fact, for geometric moduli, r is typically of order m^2_{pl}, and thus there will generically be at least one moduli with mass $\lesssim m_{3/2}$. In Appendix B we systematically discuss the moduli masses in all known (at least to us) examples where moduli stabilization is well understood. It is demonstrated that all of these examples have $r \sim m^2_{pl}$ and a modulus or modulus-like field whose mass is less than, or of order $m_{3/2}$

2.2 Non-generic Possibilities

One can discuss under what non-generic conditions moduli, or moduli-like fields, will *not* dominate the cosmic energy density prior to BBN. One possibility is that *all* moduli that have mass order $m_{3/2}$ have significant mixing with charged (under the SM gauge groups or other gauge groups) scalar fields. Then the lightest eigenvalue given by Eq. (II.7) and Eq. (II.6) can have significant charged matter components and quickly thermalizes due to it's couplings to gauge fields and matter. However, mixing between moduli and matter is proportional to vevs of the matter fields or their F-terms and are usually suppressed. It would be difficult to arrange for all moduli to have such large couplings to matter fields. But if any stabilized modulus field (or linear combination) does not mix, our results will hold. Another possibility is to have moduli stabilization unrelated to supersymmetry breaking and a stabilization mechanism that gives all moduli very large masses, but as stated earlier, non-trivial potentials that stabilize moduli will generally break supersymmetry. A third is to have r or u extremely large (and negative), e.g. $|r| \gg (30\ \mathrm{TeV}/m_{3/2})^2 m_{pl}^2$, or have very large kinetic terms so that the mass matrix (Eq. II.3) receives large scaling factors when the kinetic terms are properly normalized. At present such non-generic cases are not excluded, but any proposed model has to explain why they might occur. A final possibility is to have moduli of order $m_{3/2}$, but not oscillate in the early universe, for example, see "moduli trapping" [86, 178] or "string gas cosmology" [41, 52].

Part of the difficulty in trying to make such a model is that there is always at least one axion field present in four dimensional string theory vacua; usually, there are many axions [33, 135, 176, 181]. Some of these axions will be much lighter than the moduli, which are assumed to be heavy enough to be cosmologically irrelevant in this part of the discussion. This is because axions only gain mass via non-perturbative effects. One can calculate the relic abundance of such an axion today, as a function of its mass, m_a, decay constant, f_a and initial displacement θ_a It is given by [33, 135, 176, 181]

$$\Omega_a\, h^2 = 0.06\, \left(\frac{f_a^2}{M_{GUT}^2}\right)\left(\frac{m_a}{10^{-20}\,\mathrm{eV}}\right)^{1/2} \langle\theta_a^2\rangle \tag{II.8}$$

With a GUT scale decay constant, the axion lifetime extends into the BBN era if $m_a \leq 100$ GeV, so the above formula implies that one would have to tune the model such that there are no axions with masses between 10^{-20} eV and 100 GeV!

2.3 Non-thermal cosmological history from string theory

The arguments based on the moduli mass matrix imply that $m_\phi \leq O(m_{3/2})$, and that $m_{3/2}$ must be of order 30 TeV or larger to not conflict with BBN predictions or the observed late energy density. The moduli couple essentially universally to every Standard Model particle and their superpartners. Some moduli decays might be helicity suppressed, but decays to scalars are all present at full strength. These decays generate huge entropy, which significantly dilutes any dark matter particles that might have been present before the moduli decay. It is sufficient if only one such modulus has mass of order $m_{3/2}$ though typically many do. *Thus thermal freezeout relic densities of dark matter are not relevant to present cosmology in string models where moduli are present.* At the same time, about a quarter [16, 160] of all moduli decays will be to superpartners, and every superpartner will have a decay chain with a lightest superpartner (LSP) at the end, so a large number of LSP dark matter particles will be generated and provide a dark matter candidate if they are stable. In practice, the number density of LSP's from moduli decay is large compared to the relic density, large enough for LSP's to annihilate. The typical temperature after the moduli decay is of order 10 MeV, but the number density decreases as the Universe expands, approaching an attractor solution of the Boltzmann equations (when the number density is too small for annihilation to take place) and not a freeze-out (which occurs when the LSP's fall out of equilibrium). Surprisingly, one still finds a "WIMP miracle", where the relic number density is still given in terms of the Hubble parameter - a cosmological parameter, but now evaluated at the moduli decay temperature rather than a freezeout temperature - and a particle physics annihilation cross-section for the LSP's (appropriately averaged) [15, 160]

$$N_{LSP} \approx H(T \sim 10\,\text{MeV}) / \left\langle \sigma_{annih}^{LSP} \mathrm{v} \right\rangle . \tag{II.9}$$

All steps of this calculation have been carried out in the example of M theory compactified on a manifold of G_2 holonomy, including the moduli stabilization, calculation of the moduli masses and decays and the entropy generated, etc (see [7–9]). Importantly, in order to obtain about the right relic density, the LSP must be a wino or wino-like particle, with a large annihilation cross section of about $3 \times 10^{-24}\text{cm}^3/\sec$. Such an LSP naturally arises in the G_2-MSSM [7–9], where the tree level gaugino masses are suppressed by the supersymmetry breaking mechanism to be of order the gaugino masses from the anomaly mediation contribution. Thus the non-thermal history and a wino LSP go together and give a consistent picture for dark matter from the compactified string theory. Also encouraging is the fact that the PAMELA satellite data on positrons *and* antiprotons (which was reported after [7–9]) can be consistently described by a wino LSP [99, 112, 120, 132].

One might wonder if the moduli "reheat temperature", effectively the temperature generated by the moduli decay, could be above the thermal freezeout temperature so that the thermal history could finally take over. Unfortunately, the associated temperature is too small for a thermal history to develop. To see this let $T_{RH} \sim 10$ MeV for $m_{3/2} \sim 30$ TeV, which follows from $T_{RH} \sim \sqrt{\Gamma_\phi m_{pl}} \sim \frac{m_{3/2}^{3/2}}{m_{pl}^{1/2}}$. Then if $m_{3/2}$ were larger by even an order of magnitude, T_{RH} would grow by a factor of order $(10)^{3/2} \sim 30$, so it would still be small compared to the usual thermal freezeout temperature $T_{fr} \sim$ few GeV.

Thus very generally string theories with stabilized moduli having multi-TeV scale masses (or lighter) will have a non-thermal cosmological history, and a relic density of wino-like dark matter generated by moduli decay rather than thermal freezeout is the preferred solution with LSP dark matter. A fine-tuned period of late inflation may allow a way to evade this generic conclusion for some theories, but an inflaton with just the right properties must be found in such a theory.

2.4 Upper limit on $m_{3/2}$

In theories where the relic density is indeed generated from moduli decay, it turns out there is an upper limit on $m_{3/2}$, because the universe would be over-closed if $m_{3/2}$ were too large. Since $\Gamma_\phi \sim m_{3/2}^3/m_{pl}^2$, $H(T_{RH}) \sim \Gamma_\phi$ and $T_{RH} \sim \sqrt{\Gamma_\phi m_{pl}}$, ρ/s for the dark matter is

$$\rho/s \sim \frac{M_{LSP}}{\langle \sigma \mathrm{v} \rangle\, m_{3/2}^{3/2} m_{pl}^{1/2}} \tag{II.10}$$

M_{LSP} is the lightest eigenvalue of the neutralino mass matrix, which also contains μ and the Higgs vev, two mass scales not explicitly scaling as $m_{3/2}$. In a full theory we expect both of these to vanish when $m_{3/2}$ vanishes. For example, radiative EWSB is not possible without supersymmetry breaking. Once $m_{3/2}$ is large compared to M_Z, the off-diagonal term in the neutralino mass matrix can be neglected. If $\mu \sim m_{3/2}$ the LSP is mostly gaugino and proportional to $m_{3/2}$, while if μ is small , there is Higgsino mixing in the LSP, but the LSP mass is again essentially proportional to $m_{3/2}$. More generally, in supergravity theories all masses will be proportional to $m_{3/2}$. Since $\langle \sigma \mathrm{v} \rangle \sim M_{LSP}^{-2}$, it will decrease as $m_{3/2}^2$. Therefore

$$\rho/s \propto m_{3/2}^{3/2}/m_{pl}^{1/2} \tag{II.11}$$

If the correct relic density is obtained for, say $m_{3/2} \sim 50$ TeV, a value of order 5 times larger for $m_{3/2}$ will over-close the universe, so $m_{3/2} \lesssim 250$ TeV is required. Any theory where all masses are proportional to $m_{3/2}$ will give a similar result.

2.5 Gauge Mediation Supersymmetry Breaking

Our results suggest that any approach to supersymmetry breaking that originates in a string theory with moduli that has a gravitino mass less than about 30 TeV will have the problems described above, the moduli and gravitino problems. Thus, one would conclude that gauge mediated supersymmetry breaking, which typically has a much lighter gravitino and therefore light moduli, does not generically arise if our universe is described by a compactified string theory with stabilized moduli.

2.6 Heavy Scalars, Light Gauginos, LHC, and Rare Decays

All superpartner masses in gravity mediated supersymmetry breaking are proportional to $m_{3/2}$. Scalar masses generically will have values about equal to $m_{3/2}$, but gaugino masses are often suppressed, usually because the main source of supersymmetry breaking does not couple at tree level with the gaugino masses, and they are zero in the supersymmetry limit. Several phenomenological consequences follow from these properties of generic string theory vacua.

At LHC the scalar superpartners should not be observed directly. The gauginos, in particular the gluino, the lightest two neutralinos (including the LSP), and the lighter chargino will be observed. The gaugino spectrum is typically compressed, e.g. in pure anomaly mediation with light scalars the ratio of gluinos to winos is 9, while in the G_2 case with heavy scalars it is about half that. Because the squark masses at the weak scale are given by running from the gravitino mass scale there are effects on gluino branching ratios even though the scalars cannot be directly observed, with a large BR of a gluino into top quarks (so gluino pairs often have 4 tops per event), and in general considerably larger branching ratios to channels with final b quarks, leading to rich LHC physics [12, 98, 109]. Any effect on decays or moments of quarks and leptons that can only occur from loops should not differ from its Standard Model value significantly; in particular $g_\mu - 2$, $B_s \to \mu^+\mu^-$, a charge asymmetry from B_s-mixing like-sign di-muons, and other effects should all take on their SM values. Models can of course be constructed with scalars ~ 1 TeV, and we are not aware of any study of how non-generic or unlikely such models are to arise in string theory.

Some predictions depend on how the μ problem is solved in string theory, which is rather poorly understood. On the one hand the μ term in the superpotential must vanish so μ does not have a string scale value, presumably because of a symmetry. But μ and the supersymmetry breaking $B\mu$ terms must be non-zero so the symmetry that protects μ must be broken. When that symmetry is broken doublet-triplet splitting must be preserved, the proton must not decay too rapidly, and the LSP must have a lifetime longer than about 10^{26} sec [37,44]. If the resulting value of μ is similar to the value of M_2 that gives the wino mass, then there is a light higgsino that mixes into the LSP and the light chargino. A higgsino admixture in the neutralino mass matrix gives an off diagonal term that dilutes the wino, and necessarily mixes in some bino as well. The main observable affected by the small μ is actually the scattering cross section in direct detection experiments. For a pure wino LSP the cross-section for LSP–proton scattering is below 10^{-46} cm^2, while with a higgsino admixture it can get as large as a $few \times 10^{-44}$ cm^2. Thus the Xenon100 measurement [32] will determine the allowed higgsino mixture in the wino, and approximately measure the value of μ.

CHAPTER III

Identifying Multi-Top Events from Gluino Decay

The Large Hadron Collider (LHC) is likely to accumulate significant amounts of data in 2011. While the detector groups will be sensitive to many ways new physics could appear, it is not possible to focus equally on all possible interesting signatures, so it is valuable to examine well-motivated channels that may yield results at the initial LHC energies and luminosities. In recent years it has increasingly been recognized that considerations of new physics point toward top-quark and bottom-quark rich final states at the Large Hadron Collider (LHC), as naturalness of electroweak symmetry breaking (EWSB) typically requires the existence of a top partner to cancel the quadratic divergences in the Standard Model (SM).

Supersymmetry implies the existence of a top partner that cancels quadratic divergences. Supersymmetry also introduces a partner for the gluon, the gluino, in the low energy spectrum. At proton colliders, pair production of gluinos, and consequently their decay products, typically become the main channel of supersymmetric signals. Models with light top partners, are common and they imply that a typical signature of production of the gluino will be multiple top quarks in the final states [1]

As discussed in the previous chapter, this scenario is a generic possibility from the point of string theory. When embedding low energy supersymmetry into a string theory, moduli stabilization and cosmological constraints imply that moduli masses and gravitno mass, and consequently scalar

[1] Models looking at light gluinos at 7 TeV but not focusing on multiple tops have been studied, for example see: [25,64,98,109,127].

masses [14], must be larger than about 20 TeV [97, 134, 160, 179]. Then, standard renormalization group (RG) running of scalar masses from the unification scale down to the electroweak scale will push the third generation squark masses significantly lower than those of the other generations (see RG equations IV.52). In most cases this turns out to be right handed stop squark. Alternative models leading to multi-top final states, and corresponding analysis approaches, have been studied [35, 107, 140, 151, 166, 167] (See Ref. [12] for a more extensive list.).

The gluino decays via virtual squarks to $q\bar{q}\chi_1^0$ or $q\bar{q}\chi_1^{\pm}$. Since the rate for a given diagram scales as the virtual squark mass to the -4 power from the propagator, the lightest squarks dominate. Therefore, we are led to consider decay channels $\tilde{g} \to t\bar{t}\tilde{N}$, $\tilde{g} \to t\bar{b}\tilde{C}^-$, and $\tilde{g} \to b\bar{b}\tilde{N}$. Decays of multiple top quarks lead to b-rich and lepton rich final states, and give excellent potential for early discovery. In fact, we show that significant excesses can be observed at the early LHC-7 TeV. For example, gluino masses larger than 600 GeV can be discovered in the single-lepton plus 4 b-jets channel.

We carry out our study on several benchmark models. To study the reach of gluino pair production, with decays into third generation squarks, a detailed scan of the parameter space involving the gluino mass and LSP mass, for different branching ratios, is performed. We emphasize that the goal of this study is to demonstrate that gluino pair production with decays via third generation squarks provides an ideal channel for early discovery at the LHC, since it leads to lepton and b-quark rich final states.

3.1 Benchmark Models

Three benchmark models are considered which will form the basis for the numerical scan discussed below. The model parameters and relevant decay branching ratios are shown in Table 3.1. Model A is a simple example of multi-top physics. The spectrum would have a stop much lighter than the other squarks, and therefore gluino pair production always produces four tops in the final state. Model B is designed to include the decay channel $\tilde{g} \to b\bar{b}\chi_1^0$, which will result if the sbottom

	Branching ratios		
	$\tilde{g} \to t\bar{t}\chi_1^0$	$\tilde{g} \to b\bar{b}\chi_1^0$	$\tilde{g} \to t\bar{b}\chi_1^+ + h.c.$
A	1	0	0
B	0.5	0.5	0
C	.08	0.22	0.7

Table 3.1: Relevant branching ratios for the benchmark models considered in this Chapter. The models A and B have bino LSP. In Model C, the lightest neutralino and lightest chargino are both winos. In all models the first two generation squark masses are taken to be 8 TeV. The third generation is taken to be somewhat lighter and is chosen to generate the required branching ratios of the model. With the exception of the $t\bar{t}$ cross section, we increased all SM background cross sections by a factor of 2, to account for possible K-factor from NLO corrections and be conservative with our estimates. The $L1$ triggers can be found in Appendex A

is also lighter than the first two generation squarks, and $m_{\tilde{t}} \sim m_{\tilde{b}}$. Model B is observably different than Model A, while somewhat more difficult to discover. These models have a Bino-like LSP. In Model C, the Wino is the LSP, and is approximately degenerate with the lightest chargino, which is also Wino-like. It is designed to further include a chargino in the decay chain, which allows the decay $\tilde{g} \to t\bar{b}\chi_1^+$. Since the charged Wino is approximately degenerate with the wino LSP, it appears only as missing energy; though if one focuses on the signal events the chargino stub [101] can probably be seen in the vertex detector.

The three models are taken as a basis for 3 separate numerical scans, where $m_{\tilde{g}}$ and m_{LSP}, are varied while the branching ratios are fixed, as shown in Table 3.1. In particular, scans in model A and model B varied $m_{\tilde{g}}$ and $m_{\text{LSP}} = m_{\chi_1^0}$. while scan in model C varied $m_{\tilde{g}}$ and $m_{\text{LSP}} = m_{\chi_1^0} \simeq m_{\chi_1^\pm}$.

3.2 Signal Isolation and Backgrounds

The relatively large b-jet and lepton multiplicity associated with multiple top production provide for potentially striking signatures that are easily distinguishable above the expected SM background. By requesting multiple b-tagged jets and at least one lepton, it is possible to achieve signal significance $S/\sqrt{B} > 5$ for 1 fb^{-1} of integrated luminosity.

The most significant backgrounds from the SM for final states with many b-jets, several isolated leptons and missing energy, are from top pair production, $t\bar{t}$. The expected cross-section at the LHC for 7-TeV center-of-mass energy is $\sigma = 164$pb (NLO) [2]. Also included in the analysis are a set of SM backgrounds involving associated production of gauge bosons with third generation quarks. These contribute less significantly to the backgrounds than $t\bar{t}$, but can contribute to signals with high lepton multiplicity. All background sources considered, and their respective cross sections are given in Table 3.2. With the exception of the $t\bar{t}$ cross section, we increased all SM background cross sections by a factor of 2, to account for possible K-factor from NLO corrections. Since the relevant backgrounds for the channels considered end up small (Table 3.2), uncertainties in the cross section are not important.

All background event samples were produced with Madgraph v.4 [27], while the parton shower and hadronization were done by Pythia 6.4 [174]. Additional hard jets (up to three) were generated via Madgraph, while the MLM [26, 121, 153] matching scheme implemented in Madgraph was used to match these jets to the ones produced in the Pythia showers. The events were then passed through the PGS-4 [72] detector simulators with parameters chosen to mimic a generic ATLAS type detector. The b-tagging efficiency was changed to more closely match the expected efficiencies at ATLAS [1, 24]. For b-jets with 50 GeV $\lesssim p_T \lesssim 200$ GeV, which is typical of the b-jets in the signal, the efficiency is approximately 60% for tagging a b-quark.

The signal event samples, for gluino pair production and decay, were produced using Pythia 6.4 and have been passed through the same PGS-4 detector simulation. Basic muon isolation was applied to all samples. To reduce the number of backgrounds events are required to pass the L1-triggers as defined by PGS. We also display the effect of two possible additional selection cuts, together with the additional requirement $\not{E}_T \geq 100$ GeV,

$$\textit{cut-1} : n_j(p_T \geq 50\,\mathrm{GeV}) \geq 4 \tag{III.1}$$

$$\textit{cut-2} : n_j(p_T \geq 30\,\mathrm{GeV}) \geq 4 \tag{III.2}$$

in the last two columns of Table 3.2. The second cut (weaker than the first) is optimal for discovery

Process	σ [fb]	σ_1[fb]	σ_2[fb]
$b\bar{b}+\gamma/Z+\text{jets}$	4.69×10^5	34.0	107.8
$b\bar{b}+W^\pm+\text{jets}$	2.41×10^4	7.71	13.3
$t\bar{t}+\gamma/Z+\text{jets}$	1.54×10^3	42.3	95.4
$t\bar{t}+W^\pm+\text{jets}$	2.25×10^2	14.3	27.6
$t\bar{b}+\gamma/Z+\text{jets}+h.c.$	1.34×10^3	7.37	26.6
$b\bar{b}+VV+\text{jets}$	1.14×10^3	1.45	3.94
$t\bar{t}+\text{jets}$	1.60×10^5	2076.7	5905.6
$VV+\text{jets}$	1.03×10^5	108.6	377.7
Model A	1.19×10^3	403.8	508.1
Model B	1.19×10^3	505.2	703.1
Model C	1.19×10^3	300.5	420.5

Table 3.2: Cross sections for production of signal and backgrounds. The first column gives the total production cross section. The second gives the cross section. With the exception of the $t\bar{t}$ cross section, we increased all SM background cross sections by a factor of 2, to account for possible K-factor from NLO corrections and to be conservative with our discovery expectations. The remaining columns give the cross section after the L1 triggers (given in Appendix A) and the selection cuts in Eq. III.1 and Eq. III.2, with an additional missing energy (MET) requirement, $\not{E}_T \geq 100$ GeV. The $b\bar{b}+\text{jets}$ and $b\bar{b}b\bar{b}$-inclusive backgrounds have been considered, and after the applying the selection cuts in Eqs. III.1-III.2 and requiring at least one lepton, the number of events are negligible in the $\{b,\ell\}$ channels considered here. In this table, we set $m_{\tilde{g}} = 500$ GeV and $m_{LSP} = 100$ GeV.

signatures, such as the same-sign dilepton signature, that have relatively small SM backgrounds.

Next, the signal is searched for in multi b-jet ($n_b = 2,3,4$) and multi lepton channels ($1\ell, SS, OS, 3\ell$). All objects are required to have a minimum p_T of 20 GeV. Same sign (SS) and opposite sign (OS) di-leptons are separated as they can have different origins and sizes. We will use the possible excess in these channels to assess the discovery potential. Table 3.3 shows the expected number of events from the SM background as classified according to the number of b-tagged jets and isolated leptons in the event.

Table 3.3 shows the expected number of signal events with b-tagged jets and isolated leptons for the three benchmark models. Model A, which is predominantly a four top signal, has significantly more multi-lepton and b-jet events passing selection cuts than Model B and Model C, which have fewer four top events. In Table 3.3, the signal significance achievable with $1\ fb^{-1}$ integrated luminosity is shown. By requesting at least 4 b-tagged jets it is possible to observe signal significance $S/\sqrt{B}$ $\geq$5 for events with a single lepton. The one-lepton four-b-jet channel will prove to be robust and the best channel for discovery.

Number of Background Events $(\mathbf{B})$
Standard Model

B	*2b*	*3b*	*4b*
1ℓ	286.2	41.4	1.04
OS	32.8	5.65	0.007
SS	0.3	0.06	0
3L	0.14	0.007	0

Number of Signal Events $(\mathbf{S})$

	Model A				**Model B**				**Model C**		
S	*2b*	*3b*	*≥4b*		*2b*	*3b*	*≥4b*		*2b*	*3b*	*≥4b*
1L	47.1	39.3	19.3	*1L*	33.5	26.9	13.8	*1L*	18.0	14.4	7.4
OS	12.4	9.9	3.9	*OS*	6.4	5.0	1.7	*OS*	2.0	0.9	0.6
SS	6.6	5.1	2.3	*SS*	2.3	1.2	0.2	*SS*	0.7	0.6	0.2
3L	3.0	2.1	0.7	*3L*	0.7	1.0	0.3	*3L*	0	0.1	0.1

Significance $\left(\mathbf{S}/\sqrt{\mathbf{B+1}}\right)$

	Model A				**Model B**				**Model C**		
	2b	*3b*	*≥4b*		*2b*	*3b*	*≥4b*		*2b*	*3b*	*≥4b*
1L	2.77	6.03	13.5	*1L*	1.97	4.13	9.66	*1L*	1.06	2.21	5.18
OS	2.13	3.83	3.88	*OS*	1.10	1.93	1.69	*OS*	0.34	0.34	0.40
SS	5.75	4.95	2.30	*SS*	2.00	1.16	0.20	*SS*	0.58	0.58	0.20
3L	2.80	2.09	0.70	*3L*	0.65	0.99	0.30	*3L*	0	0.10	0.10

Table 3.3: Number of SM events, number of signal event, and signal significance, with 2, 3, or 4 b-tagged jets and OS, SS, or 3 leptons at the early LHC-7, for $1fb^{-1}$ integrated luminosity. For the 1-lepton counts, *cut-1* was applied, while for the other lepton counts *cut-2* was applied. These numbers were found for $m_{\tilde{g}} = 500$ GeV and $m_{LSP} = 100$ GeV.

3.3 Scan and Results

For each model (a fixed $m_{\tilde{g}}$ and m_{LSP}), we simulated $1fb^{-1}$ of data using Pythia and PGS. Then we searched for the models over the backgrounds for the selection cuts in Eqs. III.1-III.2 in each of the b-jet and lepton ($\{b,l\}$) channels. A statistical significance in a $\{b,\ell\}$ channel is defined as $\sigma_{\{b,\ell\}} \equiv \frac{S_{\{b,\ell\}}}{\sqrt{B_{\{b,\ell\}}+1}}$ where $S_{\{b,l\}}(B_{\{b,\ell\}})$ is the number of signal(background) events expected to be in the $\{b,\ell\}$-channel for one of the two selection cuts in Eqs. III.1-III.2. Thus, if for any of the significances, $\sigma_{cut_i,\{b,\ell\}} \geq 5$, the model can be considered discoverable at $1fb^{-1}$. In Figures 3.1-3.3 we plot $\sigma_{cut_1,\{b,\ell\}} = 5$ contours, for the channels

$$\{\geq 4b, 1\ell\}\ \{3b, 1\ell\}\ \{\geq 2b, SS\}\ \{\geq 2b, OS\}\ \{\geq 1b, 3\ell\}.$$

In the first two channels *cut-1* is used, and in the last three channel, the weaker *cut-2*, is used. As is evident from Table 3.3, the backgrounds for $\{\geq 4b, 1\ell\}$ are significantly smaller than the backgrounds for $\{3b, 1\ell\}$, and therefore it is not beneficial to combine them into the inclusive channel $\{\geq 3b, 1\ell\}$. The channels we used in this study maximize the significance.

In all case the $\{\geq 4b, 1\ell\}$- channel provides the best channel for discovery. But, the SS-dilepton channel can be a competitive mode for discovery. It is important that the 4-top final state will give signatures in several channels if it appears in any. Finding a second predicted channel would be valuable confirmation. If two or more channels are present a combined significance would be a useful construct and facilitate a claim of discovery.

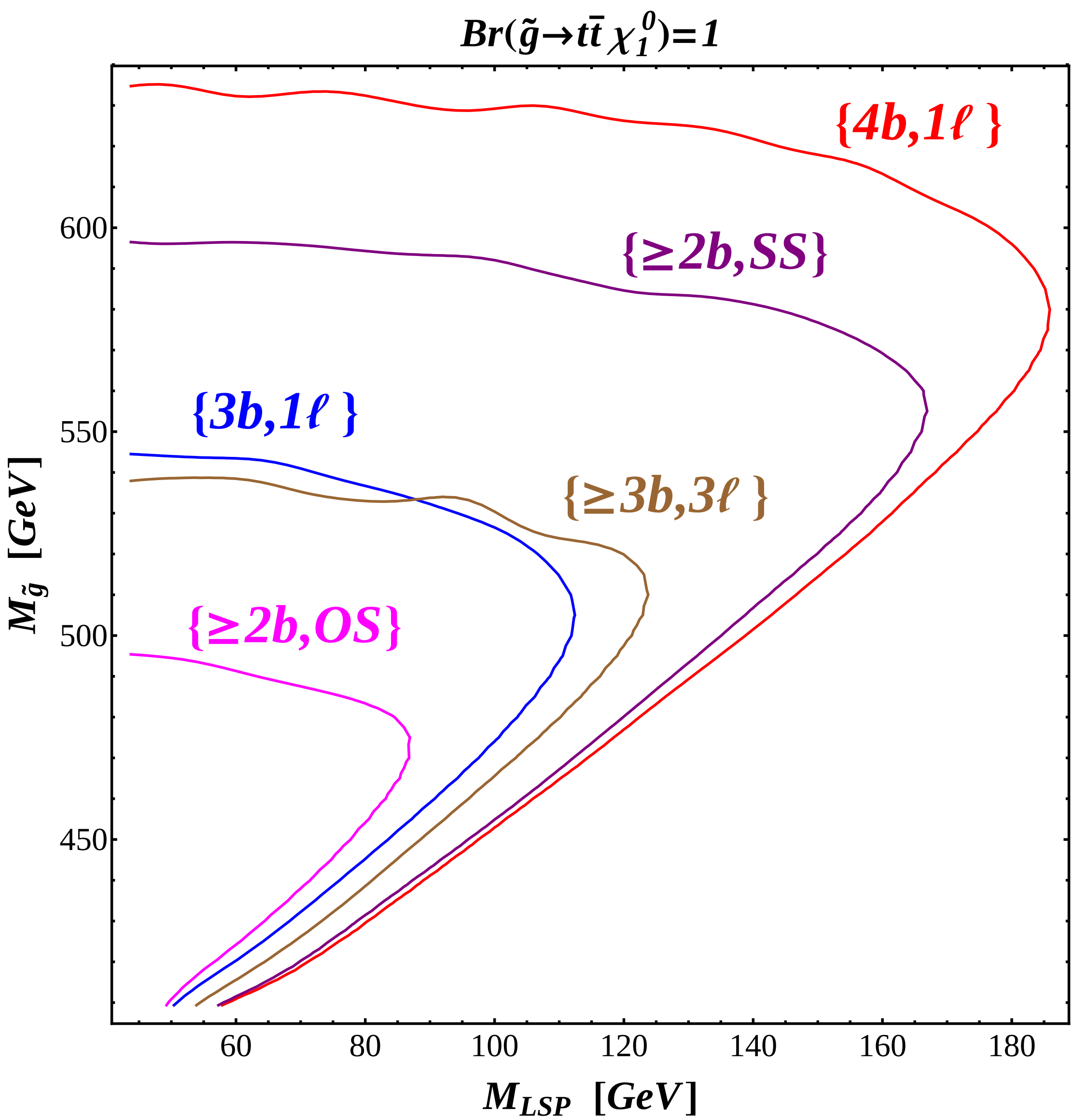

Figure 3.1: $\sigma = 5$ contours $\{b, \ell\}$-channels at LHC-7 TeV for $1fb^{-1}$ integrated luminosity of gluino pair production for Model A. The $\{4b, 1\ell\}$- channel provides the best channel for discovery. Since all events contain four tops, the SS-dilepton channel can be a competitive mode for discovery. There are other channels that will give a lower but noticeable excess, and will provide a valuable confirmation of a mutli-top signal.

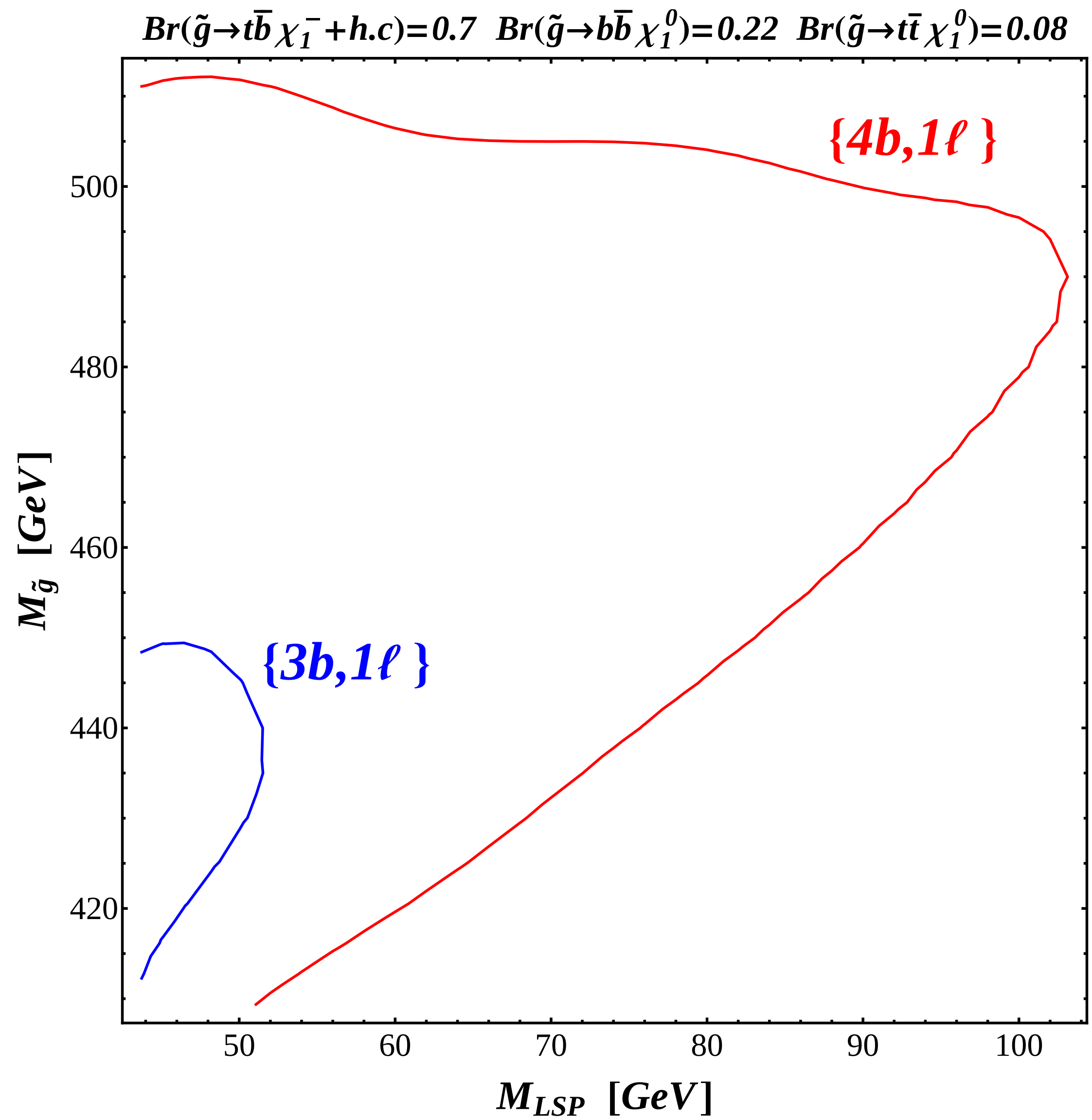

Figure 3.2: $\sigma = 5$ contours $\{b, \ell\}$-channels at LHC-7 TeV for $1fb^{-1}$ integrated luminosity of gluino pair production for Model B. The $\{4b, 1\ell\}$- channel provides the best channel for discovery. There are other channels that will give a lower but noticeable excess, and will provide a valuable confirmation of a mutli-top signal.

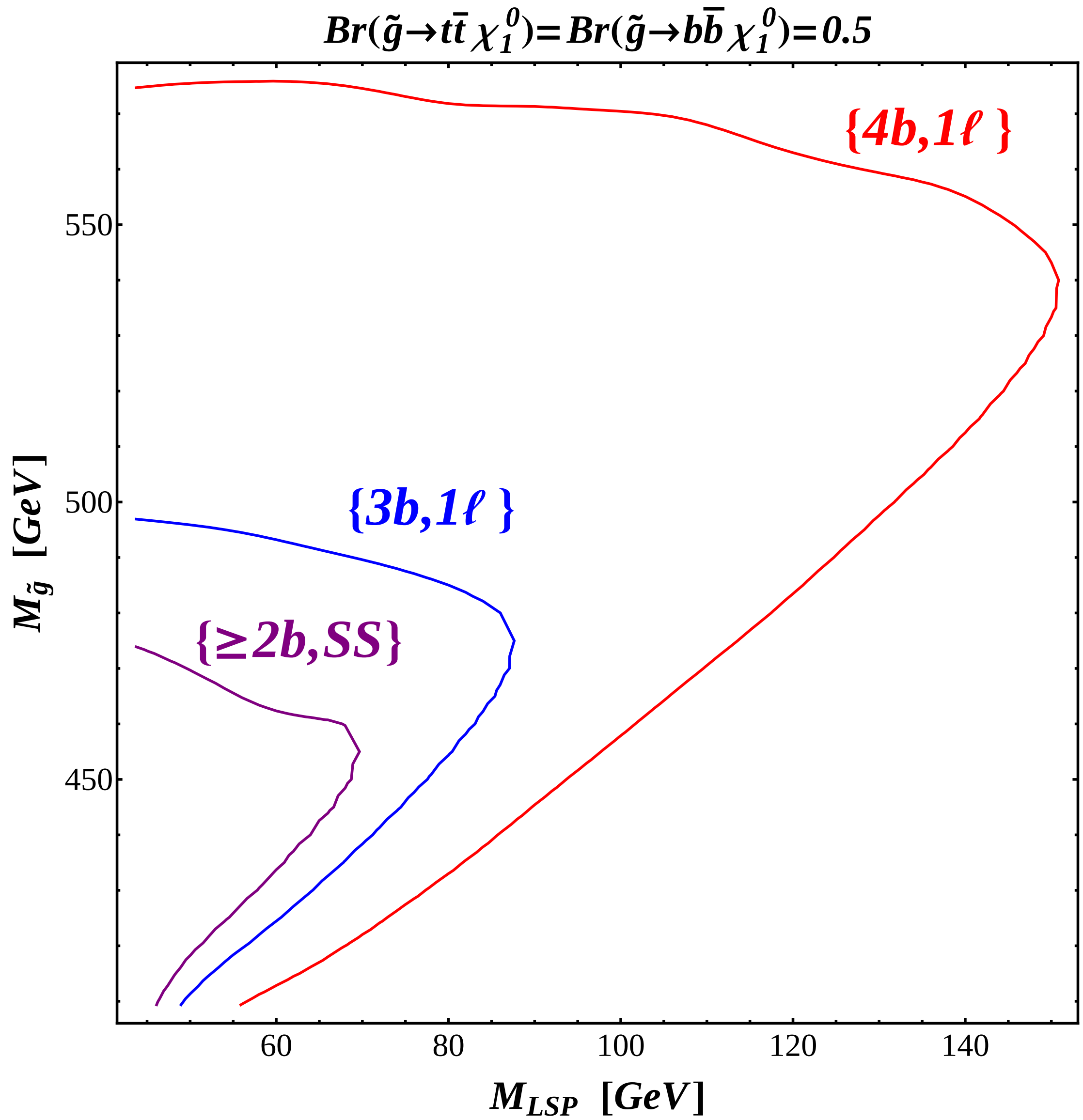

Figure 3.3: $\sigma = 5$ contours $\{b, \ell\}$-channels at LHC-7 TeV for $1fb^{-1}$ integrated luminosity of gluino pair production for Model C. The $\{4b, 1\ell\}$- channel provides the best channel for discovery. There are other channels that will give a lower but noticeable excess, and will provide a valuable confirmation of a mutli-top signal.

CHAPTER IV

Theory and Phenomenology of μ in M theory

M theory is excellent candidate for a consistent quantum theory of gravity, as it unifies the five string theories, while extending the number of spacetime dimensions from 10 to 11. Although a full formulation of M theory does not yet exist, its low energy limit is well understood as a 11 dimensional supergravity. Compactification of M theory is usually taken to be on a 7 dimensional manifold of G_2-holonomy since it contains exactly the correct number of covariantly constant spinors to leave the 4 dimensional theory with exactly $\mathcal{N} = 1$ supersymmetry.

4.0.1 Matter and Gauge Theory

Compactification of 11 dimensional supergravity on a smooth manifold (G_2 or otherwise) does not lead non-abelian symmetries and chiral matter, and thus cannot provide a description of our universe. However a realistic theory can arise if M-theory is compactified on a singular G_2-manifold. Non-abelian ADE gauge symmetries ($SU(n)$, $SO(2n)$ and E_6, E_7, E_8) are localized along three dimensional submanifolds of orbifold singularities [3, 4]. Chiral matter, charged under the ADE gauge theory, is localized at conical singularities in the seven dimensional G_2 manifold, at points where the ADE singularity is enhanced [13, 18, 183]. Matter will additionally be charged under the $U(1)$ symmetry, corresponding to the vanishing 2-cycle that enhances the singularity. Hence, all chiral matter will charged under at least one $U(1)$ symmetry. Bi-fundamental matter, charged under two non-Abelian gauge groups, is also possible, but will not be considered here.

As argued by [164], the additional $U(1)$ symmetries are never anomalous. Therefore, there is no Green-Schwarz mechanism [114] needed for anomaly cancellation, and GUT-scale FI D-terms are not present in the theory. This will be important later, since it removes a possibility for generating large scalar vacuum expectation values (vevs) for charged matter fields.

Two gauge theories will generically only have precisely the same size gauge coupling if they arise from the same orbifold singularities. Therefore, if gauge coupling unification is to be motivated theoretically, and not an approximation or accident, the gauge group of the ADE singularity should be a simple group containing the Standard Model gauge group, which we will take (for simplicity) to be $SU(5)$. Any larger group containing $SU(5)$ will give results similar to those we find below. To obtain the Standard Model gauge group, $SU(5)$ needs to be broken. Perhaps the 4D gauge symmetry can be broken spontaneously, but only representations smaller than the adjoint are realizable in M theory–the $\mathbf{10}$ and $\mathbf{5}$ representations (and their conjugates) in $SU(5)$. This leaves only "flipped $SU(5)$" [30, 40, 83] as a possible mechanism to break the GUT group and solve doublet-triplet splitting. Given the difficulty in constructing a realistic flipped $SU(5)$ model [139] (see Chapter V, it will not be considered here. The remaining possibility is to break the higher dimensional gauge theory by Wilson lines and will be discussed below.

4.0.2 Moduli Stabilization

In the mid-80's it was realized that, classically, string vacua contain a plethora of moduli fields. The standard lore was that, after supersymmetry breaking, the moduli fields would obtain masses and appropriate vacuum expectation values. Part of this lore was also the idea that strong dynamics in a hidden sector would be responsible for breaking supersymmetry at, or around, the TeV scale. Though some progress was made, it was not until recently that it has been clearly demonstrated that these ideas can be completely realized in string/M theory: in M theory compactified on a G_2-manifold (without fluxes) strong gauge dynamics can generate a potential which stabilizes all moduli and breaks supersymmetry at a hierarchically small scale [7, 8]. These vacua will be the starting point for our considerations.

In these vacua, the gravitino mass (and therefore also the moduli masses [14]) $m_{3/2} \sim \frac{\Lambda^3}{m_{pl}^2}$, where Λ is the strong coupling scale of the hidden sector gauge interaction. This is parametrically of order $\Lambda \sim e^{-2\pi/(\alpha_h b)} m_{pl}$, where α_h is the coupling constant of the hidden sector and b is a beta-function coefficient. The vacuum expectation values of the moduli fields are also determined in terms of α_h: Roughly speaking, one has:

$$\langle s^A \rangle \sim 1/\alpha_h \tag{IV.1}$$

where the modulus here is dimensionless and not yet canonically normalized. The physical meaning of the vevs of s^A is that it characterizes the volumes in eleven dimensional units of 3-cycles in the extra dimensions, e.g., the 3-cycle that supports the hidden sector gauge group. Thus, self-consistently when the hidden sector is weakly coupled in the UV, the moduli are stabilized at large enough volumes in order to trust the supergravity potential which only makes sense in this regime. In general, the rough formula exhibits the scaling with α_h and, numerically the moduli vevs in the vacua considered thus far range from about $1 \leq s^A \leq 5/\alpha_h$.

In order to incorporate the moduli vevs into the effective field theory in an M theory vacuum, we have to consider the normalized dimensionful vevs which appear in the Einstein frame supergravity Lagrangian. For obtaining the normalization it suffices to consider the moduli kinetic terms alone:

$$\mathcal{L} \supset m_{pl}^2 \frac{1}{2} g_{AB} \partial_\mu s^A \partial^\mu s^B \tag{IV.2}$$

where s^A are the dimensionless moduli described above and g_{AB} is the (Kahler) metric on the moduli space. From the fact that the extra dimensions have holonomy G_2, it follows that each component of g_{AB} is homogeneous of degree *minus* two in the moduli fields

$$g_{AB} = \partial_A \partial_B K = \partial_A \partial_B \left(-3 \ln V_7 + \dots\right) \tag{IV.3}$$

because the volume of X, V_7 is homogeneous of degree $7/3$.

For isotropic G_2-manifolds, i.e. those which receive similar order contributions to their volume from each of the N moduli, studying examples shows that, not only is the metric of order $\frac{1}{s^2}$, but also of order $1/N$:

(IV.4)
$$g \sim \frac{1}{N}\frac{1}{(s^A)^2}$$

Therefore in a given vacuum the order of magnitude of the entries of g_{AB} are

(IV.5)
$$g \sim \frac{\alpha_h^2}{N}$$

Therefore, a dimensionless modulus vev of order $1/\alpha_h$ translates into a properly normalized dimensionful vev

(IV.6)
$$\langle \hat{s^A} \rangle \sim \frac{1}{\sqrt{N}} \sim 0.1 m_{pl}$$

for $N \sim 100$, which is a typical expectation for the number of moduli [130][1].

This can lead to a suppression of the effective couplings which generate the μ-term, once the symmetry forbidding μ is broken. More precise calculations of the moduli vevs can be found in [8, 9]. Clearly, however, a G_2-manifold with less than ten or so moduli will not have suppressed, normalized moduli vevs; such cases are presumably unlikely candidates for G_2-manifolds with realistic particle spectra and will not be considered further.

We briefly also discuss the spectrum of Beyond Standard Model (BSM) particles which arise from the M theory vacuum. Classically, it is well known that string/M theory has no vacuum with a positive cosmological constant (de Sitter minimum). From the effective field theory point of view, this is the statement that moduli fields tend to have potentials which, in the classical limit have no de Sitter minimum. If we now consider quantum corrections to the moduli potential, which *only* involve the moduli fields – if they are computed in a perturbative regime – they tend to be small and hence are unlikely to generate de Sitter vacua. Positive, larger sources of vacuum energy must therefore arise from other, non-moduli fields. This is indeed the case in the M theory vacua

[1]Presumably, N is of the same order as the number of renormalizable coupling constants of the effective low energy theory.

described in [8]. Here the dominant contribution to the vacuum energy arises from a *matter* field in the hidden sector (where it can be shown that, without the matter field, no de Sitter vacuum exists). This is important for the following reasons.

Adopting supersymmetric terminology, this suggests that the fields with the dominant F-terms are not moduli. Hence, the moduli F-terms are suppressed relative to the dominant contribution (in fact, in M theory the suppression is of order α_h). This affects the spectrum of BSM particles. In string/M theory, gaugino masses are generated through F-terms of moduli vevs (because the gauge coupling function is a superfield containing volume moduli). Hence, at leading order these will be suppressed relative to, say, scalar masses which receive order $m_{3/2}$ contributions from all F-terms in the absence of accidental symmetries. Therefore, in the G_2-MSSM (and presumably other classes of string vacua) the scalar superpartners and moduli fields will have masses of order $m_{3/2}$ whereas the gaugino's will have masses which are suppressed; in fact in the G_2-MSSM the gaugino masses at the GUT scale are at least two orders of magnitude below $m_{3/2}$. This is what makes the anomaly mediated contributions to gaugino masses relevant to the G_2-MSSM and also why the models often contain a Wino LSP [9]. Important for our considerations below will be the fact that the suppression of the gaugino masses is greater than the suppression of moduli vevs discussed above by one order of magnitude (at the GUT scale), at least for G_2-manifolds with less than O(10^4) moduli.

4.0.3 Geometric Symmetries and Moduli Transformations

Compact, Ricci-flat manifolds with finite fundamental groups, such as manifolds with holonomy G_2 or $SU(3)$ can not have continuous symmetries. They can, however, have *discrete* symmetries. Witten was considering just such a discrete symmetry (G) of a G_2-manifold when he proposed the symmetry which prevents μ. Assuming the simplest possibility of an Abelian discrete symmetry, let us consider $G = \mathbf{Z_N}$, which acts on X:

$$\mathbf{Z_N} : X \longrightarrow X \tag{IV.7}$$

As a result of this, it will also act naturally on the fields on X. In particular $\mathbf{Z_N}$ will act on the set of harmonic forms on X. Our interest here is $H^3(X, \mathbf{R})$ the set of harmonic 3-forms on X, since this locally represents the moduli space of G_2-manifolds. A G_2-manifold, with moduli at a point $\langle s^S \rangle = s_0^A$ is determined by a harmonic (locally) G_2 invariant 3-form φ as

(IV.8) $$\varphi = \sum s_0^A \beta_A$$

where β_A are a basis for $H^3(X, \mathbf{R})$. If the point s_0^A is such that $\mathbf{Z_N}$ is a symmetry, then φ will be invariant under $\mathbf{Z_N}$, because invariance of φ is equivalent to invariance of the metric. The three-forms β_A transform in a representation of $\mathbf{Z_N}$, which is a real representation because the 3-forms are real on a G_2-manifold. Hence,

(IV.9) $$\mathbf{Z_N}: \quad \beta_A \to M_A^B \, \beta_B$$

where M is defined by this equation.

The fact that the particular G_2-manifold, characterized by the particular point in moduli space s_0^A, is $\mathbf{Z_N}$-invariant is simply the statement that:

(IV.10) $$s_0^B M_A^B = s_0^A$$

i.e., the s_0^A are an eigenvector of M with unit eigenvalue. Clearly, this will not be true for a generic vector s^A; hence, for a generic point in the moduli space, the entire $\mathbf{Z_N}$ symmetry will be broken. Since the representation of $\mathbf{Z_N}$ defined by the matrix M is real, it must be the sum of a complex representation plus its conjugate. Thus, the basis β_B can be chosen such that the complex representation is spanned by *complex* linear combinations of moduli fields. For instance, there might be a linear combination

(IV.11) $$S = \hat{s}^1 + i\hat{s}^2$$

which we choose to write in-terms of the dimensionful fields ($\hat{s}$), that transforms as

(IV.12) $$S \to e^{2\pi i/N} S.$$

Since we usually consider complex representations of discrete symmetries acting on the matter fields in effective field theories, it will be precisely the linear combinations of moduli (those in the form (IV.11)) which span $\mathbf{r_C}$ which will appear in the "symmetry breaking sector" of the effective Lagrangian. In other words, the moduli will appear in complex linear comibinations such as (IV.11) in the Kahler potential operators containing other fields that transform under the $\mathbf{Z_N}$. Note that in (IV.11) we are abusing notation in the sense that the "i" which appears is in general an N-by-N matrix whose square is minus the identity.

4.1 Witten's Solution

In heterotic and type-II string theories doublet-triplet splitting is often solved via orbifold compactifications [122, 182]. In these theories, higher (space-time) dimensional gauge symmetries are broken by the Wilson lines in an orbifold compactification, while the Kaluza-Klein zero mode Higgs triplets are absent due to non-trivial transformations under the orbifold symmetry. On the contrary, matter fields in M theory are co-dimension 7, that is, the fields live only in four dimensions, and are not zero modes of a KK tower of fields, so this solution to the μ-problem will not work. Other possibilities, such as NMSSM realizations or string instanton effects, will also not work since the symmetry that forbids μ (a $U(1)$ or stringy selection rules) would also forbid the triplet mass, thus spoiling doublet-triplet splitting.

One may also consider the possibility that a discrete R-symmetry can forbid the μ-term while solving doublet-triplet splitting. Requiring the symmetry to be anomaly free, and that it commutes with the gauge theory can lead to a unique symmetry [144]. However, this symmetry will also forbid the triplet mass and spoil doublet triplet splitting unless the triplets are absent from the four dimensional theory. For most string theories, this can be accomplished by a Wilson line in the higher dimensional theory, but in M theory, this is not possible since matter only exists in four dimensions.

Therefore, an alternative approach is needed to solve doublet-triplet splitting in M theory. The only known possibility, originally discussed by Witten, is to construct a discrete $\mathbf{Z_N}$ symmetry of the geometry, that will act on both matter fields and moduli-fields. When combined with a discrete Wilson line thats breaks $SU(5)$, this symmetry need not commute with the $SU(5)$, thus allowing components of a single $SU(5)$ representation to have different $\mathbf{Z_N}$ charges. Since the above arguments demonstrate that there must be a symmetry that acts differently on doublets and triplets, so far this is the only approach known to work, and maybe be the only solution.

The minimal $SU(5)$ matter content contains three generations of matter descending from three copies of $\mathbf{10_M} \oplus \mathbf{\bar{5}_M}$. There is also a $\mathbf{5_H} \oplus \mathbf{\bar{5}_H}$ pair containing the MSSM Higgs doublets, $H_u \oplus H_d$, and a vector-like pair of Higgs triplets, $T_u \oplus T_d$. Here a doublet and a triplet from one of the Higgs representations can transform differently under the $\mathbf{Z_N}$ symmetry group. Without loss of generality or phenomenology, this field is taken to be the $\mathbf{\bar{5}_H}$ field, with the following charges for the fields

(IV.13)

Field		$\mathbf{Z_N}$
$\mathbf{10}_M$		η^σ
$\mathbf{\bar{5}}_M$		η^τ
$\mathbf{5}_H$	T_u	η^α
	H_u	η^α
$\mathbf{\bar{5}}_{\overline{H}}$	T_d	η^γ
	H_d	η^δ

where $\eta \equiv e^{2\pi i/N}$.

These charges are constrained by the requirement that the $\mathbf{Z_N}$-symmetry does not forbid necessary terms in the superpotential, such as Yukawa couplings, Majorana neutrino masses, and the

Higgs triplet masses

(IV.14)

Coupling		Constraint
Up Yukawa Coupling	$\mathbf{10}_M\mathbf{10}_M H_u$	$2\sigma + \alpha = 0 \mod N$
Down Yukawa Coupling	$\mathbf{10}_M\overline{\mathbf{5}}_M H_d$	$\sigma + \tau + \delta = 0 \mod N$
Majorana Neutrino Masses	$H_d H_d \overline{\mathbf{5}}_M\overline{\mathbf{5}}_M$	$2\alpha + 2\tau = 0 \mod N$
Triplet Masses	$T_u T_d$	$\alpha + \gamma = 0 \mod N.$

The solution to these equations are

$$\begin{aligned} \alpha &= -2\sigma \\ \gamma &= 2\sigma \\ \delta &= -3\sigma + N/2 \\ \tau &= 2\sigma + N/2 \\ \sigma &= \sigma. \end{aligned} \tag{IV.15}$$

Inherently, the $\mathbf{Z_N}$ should forbid the μ-term, and if possible, other dangerous terms, such as dimension-5 proton decay operators and dimensions 3 and 4 R-parity violation.

(IV.16)

Coupling		Constraint
$\mu -$ term	$H_d H_u$	$-5\sigma + N/2 \neq 0 \mod N$
D-5 Proton Decay	$\mathbf{10}_M\mathbf{10}_M\mathbf{10}_M\overline{\mathbf{5}}_M$	$5\sigma - N/2 \neq 0 \mod N$
D-3 R-Parity	$\mathbf{5}_H\overline{\mathbf{5}}_M$	$N/2 \neq 0 \mod N$
D-4 R-Parity	$\mathbf{10}_M\overline{\mathbf{5}}_M\overline{\mathbf{5}}_M$	$5\sigma \neq 0 \mod N.$

Doublet-triplet splitting occurs if $5\sigma \neq N/2 \mod N$. If one only wants to solve doublet-triplet splitting while forbidding the μ-term, then there is a solution for $N = 2$ and $\sigma = 1$. Forbidding all the dangerous operators above can be accomplished with a $\mathbf{Z_4}$ symmetry.

An essential point is that the existing bounds coming from the LEP experiments assert that the masses of charged Higgsinos are at least 100 GeV, hence an effective μ-term must be generated. In our context here this implies that the $\mathbf{Z_N}$ symmetry must be broken, an aspect not discussed in [184]. This symmetry breaking is the subject of the next section.

4.2 Generating μ

As discussed in above, the $\mathbf{Z_N}$ symmetry is a geometric symmetry of the internal G_2 manifold, under which the moduli are charged. The G_2 moduli [7] reside in chiral supermultiplets whose complex scalar components,

$$z^j = t^j + is^j, \tag{IV.17}$$

are formed from the geometric moduli of the manifold, s_i, and axionic components of the three-from C-field, t_i. We expect the moduli to break the discrete symmetry just below Planck scale when their vevs are stabilized [8, 9],

$$\langle \hat{s}_i \rangle \sim 0.1 m_p. \tag{IV.18}$$

Likewise, the moduli F terms are expected to give gaugino masses in the usual way, so that

$$\langle F_{z^i} \rangle \simeq m_{1/2}\, m_p. \tag{IV.19}$$

where $m_{1/2}$ is the tree level gaugino mass at the GUT scale. The axion shift symmetries $t_i \to t_i + a_i$ require that only imaginary parts of the moduli appear in perturbative interactions. The superpotential, being holomorphic in the fields, will not contain polynomial terms that explicitly depend on the moduli. The μ-term can then only be generated via Kahler interactions when supersymmetry is broken via a Guidice-Massiero like mechanism [108], i.e., from Kahler potential couplings quadratic in the Higgs fields.

To understand the size of μ (and $B\mu$) we we first find a combination of moduli fields (or product of moduli fields), invariant under the axion symmetries, that transform under (a complex representation of) $\mathbf{Z_N}$ with charge $5\sigma - N/2$

$$S^1 = \hat{s}^i + i\hat{s}^j \tag{IV.20}$$

and another with charge $-5\sigma - N/2$

$$S^2 = \hat{s}^m + i\hat{s}^n. \tag{IV.21}$$

These fields have the correct charge to break the symmetry and generate the μ-term which has total $\mathbf{Z_N}$ charge $-5\sigma - N/2$.

In a general supergravity theory [53, 180] the fermion mass matrix is

$$m^{\psi}_{ij} = m^3_{pl} e^{G/2} \left(\nabla_i G_j + G_i G_j\right) \tag{IV.22}$$

and the holormorphic components of the scalar mass matrix are

$$m^{\phi\ 2}_{ij} = m^4_{pl} e^{G} \left(\nabla_i G_j + G^k \nabla_i \nabla_j G_k\right) \tag{IV.23}$$

where $G = m_{pl}^{-2} K + \ln(m_{pl}^{-6}|W|^2)$ and subscripts on G denote derivatives with respect to the scalar fields ϕ_i or their conjugates $\phi^*_{\bar{i}}$. Respectively, (IV.22) and (IV.23) can be used to find μ

$$\mu = \langle m_{3/2} K_{H_u H_d} - F^{\bar{k}} K_{H_u H_d \bar{k}} \rangle \tag{IV.24}$$

and $B\mu$

$$\begin{aligned} B\mu &= \langle 2m^2_{3/2} K_{H_u H_d} - m_{3/2} F^{\bar{k}} K_{H_u H_d \bar{k}} + m_{3/2} F^m K_{H_u H_d m} \\ &- \left(m_{3/2} F^m K^{n\bar{l}} K_{\bar{l} m H_u} K_{n H_d} + (H_d \leftrightarrow H_u)\right) \\ &- F^n F^{\bar{m}} \left(\tfrac{1}{2} K_{H_u H_d n \bar{m}} - K^{j\bar{l}} K_{\bar{l} n H_u} K_{j \bar{m} H_d} + (H_d \leftrightarrow H_u)\right)\rangle \end{aligned} \tag{IV.25}$$

where the indices run over the moduli fields and we have used that the superpotential does not contribute to either mass. Leading contributions come from Kahler potential terms

$$K \supset \alpha \frac{(S^1)^\dagger}{m_{pl}} H_u H_d + \beta \frac{(S^2)}{m_{pl}} H_u H_d + h.c. \tag{IV.26}$$

where the coefficients α, β are expected to be $\mathcal{O}(1)$. Plugging the Kahler potential (IV.26) into the formulas for μ and $B\mu$ gives the μ-term

$$\begin{aligned} \mu &= \alpha \tfrac{\langle S^1 \rangle}{m_{pl}} m_{3/2} + \alpha \tfrac{\langle F^{S^1} \rangle}{m_{pl}} \\ B\mu &= 2\alpha \tfrac{\langle S^1 \rangle}{m_{pl}} m^2_{3/2} + \alpha \tfrac{\langle F^{S^1} \rangle}{m_{pl}} m_{3/2} + \beta \tfrac{\langle F^{S^2} \rangle}{m_{pl}} m_{3/2}. \end{aligned} \tag{IV.27}$$

However, as a result of (IV.18), (IV.19) and the suppression of $m_{1/2}$ by order two orders of magnitude in the G_2-MSSM, $\langle S^i \rangle m_{3/2} \simeq 10 \quad \langle F^{S^i} \rangle$, the contribution to the masses coming from

F-terms are sub-dominant, at least if we assume that $N \ll 10^4$. Therefore, to a good approximation

$$B\mu \simeq 2\,\mu m_{3/2} \tag{IV.28}$$

a fact which will have significant phenomenological consequences[2] .

The coefficients of the operators in (IV.26) are in principle determined from M theory, but is not yet known how to calculate them precisely. It is natural to assume that the coupling coefficients are of $\mathcal{O}(1)$. When combined with a model of moduli stabilization, such as in the G_2-MSSM described in [7–9], μ and $B\mu$ can be approximately determined. Since the real and imaginary components of the complex fields, S^1 (IV.20) and S^2 (IV.21), are expected to have similar, but not necessarily identical vevs, μ will generically have a phase, that will be unrelated to the phases that enter the gaugino masses. But, $B\mu$ and μ will have the same phase since both are proportional to S^1 and the same coupling constant.

Before moving on to the next section we discuss the possibility that other matter fields may be charged under the $\mathbf{Z_N}$ symmetry, spontaneously break the $\mathbf{Z_N}$ symmetry, and generate μ. Consider an $SU(5)$-singlet matter field X that generates the μ-term via the superpotential coupling XH_uH_d. Since X is a matter field, M theory requires that it is charged under least one $U(1)$ symmetry. Then H_uH_d is not invariant under the $U(1)$, and consequently, the triplet mass term T_dT_u is not invariant, spoiling doublet-triplet splitting. Thus, such contributions should not occur.

Alternatively, the μ-term may be generated by a $U(1)$ invariant combination of two fields, for example by the operator

$$\frac{X_1X_2}{\Lambda}H_uH_d. \tag{IV.29}$$

Requiring $\mu \gtrsim 10^3$ GeV, and taking $\Lambda \sim M_{GUT}$ this would require $\sqrt{\langle X_1X_2\rangle} \gtrsim 10^{10}$ GeV. Radiative symmetry breaking will generally give a vev $\sim m_{3/2}$– usually large vevs are associated

[2] We leave the case of $N \geq 10^4$ for further study.

with FI D-terms. But since FI D-terms are absent in M theory, it may be difficult for such large vevs to arise from here. The recent results of [17] do suggest that the F-term potential can generate large matter field vevs, however in that case the vevs are too large to be relevant for the μ problem. Therefore, we very tentatively conclude that a matter field spurion is not responsible for breaking the $\mathbf{Z_N}$ symmetry and giving a physically relevant μ-term.

Finally, we comment on a potential domain wall problem. The moduli are stabilized away from a $\mathbf{Z_N}$ point, which implies that the $\mathbf{Z_N}$ symmetry was really only an approximate symmetry of the G_2-manifold. The moduli stabilization serves to parameterize the amount that the G_2-manifold differs from a $\mathbf{Z_N}$ symmetric manifold. Therefore, since the $\mathbf{Z_N}$ symmetry is not an exact symmetry of the G_2 manifold, the Lagrangian will explicitly break the $\mathbf{Z_N}$ symmetry, and domains walls would not have formed in the early universe.

4.3 Origin of R-Parity in M theory

In the Standard Model, the Yukawa couplings and Higgs potential form the most general set of renormalizable couplings consistent with the gauge symmetries. In this sense, baryon (B) and lepton (L) number are accidental symmetries of the theory. However, this is not the case in supersymmetric theories, which allow for the B and L violating renormalizable couplings[3]

(IV.31)
$$W_{\not{R}} = \lambda' L L e^c + \lambda'' L Q d^c + \lambda''' u^c d^c d^c + \kappa L h_u.$$

If the squark masses are not of order the GUT scale (which presumably they are not), these operators can lead to too rapid proton decay if not heavily suppressed. Hence one usually introduces R-parity, where the Standard Model fields have R-parity $+1$, while their supersymmetric partners have R-parity -1. This forbids all the couplings in (IV.31).

[3]The final term in (IV.31) can be rotated away in superpotential by a unitary transformation on (h_d, L). This rotation will induce additional contributions to the lepton violating coupling constants λ' and λ'' that are proportional to the Yukawa couplings. Assuming that $\mu \gtrsim \kappa$, their sizes are approximately

(IV.30)
$$\lambda' \sim y_e \frac{\kappa}{\mu} \qquad \lambda'' \sim y_d \frac{\kappa}{\mu}$$

Additionally, R-parity invariance insures the stability of the LSP, and the absence of an R-parity can eliminate the LSP as a dark matter candidate. Therefore, in this section we will discuss the origin of R-parity in M theory, or at least an approximate R-parity that leaves the proton and LSP very long lived. Of course from a theoretical point of view an R-parity or equivalent symmetry should emerge from the theory and not be put in by hand.

The $\mathbf{Z_N}$ symmetry constructed above contains R-parity, but for generic moduli charges the complete $\mathbf{Z_N}$ symmetry, including any R-parity subgroup, will be spontaneously broken. Although the $\mathbf{Z_N}$ symmetry will prevent the superpotential couplings in (IV.31) from being invariant, supersymmetry breaking will revitalize the operators just as in the case of the μ-term, from Kahler potential operators

$$K_{\not{R}} \supset \frac{\tilde{S}^\dagger}{m_{pl}^2} LLe^c + \frac{\tilde{S}^\dagger}{m_{pl}^2} LQd^c + \frac{\tilde{S}^\dagger}{m_{pl}^2} u^c d^c d^c + \frac{\tilde{S}^\dagger}{m_{pl}} Lh_u \tag{IV.32}$$

where the $\tilde{S}^\dagger$'s symbolically represent the moduli and need not all be the same.

Just as the μ-term was generated from the Kahler potential as a result of moduli stabilization, the effective superpotential can be calculated from the supersymmetry breaking contribution from (IV.32) to

$$\begin{aligned} \lambda_{ijk} &\simeq m_{pl}^{-2}(\langle \tilde{S} \rangle m_{3/2} + F_{\tilde{S}})(K_{\not{R}})_{ijk} \\ \kappa &\simeq m_{pl}^{-1}(\langle \tilde{S} \rangle m_{3/2} + F_{\tilde{S}})(K_{\not{R}})_{Lh_u} \end{aligned} \tag{IV.33}$$

for $\lambda = \lambda', \lambda'', \lambda'''$ and where i, j, k run over the matter fields. Comparing (IV.33) to (IV.27), one easily sees that $\kappa \sim \mu$, since both are generated the same way.

Then using that $\mu \sim \kappa$, the superpotential can be rewritten as

$$W_{\not{R}} \simeq \frac{\mu}{m_{pl}}(LLe^c + LQd^c + u^c d^c d^c) + \mu L h_u. \tag{IV.34}$$

The trilinear couplings are suppressed but the lepton violating bilinear coupling is large and of order the μ-term–this is simply a consequence of κ not being suppressed. After rotating away the

Lh_u term using the approximation (IV.30), the superpotential simplifies to

$$W_{\not R} \sim y_e LLe^c + y_d LQd^c + \frac{\mu}{m_p} u^c d^c d^c \tag{IV.35}$$

where smaller terms in $\lambda', \lambda'', \lambda'''$ have been dropped. Thus the lepton number violating trilinears pick up large contributions from the bilinear term, even if they were originally suppressed.

The proton lifetime for the decay mode $p \to e^+\pi^0$ is estimated to be

$$\Gamma_{p\to e^+\pi^0} \simeq \frac{\lambda''^2}{4\pi}\frac{\lambda'''^2}{4\pi}\frac{m_{\text{proton}}^5}{m_0^4}. \tag{IV.36}$$

The current bounds on this partial decay width is $\tau_{p\to e^+\pi^0} > 1.6 \times 10^{33}$ years [29]. For scalar masses in the G_2-MSSM ($\sim 10\,\text{TeV}$ see [9]) this gives the experimental bound

$$\lambda''\lambda''' \lesssim 10^{-24} \tag{IV.37}$$

which clearly excludes the superpotential (IV.35), since $\lambda'' \sim y_e \sim 10^{-5}$ and $\lambda''' \sim \mu/m_{pl} \sim 10^{-14}$. Therefore, proton stability requires an additional form of R-parity invariance beyond the discrete symmetry proposed.

One possible way to preserve the R-parity is to simply assume that the G_2-manifold in the vacuum is R-parity invariant, though not $\mathbf{Z_N}$ invariant i.e. the vacuum partially breaks $\mathbf{Z_N}$ to an R-parity subgroup. For example, take $N = 6$, then

(IV.38)

	Coupling	$\mathbf{Z_6}$ charge
$\mu -$ term	$H_d H_u$	η^4
R-Parity	$M_{10}M_{\bar 5}M_{\bar 5}$	η^5
	$M_{\bar 5}H_u$	η^3

for $\eta \equiv e^{i2\pi/6}$. If all moduli transform under the $\mathbf{Z_3}$ subgroup of $\mathbf{Z_6}$, then $\mathbf{Z_6}$ is broken to $\mathbf{Z_2}$ R-Parity, since no R-parity couplings can be generated. This is technically satisfactory, but is presumably ”non-generic”. It could certainly emerge from M theory, but we will not consider it further here.

Alternatively, R-parity may manifest itself as matter-parity, a conserved remnant of a local, continuous $U(1)$ symmetry. As is well known, matter parity arises naturally in $SO(10)$ theories. When embedded into an $SO(10)$ unified theory, the Standard Model matter fields belong to a different representation than the Higgs fields– a generation of matter is contained in a $\mathbf{16}$ of $SO(10)$, while a pair of Higgs doublets comes from a $\mathbf{10}$ of $SO(10)$.

When $SO(10)$ is broken to $SU(5)\times U(1)_\chi$, for example by a discrete Wilson line, the Higgs fields and matter fields are charged differently under $U(1)_\chi$:

$$\begin{array}{rcl} SO(10) & \rightarrow & SU(5)\times U(1)_\chi \\ \mathbf{16} & \rightarrow & \mathbf{10_{-1}}\oplus\mathbf{\bar{5}_3}\oplus\mathbf{1_{-5}} \\ \mathbf{10} & \rightarrow & \mathbf{5_2}\oplus\mathbf{\bar{5}_{-2}}. \end{array} \tag{IV.39}$$

where the subscript is the $U(1)_\chi$ charge.

The vacuum expectation values of the Higgses, which are contained in the $\mathbf{5_2}$ and $\mathbf{\bar{5}_{-2}}$ multiplets, will break the $U(1)_\chi$ symmetry into a discrete $\mathbf{Z_2}$ subgroup. This is because the Lagrangian is no longer invariant under the full local transformation $\Phi\rightarrow e^{i\alpha(x)q_d}\Phi$, but only the subgroup of transformations given by $\alpha(x)=\pi$. In terms of the $U(1)_\chi$ charges q_χ, the chiral multiplets have $\mathbf{Z_2}$-parity $e^{i\pi q_\chi}$. Thus chiral superfields with even $U(1)_\chi$ charge will have parity $+1$ and fields with odd $U(1)_\chi$ charge will have parity -1. The $\mathbf{Z_2}$ symmetry is exactly R-parity.

The only $SU(5)$ singlet with $U(1)_\chi$ charge is the $\mathbf{1_{-5}}$ field (and its conjugate), and thus this is the only field that can break $U(1)_\chi$ without breaking the SM gauge group. But since it has odd $U(1)_\chi$ charge, its vev will break R-parity. Therefore an $SO(10)$ completion of $U(1)_\chi$ will not contain an unbroken R-parity, but perhaps when combined with the $\mathbf{Z_N}$ symmetry, R-parity violating operators may be sufficiently suppressed to allow a long lived proton and LSP. Next we estimate these lifetimes.

The singlet field $\mathbf{1_{-5}}$ can be considered to be the right-handed neutrino, ν^c, since it has the right quantum numbers to make the operator $\nu^c h_u L$ invariant under $U(1)_\chi$. However, if $\langle\nu^c\rangle\neq 0$, all

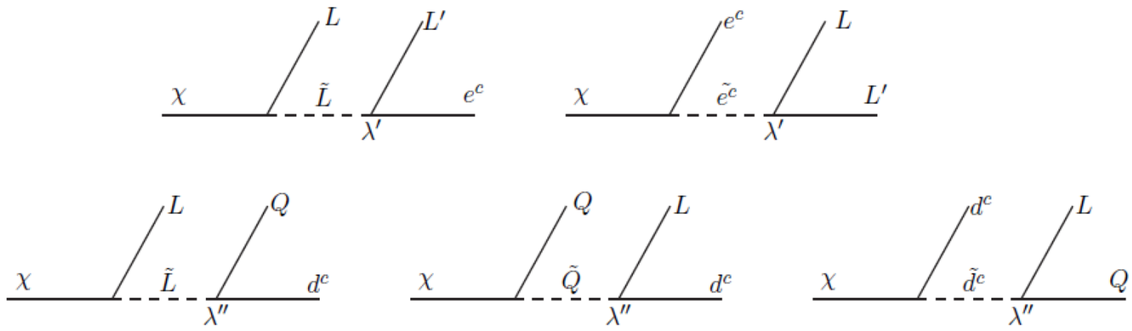

Figure 4.1: Decays of the LSP. Only the lepton number violating diagrams are shown, since the lepton number violating couplings– λ' (in the first line) and λ'' (in the second line)– receive large contributions (compared to the baryon number violating couplings) when the bilinear R-parity violating term, h_uL, is rotated away. Primes on the L indicate that the lepton flavor is different than the slepton flavor. Figures from [158].

baryon and lepton violating operators in (IV.31) will be generated via the superpotential

$$W_{\not R} \sim \nu^c LLe^c + \nu^c LQd^c + \nu^c u^c d^c d^c + \nu^c h_u L. \tag{IV.40}$$

The operators in (IV.40) should be suppressed and can be forbidden by the $\mathbf{Z_N}$ symmetry. The story will be the same as above and the Kahler potential operators will generate (IV.40) , but with additional suppression coming from $U(1)_\chi$ breaking

$$W_{\not R} = (\frac{\langle\tilde{S}\rangle m_{3/2} + F_{\tilde{S}}}{m_{pl}^2})(\frac{\langle\nu^c\rangle}{m_p})(LLe^c + LQd^c + u^c d^c d^c) + (\frac{\langle\tilde{S}\rangle m_{3/2} + F_{\tilde{S}}}{m_{pl}})(\frac{\langle\nu^c\rangle}{m_p})Lh_u. \tag{IV.41}$$

Diagonalizing away the Lh_u term, and using (IV.27) gives

$$W_{\not R} \sim y_e \frac{\langle\nu^c\rangle}{m_p} LLe^c + y_d \frac{\langle\nu^c\rangle}{m_p} LQd^c + \frac{\mu}{m_p}\frac{\langle\nu^c\rangle}{m_p} u^c d^c d^c. \tag{IV.42}$$

where again large lepton violating trilinear terms are induced by the rotation.

To be conservative in our estimates, we can take $\langle\nu^c\rangle \sim$ TeV, which may be expected from radiative symmetry breaking [28]. In this limit, proton decay constraints are safe from R-parity violation, but there are more stringent constraints coming from the LSP lifetime. Current bounds on the LSP lifetime are slightly model dependent, but for the most part are [91]

$$\tau_{LSP} \lesssim 1 \text{ sec} \qquad \text{OR} \qquad \tau_{LSP} \gtrsim 10^{25} \text{ sec} . \tag{IV.43}$$

The first bound excludes the region where the LSP decays would ruin the successful predictions of big bang nucleosynthesis on light nuclei abundances [96, 172]. The other region is excluded by indirect dark matter detection experiments that search for energetic positrons and anti-protons coming from decaying or annihilating relics [34, 37, 44, 173].

The LSP lifetime can be calculated in terms of the general R-parity violating superpotential couplings (IV.31). Diagrams in Figure 1 lead to an LSP lifetime

$$\tau \approx \frac{10^{-17}\,\text{sec}}{\lambda^2}\left(\frac{m_0}{\text{TeV}}\right)^4\left(\frac{100\ \text{GeV}}{m_{\text{LSP}}}\right)^5 \tag{IV.44}$$

where $\lambda = \lambda', \lambda'', \lambda'''$ and m_0 is the mass of the sfermion mediating the decay. Taking $\lambda = \frac{\langle \nu^c \rangle}{m_p} \sim 10^{-15}$, $m_0 \sim 10$ TeV, and $m_{LSP} \sim 100$ GeV gives

$$\tau_{\text{LSP}} \sim 10^{17}\ \text{sec}\ , \tag{IV.45}$$

about the age of the universe. The R-parity violating couplings still need to be about $10^{-4} \sim 10^{-5}$ smaller to have an LSP lifetime greater than 10^{25} seconds.

There are several ways additional suppressions might arise. We have not yet discussed the possibility of there being a horizontal family structure to the couplings. This could appear as a Froggett-Nielson symmetry, or a symmetry relating the locations of the matter singularities on the G_2 manifold, and would be responsible for forging the quark and lepton hierarchy. It may also suppress the LSP decay width pass the astrophysical bounds. Family symmetries arise naturally from the E_8 structure [137], which can also explain why the Standard Model has three generation, and this may hint towards a larger gauge theory. We leave this issue to future work..

If the family symmetry is not the answer, then it may be the case that resolution of the E_8 singularity to $SU(5)$ preserves a $U(1)$ symmetry–whose charges are necessarily given as a linear combination of four $U(1)$s belonging to the coset group $E_8/SU(5)$–and is broken to an exactly conserved R-parity. There are two well known examples, $U(1)_\chi$ and $U(1)_\psi$, defined as the symmetries coming from the breaking $SO(10) \to SU(5) \times U(1)_\chi$ and $E_6 \to SO(10) \times U(1)_\psi$.

However, $U(1)_\chi$ does not contain a field that can break $U(1)_\chi$ to R-Parity, and $U(1)_\psi$ forbids Higgs triplet masses, spoiling doublet-triplet splitting, so neither of these choices give a conserved R-parity.

However, there is a possibility that $U(1)$ symmetry is similar to $U(1)_\chi$, in that the MSSM fields and right handed handed neutrinos have the same charge assignment as in $U(1)_\chi$, but has additional $SU(5)$ singlet fields with even charges[4]. These theories can then be broken to a *conserved* R-parity, when the additional singlets get vevs. It is easy to construct such a linear combination, though it is unclear why from a purely theoretical perspective why G_2 compactifications would favor this $U(1)$ symmetry. For instance, if $U(1)_a \times U(1)_b$ is the cartan subgroup of $SU(3)$ in the breaking pattern $E_8 \to E_6 \times SU(3)$, then the $U(1)$ given by the linear combination of charges

$$q_\chi + 5(q_a - q_b)$$

allows for conical singularities that give rise to MSSM and right handed neutrino fields with $U(1)_\chi$ charges, but with additional $SU(5)$ singlets with charges $q = \pm 10$. The vevs of the additional singlets will break the $U(1)$ to a Z_{10} symmetry that contains a Z_2 R-parity.

Finally we note (for the non string duality oriented reader) that $E_8 \times E_8$ is well motivated theoretically if the G_2-manifold is a $K3$ fibration. This is because the intersection matrix of 2-cycles inside $K3$ contain the Cartan matrix of $E_8 \times E_8$. It is in this case–that the gauge-theory of M theory matches the gauge theory of $E_8 \times E_8$ Heterotic string theory– in which M theory on a $K3$-fibered $G2$-manifold and the heterotic string theory on a T^3- fibered Calabi-Yau threefold are dual.

To summarize, we find that incorporating the μ parameter into the structure of M theory compactified on a G_2-manifold, with stabilized moduli, can lead to a broken discrete symmetry allowing μ to be non-zero. R-parity is slightly broken, giving an LSP lifetime long enough to be the dark

[4]If this $U(1)$ symmetry is to be broken to R-parity, then requiring the symmetry to be flavor blind, allowing for Higgs triplet masses, and allowing an explanation for neutrino masses, basically constrains the charges of the MSSM and right handed neutrino fields to be the $U(1)_\chi$ charges.

matter, but not quite long enough to evade satellite detector constraints. The theoretical structure allows for family symmetries, or an embedding of R-parity into E_8, both of which stabilize the LSP lifetime to be consistent with the experimental constraints. An example of the latter case is given above, so this is indeed a possibility. Either case will lead to the same dark matter phenomenology. The R-parity completion of this story is an interesting avenue for further investigation.

4.4 Phenomenology

The M theory framework, along with moduli stabilization in the G_2-MSSM, allows one to estimate the high-scale SUSY breaking masses and μ to within a factor of a few. This allows M theory to make many phenomenological predictions. For some cases even small variations in the high-scale theory can have significant phenomenological consequences. In particular, the low-scale values of μ and $\tan\beta$ have significant implications for dark matter properties, and thus it is crucial to have a good understanding of their low-scale values while considering the M theory predictions of the high-scale masses.

4.4.1 Electroweak Symmetry Breaking

The first and foremost phenomenological constraint is that the theory accurately produce electroweak symmetry breaking (EWSB). That is, the theory must give a stable potential (bounded from below), break the electroweak symmetry and allow for the correct Z-boson mass. Respectively, these three conditions can be quantified by the following tree level constraints at the EWSB scale

$$\begin{aligned} |B\mu| &\leq \tfrac{1}{2}(m_{H_u}^2 + m_{H_d}^2) + |\mu|^2 \\ |B\mu|^2 &\geq (m_{H_u}^2 + |\mu|^2)(m_{H_d}^2 + |\mu|^2) \\ M_Z^2 &= -2|\mu|^2 + 2\frac{m_{H_d}^2 - m_{H_u}^2\tan^2\beta}{\tan^2\beta - 1} \end{aligned} \tag{IV.46}$$

where $\tan\beta$ is not an independent parameter, but is determined by

$$\sin 2\beta = \frac{-2B\mu}{m_A^2}. \tag{IV.47}$$

and

$$m_A^2 = m_{H_u}^2 + m_{H_d}^2 + 2|\mu|^2. \tag{IV.48}$$

where A is the pseudoscalar Higgs boson.

To get a feeling for $\tan\beta$, we plug in the expected values (at the unification scale and with degenerate scalars) of $B\mu \simeq 0.2m_{3/2}^2$ and $m_A^2 \simeq 2m_{3/2}^2$, into $(IV.47)$ which gives $\tan\beta \simeq 10$. On the other hand, RGE flow will lower the values of both $B\mu$ and m_A^2, resulting in variations around $\tan\beta \simeq 10$. A numerical scan will show a lower bound of $\tan\beta \gtrsim 5$, when scalars are taken to be degenerate at the unification scale.

The lowest values of $\tan\beta$ occur for the smallest values of m_A^2. The EW scale value for the mass depends on the running of the Higgs scalar masses, and in turn is very sensitive to the values of the squark masses. For specific non-degenerate values of scalar masses at the unification scale, m_A^2 can be of order $B\mu$ at the EW scale, resulting in values of $\tan\beta < 5$. We will consider this situation in the next section.

At tree level the mass of the Z-boson is determined by the four Higgs parameters

$$M_Z(m_{H_u}^2,\ m_{H_d}^2,\ |\mu|^2,\ \tan\beta). \tag{IV.49}$$

These parameters not only depend on their respective values at the high-scale, but also on other masses as a result of RGE-flow. Assuming that the scalar masses are much larger than the gaugino masses, M_Z has strongest dependence on the Higgs mass parameters and stop masses

$$M_Z(\hat{m}_{H_u}^2,\ \hat{m}_{H_d}^2,\ \hat{B\mu},\ |\hat{\mu}|^2,\ \hat{m}_{Q_3}^2,\ \hat{m}_{U_3}^2). \tag{IV.50}$$

where hatted ($\hat{}$) masses refer to GUT scale values.

Interestingly the cancellation between the soft scalars masses contributing to M_Z can be significant, even in the case in which the scalar masses are unified at the GUT scale

$$\hat{m}_{H_u}^2 = \hat{m}_{H_d}^2 = \hat{m}_{Q_3}^2 = \hat{m}_{U_3}^2.$$

Naively what one thought was a large fine-tuning between the Higgs soft-masses and μ in eq. (IV.46) for M_Z, is in fact smaller. This is evident (see Figure 4.2) from the fact that the scalar masses can be of order the gravitno mass at unification and μ can be an order of magnitude smaller, but the cancellation in eq. (IV.46) for M_Z still occurs. In this sense, the ratio $\mu/m_{3/2}$, shown in Figure 4.2, might be considered a measure of the fine-tuning involved in EWSB. In other words, the smaller the ratio, the less fine tuning there will be of μ against the scalar masses in order to have the correct value for M_Z.

Degenerate Scalars

A numerical scan was performed over M theory parameter space described in [9] using SOFT-SUSY [23][5]. We allow for the following variation in the G_2-MSSM parameters,

- $10\,\mathrm{TeV} \leq m_{3/2} \leq 20\,\mathrm{TeV}$ – the gravitino mass
- $10 \leq V_7 \leq 40$ – the volume of the G_2-manifold in units of the eleven-dimensional Planck length
- $-10 \leq \delta \leq 0$ – the size of the threshold corrections to the (unified) gauge coupling, $\alpha_{\mathrm{GUT}}^{-1}$. [6]

An interested reader is referred to Section V of [9] for variations in the spectra of G_2-MSSM models. In addition, order one variations are allowed for the coefficients in (IV.27) for the formula for μ, while it is imposed that $B\mu$ is in the range.

$$1\,\mu m_{3/2} < B\mu < 3\,\mu m_{3/2}. \tag{IV.51}$$

The results are shown in Figure 4.2. As is evident from the plot, values of μ much smaller than the gravitino mass are allowed under all the constraints, signaling a non-imposed cancellation among the scalars contributing to M_Z. Of note is the fact that $\tan\beta$ and μ are inversely correlated, which will play a significant role in limiting the maximum spin-independent scattering cross-section, when scalar masses are unified at the high scale.

[5] See [98] for general phenomenological discussions.

[6] see Section IV of [9] for the precise definition of δ

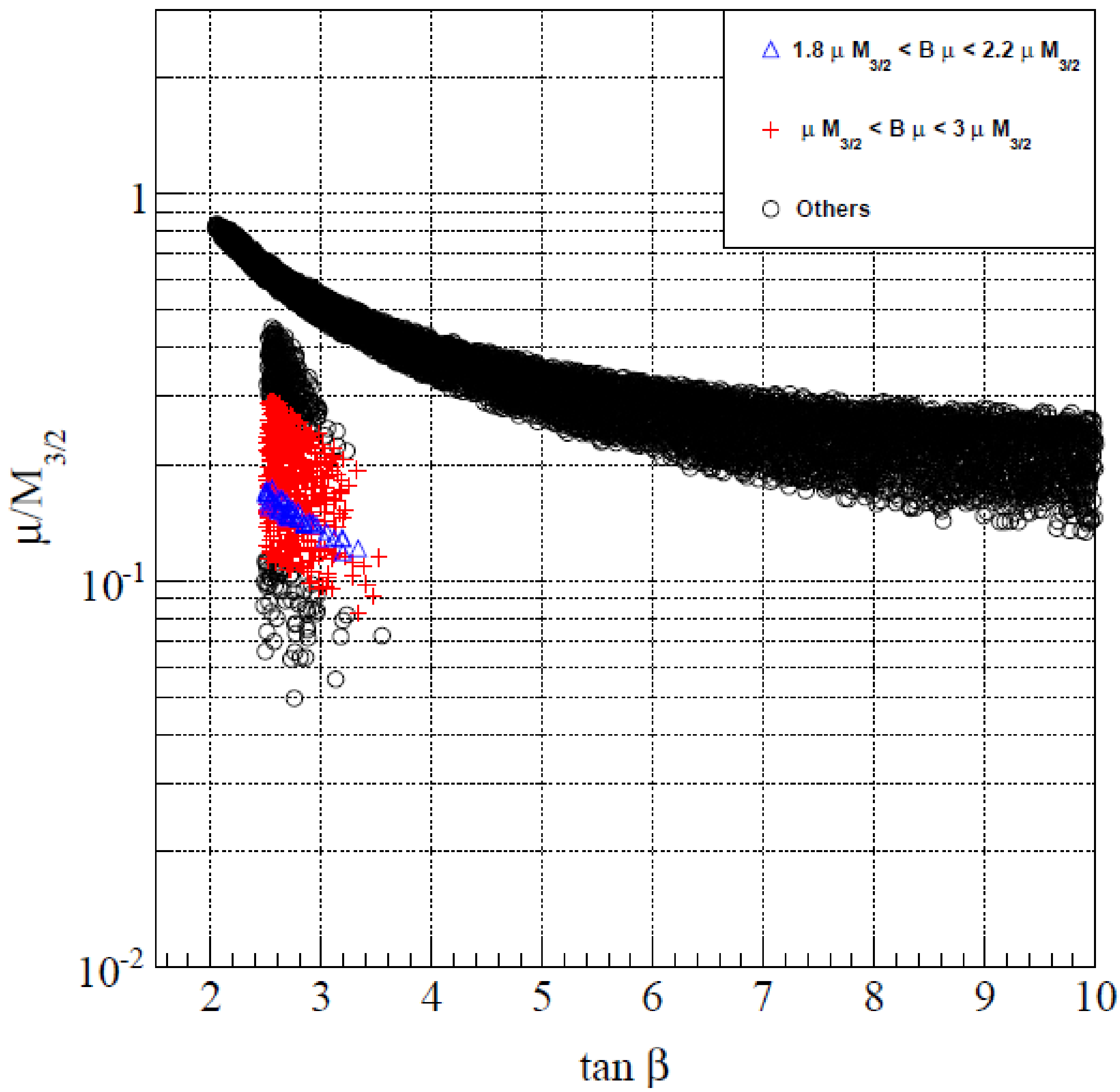

Figure 4.2: $\mu/m_{3/2}$ vs. $\tan\beta$. The upper band scans over the G_2-MSSM parameter space with *degenerate scalars* at the unification scale. The lower region on the left (low $\tan\beta$) scans over the G_2-MSSM parameter space where the scalar mass ratio $\hat{m}^2_{H_u}:\hat{m}^2_{U_3}:\hat{m}^2_{Q_3} = 3:2:1$ is required to be accurate within 20%. The black points show models that correctly break the EW symmetry, but are inconsistent with constraint $1\,\mu m_{3/2} < B\mu < 3\,\mu m_{3/2}$, so we expect them to not be valid solutions. The red points satisfy the constraint on $B\mu$ as given in the legend. The empty space on the plot, between the two regions, is expected to be filled in with complete scan over possible non-degenerate scalar mass parameter space. All points have EWSB.

Non-Degenerate Scalars and Low $\tan\beta$

We also consider the possibility that M theory allows for scalar unification to be somewhat perturbed (at the factor of two to three level). Since eventually we will be interested in calculating the largest possible spin-independent scattering cross sections we will only consider high-scale scalar masses that give $\tan\beta \lesssim 3$–since the scattering cross sections decrease with increasing $\tan\beta$

Consider the 1-Loop RGE equations, where only terms proportional to λ_t are kept, while neglecting the λ_t running. The RGE equations of the relevant scalars are:

$$\begin{aligned}
8\pi^2\frac{dm^2_{H_u}}{dt} &= 3\left|\lambda_t\right|^2\left(m^2_{H_u}+m^2_{Q_3}+m^2_{U_3}+\left|A_t\right|^2\right)\\
8\pi^2\frac{dm^2_{U_3}}{dt} &= 2\left|\lambda_t\right|^2\left(m^2_{H_2}+m^2_{Q_3}+m^2_{U_3}+\left|A_t\right|^2\right)\\
8\pi^2\frac{dm^2_{Q_3}}{dt} &= 1\left|\lambda_t\right|^2\left(m^2_{H_2}+m^2_{Q_3}+m^2_{U_3}+\left|A_t\right|^2\right)\\
8\pi^2\frac{dA_t}{dt} &= 6\lambda_t^2 A_t
\end{aligned}$$

whose solution is

(IV.52)

$$\begin{aligned}
m^2_{H_u} &= \tfrac{1}{2}\left(\hat{m}^2_{H_u}-\hat{m}^2_{U_3}-\hat{m}^2_{Q_3}+e^{\frac{3t\lambda^2}{4\pi^2}}(|\hat{A}_t|^2(-1+e^{\frac{3t\lambda^2}{4\pi^2}})+\hat{m}^2_{H_u}+\hat{m}^2_{U_3}+\hat{m}^2_{Q_3})\right)\\
m^2_{U_3} &= \tfrac{1}{3}\left(-\hat{m}^2_{H_u}+2\hat{m}^2_{U_3}-\hat{m}^2_{Q_3}+e^{\frac{3t\lambda^2}{4\pi^2}}(|\hat{A}_t|^2(-1+e^{\frac{3t\lambda^2}{4\pi^2}})+\hat{m}^2_{H_u}+\hat{m}^2_{U_3}+\hat{m}^2_{Q_3})\right)\\
m^2_{Q_3} &= \tfrac{1}{2}\left(-\hat{m}^2_{H_u}-\hat{m}^2_{U_3}+5\hat{m}^2_{Q_3}+e^{\frac{3t\lambda^2}{4\pi^2}}(|\hat{A}_t|^2(-1+e^{\frac{3t\lambda^2}{4\pi^2}})+\hat{m}^2_{H_u}+\hat{m}^2_{U_3}+\hat{m}^2_{Q_3})\right)\\
A_t^2 &= \hat{A}_t^2 e^{\frac{3t\lambda^2}{8\pi^2}}
\end{aligned}$$

where hatted () masses indicate GUT scale mass.

Since $m^2_{H_d}$ barely runs for low $\tan\beta$ and it is predicted that $\hat{\mu}^2$ is over an order of magnitude smaller than $m^2_{H_d}$, the cancellation in M_Z (IV.46) should occur between $m^2_{H_u}$ and $m^2_{H_d}$. Therefore, $m^2_{H_u}$ needs to stay positive at the EWSB scale. Ignoring the exponentially suppressed terms in (IV.52), we see that there are no choices of $\{\hat{m}^2_{H_u}, \hat{m}^2_{Q_3}, \hat{m}^2_{U_3}\}$ that leave all low-scale masses positive. On the other hand, there is a fixed point solution to the above RGEs

(IV.53) $$\hat{m}^2_{H_u} : \hat{m}^2_{U_3} : \hat{m}^2_{Q_3} = 3 : 2 : 1$$

where the non-exponentially suppressed terms are identically zero, insuring that if the trilinears are of order the scalars as expected in the G_2-MSSM, all three masses will stay positive. This fixed point is analogous to the focus point solution in minimal supergravity (mSUGRA) theories [57, 100], as it minimizes the fine tuning of EWSB. However, unlike the focus point region of mSUGRA where the Higgs scalars run small due to RGE flow, here the scalars remain heavy, and are close to the gravitino mass.

Near this region, low $\tan\beta$ parameter space with EWSB can be realized. Results on the numerical scan can be seen in Figure 4.2.

4.4.2 The Nature of the LSP

As explained in detail in [9], the G_2-MSSM framework gives rise to mostly Wino LSPs (as opposed to Bino LSPs). The tree level gaugino masses are degenerate at the GUT scale, but are suppressed by F-terms of the moduli relative to the gravitino mass to be of order the gaugino masses from the anomaly mediation contribution. The additional contribution from the anomaly lifts M_1 over M_2, leading to mostly Wino LSP models. In the original G_2-MSSM scenario, where it was simply that $\mu \sim m_{3/2}$, there were additional contributions to the gaugino masses from supersymmetric Higgs loops, proportional to μ [165], that for some choices of high scale parameters, could re-lift M_2 over M_1. These models are disfavored by precision gauge coupling unification [9], and occur less frequently in parameter space here than the original models since $\mu \not\sim m_{3/2}$. However, smaller μ will tend to introduce a small Higgsino admixture into the mostly Wino LSP - a fact which has significant implications on dark matter discovery. All these considerations combine to strongly suggest that a Wino-like LSP with mass $\sim 140 - 200$ GeV constitutes a significant fraction of the dark matter.

As emphasized in [15, 16], in order to obtain about the right relic density from the moduli decays, the LSP must be a Wino-like particle, with a large annihilation cross section of about $3 \times 10^{24}\,\mathrm{cm}^2$. A non-thermal history dominated by moduli and a wino LSP give a consistent picture for dark matter from the compactified string theory. Also encouraging is the fact that the PAMELA satellite data on positrons and antiprotons can be consistently described by a Wino LSP [64, 65, 99, 112, 120, 132]. More recently, by also considering Wino annihilations into photons and Z-bosons one finds a cross-section of about $10^{-26}\,\mathrm{cm}^2$ – a fact relevant for future Fermi data.

4.4.3 Direct Detection of Dark Matter

In December 2009, CDMS reported at most two possible WIMP candidate events, with a high likelihood of being background [21]. Combining with their previous data, this amounts to a bound on the spin-independent scattering cross-section of $\sigma_{si} \lesssim 6 \times 10^{-44}\,\mathrm{cm}^2$ for a WIMP of mass around 200 GeV. More recently, the XEXON100 experiment [32] reported observing no events after their first 11 days of running, slightly strengthening the CDMS bound. In the near future, XEXON100 is expected to report results that will probe much smaller scattering cross sections $\sigma_{SI} \sim 2 \times 10^{-45}\mathrm{cm}^2$. We will see that even this region is out of reach given the M theory predictions we calculate.

In the decoupling limit, defined when the pseudoscalar mass is much larger that the Z-boson mass, $m_{A^0} \gg M_Z$, the charged and heavy CP-even Higgses are also heavy, $m_{H^\pm} \simeq m_{H^0} \simeq m_{A^0}$. The other Higgs boson h^0 remains light and behaves in the same way as the SM Higgs boson. The lower bound on its mass, corresponds to the same bound on the SM Higgs boson, namely 114 GeV[7] [38]. All the models consistent with all the theoretical and phenomenological constraints have light Higgs mass close to thia LEP limit. Since the squarks are also heavy in G_2-MSSM, the light Higgs boson exchange will give the only substantial contribution to the spin-independent scattering cross sections. The scattering of the LSP off nuclei is via the Higgsino component. While the LSP will be mostly Wino-like, the prediction that μ is of order the TeV scale implies that the LSP wavefunction can have non-trivial Higgsino mixing.

Following [67] we estimate the size of the direct detection cross section in the decoupling limit

[7] Since there are theoretical and calculational uncertainties with calculating the Higgs mass, we will consider models with $m_h \geq 110$ GeV.

to be

(IV.54)

$$\sigma_{\rm SI}\left(\chi N\to\chi N\right)\approx 5{\times}10^{-45}{\rm cm}^2\left(\frac{115\,{\rm GeV}}{m_h}\right)^4\left(\frac{Z_{H_u}\sin\beta-Z_{H_d}\cos\beta}{0.1}\right)^2\left(Z_W-\tan\theta_W Z_B\right)^2$$

where the Z's give the composition of the LSP

$$\chi\equiv Z_B\,\tilde{B}+Z_W\,\tilde{W}\;+Z_{H_d}\,\tilde{H}_d+Z_{H_u}\,\tilde{H}_u. \tag{IV.55}$$

This gives us an estimate of the largest direct detection scattering cross sections, which naively may seem that for $Z_{H_u}\sim 0.1$ can be very close to the reach of XENON. Eq. (IV.54) can further be simplified, with the aid of analytical expressions for the neutralino mass matrix eigenvalues and eigenvectors [39,45,95]. Taking the limit $M_1=M_2$, which maximizes the scattering cross section for fixed μ and $\tan\beta$, (IV.54) becomes

$$\sigma_{\rm SI}^{\rm MSSM}\left(\chi N\to\chi N\right)\approx 6\times 10^{-45}{\rm cm}^2\left(\frac{115\,{\rm GeV}}{m_h}\right)^4\left(\frac{1\,{\rm TeV}}{\mu}\right)^2\left(\frac{\sin 2\beta+M_2/\mu}{1-(M_2/\mu)^2}\right)^2 \tag{IV.56}$$

which falls off both with $\tan\beta$ and μ. Allowing for the variation in M_1 and M_2 in the G_2-MSSM will only decrease this fraction. The value M_2/μ is typically around $.1\sim .2$. The parameters for three different models, along with their scattering cross sections, can be seen in Table 1 and are appropriately labeled in Figure 3.

However, as shown in the previous section, when considering degenerate scalar masses at the unification scale EWSB imposes that small μ corresponds to large $\tan\beta$, and small $\tan\beta$ corresponds to large μ. Hence, large cross-sections, of order the XEXON100 reach are not attainable for this region. To verify this we perform a scan of parameter space, using DarkSUSY [111]. The results are show in Figure 4.3 where it is seen that the largest scattering cross-sections are $\sim 1\times 10^{-45}{\rm cm}^2$, close to, but slightly beyond the reach of XENON100.

In Figure 4.3 we also scan over the G_2-MSSM parameter space, while requiring that the ratio $\hat{m}^2_{H_u}:\hat{m}^2_{U_3}:\hat{m}^2_{Q_3}=3:2:1$ be accurate within 20%. The spin-independent scattering cross-section reaches an upper limit of $1\times 10^{-45}\,{\rm cm}^2$, just beyond the XENON100 reach. Since this is the region where largest cross-sections appear, we can conclude that if the solution of the μ-problem proposed, along with moduli-stabilization in the G_2-MSSM, is the model of nature, the XENON100 experiment will not observe a dark matter signal soon, but its next run and upgraded detectors may do so.

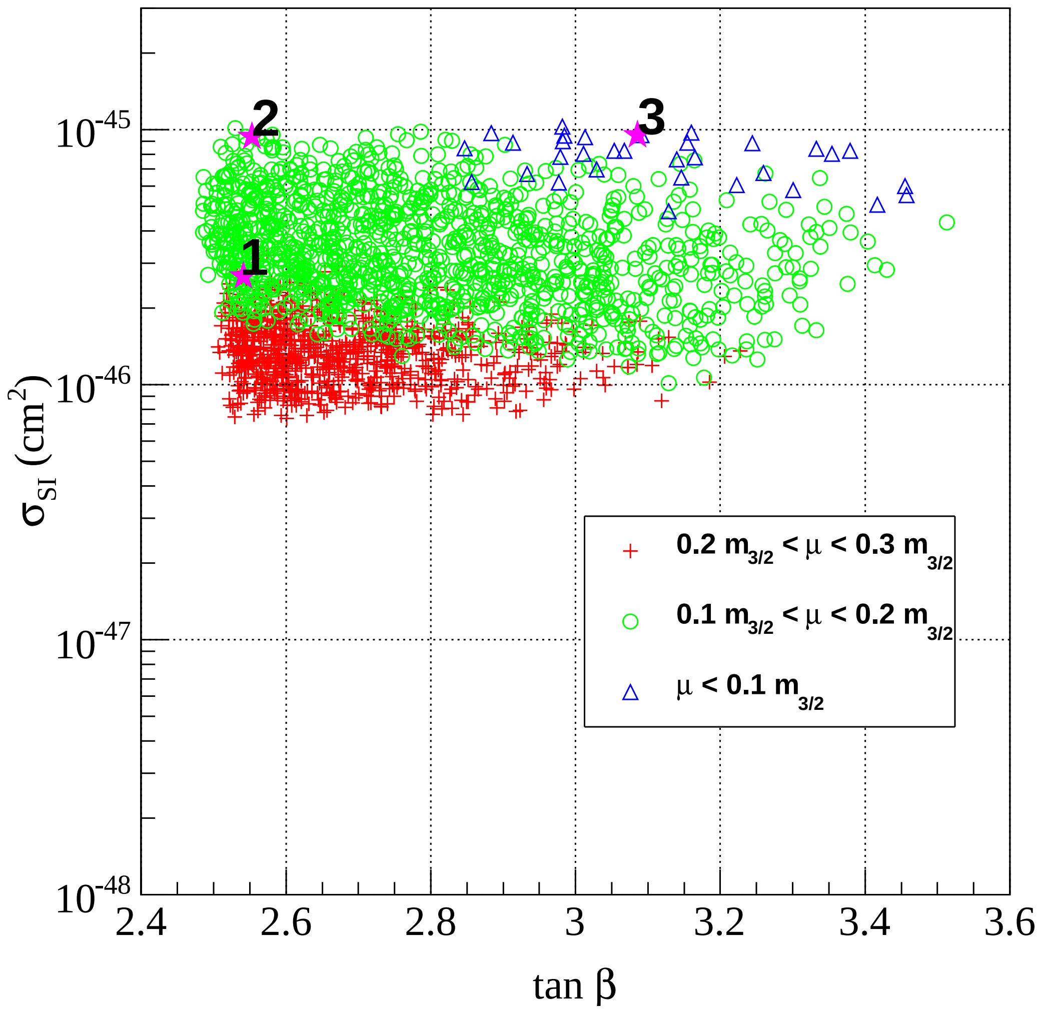

Figure 4.3: Spin-independent scattering cross-sections vs $\tan\beta$. The region shown scans over the G_2-MSSM parameter space where the scalar mass ratio $\hat{m}^2_{H_u} : \hat{m}^2_{U_3} : \hat{m}^2_{Q_3} = 3:2:1$ is required to be accurate within 20%. All points satisfy the constraint $\mu m_{3/2} < B\mu < 3\,\mu m_{3/2}$, have a SM-like Higgs with mass $m_h \geq 110$ GeV, and have EWSB. We also list the parameters for the 3 models in Table 1. In the region where EWSB, supergravity, and phenomenological constraints are satisfied, the upper limit on σ_{SI} is robust, but the lower limit can decrease if the sign of μ is reversed.

	Model 1	Model 2	Model 3
$M_{3/2}$	17.8 TeV	18.1 TeV	17.9 TeV
$\sqrt{B\mu_{\text{GUT}}}$	9.75 TeV	10.4 TeV	9.29 TeV
μ_{GUT}	3.79 TeV	2.10 TeV	1.69 TeV
M_1	151. GeV	153. GeV	150. GeV
M_2	145. GeV	143. GeV	138. GeV
μ	3.89 TeV	2.15 TeV	1.77 TeV
M_A	18.8 Tev	19.0 TeV	18.2 TeV
m_h	110. GeV	110. GeV	115. GeV
M_{χ_1}	141 GeV	143 GeV	141 GeV
M_{χ_2}	143 GeV	147 GeV	145 GeV
$M_{\chi_1^\pm}$	141 GeV	144 GeV	142 GeV
$Z_{\tilde{W}}$	0.94	0.91	0.91
$Z_{\tilde{B}}$	-0.35	-0.41	-0.41
$Z_{\tilde{H}_d}$	-0.02	-0.04	-0.05
$Z_{\tilde{H}_u}$	0.01	0.02	0.02
$\tan\beta$	2.53	2.37	2.87
$\sigma_{\text{SI}}[\text{cm}^2]$	$3. \times 10^{-46}$	$9. \times 10^{-46}$	$1. \times 10^{-45}$
$\sigma_{\text{SD}}[\text{cm}^2]$	$5. \times 10^{-45}$	$5. \times 10^{-44}$	$1. \times 10^{-43}$

Table 4.1: High scale and low scale parameters for 3 models with larger spin independent scattering cross sections. All models shown belong to the parameter space where the scalar mass ratio $\hat{m}^2_{H_u} : \hat{m}^2_{U_3} : \hat{m}^2_{Q_3} = 3 : 2 : 1$ is accurate within 20%. We assume that details of the calculation and software outputs are sufficiently uncertain to allow $m_h \gtrsim 110$ GeV to be consistent with LEP bounds.

CHAPTER V

Flipped $SU(5)$ and F-Theory

In the last few years there has been substantial interest in building flipped $SU(5)$ models in F-theory. This includes "ultra-local" constructions [129, 145] in the spirit of [43], phenomenological studies based on those constructions [146–150], and, quite recently, several "semi-local" and "global" realizations [60, 61, 66, 137][1]. At first, one might think that minimal $SU(5)$ models [42, 43, 88, 89] would be more economical in this setting; there is no problem with Yukawa suppression and many problems of ordinary 4-dimensional $SU(5)$ GUTs are avoided. Because F-theory models become effectively 8-dimensional at high scales, the GUT gauge group can be broken by turning on a nontrivial flux in the direction of $U(1)_Y$ along the internal dimensions [42,88]. This method of breaking roughly identifies the GUT scale with the compactification scale of an 8-dimensional gauge theory, hereafter referred to as the KK scale M_{KK}[2], and facilitates a simple removal of leptoquarks and Higgs triplets [43, 88].

These successes do not come for free. The $U(1)_Y$ flux, for instance, is known to distort gauge couplings at the KK scale [47,88] in a way that may be problematic. Further, if one tries to combine $U(1)_Y$ flux with the mechanism of [51, 56, 119] for generating flavor hierarchies, light charged exotic fields necessarily appear [155, 157][3]. There may be an interplay between the effects of these

[1] F-theory models describe physics near a stack of 8-dimensional branes in a nonperturbative background of type IIB string theory. "Ultra-local" models are based on intuition gained by studying physics on a single coordinate patch of the brane worldvolume. "Semi-local" models describe physics along the entire brane worldvolume and "global" models describe an embedding of the branes into a complete F-theory compactification.

[2] This identification is expected to be modified slightly as a result of contributions to gauge coupling renormalization from loops of closed string fields [68–70].

[3] As has been emphasized by J. Heckman and C. Vafa, this conclusion relies on the assumption of an underlying E_8 structure in

charged exotics, if they can be made sufficiently massive, and the distortion of unification [157]. Such a picture is significantly more complex than one might have hoped for based on the simplicity of "ultra-local" models [43], though, and therefore loses some of its appeal. For these reasons, it is important to investigate new mechanisms for breaking the GUT group or obtaining flavor hierarchies in F-theory models. Some promising ideas related to flavor include [93, 103, 137]. As for breaking the GUT group, flipped $SU(5)$ provides a natural alternative.

In this chapter, we do not focus entirely on the explicit construction of flipped $SU(5)$ models in F-theory, but rather on several phenomenological pitfalls that we encountered along the way and their implications for model building efforts. We first study the effects of nonrenormalizable operators and different choices of symmetry whose implementation can deal with them. This simple analysis is quite general and may be useful to see what is needed to embed flipped $SU(5)$ in a variety of string frameworks. After that, we center the discussion on issues specific to F-theory models. We should stress that our motivation is an alternative to GUT-breaking via hypercharge flux, to avoid disturbing gauge coupling unification. We therefore always insist that GUT-breaking and doublet-triplet splitting is accomplished via the $\mathbf{10}/\overline{\mathbf{10}}$ "GUT-Higgs" fields. We also work entirely within the framework of "minimal" flipped $SU(5)$, wherein the only light degrees of freedom are those of the MSSM and the pair of GUT-Higgs fields[4]. Models based on $SO(10)$ that utilize multiple fluxes to break the GUT group, as advocated for instance in [61, 137], do not suffer from the problems that we will discuss but will nonetheless have to deal with certain implications of $U(1)_Y$ flux[5].

When flipped $SU(5)$ models are UV completed into any particular string theory framework, physics at high scales can generate nonrenormalizable operators. Such operators can be dangerous

"semi-local" F-theory GUTs as described in Appendix D. We are not aware of any way to build semi-local or global F-theory models that avoids this so it may be that none exists. This is far from a proof, though, and it should be stressed that finding examples that do evade this structure would be very interesting.

[4]We sometimes make reference to the addition of vector-like pairs of complete $SU(5)$ multiplets as in [128]

[5]Since $SO(10)$ and $SU(5)\times U(1)_\chi$ are broken at essentially the same scale in such models, they are probably best thought of as F-theory realizations of $SO(10)$ GUTs rather than flipped $SU(5)$ "GUTs". One might also try to engineer an $SO(10)$ model that incorporates field theoretic breaking first to $SU(5)\times U(1)_\chi$ and then to the MSSM. Field theory models that do this were studied in [123].

because they arise at a scale that cannot be much larger than the roughly GUT-scale vevs of the "GUT-Higgs" fields. Innocent-looking operators of large dimension can therefore be transformed by the "GUT-Higgs" vevs into much more phenomenologically dangerous operators of dimension 4 and less that are not very strongly suppressed. The role of nonrenormalizable operators has been studied before in some specific examples, such as [152], where the resulting models are quite complicated and involve many new exotic fields. In this chapter, our interest is in the simplest type of flipped $SU(5)$ model, namely the one that exhibits a minimal particle content. That is, we include only the fields of the MSSM and the "GUT-Higgs" fields needed to break the flipped $SU(5) \times U(1)_\chi$ gauge group.

We were not able to find an exhaustive analysis in the literature of nonrenormalizable operators in flipped $SU(5)$ models, so we undertook this exercise and characterized the types of symmetries that can lead to favorable phenomenology. The most significant challenges are related to the μ problem, whose severity depends on one's attitude toward fine-tuning, although is should be noted that flipped $SU(5)$ was partially motivated to solve tuning problems. Of particular importance is a dimension 7 operator that does not seem to have been discussed in the literature before. This operator generates an enormous contribution to the μ parameter ($> 10^{10}$ GeV) and can only be controlled by an R-symmetry[6]. Our interest in F-theory makes this particularly troubling because "semi-local" F-theory models do not possess a suitable R-symmetry to deal with this. In those models, we therefore expect it to be generated and lead to a severe μ problem for which no simple solution is apparent. This issue may be important for a wider class of UV completions of flipped $SU(5)$ in string theory as well.

In addition to this, there appears to be some tension between the μ problem and generation of neutrino masses. Because the same Yukawa coupling that gives up-quark masses also contains the left and right handed neutrinos, it is well-known that a large Dirac neutrino mass will be generated.

[6]The importance of R-symmetries in flipped $SU(5)$ models has been noted before [81, 171] but we are not aware of a discussion of the operator that we study in this chapter.

Allowing the Majorana term that is needed to implement a successful type I seesaw simultaneously makes it impossible to forbid a bare μ term[7]. One can also run into trouble with dimension 5 proton decay operators here, but there are many factors such as sparticle masses and mixings that can potentially alleviate this problem [168].

Apart from the μ problem, we observe that the prevention of rapid (dimension 4-induced) proton decay requires discrete symmetries that do not have their origin as an unbroken subgroup of a continuous $U(1)$ symmetry that preserves the ordinary MSSM Lagrangian. For us, this is unfortunate because $U(1)$'s of this type are relatively easy to engineer in F-theory and represent the simplest way to generate discrete symmetries in that setting. The requisite symmetries must instead be engineered ”by hand” in F-theory models as honest discrete isometries of the compactification manifold that act in the right way on the zero modes that give rise to 4-dimensional fields. Obtaining such symmetries is conceptually straightforward but technically challenging; the only attempt we are aware of in an F-theory context was undertaken in [117].

After characterizing symmetries, we then turn to some ”F-theory-specific” challenges. Here, the most serious problems are engineering the GUT-Higgs fields and explaining their vevs. As has also been noted in the recent studies [60, 61, 66, 137], it seems very difficult to engineer only the MSSM and GUT-Higgs fields in models based on $SO(10)$ without obtaining additional exotics. The only solution seems to be realizing the GUT-Higgs as a vector-like pair which one expects to have a KK scale mass. One must then invent a mechanism by which very massive fields manage to acquire nonzero vevs. An alternative approach that we suggest is to build the gauge group $SU(5) \times U(1)_\chi$ directly. One gives up on unification here, making the proximity of α_1 to the other MSSM couplings at high scales seem like an accident, but at least the right spectrum of 4-dimensional fields can be realized. To this end, engineering $SU(5)$ is straightforward but Abelian

[7] In F-theory models, the absence of a symmetry that prevents μ essentially means that h_u and h_d must arise as a vector-like pair of zero modes on the same matter curve. While the presence of such vector-like pairs is rather generic when the matter curve has genus 1 or larger, there is no reason to expect that the pair remains massless since they can couple to moduli fields that can potentially acquire large vevs.

groups that do not embed into non-Abelian ones are somewhat subtle in F-theory. Fortunately, there has been recent progress in our understanding of these $U(1)$'s [116, 156] so it is possible to build compactifications for which we can reliably say that $U(1)_\chi$ exists as an honest gauge symmetry. In an Appendix, we provide a simple example of a compactification of this type based on the geometries of [154][8]. Several technical challenges remain, though, since we must ensure that $U(1)_\chi$ is not rendered anomalous by any of the fluxes that we use to induce chirality in the spectrum. Neither this issue, nor a simple way to count the number of ($U(1)_\chi$-charged) $SU(5)$ singlets, are well understood at the moment.

5.1 Brief Review of Flipped $SU(5)$

Flipped $SU(5)$ models are distinguished by their GUT gauge group, $SU(5) \times U(1)_\chi$, and the identification of hypercharge as a linear combination of $U(1)_\chi$ and a $U(1) \subset SU(5)$. What makes these models particularly interesting for us, though, is not the GUT gauge group itself but rather the existence of a simple, 4-dimensional mechanism for breaking $SU(5) \times U(1)_\chi$ down to the MSSM gauge group that lifts all non-MSSM fields that carry Standard Model charge (leptoquarks and Higgs triplets). Only one new set of fields is needed and, quite nicely, they transform in the $\mathbf{10}$ and $\overline{\mathbf{10}}$ representations, which are easy to engineer in string theory. Models that realize this mechanism of GUT-breaking are thus a natural alternative to consider in F-theory if one is looking for something other than internal flux to break the GUT group. Before considering this in earnest, though, we begin in this section by reviewing how this method of GUT-breaking works and the structure of flipped $SU(5)$ models in general.

In flipped $SU(5)$, hypercharge is identified as the linear combination

$$q_Y = \frac{1}{5}\left(q_\chi + q_y\right), \tag{V.1}$$

where q_χ is the $U(1)_\chi$ charge and q_y is the $SU(5)$ hypercharge (generated by diag $\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, -\frac{1}{2}, -\frac{1}{2}\right)$).

[8] There is no reason one has to use geometries like those of [154], which along with the compact models of [49, 115] were constructed with the use of $U(1)_Y$ flux in mind.

The MSSM matter fields and Higgs doublets transform under the $SU(5) \times U(1)_\chi$ as the representations

$$F_i \equiv 10_{-1} = (Q_i, d_i^c, \nu_i^c) \quad \bar{f}_i \equiv \overline{5}_3 = (u_i^c, L_i) \quad \ell_i \equiv 1_{-5} = (e_i^c)$$

(V.2) $$h \equiv 5_2 = (D_h, h_d) \quad \bar{h} \equiv \overline{5}_{-2} = (\bar{D}_h, h_u)$$

where i is a family index. Notice the "flipped" assignments of $d^c - u^c$, $e^c - \nu^c$ and $h_u - h_d$ in comparison to their typical assignment in the Georgi–Glashow $SU(5)$ model. The matter and Higgs fields participate in the typical Yukawa couplings

$$W \supset y_d^{ij} F_i F_j h + y_u^{ij} F_i \bar{f}_j \bar{h} + y_e^{ij} \bar{f}_i \ell_j h,$$

Note in "flipped" models the charged lepton and down-type quark masses need not unify, but Dirac neutrino masses and up-type quark masses do unify.

To break $SU(5) \times U(1)_\chi$, one introduces two new GUT-Higgs fields

(V.3) $$H \equiv 10_{-1} = (Q_H, D_H^c, \nu_H^c) \quad \bar{H} \equiv \overline{10}_1 = (\bar{Q}_H, \bar{D}_H^c, \bar{\nu}_H^c)$$

whose vacuum expectation values are aligned in the SM neutral directions $\langle \nu_H^c \rangle = \langle \bar{\nu}_H^c \rangle \sim M_{GUT}$. Leptoquarks are removed via the super-Higgs mechanism. To deal with Higgs triplets, one includes the superpotential couplings

(V.4) $$W_{\text{Flipped}} = \lambda_H H H h + \bar{\lambda}_H \bar{H} \bar{H} \bar{h}$$

which give masses to the Higgs color triplets

(V.5)

via mixing with the triplet components of the $H, \bar{H}$ multiplets. Since (V.4) leaves the Higgs doublets massless, flipped $SU(5)$ solves the doublet-triplet splitting problem.

The scale at which $SU(5) \times U(1)_\chi$ breaks to the MSSM is set by the $\nu_H^c, \bar{\nu}_{\bar{H}}^c$ expectation values and is typically referred to as M_{32}, since only the $SU(3)_c$ and $SU(2)_L$ couplings need unify there.

Unless the Higgs triplets are anomalously light due to small values of λ_H or $\bar{\lambda}_H$, M_{32} sits near the typical GUT scale $M_{\rm GUT} \sim 2 \times 10^{16}$ GeV.

As outlined in the introduction we will think of flipped $SU(5)$ as originating from an underlying $SO(10)$ theory. The scale at which $SU(5)$ and $U(1)_\chi$ unify into $SO(10)$ is typically referred to as the "super-unification" scale and denoted by M_{su}. In F-theory realizations that break $SO(10)$ to $SU(5) \times U(1)_\chi$ with an internal flux, M_{su} also denotes the Kaluza-Klein scale above which the physics becomes effectively 8-dimensional. For this reason, it provides us with a natural cutoff scale for 4-dimensional physics above which we expect towers of new states to generate nonrenormalizable couplings. We will use Λ to denote the cutoff scale of our 4-dimensional theory in the remainder of this chapter in order to be as general as possible, keeping in mind $\Lambda \simeq M_{su}$ in a large class of theories.

5.1.1 The Need for Symmetries

Nonrenormalizable couplings containing the GUT-Higgs fields can be a potentially serious problem in flipped $SU(5)$ models. In the presence of the large, nonzero expectation values of ν_H^c and $\bar{\nu}_H^c$, these can give rise to renormalizable couplings involving only MSSM fields. Such operators will typically be suppressed by powers of M_{32}/Λ where Λ is the cutoff scale at which the operator is generated that we usually take to be M_{su}. In the rest of this chapter we will denote the suppression factor by δ

(V.6) $$\delta \equiv \frac{\langle \nu_H^c \rangle}{\Lambda} \sim \frac{M_{32}}{g\Lambda}$$

where we used that the GUT-Higgs vev is related to the unfication scale by, $\langle \nu_H^c \rangle \sim M_{32}/g$ with g the $SU(5)$ coupling constant at M_{32}.

In general, δ cannot be too small. To get a conservative estimate for it, we can first effectively replace Λ by M_{Planck}[9] . As for M_{32}, this is not quite the standard MSSM unification scale $M_{\rm GUT} \sim 2 \times 10^{16}$ GeV because the triplets, being somewhat lighter than M_{32}, contribute to the running.

[9]One can in principle raise M_{su} up to M_{Planck} by introducing new vector like pairs at the TeV scale or above as in [128].

However, the unification scale M_{32} can be calculated from the 1-loop RGE equations for the flipped $SU(5)$ matter content, thus relating the triplet masses $m_{T,H}$ and $m_{T,\bar{H}}$, the scales M_{32} and $M_{\rm GUT}$ by

(V.7) $$M_{32}^2 \simeq \frac{M_{\rm GUT}^4}{m_{T,H} m_{T,\bar{H}}}$$

This means that the $SU(3) - SU(2)$ unification scale is actually increased relative to $M_{\rm GUT}$ by the triplets[10] allowing us to replace M_{32} by $M_{\rm GUT}$ to get a conservative estimate for δ. Putting it all together, we find that

(V.8) $$\delta \gtrsim \frac{M_{\rm GUT}}{M_{\rm Planck}} \sim 10^{-2}.$$

For many operators, this suppression will be entirely insufficient.

To control the effects of problematic nonrenormalizable operators, we must therefore introduce new symmetries. A drawback of flipped $SU(5)$ models is that one of the most useful symmetries for forbidding unwanted operators in the MSSM, $U(1)_\chi$, is strongly broken by the vevs of the GUT-Higgs fields. One reason that $U(1)_\chi$ is often so useful is that, as is well-known, it contains matter parity as a $\mathbb{Z}_2$ subgroup. Unfortunately, the GUT-Higgs fields carry odd $U(1)_\chi$ charge so not even this nice $\mathbb{Z}_2$ subgroup remains after GUT-breaking.

The situation is in fact worse because both F and H carry identical charges under any continuous symmetry that preserves the full Yukawa and flipped superpotential

(V.9) $$W_{\text{Yukawa+Flipped}} \sim FFh + F\bar{f}\bar{h} + \bar{f}\ell h + HHh + \bar{H}\bar{H}\bar{h}$$

Any attempt to realize matter parity as a subgroup of a continuous symmetry is bound to fail; H will always have the same parity as F, that is odd parity, and break it spontaneously. This is important for building string models because it means that matter parity must always be engineered on its own as an honest discrete symmetry.

[10]If the triplets become heavier than $M_{\rm GUT}$ then (V.7) indicates $M_{32} < M_{\rm GUT}$. In that case, though, the triplets would be heavier than M_{32} so would not contribute to the running. In fact, we have a bigger problem if $m_{T,a}$ is much larger than M_{32} because this would mean that the theory at M_{32} is becoming strongly coupled. We will always assume perturbativity, and hence $m_{T,a} < M_{32}$, leading to $M_{32} > M_{\rm GUT}$.

Field	$\mathbb{Z}_n$ charge	$U(1)$ charge	$U(1)_R$ charge
H	r	p	p_R
$\bar{H}$	s	q	q_R
h	$-2r \bmod n$	$-2p$	$-2p_R + 2$
$\bar{h}$	$-2s \bmod n$	$-2q$	$-2q_R + 2$
F	$r + \epsilon\frac{n}{2} \bmod n$	p	p_R
$\bar{f}$	$2s - r + \epsilon\frac{n}{2} \bmod n$	$2q - p$	$2q_R - p_R$
ℓ	$3r - 2s + \epsilon\frac{n}{2} \bmod n$	$3p - 2q$	$3p_R - 2q_R$

Table 5.1: All symmetries consistent with the full Yukawa + flipped superpotential (V.9), where r and s are taken to lie between 0 and $n-1$ and ϵ can take the value 0 or 1 if n is even but must be zero if n is odd.

Since $U(1)_\chi$ and its famous $\mathbb{Z}_2$ subgroup are unavailable, we must look to other options. In this chapter, the symmetries that we shall consider are of three types: discrete $\mathbb{Z}_n$ symmetries, continuous $U(1)$ symmetries, and $U(1)_R$ symmetries. The charges of all fields under the most general $\mathbb{Z}_n$, $U(1)$, and $U(1)_R$ symmetries that are consistent with (V.9) are listed in Table 5.1, where r and s are taken to lie between 0 and $n-1$. The parameter ϵ can take the value 0 or 1 if n is even but, obviously, must be zero if n is odd. Two common symmetries that appear in the literature are matter parity and a $\mathbb{Z}_2$ that goes by the name of H-parity. In the language of Table 5.1, these correspond to

$$\mathbb{Z}_2^{(\text{Matter parity})} \leftrightarrow n = 2,\ \epsilon = 1,\ r = s = 0 \tag{V.10}$$

and

$$\mathbb{Z}_2^{(H\text{-parity})} \leftrightarrow n = 2,\ \epsilon = 1,\ r = 1,\ s = 0 \tag{V.11}$$

In the next few sections, we will discuss ways to use symmetries of these types to address the μ problem, R-parity violation, and dimension 5 proton decay while simultaneously generating small neutrino masses[11]. The issue of controlling higher dimension operators in flipped $SU(5)$ models is of course not new but we are unaware of any previous work regarding some of the operators that we study. This is particularly true for the most troublesome operator, which has dimension 7 and

[11] We perform an operator analysis rather than studying which symmetries can be left unbroken by vevs of H and $\overline{H}$ because, in most cases, we do not need to forbid operators per se; we only need to suppress them. Often, a $\mathbb{Z}_n$ symmetry with a sufficiently large value of n will be sufficient even though what remains of it after GUT-breaking does not forbid anything

will be studied in section 5.2.

5.2 The μ Problem

5.2.1 Generalities

We begin by studying the generation of the supersymmetric Higgs mass, μ

(V.12)
$$W_\mu \sim \mu h\bar{h}$$

In addition to this bare μ term, there is an entire tower of operators that can generate a nonzero μ after GUT-breaking

(V.13)
$$\frac{1}{\Lambda^{2m-1}}\left(H\bar{H}\right)^m h\bar{h} \supset \left(\frac{\langle \nu^c_H \bar{\nu}^c_H \rangle}{\Lambda^2}\right)^m h\bar{h} \to g^{-1}\delta^{2m-1} M_{32} h\bar{h}$$

The charges of these operators under the symmetries in Table 5.1 are

(V.14)

Operator	$\mathbb{Z}_n$ charge	$U(1)$ charge	$U(1)_R$ charge
$\Lambda^{-(2m-1)}(H\bar{H})^m h\bar{h}$	$(m-2)(r+s) \bmod n$	$(m-2)(p+q)$	$(m-2)(p_R+q_R)+4$

Any continuous $U(1)$ symmetry that forbids a bare μ term has $p+q \neq 0$ and succeeds in forbidding all operators in the tower with $m \neq 2$. If we are interested in generating μ but ensuring that it is suppressed, we can instead try to use a $\mathbb{Z}_n$ symmetry with sufficiently large n since, for suitable values of r and s, the first solution other than $m = 2$ will sit at $m = n+2$. With $M_{32} \sim 10^{16}$ GeV and $\delta \sim 10^{-2}$, the operator with $m = 4$ will generate a μ of the right size $\sim 10^2$ GeV. A suitable $\mathbb{Z}_2$ symmetry is sufficient to forbid $m = 1$ and $m = 3$.

Unfortunately, the non-R symmetries always have a problem with the dimension 7 operator at $m = 2$ which, to our knowledge, has not been discussed previously in the literature

(V.15)
$$\mathcal{O}_7 = \frac{1}{\Lambda^3}(H\bar{H})^2(h\bar{h})$$

It is easy to see why $\mathbb{Z}_n$ and $U(1)$ have trouble forbidding this. The charge of $\mathcal{O}_7$ under any non-R symmetry is the sum of charges of two terms, hHH and $\bar{h}\bar{H}\bar{H}$, that are needed to lift the Higgs triplets. The only way to control it, then, is with an R-symmetry. More specifically, any $U(1)_R$ with $p_R + q_R$ neither 1 nor 2 is sufficient to eliminate the entire tower, including $\mathcal{O}_7$.

As discussed in Appendix D, though, the underlying 8-dimensional gauge theory of F-theory models does not provide a suitable R-symmetry so one always expects the operator $\mathcal{O}_7$ (V.15) to be generated. For this reason, we will spend a little more time studying it. The problem with $\mathcal{O}_7$ is that the μ term it induces is enormous

$$\mu_{\text{induced}} \gtrsim g^{-1}\delta^3 M_{32} \sim 10^{10}\text{ GeV} \tag{V.16}$$

This introduces an enormous fine-tuning problem for electroweak symmetry breaking, which defeats the purpose of building a Flipped $SU(5)$ to solve the tuning related to doublet-triplet splitting.

Recall that this estimate, which is based on taking $\Lambda \sim M_{\text{Planck}}$, is particularly conservative if we insist on realizing flipped $SU(5)$ in a semi-local F-theory model; reliability of the entire semi-local approach depends on having control over the underlying 8-dimensional gauge theory which, in turn, requires Λ to be at least an order of magnitude or two smaller than M_{Planck}.

5.2.2 Suppressing $\mathcal{O}_7$ with an approximate non-R symmetry

We cannot expressly forbid $\mathcal{O}_7$ (V.15) with a global non-R symmetry without losing the "flipped superpotential" (V.4) but we can imagine trying to suppress it with an approximate symmetry that is spontaneously broken. Since (V.4) must be generated if flipped $SU(5)$ is to elicit doublet-triplet splitting, the couplings in (V.4) are replaced by

$$W \supset \frac{S}{\Lambda}HHh + \frac{\bar{S}}{\Lambda}\bar{H}\bar{H}\bar{h}. \tag{V.17}$$

for some fields S and $\bar{S}$. Passing to expectation values, we define the dimensionless quantities λ_H and $\bar{\lambda}_H$ as

$$\lambda_H \sim \frac{\langle S\rangle}{\Lambda} \quad \bar{\lambda}_H \sim \frac{\langle \bar{S}\rangle}{\Lambda}. \tag{V.18}$$

Since the product of couplings in (V.17) is an invariant, at the very least the operator $\mathcal{O}_7$ (V.15) will be generated with suppression of $\lambda_H \bar{\lambda}_H$,

(V.19) $$\mathcal{O}_7' = \lambda_H \bar{\lambda}_H \frac{(H\bar{H})^2}{\Lambda^3} h\bar{h} \quad \to \quad \lambda_H \bar{\lambda}_H \delta^3 g^{-1} M_{32} \int d^2\theta \, h\bar{h}.$$

In this case, it naively seems that μ can be less than our previous estimate (V.16) if $\lambda_H, \bar{\lambda}_H \ll 1$. However, as $\lambda_H, \bar{\lambda}_H$ are lowered, the Higgs triplet masses, given in (V.5), will also be lowered. From (V.7), then, we see that the scale M_{32} becomes larger as we do this, and may even become super-Planckian. Taken together, it is in fact easy to see that all dependence of (V.19) on λ_H and $\bar{\lambda}_H$ cancels completely. This is because the triplet masses are related to λ_H and $\bar{\lambda}_H$ by

(V.20) $$m_{T,H} \simeq \frac{\lambda_H}{g} M_{32} \qquad m_{T,\bar{H}} \simeq \frac{\bar{\lambda}_H}{g} M_{32}$$

Using this, (V.7) becomes

(V.21) $$M_{32}^4 \sim M_{\mathrm{GUT}}^4 \times \frac{g^2}{\lambda_H \bar{\lambda}_H}$$

which leads to an induced μ term

(V.22) $$\mu_{\text{induced}} \sim \frac{\lambda_H \bar{\lambda}_H}{g^4} \frac{M_{32}^4}{\Lambda^3} \sim g^{-2} \left(\frac{M_{\mathrm{GUT}}^4}{\Lambda^3} \right) > 10^{10}\,\mathrm{GeV}$$

Introducing an approximate symmetry is, perhaps counterintuitively, not effective at lowering μ_{induced}.

5.2.3 Summary

The only way to avoid generating any contribution to the μ term after GUT-breaking is with a $U(1)_R$ symmetry that has $p_R + q_R \neq 2$. In the absence of such a symmetry, one expects $\mathcal{O}_7$ (V.15) to appear and lead to a μ term that is far too large. Provided a solution to the $\mathcal{O}_7$ problem can be found, a continuous $U(1)$ symmetry with $p + q \neq 0$ can get rid of the remaining operators in (V.13) while a $\mathbb{Z}_2$ can allow only those that give rise to $\mu \sim 10^2$ GeV or smaller. One idea for solving the $\mathcal{O}_7$ problem revolves around forbidding the terms (V.4) that generate masses for Higgs triplets with a continuous symmetry and breaking it through the vevs of suitable singlet fields. This solution does not appear to work, however, so one needs something more intricate.

5.3 R-Parity Violating Operators

Putting the μ problem aside for now, we next turn our attention to the generation of renormalizable MSSM superpotential couplings that violate R-parity. These couplings take the form

$$W_{\not R} \sim \lambda L L e^c + \lambda' Q L d^c + \lambda'' u^c d^c d^c + \kappa L h_u \tag{V.23}$$

The coupling κ can be rotated away by a field redefinition but only at the cost of inducing new contributions to the lepton violating trilinear couplings.

It is well-known that $U(1)_\chi$ contains a $\mathbb{Z}_2$ that acts like matter parity on MSSM fields, which means that none of the operators in (V.23) can arise on their own in a flipped $SU(5)$ model, which is based on gauge group $SU(5) \times U(1)_\chi$. However, the GUT-Higgses, H and $\bar{H}$, are parity-odd and will spontaneously break this $\mathbb{Z}_2$. Operators appearing in (V.23) can therefore appear in combination with suitable powers of H and $\bar{H}$

$$\begin{gathered}
\frac{1}{\Lambda^{2m}}(H\bar{H})^m H\bar{f}\bar{h} \supset \left(\frac{\nu^c_H \bar{\nu}^c_H}{\Lambda^2}\right)^m \nu^c_H L h_u \\
\frac{1}{\Lambda^{2m+1}}(H\bar{H})^m H F F \bar{f} \supset \left(\frac{\nu^c_H \bar{\nu}^c_H}{\Lambda^2}\right)^m \frac{\nu^c_H}{\Lambda} \begin{cases} QDL \\ UDD \end{cases} \\
\frac{1}{\Lambda^{2m+1}}(H\bar{H})^m H\bar{f}\bar{f}\ell \supset \frac{\nu^c_H}{\Lambda} LLE
\end{gathered} \tag{V.24}$$

In general, R-parity violating operators must be significantly suppressed, if not outright forbidden. As we have seen, the suppression factor $\delta = (\langle H_\nu \rangle / \Lambda)$ is not very small, taking values $\delta \gtrsim 10^{-2}$. This means that only operators with fairly high powers of m are safe. Additional symmetries are needed to forbid or suppress the rest.

The charges of the R-parity violating operators (V.24) under the symmetries of Table 5.1 are

(V.25)

Operator	$\mathbb{Z}_n$ charge	$U(1)$ charge	$U(1)_R$ charge
$\Lambda^{-2m}(H\bar{H})^m H\bar{f}\bar{h}$	$m(r+s)+\epsilon\frac{n}{2}$ mod n	0	2
$\Lambda^{-(2m+1)}(H\bar{H})^m HFF\bar{f}$	$(m+2)(r+s)+\epsilon\frac{n}{2}$ mod n	$(m+2)(q+p)$	$(m+2)(q_R+p_R)$
$\Lambda^{-(2m+1)}(H\bar{H})^m H\bar{f}\bar{f}\ell$	$(m+2)(r+s)+\epsilon\frac{n}{2}$ mod n	$(m+2)(q+p)$	$(m+2)(q_R+p_R)$

Notice that continuous symmetries alone are not sufficient to prevent a bilinear coupling $\kappa L H_u$ with $\kappa \sim M_{32} \gtrsim 10^{16}$ GeV. For this, we need at least one discrete symmetry.

5.3.1 A Single $\mathbb{Z}_n$ Symmetry

We begin then by discussing the simplest possibility, namely controlling R-parity violating couplings with only a single $\mathbb{Z}_n$ symmetry. Which operators are generated depends on the set of solutions for m to the equation

(V.26)
$$m(r+s)+\epsilon\frac{n}{2} = 0 \text{ mod } n$$

The simplest way to limit the number of solutions to (V.26) is to take $r+s=0$ and $\epsilon=1$. In this case, there are no solutions and all operators in (V.24) are expressly forbidden. If we set $n=2$ and $\epsilon=1$, for instance, we find two $\mathbb{Z}_2$ symmetries of this type. One of these is ordinary matter parity, $\mathbb{Z}_2^{(\text{Matter Parity})}$ (V.10). There is another $\mathbb{Z}_2$ that does the job, though, under which all MSSM fields are even while the GUT-Higgs fields are odd. It is easy to see that neither of these $\mathbb{Z}_2$'s can be embedded into a $U(1)$ symmetry that preserves the MSSM and flipped superpotentials [12]. Note also that the commonly used H-parity, $\mathbb{Z}_2^{(H\text{-Parity})}$ (V.11), allows numerous solutions starting at $m=1$ so it unable to prevent problematic R-parity violation.

We now investigate the possibility that $r+s \neq 0$ mod n. In this case, $m=n$ is always a solution for $\epsilon=0$ while, for $\epsilon=1$, we will always get a solution at one of $m=n/2$ or $m=n$.

[12] This follows imediately because the operator $H\bar{f}\bar{h}$ is neutral under any such $U(1)$ but carries odd parity under each $\mathbb{Z}_2$.

This is not a problem, though, provided n is sufficiently large that only operators from (V.24) with sufficient suppression are generated.

If we are given a solution m_0 to (V.26) then we generate the couplings λ, λ', λ'', and κ in (V.23) with the suppressions

$$\lambda, \lambda', \lambda'' \sim \delta^{2m_0-3} \qquad \kappa \sim \delta^{2m_0} M_{32} \tag{V.27}$$

where as usual $\delta = M_{32}/g\Lambda \gtrsim 10^{-2}$. Bounds from proton decay can be model dependent but the analysis of low energy SUSY in [102] suggests the order of magnitude constraint

$$\lambda'\lambda'' \lesssim 10^{-24} \qquad \Longrightarrow \; m_0 \geq 5 \tag{V.28}$$

From κ, however, we obtain an induced contribution to λ' that scales like κ/μ, where μ is the supersymmetric Higgs mass. This means that we also need

$$\frac{\kappa}{\mu}\lambda'' \gtrsim 10^{-24} \qquad \Longrightarrow \; m_0 \geq \frac{15}{4} + \frac{1}{8}\ln\left(\frac{M_{32}}{\mu}\right) \tag{V.29}$$

For $\mu \sim 100$ GeV and $M_{32} \sim 10^{16}$ GeV, this leads to the tighter constraint

$$m_0 \geq 6 \tag{V.30}$$

One would therefore need at least a $\mathbb{Z}_6$ symmetry to do the job.

5.3.2 Adding a $U(1)$ Symmetry

If we add a $U(1)$ symmetry to our $\mathbb{Z}_n$ then one can completely evade the proton decay constraints. This is because a $U(1)$ symmetry with $q + p \neq 0$ forbids $\lambda'' UDD$, which is the only source of baryon number violation in (V.23). We will still need a $\mathbb{Z}_n$ symmetry to suppress κ, though. Operators with $m = 3, 4, 5$ generate κ's of order 10^4, 1, and 10^{-4} GeV, respectively. From electroweak symmetry breaking considerarions $\kappa \sim 10^4$ GeV is much too large, but 1 GeV and 10^{-4} GeV may be ok. In each case, $\kappa/\mu \ll 1$ for $\mu \sim 100$ GeV so it seems sensible to rotate κ away, effectively replacing it with $\lambda' \sim \kappa/\mu$. Bounds on λ, λ', and λ'' individually rather than their products can be found in [22]. These bounds are model dependent, but it seems that $\lambda' \lesssim 10^{-3}$ is reasonable, leading to

$$\kappa \lesssim 10^{-3}\mu \sim 10^{-1} \text{ GeV} \tag{V.31}$$

The $m = 4$ operator seems troublesome but the operators with $m \geq 5$ should be ok. Achieving this requires a $\mathbb{Z}_n$ symmetry with $n \geq 5$. This is only a marginal improvement on the condition $n \geq 6$ that was necessary in the absence of a $U(1)$ symmetry. Introducing a $U(1)$ therefore doesn't seem to buy us very much.

The story is similar for $U(1)_R$ symmetry. We can forbid the trilinear couplings by taking $q_R + p_R > 1$ but a $\mathbb{Z}_n$ symmetry with n at least 5 is still needed.

5.3.3 Summary

Continuous $U(1)$ and $U(1)_R$ symmetries are insufficient to prevent severe R-parity violation in conflict with the measured proton lifetime. Discrete symmetries are necessary, with conventional $\mathbb{Z}_2$ matter parity one of the two most simple options. For discrete symmetries that only suppress quadratic and trilinear R-parity violation at low energies without expressly forbidding it, the order of the group can be slightly reduced if it is combined with a continuous $U(1)$ or $U(1)_R$ symmetry. The net effect of the continuous symmetries does not help us very much, though, so for our purposes we will treat R-parity violation as a problem that must be addressed by discrete symmetries.

5.4 Dimension 5 Operators: Neutrino Masses and Proton Decay

In Flipped $SU(5)$, Dirac neutrino masses

$$h_u L \nu^c \tag{V.32}$$

arise from the Yukawa couplings

$$\bar{h}\bar{f}F \supset h_u L \nu^c + h_u Q u^c. \tag{V.33}$$

Since (V.33) also supplies the up-quark masses, this limits the suppression that the Dirac neutrino masses can have, and thus Flipped $SU(5)$ requires a seesaw mechanism to generate small neutrino masses. Correspondingly, the right-handed neutrino Majorana mass

$$\nu^c \nu^c \in FF \tag{V.34}$$

must be present. But since FF is not an $SU(5)$ invariant, the above term originates from the non-renormalizable operator

$$W_{\text{Neutrino}} = \frac{1}{\Lambda} \bar{H} F \bar{H} F. \tag{V.35}$$

This is just the Type-I seesaw mechanism, and in terms of the scale Λ the light neutrino masses are

$$m_\nu = \frac{(y_u \langle h_u \rangle)^2}{\langle \bar{\nu}^c_H \rangle^2 / \Lambda} \lesssim \frac{m^2_{top}}{\delta M_{32}}. \tag{V.36}$$

The MINOS experiment [19] on neutrino oscillations is consistent with the mass splitting of two neutrino mass eigenstates, $|\Delta m^2| = (2.43 \pm .13) \times 10^{-3}$ eV^2. Requiring that the heaviest neutrino be of order the mass splitting in order to minimize the tuning in the neutrino mass matrix, (V.36) gives the correct neutrino mass for $\delta \sim 10^{-2}$. Since it has already been argued that $\delta \gtrsim 10^{-2}$, the operator (V.35) should not be further suppressed. Therefore, a *necessary* condition to generate small neutrino masses without introducing tuning into the neutrino sector is that the theory be able to generate (V.34) with only the suppression induced from (V.35).

Requiring that the Majorana mass (V.35) be invariant in addition to the flipped superpotential and Yukawa couplings in (V.9), imposes the additional constraints

$$\begin{aligned} 2r + 2s &= 0 \mod n \\ 2q + 2p &= 0 \\ 2q_R + 2p_R &= 2 \end{aligned} \tag{V.37}$$

on the charges in Table 5.1. The new charges consistent with all superpotential couplings are give in Table 5.2.

Field	$\mathbb{Z}_n$ charge	$U(1)$ charge	$U(1)_R$ charge
H	r	p	p_R
$\bar{H}$	$-r$	$-p$	$-p_R+1$
h	$-2r \bmod n$	$-2p$	$-2p_R+2$
$\bar{h}$	$2r \bmod n$	$2p$	$2p_R$
F	$r+\epsilon\frac{n}{2} \bmod n$	p	p_R
$\bar{f}$	$-3r+\epsilon\frac{n}{2} \bmod n$	$-3p$	$-3p_R+2$
ℓ	$5r+\epsilon\frac{n}{2} \bmod n$	$5p$	$5p_R-2.$

Table 5.2: Same as Table 5.1, but with the additional constraint that the Majorana neutrino mass (V.35) be invariant.

The global $U(1)$ symmetry is exactly $U(1)_\chi$ up-to a scaling, so the remaining two symmetries classify all possible (Abelian, non-family) symmetries consistent with Flipped $SU(5)$ [13]. The $\mathbb{Z}_n$ symmetry is sufficient to forbid R-parity violating operators provided $\epsilon = 1$ while the $U(1)_R$ is enough to avoid the generation of μ from nonrenormalizable operators involving H and $\overline{H}$.

5.4.1 μ Problem and Dimension 5 Proton Decay

Unfortunately, the symmetries from Table 5.2 cannot forbid the bare μ-term

(V.38) $$\bar{h}h \supset h_u h_d$$

or dimension 5 proton decay operators

(V.39) $$FFF\bar{f} + F\bar{f}\bar{f}\ell \supset QQQL + d^c u^c u^c e^c + u^c d^c d^c \nu^c$$

Consequently, one of the couplings in (V.9) and (V.35) needs to be forbidden if the μ-term (V.38) and dimension-5 proton decay operators (V.39) are to be suppressed. We focus here only on the μ problem as several factors can affect proton decay that could in principle be tuned [168]. One can consider symmetries that forbid μ but are spontaneously broken, thereby allowing (V.9) and (V.35) to arise.

A set of superpotential operators can be distinguished

(V.40) $$W \supset \bar{H}\bar{H}\bar{h} + FFh + \frac{1}{\Lambda}\bar{H}F\bar{H}F$$

[13] A discrete R-symmetry is also possible, but will not change the discussion below.

that if invariant will lead to an invariant μ term; in other words, when one is forbidden then the μ term can be forbidden. If one of the trilinear terms in (V.40) is absent, then it will necessarily have the same quantum numbers as the μ term. Then when the trilinears are generated via spontaneous symmetry breaking, so will the μ-term, and so the two will undergo similar suppression. If, on the other hand, one forbids $\bar{H}F\bar{H}F$, then this operator will have opposite charge than $h\bar{h}$. Then if the neutrino Majorana mass is generated dynamically, via an $SU(5)$ singlet S

$$\frac{S}{\Lambda^2}\bar{H}F\bar{H}F \tag{V.41}$$

then the μ-term is not generated by the vev of S. The value $\langle S\rangle$, which feeds into (V.36), needs to be close to the scale Λ, to give adequately small neutrino masses. Generating additional GUT-sized vevs will necessarily create tension when building a successful flipped $SU(5)$ model that solves the neutrino mass problem.

One can also consider models that generate effectively $\bar{H}F\bar{H}F$ when heavy fields are integrated out. The typical seesaw mechanism in Flipped $SU(5)$ [30] comes from the renormalizable superpotential couplings

$$y_u^{ij}F_i\bar{f}_j\,\bar{h} + \lambda_{ij}^{\nu}\bar{H}F^iS^j + M_{ij}^S S^iS^j \tag{V.42}$$

where there are now three $SU(5)\times U(1)_\chi$ singlets. This generates the 9×9 neutrino mass matrix

$$\begin{pmatrix} 0 & y^u\langle\overline{h}\rangle & 0 \\ y^u\langle\overline{h}\rangle & 0 & \lambda^S\langle\bar{\nu}_H^c\rangle \\ 0 & \lambda^S\langle\bar{\nu}_H^c\rangle & M^S \end{pmatrix}$$

in the (L,ν^c,S) basis. Assuming $M^S\sim\langle\bar{\nu}_H^c\rangle\sim M_{32}$ this generates light neutrino masses

$$m_\nu \lesssim \frac{\langle\overline{h}\rangle^2}{M_{32}} \tag{V.43}$$

with the same results as the Type-I seesaw mechanism described above, but without the additional factors of δ. Unfortunately, M^S has the same quantum numbers as μ, so both couplings should be of similar size, and the μ problem remains. One would need to add additional symmetries and $SU(5)$ singlet fields to make this model work.

5.4.2 Summary

Engineering neutrino masses that do not involve fine-tuning restricts the available symmetries, making it impossible to forbid either a bare μ term or operators that lead to dimension 5 proton decay. As it is a renormalizable coupling, the presence or absence of a bare μ term depends on details of the ultraviolet completion so one might hope to address this issue there without making use of an explicit symmetry. As for dimension 5 proton decay, the suppression by Λ is not sufficient in itself but the proton lifetime depends on a number of factors [168] which can allow some room for adequate suppression. One might also use family symmetries, as proposed in an F-theory context for instance in [137], to do the job. Both of these issues must be dealt with in a successful F-theory model for flipped $SU(5)$.

5.5 Challenges for Realizing Flipped $SU(5)$ in F-Theory

We now turn to a discussion of flipped $SU(5)$ in the context of semi-local F-theory models.

5.5.1 Engineering GUT-Higgs Fields

Because $SU(5) \times U(1)_\chi$ naturally embeds into $SO(10)$, one way to engineer flipped $SU(5)$ models in F-theory is to realize an $SO(10)$ gauge group and explicitly break it to $SU(5) \times U(1)_\chi$ with internal flux. The flux necessary to do this has the advantage that, unlike hypercharge flux, it does not split the gauge couplings at the high scale [129]. There has been recent interest in building GUT models in this way and a number of semi-local and global constructions have been achieved [60,61,66,137]. Some of the constructions in [61] utilize internal fluxes not only to break $SO(10) \to SU(5) \times U(1)_\chi$ but also to further break this down to the MSSM. As we are interested in alternatives to hypercharge flux in this chapter, we will insist in what follows on using GUT-Higgs fields and the flipped superpotential (V.4) to break $SU(5) \times U(1)_\chi$ to the MSSM and lift the

MSSM Higgs triplets.

One problem that has been noted by several authors [60, 61, 66] is that it is difficult to get the right spectrum including the GUT-Higgs fields. While the MSSM matter multiplets organize nicely into $\mathbf{16}$'s of $SO(10)$ and the MSSM Higgs doublets and their triplet partners fit into a $\mathbf{10}$ of $SO(10)$, the GUT-Higgs H and $\overline{H}$ do not fill out $SO(10)$ multiplets. Rather, each must come from part of a $\mathbf{16}$ of $SO(10)$ and it is here that the problems arise. In the presence of N units of $U(1)_\chi$ flux, the net chirality of multiplets that descend from $\mathbf{16}$'s follows the pattern

$$\begin{aligned} n_{\overline{\mathbf{5}}_{+3}} - n_{\mathbf{5}_{-3}} &= M + N \\ n_{\mathbf{10}_{-1}} - n_{\overline{\mathbf{10}}_{+1}} &= M \\ n_{\mathbf{1}_{-5}} - n_{\mathbf{1}_{+5}} &= M - N \end{aligned} \tag{V.44}$$

where M is the number of units of a suitable global G-flux that threads the matter curve. Any excess of $\mathbf{10}_{-1}$'s or $\overline{\mathbf{10}}_{+1}$'s is accompanied by an excess of $\overline{\mathbf{5}}_{+3}/\mathbf{5}_{-3}$'s or $\mathbf{1}_{-5}/\mathbf{1}_{+5}$'s.

To avoid introducing extra exotics, then, it becomes necessary to assume that H and $\overline{H}$ simply arise as a vectorlike pair on a single matter curve. This has two consequences. First, any $U(1)$ symmetry from Table 5.1 that happens to be preserved must give opposite charge to H and $\overline{H}$, meaning that $p+q=0$ and the global $U(1)$ charges are simply proportional to $U(1)_\chi$. Second, we must address why the GUT-Higgs fields are light or, if they sit at the KK scale, how such massive fields could possibly acquire nonzero vevs.

To make H and $\overline{H}$ light, one could start by requiring the matter curve on which they live to support a vector-like pair of the appropriate zero modes. Even then, one could not be certain that this pair does not become massive by coupling to moduli fields that acquire large nonzero vevs. Alternatively, one could imagine starting with H and $\overline{H}$ as two modes among the KK tower of $\mathbf{10}_{-1}$ fields and effectively bringing down their mass through an $SO(10)$ singlet Φ and a superpotential of the form

$$W \supset \lambda_{16}\Phi \times \mathbf{16}_H \times \overline{\mathbf{16}}_H + M_{KK}\mathbf{16}_H \times \overline{\mathbf{16}}_{\overline{H}} \tag{V.45}$$

In general, the masses of different components of the $\mathbf{16}_H/\overline{\mathbf{16}}_{\overline{H}}$ will differ by order one multiples of M_{KK}. A suitable vev of Φ could therefore render the $\mathbf{10}_H/\overline{\mathbf{10}}_{\overline{H}}$ pair very light while leaving the remaining components near the KK scale. Of course, Φ will in general couple to all KK modes on the $\mathbf{16}$ matter curve and there is no reason for this cancellation to occur only in the $\mathbf{10}_H\overline{\mathbf{10}}_{\overline{H}}$ direction and not in the others. If fact, such a cancellation is not well motivated and will likely lead to an additional enormous fine-tuning in the theory that flipped $SU(5)$ was engineered to avoid. Proceeding in this way seems quite cumbersome and will require many new assumptions.

An alternative to this, which seems particularly attractive, is to engineer $SU(5)\times U(1)_\chi$ directly. In particular, one realizes an $SU(5)$ gauge group with a $U(1)_\chi$ following the construction of semi-local $SU(5)$ GUT models [49, 115, 155, 157] and attempts to construct the global completion in such a way that the $U(1)_\chi$ survives as an honest gauge symmetry. The nature of $U(1)$'s away from the GUT-divisor is rather subtle but there has been substantial recent progress [116, 156] towards understanding them. The advantage of this approach is that one can engineer H and $\overline{H}$ directly on separate $\mathbf{10}$ curves. An example of a simple model that achieves this is constructed in Appendix E.

Unfortunately, two things remain to be resolved before realistic models can be built in this way. First, one must be wary that global fluxes may lift $U(1)_\chi$ in the same way that hypercharge flux lifts $U(1)_Y$. A necessary condition for this will be that $U(1)_\chi$ be non-anomalous, which leads to the second issue. While there has been progress towards understanding global fluxes in F-theory models [156], there is no simple procedure at the moment for counting the number of ($U(1)_\chi$-charged) $SU(5)$ singlet fields in a given model. For flipped $SU(5)$, it is crucial that the number of such fields is 3 so this must be addressed before further progress can be made in this direction.

5.5.2 Symmetries

Next, we must be sure to incorporate enough symmetry to address the phenomenological problems discussed earlier in this chapter. For dimension 4 R-parity violation, discrete symmetries seem unavoidable. Engineering these can be technically challenging and the only serious attempt we are aware of in any context is in [117]. That example already displays several pitfalls as even getting a reasonable number of generations seems difficult. This seems like a technical hurdle, though, with no conceptual obstruction blocking the way.

More troublesome is the μ problem which, as we have seen, requires a $U(1)_R$ symmetry to resolve in a satisfactory way. Unfortunately, semi-local F-theory models do not afford us this luxury. As reviewed in Appendix D, these models descend from an 8-dimensional E_8 gauge theory with $\mathcal{N} = 1$ supersymmetry in the presence of a background field configuration that breaks $E_8 \to SU(5)$. The 8-dimensional theory possesses a $U(1)_R$ symmetry and, further, additional R-symmetries could in principle follow from internal isometries of the compactification manifold that takes us from 8 down to 4 dimensions. Because we retain only $\mathcal{N} = 1$ supersymmetry in 4-dimensions, though, the supercharges are scalars with respect to the (twisted) internal isometries so only the remnant of the 8-dimensional $U(1)_R$ remains as a candidate. This symmetry, however, is broken explicitly by the background field configuration so that no continuous R-symmetry remains to control physics at the KK scale[14].

We view this μ problem as the most glaring issue for engineering flipped $SU(5)$ models in F-theory. It may be possible to avoid it phenomenologically with some intricate model building. Finding a scenario that can be easily realized within the rigid framework of F-theory, though, will be challenging.

5.5.3 Summary

We started by looking to flipped $SU(5)$ as a means to avoid some problems with minimal $SU(5)$ models in F-theory but flipped $SU(5)$ has a number of issues as well. Whether the situation is better or worse depends on one's taste but, in our opinion, the advantages of flipped $SU(5)$ are outweighed by the weaknesses. We stress, however, that all of the issues discussed here rely on the explicit use of GUT-Higgs fields to break $SU(5) \times U(1)_\chi$ and lift the Higgs triplets. Models

[14] Strictly speaking, there is a combination of topological and R-symmetries that remain unbroken by the scalar vev of the Higgs bundle. This is broken by the flux part of the Higgs bundle. Further, the 4-dimensional fields do not carry definite charge under this symmetry, so it could not constrain their physics anyway.

based on $SO(10)$ with all GUT-breaking via internal fluxes [61] do not suffer from any of the problems related to GUT-Higgs fields, including their origin and their knack for generating large contributions to the μ term.

CHAPTER VI

Conclusions

If our universe is described by a compactified string theory then the presence of stabilized moduli would generically imply that the cosmological history is non-thermal before BBN. In particular, dark matter would be produced from moduli decays and generically has to be wino-like in order to have a consistent abundance. Additionally, there is always a modulus with mass of order the gravitino mass or less in such theories. These plus cosmological considerations emphasize some difficulties in realizing gauge mediated supersymmetry breaking in string theory. All known examples of string theory vacua with stabilized moduli agree with the results shown here.

Motivated by generic string theories compactified to four dimensions with stabilized moduli, which typically have multi-TeV squarks and lighter gluinos (below a TeV), the signatures of gluino pair production at the 7 TeV LHC were studied. For 1 fb^{-1} integrated luminosity, gluinos up to about 650 GeV in mass can be detected, with larger masses accessible for higher luminosities or at higher energies. More than one signature is likely to be accessible, with one charged lepton plus two or more b-jets, and/or same-sign dileptons plus b-jets being the best channels. A non-Standard Model signal from counting is robust, and provides information on the gluino mass, cross section, and spin.

If our universe is described by M theory compactified on a manifold of G_2 holonomy, with doublet-triplet splitting solved in the way originally proposed by Witten [184], then there is a simple solution to the μ-problem: strong coupling dynamics in the the hidden sector will generate a non-perturbative potential for the moduli, which stabilizes all the moduli vevs, and breaks the symmetry forbidding μ. Then, following the numerical analysis done in the G_2-MSSM [9], the breaking will generate $\mu \sim \langle \frac{S}{m_{pl}} \rangle m_{3/2} \sim 0.1\ m_{3/2} \sim 2\,\mathrm{TeV}$. This then implies a non-zero Higgsino component of the mostly Wino LSP, with an upper limit, which in turn gives an upper limit of about $1 \times 10^{-45}\,\mathrm{cm}^2$ on the spin-independent scattering cross-section, somewhat below the reach of the XENON100 experiment, as well as a lower limit of about $10^{-46}\,\mathrm{cm}^2$. The Wino-like LSP also can account for the PAMELA positron and antiproton excesses [20, 132], and gives about the desired relic density for a non-thermal cosmological history [14], as expected in theories with moduli.

The possibility of engineering a flipped $SU(5)$ model in F-theory was explored. In particular, we show that a significant μ parameter ($\gtrsim 10^{10}$ GeV) is unavoidable in any flipped $SU(5)$ model without an R-symmetry. Since no four-dimensional R-symmetries control the superpotential in F-theory GUTs, we conclude that Flipped $SU(5)$ is not a viable mechanism to break the GUT group and solve doublet-triplet splitting in F-theory. There are other problems, although not as deadly as the lack of an R-symmetry, that can arise when trying to embed Flipped $SU(5)$ in a UV completion that has a conserved R-symmetry. At least one discrete symmetry is phenomenologically required to prevent severe R-parity violation, and that this symmetry cannot descend from a continuous $U(1)$ symmetry – which is an issue when realizing discrete symmetries in some string constructions. Additionally, if one wishes to explain the scale of the neutrino masses this will necessarily re-introduce a μ-problem regardless of whether or not there is an R-symmetry.

APPENDICES

APPENDIX A

$L1$ Triggers

Trigger Name	Object p_T
Inclusive isolated lepton	$30\,\mathrm{GeV}$
Lepton plus jet	$(20\,\mathrm{GeV}, 100\,\mathrm{GeV})$
Isolated dileptons	$15\,\mathrm{GeV}$
Dileptons plus jet	$(10\,\mathrm{GeV}, 100\,\mathrm{GeV})$
Isolated dileptons	$10\,\mathrm{GeV}$
Isolated lepton plus isolated tau	$(15\,\mathrm{GeV}, 45\,\mathrm{GeV})$
Isolated ditau	$60\,\mathrm{GeV}$
Inclusive isolated photon	$80\,\mathrm{GeV}$
Isolated diphoton	$25\,\mathrm{GeV}$
Inclusive MET	$90\,\mathrm{GeV}$
Inclusive single-jet	$400\,\mathrm{GeV}$
Jet plus MET	$(180\,\mathrm{GeV}, 80\,\mathrm{GeV})$
Acoplanar jet and MET ($1 < \Delta\phi < 2$)	$(100\,\mathrm{GeV}, 80\,\mathrm{GeV})$
Acoplanar dijets ($\Delta\phi < 2$)	$200\,\mathrm{GeV}$

APPENDIX B

The Kitano Model

For pedagogy we examine with a few toy (non-string theory) examples to demonstrate the validity of the above results.

Consider the Polonyi model, which has one field ϕ with G given by

$$G = m_{pl}^2\phi\bar{\phi} + m_{pl}^2 \log\left|\frac{\mu^2}{m_{pl}^2}(\phi-\beta)\right|^2 \tag{B.1}$$

where $\beta = \sqrt{3}-2$. The vacuum expectation value of ϕ is given by

$$\langle\phi\rangle = \langle\bar{\phi}\rangle = \sqrt{3}-1 \tag{B.2}$$

and u and r are

$$u = 2\sqrt{3}m_{pl}^2,\ \ r = 0 \tag{B.3}$$

The two eigenvalues of the mass matrix are

$$\begin{aligned} m_1^2 &= m_{3/2}^2(2\sqrt{3}) = m_{3/2}^2\tfrac{u}{m_{pl}^2} \\ m_2^2 &= m_{3/2}^2(4-2\sqrt{3}) = m_{3/2}^2(4-\tfrac{u}{m_{pl}^2}) \end{aligned} \tag{B.4}$$

in agreement with our general result Eq. (II.7). The bounds are reached since the eigenvectors are given by the sGoldstino directions.

This provides as an illustration of how one might avoid our result by going to a non-string theory withour moduli stabalization, by creating a model with large r and have scalar masses much heavier than the gravitino mass. Following [126, 138], we add a higher dimensional operator to the above Kahler potential

(B.5)
$$G = m_{pl}^2 \phi\bar{\phi} + m_{pl}^4 \frac{(\phi\bar{\phi})^2}{\Lambda^2} + m_{pl}^2 \log\left|\frac{\mu^2}{m_{pl}^2}(\phi - \frac{1}{\sqrt{3}})\right|^2 .$$

with the new scale $\Lambda \ll m_{pl}$. There is a minimum of this potential where ϕ has a small vacuum expectation value. To leading order in $\frac{\Lambda}{m_{pl}}$

(B.6)
$$\langle\phi\rangle = \langle\bar{\phi}\rangle \approx \frac{\Lambda^2}{2\sqrt{3}m_{pl}^2}.$$

In this vacuum $r = -12\frac{m_{pl}^4}{\Lambda^2}$, and one find after diagonalizing the scalar mass matrix that

(B.7)
$$m_1^2 = m_2^2 = 4\mu^2\frac{m_{pl}^2}{\Lambda^2} = 12\frac{m_{pl}^2}{\Lambda^2}m_{3/2}^2 = -\frac{r}{m_{pl}^2}m_{3/2}^2 \gg m_{3/2}^2.$$

Thus the scalar masses are at the bounds given by Eq. (II.6) and are much heavier than the gravitino mass. The field here has explicit couplings that are not moduli-like and if one tries to embed this model in string theory new problems arise – see Ex. 6 [92] in Appendix C.

APPENDIX C

String Theories with Stabalized Moduli

We examine examples we know of of string theory models in which all moduli are stabilized to gain further insight.

Ex. 1 Simple KKLT Model. This example [131] has all complex structure moduli stabilized by fluxes and a single Kahler modulus and axion stabilized by non-perturbative corrections. The vacuum energy is tuned by adding what amounts to a D-term potential. We didn't consider D-terms above, but we can incorporate them into the discussion based on these examples. Both the Kahler modulus and the axion obtain masses of order $20. \times m_{3/2}$ in this simple model. Here $r \sim 0$, but the kinetic terms for moduli are suppressed by approximately $\left(\frac{1}{10}\right)^2$, lifting the moduli masses above $\sqrt{2}m_{3/2}$. This scenario allows for a lighter gravitino $m_{3/2} \sim 1$ TeV, but the moduli will still dominate the cosmic energy density for times up to BBN and beyond. The non-thermal cosmology of mirage-mediated supersymmetry breaking in the KKLT context was discussed [162]. Late decay of moduli produce an abundance of Bino-like LSPs, which the annihilate rapidly through the pseudo-scalar Higgs resonance.

Ex. 2 LARGE Volume IIB models. These examples [36, 71] have complex structure moduli stabilized by fluxes and Kahler moduli stabilized by perturbative corrections. The vacuum energy is, naively, negative, though there might be mechanisms which generate the necessary positive contributions. The basic LARGE volume model has two Kahler moduli τ_b and τ_s. In the vacuum, $m_{3/2} \sim \frac{m_{pl}}{V}$, where V is the volume of the extra dimensions (divided by l_s^6). The masses of the moduli are given by $m_{\tau_b} \sim \frac{m_{3/2}}{V^{1/2}}$ and $m_{\tau_s} \sim m_{3/2} \log m_{pl}/m_{3/2}$. τ_b is much lighter than the gravitino and τ_s is an order of magnitude larger. Note that the suppression of the τ_b mass in this case can be shown to be from a direct cancellation in Eq. (II.6) as $r/m_{pl}^2 = 2 - O(1/V)$. In these models, unless $V \leq 10^9$, τ_b generically suffers from the cosmological moduli problem. In all cases, the early Universe is dominated by moduli oscillations. More recently, it has been realised that, by adding a third Kahler modulus, the observable sector supersymmetry breaking masses are suppressed relative to the gravitino mass, requiring 10^8 GeV $\leq m_{3/2} \leq 10^{11}$ GeV [46]. Again, in all such cases, τ_b dominates the pre-BBN cosmic energy density.

Ex. 3 M theory and Type IIA flux Vacua. These examples [5, 6, 11, 84, 177] use fluxes to stabilize all the moduli. All these vacua have a negative cosmological constant and it seems difficult to add additional sources which could change that. The moduli masses are all of order the gravitino mass.

Ex. 4 M theory on Manifolds of G_2 holonomy without flux. These examples [7–9] are based upon the idea, which goes back to Witten and others [18], that strong dynamics in the hidden sector generates a potential which breaks supersymmetry and generates a hierarchically small scale (related to the weak scale). In the M theory context it has been shown that, additionally, the potential generated by such hidden sector dynamics can stabilize *all* the moduli fields. The minimum of the potential has positive energy. The moduli spectrum for these examples has been studied in detail in [7–9]. All moduli but one have masses of order $m_{3/2}$, the remaining one having a somehwat larger mass. Hence, again, the moduli dominate the early cosmological history but decay before BBN.

Ex. 5 Type IIB flux vacua with non-perturbative effects. These examples [50] apply the ideas of [7–9] to stabilize all Kahler moduli and obtain a vacuum with positive vacuum energy self consistently. These examples all have moduli whose masses are of order $m_{3/2}$ and hence will dominate the early Universe.

Ex. 6 Gauge Mediation in String theory? In gauge mediation, the gravitino mass is relatively low and can be as small as an eV. Generically one expects that there are moduli whose masses are comparable to $m_{3/2}$. Since their lifetimes are so long, these moduli will dominate the Universe for many years and will not be able to reheat it to a temperature high enough for BBN to start (see [105] for a discussion on BBN constraints). Usually, when one considers gauge mediation, one implicitly assumes that moduli can be decoupled from the gravitino mass scale and then ignored, but our results indicate that such assumptions are perhaps too strong. Attempts at realizing a supergravity model derived from string theory with both moduli stabilization and gauge mediation are described in [92], based on earlier works of [126, 138, 141]. These models essentially couple a Type IIB Kahler moduli sector to a gauge mediation model which is assumed to arise from a configuration of branes on the Calabi-Yau of the sort described in [85]. The authors of [92] explain that it is quite difficult to find a model in which gauge mediation effects are not overcome by those of gravity mediation, when the cosmological constant is tuned to zero. In any case, if one examines the moduli masses in those examples one finds that the moduli whose masses are dominated by D-terms have masses much larger than $m_{3/2}$, but those whose masses are dominated by F-terms have masses of order $m_{3/2}$. Therefore, generally one has moduli which lead to a non-thermal comological history.

APPENDIX D

Semi-local F-theory Models

In this Appendix, we would like to address the presence or absence of (non-accidental) R-symmetries in semi-local F-theory models. For this, recall that F-theory describes nonperturbative configurations of intersecting 7-branes in type IIB string theory. Non-Abelian gauge theories can be engineered when several branes coincide. To describe the gauge degrees of freedom, it is sufficient for many purposes to consider the worldvolume theory on the branes, which is sensitive to some aspects of the local geometry but is largely independent of global details of the compactification. In all known examples for engineering SUSY GUTs, the brane worldvolume theory can be described as the maximally supersymmetric E_8 Yang-Mills theory in 8-dimensions compactified down to 4-dimensions in the presence of a nontrivial configuration for the internal gauge field and an adjoint scalar field. Aspects of the local geometry manifest themselves by specifying this configuration, which breaks E_8 down to the GUT group while giving spatially varying masses to internal wave functions that localize bifundamental matter to "matter curves". When we refer to a semi-local F-theory model, we mean precisely this 8-dimensional E_8 gauge theory with accompanying internal field configuration, which is often referred to as a Higgs bundle[1].

In general, R-symmetries of models obtained by compactifying brane worldvolumes descend either from R-symmetries of the original brane theory or internal symmetries of the compactifica-

[1] It should be noted that the assumption of a global E_8 as a starting point may not be general enough to capture all possible F-theory realizations of supersymmetric GUT models. To date, however, we know of no examples of F-theory compactifications, or even local models that manage to describe the geometry along the entire GUT divisor (as opposed to just a single coordinate patch), that engineer a GUT while avoiding this global E_8 structure.

tion. This makes it easy to see that there are no continuous R-symmetries present in semi-local F-theory models; the theory undergoes a twisting that removes any R-symmetries that could have descended from the compactification while the Higgs bundle explicitly breaks the $U(1)_R$ of the original 8-dimensional theory. In the following, we describe the twisting and the R-symmetry of the underlying 8-dimensional theory in a bit more detail to make this point clear to readers not familiar with the structure of F-theory models. This discussion very closely follows that of [42] with only a few minor emphases on R-symmetries added. For a more detailed discussion of the worldvolume theory, including not just the twisting but also an explicit construction of the action, the interested reader is referred to [42].

The worldvolume theory on a stack of 7-branes is a dimensional reduction of the 10-dimensional maximally supersymmetric Yang-Mills theory, whose field content consists of a 10-dimensional vector A_I ($I = 0, \ldots, 9$) and an $SO(9,1)$ Majorana-Weyl spinor of positive chirality ($\mathbf{16}_+$), Ψ_A. The supercharges of this theory organize themselves into the same representation, $\mathbf{16}_+$, as the fermions. In 8-dimensions, we obtain an 8-dimensional vector A_i ($i = 0, \ldots, 7$), a complex scalar $\Phi = A_8 + iA_9$, and an $SO(7,1)$ chiral spinor S_+ (along with its anti-chiral conjugate S_-). The R-symmetry of the 8-dimensional theory is the $U(1)$ that descends from $SO(9,1)$ under the reduction

$$SO(9,1) \to SO(7,1) \times U(1)_R \tag{D.1}$$

In F-theory applications, this 8-dimensional theory is compactified on a complex surface S, leaving us with a field theory 4-dimensions. Because S has a nontrivial canonical bundle in general, objects that transform as spinors under local $SO(4)$ rotations are not globally well-defined; rather, they are transformed by nontrivial transition functions as one moves from coordinate patch to coordinate patch. The lack of a globally well-defined spinor, which is needed to define 4-dimensional supercharges, clashes with our knowledge that the F-theory compactifications under study manifestly preserve $\mathcal{N} = 1$ supersymmetry in 4-dimensions. This tension tells us that the

7-brane worldvolume theory is necessarily twisted, meaning that its coupling to the background metric is altered in a way that effectively replaces the local $SO(4)$ rotation group with a combination of $SO(4)$ and $U(1)_R$. In fact, as described in [42], the twisting should respect the Kähler structure of S, which is only preserved by a $U(2)$ subgroup of $SO(4)$. This means that the twisting can be specified by a particular embedding of $U(1)_R$ into $U(2) \subset SO(4)$. To see the effect the twisting, consider first the way that 8-dimensional spinors of the theory organize into representations of $SO(3,1) \times U(2) \times U(1)_R = [SU(2) \times SU(2)] \times U(2) \times U(1)_R$. Specifying a $U(2)$ representation by an $SU(2)$ representation and $U(1)$ charge, one has that under the decomposition

(D.2) $$SO(7,1) \times U(1)_R \to SO(3,1) \times U(2) \times U(1)_R$$

the 8-dimensional chiral spinor $(S_+, +1/2)$ reduces as

(D.3) $$\left(S_+, +\frac{1}{2}\right) \to \left[(\mathbf{2},\mathbf{1}), \mathbf{2}_0, +\frac{1}{2}\right] \oplus \left[(\mathbf{1},\mathbf{2}), 1_{+1} \oplus 1_{-1}, +\frac{1}{2}\right]$$

In order to obtain one 4-dimensional chiral supercharge that transforms as a scalar under the modified internal rotation group, one must replace the generator J of the $U(1) \subset U(2)$ with one of the combinations

(D.4) $$J_{\text{top}} = J \pm 2R$$

where R is the generator of $U(1)_R$. Both of these lead to equivalent theories. Taking the + sign, the $SO(3,1) \times U(2) \times U(1)_R$ transformation properties of S_+ become

(D.5) $$\left(S_+, +\frac{1}{2}\right) \to \left[(\mathbf{2},\mathbf{1}), \mathbf{2}_{+1}, +\frac{1}{2}\right] \oplus \left[(\mathbf{1},\mathbf{2}), 1_{+2} + 1_0, +\frac{1}{2}\right]$$

where now the subscript refers to the J_{top} charge. The $[(\mathbf{1},\mathbf{2}), 1_0, +1/2]$ component gives rise to an anti-chiral supercharge in 4-dimensions that is globally well-defined on S. Decomposing the supercharges of the 8-dimensional theory in this way, these scalars give the supercharges of the resulting $\mathcal{N} = 1$ theory. Because the supercharges are scalars under the "twisted" internal rotation group, no R-symmetry can arise from there. The $U(1)_R$ that descends from the R-symmetry of the 8-dimensional theory, however, remains a global symmetry. This is the origin of a $U(1)_R$ symmetry in the 4-dimensional theory with respect to which the chiral supercharges carry charge $-\frac{1}{2}$.

Turning to the matter fields, the normalization of $U(1) \subset U(2)$ is such that it acts as $-p$ on holomorphic p-forms and p on anti-holomorphic p-forms [42]. This means that J_{top} is really a sort of topological charge, even though the theory itself is not topological. Following [42], we write the fields that descend from 8-dimensional chiral fermions in the following way, where we specify again the $SO(3,1) \times U(2) \times U(1)_R$ representations for clarity (here $m/\bar{m}$ denote holomoprhic/antiholomorphic form indices)

$$\Psi_A \to \begin{cases} \psi^{\alpha}_{\bar{m}} & \sim \left[(\mathbf{2},\mathbf{1}), \mathbf{2}_{+1}, +\frac{1}{2}\right] \\ \bar{\chi}^{\dot{\alpha}}_{\bar{m}\bar{n}} & \sim \left[(\mathbf{1},\mathbf{2}), \mathbf{1}_{+2}, +\frac{1}{2}\right] \\ \bar{\eta}^{\dot{\alpha}} & \sim \left[(\mathbf{1},\mathbf{2}), \mathbf{1}_{0}, +\frac{1}{2}\right] \end{cases} \tag{D.6}$$

along with their conjugates.

So far we have only considered fermion fields. The 8-dimensional scalar Φ, begins life as an $SO(7,1)$ singlet that carries $U(1)_R$ charge $+1$. After the twisting, its $SO(3,1) \times U(2) \times U(1)_R$ representation is

$$\Phi \sim [(\mathbf{1},\mathbf{1}), \mathbf{1}_{+2}, +1] \tag{D.7}$$

We also get scalars $A_m/A_{\bar{m}}$ with holomorphic/antiholomorphic indices $m/\bar{m}$ from dimensional reduction of the 8-dimensional vector. The scalar $A_{\bar{m}}$ has $SO(3,1) \times U(2) \times U(1)_R$ representation

$$A_{\bar{m}} \sim [(\mathbf{1},\mathbf{1}), \mathbf{2}_{+1}, 0] \tag{D.8}$$

The action of the twisted 8-dimensional gauge theory and its dimensional reduction are studied in detail in [42]. There, it is noted that the fermions (D.6) and bosons (D.7) (D.8) naturally pair up into $\mathcal{N}=1$ chiral multiplets $(A_{\bar{m}}, \psi^{\alpha}_{\bar{m}})$, $(\phi_{mn}, \chi^{\alpha}_{mn})$, and a vector multiplet (A_μ, η_α). In this language, the 4-dimensional superpotential can be written as

(D.9) $$W = \int_S d^2\theta \, \mathrm{tr}\left(F^{(0,2)} \wedge \phi\right)$$

where we denote chiral superfields by their lowest components and $F_S^{(0,2)} = \bar{\partial}_A A + A \wedge A$ is the $(0,2)$ field strength on S. We note that, by virtue of the integral over S being topological, it is necessarily invariant under the topological charge J_{top}. It is also easy to see that it is invariant under $U(1)_R$. After all, $F^{(0,2)}$ is R-invariant, ϕ carries R-charge +1, and, as we have seen, each 4-dimensional supercoordinate carries R-charge $-\frac{1}{2}$ in the present normalization. Invariance under $U(1)_R$ is not a surprise; it is a consequence of the fact that the 4-dimensional theory inherits the $U(1)_R$ symmetry of the 8-dimensional theory that we started with.

This is not the full story, though. To construct a semi-local GUT model we must add to this 8-dimensional theory a nontrivial configuration for both the scalar field ϕ and the internal field strength F_S. This configuration must satisfy the BPS equations[2]

(D.10) $$F_S^{(0,2)} = F_S^{(2,0)} \qquad \partial_A \phi = \partial_A \bar{\phi} = 0 \qquad \omega \wedge F_S + \frac{i}{2}[\phi, \bar{\phi}] = 0$$

The field ϕ carries nonzero $U(1)_R$ and $U(1)_{top}$ charges so only a single linear combination survives. This is the combination for which the superfield ϕ in (D.9) carries charge 0 while the superfield $A_{\overline{m}}$ and covariant derivatives ∂_A carry charge 1.

The Higgs bundles of interest always have nonzero $F_S^{(2,0)}$, which carries charge 2 under this symmetry, so one might conclude that even this symmetry is broken. We want to be a little careful about this because the dependence of Yukawa couplings on fluxes it is not always clear. For instance, it is known that while the spectrum depends on the gauge flux, Yukawa couplings essentially do not [56]. On the other hand, $F_S^{(2,0)}$ arises in F-theory from G-flux, which may or may not impact the Yukawas. One might argue that we can only be sure of which symmetries control the

[2] Throughout most of the literature, only Abelian configurations in which $[\phi, \bar{\phi}] = 0$ are considered. There, the flux F_S must satisfy $\omega \wedge F_S = 0$.

superpotential in the limit of vanishing $F_S^{(2,0)}$[3].

Nevertheless, it is easy to see that the combination $U(1)_R$ and $U(1)_{top}$ that is preserved by ϕ cannot descend to a symmetry that constrains the 4-dimensional effective action for massless fields. This is because of the coupled nature of the equations of motion for 4-dimensional fermions

$$
\begin{aligned}
0 &= \omega \wedge \partial_A \psi^\alpha + \frac{i}{2}[\bar{\phi}, \chi^\alpha] \\
&= \omega \wedge \partial_A \psi^\alpha - \frac{i}{2}[\phi, \bar{\chi}^{\dot{\alpha}}] \\
&= \bar{\partial}\chi^\alpha - [\phi, \psi^\alpha] \\
&= \partial_A \bar{\chi}^{\dot{\alpha}} - [\bar{\phi}, \bar{\psi}^{\dot{\alpha}}]
\end{aligned}
\tag{D.11}
$$

In the presence of a nontrivial expectation value for ϕ, these equations imply position dependent masses that cause the internal wave functions to localize along ”matter curves” where this expectation value vanishes[4]. Because ψ^α and χ^α are coupled by these equations, exciting a single mode on a matter curve corresponds to turning on nontrivial profiles for both of them. The 4-dimensional field that results does not have a well-defined charge under, $U(1)_R$, $U(1)_{top}$, or the linear combination that is preserved by ϕ, because ψ^α and χ^α carry different charges under all of these symmetries. For this reason, we do not expect any of these symmetries to control the superpotential for massless 4-dimensional fields, including both renormalizable operators and the nonrenormalizable ones that arise from integrating out KK modes.

[3]This seems kind of nonsensical because the spectrum jumps if we set $F_S^{(2,0)}$ to zero but our main point is that we are more comfortable with an argument that does not rely on explicit breaking of a $U(1)$ by $F_S^{(2,0)}$.

[4]As a meromorphic section on a complex surface, the vanishing locus of the expectation value for ϕ will generically consist of a collection curves.

APPENDIX E

Engineering $SU(5) \times U(1)_\chi$ Directly

As noted in section 5.5.1, constructing flipped $SU(5)$ models from $SO(10)$ GUTs in F-theory has some intrinsic difficulties, most notably realizing the GUT-Higgs fields without introducing new exotics into the spectrum. For this reason, it may be preferable to engineer $SU(5) \times U(1)_\chi$ directly, without using $SO(10)$ as an intermediate structure. Doing this gives up unification and introduces a fine-tuning associated with the closeness of α_1 to α_2 and α_3 at the high scale. Nevertheless, it is an alternative that may be interesting because, in such models, the $U(1)_\chi$ gauge boson will not be localized near the GUT branes but rather will correspond to a "bulk" closed string mode that can couple more readily to hidden sectors. This may make such scenarios interesting for phenomenology.

In this Appendix, we present a sample semi-local construction of an $SU(5) \times U(1)_\chi$ model that also engineers a $U(1)_{PQ}$ symmetry[1] capable of removing many, but not all, of the problematic nonrenormalizable operators involving the GUT-Higgs fields H and $\overline{H}$. This is the first explicit example we are aware of that realizes multiple $U(1)$ symmetries that generically contains no non-Kodaira type singularities[2] While we add fluxes to engineer a flipped $SU(5)$ spectrum, it should be straightforward to engineer an ordinary $SU(5)$ GUT as well in this setup. For flipped $SU(5)$,

[1] By $U(1)_{PQ}$ symmetry we mean a $U(1)$ symmetry that allows the MSSM superpotential but forbids a bare μ term.

[2] Semi-local models with multiple $U(1)$'s were recently studied in [94] but a further topologically tuning that isn't specified explicitly must be added in order to ensure the lack of non-Kodaira type singularities at isolated points where pairs of sections vanish. The construction that we describe in the following is different from those and requires no additional tuning beyond the choice $\xi_2 = \mathcal{O}$. Further, we explicitly build an example in which all objects that are used to construct the model are sections of bundles that admit holomorphic sections.

it is necessary to engineer $SU(5)$ singlet fields as well as ensure $U(1)_\chi$ remains massless in the presence of flux. Neither of these issues are sufficiently well-understood in global models to ensure that they can be solved but the parameter space of fluxes that we find in the semi-local model is large enough that it seems reasonable to expect that both of these shortcomings can be addressed in the future.

We now turn to a semi-local model for an $SU(5)$ GUT that retains a $U(1)_\chi$ and $U(1)_{PQ}$ symmetry. We provide only a brief review of semi-local models and how to construct them. For a more complete discussion, see [155].

As described in Appendix D, the starting point is an E_8 gauge theory. We must then introduce a Higgs bundle satisfying (D.10). This is done with a spectral cover $\mathcal{C}$ [90], which is a 5-sheeted cover of the complex surface, S_{GUT}, on which the gauge theory is compactified. To break $E_8 \to SU(5)_{\mathrm{GUT}}$, the scalar ϕ must take values in the adjoint of the $SU(5)_\perp$ commutant of $SU(5)_{\mathrm{GUT}}$ inside E_8. These can be parametrized by five eigenvalues that sum to zero

$$\langle\phi\rangle \sim \begin{pmatrix} t_1 & 0 & 0 & 0 & 0 \\ 0 & t_2 & 0 & 0 & 0 \\ 0 & 0 & t_3 & 0 & 0 \\ 0 & 0 & 0 & t_4 & 0 \\ 0 & 0 & 0 & 0 & t_5 \end{pmatrix} \qquad \sum_{i=1}^{5} t_i = 0 \tag{E.1}$$

Roughly speaking, one can think of each sheet of the cover as specifying one of the five eigenvalues t_i. As one moves along S_{GUT}, the t_i are mixed under monodromy. This is reflected in $\mathcal{C}$ by the manner in which the sheets are glued together. In the absence of monodromy, the $U(1)^4$ Cartan of $SU(5)_\perp$ survives as a symmetry of the theory. In the presence of monodromies, only those $U(1)$'s that are invariant survive.

Monodromies also affect the potential matter content of the theory. All matter descends from the adjoint of E_8

$$\mathbf{248} \to (\mathbf{24}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{24}) \oplus (\mathbf{10}, \mathbf{5}) \oplus (\overline{\mathbf{5}}, \mathbf{10}) \tag{E.2}$$

Without monodromy, we get 5 copies of the $\mathbf{10}$ that transform as a fundamental of $SU(5)_\perp$. We use the t_i to label these five copies, denoting them $\mathbf{10}_{t_i}$ for $i = 1, \ldots, 5$. Similarly, we get 10 copies of the $\overline{\mathbf{5}}$ labeled as $\overline{\mathbf{5}}_{t_i+t_j}$ with $i \neq j$. Finally, we get 24 singlets labeled as $\mathbf{1}_{t_i-t_j}$ with $i \neq j$. One typically doesn't discuss singlets in the context of semi-local models because their wave functions do not localize on the GUT-branes. $SU(5)$ singlets are therefore sensitive to global details of the geometry so it doesn't make sense to describe much about them in a semi-local setting other than their charges under any $U(1)$ factors that remain.

A generic monodromy group will mix all t_i's. This projects out all extra $U(1)$'s and leads to a spectrum with just one type of $\mathbf{10}$ and one type of $\overline{\mathbf{5}}$. We want to realize extra $U(1)$ symmetries to we construct a Higgs bundle with a reduced monodromy group by using a factored spectral cover $\mathcal{C}$. In order to realize both $U(1)_\chi$ and $U(1)_{PQ}$ and engineer both the MSSM superpotential and the flipped superpotential (V.4), there is in fact a unique factorization structure

$$\mathcal{C} \to \mathcal{C}^{(a)} \times \mathcal{C}^{(d)} \times \mathcal{C}^{(e)} \tag{E.3}$$

where $\mathcal{C}^{(a)}$ has two sheets, $\mathcal{C}^{(d)}$ has two sheets, and $\mathcal{C}^{(e)}$ has one sheet. The matter fields that one obtains and their charges under the two $U(1)$'s that survive are listed below

(E.4)

Field	$U(1)_\chi$	$U(1)_{PQ}$
$\mathbf{10}^{(a)} \equiv \mathbf{10}_{t_a}$	1	1
$\mathbf{10}^{(d)} \equiv \mathbf{10}_{t_d}$	1	-1
$\mathbf{10}^{(e)} \equiv \mathbf{10}_{t_e}$	-4	0
$\overline{\mathbf{5}}^{(aa)} \equiv \overline{\mathbf{5}}_{t_{a_1}+t_{a_2}}$	2	2
$\overline{\mathbf{5}}^{(dd)} \equiv \overline{\mathbf{5}}_{t_{d_1}+t_{d_2}}$	2	-2
$\overline{\mathbf{5}}^{(ad)} \equiv \overline{\mathbf{5}}_{t_a+t_d}$	2	0
$\overline{\mathbf{5}}^{(ae)} \equiv \overline{\mathbf{5}}_{t_a+t_e}$	-3	1
$\overline{\mathbf{5}}^{(de)} \equiv \overline{\mathbf{5}}_{t_d+t_e}$	-3	-1

Our identification of the first $U(1)$ as "$U(1)_\chi$" is natural once we identify the fields above with those of the MSSM in the following way

$$
\begin{aligned}
&\mathbf{10}^{(a)} \leftrightarrow F + H \\
&\overline{\mathbf{10}}^{(d)} \leftrightarrow \overline{H} \\
&\mathbf{5}^{(aa)} \leftrightarrow h \qquad\qquad \text{(E.5)}\\
&\overline{\mathbf{5}}^{(dd)} \leftrightarrow \overline{h} \\
&\overline{\mathbf{5}}^{(ae)} \leftrightarrow \overline{\mathbf{f}}
\end{aligned}
$$

To engineer the right spectrum, then, we need the following chiralities of zero modes on each matter curve

(E.6)

Curve	$\mathbf{10}^{(a)}$	$\mathbf{10}^{(d)}$	$\mathbf{10}^{(e)}$	$\overline{\mathbf{5}}^{(aa)}$	$\overline{\mathbf{5}}^{(dd)}$	$\overline{\mathbf{5}}^{(ad)}$	$\overline{\mathbf{5}}^{(ae)}$	$\overline{\mathbf{5}}^{(de)}$
Chirality	4	-1	0	-1	1	0	3	0

Spectral Cover

To construct such a model explicitly, we need a factored spectral cover. The spectral cover lives in an auxiliary space that is the total space of the canonical bundle over S_{GUT}. We refer the reader to [155] for details about this and only summarize the construction here. We write a factored cover as

$$\mathcal{C} = \mathcal{C}^{(a)}\mathcal{C}^{(d)}\mathcal{C}^{(e)} \tag{E.7}$$

with

$$\begin{aligned} \mathcal{C}^{(a)} &= a_2V^2 + a_1UV + a_0U^2 \\ \mathcal{C}^{(d)} &= d_2V^2 + d_1UV + d_0U^2 \\ \mathcal{C}^{(e)} &= e_1V + e_0U \end{aligned} \tag{E.8}$$

Here, the a_m, d_n, and e_p are sections of the bundles

(E.9)

Section	Bundle
a_m	$\eta - (m+3)c_1 - \xi_1 - \xi_2$
d_n	$= \xi_1 + (2-m)c_1$
e_p	$= \xi_2 + (1-p)c_1$

where c_1 is short for the anti-canonical bundle of S_{GUT}, $K^{-1}_{S_{\text{GUT}}}$. We can choose the bundle η, which encodes the manner in which S_{GUT} is embedded into a global model, as well as the bundles ξ_1 and ξ_2. The traceless condition on $\mathcal{C}$, which amounts to ensuring that it specifies an $SU(5)$ bundle rather than a $U(5)$ one, becomes

$$e_0d_0a_1 + e_0a_0d_1 + d_0a_0e_1 = 0 \tag{E.10}$$

We choose to solve this in a very particular way that is tailored for our ultimate choice of the complex surface S_{GUT}. First, we take ξ_2 to be a trivial bundle so that we can set $e_1 =$. Because of this, we hereafter refer to ξ_1 simply as ξ

$$\xi \equiv \xi_1, \quad \xi_2 = \mathcal{O}, \quad e_1 = 1 \tag{E.11}$$

Now, we define new sections A, B, C and set

$$a_0 = d_1 B^2 C - e_0 a_1$$
$$d_0 = a_1 A^2 C - e_0 d_1 \tag{E.12}$$
$$e_0 = ABC$$

With this parametrization, we are free to choose a bundle χ for the section C. The spectral cover now takes the form

$$\mathcal{C} = b_5 V^5 + b_4 V^4 U + b_3 V^3 U^2 + b_2 V^2 U^3 + b_1 V U^4 + b_0 U^5 \tag{E.13}$$

where

$$\begin{aligned}
b_5 &= a_2 d_2 \\
b_4 &= a_1 d_2 + a_2(d_1 + d_2 ABC) \\
b_3 &= a_1(d_1 + a_2 A^2 C) + d_1 d_2 B^2 C \\
b_2 &= C\left[d_1^2 B^2 + A(a_1 + a_2 ABC)(a_1 A - d_1 B) + d_2 A B^2 C(d_1 B - a_1 A)\right] \\
b_1 &= 0 \\
b_0 &= -A^2 B^2 C^3 (a_1 A - d_1 B)^2
\end{aligned} \tag{E.14}$$

We will tailor our construction so that it can be embedded into Calabi-Yau 4-folds based on the geometries of [154][3]. There, S_{GUT} is a dP_2 surface, whose second homology is generated by a hyperplane class, h, and two exceptional curves, e_1 and e_2[4]. In terms of these, c_1 is simply

$$c_1 = 3h - e_1 - e_2 \tag{E.15}$$

while, in the geometries of [154], η is given by

$$\eta = 17h - 6(e_1 + e_2) \tag{E.16}$$

[3]These geometries were constructed to satisfy a topological condition [43,54,88] that allows GUT-breaking via $U(1)_Y$ flux. While we will not utilize this method of GUT-breaking, we still use the geometries of [154] because of their relative simplicity.

[4]The nonzero intersections are $h^2 = 1$ and $e_i^2 = -1$. All other intersections vanish. We hope that context will avoid any confusion between the hyperplane class, h, and the up-type Higgs multiplet, which we also refer to as h.

Finally, we must be careful about our choices of ξ and χ in order to ensure that the bundles associated to all sections really do admit holomorphic sections. To that end, we take

$$\xi = h - e_1 \qquad \chi = h \tag{E.17}$$

To see that this is ok, we now list all sections that appear in (E.14), the general bundles of which they are sections, and the specific bundles for the choices (E.15), (E.16), and (E.17)

(E.18)

Section	General Bundle	Bundle in our dP_2 Construction
a_2	$\eta - 5c_1 - \xi$	$h - e_2$
a_1	$\eta - 4c_1 - \xi$	$4h - e_1 - 2e_2$
a_0	$\eta - 3c_1 - \xi$	$7h - 2e_1 - 3e_2$
d_2	ξ	$h - e_1$
d_1	$c_1 + \xi$	$4h - 2e_1 - e_2$
d_0	$2c_1 + \xi$	$7h - 3e_1 - 2e_2$
e_1	$\mathcal{O}$	$\mathcal{O}$
e_0	c_1	$3h - e_1 - e_2$
A	$-\frac{1}{2}(\eta + \chi) + 3c_1 + \xi$	$h - e_2$
B	$\frac{1}{2}(\eta - \chi) - \xi - 2c_1$	$h - e_1$
C	χ	h

For our specific choice, all of the bundles admit holomorphic sections. We now list the classes of all matter curves inside S_{GUT}

(E.19)

Field	Origin	Equation for Matter Curve in dP_2	Homology Class	Class for our choices
$\mathbf{10}^{(a)}$	$\mathcal{C}^{(a)}$	a_2	$\eta - 5c_1 - \xi$	$h - e_2$
$\mathbf{10}^{(d)}$	$\mathcal{C}^{(d)}$	d_2	ξ	$h - e_1$
$\mathbf{10}^{(e)}$	$\mathcal{C}^{(e)}$	$*$	$*$	$*$
$\overline{\mathbf{5}}^{(aa)}$	$\mathcal{C}^{(a)} - \mathcal{C}^{(a)}$	a_1	$\eta - 4c_1 - \xi$	$4h - e_1 - 2e_2$
$\overline{\mathbf{5}}^{(dd)}$	$\mathcal{C}^{(d)} - \mathcal{C}^{(d)}$	d_1	$c_1 + \xi$	$4h - 2e_1 - e_2$
$\overline{\mathbf{5}}^{(ad)}$	$\mathcal{C}^{(a)} - \mathcal{C}^{(d)}$	$(a_2d_1 + a_1d_2) + C(a_2A + d_2B)^2$	$\eta - 4c_1$	$5h - 2(e_1 + e_2)$
$\overline{\mathbf{5}}^{(ae)}$	$\mathcal{C}^{(a)} - \mathcal{C}^{(e)}$	$d_1 + a_2A^2C$	$c_1 + \xi$	$4h - 2e_1 - e_2$
$\overline{\mathbf{5}}^{(de)}$	$\mathcal{C}^{(d)} - \mathcal{C}^{(e)}$	$a_1 + d_2B^2C$	$\eta - 4c_1 - \xi$	$4h - e_1 - 2e_2$

Fluxes

The next step is to introduce suitable fluxes to engineer the desired spectrum of zero modes (E.6). In a semi-local model this amounts to twisting the Higgs bundle as described in [90]. We will make use of several fluxes. These include two non-universal fluxes that are only accommodated if we further specialize the spectral cover. To that end, we set

$$a_1 = \alpha\tilde{\alpha} - d_2B^2C$$
$$d_1 = \delta\tilde{\delta} - a_2A^2C \tag{E.20}$$

We will abuse notation in what follows and use α, δ to denote both the sections above and the bundles of which they are sections. With this in mind, the fluxes that we introduce are

$$\begin{aligned}
\gamma_a &= n_a \left(2 - p_a^* p_{a*}\right) \sigma \cdot \mathcal{C}^{(a)} \\
\gamma_d &= n_d \left(2 - p_d^* p_{d*}\right) \sigma \cdot \mathcal{C}^{(d)} \\
\tilde{\Psi}_a &= \{[V = e_0 U] \cap \alpha\} - \alpha \cdot \mathcal{C}^{(e)} \\
\tilde{\Psi}_d &= \{[V = e_0 U] \cap \delta\} - \delta \cdot \mathcal{C}^{(e)} \\
\tilde{\rho} &= \left[\mathcal{C}^{(a)} - \mathcal{C}^{(d)}\right] \cdot \rho \\
\tilde{\mu} &= \left[\mathcal{C}^{(a)} - 2\mathcal{C}^{(e)}\right] \cdot \mu \\
\tilde{\nu} &= \left[\mathcal{C}^{(d)} - 2\mathcal{C}^{(e)}\right] \cdot \nu
\end{aligned} \tag{E.21}$$

where ρ, μ, and ν denote arbitrary classes in $H_2(dP_2, \mathbb{Z})$. We also use p_a to denote the projections $p_a : \mathcal{C}^{(a)} \to S_{\mathrm{GUT}}$ and similar for p_d. All of these fluxes are constructed so that the net trace is zero but traces along individual components of $\mathcal{C}$ do not necessarily vanish. The net flux that we construct must be supersymmetric, though. The condition for supersymmetry that we impose is that the net flux, Γ, satisfies

$$\omega \cdot p_{a*}\Gamma = \omega \cdot p_{d*}\Gamma = 0 \tag{E.22}$$

for some ω in the Kähler cone of $S_{\mathrm{GUT}} = dP_2$.

To compute the spectrum from these fluxes, it is necessary to identify matter curves within the spectral cover $\mathcal{C}$ as described, for instance, in [155]. This is tedious but straightforward so we do not present the deteails here. We simply note that, with the fluxes (E.21), it is relatively easy to find a 4-parameter space of solutions that are supersymmetric and yield the proper spectrum (E.6). One sample solution from this space is

$$
\begin{aligned}
n_u &= -1 \\
n_d &= 0 \\
\delta &= -h + 7e_1 - e_2 \\
\alpha &= -5h + 3e_1 + 7e_2 \\
\rho &= -e_1 \\
\mu &= 0 \\
\nu &= 3h - 5e_1
\end{aligned}
\tag{E.23}
$$

which satisfies the supersymmetry condition for $\omega = c_1 = 3h - e_1 - e_2$.

The semi-local model presented here is only a first step. Embedding in a global model based on the geometries of [154] is straightforward since we know how to lift sections of bundles on dP_2 to sections of bundles on the 3-fold described therein. For sections that are not symmetric in e_1 and e_2 this can be a bit tricky but, as shown in [157], this can be dealt with. One must worry about ensuring that $U(1)_\chi$ survives as an honest gauge symmetry, rather than being lost due to additional effects like those described in [118], but from [116, 156] we know how to do this. We need only lift the sections appearing in (E.14) to sections on the full 3-fold of [154] and write a truncated Weierstrass from as in $y^2 = x^3 + \mathcal{C}$ [156] with no additional terms. That the fluxes (E.21) can be globally extended in this setting follows from the construction of [156].

What remains to be understood are two important ingredients. The first is engineering the proper number of $SU(5)$ singlets, which is important because these become right-handed electrons in flipped $SU(5)$ models. This will also ensure that $U(1)_\chi$ is non-anomalous, which is a necessary condition for having it remain as an honest massless gauge symmetry at low energies. As we know for experience with hypercharge flux, though, this is not enough. One must carefully ensure that the fluxes we use to induce chirality do not cause $U(1)_\chi$ to be lifted. We hope that the parameter space of fluxes we have found is large enough to allow at least some choice that does not lift $U(1)_\chi$

but we have no way of saying for certain at the moment. Further progress will require refined understanding of global fluxes and $U(1)$'s in F-theory beyond what is currently known.

BIBLIOGRAPHY

BIBLIOGRAPHY

[1] G. Aad et al. Expected Performance of the ATLAS Experiment - Detector, Trigger and Physics. 2009.

[2] Georges Aad et al. Measurement of the top quark-pair production cross section with ATLAS in pp collisions at $\sqrt{s} = 7$ TeV. 2010. * Temporary entry *.

[3] Bobby Samir Acharya. M theory, Joyce orbifolds and super Yang-Mills. *Adv. Theor. Math. Phys.*, 3:227–248, 1999.

[4] Bobby Samir Acharya. On Realizing N=1 superYang-Mills in M theory. 2000.

[5] Bobby Samir Acharya. A Moduli fixing mechanism in M theory. 2002.

[6] Bobby Samir Acharya, Francesco Benini, and Roberto Valandro. Fixing moduli in exact type IIA flux vacua. *JHEP*, 0702:018, 2007.

[7] Bobby Samir Acharya, Konstantin Bobkov, Gordon Kane, Piyush Kumar, and Diana Vaman. An M theory Solution to the Hierarchy Problem. *Phys.Rev.Lett.*, 97:191601, 2006.

[8] Bobby Samir Acharya, Konstantin Bobkov, Gordon L. Kane, Piyush Kumar, and Jing Shao. Explaining the Electroweak Scale and Stabilizing Moduli in M Theory. *Phys.Rev.*, D76:126010, 2007.

[9] Bobby Samir Acharya, Konstantin Bobkov, Gordon L. Kane, Jing Shao, and Piyush Kumar. The G(2)-MSSM: An M Theory motivated model of Particle Physics. *Phys.Rev.*, D78:065038, 2008.

[10] Bobby Samir Acharya, Konstantin Bobkov, and Piyush Kumar. An M Theory Solution to the Strong CP Problem and Constraints on the Axiverse. *JHEP*, 1011:105, 2010.

[11] Bobby Samir Acharya, Frederik Denef, and Roberto Valandro. Statistics of M theory vacua. *JHEP*, 0506:056, 2005.

[12] Bobby Samir Acharya, Phill Grajek, Gordon L. Kane, Eric Kuflik, Kerim Suruliz, et al. Identifying Multi-Top Events from Gluino Decay at the LHC. 2009.

[13] Bobby Samir Acharya and Sergei Gukov. M theory and singularities of exceptional holonomy manifolds. *Phys.Rept.*, 392:121–189, 2004.

[14] Bobby Samir Acharya, Gordon Kane, and Eric Kuflik. String Moduli Phenomenology, Cosmological History, Supersymmetry Breaking, and Dark Matter. 2010.

[15] Bobby Samir Acharya, Gordon Kane, Scott Watson, and Piyush Kumar. A Non-thermal WIMP Miracle. *Phys.Rev.*, D80:083529, 2009.

[16] Bobby Samir Acharya, Piyush Kumar, Konstantin Bobkov, Gordon Kane, Jing Shao, et al. Non-thermal Dark Matter and the Moduli Problem in String Frameworks. *JHEP*, 0806:064, 2008.

[17] Bobby Samir Acharya and Mahdi Torabian. Supersymmetry Breaking, Moduli Stabilization and Hidden U(1) Breaking in M-Theory. 2011. * Temporary entry *.

[18] Bobby Samir Acharya and Edward Witten. Chiral fermions from manifolds of G(2) holonomy. 2001.

[19] P. Adamson et al. Measurement of Neutrino Oscillations with the MINOS Detectors in the NuMI Beam. *Phys. Rev. Lett.*, 101:131802, 2008.

[20] Oscar Adriani et al. An anomalous positron abundance in cosmic rays with energies 1.5-100 GeV. *Nature*, 458:607–609, 2009.

[21] Z. Ahmed et al. Dark Matter Search Results from the CDMS II Experiment. *Science*, 327:1619–1621, 2010.

[22] B. C. Allanach, A. Dedes, and Herbert K. Dreiner. Bounds on R-parity violating couplings at the weak scale and at the GUT scale. *Phys. Rev.*, D60:075014, 1999.

[23] B.C. Allanach. SOFTSUSY: a program for calculating supersymmetric spectra. *Comput.Phys.Commun.*, 143:305–331, 2002.

[24] Baris Altunkaynak, Michael Holmes, Pran Nath, Brent D. Nelson, and Gregory Peim. SUSY Discovery Potential and Benchmarks for Early Runs at $\sqrt{s} = 7$ TeV at the LHC. *Phys.Rev.*, D82:115001, 2010.

[25] Daniele S.M. Alves, Eder Izaguirre, and Jay G. Wacker. It's On: Early Interpretations of ATLAS Results in Jets and Missing Energy Searches. 2010.

[26] Johan Alwall, Simon de Visscher, and Fabio Maltoni. QCD radiation in the production of heavy colored particles at the LHC. *JHEP*, 0902:017, 2009.

[27] Johan Alwall, Pavel Demin, Simon de Visscher, Rikkert Frederix, Michel Herquet, et al. MadGraph/MadEvent v4: The New Web Generation. *JHEP*, 0709:028, 2007.

[28] Michael Ambroso and Burt A. Ovrut. The Mass Spectra, Hierarchy and Cosmology of B-L MSSM Heterotic Compactifications. 2010.

[29] Claude Amsler et al. Review of Particle Physics. *Phys.Lett.*, B667:1, 2008.

[30] Ignatios Antoniadis, John R. Ellis, J.S. Hagelin, and Dimitri V. Nanopoulos. Supersymmetric Flipped SU(5) Revitalized. *Phys.Lett.*, B194:231, 1987.

[31] Ignatios Antoniadis, E. Gava, K.S. Narain, and T.R. Taylor. Effective mu term in superstring theory. *Nucl.Phys.*, B432:187–204, 1994.

[32] E. Aprile et al. First Dark Matter Results from the XENON100 Experiment. *Phys.Rev.Lett.*, 105:131302, 2010.

[33] Asimina Arvanitaki, Savas Dimopoulos, Sergei Dubovsky, Nemanja Kaloper, and John March-Russell. String Axiverse. *Phys.Rev.*, D81:123530, 2010.

[34] Asimina Arvanitaki et al. Decaying Dark Matter as a Probe of Unification and TeV Spectroscopy. *Phys. Rev.*, D80:055011, 2009.

[35] Yang Bai and Bogdan A. Dobrescu. Heavy octets and Tevatron signals with three or four b jets. 2010.

[36] Vijay Balasubramanian, Per Berglund, Joseph P. Conlon, and Fernando Quevedo. Systematics of moduli stabilisation in Calabi-Yau flux compactifications. *JHEP*, 0503:007, 2005.

[37] Edward A. Baltz and Paolo Gondolo. Limits on R-parity violation from cosmic ray anti-protons. *Phys.Rev.*, D57:7601–7606, 1998.

[38] R. Barate et al. Search for the standard model Higgs boson at LEP. *Phys. Lett.*, B565:61–75, 2003.

[39] Vernon D. Barger, M.S. Berger, and P. Ohmann. The Supersymmetric particle spectrum. *Phys.Rev.*, D49:4908–4930, 1994.

[40] Stephen M. Barr. A New Symmetry Breaking Pattern for SO(10) and Proton Decay. *Phys.Lett.*, B112:219, 1982.

[41] Thorsten Battefeld and Scott Watson. String gas cosmology. *Rev.Mod.Phys.*, 78:435–454, 2006.

[42] Chris Beasley, Jonathan J. Heckman, and Cumrun Vafa. GUTs and Exceptional Branes in F-theory - I. *JHEP*, 01:058, 2009.

[43] Chris Beasley, Jonathan J. Heckman, and Cumrun Vafa. GUTs and Exceptional Branes in F-theory - II: Experimental Predictions. *JHEP*, 01:059, 2009.

[44] V. Berezinsky, A. Masiero, and J.W.F. Valle. Cosmological signatures of supersymmetry with spontaneously broken R-parity. *Phys.Lett.*, B266:382–388, 1991.

[45] Gianfranco Bertone, Dan Hooper, and Joseph Silk. Particle dark matter: Evidence, candidates and constraints. *Phys. Rept.*, 405:279–390, 2005.

[46] R. Blumenhagen, J.P. Conlon, S. Krippendorf, S. Moster, and F. Quevedo. SUSY Breaking in Local String/F-Theory Models. *JHEP*, 0909:007, 2009.

[47] Ralph Blumenhagen. Gauge Coupling Unification in F-Theory Grand Unified Theories. *Phys. Rev. Lett.*, 102:071601, 2009.

[48] Ralph Blumenhagen, Mirjam Cvetic, Dieter Lust, Robert Richter, 2, and Timo Weigand. Non-perturbative Yukawa Couplings from String Instantons. *Phys. Rev. Lett.*, 100:061602, 2008.

[49] Ralph Blumenhagen, Thomas W. Grimm, Benjamin Jurke, and Timo Weigand. Global F-theory GUTs. 2009.

[50] Konstantin Bobkov, Volker Braun, Piyush Kumar, and Stuart Raby. Stabilizing All Kahler Moduli in Type IIB Orientifolds. *JHEP*, 1012:056, 2010.

[51] Vincent Bouchard, Jonathan J. Heckman, Jihye Seo, and Cumrun Vafa. F-theory and Neutrinos: Kaluza-Klein Dilution of Flavor Hierarchy. 2009.

[52] Robert H. Brandenberger and C. Vafa. Superstrings in the Early Universe. *Nucl.Phys.*, B316:391, 1989.

[53] A. Brignole, Luis E. Ibanez, and C. Munoz. Soft supersymmetry-breaking terms from supergravity and superstring models. 1997.

[54] Matthew Buican, Dmitry Malyshev, David R. Morrison, Herman Verlinde, and Martijn Wijnholt. D-branes at singularities, compactification, and hypercharge. *JHEP*, 01:107, 2007.

[55] J.A. Casas and C. Munoz. A Natural solution to the mu problem. *Phys.Lett.*, B306:288–294, 1993.

[56] Sergio Cecotti, Miranda C. N. Cheng, Jonathan J. Heckman, and Cumrun Vafa. Yukawa Couplings in F-theory and Non-Commutative Geometry. 2009.

[57] Kwok Lung Chan, Utpal Chattopadhyay, and Pran Nath. Naturalness, weak scale supersymmetry and the prospect for the observation of supersymmetry at the Tevatron and at the CERN LHC. *Phys.Rev.*, D58:096004, 1998.

[58] C. M. Chen, G. V. Kraniotis, V. E. Mayes, Dimitri V. Nanopoulos, and J. W. Walker. A K-theory anomaly free supersymmetric flipped SU(5) model from intersecting branes. *Phys. Lett.*, B625:96–105, 2005.

[59] C. M. Chen, G. V. Kraniotis, V. E. Mayes, Dimitri V. Nanopoulos, and J. W. Walker. A supersymmetric flipped SU(5) intersecting brane world. *Phys. Lett.*, B611:156–166, 2005.

[60] Ching-Ming Chen and Yu-Chieh Chung. Flipped $SU(5)$ GUTs from E_8 Singularity in F-theory. 2010.

[61] Ching-Ming Chen, Johanna Knapp, Maximilian Kreuzer, and Christoph Mayrhofer. Global SO(10) F-theory GUTs. 2010.

[62] Ching-Ming Chen, Tianjun Li, and Dimitri V. Nanopoulos. Flipped and unflipped SU(5) as type IIA flux vacua. *Nucl. Phys.*, B751:260–284, 2006.

[63] Ching-Ming Chen, V. E. Mayes, and Dimitri V. Nanopoulos. Flipped SU(5) from D-branes with type IIB fluxes. *Phys. Lett.*, B633:618–626, 2006.

[64] Ning Chen, Daniel Feldman, Zuowei Liu, Pran Nath, and Gregory Peim. Low Mass Gluino within the Sparticle Landscape, Implications for Dark Matter, and Early Discovery Prospects at LHC-7. 2010. * Temporary entry *.

[65] Ning Chen, Daniel Feldman, Zuowei Liu, Pran Nath, and Gregory Peim. Positron and Photon Compliant Higgsino Dark Matter and LHC-7. 2010. * Temporary entry *.

[66] Yu-Chieh Chung. On Global Flipped SU(5) GUTs in F-theory. 2010.

[67] Timothy Cohen, Daniel J. Phalen, and Aaron Pierce. On the Correlation Between the Spin-Independent and Spin- Dependent Direct Detection of Dark Matter. *Phys. Rev.*, D81:116001, 2010.

[68] Joseph P. Conlon. Gauge Threshold Corrections for Local String Models. *JHEP*, 04:059, 2009.

[69] Joseph P. Conlon and Eran Palti. Gauge Threshold Corrections for Local Orientifolds. *JHEP*, 09:019, 2009.

[70] Joseph P. Conlon and Eran Palti. On Gauge Threshold Corrections for Local IIB/F-theory GUTs. *Phys. Rev.*, D80:106004, 2009.

[71] Joseph P. Conlon and Fernando Quevedo. Astrophysical and cosmological implications of large volume string compactifications. *JCAP*, 0708:019, 2007.

[72] John Conway. http://www.physics.ucdavis.edu/$\sim$conway/research/software/pgs/pgs4-general.htm.

[73] G.D. Coughlan, W. Fischler, Edward W. Kolb, S. Raby, and Graham G. Ross. Cosmological Problems for the Polonyi Potential. *Phys.Lett.*, B131:59, 1983.

[74] Laura Covi, Marta Gomez-Reino, Christian Gross, Gonzalo A. Palma, and Claudio A. Scrucca. Constructing de Sitter vacua in no-scale string models without uplifting. *JHEP*, 0903:146, 2009.

[75] Mirjam Cvetic, Durmus A. Demir, J.R. Espinosa, L.L. Everett, and P. Langacker. Electroweak breaking and the mu problem in supergravity models with an additional U(1). *Phys.Rev.*, D56:2861, 1997.

[76] Mirjam Cvetic, James Halverson, and 2 Richter, Robert. Realistic Yukawa structures from orientifold compactifications. *JHEP*, 0912:063, 2009.

[77] Mirjam Cvetic and Paul Langacker. Implications of Abelian extended gauge structures from string models. *Phys.Rev.*, D54:3570–3579, 1996.

[78] Mirjam Cvetic and Paul Langacker. New grand unified models with intersecting D6-branes, neutrino masses, and flipped SU(5). *Nucl. Phys.*, B776:118–137, 2007.

[79] B. de Carlos, J.A. Casas, F. Quevedo, and E. Roulet. Model independent properties and cosmological implications of the dilaton and moduli sectors of 4-d strings. *Phys.Lett.*, B318:447–456, 1993.

[80] Andre de Gouvea, Takeo Moroi, and Hitoshi Murayama. Cosmology of supersymmetric models with low-energy gauge mediation. *Phys.Rev.*, D56:1281–1299, 1997.

[81] A. Dedes, C. Panagiotakopoulos, and K. Tamvakis. Radiative GUT symmetry breaking in a R-symmetric flipped SU(5) model. *Phys. Rev.*, D57:5493–5498, 1998.

[82] Frederik Denef and Michael R. Douglas. Distributions of nonsupersymmetric flux vacua. *JHEP*, 0503:061, 2005.

[83] J.P. Derendinger, Jihn E. Kim, and Dimitri V. Nanopoulos. Anti-SU(5). *Phys.Lett.*, B139:170, 1984.

[84] Oliver DeWolfe, Alexander Giryavets, Shamit Kachru, and Washington Taylor. Type IIA moduli stabilization. *JHEP*, 0507:066, 2005.

[85] Duiliu-Emanuel Diaconescu, Bogdan Florea, Shamit Kachru, and Peter Svrcek. Gauge: mediated supersymmetry breaking in string compactifications. *JHEP*, 0602:020, 2006.

[86] Michael Dine, Yosef Nir, and Yael Shadmi. Enhanced symmetries and the ground state of string theory. *Phys.Lett.*, B438:61–68, 1998.

[87] Michael Dine, Lisa Randall, and Scott D. Thomas. Supersymmetry breaking in the early universe. *Phys.Rev.Lett.*, 75:398–401, 1995.

[88] Ron Donagi and Martijn Wijnholt. Breaking GUT Groups in F-Theory. 2008.

[89] Ron Donagi and Martijn Wijnholt. Model Building with F-Theory. 2008.

[90] Ron Donagi and Martijn Wijnholt. Higgs Bundles and UV Completion in F-Theory. 2009.

[91] Herbert K. Dreiner. An Introduction to explicit R-parity violation. 1997. To be published in 'Perspectives on Supersymmetry', Ed. by G.L. Kane, World Scientific.

[92] E. Dudas, Y. Mambrini, S. Pokorski, Alberto Romagnoni, and M. Trapletti. Gauge versus Gravity mediation in models with anomalous U(1)'s. *JHEP*, 0903:011, 2009.

[93] Emilian Dudas and Eran Palti. Froggatt-Nielsen models from E8 in F-theory GUTs. *JHEP*, 01:127, 2010.

[94] Emilian Dudas and Eran Palti. On hypercharge flux and exotics in F-theory GUTs. 2010.

[95] M.M. El Kheishen, A.A. Aboshousha, and A.A. Shafik. Analytic formulas for the neutralino masses and the neutralino mixing matrix. *Phys.Rev.*, D45:4345–4348, 1992.

[96] John R. Ellis, G.B. Gelmini, Jorge L. Lopez, Dimitri V. Nanopoulos, and Subir Sarkar. Astrophysical constraints on massive unstable neutral relic particles. *Nucl.Phys.*, B373:399–437, 1992.

[97] John R. Ellis, Dimitri V. Nanopoulos, and M. Quiros. On the Axion, Dilaton, Polonyi, Gravitino and Shadow Matter Problems in Supergravity and Superstring Models. *Phys.Lett.*, B174:176, 1986.

[98] Daniel Feldman, Gordon Kane, Ran Lu, and Brent D. Nelson. Dark Matter as a Guide Toward a Light Gluino at the LHC. *Phys.Lett.*, B687:363–370, 2010.

[99] Daniel Feldman, Zuowei Liu, Pran Nath, and Brent D. Nelson. Explaining PAMELA and WMAP data through Coannihilations in Extended SUGRA with Collider Implications. *Phys.Rev.*, D80:075001, 2009.

[100] Jonathan L. Feng, Konstantin T. Matchev, and Takeo Moroi. Focus points and naturalness in supersymmetry. *Phys. Rev.*, D61:075005, 2000.

[101] Jonathan L. Feng, Takeo Moroi, Lisa Randall, Matthew Strassler, and Shu-fang Su. Discovering supersymmetry at the Tevatron in wino LSP scenarios. *Phys.Rev.Lett.*, 83:1731–1734, 1999.

[102] Pavel Fileviez Perez. How large could the R-parity violating couplings be? *J. Phys.*, G31:1025–1030, 2005.

[103] A. Font and L. E. Ibanez. Yukawa Structure from U(1) Fluxes in F-theory Grand Unification. *JHEP*, 02:016, 2009.

[104] Patrick Fox, Aaron Pierce, and Scott D. Thomas. Probing a QCD string axion with precision cosmological measurements. 2004.

[105] Graciela B. Gelmini and Paolo Gondolo. Neutralino with the right cold dark matter abundance in (almost) any supersymmetric model. *Phys.Rev.*, D74:023510, 2006.

[106] H. Georgi and S.L. Glashow. Unity of All Elementary Particle Forces. *Phys.Rev.Lett.*, 32:438–441, 1974.

[107] Michael Gerbush, Teng Jian Khoo, Daniel J. Phalen, Aaron Pierce, and David Tucker-Smith. Color-octet scalars at the CERN LHC. *Phys.Rev.*, D77:095003, 2008.

[108] G.F. Giudice and A. Masiero. A Natural Solution to the mu Problem in Supergravity Theories. *Phys.Lett.*, B206:480–484, 1988.

[109] Gian F. Giudice, Tao Han, Kai Wang, and Lian-Tao Wang. Nearly Degenerate Gauginos and Dark Matter at the LHC. *Phys.Rev.*, D81:115011, 2010.

[110] Marta Gomez-Reino and Claudio A. Scrucca. Locally stable non-supersymmetric Minkowski vacua in supergravity. *JHEP*, 0605:015, 2006.

[111] P. Gondolo, J. Edsjo, P. Ullio, L. Bergstrom, Mia Schelke, et al. DarkSUSY: Computing supersymmetric dark matter properties numerically. *JCAP*, 0407:008, 2004.

[112] Phill Grajek, Gordon Kane, Daniel J. Phalen, Aaron Pierce, and Scott Watson. Neutralino Dark Matter from Indirect Detection Revisited. 2008.

[113] Daniel Green and Timo Weigand. Retrofitting and the mu Problem. 2009.

[114] Michael B. Green and John H. Schwarz. Anomaly Cancellation in Supersymmetric D=10 Gauge Theory and Superstring Theory. *Phys.Lett.*, B149:117–122, 1984.

[115] Thomas W. Grimm, Sven Krause, and Timo Weigand. F-Theory GUT Vacua on Compact Calabi-Yau Fourfolds. 2009.

[116] Thomas W. Grimm and Timo Weigand. On Abelian Gauge Symmetries and Proton Decay in Global F- theory GUTs. 2010.

[117] Hirotaka Hayashi, Teruhiko Kawano, Yoichi Tsuchiya, and Taizan Watari. Flavor Structure in F-theory Compactifications. 2009.

[118] Hirotaka Hayashi, Teruhiko Kawano, Yoichi Tsuchiya, and Taizan Watari. More on Dimension-4 Proton Decay Problem in F-theory – Spectral Surface, Discriminant Locus and Monodromy. 2010.

[119] Jonathan J. Heckman and Cumrun Vafa. Flavor Hierarchy From F-theory. 2008.

[120] Junji Hisano, Masahiro Kawasaki, Kazunori Kohri, and Kazunori Nakayama. Positron/Gamma-Ray Signatures of Dark Matter Annihilation and Big-Bang Nucleosynthesis. *Phys.Rev.*, D79:063514, 2009.

[121] Stefan Hoeche, Frank Krauss, Nils Lavesson, Leif Lonnblad, Michelangelo Mangano, et al. Matching parton showers and matrix elements. 2006.

[122] Yutaka Hosotani. Dynamical Gauge Symmetry Breaking as the Casimir Effect. *Phys.Lett.*, B129:193, 1983.

[123] Chao-Shang Huang, Tianjun Li, Chun Liu, Jonathan P. Shock, Feng Wu, and Yue-Liang Wu. Embedding flipped SU(5) into SO(10). *JHEP*, 10:035, 2006.

[124] L.E. Ibanez and 2 Richter, Robert. Stringy Instantons and Yukawa Couplings in MSSM-like Orientifold Models. *JHEP*, 0903:090, 2009.

[125] L.E. Ibanez and A.M. Uranga. Instanton induced open string superpotentials and branes at singularities. *JHEP*, 0802:103, 2008.

[126] Masahiro Ibe and Ryuichiro Kitano. Gauge mediation in supergravity and gravitino dark matter. *Phys.Rev.*, D75:055003, 2007.

[127] Eder Izaguirre, Michael Manhart, and Jay G. Wacker. Bigger, Better, Faster, More at the LHC. *JHEP*, 1012:030, 2010.

[128] Jing Jiang, Tianjun Li, and Dimitri V. Nanopoulos. Testable Flipped $SU(5)$ x $U(1)_X$ Models. *Nucl. Phys.*, B772:49–66, 2007.

[129] Jing Jiang, Tianjun Li, Dimitri V. Nanopoulos, and Dan Xie. Flipped $SU(5)$ X $U(1)_X$ Models from F-Theory. *Nucl. Phys.*, B830:195–220, 2010.

[130] Dominic D. Joyce. *Compact Manifolds with Special Holonomy*. Oxford University Press, 2000.

[131] Shamit Kachru, Renata Kallosh, Andrei D. Linde, and Sandip P. Trivedi. De Sitter vacua in string theory. *Phys.Rev.*, D68:046005, 2003.

[132] Gordon Kane, Ran Lu, and Scott Watson. PAMELA Satellite Data as a Signal of Non-Thermal Wino LSP Dark Matter. *Phys.Lett.*, B681:151–160, 2009. * Brief entry *.

[133] Rolf Kappl, Hans Peter Nilles, Saul Ramos-Sanchez, Michael Ratz, Kai Schmidt-Hoberg, et al. Large hierarchies from approximate R symmetries. *Phys.Rev.Lett.*, 102:121602, 2009.

[134] M. Kawasaki and T. Moroi. Gravitino production in the inflationary universe and the effects on big bang nucleosynthesis. *Prog.Theor.Phys.*, 93:879–900, 1995.

[135] Jihn E. Kim. Light Pseudoscalars, Particle Physics and Cosmology. *Phys.Rept.*, 150:1–177, 1987.

[136] Jihn E. Kim and Hans Peter Nilles. The mu Problem and the Strong CP Problem. *Phys.Lett.*, B138:150, 1984.

[137] S.F. King, G.K. Leontaris, and G.G. Ross. Family symmetries in F-theory GUTs. *Nucl.Phys.*, B838:119–135, 2010.

[138] Ryuichiro Kitano. Gravitational Gauge Mediation. *Phys.Lett.*, B641:203–207, 2006.

[139] Eric Kuflik and Joseph Marsano. Comments on Flipped SU(5) (and F-theory). 2010.

[140] Kunal Kumar, Tim M.P. Tait, and Roberto Vega-Morales. Manifestations of Top Compositeness at Colliders. *JHEP*, 0905:022, 2009.

[141] Zygmunt Lalak, Stefan Pokorski, and Krzysztof Turzynski. Gravity in Gauge Mediation. *JHEP*, 0810:016, 2008.

[142] Paul Langacker and Ming-xing Luo. Implications of precision electroweak experiments for M(t), rho(0), sin**2-Theta(W) and grand unification. *Phys.Rev.*, D44:817–822, 1991.

[143] Oleg Lebedev and Saul Ramos-Sanchez. The NMSSM and String Theory. *Phys.Lett.*, B684:48–51, 2010.

[144] Hyun Min Lee et al. A unique Z_4^R symmetry for the MSSM. 2010.

[145] Tianjun Li. SU(5) and SO(10) Models from F-Theory with Natural Yukawa Couplings. *Phys. Rev.*, D81:065018, 2010.

[146] Tianjun Li, James A. Maxin, and Dimitri V. Nanopoulos. F-Theory Grand Unification at the Colliders. 2010.

[147] Tianjun Li, James A. Maxin, Dimitri V. Nanopoulos, and Joel W. Walker. Dark Matter, Proton Decay and Other Phenomenological Constraints in $\mathcal{F}$-SU(5). 2010.

[148] Tianjun Li, James A. Maxin, Dimitri V. Nanopoulos, and Joel W. Walker. The Golden Point of No-Scale and No-Parameter $\mathcal{F}$- $SU(5)$. 2010.

[149] Tianjun Li, Dimitri V. Nanopoulos, and Joel W. Walker. Fast Proton Decay. 2009.

[150] Tianjun Li, Dimitri V. Nanopoulos, and Joel W. Walker. Elements of F-ast Proton Decay. 2010.

[151] Ben Lillie, Jing Shu, and Timothy M.P. Tait. Top Compositeness at the Tevatron and LHC. *JHEP*, 0804:087, 2008.

[152] Jorge L. Lopez and Dimitri V. Nanopoulos. Decisive role of nonrenormalizable terms in the flipped string. *Phys. Lett.*, B251:73–82, 1990.

[153] Michelangelo L. Mangano, Mauro Moretti, Fulvio Piccinini, Roberto Pittau, and Antonio D. Polosa. ALPGEN, a generator for hard multiparton processes in hadronic collisions. *JHEP*, 0307:001, 2003.

[154] Joseph Marsano, Natalia Saulina, and Sakura Schafer-Nameki. F-theory Compactifications for Supersymmetric GUTs. 2009.

[155] Joseph Marsano, Natalia Saulina, and Sakura Schafer-Nameki. Monodromies, Fluxes, and Compact Three-Generation F-theory GUTs. *JHEP*, 08:046, 2009.

[156] Joseph Marsano, Natalia Saulina, and Sakura Schafer-Nameki. A Note on G-Fluxes for F-theory Model Building. 2010.

[157] Joseph Marsano, Natalia Saulina, and Sakura Schafer-Nameki. Compact F-theory GUTs with $U(1)_{PQ}$. *JHEP*, 04:095, 2010.

[158] Stephen P. Martin. A Supersymmetry Primer. 1997.

[159] T. Moroi, H. Murayama, and Masahiro Yamaguchi. Cosmological constraints on the light stable gravitino. *Phys.Lett.*, B303:289–294, 1993.

[160] Takeo Moroi and Lisa Randall. Wino cold dark matter from anomaly mediated SUSY breaking. *Nucl.Phys.*, B570:455–472, 2000.

[161] Hitoshi Murayama and Aaron Pierce. Not even decoupling can save minimal supersymmetric SU(5). *Phys.Rev.*, D65:055009, 2002.

[162] Minoru Nagai and Kazunori Nakayama. Nonthermal dark matter in mirage mediation. *Phys.Rev.*, D76:123501, 2007.

[163] Pran Nath and Tomasz R. Taylor. Modular invariance, soft breaking, mu and tan beta in superstring models. *Phys.Lett.*, B548:77–87, 2002.

[164] Tony Pantev and Martijn Wijnholt. Hitchin's Equations and M-Theory Phenomenology. 2009.

[165] Damien M. Pierce, Jonathan A. Bagger, Konstantin T. Matchev, and Ren-jie Zhang. Precision corrections in the minimal supersymmetric standard model. *Nucl. Phys.*, B491:3–67, 1997.

[166] Tilman Plehn and Tim M.P. Tait. Seeking Sgluons. *J.Phys.G*, G36:075001, 2009.

[167] Alex Pomarol and Javi Serra. Top Quark Compositeness: Feasibility and Implications. *Phys.Rev.*, D78:074026, 2008.

[168] Stuart Raby. Proton decay. 2002.

[169] Saul Ramos-Sanchez. The mu-problem, the NMSSM and string theory. 2010.

[170] Michael Ratz. Stringy Surprises. *Prog.Theor.Phys.Suppl.*, 180:96–111, 2010.

[171] Mansoor Ur Rehman, Qaisar Shafi, and Joshua R. Wickman. Minimal Supersymmetric Hybrid Inflation, Flipped SU(5) and Proton Decay. *Phys. Lett.*, B688:75–81, 2010.

[172] M.H. Reno and D. Seckel. Primordial Nucleosynthesis: The Effects of Injecting Hadrons. *Phys.Rev.*, D37:3441, 1988.

[173] Satoshi Shirai, Fuminobu Takahashi, and T. T. Yanagida. R-violating Decay of Wino Dark Matter and electron/positron Excesses in the PAMELA/Fermi Experiments. *Phys. Lett.*, B680:485–488, 2009.

[174] Torbjorn Sjostrand, Stephen Mrenna, and Peter Z. Skands. PYTHIA 6.4 Physics and Manual. *JHEP*, 0605:026, 2006.

[175] Daijiro Suematsu and Yoshio Yamagishi. Radiative symmetry breaking in a supersymmetric model with an extra U(1). *Int.J.Mod.Phys.*, A10:4521–4536, 1995.

[176] Peter Svrcek and Edward Witten. Axions In String Theory. *JHEP*, 0606:051, 2006.

[177] Giovanni Villadoro and Fabio Zwirner. N=1 effective potential from dual type-IIA D6/O6 orientifolds with general fluxes. *JHEP*, 0506:047, 2005.

[178] Scott Watson. Reevaluating the Cosmological Origin of Dark Matter. 2009.

[179] Steven Weinberg. Does Gravitation Resolve the Ambiguity Among Supersymmetry Vacua? *Phys.Rev.Lett.*, 48:1776–1779, 1982.

[180] J. Wess and J. Bagger. Supersymmetry and supergravity. 1992.

[181] Edward Witten. Some Properties of O(32) Superstrings. *Phys.Lett.*, B149:351–356, 1984.

[182] Edward Witten. Symmetry Breaking Patterns in Superstring Models. *Nucl.Phys.*, B258:75, 1985.

[183] Edward Witten. Anomaly cancellation on G(2) manifolds. 2001.

[184] Edward Witten. Deconstruction, G(2) holonomy, and doublet triplet splitting. pages 472–491, 2001.