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Blazars in the early Universe

M. Volonteri,[1]★ F. Haardt,[2,3] G. Ghisellini[4] and R. Della Ceca[5]

[1]*Astronomy Department, University of Michigan, Ann Arbor, MI 48109, USA*
[2]*Dipartimento di Fisica e Matematica, Università dell'Insubria, Via Valleggio 11, I-22100 Como, Italy*
[3]*INFN, Sezione di Milano-Bicocca, 20126 Milano, Italy*
[4]*INAF – Osservatorio Astronomico di Brera, Via Bianchi 46, I-23807 Merate, Italy*
[5]*INAF – Osservatorio Astronomico di Brera, Via Brera 28, I-20100 Milano, Italy*

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ABSTRACT

We investigate the relative occurrence of radio-loud and radio-quiet quasars in the first billion years of the Universe, powered by black holes heavier than one billion solar masses. We consider the sample of high-redshift blazars detected in the hard X-ray band in the 3-year all sky survey performed by the Burst Alert Telescope onboard the *Swift* satellite. All the black holes powering these blazars exceed a billion solar mass, with accretion luminosities close to the Eddington limit. For each blazar pointing at us, there must be hundreds of similar sources (having black holes of similar masses) pointing elsewhere. This puts constraints on the density of billion solar masses black holes at high redshift ($z > 4$), and on the relative importance of (jetted) radio-loud versus radio-quiet sources. We compare the expected number of high-redshift radio-loud sources with the high-luminosity radio-loud quasars detected in the Sloan Digital Sky Survey (SDSS), finding agreement up to $z \sim 3$, but a serious deficit at $z > 3$ of SDSS radio-loud quasars with respect to the expectations. We suggest that the most likely explanations for this disagreement are (i) the ratio of blazar to misaligned radio sources decreases by an order of magnitude above $z = 3$, possibly as a result of a decrease of the average bulk Lorentz factor, (ii) the SDSS misses a large fraction of radio-loud sources at high redshifts, (iii) the SDSS misses *both* radio-loud and radio-quiet quasars at high redshift, possibly because of obscuration or because of collimation of the optical–ultraviolet continuum in systems accreting near Eddington. These explanations imply very different number density of heavy black holes at high redshifts that we discuss in the framework of the current ideas about the relations of dark matter haloes at high redshifts and the black hole they host.

Key words: radiation mechanisms: non-thermal – BL Lacertae objects: general – quasars: general – X-rays: general.

1 INTRODUCTION

Ajello et al. (2009, hereafter A09) recently published the list of blazars detected in the all sky survey by *Swift*/Burst Alert Telescope (BAT), between 2005 March and 2008 March. BAT is a coded mask telescope designed to detect gamma-ray bursts (GRBs), has a large field of view ($120° \times 90°$, partially coded) and is sensitive in the (15–150 keV) energy range. This instrument was specifically designed to detect GRBs, but as GRBs are distributed isotropically in the sky, BAT performed, as a by product, an all sky survey with a reasonably uniform sky coverage, at a limiting sensitivity of the order of 1 mCrab in the 15–55 keV range (equivalent to 1.27×10^{-11} erg cm^{-2} s^{-1}) in 1-Ms exposure (see A09). Taking the period 2005 March–2008 March, and evaluating the image resulting from the superposition of all observations in this period, BAT detected 38 blazars (A09), of which 26 are flat spectrum radio quasars (FSRQs) and 12 are BL Lac objects, once the Galactic plane ($|b < 15°|$) is excluded from the analysis. A09 reported an average exposure of 4.3 Ms, and considered the (15–55 keV) energy range, to avoid background problems at higher energies. The well-defined sky coverage and sources selection criteria make the list of the found blazars a complete, flux-limited sample that enabled A09 to calculate the luminosity function (LF) and the possible cosmic evolutions of FSRQs and BL Lacs, together with their contribution to the hard X-ray background. A09 also stressed the fact that the detected BAT blazars at high redshift are among the most powerful blazars and could be associated with powerful accreting systems. Within the BAT sample, there are 10 blazars (all FSRQs) at redshift greater than 2, and five at redshift between 3 and 4. All (and

★E-mail: martav@umich.edu

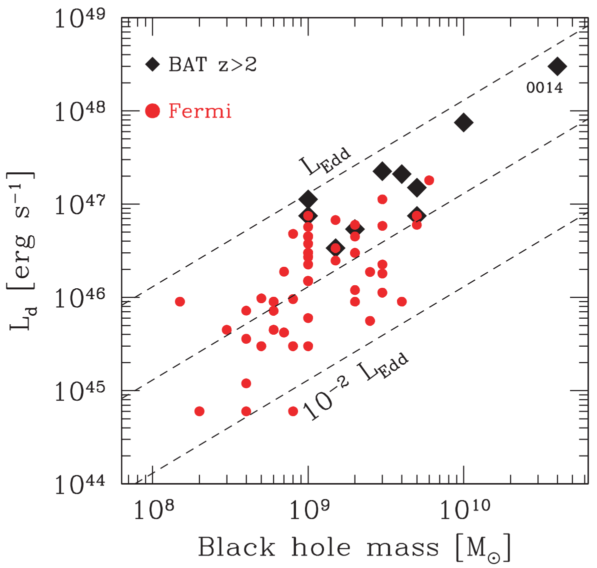

Figure 1. Accretion disc luminosity $L_{\rm d}$ as a function of black hole mass for blazars with $z > 2$ in the BAT sample (diamonds; A09) and in the 1LAC *Fermi*/LAT sample (circles; Abdo et al. 2010), as studied in G10 and in Ghisellini et al. (2011). The high-redshift BAT blazars are all characterized by black holes with $M > 10^9\,{\rm M_\odot}$ and by $L_{\rm d}/L_{\rm Edd} > 0.1$.

only) these blazars have an X-ray luminosity exceeding $L_{\rm X} = 2 \times 10^{47}\,{\rm erg\,s^{-1}}$. All these sources have been studied by Ghisellini et al. (2010, hereafter G10) that showed that their optical–ultraviolet (UV) emission is dominated by the emission of their accretion disc, with no contamination from the beamed non-thermal continuum, even if the latter is dominating the total bolometric luminosity. Fitting the optical–UV emission with a standard Shakura & Sunyaev (1973) accretion disc, it was possible to estimate both the mass and the disc luminosity. These high-redshift blazars are shown in Fig. 1 (diamonds) together with all the blazars with $z > 2$ detected above 100 MeV in the 11 months all sky survey performed by *Fermi*/Large Area Telescope (LAT; Abdo et al. 2010) and studied in Ghisellini et al. (2011). Fig. 1 shows that all high-redshift BAT blazars are characterized by large Eddington ratio of order 0.2–1 and by black holes heavier than $10^9\,{\rm M_\odot}$.

Since these objects are at high redshifts, our finding has important implications on the number density of heavy black holes, especially if we consider that for each blazar pointing at us, there must be hundreds of similar sources (having black holes of similar masses) pointing elsewhere. In fact, if the emitting plasma is moving with a bulk Lorentz factor Γ in one direction, the number of sources observed within the beaming angle $1/\Gamma$ is only a fraction $1/(2\Gamma^2)$ of the sources pointing in other directions.

Taking the LF of A09 at face value and calculating the expected number of luminous sources (i.e. $L_{\rm X} > 2 \times 10^{47}\,{\rm erg\,s^{-1}}$, likely hosting a black hole with $M > M_9$, where $M_9 = 10^9\,{\rm M_\odot}$) at $z > 4$, one finds that the number density of heavy black holes of jetted sources is close to or even greater than the upper limit defined by standard 'dark matter halo–black holes' relationships at the largest redshifts. G10 then corrected the original A09 LF by assuming an evolutionary model that is equal to the A09 one up to $z \sim 4.3$ (where they measure the peak of the density of high X-ray luminosity blazars), and then cuts off exponentially ('minimal' LF). This 'minimal LF' was consistent with the constraints posed by standard 'dark matter halo–black holes' relationships and consistent with the constraints given by the existence of a few blazars (discovered serendipitously) at $z > 4$ (see below).

In this paper we explore the implications of G10 results in view of the properties of the radio-quiet and radio-loud populations, their redshift evolution and their connection to host dark matter haloes. First we check if the expected number of radio-loud sources calculated from the A09 and G10 LFs agrees with those detected in the Sloan Digital Sky Survey (SDSS) of quasars. As discussed in Section 2, while a rough agreement is found up to $z \sim 3$, there is a serious deficit of SDSS radio-loud sources above this redshift. We then investigate the possible reasons of this discrepancy, discussing three possible solutions. Although we cannot confidently select one of these, we point out the consequences they have on our understanding of the physics of jets and on the relationship between dark matter haloes and the mass of the black holes they host.

2 RADIO-LOUD HIGH-REDSHIFT SOURCES

One can estimate the volume density of high-redshift blazars hosting a black hole of mass larger than M_9 using the cosmological evolution model of A09 along with its high-z cut-off (i.e. 'minimal') version, assuming that all blazars with $L_{\rm X} > 2 \times 10^{47}\,{\rm erg\,s^{-1}}$ have a $M > M_9$ black hole (G10). We cannot exclude that blazars with lower X-ray luminosity also host massive black holes, so the 'observational' points, strictly speaking, are *lower limits*.

Lower limits to the density of high-redshift blazars powered by black holes with $M > M_9$ are placed by the existence of at least four blazars at $4 < z < 5$ for which G10 have estimated a black hole mass larger than $10^9\,{\rm M_\odot}$. These blazars are RXJ 1028.6−0844 ($z = 4.276$; Yuan et al. 2005), GB 1508+5714 ($z = 4.3$; Hook et al. 1995), PMN J0525−3343 ($z = 4.41$; Worsley et al. 2004b) and GB 1428+4217 ($z = 4.72$; Worsley et al. 2004a). The lower limit in the 5–6 redshift range corresponds to the existence of at least one blazar, Q0906+6930 at $z = 5.47$, with an estimated black hole mass of $2 \times 10^9\,{\rm M_\odot}$ (Romani 2006).

These are all sources pointing at us. The real number density of heavy black holes in jetted sources, $\Phi_{\rm RL}(z, M > M_9)$, must account for the much larger population of misaligned sources. We have then multiplied the mass function of blazars by $2\Gamma^2 = 450$, i.e. we have assumed an average Γ factor of 15, appropriate for the BAT blazars analysed in G10.

Summarizing, the BAT blazar survey allowed to meaningfully construct the hard X-ray LF of blazars. G10 have also constructed the minimal evolution consistent with the existing data and the (few) existing lower limits. At the high-luminosity end the LF can be translated into the mass function of black holes with more than one billion solar masses. In Fig. 2 we show $\Phi_{\rm RL}(z, M > M_9)$ as derived from the cosmological evolution model of A09 as a red stripe, and that derived from the 'minimal' LF as a green stripe.

The BAT blazars described above can be compared to the radio-loud sources in the quasar catalogue of the SDSS (Schneider et al. 2010) Data Release 7 (DR7) that includes information on radio detection in and the Faint Images of the Radio Sky at Twenty-cm survey (FIRST; Becker, White & Helfand 1995). The region of the sky covered by both surveys is $\sim$8770 deg^2. We adopt the public catalogue with quasar properties described in Shen et al. (2011), which includes quasars bolometric luminosity (using bolometric corrections derived from the composite spectral energy distributions from Richards et al. 2006). The catalogue also provides the radio flux density at rest frame 6 cm and the optical flux density at rest frame 2500 Å that can be used to calculate the radio loudness.

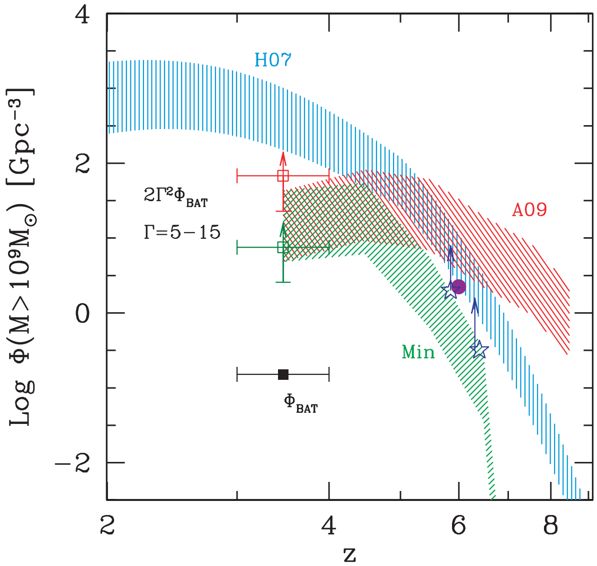

Figure 2. Number density of black holes with $M > 10^9\,M_\odot$ as a function of redshift. The filled black square in the $3 < z < 4$ bin is taken directly from fig. 10 of A09. The two empty squares account for the population of misaligned sources, multiplying by $2\Gamma^2$, with $\Gamma = 5$ and 15. Red hatched area: number density of heavy black holes in radio-loud sources derived from the blazar LF studied in A09, assuming $\Gamma = 5$ (lower bound) or $\Gamma = 15$ (upper bound). Green hatched area: number density of heavy black holes assuming the 'minimal' cosmic evolution of the LF of blazars (G10), with $\Gamma = 5$ (lower bound) or $\Gamma = 15$ (upper bound). Blue hatched area: number density of heavy black holes in radio-quiet quasars (assuming the LF and its evolution of H07), assuming an average Eddington fraction of $f_{\rm Edd} = 0.3$ (upper boundary) or $f_{\rm Edd} = 1$ (lower boundary). Blue stars: number density of $M > 10^9\,M_\odot$ derived from the existence of the black holes analysed in Kurk et al. (2007). Purple dot: number density of $M > 10^9\,M_\odot$ derived from the mass function of black holes at $z = 6$ proposed by Willott et al. (2010).

Following Jiang et al. (2007) we define a source radio loud if it has radio to optical flux ratio, R, larger than 10. For a handful of sources where optical quantities are not provided, we supplement the 'raw' catalogue by calculating the bolometric luminosity from the absolute i-band magnitude, assuming a bolometric correction of 2.5. This bolometric correction is derived by matching the average bolometric luminosity provided by Shen et al. (2011) with the bolometric luminosity calculated from the absolute i-band magnitude for sources where the catalogue lists both quantities. We then calculate the rest-frame optical flux from the luminosity in order to derive an estimate of the radio loudness. This 'extended' catalogue will be our reference.

We select all sources that are in the FIRST+SDSS footprint, have an optical bolometric luminosity $>10^{47}$ erg s^{-1} and $R > 10$. We also require the quasars to be selected uniformly using the final quasar target selection algorithm described in Richards et al. (2002). These amount to e.g. 21 radio-loud sources in the $4 < z < 5$ redshift bin. We note that this number *is not* representative of a complete, volume-limited sample (i.e. of the true LF). To derive a simple estimate of the statistical incompleteness, we compare the number of quasars in Table 1 (column b) to the number predicted by the bolometric LF, which is derived from surveys that include the SDSS (Hopkins et al. 2007, hereafter H07), in the SDSS+FIRST area. For example, in the same redshift bins, approximately a factor of $\simeq$10 more objects are expected from the LF proposed by H07.

On the other hand, such 'incompleteness' biases should not affect number ratios, such as the radio-loud fraction (RLF) derived from our FIRST+SDSS sample. As a matter of fact, our derived RLF (column d of Table 1) is completely consistent with the RLF derived by Jiang et al. (2007). This allows us to estimate the 'expected number' of RL objects simply multiplying the values obtained from the H07 LF by the RLF we have derived from our sample. Such numbers are reported in column (e) of Table 1, and should be compared to the expectations from detected BAT blazars (columns g and h). It is immediately clear that, at least at $z \gtrsim 3$, the expectations largely exceed what derived from the observed LF if $\Gamma = 15$, while $\Gamma = 5$ seems to be quantitatively consistent with data. For increasing redshift, the corrected fraction of RL objects (column e) is progressively lower, and its ratio with respect to columns (g) and (h) is decreasing (the figure in column e exceeds both columns g and h for $z = 2$–3, in between the values of columns g and h for $z =$ 3–4 and lower for $z = 4$–5 and 5–6; in the latter case, the corrected fraction of RL objects just coincides with the lower boundary of the range from column h that adopts $\Gamma = 5$). We note that a discrepancy between the observed number of radio-loud quasars and theoretical predictions was first noted by Haiman et al. (2004) and confirmed by McGreer et al. (2009).

Table 1. Column (a): redshift bin. Column (b): number of uniformly selected quasars with $\log L_{\rm bol} > 47$ (erg s^{-1}) in the SDSS+FIRST survey (8770 deg^2 in common). Column (c): number of objects with the radio (5 GHz) to optical (2500 Å) monochromatic flux ratio larger than 10. Column (d): column (c) divided by column (b). Column (e): number of radio-loud objects expected in the SDSS+FIRST footprint, ~8770 deg^2. The number is obtained from the SDSS quasi-stellar object LF of H07 rescaled by the RLF given in column (d). Column (f): the number of blazars detected by *Swift*/BAT with an estimated black hole mass exceeding $10^9\,M_\odot$, from the minimal and the A09 evolutions, when relevant. Column (g): the expected number of radio-loud sources calculated by multiplying the number of detected blazars (column f) in the given redshift bin by $450 \times 8770/40\,000 \sim 10^2$ (i.e. assuming $\Gamma = 15$). Column (h): same as column (g), but for $\Gamma = 5$ (the number scales as Γ^2).

z	All	Radio loud $R > 10$	*Radioloud percent*	$R > 10$ corrected	*Swift*/BAT	Expected, $\Gamma = 15$	Expected, $\Gamma = 5$
(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)
All	6194	576	9.30				
1–2	1342	160	11.92				
2–3	2541	260	10.23	5200	20	2000	222
3–4	1706	129	7.56	1800	45	4500	500
4–5	550	21	3.81	252	52–78	5200–7800	580–870
5–6	36	2	5.55	56	5–52	500–5200	55–580

Before discussing the possible nature of such discrepancy, it is useful to analyse the predictions concerning the fraction of radio-loud sources at high redshifts.

2.1 Radio-loud versus radio-quiet quasars at high redshift

We now estimate the ratio of radio-loud to radio-quiet quasars having black holes exceeding M_9 as a function of redshift. We must therefore derive the number density $\Phi_{\rm RQ}(z, M > M_9)$ of radio-quiet quasars hosting black holes of $M > M_9$ in different redshift bins. Consider the bolometric LF of radio-quiet sources (H07) and simply assume that quasars radiate at an average fraction $f_{\rm Edd}$ of the Eddington limit so that

$$\frac{M}{10^9\,{\rm M}_\odot} = 3 \times 10^{-14} \frac{1}{f_{\rm Edd}} \frac{L}{{\rm L}_\odot}. \quad (1)$$

To derive $\Phi_{\rm RQ}(z, M > M_9)$ one then integrates the LF above the luminosity threshold, $L_{\rm min}$, corresponding to $M = M_9$.

Estimates of $\Phi_{\rm RQ}(z, M > M_9)$ for radio-quiet quasars are shown in Fig. 2 for $f_{\rm Edd} = 0.3$ ($L_{\rm min} = 10^{13}\,{\rm L}_\odot$) and $f_{\rm Edd} = 1$ ($L_{\rm min} = 3.4 \times 10^{13}\,{\rm L}_\odot$). Note that the lower the average $f_{\rm Edd}$ the lower the luminosity of a quasar that hosts a $M > M_9$ black hole is. Therefore, decreasing $f_{\rm Edd}$ allows one to integrate the LF down to lower luminosities, thus increasing the overall $\Phi_{\rm RQ}(z, M > M_9)$. However, if we were to assume, say, $f_{\rm Edd} = 0.1$, then the range of luminosities where we compare radio-loud and radio-quiet quasars would differ, as in blazars the (observed) accretion disc component is always more luminous than several $\times\, 10^{46}$ erg s^{-1} (cf. Fig. 1).

Finally, if the SDSS misses quasars because of obscuration biasing optical selection (see below and Treister et al. 2011), then this mass function is in reality a lower limit, as more active black holes might exist.

We quantify the redshift evolution of the ratio of radio-loud versus radio-quiet sources in Figs 2 and 3. *We stress that up to $z = 4$, where we do see blazars, the cosmological evolution model, as derived by A09, is secure*. Beyond $z = 4$ it depends strongly on the assumed evolution. Since the 'minimal' evolution provides a lower limit to the number of radio-loud systems, we can be assured that *the radio-loud versus radio-quiet fraction remains at least close to constant, and near unity, up until $z \simeq 6$*. We find that the fraction of jetted sources increases from $z = 3.5$ to 4.5 by roughly an order of magnitude. Fig. 2 also shows that for $M > M_9$ and $L > 10^{13}\,{\rm L}_\odot$ ($L \gtrsim 10^{47}$ erg s^{-1}) the number density of radio-loud quasars approaches and possibly prevails over that of radio-quiet quasars, if we take face value the extrapolation of the cosmic evolution suggested by A09.

We further check our results via a comparison of the RLF that we derive from the FIRST+SDSS sample we uniformly selected. This RLF is shown in Fig. 3 (blue squares). We compare our estimate with the results for quasars of similar luminosities of Jiang et al. (2007), shown as grey triangles (their fig. 3, lower left-hand panel). The point at $z = 1.5$ is based on the *Fermi* blazars analysed in Ghisellini et al. (2011), in order to obtain a RLF at low redshift and maximize the range where we can compare our results to Jiang et al. (2007). The agreement between our selection in the SDSS+FIRST and Jiang's is excellent where the analyses overlap ($z \le 4$). A striking result we find that while at $z \le 2.5$ the 'parent population' of SDSS+FIRST radio-loud quasars traces almost perfectly the BAT blazars, assuming $\Gamma = 15$ and $f_{\rm Edd} = 1$, the two selections deviate at higher redshift.

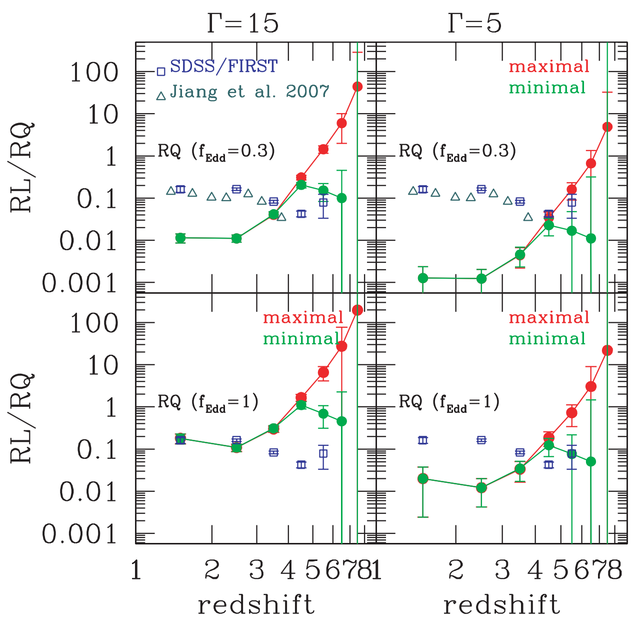

Figure 3. Ratio of the number density of radio-loud to radio-quiet quasars with black hole masses exceeding $10^9\,{\rm M}_\odot$. Blue and grey symbols refer to the ratio calculated for the SDSS sources (this work and Jiang et al. 2007, respectively), while green (lower set) and red (upper set) dots refer to the ratio of the radio-loud population inferred from the *Fermi* (at $z = 1.5$) and BAT blazars and the radio-quiet quasars as derived from the SDSS (H07). Red points have been calculated assuming the A09 cosmic evolution of blazars; green points correspond to the 'minimal' cosmic evolution of blazars suggested in G10. The radio-quiet population is estimated from the LF of radio-quiet quasars and its redshift evolution, from H07. Top panel: average Eddington fraction of radio-quiet quasars is $f_{\rm Edd} = 0.3$. Bottom panel: average Eddington fraction of radio-quiet quasars is $f_{\rm Edd} = 1$. Left-hand panels: $\Gamma = 15$. Right-hand panels: $\Gamma = 5$.

This analysis, which relies on a uniformly selected sample, does not require a volume complete sample, as we are now dealing with fractions. We still find a dearth of radio-loud sources at high redshift.

We stress once again that at $z < 2.5$ the blazar population, with $\Gamma = 15$, joined with the radio-quiet population with $f_{\rm Edd} = 1$, is in excellent agreement with SDSS+FIRST data (both our analysis and Jiang et al. 2007 analysis). At $z = 3.5$ the number density of blazars is derived from observed sources (no redshift extrapolation), and our only assumption is the value of Γ. We are therefore confident that there *must* be either a transition in the astrophysical properties of the population or a selection bias. Below we discuss some possibilities to explain the found discrepancy.

2.2 Where are radio-loud high-redshift quasars?

The problem we face is simply that BAT blazars indicate that there must be many radio-loud sources that instead the SDSS+FIRST survey does not detect. In the following we list possible solutions.

(i) The value $\Gamma \sim 15$ for the average bulk Lorentz factor of blazars is too large. A value of $\Gamma \sim 5$ would make the predicted numbers of misaligned radio sources to decrease by an order of magnitude (it is proportional to Γ^2), becoming then consistent with the SDSS+FIRST detected radio-loud sources. We have checked that fitting the spectral energy distribution of our blazars with $\Gamma = 5$ gives reasonable results, but this value of Γ cannot account for all the measured apparent superluminal velocity and, furthermore, it implies that the jet is away from the conditions of minimum jet power requirements (see Ghisellini & Tavecchio 2010). Since the discrepancy appears as redshift increases, we need that Γ evolves with cosmic time (Γ must decrease with increasing redshift).

A similar solution is that the jet has a radial velocity structure, similar to GRBs, thus emitting (at approximately the same level) within an angle larger than 1/Γ. We thus observe preferentially that part of the source exactly pointing at us (i.e. with a viewing angle close to zero) maximally beamed. The other slightly misaligned part of the source contribute less to the observed flux. This implies that we underestimate the jet power that refers to the part of the jet mostly contributing to the flux, the one pointing at us. The other parts, viewed at angles larger than 1/Γ, are not accounted for when estimating the jet power. In other words, we calculate the power of only a part of the jet, of solid angle $\sim 1/\Gamma^2$, which is smaller than the jet solid angle $\Omega_j \sim \theta_j^2$, where θ_j is the jet half opening angle. Therefore, very approximately, we underestimate the jet power by the factor $(\theta_j/\Gamma)^2$. To account for the disagreement in number, $(\theta_j/\Gamma)^2$ should be one order of magnitude.

(ii) There is a bias, in the SDSS+FIRST survey, against the detection of powerful radio-loud sources at high z, but not against radio-quiet quasars. This (yet unknown) bias could be due to the compactness of the radio halo in radio sources that are still too young to have developed an extended structure. As a consequence, the flux emitted in this compact and isotropic structure is self-absorbed up to the GHz frequency range. Further study is however needed to verify this possibility that now is only a speculation.

(iii) The SDSS+FIRST selection misses a large fraction of powerful quasars (both radio loud and radio quiet) at $z > 3$. This may be the result of optical absorption, or else of collimation of the optical emission of the disc (making the apparent disc luminosity much dimmer if the disc is observed edge on). An absorbed (or optically anisotropic) source intrinsically emitting more than 10^{47} erg s^{-1} in the optical (our selection limit) would not pass our selection. This would occur also for radio-loud objects, because, even if the radio emission is unaffected, the primary selection (SDSS) is on the optical luminosity.

The RLF could be right (if the bias applies equally to both kind of sources), but both classes are under-represented by an order of magnitude, at least at $z > 3$.

These possibilities are listed in order of increasing demands of heavy black holes at large redshifts. In fact, case (i) would minimize the total number of high z powerful radio-loud sources (and their associated heavy black hole) that would be well described by the SDSS+FIRST survey. The price to pay is a correspondingly larger requirement on the jet power of each quasar (i.e. we have a factor 10 less radio loud powerful sources, and then less heavy black holes, but each jet is 10 times more powerful). In this case the total number of heavy black holes at high z is practically given by the radio-quiet quasars (the radio loud contributing by less than 10 per cent).

Case (ii) is intermediate, since it implies that the number of powerful radio-loud sources is correctly described by the BAT blazars (with $\langle\Gamma\rangle \sim 15$) that should be as numerous as the radio-quiet sources (with $M > M_9$) detected by the SDSS. Therefore, the number of heavy and early black holes is roughly twice as much as the one derived by the radio-quiet quasars LF. The most important consequence in this case is the strong evolution of the RLF, becoming close to unity at $z > 3$, with interesting consequences on our understanding of the growth of the early supermassive black holes.

Case (iii) is the more demanding: it implies that the SDSS misses 90 per cent of the most powerful quasars (no matter their radio loudness) and thus that the number of heavy black holes is a whole order of magnitude more than the mass function derived from the H07 LF would predict.

In the following we will examine the current ideas about the relationship between dark matter haloes and black hole mass in order to estimate the expected number of heavy black holes as a function of redshift.

3 BLACK HOLE–DARK HALO CONNECTION

Empirical correlations have been found between the central stellar velocity dispersion and the asymptotic circular velocity (V_c) of galaxies (Ferrarese 2002; Baes et al. 2003; Pizzella et al. 2005):

$$\sigma = 200\,\mathrm{km\,s^{-1}} \left(\frac{V_c}{320\,\mathrm{km\,s^{-1}}}\right)^{1.35} \quad (2)$$

and

$$\sigma = 200\,\mathrm{km\,s^{-1}} \left(\frac{V_c}{339\,\mathrm{km\,s^{-1}}}\right)^{1.04} \quad (3)$$

as suggested by Pizzella et al. (2005) and Baes et al. (2003), respectively. Some of these relationships (Ferrarese 2002; Baes et al. 2003) mimic closely the simple $\sigma = V_c/\sqrt{3}$ definition that one derives assuming complete orbital isotropy. We note that in an isothermal sphere $\sigma = V_c/\sqrt{2}$.

Since the asymptotic circular velocity (V_c) of galaxies is a measure of the total mass of the dark matter halo of the host galaxies, one can relate in simple ways the mass of the central black hole to the mass of its host halo ('hole–halo' connection; e.g. Ferrarese 2002; Volonteri, Haardt & Madau 2003; Wyithe & Loeb 2003; Rhook & Wyithe 2005; Croton 2009). A halo of mass M_h collapsing at redshift z has a circular velocity

$$V_c = 142\,\mathrm{km\,s^{-1}} \left[\frac{M_h}{10^{12}\,\mathrm{M_\odot}}\right]^{1/3} \left[\frac{\Omega_m}{\Omega_m^z}\,\frac{\Delta_c}{18\pi^2}\right]^{1/6} (1+z)^{1/2}, \quad (4)$$

where Δ_c is the overdensity at virialization relative to the critical density. For a *Wilkinson Microwave Anisotropy Probe* 5-year data (*WMAP*5) cosmology we adopt here the fitting formula $\Delta_c = 18\pi^2 + 82d - 39d^2$ (Bryan & Norman 1998), where $d \equiv \Omega_m^z - 1$ is evaluated at the collapse redshift. In this case we obtain

$$\Omega_m^z = \frac{\Omega_m(1+z)^3}{\Omega_m(1+z)^3 + \Omega_\Lambda + \Omega_k(1+z)^2}. \quad (5)$$

We will further assume that the black hole–σ (M–σ) scaling is

$$\frac{M}{M_9} = \left(\frac{\sigma}{356\,\mathrm{km\,s^{-1}}}\right)^4 \quad (6)$$

(Tremaine et al. 2002). We also assume that these scaling relations observed in the local universe hold at all redshifts. Therefore, we derive relationships between black hole and dark matter halo mass:

$$\frac{M_h}{10^{13}\,\mathrm{M_\odot}} = 4.1\,(M/M_9)^{0.56} \left[\frac{\Omega_m}{\Omega_m^z}\,\frac{\Delta_c}{18\pi^2}\right]^{-1/2} (1+z)^{-3/2} \quad (7)$$

if we adopt the relationship in Pizzella et al. (2005);

$$\frac{M_h}{10^{13}\,\mathrm{M_\odot}} = 8.2\,(M/M_9)^{0.75} \left[\frac{\Omega_m}{\Omega_m^z}\,\frac{\Delta_c}{18\pi^2}\right]^{-1/2} (1+z)^{-3/2} \quad (8)$$

if we adopt $\sigma = V_c/\sqrt{3}$ (almost equivalent to what one would calculate using the relationship in Baes et al. 2003).

3.1 Black hole mass functions

We can therefore estimate the mass function of black holes by convolving equations (7) and (8) with the mass density of dark matter haloes with mass $M_{\rm h}$ derived from the Press–Schechter formalism (Sheth & Tormen 1999). The number density of black holes with $M > M_9$, $\Phi(z, M > M_9)$, therefore, corresponds to the number density of haloes with mass $M_{\rm h} > M_{\rm thr}$, if $M_{\rm thr}$ is the mass of a halo that hosts a billion solar masses black hole, and we assume that all haloes host black holes.

In Fig. 4 we show the mass functions derived via equations (7) and (8) coupled to the Press–Schechter function at $z = 0$ (bottom panel) and $z = 6$ (top panel). As an exercise, one can also assume that the black hole mass scales linearly with the halo mass. We can derive a plausible upper limit to this scaling assuming $M = f_{\rm b} 10^{-3} M_{\rm h}$, where $f_{\rm b} = \Omega_{\rm b}/\Omega_{\rm M} \simeq 0.14$ is the universal baryon fraction, and $M \simeq 10^{-3} M_{\rm bulge}$ is the empirical correlation between black hole and bulge mass in elliptical galaxies (Marconi & Hunt 2003; Häring & Rix 2004). This assumption corresponds to assuming that galaxies do not loose any baryon because of feedback effects, and all baryons end up in a stellar bulge. At $z = 0$ this relationship is obviously wrong (the baryon content in galaxies is much less than $f_{\rm b}$ and not all baryons end up in stellar bulges), and we find that $M = 10^{-5} M_{\rm h}$ provides a more acceptable solution, as shown in the upper panel of Fig. 4. At $z = 0$ the mass function of black holes is estimated in the literature by coupling the empirical correlations found between black hole mass and host properties (bulge mass, luminosity and velocity dispersion; see Marconi et al. 2004; Gültekin et al. 2009 and references therein) with the distribution of galaxies as a function of these properties. In the top panel of Fig. 4 we show with a vertical bar the current limits on the mass density of black holes with $M > M_9$ at $z = 0$. The 'hole–halo' connection coupled to the Press–Schechter function might possibly be slightly overestimating the number density of large black holes at $z = 0$. This is due to the LF of galaxies being steeper at the high-mass end than the halo mass function at the high-mass end.

We stress here that at least two biases do exist that affect the comparison between the 'hole–halo' connection and observational samples: first, Lauer et al. (2007a) suggest that the M–σ relation compared to M–L relation *underestimate* the number of black holes with $M > M_9$ (but see Bernardi et al. 2007; Tundo et al. 2007). Secondly, going in the same direction, the intrinsic scatter in the M–σ relation allows a larger number of possible haloes hosting massive black holes (see Lauer et al. 2007b; Gültekin et al. 2009). We discuss the importance of the intrinsic scatter in Section 3.3.

Finally, Kormendy & Bender (2011) question any correlation between black holes and dark matter haloes (but see Volonteri, Natarajan & Gultekin 2011). We notice that Kormendy's argument is not a concern here, as at large masses Kormendy, Bender & Cornell (2011) suggest that a 'cosmic conspiracy' causes σ and $V_{\rm c}$ to correlate, thus making the link between M and $V_{\rm c}$ adequate. Although estimates we derive from the halo–hole connection are therefore *extremely* uncertain, they can still provide some sense of the possible hosts of these massive black holes at high redshift.

3.2 Number density of high-redshift $M > 10^9$ M$_\odot$ black holes powering blazars

We now turn to compare the number density of $M > M_9$ and $L > 10^{13}$ L$_\odot$ ($L \gtrsim 10^{47}$ erg s^{-1}) blazars to the upper limits defined by 'hole–halo' connection. The various assumptions for the 'hole–halo' connection discussed in Section 3 are shown in Fig. 5. The number density of heavy black holes powering jetted sources is now close to or even greater (if we extend the cosmological evolution model of A09 beyond $z \sim 4$) than the upper limit defined by 'hole–halo' connections at the largest redshifts. The mass function derived by the 'minimal' LF is instead consistent.

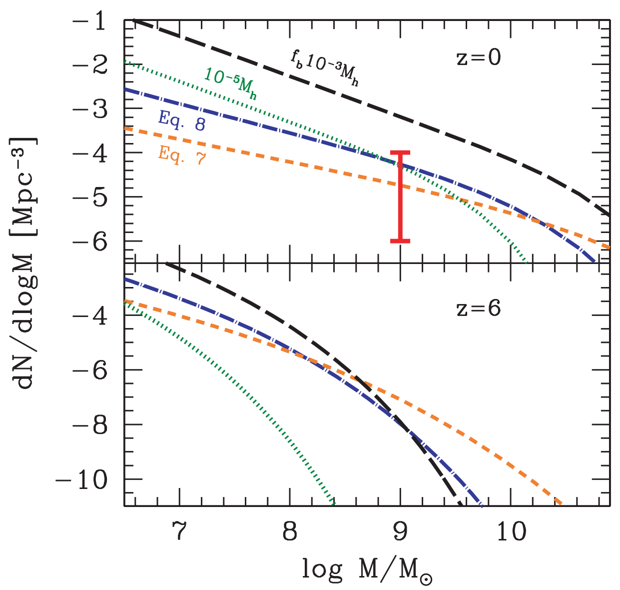

Figure 4. Mass functions of black holes at $z = 0$ (top) and at $z = 6$ (bottom). Vertical (red) bar: constraints at $z = 0$ (Gültekin et al. 2009). Dash–dotted (blue) curve: $\sigma = V_{\rm c}/\sqrt{3}$ (equation 8) + M–σ; short dashed (orange) curve: Pizzella (equation 7) + M–σ. As a reference, we also show the mass function obtained by assuming $M = f_{\rm bar}\, 10^{-3} M_{\rm h}$ (black long dashed curve) and the case $M = 10^{-5} M_{\rm h}$ (green dotted curve).

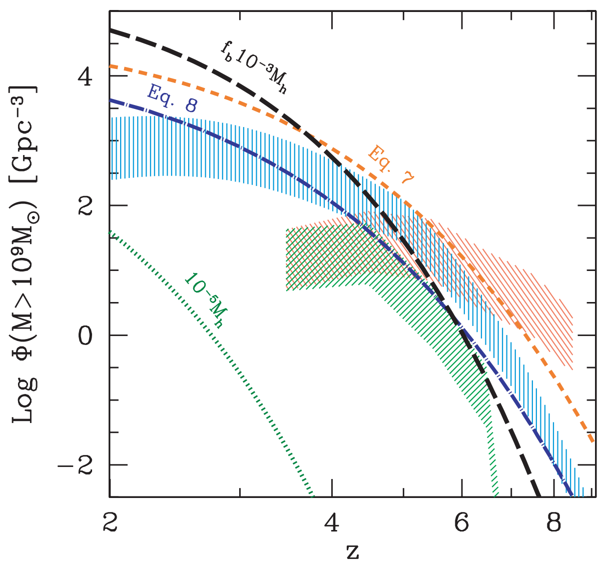

Figure 5. Number density of black holes with $M > 10^9$ M$_\odot$ as a function of redshift. Red hatched area: number density of black holes in radio-loud sources, derived from the blazar LF of A09, assuming $\Gamma = 5$ (lower bound) or $\Gamma = 15$ (upper bound). Green hatched area: 'minimal' number density (studied in G10), assuming with $\Gamma = 5$ (lower bound) or $\Gamma = 15$ (upper bound). Blue hatched area: black hole number density of radio-quiet quasars (from the LF and its evolution studied in H07). Line styles and colours of the 'hole–halo' connection predictions as in Fig. 4.

Despite the uncertainties, we can make some simple inferences on the 'hole–halo' connection at high redshift: the number density of $M > M_9$ black holes cannot be below the limit imposed by the number density of blazars derived from the 'minimal' evolution. For instance, it can be ruled out that at high redshift the black hole mass scales as $M = 10^{-5} M_{\rm h}$, if scatter is negligible. This implies that high-redshift black holes represent a higher fraction of the mass of a dark matter halo (at least, this is the case for the most massive active black holes, $M > M_9$ and $L > 10^{13}$ L$_\odot$, this might not be the case for lower mass holes, see e.g. the discussion in Willott et al. 2010). Including the effect of scatter eases such constraints, as we show below.

3.3 Importance of scatter

We can assume that at fixed σ the logarithmic scatter in black hole mass is $\Delta = 0.3$–0.5 dex ($M_{\rm BH} = M_{{\rm BH},\sigma} \times 10^{\Delta\delta}$, where δ is normally distributed, see e.g. Gültekin et al. 2009). We include scatter, at various levels of Δ, by performing a Monte Carlo simulation, where for each black hole mass we create 500 realizations of the host mass. Fig. 6 shows two examples of Monte Carlo realizations of the mass function and number density that include scatter at the level of $\Delta =$ 0.3–0.5 dex (Merloni et al. 2010). By comparing Fig. 6 to Fig. 5 one clearly sees how scatter dramatically increases the number of black holes with $M > M_9$, and for most 'hole–halo' connection the number density of high-redshift radio-loud sources can be accommodated. For instance, the M–σ relations can accommodate the radio-loud population as long as scatter is around 0.3 dex. Notice that in the logarithmic scale of Fig. 6 today's number density of $M > M_9$ black holes (red vertical bar in the upper panel of Fig. 4) is between 3 and 5 (grey stripe in Fig. 6), comparable to the number density at $z \simeq 5$ in the cases with significant scatter. For instance, if we assume the scaling of equation (8), the number density of $M > M_9$ black holes reaches 10^3 Gpc^{-3} at $z = 4$ for $\Delta = 0.3$ and $z = 7$ for $\Delta = 0.5$. If $\Delta = 0.5$ the upper limit to today's number density, 10^5 Gpc^{-3}, is reached at $z = 2$, implying that after that cosmic time the number density of $M > M_9$ black holes cannot increase any more.

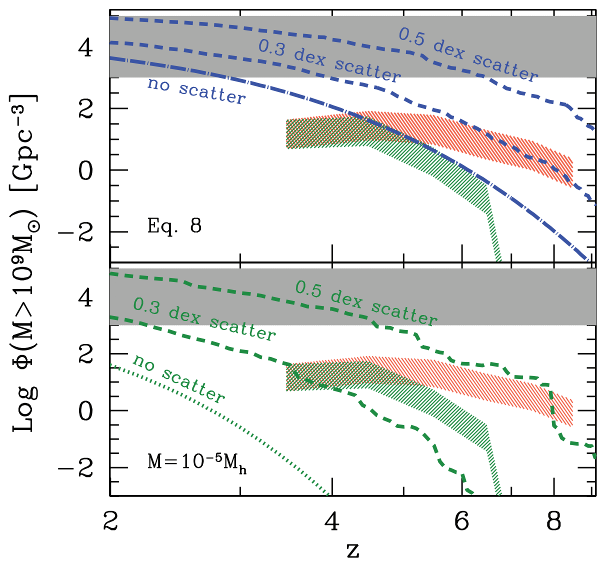

Figure 6. Importance of scatter for the prediction of the number density of black holes with $M > 10^9$ M$_\odot$ as a function of redshift. Top panel: the $\sigma = V_{\rm c}/\sqrt{3}$ (equation 8) + M–σ curve (blue dashed dotted) increases by assuming a scatter of 0.3 and 0.5 dex (dashed blue curves). Bottom panel: even the $M = 10^{-5} M_{\rm h}$ curve (green dotted) can become consistent with the LF of radio-loud sources (hatched areas) assuming a scatter of 0.5 dex. The grey shaded area in both panels indicate the limits on today's number density of $M > M_9$ black holes.

We have hitherto assumed that the duty cycle, $x_{\rm dc} = 1$, corresponding to the fraction of black holes that are active (related to the ratio of the lifetime of quasars to the Hubble time), i.e. all haloes host an active black hole. We have further assumed that radio-loud quasars dominate at high redshift. If the active fraction $x_{\rm dc}$ is less than unity, implying that not all black holes with $M > M_9$ are active and accreting close to the Eddington rate, then the requirements become stricter. Similarly, if radio-quiet quasars dominate the population of active black holes, their number has to be accounted for as well. In the next section we expand our analysis to include radio-quiet sources and allow for an active fraction, or duty cycle, below unity.

3.4 Active fraction of high-redshift $M > 10^9$ M$_\odot$ black holes

Until now we have focused only on the constraints that high-redshift blazars impose on the number density of $M > M_9$ black holes. However, this is clearly a lower limit to the number of massive black holes that have to exist at high redshift, as we have to take into account radio-quiet sources, for instance including optically selected quasars as described in Section 2.1. If the bolometric LF of radio-quiet sources (H07) is a good tracer of the quasar population, then there are roughly 0.1 radio-loud quasars for each radio-quiet one (assuming the minimal evolution of the LF of blazars), up to one or more, if we take the evolution of A09 face value.

However, based on the arguments of Sections 2.1 and 2.2, there might be a large population of radio-quiet quasars that are not accounted for in the bolometric LF of radio-quiet sources of H07 (see case iii in Section 2.2). Since we have no information on this putative hidden population we do not here include this speculation in our analysis, but if in reality most quasars are missed by current surveys, then all problems we discuss below are exacerbated.

We can assess the requirements on the population of high-redshift black holes hosts by investigating the fraction of $M > M_9$ black holes that are actively accreting, i.e. the active fraction of black holes. We define the active fraction as

$$x_{\rm dc}({\rm RL}) = \frac{\Phi_{\rm RL}(z, M > M_9)}{\Phi(z, M > M_9)} \qquad (9)$$

for radio-loud blazars only, and

$$x_{\rm dc}({\rm RL+RQ}) = \frac{\Phi_{\rm RL}(z, M > M_9) + \Phi_{\rm RQ}(z, M > M_9)}{\Phi(z, M > M_9)} \qquad (10)$$

for all active sources, where $\Phi_{\rm RL}(z, M > M_9)$ and $\Phi_{\rm RQ}(z, M > M_9)$ are defined in Section 2. $\Phi_{\rm RQ}(z, M > M_9)$ depends on one parameter, the typical Eddington ratio of radio-quiet black holes. $\Phi(z, M > M_9)$ is defined in Section 3.1, and depends on the 'hole–halo' connection and on the level of scatter that this relationship suffers. The numerator and denominator in both equations (9) and (10) are derived independently, hence a priori the active fraction can apparently assume values above unity. When $x_{\rm dc} > 1$ the result is however unphysical, and it allows us to rule out a given model.

The active fraction is shown in Fig. 7 for $f_{\rm Edd} = 1$, 0.3 and negligible scatter in the 'hole–halo' connection. We here show both the total active fraction, $x_{\rm dc}$(RL+RQ), and the active fraction in radio-loud sources only, $x_{\rm dc}$(RL). In both cases we have assumed $\Gamma = 15$. Face value, by $z \simeq 5$ almost all 'hole–halo' connections, except for the case of equation (7), require an active fraction (or a duty cycle) of unity. A significant amount of scatter might however

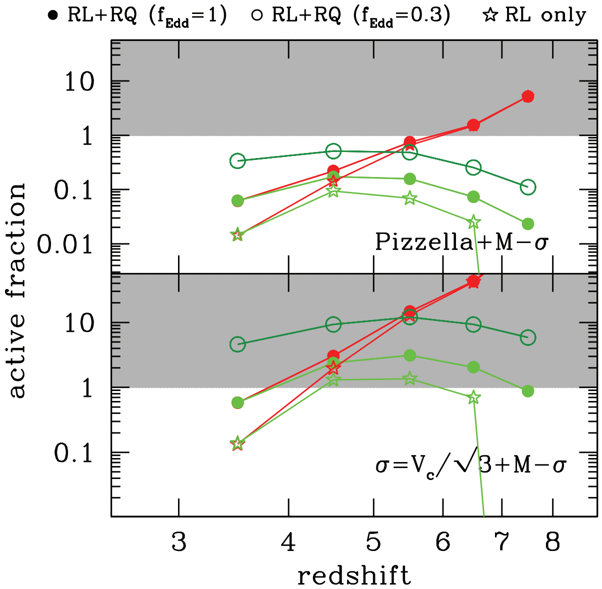

Figure 7. Fraction of $M > 10^9\,M_\odot$ that is active. Red: A09 cosmic evolution inferred from the LF of blazars (this curve is basically unaffected by different choices of f_{Edd} as radio-loud quasars dominate the active population). Green: 'minimal' cosmic evolution inferred from the LF of blazars. Stars: radio-loud systems only. Filled circles: all active quasars, where we assumed $f_{Edd} = 1$. Empty circles: all active quasars, where we assumed $f_{Edd} = 0.3$. Bottom: $\sigma = V_c/\sqrt{3} + M$–$\sigma$. Top: Pizzella + M–σ. In all cases we ignore scatter in the M–σ relation. The grey shaded area is not permitted as the active fraction becomes larger than unity.

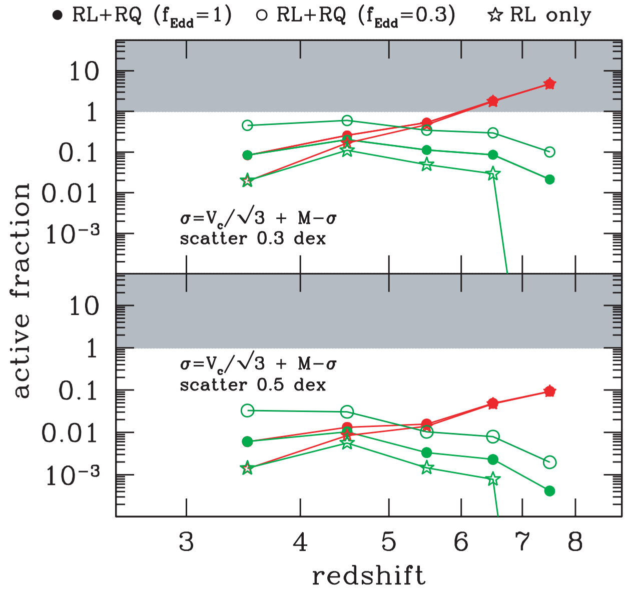

Figure 8. Same as Fig. 7, for the case $\sigma = V_c/\sqrt{3} + M$–$\sigma$, but including scatter.

alleviate the issue, as shown in Fig. 8. As discussed above, scatter increases very significantly the number density in $M > M_9$ black holes, and pushes the active fraction to much lower values, at the cost however of having already built up almost all $M > M_9$ black holes by $z \simeq 5$ (cf. Fig. 6).

Finally, if black holes accreted at rates significantly below the Eddington rate, the estimate of the number density of radio-quiet quasars that we derive from the LF would increase (Section 3). This decreases the RLF (see Fig. 5), but at the same time the *total* active fraction increases, and the active fraction becomes close to unity even for a significant scatter of 0.3 dex.

4 DISCUSSION AND CONCLUSIONS

We investigated the relative occurrence of radio-loud and radio-quiet quasars in the first billion years of the Universe, based on the sample of high-redshift blazars detected in the 3-year all sky survey performed by the BAT onboard the *Swift* satellite (A09). The masses M of the black holes powering these quasars exceeds a few billions solar masses, with accretion luminosities being a large fraction of the Eddington limit (G10). For each blazar pointing at us, there must be hundreds of similar sources (having black holes of similar masses) pointing elsewhere. This can set constraints on the number density of dark matter haloes that can host massive black holes at high redshift.

We first compared the number of radio sources hosting heavy black holes estimated from the BAT detected blazars to the SDSS+FIRST survey to explore the relative importance of (jetted) radio-loud versus radio-quiet sources. We find a rough agreement up to $z \sim 3$, but beyond this redshift there is a deficit of radio sources (detected by the FIRST and present in the SDSS surveys) with respect to the expectations (see also Haiman et al. 2004; McGreer et al. 2009). We found no obvious explanation for this deficit, and have suggested three possibilities: (i) the bulk Lorentz factor of the jet (controlling the number of misaligned sources) decreases beyond $z = 3$ (from $\Gamma \sim 15$ to $\sim$5); (ii) there is a bias against detecting distant (and therefore possibly young) radio sources with the FIRST survey (i.e. at 1.4 GHz) and (iii) there is a bias against optical selection of distant and powerful quasars, both radio quiet and radio loud, due to absorption or collimation of the disc emission. These possibilities affect our estimates of the number density of heavy black holes ($M > 10^9\,M_\odot$) in an increasing way (from possibility i to iii).

In the first case the majority of quasars are radio quiet at all redshifts, and the number density of high-redshift $M > 10^9\,M_\odot$ black holes can be safely derived from the observed LF of radio-quiet quasars. In the second case radio-loud and radio-quiet quasars powered by heavy black holes become comparable in number beyond $z \sim$ 3–4, doubling the number of heavy and high-z black holes estimated from radio-quiet quasars only. This would also mean that the RLF increases with z for sources with heavy black holes, suggesting that that a radio phase is perhaps a necessary ingredient for fast black hole growth at early cosmic times. The last possibility is the most demanding, since it implies that we see only a minor fraction of the intrinsically luminous high-z quasars (both radio loud and radio quiet), implying that although the RLF is always of the order of 10 per cent (i.e. at all redshifts), the number of heavy black holes is now severely underestimated (by one order of magnitude).

We re-iterate that there is a good agreement between the number density of $M > 10^9\,M_\odot$ black holes found with blazars and the total number of radio-loud quasars up to $z \sim 3$, but not beyond. We then conclude that even if we do not know the cause for this disagreement, there must be some change occurring at $z \sim 3$.

We then studied plausible 'hole–halo' connections in order to predict the number of supermassive black hole at high redshifts. We found that, unfortunately, the predicting power of these relations is weak, mainly because of the large effect that scatter can have: since we are dealing with fastly declining functions (corresponding to the high end of the distributions of luminosities and/or black hole masses), it is possible that even a few large black hole inhabiting halo slightly less massive than implied by the adopted relation can dominate the number density at a given redshift.

On the other hand, despite the rather large uncertainties, the sources that have been already observed (cf. Section 2) suggest that large and distant black holes are all active (or nearly so), and that they are all Eddington limited (or nearly so). If not, the number of heavy black holes would be larger, and the simple theoretical ideas we have adopted in this paper would start to have some difficulties to account for them.

Finding more luminous radio-quiet quasars at $z > 5$ is obviously mandatory to study the high-mass end of the black hole mass density, but we would like to stress that finding high-redshift blazars might be, in the end, even more important, since each one of those implies the existence of many more misaligned sources. A few blazars detected at $z \sim 6$ would be very challenging for structure formation, very constraining and possibly illuminating for the understanding the early growth of very massive black holes and its feedback on the host. The existence of these blazars possibly implies that normal 'feedback' might not be at play at the highest redshifts. A possible explanation is that high accretion rate events, distinctively possible during the violent early cosmic times, trigger the formation of collimated outflows that do not cause feedback directly on the host, which is pierced through. These jets will instead deposit their kinetic energy at large distances, leaving the host unscathed. This is likely if at large accretion rates photon trapping decreases the disc luminosity, while concurrently the presence of a jet helps dissipating angular momentum, thus promoting efficient accretion. This picture may explain why high-redshift massive black holes can accrete at very high rates without triggering self-regulation mechanisms.

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