

MASS PREDICTIONS, PARTIAL DIFFERENCE EQUATIONS
AND HIGHER-ORDER ISOSPIN EFFECTS

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ABSTRACT

The Garvey-Kelson mass relation has been extended by introducing inhomogeneous source terms to improve problems with long-range extrapolations. Such mass relations are third-order partial difference equations with solutions representing mass equations. It was found that inhomogeneous source terms based on shell-dependent Coulomb and symmetry energy terms are not sufficient to improve upon extrapolations. However, contributions from higher-order perturbations in isospin (mostly cubic) have a significant effect. A many-parameter mass equation was constructed as the solution of an inhomogeneous difference equation with properly adjusted shell-dependent source terms. The standard deviation for reproducing the experimental mass values is $\sigma_m = 194$ keV. Nuclear contributions were subjected to the constraint of charge symmetry, and Coulomb displacement energies are reproduced with $\sigma_c = 41$ keV. Mass predictions for over 4000 nuclei with $A > 16$ and both $N > Z$ and $N < Z$ (except $N = Z = \text{odd}$ for $A < 40$) are reported.

Mass relations are recursion relations which can be used to predict unknown masses. When viewed as partial difference equations they can also be used to construct mass equations. The Garvey-Kelson relation¹ has been very successful to predict masses and binding energies of nuclei close to the known nuclei. However, long-range extrapolations display systematic effects. Figs. 1 and 2 show systematic deviations when a band of nuclei along the line of β -stability is used as data base to predict the masses of the known nuclei outside this band. Fig. 1 displays the average deviations for bands of different widths. Cubic contributions in isospin are suggested. Fig. 2 displays the individual residuals for one specific example. Coulomb and symmetry energy contributions to nuclear binding energies are known to generate inhomogeneous source terms.^{2,3} Detailed shell-model expressions were therefore derived and included in the mass equation. Fig. 3 shows again the comparison between the experimental and predicted mass values outside the data base along the line of β -stability. Only minor improvements are observed. It was therefore concluded that shell-dependent terms in isospin of the type $f(A)(T - T_{\text{stab}})^3$ must be included in the description of the symmetry energy. Such terms arise from higher-order perturbations in isospin due to subshell mixing, core excitations or departures from simple coupling schemes and also indirectly from deformation effect. Fig. 4 shows a graphical representation of these higher-order shell-dependent contributions which were constructed from the data taking

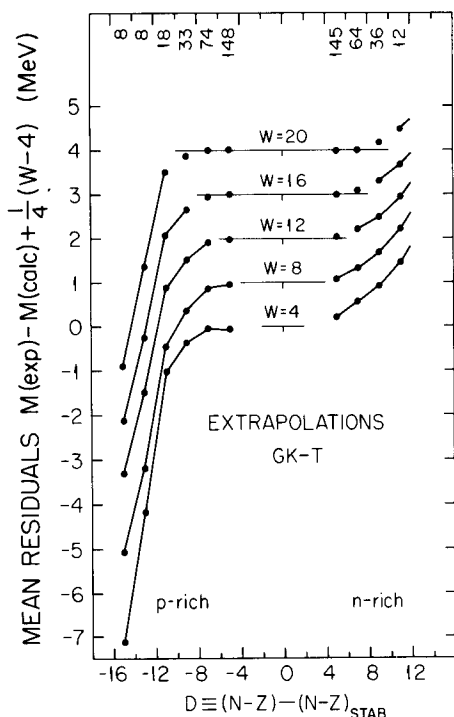


Fig. 1. Differences between experimental and extrapolated masses as function of the distance from the line of β -stability averaged over all mass numbers A . The horizontal bars (displaced) represent the widths of the data bases.

into consideration the combined effects of the homogeneous and inhomogeneous parts of the partial difference equation. Fig. 5 shows greatly improved extrapolations when these higher-order terms are included. The solutions were subjected to the constraint of charge symmetry of nuclear forces. They are therefore valid for nuclei with both $N > Z$ and $Z > N$, and the accuracy of the Coulomb energies ($\sigma_c = 41$ keV for the Coulomb displacement energies⁴) is preserved. The final many-parameter mass equation (see Ref. 5 for details) was derived using all available data as input. The standard deviation for reproducing the data is $\sigma_m = 194$ keV, and mass predictions for over 4000 nuclei with $A > 16$ were made.

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1. G. T. Garvey, W. J. Gerace, R. L. Jaffe, I. Talmi and I. Kelson, *Rev. Mod. Phys.* **41**, S1 (1969).
2. J. Jänecke, in *Atomic Masses and Fundamental Constants 7*, ed. O. Klepper (THD Schriftenreihe 26, Darmstadt 1984), p. 420.
3. J. Jänecke and E. Comay, *Nucl. Phys.* **A346**, 108 (1985).
4. E. Comay and J. Jänecke, *Nucl. Phys.* **A410**, 103 (1983); J. Jänecke and E. Comay, *Phys. Lett.* **140B**, 1 (1984).
5. J. Jänecke and P. J. Masson, *Atomic Data Nucl. Data Tables* (1987); P. J. Masson and Jänecke, *ibidem*.

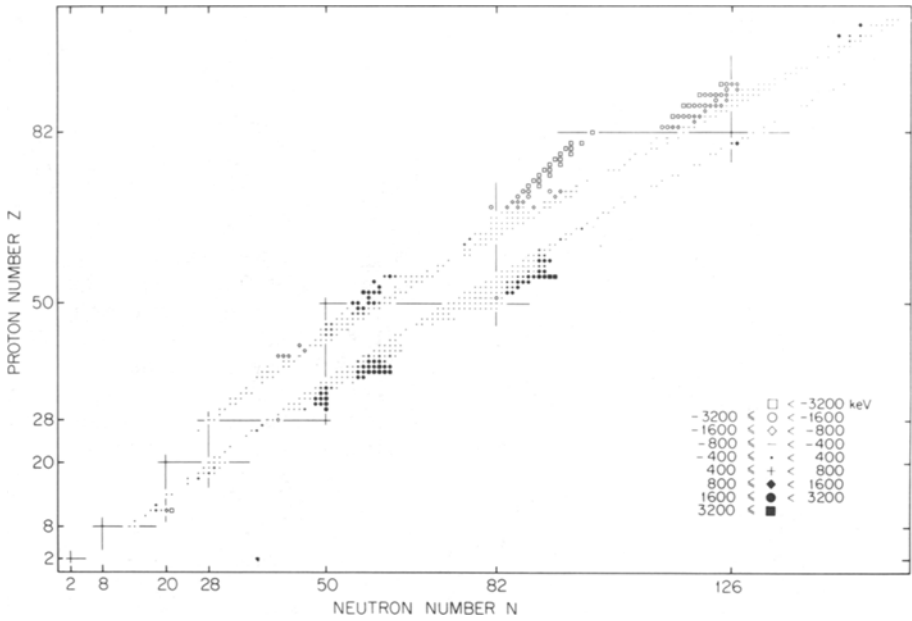


Fig. 2. Differences between experimental and calculated (extrapolated) masses for the transverse Garvey-Kelson relation. Only the empty band of nuclei along the line of β -stability is used as data base in the fitting procedure. (See also Figs. 3 and 5.)

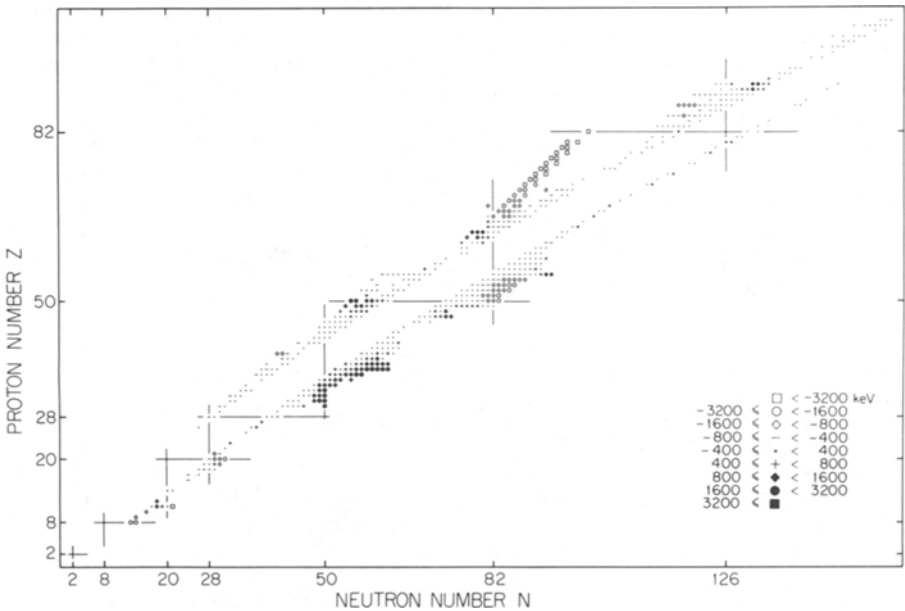


Fig. 3. Differences between experimental and calculated (extrapolated) masses. Shell-dependent Coulomb and symmetry energy terms (quadratic in isospin) are included as inhomogeneous source terms.

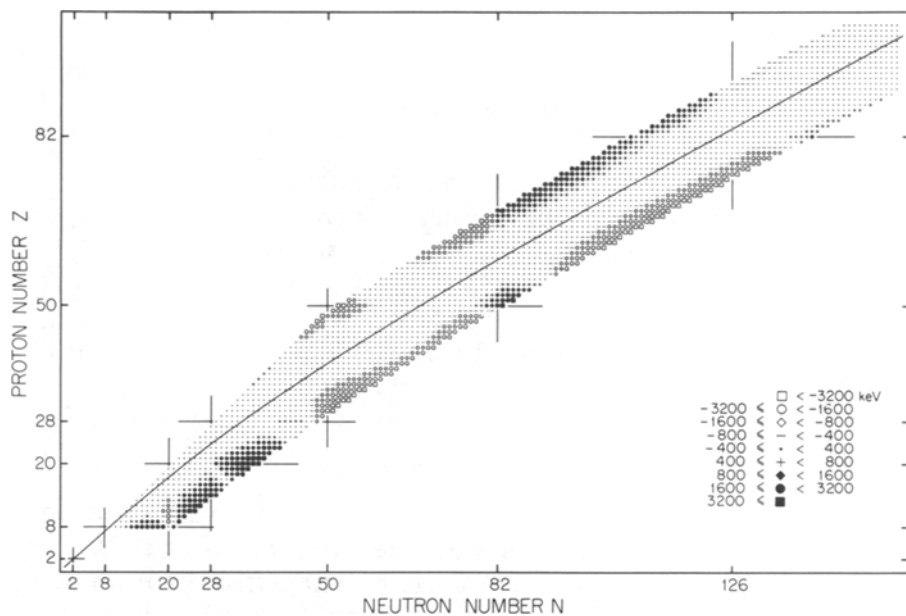


Fig. 4. Shell-dependent symmetry-energy contributions of higher order in isospin ($T > 2$).

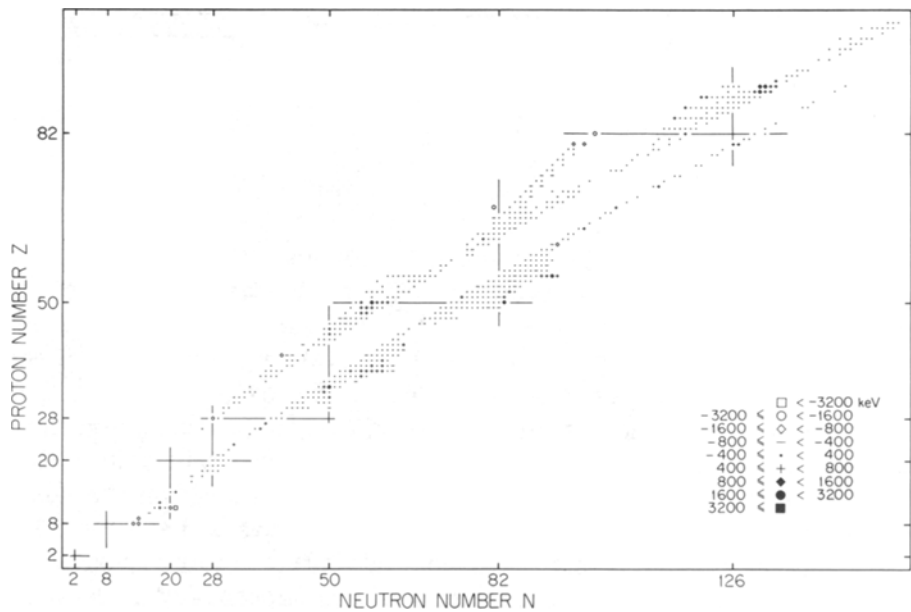


Fig. 5. Differences between experimental and calculated (extrapolated) masses. The inhomogeneous source terms include those used in Fig. 3 and the higher-order terms of Fig. 4.