Braneworlds and Large Extra Dimensions

James T. Liu

Michigan Center for Theoretical Physics
Randall Laboratory, Department of Physics, University of Michigan
Ann Arbor, MI 48109–1120, USA

Abstract. The idea of extra dimensions has always played an important rôle in string theory. However, until recently, most of the interest has been on small extra dimensions. With the introduction of braneworlds, this view has been turned around, and many investigations have been made of incorporating large extra dimensions into string theory. Here I provide a brief survey of how such large dimensions may arise from string theory, and additionally discuss some experimental signatures for extra dimensions.

1. INTRODUCTION

Since the time when Oskar Klein proposed the idea of a fifth dimension, the notion that we may live in a universe with extra dimensions has been an especially fascinating and important one. From a modern perspective, extra dimensions arise naturally in M-theory, which encompasses both eleven dimensional spacetimes and ten dimensional strings. Although traditionally, extra dimensions in M-theory were intended to be compactified at a very small scale, and hence were too small to be observed, much recent excitement has focused on an intriguing possibility of our universe having large (and even infinite) extra dimensions.

Although Klein's initial proposal for the fifth dimension was intended to unify electromagnetism with gravity, this goal still remains elusive, as gravity physically appears quite different from the other three forces. However, this led Arkani-Hamed, Dimopoulos and Dvali (ADD), to propose that gravity is in fact different in that it lives in a higher dimensional bulk, whereas the matter fields are constrained to live on a 3-brane [1, 2]. In such a braneworld scenario, the extra dimensions, while large, remain compact. Thus it was a rather surprising result of Randall and Sundrum when they demonstrated that gravity can be trapped on a 3-brane, even with an infinite extra dimension [3, 4].

This Randall-Sundrum braneworld evades conventional Kaluza-Klein lore by using a warped compactification, where the 3-brane directions (our universe) and the extra dimensions are no longer treated independently in the metric. While this model appears initially unrelated to M-theory, it has now been shown that there is a close connection between the Randall-Sundrum braneworld and AdS/CFT duality [5, 6, 7] in M-theory. Nevertheless, the precise connection between the models is not yet clear, although much has been written on the subject. In any case, this model at the very least hints at new approaches to string compactifications yielding large extra dimensions.

In this review, I would like to make the case that string theory naturally accommodates

CP655, *Particle Physics and Cosmology: Third Tropical Workshop–Neutrinos, Branes and Cosmology*, edited by J. F. Nieves and C. N. Leung

the possibility of large extra dimensions, and furthermore that the study of braneworld models may in fact yield new results pertinent to string theory.

2. LARGE EXTRA DIMENSIONS IN STRING THEORY?

When the notion of a ten-dimensional spacetime arose in string theory, it forced the community to confront the question of how the very nature of spacetime arises, and why our universe appears to have one time and three space dimensions. While a natural reaction would be to hide the extra dimensions through compactification, this possibility perhaps raises as many questions as it was meant to address. Is there a dynamical reason for precisely six of the ten dimensions to become very small, while the remaining four grow large? Or, even taking this as granted, what mechanism selects the proper vacuum out of all the possibilities? Compactifications even exist without any obvious geometric interpretation of what would be the extra dimensions.

Investigating large extra dimensions may provide new approaches to addressing such questions. While it is not yet clear what the outcome of this program will be, nevertheless many novel features have already been uncovered from the investigation of braneworld models.

To see how large extra dimensions may arise in string theory, we first consider a traditional compactification. For a model with n extra dimensions, we may take a direct product space of the form

$$\mathscr{M}_{4+n} = M^{1,3} \otimes K_n, \tag{1}$$

where $M^{1,3}$ represents our spacetime, while K_n is a *compact* internal manifold. This direct product form is sufficient to determine a generic relation between the coupling strengths of the original $(4+n)$-dimensional theory and the 4-dimensional one. Consider

$$S \sim \int d^4x d^ny \left[-\frac{1}{4g_{4+n}^2} F_{\mu\nu}^2 \right] = \int d^4x \left[- \left(\int d^ny \frac{1}{4g_{4+n}^2} \right) F_{\mu\nu}^2 + \cdots \right]. \tag{2}$$

Identifying the term in parenthesis with a four-dimensional effective coupling yields

$$\frac{1}{g_4^2} = \frac{1}{g_{4+n}^2} \mathrm{Vol}(K_n). \tag{3}$$

In addition, the four-dimensional spectrum would typically be composed of a set of zero modes (light fields), plus a tower of Kaluza-Klein modes, with masses on the order of $1/R$, where R is a typical size of K_n.

Consider the case of a heterotic string compactified on a six-torus, T^6. While this is not particularly realistic, it provides a simple model with many of the important features of a more involved compactification. The ten-dimensional string parameters include:

$$\begin{array}{ll} M_s = (2\pi\alpha')^{-1/2} & \text{String scale,} \\ g_s = \langle e^{\phi} \rangle & \text{String coupling constant,} \end{array} \tag{4}$$

while the four-dimensional parameters of interest are instead:

$$\begin{array}{ll} M_{\rm pl} \approx 10^{19}\,\text{GeV} & \text{Planck mass,} \\ g_{\rm YM} & \text{gauge coupling constant.} \end{array} \tag{5}$$

As indicated above, these parameters are related through the volume of the compact manifold. Assuming for simplicity a single radius parameter, R, so that the compactification volume is $V_6 = R^6$, we may use the Kaluza-Klein relations

$$M_{\rm pl}^2 = \frac{1}{g_s^2}(M_s R)^6 M_s^2, \qquad \frac{1}{g_{\rm YM}^2} = \frac{1}{g_s^2}(M_s R)^6, \tag{6}$$

to obtain

$$M_s = g_{\rm YM} M_{\rm pl} \approx 10^{18}\,\text{GeV}, \tag{7}$$

$$\frac{1}{R} = M_s (g_{\rm YM}/g_s)^{1/3} \approx M_s \approx 10^{18}\,\text{GeV}. \tag{8}$$

Here, we have assumed that the couplings remain perturbative, $g_{\rm YM} \sim g_s \sim \mathcal{O}(1)$.

As a result, both the string scale and the compactification scale are necessarily located near the Planck scale. This is the conventional perturbative string picture with small extra dimensions. If this were the only possibility, then it would be difficult to foresee the importance of large extra dimensions to string models. However, there is a way out, and that is to incorporate different effective dimensions for matter (including gauge fields) and gravity.

For example, in a D-brane model, gauge fields arise from open strings, and live on the world-volume of the D-brane, while gravity fields are composed of closed string states, and propagate in the bulk. D3-branes, in particular, are described by four-dimensional super-Yang-Mills fields with coupling $g_{\rm YM}^2 = g_s$. Although these gauge fields are already four-dimensional, gravity must still be compactified on a six dimensional manifold. Thus, for this D3-brane toy model, the Kaluza-Klein relations (6), now read instead

$$M_{\rm pl}^2 = \frac{1}{g_s^2}(M_s R)^6 M_s^2, \qquad \frac{1}{g_{\rm YM}^2} = \frac{1}{g_s}. \tag{9}$$

These relations are no longer sufficient for determining the string scale. In particular, this example allows the possibility

$$M_s \approx 10^3\,\text{GeV}, \tag{10}$$

$$\frac{1}{R} \approx 10^{-2}\,\text{GeV} \qquad (\approx 20\,\text{fm}). \tag{11}$$

So large extra dimensions may exist for the gravity sector of string models. Having such large dimensions has the added consequence of bringing down the fundamental (string) scale to as low as the TeV range. This feature was extensively promoted as a means of solving the hierarchy problem (although, in a way, the problem has instead been shifted to that of stabilizing the size of the extra dimensions).

3. SOME SIGNATURES FOR LARGE EXTRA DIMENSIONS

The presence of extra dimensions naturally lead to deviations from four-dimensional behavior at distances shorter than the compactification scale. For extra dimensions near the Planck scale, this is essentially unimportant, as the dimensions are very well hidden. However, for large extra dimensions, many stringent bounds may arise.

A natural signature of extra dimensions would be deviations from the inverse square force law at short distances; one would expect $1/r^{2+n}$ forces at distances shorter than the size R of the extra dimensions (here n is the number of 'large' extra dimensions). Experimentally, this is quite constraining on gauge fields, as no deviations from Coulomb behavior have been observed up to accelerator energies ('small' extra dimensions are of course still allowed). However, for gravity, this is an entirely different situation. Until recently, laboratory experiments have only been able to probe Newtonian behavior of gravity down to centimeter or so scales, thus leaving potentially large dimensions open for gravity.

Motivated by the ADD picture of large extra dimensions for gravity, several torsion balance tests of the inverse square law have since been performed. The current limit is $200\,\mu$m (or $150\,\mu$m for $n=2$) for the size of gravitational extra dimensions [8, 9] (see also Ref. [10]). The implications of this may be seen by considering the relation between the size R of the extra dimensions and the fundamental scale, which we denote M_s (we always take $M_{\text{pl}} \approx 10^{-19}\,$GeV to be the four-dimensional Planck scale). For a model with n extra toroidal dimensions, we have $M_{\text{pl}} \approx (M_s R)^{n/2} M_s$, or

$$R \approx \frac{1}{M_{\text{pl}}} \left(\frac{M_{\text{pl}}}{M_s} \right)^{1+2/n}. \tag{12}$$

Once we fix n, this then relates M_s with R.

Motivated by the desire to remove unnatural hierarchies, we may consider setting $M_s \approx M_{\text{electroweak}} \approx 1\,$TeV. This results in extra dimensions of sizes $10^{10}\,$km (for $n=1$) to $10\,$fm (for $n=6$). Clearly having one extra dimension in this case is absurd. However, for $n=2$, we find $R \approx 1\,$mm, which is right in the range of the above experimental limit. It is for this reason that the case of two large extra dimensions has attracted widespread attention. However, it should be noted that nothing *a priori* forces M_s to be at the TeV scale. Thus even the single extra dimension case cannot be ruled out, provided one chooses M_s accordingly.

For models with gauge fields living on a single brane and gravitons in the bulk, their phenomenology may generally be understood in terms of the conventional (or supersymmetric) Standard Model coupled to gravity plus a tower of Kaluza-Klein gravitons. Since couplings to the KK tower occur with (four-dimensional) gravitational strength, one may at first be tempted to ignore their presence. However, for large extra dimensions, the KK level spacing is extremely tiny. Hence there are an enormous number of KK modes, and this large number effectively compensates for their weak coupling. Typically, the effects of the KK modes may be felt at the scale M_s, which would not be surprising, as that is after all the fundamental scale of the underlying theory. Signatures for extra dimensions thus involve the detection of the effects arising from KK gravitons, and for the most part may be divided among gravity experiments (mentioned above), collider signatures and astrophysical (and cosmological) observations.

3.1. Collider signatures

Standard Feynman rules for the propagation and interaction of KK gravitons have been worked out in Refs. [11, 12]. Looking for large extra dimensions at colliders is then a matter of identifying the most promising channels for observation of the effects of the KK gravitons.

Perhaps the most dramatic signature four extra dimensions would be the direct production of KK gravitons, which then escape into the bulk. This shows up as missing energy, and typical signatures at LEP include [11, 12, 13]

$$e^+e^- \to \gamma + \text{(missing)}, \qquad e^+e^- \to Z + \text{(missing)}. \tag{13}$$

Similarly, at hadron colliders, one may look for

$$q\bar{q} \to \text{jet} + \text{(missing)}. \tag{14}$$

For two extra dimensions, the combined LEP limit is of the order [14, 15, 16, 17, 18]

$$M_s > 1.45\,\text{TeV} \quad \text{or} \quad R < 2.3 \times 10^{-4}\,\text{m} \tag{15}$$

(care must be taken when comparing results, as various conventions have been used).

In addition, one may search indirectly for the effects of virtual exchange of KK gravitons [11, 12, 19]. Since the KK gravitons carry spin-2, as opposed to ordinary spin-1 gauge bosons, useful signatures may especially be obtained from the angular distribution of the final state particles. Corrections to Bhabha scattering at LEP, $e^+e^- \to e^+e^-$, are particularly important, although searches have also been conducted in the $e^+e^- \to \gamma\gamma$ channel [20, 21]. The indirect limits from LEP are of the order [22, 14, 16, 23]

$$M_s > 1.13\,\text{TeV} \tag{16}$$

(again for two extra dimensions). Searches have also been conducted both at HERA [24] and at the Tevatron [25].

3.2. Astrophysical bounds on extra dimensions

Turning to the astrophysical realm, there is a danger that KK gravitons may be copiously produced in supernova explosions, and as a result carry off too large a fraction of the energy. Most of the energy from the supernova is expected to be carried away by neutrinos escaping the core. Since essentially the expected flux was observed for SN1987A, this leaves little room for the KK gravitons [26]. For two extra dimensions, the bounds are [26, 27, 28]

$$M_s > 50\,\text{TeV} \quad \text{or} \quad R < 3 \times 10^{-7}\,\text{m}. \tag{17}$$

Additional astrophysical limits involving observed neutrino fluxes have been investigated in Ref. [29].

In any case, assuming supernova explosions produce a cloud of KK gravitons, it turns out that a large fraction of these particles may remain in the vicinity of the resulting neutron star. As a result, their decay products may overly heat the neutron star, essentially preventing it from cooling to observed luminosities. The analysis of Ref. [30] yields (for $n = 2$)

$$M_s > 1700\,\mathrm{TeV}, \tag{18}$$

although several uncertainties enter such an analysis. Regardless, this essentially rules out the phenomenologically interesting range of M_s for the $n = 2$ model.

It is also worth noting that, with a low fundamental string scale, quantum gravity effects become important at accessible energies. This would lead to potentially dangerous effects such as black hole catalyzed proton decay [31] (which however can be avoided via several mechanisms). Furthermore, one must consider seriously the possibility that black holes would be copiously produced at the TeV scale. In addition to production in cosmic ray events, much has been made of the possibility of creating black holes at the LHC. Such black holes, if produced, would decay rapidly by Hawking radiation, giving rise to potentially visible events. Although production cross sections are not under complete control, reasonable estimates may be made; see *e.g.* Refs. [32, 33, 34, 35].

Bounds on extra dimensions may also be obtained from cosmology. For example, the decays of the KK gravitons in the early universe may affect the cooling rate of the cosmic microwave background [36]. Decays of KK gravitons may also contribute to the cosmic diffuse gamma ray background; observations yield an bound of approximately [37]

$$M_s > 100\,\mathrm{TeV}. \tag{19}$$

Putting together these above constraints, we find that, for three or fewer extra dimensions, the fundamental scale must be sufficiently high above the electroweak scale that this model loses some of its original attractiveness. However, this is really only one of the the simplest frameworks for incorporating large extra dimensions.

4. BRANEWORLD GENERALIZATIONS

Most of the experimental limits given above have been given in terms of toroidal extra dimensions. However it is certainly possible to replace T^n by a more general internal manifold. One possibility, for example, is the model of compact hyperbolic dimensions of Refs. [38, 39, 40]. By changing the geometry of the internal manifold, one may modify the relation (12) sufficiently to relax the conditions on having a TeV fundamental scale.

Another possibility would be, for example, to have an intersecting brane model, with different gauge groups and matter fields realized on different branes. Such models may also provide extra dimensions for gauge bosons (so that there would be KK excitations of *e.g.* γ, Z, $W^{\pm}$, g). Intersecting branes and branes at orbifold points may arise from string models.

Finally, I have not even touched upon possibility of warped compactifications, which include the Randall-Sundrum model. Warped compactifications open up a large set of possibilities, and have attracted attention, as they naturally arise in many interesting classes of string vacua. From an effective gravity point of view, such models are the most general possibilities consistent with the low energy Poincaré symmetry of spacetime. Yet they are often predictive enough that non-trivial results may be obtained using some of the tools of string theory.

While this is an admittedly incomplete survey, I hope it presents some of the flavor of braneworld physics as a completely new way of looking at our universe. While it may be as yet premature to speculate on the nature of the spacetime dimensions, braneworlds at least force us to confront this issue in a much more direct manner. Finally, as an essential part of string theory, the very presence of extra dimensions (small and large) may provide the elusive connection between physics of the highest energies and that of the longest distances.

Acknowledgments

This research was supported in part by DOE Grant DE-FG02-95ER40899.

REFERENCES

1. N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, *The hierarchy problem and new dimensions at a millimeter*, Phys. Lett. B **429**, 263 (1998) [arXiv:hep-ph/9803315].
2. I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, *New dimensions at a millimeter to a Fermi and superstrings at a TeV*, Phys. Lett. B **436**, 257 (1998) [arXiv:hep-ph/9804398].
3. L. Randall and R. Sundrum, *A large mass hierarchy from a small extra dimension*, Phys. Rev. Lett. **83**, 3370 (1999) [arXiv:hep-ph/9905221].
4. L. Randall and R. Sundrum, *An alternative to compactification*, Phys. Rev. Lett. **83**, 4690 (1999) [arXiv:hep-th/9906064].
5. J. Maldacena, *The large N limit of superconformal field theories and supergravity*, Adv. Theor. Math. Phys. **2**, 231 (1998) [Int. J. Theor. Phys. **38**, 1113 (1998)] [arXiv:hep-th/9711200].
6. S. S. Gubser, I. R. Klebanov and A. M. Polyakov, *Gauge theory correlators from non-critical string theory*, Phys. Lett. B **428**, 105 (1998) [arXiv:hep-th/9802109].
7. E. Witten, *Anti-de Sitter space and holography*, Adv. Theor. Math. Phys. **2**, 253 (1998) [arXiv:hep-th/9802150].
8. C. D. Hoyle, U. Schmidt, B. R. Heckel, E. G. Adelberger, J. H. Gundlach, D. J. Kapner and H. E. Swanson, *Sub-millimeter tests of the gravitational inverse-square law: A search for 'large' extra dimensions*, Phys. Rev. Lett. **86**, 1418 (2001) [arXiv:hep-ph/0011014].
9. E. G. Adelberger [EOT-WASH Group Collaboration], *Sub-millimeter tests of the gravitational inverse square law*, arXiv:hep-ex/0202008.
10. J. C. Long, H. W. Chan, A. B. Churnside, E. A. Gulbis, M. C. Varney and J. C. Price, *New experimental limits on macroscopic forces below 100-microns*, arXiv:hep-ph/0210004.
11. G. F. Giudice, R. Rattazzi and J. D. Wells, *Quantum gravity and extra dimensions at high-energy colliders*, Nucl. Phys. B **544**, 3 (1999) [arXiv:hep-ph/9811291].
12. T. Han, J. D. Lykken and R. J. Zhang, *On Kaluza-Klein states from large extra dimensions*, Phys. Rev. D **59**, 105006 (1999) [arXiv:hep-ph/9811350].
13. E. A. Mirabelli, M. Perelstein and M. E. Peskin, *Collider signatures of new large space dimensions*, Phys. Rev. Lett. **82**, 2236 (1999) [arXiv:hep-ph/9811337].
14. M. Acciarri *et al.* [L3 Collaboration], *Search for low scale gravity effects in e^+e^- collisions at LEP*, Phys. Lett. B **464**, 135 (1999) [arXiv:hep-ex/9909019].

15. M. Acciarri *et al.* [L3 Collaboration], *Single and multi-photon events with missing energy in e^+e^- collisions at $\sqrt{s} = 189\,\mathrm{GeV}$*, Phys. Lett. B **470**, 268 (1999) [arXiv:hep-ex/9910009].
16. M. Acciarri *et al.* [L3 Collaboration], *Search for extra dimensions in boson and fermion pair production in e^+e^- interactions at LEP*, Phys. Lett. B **470**, 281 (1999) [arXiv:hep-ex/9910056].
17. G. Abbiendi *et al.* [OPAL Collaboration], *Photonic events with missing energy in e^+e^- collisions at $\sqrt{s} = 189\,\mathrm{GeV}$*, Eur. Phys. J. C **18**, 253 (2000) [arXiv:hep-ex/0005002].
18. P. Abreu *et al.* [DELPHI Collaboration], *Photon Events with Missing Energy at $\sqrt{s} = 183$ to 189 GeV*, Eur. Phys. J. C **17** (2000) 53 [arXiv:hep-ex/0103044].
19. J. L. Hewett, *Indirect collider signals for extra dimensions*, Phys. Rev. Lett. **82**, 4765 (1999) [arXiv:hep-ph/9811356].
20. G. Abbiendi *et al.* [OPAL Collaboration], *Multi-photon production in e^+e^- collisions at $\sqrt{s} = 189\,\mathrm{GeV}$*, Phys. Lett. B **465**, 303 (1999) [arXiv:hep-ex/9907064].
21. P. Abreu *et al.* [DELPHI Collaboration], *Determination of the $e^+e^- \to \gamma\gamma(\gamma)$ cross-section at centre-of-mass energies ranging from 189 GeV to 202 GeV*, Phys. Lett. B **491**, 67 (2000) [arXiv:hep-ex/0103005].
22. G. Abbiendi *et al.* [OPAL Collaboration], *Tests of the standard model and constraints on new physics from measurements of fermion pair production at 189 GeV at LEP*, Eur. Phys. J. C **13**, 553 (2000) [arXiv:hep-ex/9908008].
23. P. Abreu *et al.* [DELPHI Collaboration], *Measurement and Interpretation of Fermion-Pair Production at LEP Energies of 183 and 189 GeV*, Phys. Lett. B **485**, 45 (2000) [arXiv:hep-ex/0103025].
24. C. Adloff *et al.* [H1 Collaboration], *Search for compositeness, leptoquarks and large extra dimensions in eq contact interactions at HERA*, Phys. Lett. B **479**, 358 (2000) [arXiv:hep-ex/0003002].
25. B. Abbott *et al.* [D0 Collaboration], *Search for large extra dimensions in dielectron and diphoton production*, Phys. Rev. Lett. **86**, 1156 (2001) [arXiv:hep-ex/0008065].
26. S. Cullen and M. Perelstein, *SN1987A constraints on large compact dimensions*, Phys. Rev. Lett. **83**, 268 (1999) [arXiv:hep-ph/9903422].
27. V. D. Barger, T. Han, C. Kao and R. J. Zhang, *Astrophysical constraints on large extra dimensions*, Phys. Lett. B **461**, 34 (1999) [arXiv:hep-ph/9905474].
28. S. Hannestad and G. Raffelt, *New supernova limit on large extra dimensions*, Phys. Rev. Lett. **87**, 051301 (2001) [arXiv:hep-ph/0103201].
29. L. A. Anchordoqui, J. L. Feng, H. Goldberg and A. D. Shapere, *Neutrino bounds on astrophysical sources and new physics*, arXiv:hep-ph/0207139.
30. S. Hannestad and G. G. Raffelt, *Stringent neutron-star limits on large extra dimensions*, Phys. Rev. Lett. **88**, 071301 (2002) [arXiv:hep-ph/0110067].
31. F. C. Adams, G. L. Kane, M. Mbonye and M. J. Perry, *Proton decay, black holes, and large extra dimensions*, Int. J. Mod. Phys. A **16**, 2399 (2001) [arXiv:hep-ph/0009154].
32. S. B. Giddings and S. Thomas, *High energy colliders as black hole factories: The end of short distance physics*, Phys. Rev. D **65**, 056010 (2002) [arXiv:hep-ph/0106219].
33. S. Dimopoulos and G. Landsberg, *Black holes at the LHC*, Phys. Rev. Lett. **87**, 161602 (2001) [arXiv:hep-ph/0106295].
34. J. L. Feng and A. D. Shapere, *Black hole production by cosmic rays*, Phys. Rev. Lett. **88**, 021303 (2002) [arXiv:hep-ph/0109106].
35. L. A. Anchordoqui, J. L. Feng, H. Goldberg and A. D. Shapere, *Black holes from cosmic rays: Probes of extra dimensions and new limits on TeV-scale gravity*, Phys. Rev. D **65**, 124027 (2002) [arXiv:hep-ph/0112247].
36. M. Fairbairn, *Cosmological constraints on large extra dimensions*, Phys. Lett. B **508**, 335 (2001) [arXiv:hep-ph/0101131].
37. L. J. Hall and D. R. Smith, *Cosmological constraints on theories with large extra dimensions*, Phys. Rev. D **60**, 085008 (1999) [arXiv:hep-ph/9904267].
38. N. Kaloper, J. March-Russell, G. D. Starkman and M. Trodden, *Compact hyperbolic extra dimensions: Branes, Kaluza-Klein modes and cosmology*, Phys. Rev. Lett. **85**, 928 (2000) [arXiv:hep-ph/0002001].
39. G. D. Starkman, D. Stojkovic and M. Trodden, *Large extra dimensions and cosmological problems*, Phys. Rev. D **63**, 103511 (2001) [arXiv:hep-th/0012226].
40. G. D. Starkman, D. Stojkovic and M. Trodden, *Homogeneity, flatness and 'large' extra dimensions*, Phys. Rev. Lett. **87**, 231303 (2001) [arXiv:hep-th/0106143].