Physics Goals of a $\mu^+\mu^-$ Collider*

V. Barger[a], M.S. Berger[b], K. Fujii[c], J.F. Gunion[d], T. Han[d], C. Heusch[e],
W. Hong[f], S.K. Oh[g], Z. Parsa[h], S. Rajpoot[i], R. Thun[j], and B. Willis[k]

[a]*Physics Department, University of Wisconsin, Madison, WI 53706, USA*
[b]*Physics Department, Indiana University, Bloomington, IN 47405, USA*
[c]*Physics Division, KEK, 1-1 Oho, Tsukuba, Ibaraki, Japan*
[d]*Physics Department, University of California, Davis, CA 95616, USA*
[e]*Physics Department, University of California, Santa Cruz, CA 95064, USA*
[f]*Physics Department, University of California, Los Angeles, CA 90024, USA*
[g]*Physics Department, Kon-kuk University, Seoul 133-701, South Korea*
[h]*Physics Department, Brookhaven National Laboratory, Upton, NY 11973, USA*
[i]*Physics Department, California State University, Long Beach, CA 90840, USA*
[j]*Physics Department, University of Michigan, Ann Arbor, MI 48109, USA*
[k]*Physics Department, Columbia University, New York, NY 10027, USA*

Abstract

This working group report focuses on the physics potential of $\mu^+\mu^-$ colliders beyond what can be accomplished at linear e^+e^- colliders and the LHC. Particularly interesting possibilities include (i) s-channel resonance production to discover and study heavy Higgs bosons with ZZ and WW couplings that are suppressed or absent at tree-level (such as the H and A Higgs bosons of supersymmetric models), (ii) measurements of the masses and properties of heavy supersymmetric particles, and (iii) study of the strongly interacting electroweak sector, where higher energies give larger signals.

I. INTRODUCTION

The physics working group focused on the new physics potential of a $\mu^+\mu^-$ collider[1]. Since the time available for analysis was limited, this report is largely directed to two areas of major current interest in particle physics:

- finding Higgs bosons or detecting strong WW scattering and thereby understanding the origin of electroweak symmetry breaking (EWSB);
- finding and studying supersymmetric (SUSY) particles.

For the same energy and integrated luminosity, it should be possible to do anything at a $\mu^+\mu^-$ collider that can be done at an e^+e^- collider. Moreover, a $\mu^+\mu^-$ collider opens up two particularly interesting possibilities for improving the physics reach over that of e^+e^- colliders:

---

*Report presented by V. Barger. Working group leaders V. Barger and J.F. Gunion

© 1996 American Institute of Physics

- s-channel Higgs production;
- higher center-of-mass energy with reduced backgrounds.

Both possibilities are simply a result of the large muon mass as compared to the electron mass. Direct s-channel Higgs production is greatly enhanced at a $\mu^+\mu^-$ collider because the coupling of any Higgs boson to the incoming $\mu^+\mu^-$ is proportional to m_μ, and is therefore much larger than in e^+e^- collisions. Higher energy is possible at a $\mu^+\mu^-$ collider due to the ability to recirculate the muons through a linear acceleration array without an overwhelming radiative energy loss. Current estimates are that an e^+e^- collider with energy above 1.5 TeV will be very difficult to construct (with adequate luminosity), whereas a 4 TeV $\mu^+\mu^-$ collider would appear to be well within the range of possibility [2]. Meanwhile, backgrounds from processes arising from photon radiation from one or both beams will be suppressed by the higher muon mass. As we shall discuss, higher energy could be crucial in at least two ways:

- improved signals for strong WW scattering — since higher energies are achievable the signal level is increased, while backgrounds are reduced due to there being less photon radiation;
- the kinematical reach for pair production of SUSY particles is extended to a possibly crucial higher mass range.

Very briefly, what new physics can an e^+e^- collider[3] do? First, neutral Higgs bosons can be discovered that are coupled to the Z-boson. The Standard Model Higgs boson or the lightest Higgs boson of the Minimal Supersymmetric Standard Model (MSSM) can be discovered (in $Z^* \to Zh$ production) if $m_h \lesssim 0.7\sqrt{s}$. Since there is a theoretical upper bound[4,5] on the lightest Higgs in the MSSM of $m_h \lesssim 130$ to 150 GeV, an e^+e^- machine with CM energy $\sqrt{s} = 300$ GeV can exclude or confirm supersymmetric theories based on grand unified theories (GUTs). However, if heavy, the other neutral SUSY Higgs bosons H, A can only be discovered via $Z^* \to HA$ production for $m_H \sim m_A < \sqrt{s}/2$; $Z^* \to ZH$ (ZA) production is not useful since the H (A) coupling to ZZ is suppressed (absent) at tree-level.

Discovery of SUSY sparticles is also possible at an e^+e^- collider for sparticle masses $m < \sqrt{s}/2$. While the energy reach of an e^+e^- collider will probably be adequate for pair producing the lightest chargino $\chi_1^\pm$, the neutralino combination $\chi_1^0\chi_2^0$ and possibly the selectron, smuon and stau $\tilde{e}, \tilde{\mu}, \tilde{\tau}$, and the lighter stop eigenstate, $\tilde{t}_1$, it could be inadequate for the heavier chargino and neutralinos and other squarks.

The figure of merit in physics searches at an e^+e^- or $\mu^+\mu^-$ collider is the QED point cross section for $e^+e^- \to \mu^+\mu^-$, which has the value

$$\sigma_{QED}(\sqrt{s}) = \frac{100 \text{ fb}}{s \text{ (TeV}^2)} \left(\frac{\alpha(s)}{\alpha(M_Z^2)}\right)^2 . \tag{1}$$

Henceforth we neglect the factor $(\alpha(s)/\alpha(M_Z^2))^2$, which varies slowly with s. As a rule of thumb, the integrated luminosity needed for the study of new physics signals is

$$\left(\int \mathcal{L}dt\right) \sigma_{QED} \gtrsim 1000 \text{ events} . \tag{2}$$

Thus $\mu^+\mu^-$ machine designs should be able to deliver an integrated luminosity of

$$\int \mathcal{L}dt \gtrsim 10 \cdot s \ (\text{fb})^{-1} . \tag{3}$$

If this is to be accumulated in one year's running, the luminosity requirement is

$$\mathcal{L} \gtrsim 10^{33} \cdot s \ (\text{cm})^{-2}(\text{sec})^{-1} . \tag{4}$$

Two possible $\mu^+\mu^-$ machines were considered at this meeting:

- Low $\sqrt{s} \simeq 400$ GeV, requiring

$$\int \mathcal{L}dt \gtrsim 1 \ (\text{fb})^{-1}, \quad \mathcal{L} \gtrsim 10^{32} \ (\text{cm})^{-2}(\text{sec})^{-1} \tag{5}$$

- High $\sqrt{s} \gtrsim 4$ TeV, requiring

$$\int \mathcal{L}dt \gtrsim 100 \ (\text{fb})^{-1}, \quad \mathcal{L} \gtrsim 10^{34} \ (\text{cm})^{-2}(\text{sec})^{-1} \tag{6}$$

Fortunately these luminosity requirements may be achievable with the designs under consideration [2]. Indeed, the luminosity of a $\sqrt{s} \sim 400$ GeV machine could be as large as $\mathcal{L} \gtrsim 10^{33}$ $(\text{cm})^{-2}(\text{sec})^{-1}$.

II. FIRST MUON COLLIDER (FMC)

Consider a $\mu^+\mu^-$ machine with CM energy $\sqrt{s}$ in the range 250 to 500 GeV. As noted above, the yearly integrated luminosity for this machine would be at least $L \geq 1$ $(\text{fb})^{-1}$ and possibly as much as $L \geq 10-20$ $(\text{fb})^{-1}$. The most interesting physics at such a $\mu^+\mu^-$ collider that goes beyond that accessible at an e^+e^- collider of similar energy is the possibility of s-channel heavy Higgs production, as illustrated in Fig. 1.

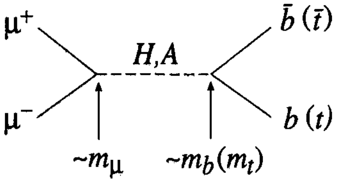

Fig. 1: s-channel diagrams for production of H, A MSSM Higgs bosons.

To discover a narrow resonance, one would ideally like to have a broad $\sqrt{s}$ spectrum. (Alternatively, a scan with limited luminosity at discretely spaced energies could be employed; our results for the case of a broad spectrum are easily altered to this latter procedure.) Once a Higgs boson is discovered, it would be best to change to as narrow as possible a spectrum and then sit on the peak to study the resonance properties.

The s-channel Higgs resonance cross section is

$$\sigma_h = \frac{4\pi\Gamma(h \to \mu\mu)\,\Gamma(h \to X)}{\left(s - m_h^2\right)^2 + m_h^2\Gamma_h^2}\,, \tag{7}$$

where X denotes a final state and Γ_h is the total width. The sharpness of the resonance peak is determined by Γ_h. For a broad energy spectrum, the relevant energy resolution is that determined by the detector. The widths of the Higgs bosons under consideration will be seen to be quite small, such that it is reasonable to consider

$$\Delta E(\text{resolution}) \geq \Gamma(\text{Higgs})\,. \tag{8}$$

Then the integrated signal over the narrow resonance is

$$S_h = \int \sigma_h d\sqrt{s} = \frac{\pi}{2}\Gamma_h \sigma_h^{\text{peak}}\,, \tag{9}$$

where the peak cross section is

$$\sigma_h^{\text{peak}} = \frac{4\pi}{m_h^2}\text{BF}(h \to \mu\mu)\text{BF}(h \to X)\,, \tag{10}$$

leading to

$$S_h = \frac{2\pi^2}{m_h^2}\Gamma(h \to \mu\mu)\,\text{BF}(h \to X)\,. \tag{11}$$

With the integrated luminosity $L = \int \mathcal{L}dt$ spread over an energy band ΔE in the search mode, the event rate for the Higgs signal is

$$N_h = S_h \frac{L}{\Delta E}\,. \tag{12}$$

In the above, we have used the general notation h for the Higgs boson.

In applying the above general formulae to the Minimal Supersymmetric Model (MSSM) Higgs bosons we note the following important facts. The couplings to fermions and vector bosons depend on the SUSY parameter $\tan\beta = v_2/v_1$ and on the mixing angle α between the neutral Higgs states (α is determined by the Higgs masses, $\tan\beta$, the top and the stop masses). SUSY GUT models predict large m_A and $\alpha \approx \beta - \pi/2$. In this case, the coupling factors of the Higgs bosons are approximately[6]

$$\begin{array}{cccc} & \mu^+\mu^-, b\bar{b} & t\bar{t} & ZZ, W^+W^- \\ h & 1 & -1 & 1 \\ H & \tan\beta & -1/\tan\beta & 0 \\ A & -i\gamma_5\tan\beta & -i\gamma_5/\tan\beta & 0 \end{array} \tag{13}$$

times the Standard-Model factor of $gm/(2m_W)$ in the case of fermions (where m is the relevant fermion mass), or $gm_W, gm_Z/\cos\theta_W$ in the case of the W, Z. The broad spectrum inclusive signal rate S_h is proportional to $\Gamma(h \to \mu^+\mu^-)$ and since the coupling of $h = H, A$ to the $\mu^+\mu^-$ channel is proportional to $\tan\beta$, larger $\tan\beta$ values give larger production.

To obtain the rate in a given final state mode X we multiply the inclusive rate by $\mathrm{BF}(h \to X)$. Here, we consider only the $b\bar{b}$ and $t\bar{t}$ decay modes for $h = H, A$, although the relatively background free $H \to hh \to b\bar{b}b\bar{b}$ and $A \to Zh \to Zb\bar{b}$ modes might also be useful for discovery. Figure 2 shows the dominant branching fractions to $b\bar{b}$ and $t\bar{t}$ of Higgs bosons of mass $m_A = 400$ GeV $\approx m_H$ versus $\tan\beta$, taking $m_t = 170$ GeV. The $b\bar{b}$ decay mode is dominant for $\tan\beta > 5$, which is the region where observable signal rates are obtained. From the figure we see that $\mathrm{BF}(h \to b\bar{b})$ grows rapidly with $\tan\beta$ for $\tan\beta \lesssim 5$, while $\mathrm{BF}(h \to t\bar{t})$ falls slowly. For low to moderate $\tan\beta$ values, the event rates behave as

$$N(\mu^+\mu^- \to H, A \to b\bar{b}) \propto m_\mu^2 m_b^2 \left[(\tan\beta)^6 \text{ to } (\tan\beta)^4\right] \tag{14}$$

$$N(\mu^+\mu^- \to H, A \to t\bar{t}) \propto m_\mu^2 m_t^2 \left[(\tan\beta)^2 \text{ to } (\tan\beta)\right] . \tag{15}$$

It is this growth with $\tan\beta$ that makes H, A discovery possible for relatively modest values of $\tan\beta$ larger than 1. At high $\tan\beta$ the $b\bar{b}$ branching ratio asymptotes to a constant value while the $t\bar{t}$ branching ratio falls as $1/(\tan\beta)^4$, so that

$$N(\mu^+\mu^- \to H, A \to b\bar{b}) \propto m_\mu^2 m_b^2 (\tan\beta)^2 \tag{16}$$

$$N(\mu^+\mu^- \to H, A \to t\bar{t}) \propto m_\mu^2 m_t^2 (\tan\beta)^{-2} . \tag{17}$$

Consequently, the $t\bar{t}$ channel will not be useful at large $\tan\beta$, whereas the $b\bar{b}$ channel continues to provide a large event rate.

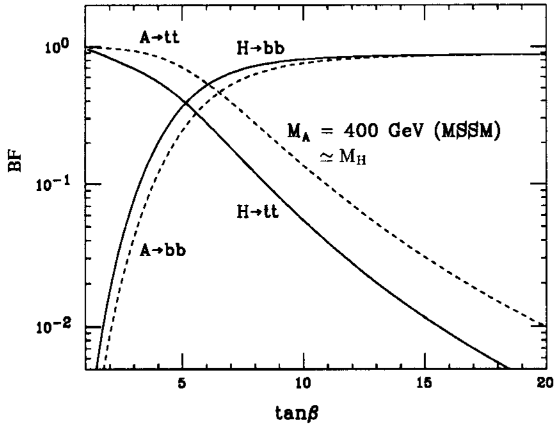

Fig. 2: Dependence of the branching fractions for the heavy supersymmetric Higgs bosons on $\tan\beta$ (from Ref. [7]).

The calculated Higgs boson widths are shown in Fig. 3 versus m_h for $\tan\beta = 5$. As promised, the H and A are typically narrow resonances ($\Gamma_{H,A} \sim 0.1$ to 2 GeV), and our approximation of $\Delta E \geq \Gamma_{H,A}$ will generally be valid; see below.

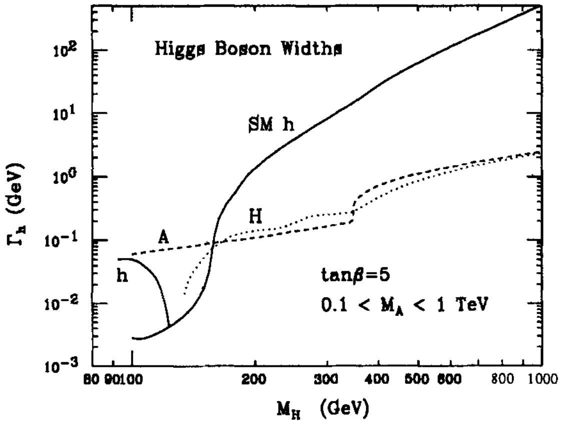

Fig. 3: The Standard Model Higgs boson and the supersymmetric Higgs boson widths (from Ref. [7]).

The irreducible backgrounds to the Higgs signals are

$$\mu^+\mu^- \to \gamma^*, Z^* \to b\bar{b}, t\bar{t}\,. \tag{18}$$

The final-state energy resolution can be estimated by taking the heavy quark energy resolution to be that of the hadron energy resolution in NLC design studies [3]

$$\frac{\Delta E_Q}{E_Q} = \frac{50\%}{\sqrt{E_Q}} + 2\%\,. \tag{19}$$

Then for example at $m_h = 400$ GeV the final state mass resolution is

$$\Delta m(b\bar{b}) \simeq 2\% m(b\bar{b})\,. \tag{20}$$

The background cross section is integrated over the bin $\sqrt{s} = m_h \pm \frac{1}{2}\Delta m(b\bar{b})$

$$B = \int \sigma_B \, d\sqrt{s}\,. \tag{21}$$

The light-quark backgrounds can be rejected using b-tagging. As a single b-tag efficiency, we assume $\epsilon_b \simeq 0.5$ and neglect mistags.

The H signal (for $\tan\beta = 5$) and the backgrounds integrated over the resolution are shown in Fig. 4 versus m_H. As a concrete example let us assume the optimistic integrated luminosity of $L = 20$ $(\text{fb})^{-1}$ spread over an energy band $\Delta E = 50$ GeV, giving $L/\Delta E = 0.4$ (fb)$^{-1}$/GeV. The signal in the $b\bar{b}$ channel for $m_H = 400$ GeV with $\tan\beta = 5$ is

$$N(\text{signal}) = 250 \text{ events} \tag{22}$$

while the $b\bar{b}$ background is

$$B(b\bar{b}\,\text{background}) = 2000 \text{ events}\ , \tag{23}$$

where both numbers include the single b-tag efficiency ϵ_b. The significance of the signal is

$$n_{SD} \simeq 5.6\ . \tag{24}$$

Thus, discovery is possible here!

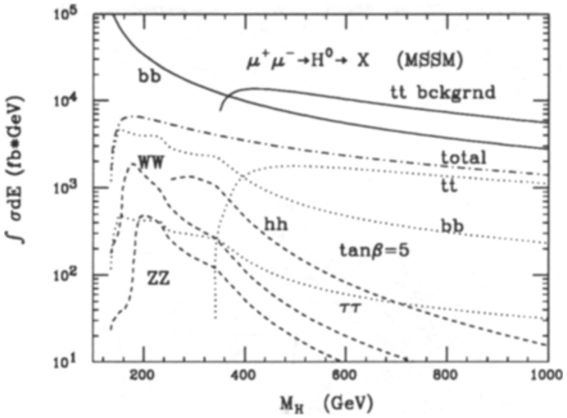

Fig. 4: Typical signals and backgrounds for $\mu^+\mu^- \to H \to X$ in the MSSM (from Ref. [7]).

Figure 5 shows the significance for H detection versus $\tan\beta$ for a variety of $L/\Delta E$ possibilities: 1 (fb)$^{-1}$/GeV, 0.2 (fb)$^{-1}$/GeV, and 0.02 (fb)$^{-1}$/GeV. Single b-tagging with $\epsilon_b = 0.5$ is assumed. Due to the sharp rise of n_{SD} with $\tan\beta$ a decrease of $L/\Delta E$ by a factor of 50 only changes the lowest $\tan\beta$ for which discovery is possible from $\tan\beta \sim 4$ to $\tan\beta \sim 7$.

Figure 6 provides an overall view of the possibilities for MSSM Higgs boson detection in terms of regions in the $\tan\beta, m_A$ plane for which $n_{SD} \geq 4$ in $h = h, H, A$ searches. This figure assumes a moderately conservative value of $L/\Delta E = 0.08$ (fb)$^{-1}$/GeV. Further, *double* b-tagging (with $\epsilon_b = 0.6$) is required in the $b\bar{b}$ mode and an additional general efficiency factor of 0.5 is included for both the $b\bar{b}$ and $t\bar{t}$ final states. Regions in which h, H, or A detection is individually possible are shown, as well as the additional region that is covered by combining the H and A signals when they are degenerate within the final state resolution. The figure shows that in the $b\bar{b}$ mode either H and A detection (for $m_A \gtrsim 140$ GeV — as preferred in GUT scenarios), or h and A detection (for $m_A \lesssim 140$ GeV), will be possible provided $\tan\beta > 3$–5. Detection of $H + A$ in the $t\bar{t}$ mode is limited to $m_H \sim m_A > 2m_t$ and $\tan\beta$ values lying between 3 and about 12.

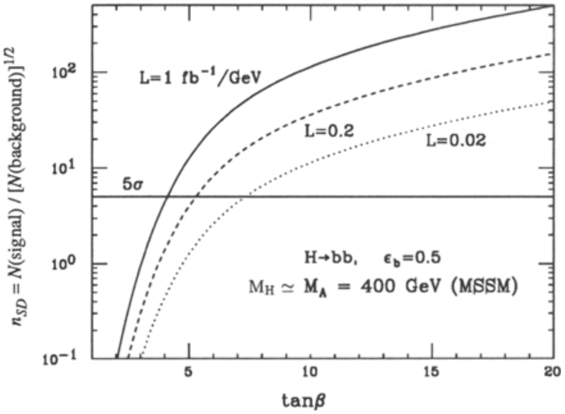

Fig. 5: The statistical significance of the Higgs H signal versus $\tan\beta$. Since in SUSY GUT models one finds H and A to be approximately mass degenerate, the combined signal will be larger (from Ref. [7]).

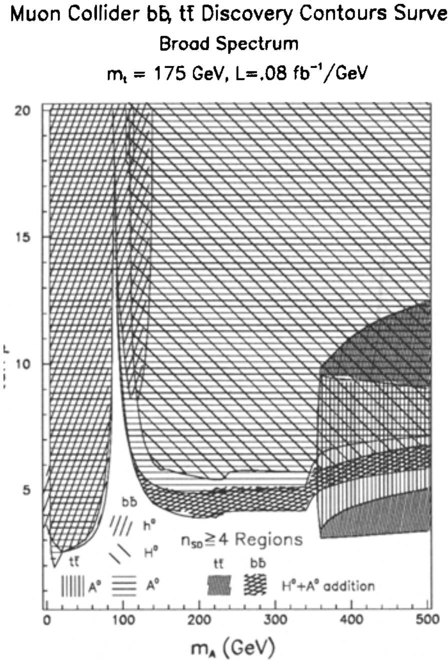

Fig. 6: Higgs boson H, A discovery regions at a muon collider in the modes $H, A \to b\bar{b}$ and $t\bar{t}$ (from Ref. [8]).

Thus, direct s-channel production allows H, A discovery up to the machine kinematical limit, so long as $\tan\beta$ is not small. This is a very important extension as compared to an e^+e^- collider. The above results assume the absence of SUSY decay modes for the H and A. Once the H, A mass becomes large it could happen that SUSY decay modes become kinematically allowed. However, at large $\tan\beta$ the enhanced $b\bar{b}$ coupling guarantees that the $b\bar{b}$ mode branching ratio will still be large. In practice, SUSY decay modes would only shift the discovery regions to slightly higher $\tan\beta$ values.

Once a signal is identified in the search mode, the luminosity can be concentrated over the Higgs peak, $\Delta E = 2\Gamma_H \simeq 1$ GeV, to study the resonance properties. Then the signal and background each increase by a factor of the luminosity energy spread ΔE_S in the broad-spectrum/scanning mode, giving an increase in the significance of a factor of $\sqrt{\Delta E_S}$ [a factor of 7 in the example of Eqs. (22–24)].

High polarization P of both beams would be useful to suppress the background to s-channel Higgs production if the luminosity reduction is less than a factor of $\left(1+P^2\right)^2 / \left(1-P^2\right)$, which would leave the significance of the signal unchanged [10]. For example, $P = 0.84$ would compensate a factor of 10 reduction in luminosity.

It could also be possible to detect the lightest SUSY Higgs boson via s-channel production if m_h becomes known within a few GeV from electroweak radiative corrections or from studies of $\mu^+\mu^- \to Z^* \to Zh$ production, provided that the machine could be operated at $\sqrt{s} = m_h$. However, the $\mu^+\mu^- \to h \to b\bar{b}$ channel suffers a formidable background from $\mu^+\mu^- \to \gamma^*, Z^* \to b\bar{b}$, so the direct channel h search might well require polarized muon beams. Nonetheless, detection of the light h in direct s-channel production would be very interesting in that it would allow a determination of the $\mu^+\mu^-$ coupling of the h.

If a 0.1% energy resolution could be achieved, another interesting application of the FMC could be a precision determination of the top mass by measuring the $t\bar{t}$ threshold cross section[9]. The muon collider could present an improvement over the electron collider from the reduced initial state radiation.

III. NEXT MUON COLLIDER (NMC)

The reduced synchrotron radiation for muons, as compared to electrons, allows the possibility of a recirculating muon colliding beam accelerator with higher energy than is realizable at future linear e^+e^- colliders. A design goal under consideration is a $\sqrt{s} = 4$ TeV $\mu^+\mu^-$ machine with an integrated luminosity $L \geq 100\ (\text{fb})^{-1}$. The construction of the higher energy $\mu^+\mu^-$ collider could occur at the same time as the $\sqrt{s} = 0.4$ TeV machine [2].

A. Sparticle Studies

An exciting possibility is that the NMC could be a SUSY factory, producing squark pairs, slepton pairs, chargino pairs, associated neutralinos, associated $H + A$ Higgs, and gluinos from squark decay if kinematically allowed. If the

SUSY mass scale is $M_{\rm SUSY} \sim 1$ TeV, many sparticles could be beyond the reach of the NLC. The LHC can produce them, but disentangling the SUSY spectrum and measuring the sparticle masses will be a real challenge at a hadron collider, due to the complex nature of the sparticle cascade decays and the presence of QCD backgrounds. The measurement of the sparticle masses is important since they are a window to GUT scale physics.

The p-wave suppression of squark pair production in e^+e^- or $\mu^+\mu^-$ collisions, illustrated in Fig. 7, means that energies well above the threshold are needed. The threshold dependence of the cross section may be useful in sparticle mass measurements.

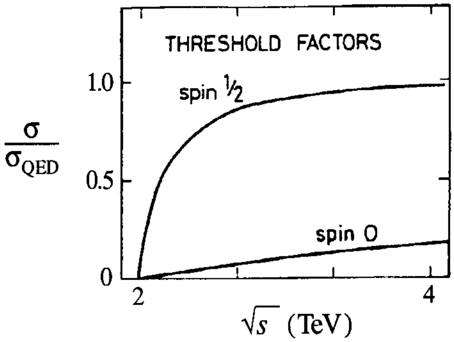

Fig. 7: Comparison of kinematic suppression for fermion pairs and squark pair production at e^+e^- or $\mu^+\mu^-$ colliders.

The cross sections for squarks (of one flavor in the approximation of L, R degeneracy), charginos, top and three generations of singlet quarks (from an E_6 GUT model, for example) are respectively

$$\begin{aligned} \sigma_{\tilde{u}_{L,R}} &= 4\beta^3 \text{fb} \rightarrow 250 \text{ events} \\ \sigma_{\tilde{d}_{L,R}} &= 1\beta^3 \text{fb} \rightarrow 60 \text{ events} \\ \sigma_{\chi^\pm} &= 6\beta \text{ fb} \rightarrow 500 \text{ events} \\ \sigma_t &= 8 \text{ fb} \rightarrow 800 \text{ events} \\ \sigma_{Q_{E_6}} &= 6\beta \text{ fb} \rightarrow 600 \text{ events} \end{aligned} \tag{25}$$

where the event rates given are for sparticle masses of 1 TeV with an integrated luminosity of 100 fb^{-1}. The production of heavy SUSY particles will give spherical events near threshold characterized by

- multijets
- missing energy (associated with the LSP)
- leptons

There should be no problem with backgrounds from SM processes.

A supergravity model with $\tan\beta = 5$, universal scalar mass $m_0 = 1000$ GeV and gaugino mass $m_{1/2} = 150$ GeV provides an illustration of a heavy sparticle spectrum, as follows:

$$\begin{array}{ccc}
\underline{\text{sparticle}} & \underline{\text{mass}} & \\
\tilde{u} & 1000 \text{ GeV} & \\
\tilde{g} & 500 & \\
\tilde{\ell} & 1000 & \\
\chi_4^0,\, \chi_3^0,\, \chi_2^+ & 350 & \\
\chi_2^0,\, \chi_1^+ & 130 & \\
\chi_1^0 & 60 & \text{(LSP)}
\end{array} \tag{26}$$

Consider $\tilde{u}\bar{\tilde{u}}$ production at the NMC. The dominant cascade chain for the decays is

$$\begin{array}{rcl}
\tilde{u}\bar{\tilde{u}} & \to & (\tilde{g}u)(\tilde{g}\bar{u}) \\
\tilde{g} & \to & \chi_1^{\pm} q\bar{q} \\
\chi_1^{\pm} & \to & \chi_1^0 \ell\nu,\, \chi_1^0 q\bar{q}
\end{array} \tag{27}$$

The dominant branching fractions of the $\tilde{u}\bar{\tilde{u}}$ final state are

$$\begin{array}{lr}
10 \text{ jets} + \not{p}_T & 10\% \\
8 \text{ jets} + 1\ell + \not{p}_T & 10\% \\
6 \text{ jets} + 2\ell + \not{p}_T & 2\%
\end{array} \tag{28}$$

Of the two lepton events, one half will be like-sign dileptons ($\ell^+\ell^+$, $\ell^-\ell^-$). The environment of a $\mu^+\mu^-$ collider may be better suited than the LHC to the study of the many topologies of sparticle events.

Turning to the SUSY Higgs sector, we simply emphasize the fact that $Z^* \to HA, H^+H^-$ will allow $H, A, H^{\pm}$ discovery up to $m_H \sim m_A \sim m_{H^\pm}$ values somewhat below $\sqrt{s}/2 \sim 2$ TeV. While GUT scenarios prefer $H, A, H^{\pm}$ masses above 200 to 250 GeV, such that HA and H^+H^- pair production is beyond the kinematical reach of a 400 to 500 GeV collider, even the most extreme GUT scenarios do not yield Higgs masses beyond 2 TeV. Thus, a 4 TeV $\mu^+\mu^-$ collider is guaranteed to find all the SUSY Higgs bosons.

B. Strong *WW* Scattering

If Higgs bosons with $m_H \leq \mathcal{O}(800$ GeV$)$ do not exist, then the interactions of longitudinally polarized weak bosons W_L, Z_L became strong. This means that new physics must be present at the TeV energy scale[11]. The high reach in energy of the NMC is of particular interest for study of a strongly interacting electroweak sector (SEWS) at a $\mu^+\mu^-$ collider via the WW fusion graphs in Fig. 8.

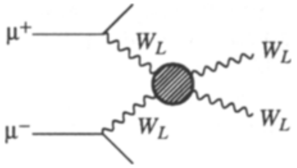

Fig. 8: Strong $W_L^+W_L^-$ scattering in $\mu^+\mu^-$ collisions.

The SEWS signals depend on the model for $W_L^+W_L^-$ scattering. An estimate of the size of these signals can be obtained by taking the difference of the cross section due to a heavy Higgs boson ($m_H = 1$ TeV) and that with a massless Higgs particle

$$\Delta\sigma_{\text{SEWS}} = \sigma(m_H = 1\ \text{TeV}) - \sigma(m_H = 0)\ . \tag{29}$$

The subtraction of the $m_H = 0$ result removes the contributions due to scattering of transversely polarized W-bosons. Figure 9 shows the growth of $\Delta\sigma_{\text{SEWS}}$ with energy. The Table below gives results at $\sqrt{s} = 1.5$ TeV (for the NLC e^+e^- collider) and at $\sqrt{s} = 4$ TeV (for the NMC).

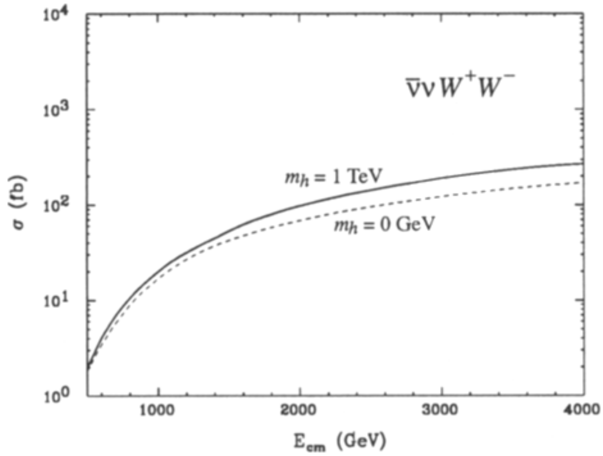

Fig. 9: The growth of the SEWS signal with E_{cm} (from Ref. [12]).

Table I: SEWS signals at future colliders

$\sqrt{s}$	$\Delta\sigma(W^+W^-)$	$\Delta\sigma(ZZ)$
1.5 TeV	8 fb	6 fb
4 TeV	80 fb	50 fb

The energy reach is thereby a critical consideration in the study of SEWS. Additionally, the backgrounds from the photon exchange process of Fig. 10 will be a factor of 3 less at a $\mu^+\mu^-$ machine compared to an e^+e^- machine[12]. The probability for γ radiation from a charged lepton

$$P_{\gamma/\ell}(x) = \frac{\alpha}{\pi}\frac{1+(1-x)^2}{x}\ln(E_\ell/m_\ell) \tag{30}$$

is logarithmically dependent on the charged lepton mass. Figure 11 shows SM cross sections with $m_H = 0$ versus CM energy. The background to SEWS from $e^+e^- \to e^+e^-W^+W^-$ can be rejected by requiring[13]

$$\text{no } e^\pm \text{ with } E_\ell > 50 \text{ GeV and } |\cos\theta_\ell| < |\cos(0.15 \text{ rad})| \, . \tag{31}$$

Some similar rejection would be necessary to suppress the $\mu^+\mu^- \to \mu^+\mu^-W^+W^-$ background at the NMC.

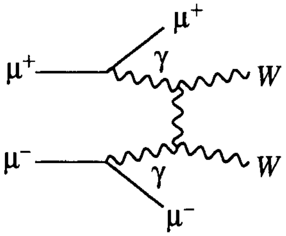

Fig. 10: The background to the SEWS signal from the photon exchange process.

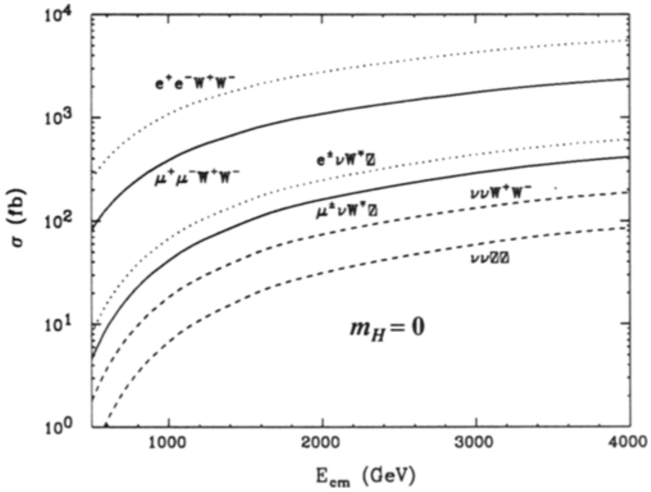

Fig. 11: The cross sections for Standard Model processes at a muon collider (from Ref. [12]).

The various SM cross sections grow with energy, for example,

$$\sigma(\mu^+\mu^- \to \mu^+\mu^- W^+W^-) = 2000 \text{ fb} \tag{32}$$

at $\sqrt{s} = 4$ TeV. These cross sections would be higher if the W is composite. Thus precision studies of WW self couplings should be possible at the NMC.

There are many other new possibilities of interest for the NMC that we have not yet addressed, including

- extra neutral gauge bosons (the NMC could be a Z' factory, with its decays giving Higgses and W^+W^- along with particle and sparticle pairs)
- right-handed weak bosons (the present limit on the right-handed weak boson of left-right symmetric models is $M_{W_R} \gtrsim 1.5$ TeV)
- vector-like quarks and leptons (present in E_6 models)
- horizontal gauge bosons X (whose presence may be detected as an interference between t-channel X exchange and s-channel γ, Z exchanges; present limits are $M_X \gtrsim 1$ TeV)
- leptoquarks

The list goes on with other exotica.

IV. CONCLUSION

In conclusion, $\mu^+\mu^-$ colliders seem to offer unparalleled new opportunities at both the low ($\sqrt{s} \simeq 400$ GeV) and high ($\sqrt{s} \simeq 4$ TeV) energy frontiers. The primary advantages of such a collider are:

- to discover and study Higgs bosons that are not coupled to ZZ or WW (e.g. the heavy SUSY Higgs bosons H, A) by employing either direct s-channel resonance production (at a low energy machine) or $Z^* \to HA$ (at a high energy machine);
- to measure the masses and properties of heavy SUSY particles in the improved background environment of a lepton collider;
- to study a strongly interacting electroweak sector with higher signal rates at higher energies.

Along with accelerator development, much work remains to be done on the physics for such machines and a continuing investigation is underway[14].

ACKNOWLEDGMENTS

This work was supported in part by the U.S. Department of Energy under Grant numbers DE-FG02-95ER40896, DE-FG03-91ER40674 and DE-AC02-76CH00016. Further support was provided by the University of Wisconsin Research Committee, with funds granted by the Wisconsin Alumni Research Foundation, and by the Davis Institute for High Energy Physics.

REFERENCES

1. *Proceedings of the First Workshop on the Physics Potential and Development of $\mu^+\mu^-$ Colliders*, Napa, California, 1992, Nucl. Instr. and Meth. **A350**, 24-5-6 (1994).
2. See summary talks in these proceedings, by R. Fernow, D. Miller, F. Mills and D. Neuffer.
3. For references to in-depth studies of physics at future e^+e^- colliders, see e.g. *Proceedings of the Workshop on Physics and Experiments with Linear Colliders*, ed. F.A. Harris, *et al.*, World Scientific (1993); *Proceedings of the Workshop on Physics and Experiments with Linear e^+e^- Colliders*, Waikoloa, Hawaii (April 1993), ed. F. Harris *et al.* (World Scientific, 1993); JLC Group, KEK Report 92-16 (1992); *Proceedings of the Workshop on Physics and Experiments with Linear Colliders*, Saariselkä, Finland (Sept. 1991), ed. R. Orava *et al.*, World Scientific (1992).
4. M. Drees, Int. J. Mod. Phys. **A4**, 3635 (1989); J. Ellis *et al.*, Phys. Rev. **D39**, 844 (1989); L. Durand and J.L. Lopez, Phys. Lett. **B217**, 463 (1989); J.R. Espinosa and M. Quirós, Phys. Lett. **B279**, 92 (1992); P. Binétruy and C.A. Savoy, Phys. Lett. **B277**, 453 (1992); T. Morori and Y. Okada, Phys. Lett. **B295**, 73 (1992); G. Kane, *et al.*, Phys. Rev. Lett. **70**, 2686 (1993); J.R. Espinosa and M. Quirós, Phys. Lett. **B302**, 271 (1993); U. Ellwanger, Phys. Lett. **B303**, 271 (1993); J. Kamoshita, Y. Okada, M. Tanaka*et al.*, Phys. Lett. **B328**, 67 (1994).
5. V. Barger, *et al.*, Phys. Lett. **B314**, 351 (1993); P. Langacker and N. Polonshy, University of Pennsylvania preprint UPR-0594-T, hep-ph 9403306.
6. J.F. Gunion and H.E. Haber, Nucl. Phys. **B272**, 1 (1986).
7. V. Barger and T. Han, results presented in the introductory talk at this workshop.
8. J.F. Gunion, results prepared in advance for this workshop.
9. For discussions of the $t\bar{t}$ threshold behavior at e^+e^- colliders, see V.S. Fadin and V.A. Khoze, JETP Lett. **46** 525 (1987); Sov. J. Nucl. Phys. **48** 309 (1988); M. Peskin and M. Strassler, Phys. Rev. **D43**, 1500 (1991); G. Bagliesi, *et al.*, CERN Orange Book Report CERN-PPE/92-05; Y. Sumino ,*et al.*, KEK-TH-284, Phys. Rev. **D47**, 56 (1992); M. Jezabek, J.H. Kühn, T. Teubner, Z. Phys. **C56**, 653 (1992).
10. Z. Parsa, *$\mu^+\mu^-$ Collider and Physics Possibilities*, to be published.
11. M.S. Chanowitz and M.K. Gaillard, Nucl. Phys. **B261**, 379 (1985); J. Bagger et al., Phys. Rev. **D49**, 1246 (1994).
12. V. Barger, K. Cheung, T. Han, R.J.N. Phillips, University of Wisconsin preprint MADPH-95-865.
13. K. Hagiwara, J. Kanzaki and H. Murayama, DTP/91/18.
14. V. Barger, M.S. Berger, J.F. Gunion and T. Han, in preparation.