

Radio-Frequency Polarimetry

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Abstract. A method of fast non-destructive absolute spin monitoring for a bunched beam in an accelerator ring based on use of the RF techniques is considered. The coherent spin of the beam is driven by RF magnets in the spin echo regime. A passive superconducting resonator is proposed to respond to the flipping spin. A possibility is established to enhance the spin-related excitation of the resonator using the charge-resonator dipole interaction and spin-orbit coupling induced by the quadrupoles. It is shown that the spin impedance can be gauged via measurements of the beam dipole impedance. The noise demands are evaluated. Numerical examples are given.

INTRODUCTION

The idea of the radio-frequency method of spin monitoring for a polarized beam circulating in an accelerator or storage ring was proposed in recent years [1–3]. It is based on a consideration that, if the coherent spin of the beam is declined from the equilibrium direction, \vec{n} (vertical or, in general case, periodical along the beam orbit), then the electromagnetic field of the beam “observed” by a cavity located at a point of the orbit has to have a modulation with periodicity of the spin precession in the accelerator, which is different from the periodicity of the particle orbital motion. Apparently, this signal is very small. However, if the spin tune spread, $\delta\nu$, is small enough, the beam current, J , is sufficiently high, and the quality Q_R of the cavity tuned in resonance with the spin free precession is also high enough, then the beam could excite the cavity resonance mode to a measurable level while an unpolarized beam would not be able to do this.

The attractiveness of the RF method is increased by the fact that its efficiency does not drop with the beam energy. It should be also noted that it is equally efficient for proton and electron beams although in the case of electron beams there are such well-developed and tested non-destructive methods as Compton backscattering [4] and “spin light” [5]. The most critical issue of

the RF polarimetry is spin decoherency due to the spin tune spread.

Below we will discuss the principles and recent progress in the RF polarimetry concepts.

GENERAL SCHEME OF THE RF SPIN MEASUREMENT

RF arrangement for non-touching beam spin monitoring involves the following basic elements:

- the spin-resonance RF magnets to cause spin precession around a horizontal (rotating) axis, similarly to spin motion in NMR;
- a passive superconducting resonator (single or a few in a row) to be excited by the precessing coherent spin by mean of the electromagnetic interaction of the polarized beam with a resonance standing wave;
- monitoring RF circuit (a loop, filter, amplifier, scope) to determine the spin-related high frequency voltage accumulated in the resonator;
- optionally, the spin-orbital amplifier (SOA), i.e. RF quadrupoles can be introduced in the ring to induce the spin-correlated beam dipole moment for an enhancement of spin-related excitation of the resonator.

THE DYNAMIC BACKGROUNDS

There are shown in Table 1 the quasiclassical Hamiltonian and ground equations those have been used to calculate the spin-related effects in polarized beam dynamics and electrodynamics.

The Hamiltonian is simply combined as a sum of the conventional spinless Hamiltonian and spin term defined to reproduce the Thomas-BMT equations for spin [6–8]. This composition coincides with the Hamiltonian that can be derived from Dirac-Maxwell equations for a particle with an anomalous magnetic moment, when splitting the Dirac equation in two branches with positive and negative energies according to Foldi-Wouthuysen [9,10]. Thus, the treatment presented in Table 1 can be considered as adequate to the fundamental theory, complete, and sufficient for quasiclassical conditions.

Earlier, this approach was used [11] to reproduce the Sokolov-Ternov polarization effect for ultrarelativistic electrons; in this case, the contribution of the Lorentz force term in \vec{W} is negligible. In contrary, this term dominates in case of ultrarelativistic particles ($\gamma G \gg 1$) in external fields, static or alternating.

TABLE 1. Hamiltonian and basic equations ($c=1$).

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- (1) Total Hamiltonian: $H = \Sigma(H_o + \vec{W} \cdot \vec{S}) + H_R$
(2) Spinless energy: $H_o = \sqrt{(\vec{p} - e\vec{A})^2 + m^2} + eA_o$
(3) Resonator energy: $H_R = \frac{1}{2}(P^2 + \omega_R^2 Q^2)$
(4) Spin precession velocity ($\gamma^{-2} = 1 - v^2$):
 $\vec{W} = -\frac{e}{m}[(G + \frac{1}{\gamma+1})(\vec{E} + \vec{v} \times \vec{B}) \times \vec{v} + \frac{1+\gamma}{\gamma}(\vec{B}_v + \frac{1}{\gamma}\vec{B}_{tr})]$
(5) Electromagnetic field: $\vec{A} = \vec{A}_{ext} + Q(t)\vec{e}(\vec{r})$
 $\vec{E} = -\frac{\partial \vec{A}}{\partial t} - \vec{\nabla}A_o; \vec{B} = \vec{\nabla} \times \vec{A}; \int \vec{e}^2 d^3r = 4\pi$
(6) Non-zeroth Poisson's brackets: $\{\vec{p}_\alpha, r_\alpha\} = 1; \{P, Q\} = 1; \{S_\alpha, S_\beta\} = -\epsilon_{\alpha\beta\gamma} S_\gamma$
(7) Orbit equations ($\vec{p} \equiv \vec{p} - e\vec{A}, \vec{v} \equiv \vec{p}/\sqrt{p^2 + m^2}$):
 $\dot{r}_\alpha = v_\alpha + \frac{\partial}{\partial p_\alpha}(\vec{W} \cdot \vec{S}); \dot{p}_\alpha = -e\frac{dA_\alpha}{dt} - e\vec{v} \cdot \frac{\partial \vec{A}}{\partial r_\alpha} - \frac{\partial}{\partial r_\alpha}(\vec{W} \cdot \vec{S})$
(8) Resonator field equations: $\dot{Q} = P + \frac{\partial}{\partial P}\Sigma(\vec{W} \cdot \vec{S}); \dot{P} = -\omega_R^2 Q + e\vec{v} \cdot \vec{e} - \frac{\partial}{\partial Q}\Sigma(\vec{W} \cdot \vec{S})$
(9) Thomas-BMT equation: $\dot{\vec{S}} = \vec{W} \times \vec{S}$
Here the symbol Σ means summation on particles of a polarized beam.
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SPIN DRIVER

In the earlier RF polarimetry concept [1], the free precession of the coherent spin in magnetic field of an accelerator ring was supposed to be excited (by a pulse RF magnet) to effect a superconducting cavity. Then, the maximum time of RF voltage accumulation in the cavity was limited by the spin tune spread, $\delta\nu_o$, where ν_o is number of spin precessions per particle turn in a ring. The $\delta\nu_o$ value is defined by the beam emittances and magnetic lattice imperfections, it grows rapidly with particle energy. It is relatively small in electron storage rings (after averaging on energy oscillation), thank to a small value of the vertical beam emittance. The $\delta\nu_o$ value becomes essentially reduced in a ring with Siberian snakes [12]. However, it seems difficult to reduce $\delta\nu_o$ in hadron colliders to a value less than $(1-0.3) \cdot 10^{-3}$. The RF polarimetry with this coherent spin quality value seems possible but not highly efficient.

A simple step can improve the spin quality: apply the spin-resonance RF field of a relatively large strength $\varepsilon \equiv GeBl/2\pi m \gg \delta\nu_o$ and let the spin precess around this field during a possibly long time (Fig. 1). Then the effective spin tune spread becomes reduced by a factor $\sim \delta\nu_o/2\varepsilon$, with the correspondent increase of the spin coherency time, τ . Optionally, a SC resonator tuned to frequency $(k + \varepsilon)\omega_o$, or $(k + \nu_o + \varepsilon)\omega_o$, can be choosen. Note, that the reduction of the spin tune shift and spread also helps to stabilize the spin against the weak depolarizing resonances.

Next, consider the spin echo regime when the RF field changes the phase on $\pi/2$ at the moments when the spin is close to the vertical direction, twice a single precession (Fig. 2). This cycle can be perfectly synchronized with the beam revolution. The spin dynamics here is similar to spin motion in a

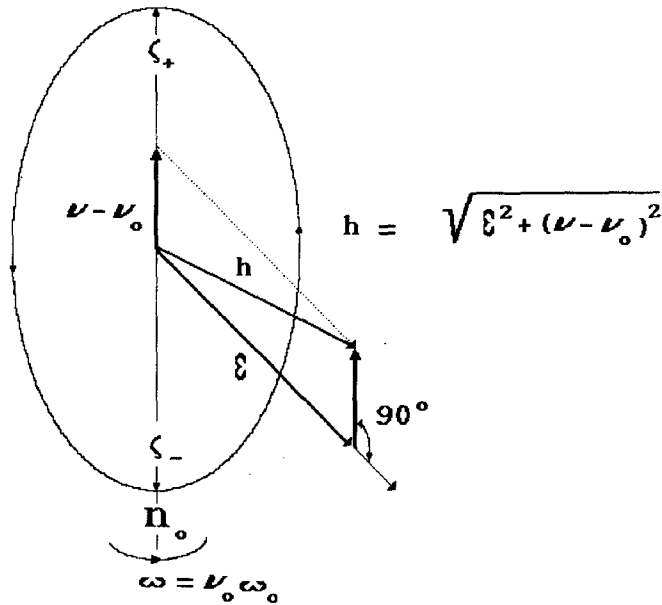


FIGURE 1. Reduction of the spin tune dispersion in resonance RF field.

ring with two Siberian snakes [13]. Thus, the spin precession in the RF field becomes stable (periodical of a period $T = 2\pi(2q + 1)/\omega_0\epsilon$, where q is an integer) motion with no phase divergence: the free precession now moved in the plane transverse to the periodical spin, with effective tune $\hat{\nu} = 1/2\omega_0 T$ and tune divergence $\Delta\hat{\nu} \sim (\Delta\nu_0)^2/2\epsilon$, at $q \ll (\epsilon/\Delta\nu_0)^2$.

Now, we nominate the vertical spin oscillating (with infinite quality) with frequency $(\omega_0\epsilon/2\pi) \equiv (2q + 1)/T$ to be in charge to excite the SC resonator. This frequency effectively becomes a characteristic spin tune, inspite it is a harmonic (high order, $2q + 1$) of the RF field phase switching frequency. The demands related to the parasitic beam excitation (of period T) because of the RF field misalignments are discussed in section 7.

SPIN-INDUCED CHARGE-RESONATOR INTERACTION

The precessing or flipping coherent spin will effect the beam orbit via the spin-orbit interaction in quadrupoles according to the general orbital equations shown in Table 1; the reduced equations are given in Table 2.

Here $\vec{\zeta}(t)$ is the coherent beam polarization, n is the focusing field index, and g_x, g_y are the factors due to the coherent beam dipole interaction. The spin-

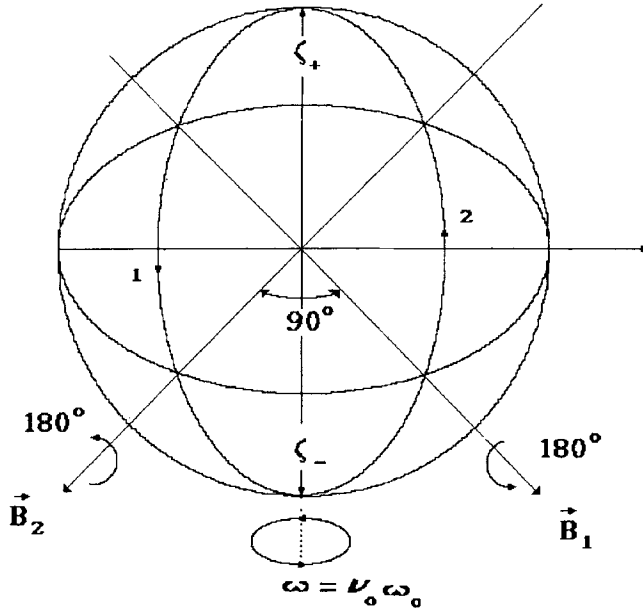


FIGURE 2. Spin motion schematic in RF spin echo regime.

correlated beam dipole moment, \hat{d}_j , contributes to the resonance excitation of the SC cavity, effectively renormalizing the spin-related interaction. The effect grows near the spin-orbit resonances $k \pm \nu_{sp} = \pm \nu_b$, where ν_b is one of two tunes of beam coherent oscillations; this mechanism can be used to raise the RF polarimeter efficiency. Use of this effect is complicated by the fact that it alternates with energy. But, observation of the resonance behavior of the spin-orbital effect leads to idea to introduce the RF quadrupoles of the bridge frequency $\omega_q = \omega_R - k\omega_o \pm \omega_{sp}$, at $\omega_R \approx (k_1 + \nu_b)\omega_o$. The RF quadrupoles can be skew, if needed. With the RF spin-orbital amplification (SOA), the immediate spin-resonator interaction becomes non-resonant, hence, negligible. The SOA is limited by the depolarization near spin-orbital resonances due to the beam emittances. Other limiting factors are betatron tune spread and (or) beam damping, coherent and incoherent. Estimations show that the maximum gain varies from a factor of about 50 in high energy hadron rings to $\sim 10^3$ in e^\pm storage rings and low energy proton synchrotrons with cooled beams, although one can meet a lower practical limit because of necessity to control the beam tunes with a high precision.

TABLE 2. RF polarimetry equations at $\gamma G \gg 1$.

(1)	$\hat{d} = GNe\hbar\hat{\rho}/2m; \quad \hat{\rho} = (\hat{x}, \hat{y}) \equiv \hat{g}\zeta(t)$
(2)	$\hat{y} + 2\lambda_y\dot{\hat{y}} + (n + g_y)\hat{y} = \frac{e}{\gamma m} \frac{\partial \vec{B}_{sr}}{\partial y} \zeta$
(3)	$\hat{x} + 2\lambda_x\dot{\hat{x}} + (K^2 - n + g_x)\hat{x} = \frac{e}{\gamma m} \frac{\partial \vec{B}_{sr}}{\partial x} \zeta$
(4)	Spin-induced charge-resonator interaction: $(-e\vec{v}\vec{A})_{sp} = -Q\hat{d} \frac{\partial e_v}{\partial \hat{\rho}}$
(5)	Resonator equation: $\dot{Q} + 2\lambda_R\dot{Q} + \omega_R^2 Q = -\frac{NeG\hbar}{2m} (\vec{v} \times \vec{\nabla} e_v + \vec{\nabla} e_v \cdot \hat{g}) \zeta(t)$

SPIN IMPEDANCE

Spin responding RF modes. There are three basic types of standing waves to interact with spin:

- a) solenoidal TE_{011} mode of an axial resonator. Use of this mode may be reasonable at $\gamma G \leq 1$.
- b) magnetic dipole TM_{110} mode.
- c) standing (TEM $\frac{\lambda}{4}$) wave of feeder type (Fig. 3).

In the area $\gamma G \gg 1$, spin interaction with modes b) and c) does not drop with energy. An additional advantage of this modes (at all energies) is that they also interact with the beam dipole motion (excited by the coherent spin). The specific feature of the feeder mode is that the transverse size of the resonator, hence, the interaction strength, is not limited by the wave length (i.e., by the bunch length).

Spin impedance gauging. Spin impedance can be precisely calculated by mapping the RF modes. At high energies ($\gamma G \gg 1$) and short bunches ($\ell \ll \beta$, where β is the beam focussing parameter) it also can be gauged via measurement of the beam dipole impedance [14]. To prove this possibility, note that the Lorentz force in \vec{W} can be represented as $\vec{F} = -e(d\vec{A}/dt) - e\vec{\nabla}(\vec{v}\vec{A})$; since the spin does not precess in the resonator, the first term can be omitted. Then, one can see from the comparison with the charge interaction Hamiltonian, $-e\vec{v}\vec{A}$, that the ratio between spin and beam dipole impedances is simply $G + \frac{1}{\gamma}$. The final equation for the resonator field is shown in Table 2.

The accumulated HF voltage. Table 3 illustrates the estimations of the accumulation rate and maximum value of spin-related voltage in the feeder type $\frac{\lambda}{4}$ resonator based on two simple formulas: $\dot{V} = 2\pi\sqrt{2}(\hbar/m)(J/\ell d)(G + \frac{1}{\gamma})(1 + g)$, and $V_{max} = \dot{V}/\lambda_R$, at polarization degree $\zeta = 1$, $d = 2cm$, $\lambda_R^{-1} = 2 \cdot 10^{-2} s(Q_R = (1 - 3) \cdot 10^7)$, and spin-orbital gain $g = 30 - 250$ (optional). At number N_R resonators in a row, the integrated voltage is increased by a factor N_R .

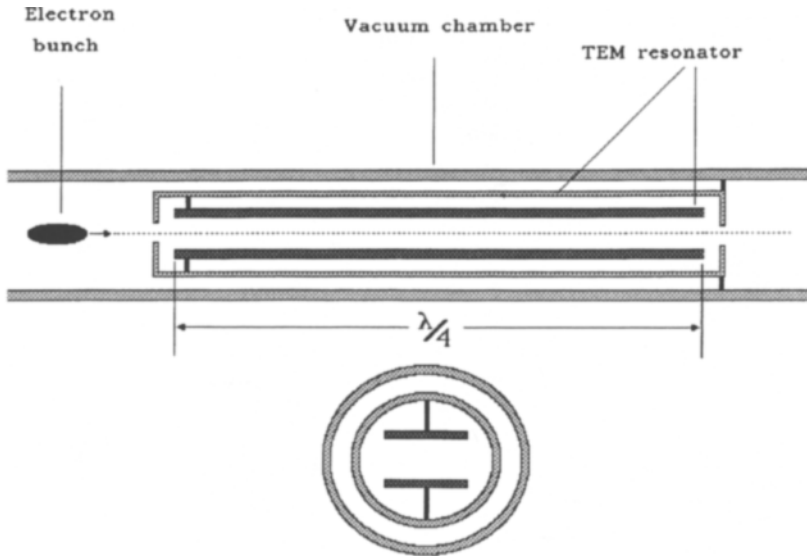


FIGURE 3. Superconducting $\lambda/4$ spin-responding resonator.

MEASUREMENT AND NOISE DEMANDS

There is a number of different kind of noise signals which can overlap with the spin-related signal. They have to be filtered or reduced to a sufficiently low level.

Thermal noise of the resonator [1] does not present a serious problem for a precision spin monitoring in most cases shown in Table 3 ($V_T \sim \sqrt{T_R/\ell} \sim 10^{-3}mV$ at $T_R = 1^{\circ}K$), especially at use of the spin-orbital amplification.

The input thermal voltage of a voltmeter can be as low as $10^{-5}mV$ or less value, i.e. negligible [1].

The incoherent beam noise can limit closeness to the spin-orbital resonance that is used for an enhancement of the spin-resonator interaction. However, the dipole noise of an intense bunched beam is absent if the coherent tune shift exceeds the tune spread.

Estimations of the beam overtone thermal noise indicates that all the non-linear harmonics of order less then 5-6 have to be tuned off the voltmeter frequency band, $\Delta\omega$.

Coherent noise due to the orbit-resonator misalignments can be filtered at condition $\Delta\omega \ll \epsilon\omega_0$.

A special issue is filtering of the transient signal of the beam caused by the

TABLE 3. Estimations of HF voltage accumulated in $\lambda/4$ resonator.

Machine particles	Energy γ	Beam current mA	Resonator length cm	Voltage rate mV/s		Maximum voltage mV	
				SOA	no SOA	SOA	no SOA
IUCF Cooler p	1.3	1	100	0.6	$2.4 \cdot 10^{-3}$	0.01	$0.4 \cdot 10^{-4}$
RHIC p	270	50	30	36	0.36	0.7	$0.7 \cdot 10^{-2}$
HERA $e\pm$	$6 \cdot 10^4$	30	10	70	0.7	1.4	$1.4 \cdot 10^{-2}$
HERA p	900	100	30	20	0.7	0.4	$1.4 \cdot 10^{-2}$
Tevatron p	900	100	30	20	0.7	0.4	$1.4 \cdot 10^{-2}$

misalignments of the spin driver (see section 4) and (in product) the spin-orbital amplifier (RF quadrupoles). To eliminate the transient echo from the observation, the beam, resonator, and filter have to relax with exponent λ^{-1} short with respect to the spin driver switching period ($\lambda \gg 2\pi q\epsilon/\omega_0$). The shunting measures can be undertaken, if needed.

CONCLUSIONS

Recent investigations of the RF polarimetry method resulted in a few important improvements.

The most significant step is that the RF spin echo technique is proposed to drive the coherent spin. It removes the spin decoherence and allows, at the same time, the spin to manifest in a signal of a characteristic frequency, in accordance with the basic principle of the RF polarimetry.

The proposal of the feeder $\lambda/4$ superconducting resonator increases the polarized beam-resonator interaction and makes the RF polarimeter efficient and practical at relatively long bunches (0.5 - 1 m in length), as well.

Involvement of the resonance spin-orbit interaction, intrinsic or RF-induced, makes it possible to frequently increase the spin-related excitation of the superconducting resonator.

Today estimations of the RF polarimeter efficiency encourage us to call for practical design of the involved RF elements and test experiments.

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