

INTEGRAL TRANSFORMATIONS, GENERATOR COORDINATES
AND THE EVALUATION OF RESONATING GROUP KERNELS.

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Microscopic calculations of clustering in nuclei for many years developed along two lines, usually known as resonating group method (R.G.M.) (1) and generator coordinate method (G.C.M.) (2). Recently the use of integral transforms (IT) to calculate R.G.M. kernels (3,4,5) has clarified the intimate relation between those methods and made possible microscopic reaction calculations of larger systems.

It thus becomes important to understand the underlying mechanisms of the applications of IT's, independent of the numerous specific IT's used. In this context two aspects are of overriding interest: a) what properties must an IT satisfy to be useful in this context? b) how do we proceed to evaluate the R.G.M. kernel in a rational way?

a) This problem has been discussed from a mathematical point of view in (6) and from a more practical point of view in (5). Basically one finds that an IT must be invertible on an adequate dense subset of Hilbert space and that upon folding the conjugate kernel, the relevant operators and the original kernel of the IT we get a bona fide integral kernel. These kernels can be obtained explicitly if the IT is gaussian and the operators are given in terms of derivatives, powers, spherical harmonics and gaussians of the variables x_j for the i^{th} particle. Clearly the IT's are in general not unitary and in particular if we choose the standard measure dx_j they cannot be unitary.

b) The next step involves integration over the internal coordinates of the clusters which again can be performed analytically if the internal functions are of oscillator type. This problem used to be treated for the case when all fragments had equal oscillator frequency (3); an assumption that is very restrictive. Indeed this restriction is not essential and has been overcome (4,5). The basic trick to do this seems to be based on using three systems of coordinates rather than two. We start as usual by obtaining a kernel in terms

of the single particle coordinates. From there we pass by an orthogonal transformation to relative and c.m. coordinates of the particles constituting each cluster; at this stage we do not introduce relative coordinates between the clusters. We can now integrate over the internal coordinates of the clusters without any interference from the different frequencies for different clusters, as no transformation connects them as yet. We thus obtain a kernel for the k c.m. coordinates of the clusters. The passage to relative coordinates is then no longer a problem as the dimension of the matrices involved has been reduced to just the total number of clusters.

A useful simplification is achieved if the IT is chosen to be a generating function for harmonic oscillator functions of a given width for the single particle coordinates associated with each cluster. Then the integration over the internal coordinates in the second coordinate system becomes trivial (4,5). The alternative is to interpret the IT kernel as a single particle basis and to use the corresponding techniques. This leads to the G.C.M.

We may summarize that, using -often singular- integral transforms fulfilling conditions a), we can obtain R.G.M. kernels in closed form if trial functions and interaction are given in adequate analytic form, and this indeed is possible without assuming equal width parameters for the internal oscillator functions of all clusters.

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